Corporate Leniency Programs when Firms have Private Information: The Push of Prosecution and the Pull of Pre-emption*

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November 2011

Abstract

A corporate leniency program provides relief from government penalties to the first member of a cartel to come forward and cooperate with the authorities. This study explores the incentives to apply for leniency when each cartel member has private information as to the likelihood that the competition authority will be able to convict them without a cooperating firm. A firm may apply for leniency because it fears being convicted ("prosecution effect") or because it fears another firm will apply ("pre-emption effect"). Policies by the competition authority to magnify concerns about pre-emption - and thereby induce greater use of the leniency program - are also explored.

1 Introduction

One of the most important policy developments in U.S. antitrust policy in recent decades is the 1993 revision of the Corporate Leniency Program by the Department of Justice (DOJ). This program allows corporations, who are engaging in illegal antitrust activity (such as price-fixing), to receive amnesty from government penalties if they come forward and cooperate. The appeal of such incentives for discovering cartels and acquiring the evidence to effectively prosecute has resulted in more than 50 countries and jurisdictions having adopted some form of a corporate leniency program.

In light of the importance of leniency programs in practice, there has been a considerable amount of research exploring how leniency programs affect the incentives

*I very much appreciate the comments and suggestions of Faruk Gul, Martin Peitz, seminar participants at the U. of Copenhagen, and conference participants at CRESSE 2011 (Rhodes, Greece), and the research assistance of Weining Bao.

to collude and to report cartels.[1] Beginning with the pioneering paper of Motta and Polo (2003), the primary force in theoretical analyses is that the competition authority may catch the colluding firms and, in anticipation of that prospect, firms may apply for leniency; this I will refer to as the prosecution effect. For example, in Harrington (2008a), the probability of the competition authority discovering and successfully prosecuting the cartel varies over time and, when it is sufficiently high, collusion collapses and all firms race for leniency. While the threat of the competition authority catching the cartel is indeed critical, there is another first-order effect which is absent in previous analyses. Referring to it as the pre-emption effect, this is when a firm - which doesn't necessarily believe the competition authority is likely to catch the cartel - is still concerned that another cartel member may apply for leniency and, because of that concern, applies itself. Indeed, in practice, it is typical that one firm pre–empts its rivals by applying for leniency, as opposed to multiple firms racing for leniency. This outcome - whereby a single firm turns in its fellow colluders - runs contrary to the prediction of all previous models which is that either all or no firms apply.

The objective of this paper is to develop and explore a model that encompasses the pre-emption effect in order to both better understand the incentives of firms in an environment with a leniency program and to investigate how the competition authority can manipulate those incentives through ancillary instruments in order to make a leniency program more effective. Obviously, the key modelling modification is to allow cartel members to have private information regarding the likelihood that the competition authority may be able to effectively prosecute them. As the introduction of private information is a substantive complication, I explore its role in the post-cartel environment; that is, the cartel has collapsed for internal reasons and the objective of each firm is to minimize its expected penalties. Future work will consider embedding this setting into an infinitely repeated game so that the impact of the leniency program on the stability of collusion can be formally explored.

As current intuition regarding the incentive effects of leniency programs is predicated upon firms having common information, I begin by comparing equilibria with private information to equilibria when there is public information. That analysis identifies and investigates the prosecution and pre-emption effects. Whether leniency usage and the frequency of conviction is more or less likely with private signals, compared to when signals are public, depends on the details of the leniency program. When leniency is sufficiently generous (that is, a high fraction of fines are waived), firms are more likely to apply when they have private signals; however, that need not be the case when leniency is stingy. More interestingly, if enough fines are waived with leniency, firms apply for leniency for sure with private signals. Even if the prosecution effect is very weak (that is, a firm believes the competition authority's case

[1] For example, theoretical research includes Motta and Polo (2003), Spagnolo (2003), Aubert, Rey, and Kovacic (2006), Chen and Harrington (2007), Chen and Rey (2007), Harrington (2008a), and Choi and Gerlach (2010); experimental research includes Apesteguia et al, (2007), Hinloopen and Soetevant (2008), and Bigoni et al (2010); and empirical research includes Brenner (2009), Miller (2009), Klein (2010), and Zhou (2010). For a review of some of the research on leniency programs, see Spagnolo (2008).

is unlikely to be adequate to convict), it will still apply for leniency out of concern that its rival will apply. In comparison, when there are public signals, firms do not apply when the prosecution effect is very weak. It is also shown, using numerical analysis, that a more aggressive competition authority can have a significant effect on the probability of conviction through the manner in which the prosecution and pre-emption effects interact. A more aggressive competition authority enhances the prosecution effect which makes firms more likely to apply for leniency. In addition, the prospect of a firm's rival being more likely to apply for leniency (because of the prosecution effect) then makes a firm yet more likely to apply (pre-emption effect) which then makes its rival yet more likely and so forth. Thus, a rise in the prosecution effect is magnified by the pre-emption effect so there is a big return in terms of a rise in the probability of securing a conviction.

After describing the model in Section 2, the theoretical literature on leniency programs is reviewed in Section 3 as it pertains to the issue of information. Equilibria are characterized in Section 4 with a comparison of the cases of public and private information in Section 5. I then analyze the effects of a more aggressive competition authority when it comes to discovering and prosecuting cartels (Section 6). Section 7 concludes.

2 General Model

Consider a cartel composed of two firms for which collusion has ended and firms are independently deciding whether or not to apply for leniency. If a firm is convicted without having received leniency, it pays a fine $F > 0$; while if it receives leniency then its fine is θF where $\theta \in [0, 1)$, so more leniency is associated with a lower value of θ. A firm's only decision is whether or not to apply for leniency and its objective is to minimize expected penalties.

Regarding the decision to apply for leniency, a primary source of uncertainty for a firm is the likelihood that it'll be prosecuted and convicted by the competition authority (CA) when no firm has cooperated through the leniency program; that is, enforcement without assistance of the leniency program. Let ρ denote the probability of a conviction when no firm has applied for leniency. ρ is a random variable from the perspective of firms and, prior to making a leniency decision, firm i receives a private signal $s_i \in [\underline{s}, \overline{s}]$ of ρ. After learning their signals, firms simultaneously decide whether or not to apply for leniency. A strategy for a firm is then of the form: $\phi : [\underline{s}, \overline{s}] \rightarrow \{\text{Apply, Do not apply}\}$. Though firm i does not get to observe firm j's signal, it will have some information if s_i and s_j are correlated, which will be the case if both signals are informative with respect to ρ. Let $H(s_j | s_i)$ be firm i's cdf on firm j's signal conditional on its own signal, $i, j = 1, 2$ and $i \neq j$. To capture the positive correlation between firms' signals, assume A1.

A1 $H(s_j | s_i)$ $(j \neq i)$ is continuously differentiable in s_i and s_j. If $s'' > s'$ then $H(\cdot | s_i = s'')$ weakly first-order stochastically dominates (FOSD) $H(\cdot | s_i = s')$.

A higher signal for a firm results in it attaching more probability to high signals for its rival.

