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A third sector in the core-periphery model: non-tradable goods*

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Abstract. We extend an analytically solvable core-periphery model by introducing a monopolistically competitive sector of non-tradable goods. We study how trade costs affect the spatial distribution of economic activity. Trade costs have no effect when the elasticity of substitution among non-tradable goods is low. In this case, concentration of all production (of tradable and non-tradable goods) is the unique equilibrium. When the elasticity of substitution among non-tradable goods is high, we find two equilibrium configurations: symmetric dispersion of the production of tradable and non-tradable goods, if trade costs are high; and concentration of production of tradable goods with asymmetric dispersion of production of non-tradable goods, if trade costs are low.

Keywords: New economic geography, Core-periphery model, Footloose entrepreneur, Non-tradable goods.

JEL Classification Numbers: F12, R12.

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1 Introduction

The literature on new economic geography has grown extensively over the two last decades. The idea of trade and geography in general equilibrium models was introduced for the first time by Krugman (1991), who developed a model illustrating how a country can endogenously become differentiated into an industrialized core and an agricultural periphery. In this model, trade costs are crucial to explain the spatial distribution of economic activity. If trade costs are high, industrial activity is dispersed across regions, while if trade costs are low, then industrial activity becomes concentrated in one region.

Despite it being a stylized fact that services (which are mainly non-tradable) have a very significant weight in the developed economies, representing more than two thirds of the total employment in the EU27, the standard literature in new economic geography assumes that regions have an "agricultural" sector (which produces perfectly tradable goods) and an "industrial" sector (which produces partially tradable goods).[1] A notable exception is the work of Helpman (1998) who substituted the agricultural sector by a perfectly competitive non-tradable goods sector (housing). Assuming that the location of this sector is exogenous, Helpman showed that housing acts as a dispersion force, by increasing the cost-of-living in a more populated region.

This result also appears in the economic geography model developed by Pflüger and Südekum (2008), in which agents are assumed to have a logarithmic quasi-linear utility function and housing costs act in the spirit of Helpman (1998). They show that, starting from a situation of dispersion of industrial activity, falling trade costs lead to agglomeration. However, when trade costs become sufficiently low, the relative importance of housing prices dominates the agglomeration forces, and dispersion occurs again. Contrasting with most new economic geography models, which feature 'bang-bang' phenomena (either symmetric dispersion or full agglomeration of the industrial activity in one of two regions), their model can generate partial agglomeration.

In the model of Behrens (2004), the absence of interregional trade is an endogenous outcome. Firms want to sell in the locations that allow them to make a positive profit. Depending on the level of trade costs and on the degree of competition, each good may be effectively traded in equilibrium or not. Behrens (2004) shows that when the trade costs are higher than a threshold value, all the industrial goods are non-tradable. In such an environment, the economy comprises only an agricultural (traditional) sector and a non-tradable goods sector. For this particular case, Behrens (2004) also shows that full and partial agglomeration in the

[1] See, for example, the works of Puga (1999), Fujita, Krugman and Venables (2001), Forslid and Ottaviano (2003) and Baldwin *et al.* (2003).

non-tradable sector arises in a completely autarchic world, and the structure of the spatial economy is determined by the ratio of the mobile to immobile factor.

To the best of our knowledge, there is no model that explains the spatial distribution of the production of both tradable and non-tradable goods. In order to fill this gap, this paper generalizes the analytically solvable core-periphery model of Forslid and Ottaviano (2003) by considering a third sector, which produces non-tradable goods (services). Like the industrial sector, this service sector is assumed to be monopolistically competitive and mobile across regions.

Workers and firms operating in the industrial and service sectors move to the region with the highest utility level, until a spatial equilibrium is reached. We find that the resulting configuration may consist in full agglomeration, symmetric dispersion, or a combination of full agglomeration of industry with partial agglomeration of the service sector.

A strong preference for variety in the service sector is a very strong agglomeration force. For any value of the trade costs, full agglomeration of industry and services in one region is an equilibrium whenever the elasticity of substitution among services is lower than a threshold value.

If the preference for variety of services is relatively weak, trade costs become crucial to explain the location of the economic activity. If trade costs are high, the industry and services become symmetrically dispersed across regions. If trade costs are low, then the industry becomes agglomerated in one region, while the services become only partially agglomerated. In this case, the region where all the industrial activity takes place will have more than one-half of the service sector activity.

2 The Model

2.1 Basic setup

The model is an extension of the analytically solvable core-periphery model of Forslid and Ottaviano (2003) that incorporates a third sector of services (non-tradable goods).[2]

[2] It is straightforward to verify that by considering that the size of the service sector in null ($\mu_s = 0$), we obtain the model of Forslid and Ottaviano (2003).

The economy comprises two regions and three sectors: an agricultural sector (perfectly tradable goods), an industrial sector (partially tradable goods) and a service sector (non-tradable goods). There are three factors of production: unskilled workers (L), industrial sector workers (M) and service sector workers (S). The unskilled workers are immobile across regions, while the industrial and service workers are mobile.

We denote by M_1 and M_2, with $M_1 + M_2 = M$, the supply of industrial workers in regions 1 and 2, respectively, and by S_1 and S_2, with $S_1 + S_2 = S$, the supply of service workers in regions 1 and 2, respectively. The supply of unskilled workers is the same in each region, $L_1 = L_2 = L/2$, and the total population is normalized to unity, $L + M + S = 1$.

The agricultural sector is perfectly competitive and produces a homogeneous good under constant returns to scale using only unskilled labor. Transportation of agricultural output across regions is costless. The industrial sector and the service sector produce a horizontally differentiated product using sector-specific labor (fixed cost) and unskilled labor (variable cost).

Transportation of industrial goods and services is subject to iceberg transportation costs. For each unit of industrial good that is shipped to the other region, only a fraction $\tau_m \in (0, 1)$ arrives. The trade of services across regions is more costly: only a fraction $\tau_s \in [0, \tau_m]$ arrives. We will give particular attention to the case in which services are non-tradable across regions ($\tau_s = 0$).

All the agents have the same preferences for consumption of industrial goods (C_M), services (C_S) and agricultural goods (C_A). A natural extension of the utility function used by Krugman (1991) and by Forslid and Ottaviano (2003) to an economy with three sectors is the following:

$$U = C_M^{\mu_m} C_S^{\mu_s} C_A^{1-\mu_m-\mu_s}, \tag{1}$$

$$C_M = \left(\sum_{i=1}^{n_m} c_{m_i}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}}, \tag{2}$$

$$C_S = \left(\sum_{i=1}^{n_s} c_{s_i}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{3}$$

where $\mu_m \in (0, 1)$ and $\mu_s \in (0, 1)$, with $\mu_m + \mu_s < 1$, are the shares of spending on industrial products and on services; n_m and n_s are the number of varieties of industrial goods and of services; c_{m_i} and c_{s_j} are the consumption of the industrial good produced by firm i and of the service provided by the firm j; and, finally, $\sigma_m > 1$ and $\sigma_s > 1$ are the elasticities of substitution among industrial goods and among services.

2.2 Supply

2.2.1 Agricultural sector

In the agricultural sector, firms use unskilled labor to produce a homogeneous good under constant returns to scale. The production function is $q^A = L$, where q^A is the amount of agricultural goods produced and L is the quantity of unskilled labor employed. The cost function is $CT^A = W_a q^A$, where W_a is the nominal wage of the unskilled workers employed. The profit function is:

$$\Pi^A = (p_a - W_a)q^A,$$

where p_a is the price of an agricultural good, taken as given by the firms (perfect competition) and chosen to be the numeraire ($p_a = 1$).

The sector is perfectly competitive, therefore: $W_a = p_a = 1$.

2.2.2 Industrial sector

Firms in the industrial sector support a fixed cost of α_m units of industrial labor, and a variable cost of β units of unskilled labor per unit of good produced. Since $W_a = 1$, the cost function is $CT^M = \alpha_m W_m + \beta q^M$, where q^M is the quantity of industrial goods produced by an industrial firm and W_m is the nominal wage of the industrial workers employed by the firm. The profit function is:

$$\Pi^M = p^M(q^M)q^M - \beta q^M - \alpha_m W_m. \tag{4}$$

Firms choose q^M to maximize profit. This implies that:

$$p^M = \frac{\epsilon}{\epsilon - 1}\beta,$$

where ϵ is the price-elasticity of demand.

Since there is a large number of firms in the industrial sector, $\epsilon \approx \sigma_m$ (we have equality if there is a continuum of firms). Thus:[3]

$$p^M = \frac{\sigma_m}{\sigma_m - 1}\beta. \tag{5}$$

[3] In the case of Cournot competition: $\frac{1}{\epsilon} = \frac{1}{\sigma_m} + s(1 - \frac{1}{\sigma_m})$, where s is the market share of each firm. With many firms in the economy ($s \approx 0$), the price elasticity of demand, ϵ, is approximately equal to the elasticity of substitution among the differentiated goods, σ_m.

Given the assumption of free entry, the profit of each firm must be zero. Substituting (5) in (4), we obtain:

$$q^M = \frac{\alpha_m}{\beta}(\sigma_m - 1)W_m \tag{6}$$

Since an industrial firm employs α_m units of skilled labor, the total demand for skilled labor is $n_m\alpha_m$. Therefore, the number of firms must be:

$$n_m = \frac{M}{\alpha_m}. \tag{7}$$

2.2.3 Service sector

Firms in the service sector use α_s units of service workers as a fixed cost, and β units of unskilled labor per unit of product.

As in the industrial sector, the price chosen by each firm is:

$$p^S = \frac{\sigma_s}{\sigma_s - 1}\beta.$$

The quantity produced by each firm is:

$$q^S = \frac{\alpha_s}{\beta}(\sigma_s - 1)W_s, \tag{8}$$

where W_s is the nominal wage of the service workers employed by the firm.

And the number of firms is:

$$n_s = \frac{S}{\alpha_s}. \tag{9}$$

2.3 Demand

2.3.1 Industrial sector

Individual demand for each industrial variety is obtained from utility maximization (2) with respect to c_{m_j}. It can be shown that (Baldwin et al., 2003, pp. 38-39):

$$c_{m_j} = p_j^{-\sigma_m}\frac{\mu_m y}{\sum_{i=1}^{n_m} p_i^{1-\sigma_m}},$$

where y is the income of the agent.

The industrial price index can be defined as:

$$P_m = \left(\sum_{i=1}^{n_m} {p_i}^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}} \tag{10}$$

Using (10), the individual demand for the industrial variety j becomes:

$$c_{m_j} = \frac{p_j^{-\sigma_m}}{P_m^{1-\sigma_m}} \mu_m y. \tag{11}$$

Each firm sells its products in both regions. The price of a representative local industrial good is $p_{ii} = \frac{\sigma_m}{\sigma_m - 1}\beta$, and the price of a product that is exported from region i to region j is $p_{ij} = \tau_m^{-1} \frac{\sigma_m}{\sigma_m - 1}\beta$.

Since all manufacturing firms of a region set the same price, the industrial price index in region i is:

$$\begin{aligned} P_{mi} &= \left(n_{mi} {p_{ii}}^{1-\sigma_m} + n_{mj} {p_{ij}}^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}} = \\ &= \frac{\beta \sigma_m}{\sigma_m - 1} \left(n_{mi} + n_{mj} \tau_m^{\sigma_m - 1} \right)^{\frac{1}{1-\sigma_m}}, \end{aligned} \tag{12}$$

where n_{mi} and n_{mj} are the number of industrial firms in regions i and j, respectively.

Defining $\phi_m = \tau_m^{\sigma_m - 1}$ as the degree of economic integration for the industrial sector (Baldwin et al., 2003), we obtain:

$$P_{mi} = \frac{\beta \sigma_m}{\sigma_m - 1} \left(n_{mi} + n_{mj} \phi_m \right)^{\frac{1}{1-\sigma_m}}. \tag{13}$$

Denoting the total demand of an industrial product that is produced in region i and consumed in region j by $C_{m_{ij}}$, we have:

$$C_{m_{ii}} = \frac{p_{ii}^{-\sigma_m}}{P_{mi}^{1-\sigma_m}} \mu_m Y_i \quad \text{and} \quad C_{m_{ij}} = \frac{p_{ij}^{-\sigma_m}}{P_{mj}^{1-\sigma_m}} \mu_m Y_j,$$

where Y_i and Y_j are the nominal incomes in regions i and j, respectively.

Since $p_{ii} = \frac{\beta \sigma_m}{\sigma_m - 1}$ and $p_{ij} = \tau_m^{-1} \frac{\beta \sigma_m}{\sigma_m - 1}$, the above equations become:

$$C_{m_{ii}} = \frac{\left(\frac{\beta \sigma_m}{\sigma_m - 1} \right)^{-\sigma_m}}{P_{mi}^{1-\sigma_m}} \mu_m Y_i \quad \text{and} \quad C_{m_{ij}} = \frac{\tau_m^{\sigma_m} \left(\frac{\beta \sigma_m}{\sigma_m - 1} \right)^{-\sigma_m}}{P_{mj}^{1-\sigma_m}} \mu_m Y_j. \tag{14}$$

Denoting the output of an industrial firm in region i by q_{mi}, we have:

$$q_{mi} = C_{m_{ii}} + \tau_m^{-1} C_{m_{ij}}. \tag{15}$$

Substituting (14) in (15), we obtain:

$$q_{mi} = \mu_m \left(\frac{\beta \sigma_m}{\sigma_m - 1} \right)^{-\sigma_m} \left(\frac{Y_i}{P_{mi}^{1-\sigma_m}} + \frac{\tau_m^{\sigma_m - 1} Y_j}{P_{mj}^{1-\sigma_m}} \right). \tag{16}$$

Replacing (13) in (16):

$$q_{m_i} = \mu_m \frac{\sigma_m - 1}{\beta \sigma_m} \left(\frac{Y_i}{n_{m_i} + \phi_m n_{m_j}} + \frac{\phi_m Y_j}{\phi_m n_{m_i} + n_{m_j}} \right).$$

Substituting (6) and (7) above, we obtain the nominal wage of the skilled workers in each region:

$$W_{mi} = \frac{\mu_m}{\sigma_m} \left(\frac{Y_i}{M_i + \phi_m M_j} + \frac{\phi_m Y_j}{\phi_m M_i + M_j} \right). \tag{17}$$

2.3.2 Service sector

All the expressions obtained in the previous subsection apply.

The individual demand for a service, c_{s_j}, is:

$$c_{s_j} = \frac{p_{s_j}^{-\sigma_s}}{P_s^{1-\sigma_s}} \mu_s y, \tag{18}$$

where $P_s = \left(\sum_{i=1}^{n_s} p_i{}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$.

The internal and external demand for a service produced in region i are:

$$C_{s_{ii}} = \frac{\left(\frac{\beta \sigma_s}{\sigma_s - 1} \right)^{-\sigma_s}}{P_{si}^{1-\sigma_s}} \mu_s Y_i \quad \text{and} \quad C_{s_{ij}} = \frac{\tau_s^{\sigma_s} \left(\frac{\beta \sigma_s}{\sigma_s - 1} \right)^{-\sigma_s}}{P_{sj}^{1-\sigma_s}} \mu_s Y_j. \tag{19}$$

The output of a service provider in region i is:

$$q_{si} = \mu_s \left(\frac{\beta \sigma_s}{\sigma_s - 1} \right)^{-\sigma_s} \left(\frac{Y_i}{P_{si}^{1-\sigma_s}} + \frac{\phi_s Y_j}{P_{sj}^{1-\sigma_s}} \right), \quad \text{for } i, j = 1, 2, \tag{20}$$

where $\phi_s = \tau_s^{\sigma_s - 1}$ is the degree of economic integration in the service sector.

The price index of services in region i is:

$$P_{si} = \frac{\beta\sigma_s}{\sigma_s - 1} \left(n_{si} + n_{sj}\phi_s\right)^{\frac{1}{1-\sigma_s}}, \quad \text{for } i, j = 1, 2, \tag{21}$$

where n_{si} and n_{sj} are the number of service providers in region i and j, respectively.

The nominal wage of the skilled workers in the service sector in region i is:

$$W_{si} = \frac{\mu_s}{\sigma_s} \left(\frac{Y_i}{S_i + \phi_s S_j} + \frac{\phi_s Y_j}{\phi_s S_i + S_j} \right), \quad \text{for } i, j = 1, 2. \tag{22}$$

2.3.3 Regional income, perfect price index

The nominal income in region i, is equal to the sum of the incomes in the agricultural, industrial and service sector:

$$Y_i = \frac{1 - M - S}{2} + W_{mi} M_i + W_{si} S_i, \quad \text{for } i = 1, 2. \tag{23}$$

The perfect price index of region, P_i, aggregates three price indices: the price index of the agricultural sector (normalized to 1), the price index of the industrial sector, P_{mi}, and the price index of the service sector, P_{si}.

We obtain the price index of industrial goods in region i, by substituting (7) into (13):

$$P_{mi} = \frac{\beta\sigma_m}{\sigma_m - 1} \left(\frac{M_i}{\alpha_m} + \frac{\phi_m M_j}{\alpha_m} \right)^{\frac{1}{1-\sigma_m}}, \quad \text{for } i, j = 1, 2.$$

Denote the share of industrial workers in region 1 by $f_m = \frac{M_1}{M}$. Substituting above:

$$P_{m1} = \frac{\beta\sigma_m}{\sigma_m - 1} \left(\frac{M}{\alpha_m} \right)^{\frac{1}{1-\sigma_m}} \left[f_m + (1 - f_m)\phi_m\right]^{\frac{1}{1-\sigma_m}},$$

and

$$P_{m2} = \frac{\beta\sigma_m}{\sigma_m - 1} \left(\frac{M}{\alpha_m} \right)^{\frac{1}{1-\sigma_m}} \left[\phi_m f_m + (1 - f_m)\right]^{\frac{1}{1-\sigma_m}}.$$

Substituting (9) into (21), and defining $f_s = \frac{S_1}{S}$ as the share of service sector workers in region 1, we obtain:

$$P_{s1} = \frac{\beta\sigma_s}{\sigma_s - 1}\left(\frac{S}{\alpha_s}\right)^{\frac{1}{1-\sigma_s}}\left[f_s + (1-f_s)\phi_s\right]^{\frac{1}{1-\sigma_s}},$$

and

$$P_{s2} = \frac{\beta\sigma_s}{\sigma_s - 1}\left(\frac{S}{\alpha_s}\right)^{\frac{1}{1-\sigma_s}}\left[\phi_s f_s + (1-f_s)\right]^{\frac{1}{1-\sigma_s}}.$$

Using the last four expressions, we obtain the perfect price indices for each region:

$$P_1 = \rho\left[f_m + (1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\left[f_s + (1-f_s)\phi_s\right]^{\frac{\mu_s}{1-\sigma_s}}, \tag{24}$$

and

$$P_2 = \rho\left[\phi_m f_m + (1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left[\phi_s f_s + (1-f_s)\right]^{\frac{\mu_s}{1-\sigma_s}}, \tag{25}$$

where $\rho = \left(\frac{\beta\sigma_m}{\sigma_m - 1}\right)^{\mu_m}\left(\frac{\beta\sigma_s}{\sigma_s - 1}\right)^{\mu_s}\left(\frac{M}{\alpha_m}\right)^{\frac{\mu_m}{1-\sigma_m}}\left(\frac{S}{\alpha_s}\right)^{\frac{\mu_s}{1-\sigma_s}}$.

2.4 Short-run equilibrium

In the short-run, workers are immobile across regions. A short-run equilibrium consists in the equality of supply and demand. Aggregate prices, output and wages are endogenously determined.

Equations (17), (22), (23), (24) and (25) determine the short-run equilibrium of the model. We recall these equations:

$$\begin{aligned}
W_{m1} &= \frac{\mu_m}{\sigma_m}\left(\frac{Y_1}{M_1 + \phi_m M_2} + \frac{\phi_m Y_2}{\phi_m M_1 + M_2}\right), \\
W_{m2} &= \frac{\mu_m}{\sigma_m}\left(\frac{Y_2}{M_2 + \phi_m M_1} + \frac{\phi_m Y_1}{\phi_m M_2 + M_1}\right), \\
W_{s1} &= \frac{\mu_s}{\sigma_s}\left(\frac{Y_1}{S_1 + \phi_s S_2} + \frac{\phi_s Y_2}{\phi_s S_1 + S_2}\right), \\
W_{s2} &= \frac{\mu_s}{\sigma_s}\left(\frac{Y_2}{S_2 + \phi_s S_1} + \frac{\phi_s Y_1}{\phi_s S_2 + S_1}\right), \\
Y_1 &= \frac{1 - M - S}{2} + W_{m1}M_1 + W_{s1}S_1, \\
Y_2 &= \frac{1 - M - S}{2} + W_{m2}M_2 + W_{s2}S_2, \\
P_1 &= \rho\left[f_m + (1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}\left[f_s + (1-f_s)\phi_s\right]^{\frac{\mu_s}{1-\sigma_s}}, \\
P_2 &= \rho\left[\phi_m f_m + (1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left[\phi_s f_s + (1-f_s)\right]^{\frac{\mu_s}{1-\sigma_s}}.
\end{aligned}$$

Solving these equations, we find the nominal wages of the workers in each region:[4]

$$
\begin{aligned}
W_{m1} &= \overline{C}\sigma_m 2\phi_m M_1 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1-\phi_s^2\right)}{S_1 S_2 \left(1+\phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] + \\
&+ \overline{C}\sigma_m M_2 \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} + \frac{S_2 - \phi_s S_1}{\phi_s S_1 + S_2}\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
W_{m2} &= \overline{C}\sigma_m 2\phi_m M_2 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1-\phi_s^2\right)}{S_1 S_2 \left(1+\phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] + \\
&+ \overline{C}\sigma_m M_1 \left\{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s \left(\phi_m^2 \frac{S_2 - \phi_s S_1}{\phi_s S_1 + S_2} + \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2}\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
W_{s1} &= \overline{D}\sigma_s 2\phi_s S_1 \left[\sigma_m - \mu_m \frac{M_1 M_2 \left(1-\phi_m^2\right)}{M_1 M_2 \left(1+\phi_m^2\right) + \phi_m \left(M_1^2 + M_2^2\right)}\right] + \\
&+ \overline{D}\sigma_s S_2 \left\{\sigma_m \left[\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}(\phi_s^2 - 1)\right] - \mu_m \left(\phi_s^2 \frac{M_1 - \phi_m M_2}{M_1 + \phi_m M_2} + \frac{M_2 - \phi_m M_1}{\phi_m M_1 + M_2}\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
W_{s2} &= \overline{D}\sigma_s 2\phi_s S_2 \left[\sigma_m - \mu_m \frac{M_1 M_2 \left(1-\phi_m^2\right)}{M_1 M_2 \left(1+\phi_m^2\right) + \phi_m \left(M_1^2 + M_2^2\right)}\right] + \\
&+ \overline{D}\sigma_s S_1 \left\{\sigma_m \left[\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}(\phi_s^2 - 1)\right] - \mu_m \left(\phi_s^2 \frac{M_2 - M_1 \phi_m}{\phi_m M_1 + M_2} + \frac{M_1 - \phi_m M_2}{M_1 + \phi_m M_2}\right)\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{C} &= \frac{\mu_m \sigma_s \frac{1-M-S}{2}(S_1 + \phi_s S_2)(\phi_s S_1 + S_2)}{R}, \\
\overline{D} &= \frac{\mu_s \sigma_m \frac{1-M-S}{2}(M_1 + \phi_m M_2)(\phi_m M_1 + M_2)}{R},
\end{aligned}
$$

and

$$
\begin{aligned}
R &= \left\{\sigma_m \left(M_1 + \phi_m M_2\right)\left[\sigma_s \left(S_1 + \phi_s S_2\right) - S_1 \mu_s\right] - M_1 \mu_m \sigma_s \left(S_1 + \phi_s S_2\right)\right\} \times \\
&\times \left\{\sigma_m \left(\phi_m M_1 + M_2\right)\left[\sigma_s \left(\phi_s S_1 + S_2\right) - S_2 \mu_s\right] - M_2 \mu_m \sigma_s \left(\phi_s S_1 + S_2\right)\right\} - \\
&- \left[M_1 \mu_m \phi_m \sigma_s \left(\phi_s S_1 + S_2\right) + S_1 \mu_s \phi_s \sigma_m \left(\phi_m M_1 + M_2\right)\right] \times \\
&\times \left[M_2 \mu_m \phi_m \sigma_s \left(S_1 + \phi_s S_2\right) + S_2 \mu_s \phi_s \sigma_m \left(M_1 + \phi_m M_2\right)\right].
\end{aligned}
$$

The real wages of the industrial sector workers in regions 1 and 2 are $\omega_{m1} = \dfrac{W_{m1}}{P_1}$ and $\omega_{m2} = \dfrac{W_{m2}}{P_2}$, and the real wages of the service sector workers in regions 1 and 2 are $\omega_{s1} = \dfrac{W_{s1}}{P_1}$ and $\omega_{s2} = \dfrac{W_{s2}}{P_2}$.