If only one firm applied for leniency then it pays a penalty of θF and the other firm pays F (hence, it is assumed conviction occurs for sure because of a cooperating cartel member). If both firms apply for leniency then each has an equal chance of being the one to receive leniency, so the expected fine is $\left(\frac{1+\theta}{2}\right)F$. If no one applied for leniency then firms are convicted with probability ρ and each pays F, which means firm i's expectation on its penalty is $E\left[\rho\left|s_i\right.\right]F$ where $E\left[\rho\left|s_i\right.\right]$ is its expectation on ρ conditional on its signal. It is assumed that:

A2 $E\left[\rho\left|s_i\right.\right] : [\underline{s}, \overline{s}] \rightarrow (0,1)$ is continuously differentiable and increasing in s_i.

In light of preceding research focusing on the case of public signals, Section 4 begins by comparing the incentives between when signals are private and they are public in order to identify the unique features introduced by allowing firms to have private information. To do so, we will also explore a game when firms' signals (s_1, s_2) are common knowledge so that, when they simultaneously decide whether or not to apply for leniency, they have a common expectation $E\left[\rho\left|s_1, s_2\right.\right]$ on ρ. For that game, a firm's strategy is then of the form $\psi : [\underline{s}, \overline{s}]^2 \rightarrow \{\text{Apply, Do not apply}\}$.

A3 $E\left[\rho\left|s_1, s_2\right.\right] : [\underline{s}, \overline{s}] \rightarrow (0,1)$ is continuously differentiable, responds symmetrically to s_1 and s_2, and is increasing in s_1 and s_2.

3 Literature Review

Since the main modelling innovation of this paper is in terms of information, the literature review will focus on the informational assumptions of previous work and their implications. The initial class of models examining the effect of a leniency program on cartel stability modified the standard infinitely repeated oligopoly game (usually, the Prisoners' Dilemma) by assuming that, in each period that firms are colluding, there is a fixed probability ρ that firms are caught by the CA - in which case they pay a fixed penalty - and firms have the option to apply for leniency to receive reduced penalties. Firms make the leniency decision simultaneously and models differ in terms of whether it occurs after the current period's prices are revealed (e.g., Motta and Polo, 2003) or a firm makes its price and leniency decisions simultaneously, in which case it can simultaneously undercut the collusive price and apply for leniency (e.g., Harrington and Chang, 2010).[2] With ρ being fixed and known over time, the stationarity of the environment implies that if equilibrium involves firms colluding then firms never apply for leniency; if leniency is used as part of an equilibrium then, given stationarity, it occurs in the first period but if conviction prevents reformation of the cartel (for a sufficiently long time) then firms would cheat in the initial period (as they anticipate collusion ending because a firm applied for leniency) which then

[2] Bigoni et al (2010) conducts experiments and allow subjects both to apply for leniency when setting its price or, in the event that no firm has applied for leniency, to apply after prices are revealed and firms learn whether or not there was a deviation.

makes collusion unstable. In brief, a fixed and known value for ρ implies that leniency is not used in equilibrium.[3]

In practice, firms do form a cartel and then, at some point in time, apply for leniency. This can occur if the environment changes so that it was initially optimal for firms to collude and not apply for leniency, and at some later time it becomes optimal to apply. This possibility is explored in Harrington (2008a) where it is assumed ρ is not fixed - it is *iid* over time - but the assumption that ρ is public information among firms is maintained. A Pareto-efficient equilibrium is now characterized by a cut-off value for ρ such that if ρ exceeds that cut-off then firms stop colluding and all apply for leniency - because the prospects of being caught by the CA are sufficiently great so as to cause the cartel to collapse - and otherwise firms collude and do not apply.[4]

There have been some recent analyses to allow for private information in ways distinct from how it is modelled here. Silbye (2010b) assumes ρ is common knowledge but each firm possesses evidence that it could submit to convict the other firm if it applied for leniency. $\varepsilon_i \in [0, 1-\rho]$ is the evidence possessed by firm i to assist in convicting firm j and is private information to firm i. If firm i receives leniency then firm j's expected penalty is $(\rho + \varepsilon_i) F$. If no one applies then each has an expected fine of ρF, which, as noted, is common knowledge. Sauvagnat (2010) allows the CA to have private information about the strength of its case and it is a strategic decision whether to open an investigation. (Previous work implicitly allowed for such private information but assumed the start of an investigation was exogenous.) Of particular relevance is that the CA may open an investigation even when its case is weak, as doing so may induce firms to apply for leniency. Finally, Pinna (2010) considers the strategic choice of competition policy when the firms know whether or not they are colluding but the CA does not.

4 Characterization of Equilibrium

4.1 Public Signals

To appreciate the new forces introduced with private information, let us begin by characterizing equilibrium when firms' signals (s_1, s_2) are public information. First note that there are only symmetric equilibria since applying for leniency is optimal when the other firm applies. Hence, either both apply or neither apply. The set of Bayes-Nash equilibria is:

$$\psi(s_1, s_2) = \begin{cases} \text{Apply} & \text{if } (s_1, s_2) \notin \Omega \\ \\ \text{Do not apply} & \text{if } (s_1, s_2) \in \Omega \end{cases} \qquad (1)$$

[3] There is also an equilibrium in Motta and Polo (2003) - referred to as "collude and report" - for which firms collude and apply for leniency in every period. This occurs when the cartel can immediately reform and leniency is sufficiently generous. Though it can be an equilibrium, it would not seem to be an empirically relevant solution.

[4] Harrington (2008a) also considers a policy space for which amnesty is awarded if and only if ρ is sufficiently low (i.e., the competition authority's case is not too strong). Silbye (2010a) enriches the policy space to when the amount of leniency can depend continuously on ρ.

where

$$E\left[\rho\,|s_1,s_2\right]\leq\theta\ \forall\ \ (s_1,s_2)\in\Omega. \tag{2}$$

If $(s_1,s_2)\notin\Omega$, so that a firm's rival is going to apply for leniency, it is optimal for a firm to apply as well, as an expected penalty of $\left(\frac{1+\theta}{2}\right)F$ is preferred to a sure penalty of F. Now consider $(s_1,s_2)\in\Omega$ so that a firm's rival is not expected to apply for leniency. If a firm does not apply as well then its expected penalty is $E\left[\rho\,|s_1,s_2\right]F$, while it is θF from applying. Hence, not applying is optimal if and only if (iff) $E\left[\rho\,|s_1,s_2\right]\leq\theta$. As long as

$$\theta>E\left[\rho\,|s_1=\underline{s},s_2=\underline{s}\right],$$

so there are some signals for which it is an equilibrium for both not to apply, there are an infinite number of equilibria as there are an infinite number of sets Ω satisfying (2).

Notice that if $(s_1,s_2)\notin\Omega$ and $E\left[\rho\,|s_1,s_2\right]<\theta$ then firms are incurring higher penalties by both applying for leniency than if both did not, as

$$E\left[\rho\,|s_1,s_2\right]F<\theta F<\left(\frac{1+\theta}{2}\right)F,$$

where the term to the left of the first inequality is the expected penalties from both not applying and the term to the right of the second inequality is from both applying. This makes it useful to define the Pareto-efficient equilibrium,

$$\Omega^*\equiv\left\{(s_1,s_2):E\left[\rho\,|s_1,s_2\right]\leq\theta\right\},$$

which has firms not apply whenever it is an equilibrium. This equilibrium minimizes expected penalties. Note that if

$$\theta<E\left[\rho\,|s_1,s_2\right]<\frac{1+\theta}{2},$$

expected penalties are higher with both applying compared to both not applying but it is not an equilibrium for both not to apply given those signals.