[4] See Appendix 7.1 for detailed calculations.

2.5 Long-run equilibrium

In the long-run, the skilled workers of the industrial and service sectors choose their location with the objective of maximizing their utility (equivalently, they move to the region with the highest real wage).

Migration is assumed to be determined by the following processes:

$$\dot{f}_m = \frac{df_m}{dt} = \begin{cases} \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m), & \text{if } 0 < f_m < 1 \\ \min\{0, \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m)\}, & \text{if } f_m = 1 \\ \max\{0, \omega_{m1}(f_s, f_m) - \omega_{m2}(f_s, f_m)\}, & \text{if } f_m = 0 \end{cases}$$

and

$$\dot{f}_s = \frac{df_s}{dt} = \begin{cases} \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m), & \text{if } 0 < f_s < 1 \\ \min\{0, \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m)\}, & \text{if } f_s = 1 \\ \max\{0, \omega_{s1}(f_s, f_m) - \omega_{s2}(f_s, f_m)\}, & \text{if } f_s = 0, \end{cases}$$

where f_s and f_m are functions of time, t, which is left implicit to simplify notation. The derivatives of f_s and f_m with respect to t are denoted by $\dot{f}_s$ and $\dot{f}_m$, respectively.

A distribution of economic activity, (f_s^*, f_m^*), is a steady-state if and only if $\dot{f}_m = \dot{f}_s = 0$ at (f_s^*, f_m^*). A long-run equilibrium is a stable steady-state.

The sufficient conditions for stability are the following:

(i) $f_x^* = 0 \;\Rightarrow\; (\omega_{x1} - \omega_{x2})\,|_{(f_s^*, f_m^*)} < 0$, for $x \in \{s, m\}$;

(ii) $f_x^* = 1 \;\Rightarrow\; (\omega_{x1} - \omega_{x2})\,|_{(f_s^*, f_m^*)} > 0$, for $x \in \{s, m\}$;

(iii) $f_x^* \in (0,1) \wedge f_y^* \in \{0,1\} \;\Rightarrow\; \frac{\partial(\omega_{x1} - \omega_{x2})}{\partial f_x}\,|_{(f_s^*, f_m^*)} < 0$, for $(x, y) \in \{(s, m), (m, s)\}$;

(iv) $(f_s^*, f_m^*) \in (0,1)^2 \;\Rightarrow\; det(J)|_{(f_s^*, f_m^*)} > 0$ and $tr(J)|_{(f_s^*, f_m^*)} < 0$, where:

$$J = \begin{bmatrix} \frac{\partial(\omega_{m1} - \omega_{m2})}{\partial f_m} & \frac{\partial(\omega_{m1} - \omega_{m2})}{\partial f_s} \\ \frac{\partial(\omega_{s1} - \omega_{s2})}{\partial f_m} & \frac{\partial(\omega_{s1} - \omega_{s2})}{\partial f_s} \end{bmatrix}.$$

3 The case in which services are non-tradable

In this section, we study the case in which services are asymptotically non-tradable ($\tau_s \to 0$ and, thus, $\phi_s \to 0$).[5] This is the case for most services related to arts, entertainment, real estate, rental, wholesale trade, education and health services.

3.1 Short-run equilibrium

We start by computing, for each sector, the difference between the real wages of the skilled workers in region 1 and region 2 (see Appendix 7.2). We obtain:

$$\omega_{m1} - \omega_{m2} = \frac{\mu_m K}{\phi_m \left[f_m^2 + (1-f_m)^2\right] K_1 + (1-f_m) f_m K_2} \times (\bar{\omega}_{m1} - \bar{\omega}_{m2}) \tag{26}$$

and

$$\omega_{s1} - \omega_{s2} = \frac{\mu_s K \left\{\phi_m \left[f_m^2 + (1-f_m)^2\right] + f_m (1-f_m)(1+\phi_m^2)\right\}}{\left\{\phi_m \left[f_m^2 + (1-f_m)^2\right] K_1 + (1-f_m) f_m K_2\right\}} (\bar{\omega}_{s1} - \bar{\omega}_{s2}), \tag{27}$$

where:

$$\begin{aligned}
K_1 &= \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s) - \sigma_s \mu_m\right], \\
K_2 &= \sigma_m (\sigma_s - \mu_s) \left[\sigma_m (\sigma_s - \mu_s)\left(1+\phi_m^2\right) - 2\mu_m \sigma_s\right] + \mu_m^2 \sigma_s^2 \left(1 - \phi_m^2\right), \\
K &= \frac{\sigma_m^{1-\mu_m} \sigma_s^{1-\mu_s} (1-M-S)(\sigma_m - 1)^{\mu_m} (\sigma_s - 1)^{\mu_s}}{2\alpha_m^{\frac{\mu_m}{\sigma_m - 1}} \alpha_s^{\frac{\mu_s}{\sigma_s - 1}} \beta^{\mu_m + \mu_s} M^{1-\frac{\mu_m}{\sigma_m - 1}} S^{-\frac{\mu_s}{\sigma_s - 1}}}, \\
\bar{\omega}_{m1} &= \frac{2\phi_m f_m (\sigma_s - \mu_s) + (1-f_m)\left[\sigma_s(\phi_m^2+1) + \frac{\sigma_s \mu_m}{\sigma_m}(\phi_m^2 - 1) - \mu_s (\phi_m^2+1)\right]}{\left[f_m + (1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{\frac{\mu_s}{1-\sigma_s}}}, \\
\bar{\omega}_{m2} &= \frac{2\phi_m (1-f_m)(\sigma_s - \mu_s) + f_m\left[\sigma_s(\phi_m^2+1) + \frac{\sigma_s \mu_m}{\sigma_m}(\phi_m^2 - 1) - \mu_s (\phi_m^2+1)\right]}{\left[\phi_m f_m + (1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{\frac{\mu_s}{1-\sigma_s}}}, \\
\bar{\omega}_{s1} &= \frac{\sigma_m \left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}}{\left[f_m + (1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s - 1}}}, \\
\bar{\omega}_{s2} &= \frac{\sigma_m \left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{f_m(1+\phi_m)-\phi_m}{f_m(1-\phi_m)+\phi_m}}{\left[\phi_m f_m + (1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{1-\frac{\mu_s}{\sigma_s - 1}}}.
\end{aligned}$$

[5]We do not consider the limit case because, with $\tau_s = 0$, the demand of the agricultural workers is indeterminate when services are concentrated in the other region. When restricted to $C_S = 0$, the agricultural workers are indifferent between any attainable consumption vector.

3.2 Long-run equilibrium

In this section, we study the long-run equilibrium of the model. The following are possible spatial equilibrium configurations:

(i) concentration of industry and services in the same region;

(ii) concentration of industry in one region with asymmetric dispersion of services;

(iii) symmetric dispersion of industry and services.

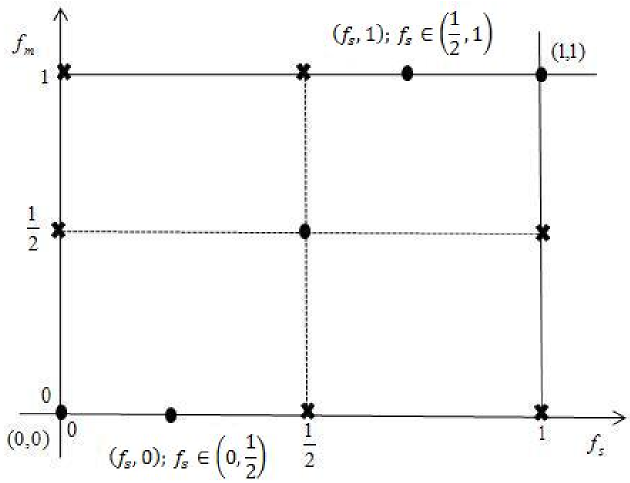

Figure 1: Possible equilibrium configurations

Figure 1 illustrates the possible equilibrium solutions. In the vertical axis we have the share of industrial workers in region 1 and in the horizontal axis we have the share of service workers in region 1. The possible equilibrium solutions are signalled by black dots. The black crosses signal configurations which are never an equilibrium.

3.2.1 Concentration of industry and services in the same region

Concentration of the industrial and service activity in region 1, $(f_s^*, f_m^*) = (1, 1)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(1,1)} \geq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(1,1)} \geq 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in (1-\epsilon, 1]^2$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \geq 0, \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)} \geq 0. \end{cases}$$

Similarly, concentration of the industrial and service activity in region 2, $(f_s^*, f_m^*) = (0,0)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(0,0)} \leq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(0,0)} \leq 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in [0, \epsilon)^2$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \leq 0, \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)} \leq 0. \end{cases}$$

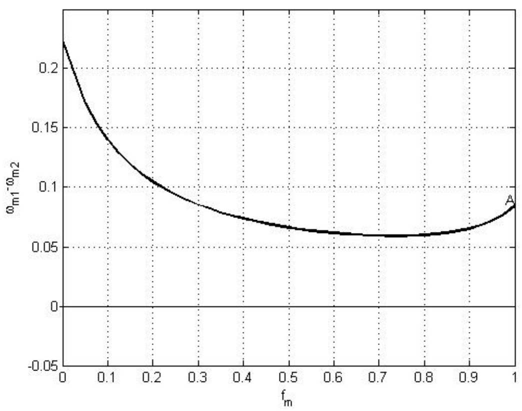

Figure 2: Concentration of industry.

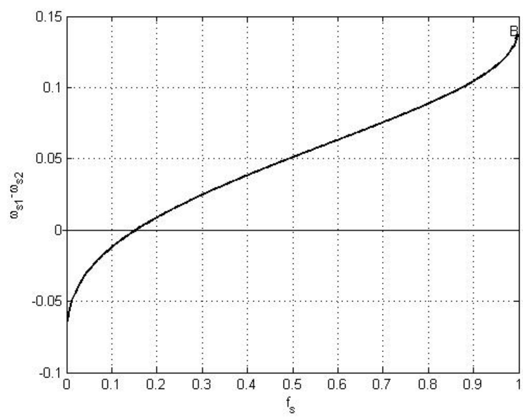

Figure 3: Concentration of services.

Figures 2 and 3 illustrate the existence of full agglomeration of the industrial and service activity in one region, when $\sigma_s \leq \mu_s + 1$.[6]

In figure 2, we assume that all the services are concentrated in region 1, $f_s = 1$, and we study the relationship between the spatial distribution of industry and the difference between the real wages of the industrial workers across regions. We find that the real wage is always higher in region 1. Thus, all industrial workers migrate to region 1.

In figure 3, we assume that all the industry is concentrated in region 1, $f_m = 1$, and we study how the spatial distribution of services affects the difference between the real wages

[6]To plot these figures, we have set $\tau_m = 0.5$, $\mu_m = 0.4$, $\sigma_m = 4$, $\mu_s = 0.4$ and $\sigma_s = 1.3$.

of the service sector workers across regions. We find that if the service sector activity in region 1 is high enough, the service sector workers obtain a higher real wage in region 1. We conclude that full agglomeration of industry and services is an equilibrium.