4.2 Private Signals

Now suppose firms' signals are private information, and consider a symmetric strategy profile which is a cut-off strategy:

$$\phi(s_i)=\begin{cases}\text{Do not apply} & \text{if } s_i\in[\underline{s},x]\\ \\ \text{Apply} & \text{if } s_i\in(x,\overline{s}]\end{cases} \tag{3}$$

A firm applies for leniency iff its signal exceeds x. The set of symmetric cut-off Bayes-Nash equilibria can then be characterized by the set of values for x such that

ϕ is a Bayes-Nash equilibrium. Given firm 2 uses this strategy, the expected penalty to firm 1 from not applying is

$$\int_{\underline{s}}^{x} E\left[\rho \left| s_1, s_2\right.\right] F H'\left(s_2 \left| s_1\right.\right) ds_2 + \left[1 - H\left(x \left| s_1\right.\right)\right] F$$

or

$$H\left(x \left| s_1\right.\right) E\left[\rho \left| s_1, s_2 \leq x\right.\right] F + \left[1 - H\left(x \left| s_1\right.\right)\right] F. \tag{4}$$

If $s_2 \leq x$ then firm 2's signal is sufficiently low that it does not apply in which case firm 1's expected penalty from not applying is its expectation on ρ multiplied by F. This expectation, $E\left[\rho \left| s_1, s_2 \leq x\right.\right]$, is conditional on firm 1's signal and firm 2 having a signal that induces it not to apply for leniency. If $s_2 > x$ then firm 2 applies for leniency in which case, by not applying, firm 1 is convicted and pays F. If instead firm 1 applies for leniency then its expected penalty is

$$H\left(x \left| s_1\right.\right) \theta F + \left[1 - H\left(x \left| s_1\right.\right)\right] \left(\frac{1+\theta}{2}\right) F. \tag{5}$$

If $s_2 \leq x$ then it is the only firm to apply for leniency so its penalty is θF, while if $s_2 > x$ then both firms have applied in which case firm 1's expected penalty is $\left(\frac{1+\theta}{2}\right) F$.

Firm 1 strictly prefers to apply for leniency iff (4) exceeds (5):

$$\begin{aligned} & H\left(x \left| s_1\right.\right) E\left[\rho \left| s_1, s_2 \leq x\right.\right] F + \left[1 - H\left(x \left| s_1\right.\right)\right] F \\ > \; & H\left(x \left| s_1\right.\right) \theta F + \left[1 - H\left(x \left| s_1\right.\right)\right] \left(\frac{1+\theta}{2}\right) F, \end{aligned} \tag{6}$$

where recall that each firm is making a choice to *minimize* expected penalties. This expression can be re-arranged to

$$E\left[\rho \left| s_1, s_2 \leq x\right.\right] > \theta - \left(\frac{1-\theta}{2}\right) \left[\frac{1 - H\left(x \left| s_1\right.\right)}{H\left(x \left| s_1\right.\right)}\right]. \tag{7}$$

It is optimal to apply for leniency when the expected probability of being caught by the CA, $E\left[\rho \left| s_1, s_2 \leq x\right.\right]$, is sufficiently large relative to the leniency parameter θ. The relevant expectation on being caught by the CA is for when no one applies for leniency, which is the expectation on ρ conditional on a firm's own signal *and* that its rival's signal is sufficiently low that it does not apply for leniency.[5]

Using (7), define

$$\Delta\left(s_1, x\right) \equiv E\left[\rho \left| s_1, s_2 \leq x\right.\right] - \theta + \left(\frac{1-\theta}{2}\right) \left[\frac{1 - H\left(x \left| s_1\right.\right)}{H\left(x \left| s_1\right.\right)}\right].$$

[5] The intuition is analogous to that for auctions when bidders have affiliated values. A bidder selects its bid based on beliefs over the true value when its bid matters for its payoff, which is when its signal is the highest as then its bid is the highest so it wins the item. Analogously, a firm evaluates the payoff from not applying for leniency using beliefs as to the likelihood of being caught by the CA when such beliefs matter, which is when the other firm's signal is sufficiently low that it does not apply for leniency.

Given signal s_1 and threshold x, applying is optimal for firm 1 iff $\Delta(s_1, x) > 0$. Next note that leniency becomes relatively more attractive when a firm's own signal is higher since a higher signal makes it more likely it'll be caught by the CA:

$$\frac{\partial \Delta(s_1, x)}{\partial s_1} = \frac{\partial E[\rho\,|s_1, s_2 \leq x]}{\partial s_1} - \left(\frac{1-\theta}{2}\right)\left[\frac{\partial H(x\,|s_1)/\partial s_1}{H(x\,|s_1)^2}\right] > 0 \tag{8}$$

as

$$\frac{\partial E[\rho\,|s_1, s_2 \leq x]}{\partial s_1} > 0$$

by A2 and

$$\frac{\partial H(x\,|s_1)}{\partial s_1} \leq 0$$

by A1. Hence, applying for leniency is optimal when $s_1 > x$ iff $\Delta(x, x) \geq 0$, as then $\Delta(s_1, x) > 0, \forall s_1 > x$. Not applying is optimal when $s_1 < x$ iff $\Delta(x, x) \leq 0$, as then $\Delta(s_1, x) < 0, \forall s_1 < x$. Therefore, if $x \in (\underline{s}, \overline{s})$ then ϕ is a Bayes-Nash equilibrium iff $\Delta(x, x) = 0$.

Define

$$\begin{aligned} \Phi(x) &\equiv \Gamma(x)\,\Delta(x, x) \\ &= \Gamma(x)\{E[\rho\,|s_1 = x, s_2 \leq x] - \theta\} + \left(\frac{1-\theta}{2}\right)[1 - \Gamma(x)], \end{aligned}$$

where $\Gamma(x) \equiv H(x\,|x)$ and is the probability that $s_2 \leq x$ conditional on $s_1 = x$. In some of the analysis, it'll be easier to work with $\Phi(x)$ because it is bounded as $x \to \underline{s}$. Here are some sufficient conditions for equilibrium, with the first condition summarizing the preceding analysis.

- If $\Phi(s') = 0$ then $x = s'$ is an equilibrium cut-off.

- $x = \underline{s}$ is an equilibrium cut-off. Note that

  $$\begin{aligned} \Phi(\underline{s}) &= H(\underline{s}\,|s_1 = \underline{s})\{E[\rho\,|s_1 = \underline{s}, s_2 \leq \underline{s}] - \theta\} + \left(\frac{1-\theta}{2}\right)[1 - H(\underline{s}\,|s_1 = \underline{s})] \\ &= \frac{1-\theta}{2} > 0, \end{aligned}$$

  which, by (8), means $\Delta(s_1, \underline{s}) > 0\ \forall\ s_1$, and, therefore, applying is strictly preferred to not applying for all signals. In other words, if a rival is going to apply for leniency for sure (that is, for every signal) then it is optimal to do so as well.

- If $\Phi(\overline{s}) \leq 0$ then $x = \overline{s}$ is an equilibrium cut-off. Again by (8), $\Delta(s_1, \overline{s}) \leq 0\ \forall\ s_1 < \overline{s}$, which implies it is never optimal to apply. Thus, not applying for all signals is an equilibrium if, conditional on the other firm never applying, a firm prefers not to apply even when it receives the strongest signal $\overline{s}$.