Lemma 3.1. *Concentration of both sectors in a single region is an equilibrium if and only if* $\sigma_s \leq \mu_s + 1$.

This result (which is proved in Appendix 7.3) shows that trade costs in the industrial sector are irrelevant to explain concentration of industry and services activity in a single region, when the degree of differentiation in the service sector is high and services are non-tradable. For any value of the trade costs in the industrial sector, a high preference for variety of services is a sufficient condition to induce full concentration of industry and services. Under this condition, even if industrial goods are perfectly tradable, all the industrial firms locate their production in the same region. This contrasts with the results obtained in the classical model.

From now on, we will frequently assume that $\sigma_s > \mu_s + 1$ (no black-hole condition).

3.2.2 Concentration of industry and asymmetric dispersion of services

Concentration of industrial activity in region 1 with asymmetric dispersion of the service activity, $(f_s^*, f_m^*) = (s, 1)$, with $s \in (0.5, 1)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(s,1)} \geq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(s,1)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in (s - \epsilon, s + \epsilon) \times (1 - \epsilon, 1]$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \geq 0 \\ \left.\frac{\partial(\omega_{s1} - \omega_{s2})}{\partial f_s}\right|_{(f_s,f_m)} < 0 \end{cases}$$

Similarly, concentration of industrial activity in region 2 with asymmetric dispersion of the service activity, $(f_s^*, f_m^*) = (s, 0)$, with $s \in (0, 0.5)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(s,0)} \leq 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(s,0)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if there exists an $\epsilon > 0$ such that, $\forall (f_s, f_m) \in (s - \epsilon, s + \epsilon) \times [0, \epsilon)$:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)} \leq 0 \\ \left.\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s}\right|_{(f_s,f_m)} < 0 \end{cases}$$

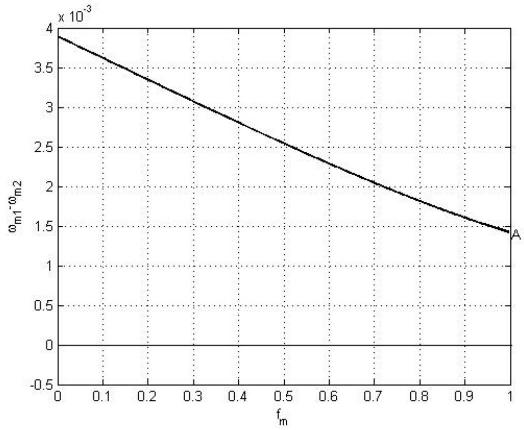

Figure 4: Concentration of industry.

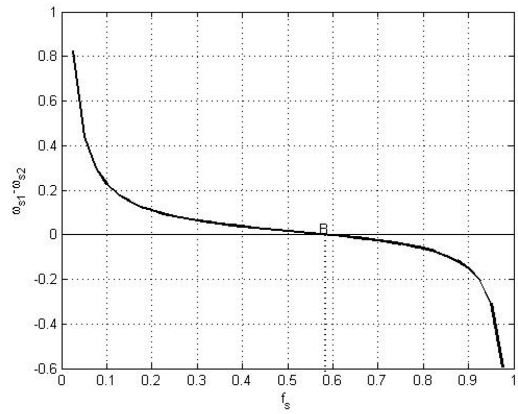

Figure 5: Asymmetric dispersion of services.

Figures 4 and 5 illustrate an equilibrium with full concentration of industry and asymmetric dispersion of services.[7]

With $f_s = 0.586$, we can see from figure 4 that the real wage of the industrial workers is always higher in region 1. Thus, industrial workers locate in region 1.

In figure 5, we assume that all the industrial activity is concentrated in region 1, $f_m = 1$, and study how the spatial distribution of the service sector activity affects the difference between the real wages of the service sector workers across regions. The migration of the service sector workers leads to an equilibrium with $f_s = 0.586$, as the real wages of the service sector workers coincide.

We conclude that the concentration of industry in region 1 and the asymmetric dispersion of services (58.6% in region 1) constitutes an equilibrium.

The following lemma describes the conditions for the existence of an equilibrium in which the industry is concentrated while the service sector is asymmetrically dispersed.

[7]To plot these figures, we have set $\tau_m = 0.825$, $\mu_m = 0.4$, $\sigma_m = 4$, $\mu_s = 0.4$ and $\sigma_s = 4$. We have also set $f_s = 0.586$ in figure 4 and $f_m = 1$ in figure 5.

Lemma 3.2. *Concentration of industrial activity in region 1 or 2 and asymmetric dispersion of the service activity is an equilibrium if $\sigma_s > \mu_s + 1$ and:*
$\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s})-\mu_m}{\sigma_m(1-\frac{\mu_s}{\sigma_s})+\mu_m}\right]^{\frac{\mu_s}{\mu_s+1-\sigma_s}} \phi_m^{1-\frac{\mu_m(\sigma_s-1)}{(\sigma_m-1)(\sigma_s-1-\mu_s)}} - \frac{\phi_m^2\left[\sigma_s(1+\frac{\mu_m}{\sigma_m})-\mu_s\right]+\sigma_s(1-\frac{\mu_s}{\sigma_m})-\mu_s}{2(\sigma_s-\mu_s)} > 0.$

To prove lemma 3.2, we use the following result. It states that when the elasticity of substitution of services is high enough to satisfy what we may call the black-hole condition, then a movement of service workers to a region decreases the attractiveness of this region to the service workers.

Lemma 3.3. *When $\sigma_s > \mu_s + 1$, an increase in the share of services in region 1 (f_s) decreases the difference between the real wages in the service sector ($\omega_{s1} - \omega_{s2}$).*

3.2.3 Symmetric dispersion of industry and services

Symmetric dispersion of the industrial activity and service activity, $(f_s^*, f_m^*) = (0.5, 0.5)$, is a steady-state if:

$$\begin{cases} (\omega_{m1} - \omega_{m2})\,|_{(f_s,f_m)=(0.5,0.5)} = 0 \\ (\omega_{s1} - \omega_{s2})\,|_{(f_s,f_m)=(0.5,0.5)} = 0, \end{cases}$$

and it is an equilibrium, that is, a stable steady-state, if $det(J)|_{(f_s,f_m)=(0.5,0.5)} > 0$ and $tr(J)|_{(f_s,f_m)=(0.5,0.5)} < 0$, where J is the Jacobian matrix of the model described by expressions (26) and (27):

$$J = \begin{bmatrix} \frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_m} & \frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_s} \\ \frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} & \frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s} \end{bmatrix}.$$

The following result implies that all these derivatives are well defined.

Claim 3.4. *The differences $\omega_{m1} - \omega_{m2}$ and $\omega_{s1} - \omega_{s2}$ are continuous and differentiable functions of f_m and f_s, for $(f_s, f_m) \in (0,1)^2$.*

Symmetric dispersion is always a steady-state.

Lemma 3.5. *When $(f_s, f_m) = (0.5, 0.5)$, the real wages in the industrial sector and in the service sector are equal across regions ($\omega_{m1} = \omega_{m2}$ and $\omega_{s1} = \omega_{s2}$).*

We already know that $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s} < 0$. Calculating the remaining elements of the Jacobian matrix, we obtain the following results.

Lemma 3.6. *When $(f_s, f_m) = (0.5, 0.5)$, the migration of service sector workers to region 1 increases the difference between the real wages of the industrial sector workers in region 1 and region 2, that is,* $\frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_s} > 0$.

Lemma 3.7. *When $(f_s, f_m) = (0.5, 0.5)$, the migration of industrial sector workers to region 1 increases the difference between the real wages of the service sector workers in region 1 and region 2, that is,* $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} > 0$.

These lemmas are important to explain the stability of the dispersion configuration and the following figures are useful for an intuitive understanding of the dynamics.[8]

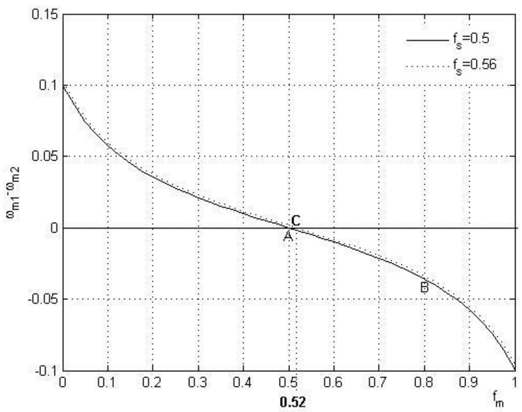

Figure 6: Dispersion of industry.

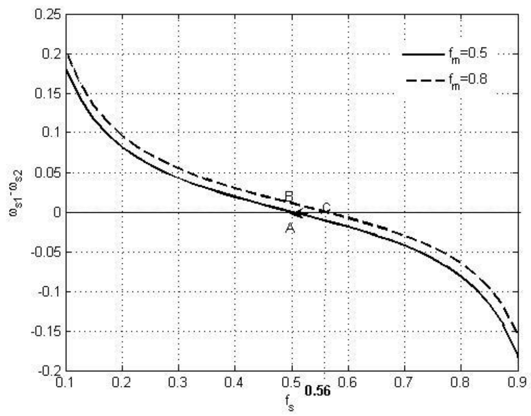

Figure 7: Dispersion of services.

Assume that the economy is initially located in point A, with $(f_s, f_m) = (0.5, 0.5)$. We have $\omega_{m1} = \omega_{m2}$ (figure 6) and $\omega_{s1} = \omega_{s2}$ (figure 7). Consider an increase in the number of industrial workers in region 1, to $f_m = 0.8$ (point B). From Lemma 3.7, an increase in f_m increases $\omega_{s1} - \omega_{s2}$, and thus the curve in figure 7 moves up. The service sector workers would tend to move to region 1, until $f_s = 0,56$. On the other hand, with $f_s = 0.56$, the curve in figure 6 moves up, and the industrial workers would also migrate, until $f_m = 0.52$. With $f_m = 0.52$, the resulting f_s would be lower than 0.56, giving rise to a new f_m, lower than 0.52. It seems that this process continues until $(f_s, f_m) = (0.5, 0.5)$, suggesting that symmetric dispersion is an equilibrium.

We can see from figure 8 that dispersion is unstable when $\tau_m = 0.9$.[9] An increase in f_m increases the difference between the real wages of the industrial workers in region 1 and

[8] To plot figures 6 and 7, we have set $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 4$ and $\tau_m = 0.5$.

[9] To plot figures 8 and 9, we have set $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 4$ and $\tau_m = 0.9$. Additionally, we have set $f_s = 0.5$ to plot figure 8 and $f_m = 1$ to plot figure 9.

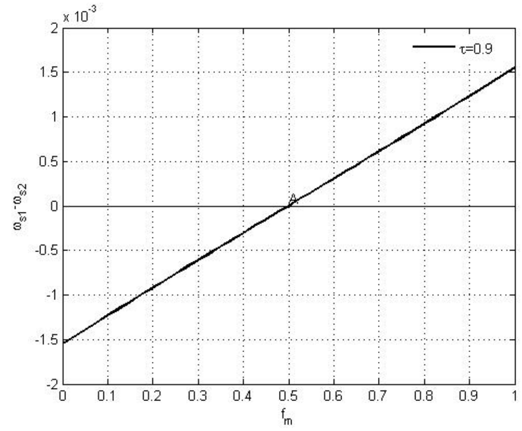

Figure 8: Dispersion becomes unstable.

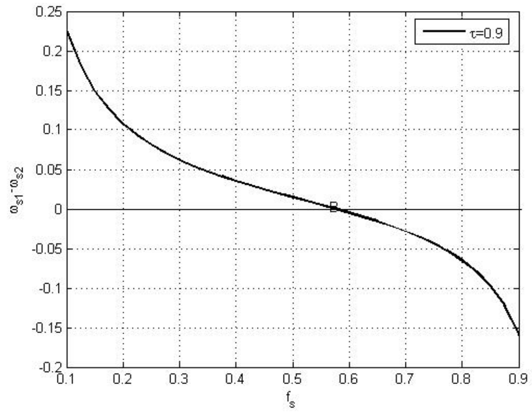

Figure 9: Asymmetric Dispersion.

region 2, attracting workers from region 2 to region 1. From lemma 3.7, an increase in f_m also increases the difference between the real wages in the service sector Therefore, in the long-run, we will have an asymmetric dispersion of the service sector activity and full concentration of the industrial activity.

In Appendix 7.3 we calculate the Jacobian matrix. Here, we compute $det(J)$ and $tr(J)$ using a numerical example. Figure 10 illustrates a case in which symmetric dispersion of the industrial and service activity is an equilibrium for low values of ϕ_m.[10]

For the parameter values in our numerical example, $tr(J)$ is negative for any $\phi_m \in (0, 1)$. Therefore, the sign of the determinant is crucial for the stability. From figure 10, we can see that $det(J)$ is positive for low values of ϕ_m. This means that the eigenvalues of J have negative real parts and symmetric dispersion of industry and services is an equilibrium.

For high values of ϕ_m, the symmetric dispersion becomes unstable. In this case, asymmetric dispersion of the service activity with full concentration of the industrial activity in one region becomes the equilibrium.

This result can be viewed in figure 11, where we also plot the "asymmetric condition" (49) as a function of ϕ_m. When ϕ_m is higher than ϕ_m^*, concentration of all industrial activity in one region with asymmetric dispersion of the service activity becomes an equilibrium. Therefore, point A is a threshold value for ϕ_m.

In particular, when $\sigma_s > \mu_s + 1$, the economy can have two distinct equilibrium configurations. For high trade costs (low ϕ_m), we find that symmetric dispersion of services and

[10]In figures 10 and 11, it is also assumed that $\sigma_m = 4$, $\sigma_s = 4$, $\mu_m = 0.4$ and $\mu_s = 4$.

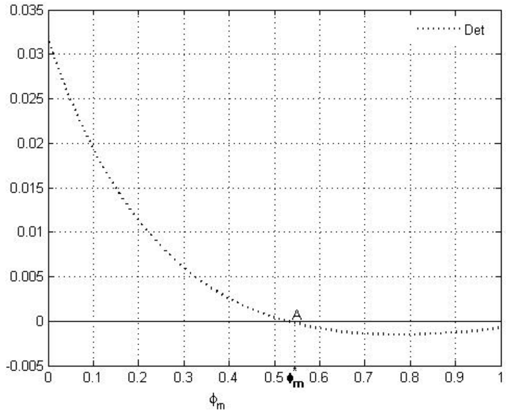

Figure 10: $Det(J)$ as a function of ϕ_m.

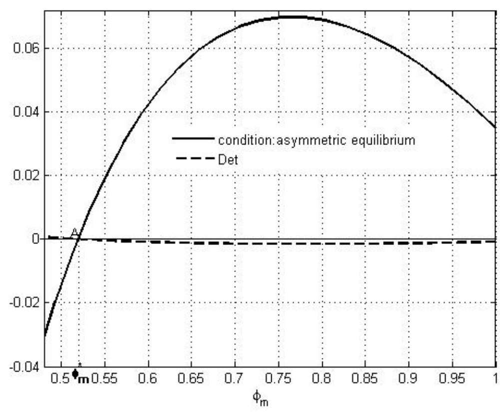

Figure 11: Threshold value for ϕ_m.

industry is an equilibrium, while for low trade costs (high ϕ_m), we find that concentration of industry with asymmetric dispersion of services is an equilibrium.

3.2.4 Configurations which are never an equilibrium

We also show that the following configurations are never an equilibrium:
(i) concentration of services and concentration of industry in different regions;
(ii) symmetric dispersion of services and concentration of industry;
(iii) concentration of services together with dispersion of industry.

Lemma 3.8. *Concentration of each sector in different regions is never an equilibrium.*

Lemma 3.9. *Symmetric dispersion of services and concentration of industry is never an equilibrium.*

Lemma 3.10. *Concentration of services together with dispersion of industry is never an equilibrium.*

4 The case in which services are tradable

In this section we show, numerically, that a different spatial configuration of economic activity, concentration of services with dispersion of industry, can appear when services are tradable ($0 < \phi_s \leq \phi_m < 1$).

Consider an initial equilibrium, in which both industry and services are concentrated in region 1. Figure 12 illustrates how an decrease in the trade cost of services (an increase in τ_s) affects the spatial distribution of industry, when all services are concentrated in region 1 (point A). The main result is that a fall in the trade cost of services leads to an equilibrium in which industry becomes asymmetric dispersed (point C), while the service sector remains concentrated in region 1 (see figure 13).[11]

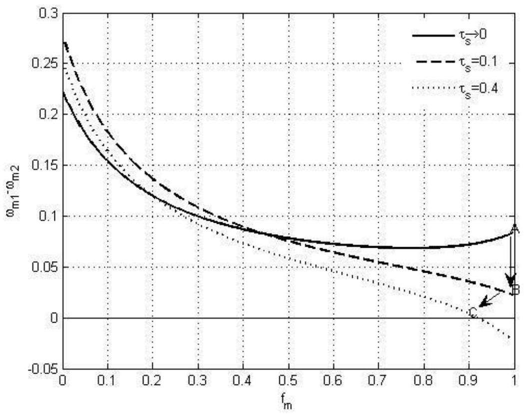

Figure 12: Industry becomes dispersed.

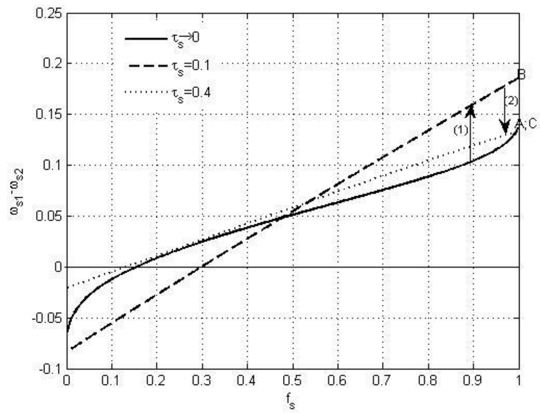

Figure 13: Concentration of services.

The same decrease in the trade cost of services (from $\tau_s \rightarrow 0$ to $\tau_s = 0.4$) may not change the initial equilibrium configuration, being compatible with symmetric dispersion of both sectors (set $\tau_m = 0.5$, keeping fixed the remaining parameters) or asymmetric dispersion of services with full concentration of industry (set $\tau_m = 0.825$).

5 Conclusion

We have extended the footloose entrepreneur model (Forslid and Ottaviano, 2003) to allow for a third sector: a monopolistic competitive sector of services, assumed to be non-tradable across regions.

We find that the strength of the preference for variety of services is crucial to explain the spatial distribution of the industrial and service activity. When the elasticity of substitution among services is below a certain threshold, full concentration of industry and services in

[11]To plot figures 12 and 13, we have set $\tau_m = 0.5$, $\mu_m = 0.4$ $\sigma_m = 4$, $\mu_s = 0.4$, $\sigma_s = 1.3$. We also set $f_s = 1$ in figure 12 and $f_m = 1$ in figure 13.

a single region is always an equilibrium. But this threshold value for the elasticity of substitution among services seems a bit too low to be attainable in modern economies, as it corresponds to a very high price markup over marginal cost.[12] Based on this model, we should not expect, therefore, full concentration of industry and services in a single region.

With a higher elasticity of substitution among services, which is more likely, the spatial distribution of economic activity depends on the trade costs in the industrial sector. If these trade costs are high, symmetric dispersion of the services and industrial activity is an equilibrium. If they are low, concentration of industry with asymmetric dispersion of services is an equilibrium (in this case, the industrialized region has more than 50% of the service sector activity).

Taking into account the existence of non-tradable goods, with specialized workers which are mobile across regions, should provide new insights about the determinants of the spatial organization of economic activity. We hope that this model may be seen as a step in this direction.

6 Bibliography

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[12] With services representing 50% of the economic activity ($\mu_s = 0.5$), at this threshold ($\sigma_s = 1 + \mu_s$), we obtain $p_s = \frac{\sigma_s}{\sigma_s - 1}\beta = 3\beta$ (the price of a representative service is equal to 3 times the marginal cost). A lower price markup is obtained if we consider a higher elasticity of substitution among services.

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7 Appendix

7.1 Short-run equilibrium

The nominal wages of the industrial and service sector workers are:

$$W_{mi} = \frac{\mu_m}{\sigma_m}\left(\frac{Y_i}{M_i + \phi_m M_j} + \frac{Y_j \phi_m}{\phi_m M_i + M_j}\right), \tag{28}$$

$$W_{si} = \frac{\mu_s}{\sigma_s}\left(\frac{Y_i}{S_i + \phi_s S_j} + \frac{Y_j \phi_s}{\phi_s S_i + S_j}\right). \tag{29}$$

The regional nominal incomes are:

$$Y_i = \frac{1 - H - S}{2} + W_{mi} M_i + W_{si} S_i. \tag{30}$$

Our goal in this section is to find W_{mi} and W_{si} as functions of the parameters of the model. First, we determine Y_i and Y_j. Then, we substitute these into W_{mi} and W_{si}.

Substituting (28) and (29) in (30), and using $a = \frac{1-H-S}{2}$, we obtain:

$$Y_i = a + M_i \frac{\mu_m}{\sigma_m}\left(\frac{Y_i}{M_i + \phi_m M_j} + \frac{Y_j \phi_m}{\phi_m M_i + M_j}\right) + S_i \frac{\mu_s}{\sigma_s}\left(\frac{Y_i}{S_i + \phi_s S_j} + \frac{Y_j \phi_s}{\phi_s S_i + S_j}\right).$$

Rearranging, we obtain Y_i as a function of Y_j:

$$\begin{aligned} Y_i \quad & \left[1 - \frac{\mu_m M_i}{\sigma_m (M_i + \phi_m M_j)} - \frac{\mu_s S_i}{\sigma_s (S_i + \phi_s S_j)}\right] = \\ = \quad & a + \left[\frac{\mu_m M_i \phi_m}{\sigma_m (\phi_m M_i + M_j)} + \frac{\mu_s S_i \phi_s}{\sigma_s (\phi_s S_i + S_j)}\right] Y_j. \end{aligned}$$

For convenience, define:

$$\begin{aligned} b_m &= \phi_m M_i + M_j, \\ c_m &= M_i + \phi_m M_j, \\ b_s &= \phi_s S_i + S_j, \\ c_s &= S_i + \phi_s S_j. \end{aligned}$$

With some manipulation, we obtain:

$$Y_i \frac{\sigma_m c_m \sigma_s c_s - \mu_m M_i \sigma_s c_s - \mu_s S_i \sigma_m c_m}{\sigma_m c_m \sigma_s c_s} = a + \frac{\mu_m M_i \phi_m \sigma_s b_s + \mu_s S_i \phi_s \sigma_m b_m}{\sigma_s b_s \sigma_m b_m} Y_j.$$

Which is equivalent to:

$$Y_i = \frac{a\sigma_m \sigma_s c_m c_s + \left(\mu_m M_i \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m\right) c_m c_s b_m^{-1} b_s^{-1} Y_j}{\sigma_m c_m \left(\sigma_s c_s - S_i \mu_s\right) - M_i \mu_m \sigma_s c_s}. \tag{31}$$

The above equation yields Y_i as a function of Y_j. By symmetry, we can write Y_j as function of Y_i as follows:

$$Y_j = \frac{a\sigma_m \sigma_s b_m b_s + \left(\mu_m M_j \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m\right) b_m b_s c_m^{-1} c_s^{-1} Y_i}{\sigma_m b_m \left(\sigma_s b_s - S_j \mu_s\right) - M_j \mu_m \sigma_s b_s}. \tag{32}$$