- If
$$E\left[\rho\left|s_1=\overline{s},s_2=\overline{s}\right.\right]>\theta>E\left[\rho\left|s_1=\overline{s}\right.\right]$$
then there are at least three equilibria: $x\in\{\underline{s},s',\overline{s}\}$ for some $s'\in(\underline{s},\overline{s})$.This follows from:
$$\Phi\left(\underline{s}\right)=\frac{1-\theta}{2}>0>E\left[\rho\left|s_1=\overline{s}\right.\right]-\theta=\Phi\left(\overline{s}\right).$$
By continuity, $\exists s'\in(\underline{s},\overline{s})$ such that $\Phi(s')=0$.

It will be useful to consider the Pareto-efficient symmetric cut-off Bayes-Nash equilibrium which is the equilibrium with the highest threshold x, as I will show it minimizes expected penalties for firms. Given the *other* firm's threshold is x, a firm expected penalty from applying for leniency is decreasing in x,

$$H'\left(x\left|s_1\right.\right)\theta F-H'\left(x\left|s_1\right.\right)\left(\frac{1+\theta}{2}\right)F=-H'\left(x\left|s_1\right.\right)\left(\frac{1-\theta}{2}\right)F<0,$$

and from not applying for leniency is decreasing in x,

$$-F\left\{1-E\left[\rho\left|s_1,s_2=x\right.\right]\right\}H'\left(x\left|s_1\right.\right)<0.$$

For any strategy of this firm, its expected penalty is lower when the other firm's threshold is higher. Hence, its optimal strategy must result in lower expected penalties when x is higher. Thus, firms rank equilibria according to the threshold x, and equilibria with higher x are more preferred.

5 Comparison of Equilibria with Public and Private Signals

With public signals and assuming the other firm does not apply for leniency, a firm prefers to apply iff

$$E\left[\rho\left|s_1,s_2\right.\right]-\theta>0. \tag{9}$$

Focusing on the Pareto-efficient equilibrium, leniency is used only when the likelihood attached to the CA prosecuting and convicting them is sufficiently high relative to the leniency parameter. Behavior is entirely driven by beliefs as to CA behavior.

By comparison, consider the situation when firms' signals are private. From (7), firm 1 prefers to apply for leniency iff

$$\underbrace{E\left[\rho\left|s_1,s_2\leq x\right.\right]-\theta}_{\text{Prosecution effect}}>\underbrace{-\left(\frac{1-\theta}{2}\right)\left[\frac{1-H\left(x\left|s_1\right.\right)}{H\left(x\left|s_1\right.\right)}\right]}_{\text{Pre-emption effect}} \tag{10}$$

In contrasting, (9) and (10), first note that the LHS is different, which encompasses what is referred to as the *prosecution effect* for it deals with beliefs as to the CA's probability of a successful prosecution (without use of the leniency program) relative to the leniency parameter. With public signals, the likelihood attached to the CA levying penalties is based on firms' common signals. With private signals, a firm doesn't know its rival's signal and so its expectation is based on its own signal and its rival's signal being sufficiently low that it chooses not to apply. The relationship between these two expectations - $E[\rho | s_1, s_2]$ and $E[\rho | s_1, s_2 \leq x]$ - is ambiguous. What is not ambiguous is the relationship between the RHS of these two conditions. With private signals, a firm is not assured as to what the other firm will do. Even if firm 1's signal is very low - suggesting that being caught by the CA is unlikely and thus firms should not apply for leniency (that is, the prosecution effect is weak) - it realizes that firm 2's signal could be high in which case it would apply. Note that $1 - H(x | s_1)$ is the probability that a rival applies for leniency conditional on a firm's signal in which case the RHS of (10) is lower, the more likely it is that the other firm will apply for leniency. This provides a second reason for firm 1 to apply for leniency, quite independent of whether it thinks the CA will catch them. It is referred to as the *pre-emption effect* because it captures a firm's concern with its rival applying for leniency prior to the firm itself having information that the CA is a serious threat.

In sum, it is not immediately clear whether information being private makes firms more or less inclined to apply for leniency. While the pre-emption effect is present only with private signals - and clearly serves to enhance the attractiveness of applying for leniency - whether the prosecution effect is stronger or weaker with private signals is not determined. To clarify matters, Theorem 1 shows that a sufficiently generous leniency program induces firms to always apply for leniency when they have private signals. All proofs are in Appendix A.

Theorem 1 *For the case of private signals, $\exists \theta' \in (0,1)$ such that, $\forall \theta \in [0, \theta')$, the unique symmetric cut-off Bayes-Nash equilibrium is for firms to apply for leniency for all signals.*

No matter how generous is leniency, as long as some penalties are not waived (that is, $\theta > 0$), it is possible that firm 1 could receive a sufficiently weak signal that it would prefer not to apply for leniency on the basis that the CA is sufficiently unlikely to convict; that is, $E[\rho | s_1] < \theta$. As θ gets smaller, the requisite signal for that to be true must be lower but, at least when $E[\rho | s_1 = \underline{s}] = 0$, such signals exist. Of course, firm 1 is also concerned with the prospect of firm 2 applying and if, by the same argument, it takes a really weak signal for firm 2 not to apply then firm 1 attaches low probability to that event. That is, even if firm 1's signal is extremely weak, it is very unlikely that firm 2's signal is also extremely weak. Given that firm 1 then believes firm 2 is likely to apply, firm 1 finds it optimal to apply as well, *regardless of the signal that firm 1 receives.* The prosecution effect can be very weak but, due to the strength of the pre-emption effect, firms apply for leniency. Hence, when leniency is sufficiently generous (even though it is not complete) and signals are private, firms always apply.

Theorem 1 does not by itself prove that, when leniency is sufficiently generous, the program is used more often with private signals than with public signals. It shows that when θ is sufficiently low, firms use the leniency program with probability one when signals are private. For usage to exceed that when signals are public, the leniency program must be used with probability less than one when signals are public which is the case only if $\theta > E[\rho \,| s_1 = \underline{s}, s_2 = \underline{s}]$, but then there is the issue of whether θ is low enough for Theorem 1 to apply. This matter is resolved with the next result.

Theorem 2 *There exists $\varepsilon > 0$ such that if*

$$E[\rho \,| s_1 = \underline{s}, s_2 = \underline{s}] < \theta < E[\rho \,| s_1 = \underline{s}, s_2 = \underline{s}] + \varepsilon$$

then the probability of leniency usage and the probability of conviction is higher for all symmetric cut-off Bayes-Nash equilibria with private signals than for the Pareto-efficient symmetric cut-off equilibrium with public signals.

The next result presumes the condition $E[\rho \,| s_1 = \overline{s}, s_2 = \overline{s}] > E[\rho \,| s_1 = \overline{s}]$, which is quite natural if $E[\rho \,| s_1 = \overline{s}] > E[\rho]$, so that the highest signal causes firms to increase their expectation on ρ, and firms' signals are positively but not perfectly correlated. If leniency is sufficiently weak - $E[\rho \,| s_1 = \overline{s}] < \theta$ - then the Pareto-efficient equilibrium with private signals is to never apply for leniency because, given the other firm never applies, a firm prefers not to apply even if it receives the strongest signal. If, in addition, leniency is not too weak - so that $\theta < E[\rho \,| s_1 = \overline{s}, s_2 = \overline{s}]$ - then leniency is used with positive probability in all equilibria when there are public signals; both firms receiving the strongest signals induces usage of the program. Under these conditions, leniency is used more frequently when there are public signals because, given firms are initially doubtful about the prospects of being caught, inducing them to apply for leniency requires enough information to counteract those prior beliefs. A single signal is inadequate but two signals could be sufficient.