Substituting (32) in (31) and simplifying, we obtain:

$$Y_i = \frac{c_m c_s \left[\sigma_m b_m \left(\sigma_s b_s - S_j \mu_s\right) - M_j \mu_m \sigma_s b_s + M_i \mu_m \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m\right]}{a^{-1} \sigma_m^{-1} \sigma_s^{-1} R}, \tag{33}$$

and, by symmetry:

$$Y_j = \frac{b_m b_s \left[\sigma_m c_m \left(\sigma_s c_s - S_i \mu_s\right) - M_i \mu_m \sigma_s c_s + M_j \mu_m \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m\right]}{a^{-1} \sigma_m^{-1} \sigma_s^{-1} R}, \tag{34}$$

where:

$$\begin{aligned} R \;=\; & \left[\sigma_m c_m \left(\sigma_s c_s - S_i \mu_s\right) - M_i \mu_m \sigma_s c_s\right] \left[\sigma_m b_m \left(\sigma_s b_s - S_j \mu_s\right) - M_j \mu_m \sigma_s b_s\right] - \\ & - \left(M_i \mu_m \phi_m \sigma_s b_s + S_i \mu_s \phi_s \sigma_m b_m\right) \left(M_j \mu_m \phi_m \sigma_s c_s + S_j \mu_s \phi_s \sigma_m c_m\right). \end{aligned}$$

Denoting by Y_i^N and Y_j^N the numerators of Y_i and Y_j in equations (33) and (34), we can rewrite (28) in the following way:

$$W_{mi} = \frac{\mu_m \sigma_s a}{c_m b_m R} \left(Y_i^N b_m + \phi_m Y_j^N c_m\right).$$

Replacing the expressions for Y_i and Y_j, and setting $i = 1$ and $j = 2$, we obtain:

$$\begin{aligned} W_{m1} &= \frac{\mu_m a \sigma_m \sigma_s}{R} \left[\, c_s b_m \left(\sigma_s b_s - S_2 \mu_s\right) - M_2 c_s \sigma_m^{-1} \mu_m \sigma_s b_s + M_1 c_s \sigma_m^{-1} \mu_m \phi_m \sigma_s b_s + \right. \\ &+ S_1 \mu_s \phi_s c_s b_m + \phi_m b_s c_m \left(\sigma_s c_s - S_1 \mu_s\right) - M_1 \phi_m b_s \mu_m \sigma_s \sigma_m^{-1} c_s + \\ &+ \left. M_2 \phi_m^2 b_s \mu_m \sigma_s \sigma_m^{-1} c_s + S_2 \phi_m b_s \mu_s \phi_s c_m \,\right]. \end{aligned}$$

Denoting $\overline{C} = \frac{\mu_m c_s b_s \sigma_s a}{R}$, and replacing b_m, c_m, b_s and c_s by the corresponding expressions:

$$\begin{aligned} \frac{W_{m1}}{\overline{C}} &= \sigma_m \left(\phi_m M_1 + M_2\right) \left(\sigma_s - \mu_s \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right) + \mu_m \sigma_s \left(\phi_m M_1 - M_2\right) + \\ &+ \phi_m \left[\sigma_m \left(M_1 + \phi_m M_2\right) \left(\sigma_s - \mu_s \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2}\right) + \mu_m \sigma_s (\phi_m M_2 - M_1)\right]. \end{aligned}$$

Manipulating:

$$\begin{aligned} &\frac{W_{m1}}{\overline{C}} = \phi_m \sigma_m M_1 \left[2\sigma_s - \mu_s \left(\frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1} + \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2}\right)\right] + \\ &+ M_2 \left[\sigma_m \sigma_s \left(\phi_m^2 + 1\right) + \sigma_s \mu_m \left(\phi_m^2 - 1\right) - \sigma_m \mu_s \left(\phi_m^2 \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} + \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right)\right]. \end{aligned}$$

It is easy to show that:

$$\frac{S_2 - \phi_s S_1}{\phi_s S_1 + S_2} + \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} = \frac{2 S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}.$$

Substituting this expression, we determine W_{m1} and (by symmetry) W_{m2}:

$$\begin{aligned} &\frac{W_{m1}}{\overline{C} \sigma_m} = 2 \phi_m M_1 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] + \\ &+ M_2 \left[\sigma_s \left(\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m} \phi_m^2 - \frac{\mu_m}{\sigma_m}\right) - \mu_s \left(\phi_m^2 \frac{S_1 - \phi_s S_2}{S_1 + \phi_s S_2} + \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1}\right)\right] \end{aligned} \tag{35}$$

and

$$\begin{aligned} &\frac{W_{m2}}{\overline{C} \sigma_m} = 2 \phi_m M_2 \left[\sigma_s - \mu_s \frac{S_1 S_2 \left(1 - \phi_s^2\right)}{S_1 S_2 \left(1 + \phi_s^2\right) + \phi_s \left(S_1^2 + S_2^2\right)}\right] + \\ &+ M_1 \left[\sigma_s \left(\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m} \phi_m^2 - \frac{\mu_m}{\sigma_m}\right) - \mu_s \left(\phi_m^2 \frac{S_2 - \phi_s S_1}{S_2 + \phi_s S_1} + \frac{S_1 - \phi_s S_2)}{S_1 + \phi_s S_2}\right)\right], \end{aligned} \tag{36}$$

where:

$$\overline{C} = \frac{\mu_m c_s b_s \sigma_s a}{R} = \frac{\mu_m \sigma_s a}{R} (S_1 + \phi_s S_2)(S_2 + \phi_s S_1),$$

and:

$$\begin{aligned}
R &= \left\{\sigma_m \left(M_1 + \phi_m M_2\right)\left[\sigma_s \left(S_1 + \phi_s S_2\right) - S_1 \mu_s\right] - M_1 \mu_m \sigma_s \left(S_1 + \phi_s S_2\right)\right\} \times \\
&\times \left\{\sigma_m \left(\phi_m M_1 + M_2\right)\left[\sigma_s \left(\phi_s S_1 + S_2\right) - S_2 \mu_s\right] - M_2 \mu_m \sigma_s \left(\phi_s S_1 + S_2\right)\right\} - \\
&- \left[M_1 \mu_m \phi_m \sigma_s \left(\phi_s S_1 + S_2\right) + S_1 \mu_s \phi_s \sigma_m \left(\phi_m M_1 + M_2\right)\right] \times \\
&\times \left[M_2 \mu_m \phi_m \sigma_s \left(S_1 + \phi_s S_2\right) + S_2 \mu_s \phi_s \sigma_m \left(M_1 + \phi_m M_2\right)\right].
\end{aligned}$$

Equations (35) and (36) are explicit functions of the parameters of the model.

By analogy, we find the nominal wages in the service sector:

$$\begin{aligned}
&\frac{W_{s1}}{\overline{D}\sigma_s} = 2\phi_s S_1 \left[\sigma_m - \mu_m \frac{M_1 M_2 \left(1 - \phi_m^2\right)}{M_1 M_2 \left(1 + \phi_m^2\right) + \phi_m \left(M_1^2 + M_2^2\right)}\right] + \\
&+ S_2 \left[\sigma_m \left(\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}\phi_s^2 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \left(\phi_s^2 \frac{M_1 - \phi_m M_2}{M_1 + \phi_m M_2} + \frac{M_2 - \phi_m M_1}{M_2 + \phi_m M_1}\right)\right]
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
&\frac{W_{s2}}{\overline{D}\sigma_s} = 2\phi_s S_2 \left[\sigma_m - \mu_m \frac{M_1 M_2 \left(1 - \phi_m^2\right)}{M_1 M_2 \left(1 + \phi_m^2\right) + \phi_m \left(M_1^2 + M_2^2\right)}\right] + \\
&+ S_1 \left[\sigma_m \left(\phi_s^2 + 1 + \frac{\mu_s}{\sigma_s}\phi_s^2 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \left(\phi_s^2 \frac{M_2 - \phi_m M_1}{M_2 + \phi_m M_1} + \frac{M_1 - \phi_m M_2)}{M_1 + \phi_m M_2}\right)\right],
\end{aligned} \tag{38}$$

where:

$$\overline{D} = \frac{\mu_s c_m b_m \sigma_m a}{R} = \frac{\mu_s \sigma_m a}{R}(M_1 + \phi_m M_2)(M_2 + \phi_m M_1),$$

and:

$$\begin{aligned}
R &= \left\{\sigma_m \left(M_1 + \phi_m M_2\right)\left[\sigma_s \left(S_1 + \phi_s S_2\right) - S_1 \mu_s\right] - M_1 \mu_m \sigma_s \left(S_1 + \phi_s S_2\right)\right\} \times \\
&\times \left\{\sigma_m \left(\phi_m M_1 + M_2\right)\left[\sigma_s \left(\phi_s S_1 + S_2\right) - S_2 \mu_s\right] - M_2 \mu_m \sigma_s \left(\phi_s S_1 + S_2\right)\right\} - \\
&- \left[M_1 \mu_m \phi_m \sigma_s \left(\phi_s S_1 + S_2\right) + S_1 \mu_s \phi_s \sigma_m \left(\phi_m M_1 + M_2\right)\right] \times \\
&\times \left[M_2 \mu_m \phi_m \sigma_s \left(S_1 + \phi_s S_2\right) + S_2 \mu_s \phi_s \sigma_m \left(M_1 + \phi_m M_2\right)\right].
\end{aligned}$$

7.2 Short-run equilibrium with $\tau_s \to 0$

With $\tau_s \to 0$, equations (17), (22), (23), (24) and (25) become:

$$
\begin{aligned}
W_{m1} &= \frac{\mu_m}{\sigma_m M}\left[\frac{Y_1}{f_m+\phi_m(1-f_m)}+\frac{\phi_m Y_2}{\phi_m f_m+1-f_m}\right]\\
W_{m2} &= \frac{\mu_m}{\sigma_m M}\left[\frac{Y_2}{1-f_m+\phi_m f_m}+\frac{\phi_m Y_1}{\phi_m(1-f_m)+f_m}\right]\\
W_{s1} &= \frac{\mu_s Y_1}{\sigma_s S f_s}\\
W_{s2} &= \frac{\mu_s Y_2}{\sigma_s S(1-f_s)}\\
Y_1 &= \frac{1-M-S}{2}+W_{m1}Mf_m+W_{s1}Sf_s\\
Y_2 &= \frac{1-M-S}{2}+W_{m2}M(1-f_m)+W_{s2}S(1-f_s)\\
P_1 &= \rho\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{\frac{\mu_s}{1-\sigma_s}}\\
P_2 &= \rho\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{\frac{\mu_s}{1-\sigma_s}}.
\end{aligned}
$$

From (35)-(38), we compute the nominal wages of the workers in each region:

$$\frac{W_{m1}}{\overline{C}_0\sigma_m} = 2\phi_m M_1(\sigma_s-\mu_s)+M_2\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}, \tag{39}$$

$$\frac{W_{m2}}{\overline{C}_0\sigma_m} = 2\phi_m M_2(\sigma_s-\mu_s)+M_1\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}, \tag{40}$$

$$\frac{W_{s1}}{\overline{D}_0\sigma_s} = S_2\left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{M_2-\phi_m M_1}{\phi_m M_1+M_2}\right], \tag{41}$$

$$\frac{W_{s2}}{\overline{D}_0\sigma_s} = S_1\left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{M_1-\phi_m M_2}{M_1+\phi_m M_2}\right], \tag{42}$$

where

$$\overline{C}_0 = \frac{\mu_m S_1 S_2}{R_0\sigma_m},$$

$$\overline{D}_0 = \frac{\mu_s(M_1+\phi_m M_2)(\phi_m M_1+M_2)}{R_0\sigma_s},$$

and

$$
\begin{aligned}
R_0 &= \frac{S_1 S_2}{a\sigma_m\sigma_s}\left\{\,\left[\sigma_m\left(M_1+\phi_m M_2\right)\left(\sigma_s-\mu_s\right)-M_1\mu_m\sigma_s\right]\times\right.\\
&\times\ \left.\left[\sigma_m\left(\phi_m M_1+M_2\right)\left(\sigma_s-\mu_s\right)-M_2\mu_m\sigma_s\right]-M_1M_2\mu_m^2\phi_m^2\sigma_s^2\,\right\}.
\end{aligned}
$$