Theorem 3 *If*

$$E[\rho \,| s_1 = \overline{s}] < \theta < E[\rho \,| s_1 = \overline{s}, s_2 = \overline{s}]$$

then the probability of leniency usage and conviction is higher for all symmetric cut-off Bayes-Nash equilibria with public signals compared to the Pareto-efficient symmetric cut-off Bayes-Nash equilibrium with private signals.

In Harrington (2011), an example is provided which illustrates the main results of this section. There it is assumed the probability of conviction equals the sum of the firms' signals, $\rho = s_1 + s_2$, where s_1 and s_2 are independent with a uniform distribution on $[0, 1/2]$. It is shown that the probability of conviction with private signals is higher (lower) than that with public signals when $\theta < .715$ ($\theta > .715$). In fact, conviction can be significantly more likely when θ is moderately low and is at most mildly more likely when θ is high.

In concluding this section, it is worth noting that these results have some implications for optimal competition policy. Suppose a CA has some evidence which it can share with two firms suspected of having formed a cartel. Does it want to have a policy of sharing the same evidence with both firms or instead a policy in which it shares different parts of the evidence with different firms? To be more specific, suppose one batch of evidence is represented by signal s_1 and a second batch is represented by signal s_2. The case of public signals is then a policy of sharing all evidence (both signals) with both firms, while the case of private signals is a policy of sharing some evidence with one firm (signal s_1) and different evidence with the other firm (signal s_2). What the preceding analysis suggests is that, as long as the leniency program is sufficiently generous, a CA wants to show firms different evidence because, by resulting in firms having private information, a CA can enhance the pre-emption effect and induce greater use of the leniency program and a higher probability of conviction. Asymmetric beliefs among firms is complementary to a generous leniency program.

6 Impact of a More Aggressive Competition Authority

In this section, we explore the impact of a CA being seen as more aggressive in the sense that firms initially believe the CA is more likely to discover and successfully prosecute a cartel. In order to perform this comparative static, the model is simplified by assuming the support of ρ is $\{\underline{\rho}, \overline{\rho}\}$ where $0 \leq \underline{\rho} < \overline{\rho} \leq 1$. I will speak of the CA having either a strong case ($\rho = \overline{\rho}$) or a weak case ($\rho = \underline{\rho}$). Let γ be the prior probability that the CA is strong, and the CA is said to be more aggressive when γ is higher. A CA could be more aggressive by instituting the practice of screening data for evidence of collusion or having a bigger budget for prosecuting cases.[6] $f(s\,|\rho)$ is the density function on a firm's signal conditional on the strength of the CA's case (as summarized by ρ). The associated cdf is $F(s\,|\rho)$, and assume $F(\cdot\,|\overline{\rho})$ FOSD $F(\cdot\,|\underline{\rho})$. Finally, assume that firms' signals are independent conditional on ρ.

Define x^* as the maximal equilibrium:

$$x^* = \begin{cases} \underline{s} & \text{if } \Phi(x) > 0, \forall x \in [\underline{s}, \overline{s}] \\ \max\{x : \Phi(x) = 0\} & \text{if } \exists x \in (\underline{s}, \overline{s}) \text{ s.t. } \Phi(x) \leq 0 \text{ and } \Phi(\overline{s}) > 0 \\ \overline{s} & \text{if } \Phi(\overline{s}) \leq 0. \end{cases}$$

where $\Phi(x) = \Gamma(x)\Delta(x)$ and recall:

$$\Delta(x) = E[\rho\,|s_1 = x, s_2 \leq x] - \theta + \left(\frac{1-\theta}{2}\right)\left[\frac{1-\Gamma(x)}{\Gamma(x)}\right]. \tag{11}$$

Assuming the two-point distribution and conditional independence of signals, it is derived in Appendix B that:

$$\Gamma(x) = \frac{\gamma f(x\,|\overline{\rho})\,F(x\,|\overline{\rho}) + (1-\gamma)\,f(x\,|\underline{\rho})\,F(x\,|\underline{\rho})}{\gamma f(x\,|\overline{\rho}) + (1-\gamma)\,f(x\,|\underline{\rho})} \tag{12}$$

and

$$E[\rho\,|s_1 = x, s_2 \leq x] = \frac{\gamma f(x\,|\overline{\rho})\,F(x\,|\overline{\rho})\,\overline{\rho} + (1-\gamma)\,f(x\,|\underline{\rho})\,F(x\,|\underline{\rho})\,\underline{\rho}}{\gamma f(x\,|\overline{\rho})\,F(x\,|\overline{\rho}) + (1-\gamma)\,f(x\,|\underline{\rho})\,F(x\,|\underline{\rho})}. \tag{13}$$

[6] The case for screening is presented in Harrington (2007, 2008b).

The next result shows that a more aggressive CA induces greater usage of the leniency program. Recall that a firm applies for leniency iff its signal exceeds x^*.

Theorem 4 *If $x^* \in (\underline{s}, \overline{s})$ then, generically, $\frac{\partial x^*}{\partial \gamma} < 0$.*

By Theorem 4, a more aggressive CA results in a greater likelihood of a leniency application coming from a cartel, as the requisite signal to induce a leniency application is not as high. To consider its effect on the probability of conviction, first note that this probability equals

$$\gamma \left[\overline{\rho} + (1-\overline{\rho}) \left(1 - F\left(x^*(\gamma) | \overline{\rho}\right)^2\right)\right] + (1-\gamma) \left[\underline{\rho} + \left(1-\underline{\rho}\right) \left(1 - F\left(x^*(\gamma) |\underline{\rho}\right)^2\right)\right]. \tag{14}$$

When the CA has a strong case (which occurs with probability γ), the CA successfully convicts with probability $\overline{\rho}$ regardless of whether firms apply for leniency and, in the event that they do not have the evidence to achieve a guilty verdict (which occurs with probability $1-\overline{\rho}$), still convict because a firm applies for leniency, which occurs with probability $1-F\left(x^*(\gamma) | \overline{\rho}\right)^2$ (which is the probability at least one firm receives a signal exceeding x^*). There is an analogous description when, with probability $1-\gamma$, the CA has a weak case.

In order to assess the effect of a more aggressive CA, take the derivative of the probability of conviction in (14) with respect to γ:

$$\begin{aligned}&\left[\left(1-\underline{\rho}\right) F\left(x^*(\gamma) |\underline{\rho}\right)^2 - (1-\overline{\rho}) F\left(x^*(\gamma) | \overline{\rho}\right)^2\right] \\ &-2\left[\gamma (1-\overline{\rho}) F\left(x^*(\gamma) | \overline{\rho}\right) f\left(x^*(\gamma) | \overline{\rho}\right) + (1-\gamma)\left(1-\underline{\rho}\right) F\left(x^*(\gamma) |\underline{\rho}\right) f\left(x^*(\gamma) |\underline{\rho}\right)\right] \left(\frac{\partial x^*}{\partial \gamma}\right).\end{aligned}$$

The first bracketed term is positive because $1-\underline{\rho} > 1-\overline{\rho}$ and that a higher value for ρ causes a FOSD shift in F so $F\left(x^*(\gamma) |\underline{\rho}\right)^2 \geq F\left(x^*(\gamma) | \overline{\rho}\right)^2$. That expression captures the direct effect from a more aggressive CA in that it is more likely to have a strong case and thus more likely to get a conviction, either because it has the evidence to achieve a guilty plea or it results in a firm receiving a signal that induces it to apply for leniency. The second bracketed term is also positive and is the indirect effect coming from increased usage of the leniency program due to a fall in the requisite signal required to go to the CA: $\partial x^*/\partial \gamma < 0$. Given stronger prior beliefs that the CA is aggressive, the signal that a firm must receive to induce it to apply for leniency does not have to be as supportive of the CA having a convincing case.