Dividing the nominal wage in the industrial sector (39) by the regional price level, we obtain the real wages of the industrial workers:

$$
\omega_{m1}=\frac{2\phi_m f_m\left(\sigma_s-\mu_s\right)+\left(1-f_m\right)\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}}{R_m\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}f_s^{\frac{\mu_s}{1-\sigma_s}}}
\tag{43}
$$

and

$$
\omega_{m2}=\frac{2\phi_m(1-f_m)\left(\sigma_s-\mu_s\right)+f_m\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(1+\phi_m^2\right)\right\}}{R_m\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left(1-f_s\right)^{\frac{\mu_s}{1-\sigma_s}}},
\tag{44}
$$

where

$$
\begin{aligned}
R_m &= \frac{\rho M}{a\mu_m\sigma_m\sigma_s}\left\{\,\left\{\sigma_m\left[f_m+\phi_m(1-f_m)\right]\left(\sigma_s-\mu_s\right)-f_m\mu_m\sigma_s\right\}\times\right.\\
&\times\ \left.\left\{\sigma_m\left[\phi_m f_m+(1-f_m)\right]\left(\sigma_s-\mu_s\right)-(1-f_m)\mu_m\sigma_s\right\}-f_m(1-f_m)\mu_m^2\phi_m^2\sigma_s^2\,\right\}.
\end{aligned}
$$

With some manipulation, we find that the real wage differential in the industrial sector can be written as:

$$
\omega_{m1}-\omega_{m2}=\frac{\mu_m K}{\phi_m\left[f_m^2+(1-f_m)^2\right]K_1+\left(1-f_m\right)f_m K_2}\times\left(\bar\omega_{m1}-\bar\omega_{m2}\right),
\tag{45}
$$

where:

$$
\begin{aligned}
K &= \frac{\sigma_m^{1-\mu_m}\sigma_s^{1-\mu_s}\left(1-M-S\right)\left(\sigma_m-1\right)^{\mu_m}\left(\sigma_s-1\right)^{\mu_s}}{2\alpha_m^{\frac{\mu_m}{\sigma_m-1}}\alpha_s^{\frac{\mu_s}{\sigma_s-1}}\beta^{\mu_m+\mu_s}M^{1-\frac{\mu_m}{\sigma_m-1}}S^{-\frac{\mu_s}{\sigma_s-1}}},\\
K_1 &= \sigma_m\left(\sigma_s-\mu_s\right)\left[\sigma_m\left(\sigma_s-\mu_s\right)-\sigma_s\mu_m\right],\\
K_2 &= \sigma_m\left(\sigma_s-\mu_s\right)\left[\sigma_m\left(\sigma_s-\mu_s\right)\left(1+\phi_m^2\right)-2\mu_m\sigma_s\right]+\mu_m^2\sigma_s^2\left(1-\phi_m^2\right),\\
\bar\omega_{m1} &= \frac{2\phi_m f_m\left(\sigma_s-\mu_s\right)+\left(1-f_m\right)\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2+1\right)\right\}}{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}f_s^{\frac{\mu_s}{1-\sigma_s}}},\\
\bar\omega_{m2} &= \frac{2\phi_m(1-f_m)\left(\sigma_s-\mu_s\right)+f_m\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2+1\right)\right\}}{\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left(1-f_s\right)^{\frac{\mu_s}{1-\sigma_s}}}.
\end{aligned}
$$

Similarly, for the service sector, we obtain:

$$\omega_{s1} = \frac{[f_m + \phi_m(1 - f_m)]\,[\phi_m f_m + 1 - f_m]\left[\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{1 - f_m - \phi_m f_m}{1 - f_m + \phi_m f_m}\right]}{R_s\,[f_m + (1 - f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}}\; f_s^{1 - \frac{\mu_s}{\sigma_s - 1}}}$$

and:

$$\omega_{s2} = \frac{[f_m + \phi_m(1 - f_m)]\,[\phi_m f_m + 1 - f_m]\left[\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{f_m - \phi_m(1 - f_m)}{f_m + \phi_m(1 - f_m)}\right]}{R_s\,[\phi_m f_m + (1 - f_m)]^{\frac{\mu_m}{1-\sigma_m}}\; (1 - f_s)^{1 - \frac{\mu_s}{\sigma_s - 1}}},$$

where

$$\begin{aligned} R_s &= \frac{\rho S}{a\sigma_m\sigma_s}\left\{\,[\sigma_m\,[f_m + \phi_m(1 - f_m)]\,(\sigma_s - \mu_s) - f_m\mu_m\sigma_s] \times \right. \\ &\times \left. [\sigma_m\,(\phi_m f_m + 1 - f_m)\,(\sigma_s - \mu_s) - (1 - f_m)\mu_m\sigma_s] - f_m(1 - f_m)\mu_m^2\phi_m^2\sigma_s^2\,\right\}. \end{aligned}$$

Again, after some manipulation, we write the real wage differential in the service sector as:

$$\omega_{s1} - \omega_{s2} = \frac{\mu_s K\left\{\phi_m\left[f_m^2 + (1 - f_m)^2\right] + f_m\,(1 - f_m)\,(1 + \phi_m^2)\right\}}{\phi_m\left[f_m^2 + (1 - f_m)^2\right] K_1 + (1 - f_m)\,f_m K_2}\,(\bar{\omega}_{s1} - \bar{\omega}_{s2}) \qquad (46)$$

where:

$$\begin{aligned} \bar{\omega}_{s1} &= \frac{\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{1 - f_m(1 + \phi_m)}{1 - f_m(1 - \phi_m)}}{[f_m + (1 - f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}}\; f_s^{1 - \frac{\mu_s}{\sigma_s - 1}}}; \\ \bar{\omega}_{s2} &= \frac{\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m \frac{f_m(1 + \phi_m) - \phi_m}{f_m(1 - \phi_m) + \phi_m}}{[\phi_m f_m + (1 - f_m)]^{\frac{\mu_m}{1-\sigma_m}}\; (1 - f_s)^{1 - \frac{\mu_s}{\sigma_s - 1}}}. \end{aligned}$$

7.3 Long-run equilibrium

Proof of Lemma 3.1.

Since regions are symmetric, we only study concentration in region 1.

When the workers become concentrated in region 1, R_m converges to:

$$\lim_{f_m \to 1^-} R_m = \frac{\rho M \phi_m}{a\mu_m\sigma_s}\,(\sigma_m\sigma_s - \mu_s\sigma_m - \mu_m\sigma_s)\,(\sigma_s - \mu_s) > 0.$$

Therefore, we have:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m1} = \frac{2\phi_m\left(\sigma_s - \mu_s\right)}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_m} = \frac{2a\mu_m\sigma_s}{\rho M\left(\sigma_m\sigma_s - \sigma_m\mu_s - \mu_m\sigma_s\right)}.$$

While:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2} = \frac{\sigma_s\left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2 - 1)\right] - \mu_s\left(1 + \phi_m^2\right)}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_m\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}}.$$

The numerator is positive, while the denominator goes to infinity. Thus:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2} = 0.$$

We conclude that the industrial workers remain concentrated, as:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m1} > \lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{m2}.$$

In the service sector, real wages in region 1 tend to:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s1} \quad = \quad \lim_{(f_m,f_s)\to(1^-,1^-)} \frac{\phi_m(1-f_s)\left[\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) + \mu_m\right]}{R_s}.$$

Notice that:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \frac{R_s}{\phi_m(1-f_s)} \quad = \quad \frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s - \sigma_m\mu_s - \mu_m\sigma_s\right)\left(\sigma_s - \mu_s\right).$$

Therefore:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s1} \quad = \quad \lim_{(f_m,f_s)\to(1^-,1^-)} \frac{\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) + \mu_m}{\frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s - \sigma_m\mu_s - \mu_m\sigma_s\right)\left(\sigma_s - \mu_s\right)} > 0.$$

While:

$$\lim_{(f_m,f_s)\to(1^-,1^-)} \omega_{s2} = \frac{\phi_m^{\frac{1-\sigma_m-\mu_m}{1-\sigma_m}}\left[\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\right]}{\lim_{(f_m,f_s)\to(1^-,1^-)} R_s(1-f_s)^{\frac{\mu_s}{1-\sigma_s}}} =$$

$$= \frac{\tau_m^{\mu_m}\left[\sigma_m\left(1 - \frac{\mu_s}{\sigma_s}\right) - \mu_m\right]}{\frac{\rho S}{a\sigma_s}\left(\sigma_m\sigma_s - \sigma_m\mu_s - \mu_m\sigma_s\right)\left(\sigma_s - \mu_s\right)} \lim_{(f_m,f_s)\to(1^-,1^-)} (1-f_s)^{\frac{\sigma_s-1-\mu_s}{1-\sigma_s}}.$$

The real wage of the service workers in the empty region tends to zero if $\sigma_s < 1 + \mu_s$ and to plus infinity if $\sigma_s > 1 + \mu_s$. Notice also that with $\sigma_s = 1 + \mu_s$, we have $\omega_{s1} > \omega_{s2}$ (in the limit).

We conclude that concentration of both sectors in a single region is an equilibrium if and only if $\sigma_s \leq 1 + \mu_s$. □

Proof of Lemma 3.2.

Since regions are symmetric, we only study the case in which all the industrial activity is concentrated in region 1.

When $f_m = 1$, the difference between the real wages of the service sector workers is:

$$\omega_{s1} - \omega_{s2} = \frac{\mu_s K}{K_1}\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m}{f_s^{1-\frac{\mu_s}{\sigma_s-1}}} - \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{1-\frac{\mu_s}{\sigma_s-1}}}\right].$$

We begin our proof by calculating the values of $f_s \in (0,1)$ for which $\omega_{s1} - \omega_{s2} = 0$. Since K and K_1 are strictly positive and finite:

$$\begin{aligned} 0 &= \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m}{f_s^{1-\frac{\mu_s}{\sigma_s-1}}} - \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}(1-f_s)^{1-\frac{\mu_s}{\sigma_s-1}}} \Leftrightarrow \\ \Leftrightarrow \frac{1-f_s}{f_s} &= \left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m\right]}\right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}}. \end{aligned} \tag{47}$$

Simplifying:

$$f_s = \frac{1}{\left\{\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m\right]}\right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}} + 1}. \tag{48}$$

Notice that since $\phi_m^{\frac{\mu_m}{1-\sigma_m}} > 1$, we have:

$$0 < \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}}\left[\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m\right]} < 1.$$

Therefore, since $\sigma_s > \mu_s + 1$, we have:

$$0 < \left\{ \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}} \left[\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m\right]} \right\}^{\frac{\sigma_s-1}{\sigma_s-1-\mu_s}} < 1$$

and

$$\frac{1}{2} < f_s < 1.$$

It remains to verify the equilibrium condition for the industrial sector.

When the skilled workers in the industrial sector are concentrated in region 1, the difference between the real wages of the industrial workers is:

$$\omega_{m1} - \omega_{m2} =$$

$$= \frac{\mu_m K}{\phi_m K_1} \left\{ \frac{2\phi_m(\sigma_s - \mu_s)}{f_s^{\frac{\mu_s}{1-\sigma_s}}} - \frac{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right] - \mu_s(\phi_m^2+1)}{\phi_m^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{\frac{\mu_s}{1-\sigma_s}}} \right\}.$$

Then, $\omega_{m1} - \omega_{m2} > 0$ if and only if:

$$\begin{aligned} \frac{2\phi_m(\sigma_s - \mu_s)}{f_s^{\frac{\mu_s}{1-\sigma_s}}} &> \frac{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right] - \mu_s(\phi_m^2+1)}{\phi_m^{\frac{\mu_m}{1-\sigma_m}} (1-f_s)^{\frac{\mu_s}{1-\sigma_s}}} \Leftrightarrow \\ \Leftrightarrow \left(\frac{1-f_s}{f_s}\right)^{\frac{\mu_s}{1-\sigma_s}} &> \frac{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right] - \mu_s(\phi_m^2+1)}{2\phi_m(\sigma_s-\mu_s)\phi_m^{\frac{\mu_m}{1-\sigma_m}}} \end{aligned}$$

Replacing equation (47), we obtain:

$$\left\{ \frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\phi_m^{\frac{\mu_m}{1-\sigma_m}} \left[\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m\right]} \right\}^{\frac{-\mu_s}{\sigma_s-1-\mu_s}} > \frac{\sigma_s \left[\phi_m^2 + 1 + \frac{\mu_m}{\sigma_m}(\phi_m^2+1)\right] - \mu_s(\phi_m^2+1)}{2(\sigma_s-\mu_s)\phi_m^{\frac{\mu_m}{1-\sigma_m}+1}}.$$

Rearranging:

$$\begin{aligned} &\left[\frac{\sigma_m(1-\frac{\mu_s}{\sigma_s}) - \mu_m}{\sigma_m(1-\frac{\mu_s}{\sigma_s}) + \mu_m}\right]^{\frac{-\mu_s}{\sigma_s-1-\mu_s}} \phi_m^{1-\frac{\mu_m(\sigma_s-1)}{(\sigma_m-1)(\sigma_s-1-\mu_s)}} \\ &- \frac{\phi_m^2\left[\sigma_s(1+\frac{\mu_m}{\sigma_m}) - \mu_s\right] + \sigma_s(1-\frac{\mu_s}{\sigma_m}) - \mu_s}{2(\sigma_s-\mu_s)} > 0. \end{aligned} \tag{49}$$

Concentration of all the industrial activity in a region and asymmetric dispersion of the service activity is a steady-state when the above condition is satisfied. Lemma 3.3. provides the stability condition, guaranteeing that it is an equilibrium. □

Proof of Lemma 3.3.

From equation (46), the sign of $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_s}$ is equal to the sign of $\frac{\partial(\bar{\omega}_{s1}-\bar{\omega}_{s2})}{\partial f_s}$.

Calculating the partial derivatives:

$$\begin{aligned}
\frac{\partial \bar{\omega}_{s1}}{\partial f_s} &= \left(-1+\frac{\mu_s}{\sigma_s-1}\right)\frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}}{\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}}} f_s^{-2+\frac{\mu_s}{\sigma_s-1}}, \\
\frac{\partial \bar{\omega}_{s2}}{\partial f_s} &= \left(1-\frac{\mu_s}{\sigma_s-1}\right)\frac{\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{f_m(1+\phi_m)-\phi_m}{f_m(1-\phi_m)+\phi_m}}{\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}}(1-f_s)^{-2+\frac{\mu_s}{\sigma_s-1}}.
\end{aligned}$$

With $\sigma_s > \mu_s + 1$, we find that, for $f_s \in (0,1)$:

$$\begin{aligned}
\frac{\partial \bar{\omega}_{s1}}{\partial f_s} &< 0, \\
\frac{\partial \bar{\omega}_{s2}}{\partial f_s} &> 0.
\end{aligned}$$

When $f_s \in \{0,1\}$, one of these partial derivatives is null, but we still have $\frac{\partial \bar{\omega}_{s1}}{\partial f_s} < \frac{\partial \bar{\omega}_{s2}}{\partial f_s}$. □

Proof of Claim 3.4.

Inspection of the expressions for $\omega_{m1}-\omega_{m2}$ and $\omega_{s1}-\omega_{s2}$ shows that continuity of these differences and their derivatives depends on the denominator in (45) not being zero. This is the case since K_1 and K_2 are positive. □

Proof of Lemma 3.5.

When $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, we have $\bar{\omega}_{m1} = \bar{\omega}_{m2}$ and $\bar{\omega}_{s1} = \bar{\omega}_{s2}$.

Therefore, we also have $\omega_{m1} = \omega_{m2}$ and $\omega_{s1} = \omega_{s2}$. □

Proof of Lemma 3.6.

We want to prove that $\frac{\partial(\omega_{m1}-\omega_{m2})}{\partial f_s} > 0$.

From (45), we know that the sign of $\frac{\partial(\bar{\omega}_{m1}-\bar{\omega}_{m2})}{\partial f_s}$ is the same.

Calculating the partial derivatives:

$$\frac{\partial \bar{\omega}_{m1}}{\partial f_s} = \frac{2\phi_m f_m\left(\sigma_s-\mu_s\right)+(1-f_m)\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2+1\right)\right\}}{\frac{\sigma_s-1}{\mu_s}\left[f_m+(1-f_m)\phi_m\right]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s-1}}},$$

$$\frac{\partial \bar{\omega}_{m2}}{\partial f_s} = -\frac{2\phi_m(1-f_m)\left(\sigma_s-\mu_s\right)+f_m\left\{\sigma_s\left[\phi_m^2+1+\frac{\mu_m}{\sigma_m}(\phi_m^2-1)\right]-\mu_s\left(\phi_m^2+1\right)\right\}}{\frac{\sigma_s-1}{\mu_s}\left[\phi_m f_m+(1-f_m)\right]^{\frac{\mu_m}{1-\sigma_m}}\left(1-f_s\right)^{1-\frac{\mu_s}{\sigma_s-1}}}.$$

Substituting $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, we find that:

$$\frac{\partial\left(\bar{\omega}_{m1}-\bar{\omega}_{m2}\right)}{\partial f_s} = \frac{2\phi_m\left(\sigma_s-\mu_s\right)}{\frac{\sigma_s-1}{\mu_s}\left[\frac{1}{2}(1+\phi_m)\right]^{\frac{\mu_m}{1-\sigma_m}} 2^{-1+\frac{\mu_s}{\sigma_s-1}}} > 0.$$

$\square$

Proof of Lemma 3.7.

We want to prove that $\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} > 0$. Using equation (46), we determine the partial derivative:

$$\begin{aligned}\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} &= \frac{\partial}{\partial f_m}\left[\frac{\mu_s K\left\{\phi_m\left[f_m^2+(1-f_m)^2\right]+f_m\left(1-f_m\right)\left(1+\phi_m^2\right)\right\}}{\phi_m\left[f_m^2+(1-f_m)^2\right]K_1+\left(1-f_m\right)f_m K_2}\right]\left(\bar{\omega}_{s1}-\bar{\omega}_{s2}\right)+\\ &+\frac{\partial\left(\bar{\omega}_{s1}-\bar{\omega}_{s2}\right)}{\partial f_m}\left[\frac{\mu_s K\left\{\phi_m\left[f_m^2+(1-f_m)^2\right]+f_m\left(1-f_m\right)\left(1+\phi_m^2\right)\right\}}{\phi_m\left[f_m^2+(1-f_m)^2\right]K_1+\left(1-f_m\right)f_m K_2}\right].\end{aligned}$$

Since we are evaluating the derivative at $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$, the first term disappears:

$$\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m} = \frac{\partial\left(\bar{\omega}_{s1}-\bar{\omega}_{s2}\right)}{\partial f_m}\left[\frac{\mu_s K\left(2\phi_m+1+\phi_m^2\right)}{2\phi_m K_1+K_2}\right].$$

The partial derivative of $\bar{\omega}_{s1}$ with respect to f_m is:

$$\begin{aligned}
\frac{\partial \bar{\omega}_{s1}}{\partial f_m} &= \frac{1}{[f_m+(1-f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s-1}}} \times \\
&\times \mu_m \frac{(1+\phi_m)[1-f_m(1-\phi_m)]+(\phi_m-1)[1-f_m(1+\phi_m)]}{[1-f_m(1-\phi_m)]^2} + \\
&+ \frac{1}{\left\{[f_m+(1-f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}} f_s^{1-\frac{\mu_s}{\sigma_s-1}}\right\}^2} \times \\
&\times \frac{\mu_m}{\sigma_m-1}[f_m+(1-f_m)\phi_m]^{\frac{\mu_m}{1-\sigma_m}-1} f_s^{1-\frac{\mu_s}{\sigma_s-1}}(1-\phi_m) \times \\
&\times \left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{1-f_m(1+\phi_m)}{1-f_m(1-\phi_m)}\right].
\end{aligned}$$

Substituting $(f_s, f_m) = (\frac{1}{2}, \frac{1}{2})$:

$$\begin{aligned}
\left.\frac{\partial \bar{\omega}_{s1}}{\partial f_m}\right|_{(f_s,f_m)=\left(\frac{1}{2},\frac{1}{2}\right)} &= \frac{\mu_m\left[(1+\phi_m)\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)+(\phi_m-1)\left(\frac{1}{2}-\frac{1}{2}\phi_m\right)\right]}{\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}} 2^{-1+\frac{\mu_s}{\sigma_s-1}}\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^2} + \\
+ \frac{\frac{\mu_m}{\sigma_m-1}\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}-1} 2^{-1+\frac{\mu_s}{\sigma_s-1}}(1-\phi_m)}{\left[\left(\frac{1}{2}+\frac{1}{2}\phi_m\right)^{\frac{\mu_m}{1-\sigma_m}} 2^{-1+\frac{\mu_s}{\sigma_s-1}}\right]^2} & \left[\sigma_m\left(1-\frac{\mu_s}{\sigma_s}\right)-\mu_m\frac{\frac{1}{2}-\frac{1}{2}\phi_m}{\frac{1}{2}+\frac{1}{2}\phi_m}\right].
\end{aligned}$$

Both terms are positive, therefore, $\frac{\partial \omega_{s1}}{\partial f_m} > 0$.

By symmetry, $\frac{\omega_{s2}}{\partial f_m} = -\frac{\partial \omega_{s1}}{\partial f_m}$. Hence, we conclude that $\left.\frac{\partial(\omega_{s1}-\omega_{s2})}{\partial f_m}\right|_{f_m=f_s=\frac{1}{2}}$ is positive. □

Proof of Lemma 3.8.

Since regions are symmetric, we only need to study the case in which industry is concentrated in region 1 and services are concentrated in region 2. We look at points near $(f_m, f_s) = (1, 0)$ and at what happens when first $f_m \to 1$ and then $f_s \to 0$.

The difference between the real wages in the industrial sector when all the industry is located in region 1 and all the services are located in region 2 is:

$$\omega_{m1} - \omega_{m2} = \frac{K_m}{K_1}(\bar{\omega}_{m1} - \bar{\omega}_{m2}).$$

Notice that the constants K_s and K_1 are strictly positive and finite.

Observe also that when $(f_m, f_s) = (1, 0)$, we have $\bar{\omega}_{m1} = 0$ and $\bar{\omega}_{m2} > 0$.

Therefore, $\omega_{m1} < \omega_{m2}$.

Concentration of each sector in a different region is never an equilibrium. $\square$

Proof of Lemma 3.9.

Since regions are symmetric, it is enough to study the case in which services are symmetrically dispersed while the industry is concentrated in region 1.

The difference between the real wages in the service sector when $f_s = 0.5$ and $f_m = 1$ is:

$$\omega_{s1} - \omega_{s2} = \frac{4\mu_s K}{K_1} \left(\bar{\omega}_{s1} - \bar{\omega}_{s2}\right).$$

The constants K and K_1 are strictly positive and finite.

With $(f_s, f_m) = \left(\frac{1}{2}, 1\right)$, we have $\bar{\omega}_{s2} = \bar{\omega}_{s1} \phi_m^{\frac{\mu_m}{\sigma_m - 1}} < \bar{\omega}_{s1}$.

Therefore, $\omega_{s2} < \omega_{s1}$.

Symmetric dispersion of services with concentration of industry cannot be an equilibrium. $\square$

Proof of Lemma 3.10.

Since regions are symmetric, it is enough to study the case in which industry is dispersed while services are concentrated in region 2 ($f_s = 0$).

From expression (45), the difference between the real wages in the industrial sector is:

$$\omega_{m1} - \omega_{m2} = \frac{\mu_m K}{\phi_m \left[f_m^2 + (1 - f_m)^2\right] K_1 + (1 - f_m) f_m K_2} \left(\bar{\omega}_{m1} - \bar{\omega}_{m2}\right).$$

With $f_s = 0$, we have $\bar{\omega}_{m1} = 0$ and $\bar{\omega}_{m2} > 0$.

This implies that $\omega_{m1} < \omega_{m2}$, because K, K_1 and K_2 are strictly positive and finite.

Dispersion of industry with concentration of services cannot be an equilibrium. $\square$

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