In assessing the impact of a more aggressive CA, it is important to recognize a feedback effect that arises in how the prosecution and pre-emption effects interact. A more aggressive CA enhances the prosecution effect which makes, say, firm 1 more inclined to apply for leniency because it thinks it is more likely the CA will be able to convict them without a firm in the leniency program. By the same argument, firm 1 realizes that a stronger prosecution effect makes it more likely that firm 2 will apply; hence, firm 1 is yet more inclined to apply because firm 2 is more likely to do so. Again, firm 2 goes through the same calculus so firm 2 is more apt to apply because firm 1 is more apt to do so which makes firm 1 even more inclined to apply and so forth. In this manner, there is a multiplier effect from a more aggressive CA in that it raises the prosecution effect which then leads to a series of pre-emption effects.

The preceding discussion suggests that a more aggressive CA should have an increasing *convex* effect on the probability of conviction. Numerical analysis is conducted to explore this conjecture. Assume the following density function on a firm's signal $s \in [0,1]$,

$$f(s\,|\rho) = 2(1-\rho) + 2(2\rho - 1)s.$$

Recall that firms' signals are independent conditional on ρ.[7] Note that $f(s\,|\rho)$ is increasing in s when $\rho < 1/2$, is the uniform density when $\rho = 1/2$, and is decreasing in s when $\rho > 1/2$.[8] There are four parameters in the model, $\left(\underline{\rho}, \overline{\rho}, \gamma, \theta\right)$. Numerical analysis was conducted for a wide array of parameterizations and the results reported here are for:

$$\left(\underline{\rho}, \overline{\rho}\right) \in \{(0,1), (0,.75), (.25,1)\}, \theta \in \{.3,.4,...,.7\}, \gamma \in \{.10,.11,...,.90\}$$

Figure 1 is when a strong CA convicts for sure ($\overline{\rho} = 1$) and a weak CA fails to convict for sure ($\underline{\rho} = 0$). The probability of conviction is increasing and convex in the prior probability that firms assign to the CA being strong (γ). It is also increasing in the extent of leniency; note that the curves shift up for smaller values of θ. Consider, for example, $\theta = .5$. A .10 rise in γ from .2 to .3 increases the probability of conviction by .135 (rising from .251 to .398), while an additional .10 rise in γ from .3 to .4 increases the probability of conviction by .147 (as it rises to .553). The convexity is yet more apparent when the strength of a strong CA is reduced to convicting with probability .75 (Figure 2). Similar conclusions are drawn from Figure 3 which reports when the strength of a weak CA is raised to convicting with probability .25.[9] The way in which the pre-emption effect magnifies a rise in the prosecution effect argues to putting more resources into a competition authority after a leniency program is instituted.

7 Concluding Remarks

This paper has provided the first analysis of the incentive effects of leniency program when former cartel members have private information about the likelihood that a competition authority can convict. The presence of private information is compelling and was shown to have a substantive effect on how leniency program generate convictions. A firm's decision to apply for leniency is driven not only by concerns that the competition authority has a strong case (prosecution effect) but also with concerns that its rival believes the competition authority has a strong case and thus will apply (pre-emption effect). The pre-emption effect can cause firms to apply for sure when

[7] This density function is a generalization of an example suggested by Faruk Gul.

[8] If $\rho' > \rho''$ then $f(s\,|\rho')/f(s\,|\rho'')$ is increasing in s and therefore satisfies the Monotone Likelihood Ratio Property.

[9] The cases of $\theta \in \{.3,.4\}$ are not reported as the probability of conviction is one for all $\gamma \in \{.1,...,.9\}$.

the leniency program is sufficiently generous, and can create a multiplier effect with a rise in the prosecution effect.

The model formulated and investigated in this paper was very simple and, as a result, there are a variety of ways in which to enrich it to tackle other issues related to collusion and competition policy. Thus far, the analysis has focused on the incentives to apply for leniency after a cartel has collapsed. By embedding this end game into an infinitely repeated game of collusion, one can assess how leniency programs - when firms have private information - influence the decision to form a cartel and the expected duration of the cartel. Previous theoretical research, beginning with Motta and Polo (2003), has developed some understanding as to how leniency programs destabilize cartels or, in some cases, stabilize them (Chen and Harrington, 2007). How is that understanding changed when firms have private information when it comes to deciding whether to apply for leniency?

In all previous work - including the current paper - the post-cartel environment is static (or stationary) so that, in equilibrium, firms either apply for leniency immediately upon collapse of the cartel or never apply. In reality, the post-cartel environment is far richer and more nuanced. A better description is that firms receive information over time as to whether the competition authority will open an investigation and, if they have already done so, the strength of the case. Firms are then engaged in a multi-period game in which they receive signals according to some stochastic process and update their beliefs over time. Now, a firm that decides not to apply for leniency has an option value associated with applying later, at least as long as a rival does not go to the competition authority in the meantime. Given that set-up, it would be interesting to consider the strategic role of the competition authority in encouraging firms to apply for leniency. The competition authority could also receive signals - in the form of complaints and other forms of evidence - and can choose whether and how to share them with the firms. More generally, it is worthwhile to engage in a richer investigation of what instruments are available to the competition authority and how it can best use them to enhance the pre-emption effect and thereby make leniency programs more effective at producing convictions.

8 Appendix A

Proof of Theorem 1. For this theorem to hold, it must be true that

$$\Phi(x) = \Gamma(x)\{E[\rho\,|s_1 = x, s_2 \leq x] - \theta\} + \left(\frac{1-\theta}{2}\right)[1-\Gamma(x)] > 0,\ \forall x.$$

Set $\Phi(x) = 0$ and solve for θ :

$$\Gamma(x)\{E[\rho\,|s_1 = x, s_2 \leq x] - \theta\} + \left(\frac{1-\theta}{2}\right)[1-\Gamma(x)] = 0$$

$$\theta = \frac{1+\Gamma(x) - 2\Gamma(x)\{1 - E[\rho\,|s_1 = x, s_2 \leq x]\}}{1+\Gamma(x)} \equiv \theta^o(x) \in (0,1), \forall x \in (\underline{s}, \overline{s}).$$

Since $\lim_{x\to\underline{s}} \Gamma(x) = 0$ and $\lim_{x\to\overline{s}} \Gamma(x) = 1$ then

$$\lim_{x\to\underline{s}} \theta^o(x) = 1, \lim_{x\to\overline{s}} \theta^o(x) = E[\rho\,|s_1 = \overline{s}] > 0,$$

which is positive by A1. Hence,

$$\theta^o(x) \in (0,1], \forall x \in [\underline{s}, \overline{s}].$$

Since

$$\frac{\partial \Phi(x)}{\partial \theta} = -\left(\frac{1+\Gamma(x)}{2}\right) < 0,$$

then

$$\Phi(x) > 0,\ \forall \theta \in [0, \theta^o(x)).$$

Defining

$$\theta' \equiv \min\{\theta^o(x) : x \in [\underline{s}, \overline{s}]\} \in (0,1)$$

then

$$\Phi(x) > 0,\ x \in [\underline{s}, \overline{s}],\ \forall \theta \in [0, \theta').$$

■

Proof of Theorem 2. With public signals, the Pareto-efficient equilibrium has firms not applying for leniency when $E[\rho\,|s_1, s_2] < \theta$. If $E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}] < \theta$ then the event, $E[\rho\,|s_1, s_2] < \theta$, occurs with positive probability. Therefore, the probability of leniency usage and the probability of conviction are both less than one.

The next step is to provide sufficient conditions for the unique equilibrium with private signals to involve firms applying for leniency for all signals, in which case the probability of leniency usage and the probability of conviction both equal one. The unique equilibrium has firms always applying for leniency iff

$$(\Phi(x) =)\, \Gamma(x)\{E[\rho\,|s_1 = x, s_2 \leq x] - \theta\} + \left(\frac{1-\theta}{2}\right)[1-\Gamma(x)] > 0,\ \forall x. \quad (15)$$

As an intermediate step, let us show that $E[\rho|s_1 = x, s_2 \leq x]$ is increasing in x. Let G be the cdf on a firm's signal

$$E[\rho|s_1 = x, s_2 \leq x] = \int_{\underline{s}}^{x} E[\rho|s_1 = x, s_2] \left(\frac{G'(s_2)}{G(x)} \right) ds_2$$

$$\begin{aligned} \frac{\partial E[\rho|s_1 = x, s_2 \leq x]}{\partial x} &= E[\rho|s_1 = x, s_2 = x] \left(\frac{G'(x)}{G(x)} \right) + \int_{\underline{s}}^{x} \left(\frac{\partial E[\rho|s_1 = x, s_2]}{\partial s_1} \right) \left(\frac{G'(s_2)}{G(x)} \right) ds_2 \\ &\quad - \int_{\underline{s}}^{x} E[\rho|s_1 = x, s_2] \left(\frac{G'(s_2) G'(x)}{G(x)^2} \right) ds_2. \end{aligned}$$

Since $\partial E[\rho|s_1 = x, s_2] / \partial s_1 > 0$ then $\partial E[\rho|s_1 = x, s_2 \leq x] / \partial x > 0$ if

$$E[\rho|s_1 = x, s_2 = x] \left(\frac{G'(x)}{G(x)} \right) - \int_{\underline{s}}^{x} E[\rho|s_1 = x, s_2] \left(\frac{G'(s_2) G'(x)}{G(x)^2} \right) ds_2 > 0$$

$$E[\rho|s_1 = x, s_2 = x] - \int_{\underline{s}}^{x} E[\rho|s_1 = x, s_2] \left(\frac{G'(s_2)}{G(x)} \right) ds_2 > 0,$$

which is true since $E[\rho|s_1 = x, s_2]$ is increasing in s_2.

Assume

$$\theta \in (E[\rho|s_1 = \underline{s}, s_2 = \underline{s}], E[\rho|s_1 = \overline{s}, s_2 \leq \overline{s}]). \tag{16}$$

By continuity and that $E[\rho|s_1 = x, s_2 \leq x]$ is increasing in x, there exists unique $s' \in (\underline{s}, \overline{s})$ defined by

$$E[\rho|s_1 = s', s_2 \leq s'] - \theta = 0.$$

It follows that $\Phi(x) > 0 \; \forall x \geq s'$. To prove (15), we then need to show:

$$\Gamma(x) \{E[\rho|s_1 = x, s_2 \leq x] - \theta\} + \left(\frac{1-\theta}{2} \right) [1 - \Gamma(x)] > 0, \; \forall x < s'. \tag{17}$$

Given this expression is increasing in $E[\rho|s_1 = x, s_2 \leq x]$ and $E[\rho|s_1 = x, s_2 \leq x]$ is increasing in x then a lower bound is

$$\Gamma(x) \{E[\rho|s_1 = \underline{s}, s_2 = \underline{s}] - \theta\} + \left(\frac{1-\theta}{2} \right) [1 - \Gamma(x)]. \tag{18}$$

Since, by assumption $E[\rho|s_1 = \underline{s}, s_2 = \underline{s}] - \theta < 0$, (18) is decreasing in $\Gamma(x)$. Define

$$\widetilde{x} = \arg \max_{x \in [\underline{s}, s']} \Gamma(x), \tag{19}$$

and note that $\Gamma(\widetilde{x}) < 1$. Hence, a lower bound to (19) is achieved by replacing $\Gamma(x)$ with $\Gamma(\widetilde{x})$. Thus, a sufficient condition for (17) to be true is

$$\Gamma(\widetilde{x}) \{E[\rho|s_1 = \underline{s}, s_2 = \underline{s}] - \theta\} + \left(\frac{1-\theta}{2} \right) [1 - \Gamma(\widetilde{x})] > 0. \tag{20}$$

Re-arranging (20),

$$\theta < \left(\frac{2\Gamma(\widetilde{x})}{1+\Gamma(\widetilde{x})}\right) E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}] + \frac{1-\Gamma(\widetilde{x})}{1+\Gamma(\widetilde{x})}. \tag{21}$$

As $E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}] < 1$ and the RHS is a convex combination of $E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}]$ and 1, then the RHS exceeds $E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}]$. Hence, if

$$\theta \in \left(E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}], \left(\frac{2\Gamma(\widetilde{x})}{1+\Gamma(\widetilde{x})}\right) E[\rho\,|s_1 = \underline{s}, s_2 = \underline{s}] + \frac{1-\Gamma(\widetilde{x})}{1+\Gamma(\widetilde{x})}\right)$$

then (17) is true, which implies the unique equilibrium with private signals has leniency and conviction with probability one. ■

Proof of Theorem 3. First note that

$$E[\rho\,|s_1 = \overline{s}] = E[\rho\,|s_1 = \overline{s}, s_2 \leq \overline{s}],$$

from which it follows that

$$\Phi(\overline{s}) = E[\rho\,|s_1 = \overline{s}] - \theta > 0.$$

Therefore, $x = \overline{s}$ is an equilibrium with private signals, and leniency is never used. For public signals, since $E[\rho\,|s_1 = \overline{s}, s_2 = \overline{s}] > \theta$ then leniency is used for a positive measure of signals for all equilibria. ■

Proof of Theorem 4. If $x^* \in (\underline{s}, \overline{s})$ then

$$\Delta(x) \geq 0 \text{ as } x \geq x^*,$$

and, generically,

$$\Delta(x) \gtreqless 0 \text{ as } x \gtreqless x^*,\ x \in [x^* - \varepsilon, 1] \text{ for some } \varepsilon > 0.$$

By the differentiability of $\Delta(x)$, this implies $\partial\Delta(x^*)/\partial x > 0$. Take the total derivative of $\Delta(x^*) = 0$ with respect to γ:

$$\left(\frac{\partial\Delta(x^*)}{\partial x}\right)\left(\frac{\partial x^*}{\partial\gamma}\right) + \frac{\partial\Delta(x^*)}{\partial\gamma} = 0 \Rightarrow \frac{\partial x^*}{\partial\gamma} = -\frac{\partial\Delta(x^*)/\partial\gamma}{\partial\Delta(x^*)/\partial x}.$$

Therefore,

$$\text{if } \frac{\partial\Delta(x^*)}{\partial\gamma} > 0 \text{ then } \frac{\partial x^*}{\partial\gamma} < 0.$$

Consider

$$\frac{\partial\Delta(x)}{\partial\gamma} = \frac{\partial E[\rho\,|s_1 = x, s_2 \leq x]}{\partial\gamma} - \left(\frac{1-\theta}{2}\right)\left[\frac{\partial\Gamma(x)/\partial\gamma}{\Gamma(x)^2}\right]. \tag{22}$$

Using (12),

$$\begin{aligned}
\frac{\partial \Gamma(x)}{\partial \gamma} &= \left(\frac{1}{\gamma f(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right)}\right)^2 \times \\
&\left\{\left[f(x|\overline{\rho}) F(x|\overline{\rho}) - f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\right]\left[\gamma f(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right)\right]\right. \\
&\left. - \left[\gamma f(x|\overline{\rho}) F(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\right]\left[f(x|\overline{\rho}) - f\left(x\left|\underline{\rho}\right.\right)\right]\right\}
\end{aligned}$$

and, after performing some manipulations,

$$sign\left\{\frac{\partial \Gamma(x)}{\partial \gamma}\right\} == sign\{-f\left(x\left|\underline{\rho}\right.\right) f(x|\overline{\rho})\left[F\left(x\left|\underline{\rho}\right.\right) - F(x|\overline{\rho})\right]\} \leq 0.$$

It is non-positive because $F(x|\overline{\rho})$ FOSD $F\left(x\left|\underline{\rho}\right.\right)$. Hence, the second term in (22) is non-negative.

Using (13), consider

$$\begin{aligned}
\frac{\partial E(\rho|s_1 = x, s_2 \leq x)}{\partial \gamma} &= \left(\frac{1}{\gamma f(x|\overline{\rho}) F(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)}\right)^2 \times \\
&\left\{\left[f(x|\overline{\rho}) F(x|\overline{\rho})\overline{\rho} - f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\underline{\rho}\right] \times\right. \\
&\left[\gamma f(x|\overline{\rho}) F(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\right] \\
&- \left[\gamma f(x|\overline{\rho}) F(x|\overline{\rho})\overline{\rho} + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\underline{\rho}\right] \times \\
&\left.\left[f(x|\overline{\rho}) F(x|\overline{\rho}) - f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\right]\right\}
\end{aligned}$$

and, after performing some manipulations,

$$sign\left\{\frac{\partial E(\rho|s_1 = x, s_2 \leq x)}{\partial \gamma}\right\} = sign\{f(x|\overline{\rho}) F(x|\overline{\rho}) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)\left(\overline{\rho} - \underline{\rho}\right)\} > 0$$

Therefore, (22) is positive. ■

9 Appendix B

First note that

$$\Gamma(x) = \Pr(s_2 \leq x|s_1 = x) = \Pr(s_2 \leq x, \overline{\rho}|s_1 = x) + \Pr\left(s_2 \leq x, \underline{\rho}\left|s_1 = x\right.\right). \quad (23)$$

Given firms' signals are conditionally independent then

$$\Pr(s_2 \leq x, \rho|s_1 = x) = \Pr(\rho|s_1 = x)\Pr(s_2 \leq x|\rho). \quad (24)$$

Inserting (24) into (23),

$$\begin{aligned}
\Gamma(x) &= \Pr(\overline{\rho}|s_1 = x)\Pr(s_2 \leq x|\overline{\rho}) + \Pr\left(\underline{\rho}\left|s_1 = x\right.\right)\Pr\left(s_2 \leq x\left|\underline{\rho}\right.\right) \\
&= \left(\frac{\Pr(\overline{\rho})\Pr(s_1 = x|\overline{\rho})}{\Pr(s_1 = x)}\right)\Pr(s_2 \leq x|\overline{\rho}) + \left(\frac{\Pr\left(\underline{\rho}\right)\Pr\left(s_1 = x\left|\underline{\rho}\right.\right)}{\Pr(s_1 = x)}\right)\Pr\left(s_2 \leq x\left|\underline{\rho}\right.\right) \\
&= \left(\frac{\gamma f(x|\overline{\rho}) F(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right) F\left(x\left|\underline{\rho}\right.\right)}{\gamma f(x|\overline{\rho}) + (1-\gamma) f\left(x\left|\underline{\rho}\right.\right)}\right).
\end{aligned}$$

Next, let us derive

$$E\left[\rho\left|s_1=x,s_2\leq x\right.\right]=\Pr\left(\overline{\rho}\left|s_1=x,s_2\leq x\right.\right)\overline{\rho}+\left[1-\Pr\left(\overline{\rho}\left|s_1=x,s_2\leq x\right.\right)\right]\underline{\rho}.$$

Given

$$\Pr\left(\rho,s_1=x,s_2\leq x\right)=\Pr\left(\rho\right)\Pr\left(s_1=x\left|\rho\right.\right)\Pr\left(s_2\leq x\left|\rho\right.\right)$$

and

$$\Pr\left(s_1=x,s_2\leq x\right)=\Pr\left(\overline{\rho}\right)\Pr\left(s_1=x\left|\overline{\rho}\right.\right)\Pr\left(s_2\leq x\left|\overline{\rho}\right.\right)+\Pr\left(\underline{\rho}\right)\Pr\left(s_1=x\left|\underline{\rho}\right.\right)\Pr\left(s_2\leq x\left|\underline{\rho}\right.\right).$$

then

$$\begin{aligned}\Pr\left(\rho\left|s_1=x,s_2\leq x\right.\right) &= \frac{\Pr\left(\rho,s_1=x,s_2\leq x\right)}{\Pr\left(s_1=x,s_2\leq x\right)}\\ &= \frac{\Pr\left(\rho\right)\Pr\left(s_1=x\left|\rho\right.\right)\Pr\left(s_2\leq x\left|\rho\right.\right)}{\Pr\left(\overline{\rho}\right)\Pr\left(s_1=x\left|\overline{\rho}\right.\right)\Pr\left(s_2\leq x\left|\overline{\rho}\right.\right)+\Pr\left(\underline{\rho}\right)\Pr\left(s_1=x\left|\underline{\rho}\right.\right)\Pr\left(s_2\leq x\left|\underline{\rho}\right.\right)}.\end{aligned}$$

Hence,

$$E\left[\rho\left|s_1=x,s_2\leq x\right.\right]=\frac{\gamma f\left(x\left|\overline{\rho}\right.\right)F\left(x\left|\overline{\rho}\right.\right)\overline{\rho}+\left(1-\gamma\right)f\left(x\left|\underline{\rho}\right.\right)F\left(x\left|\underline{\rho}\right.\right)\underline{\rho}}{\gamma f\left(x\left|\overline{\rho}\right.\right)F\left(x\left|\overline{\rho}\right.\right)+\left(1-\gamma\right)f\left(x\left|\underline{\rho}\right.\right)F\left(x\left|\underline{\rho}\right.\right)}.$$

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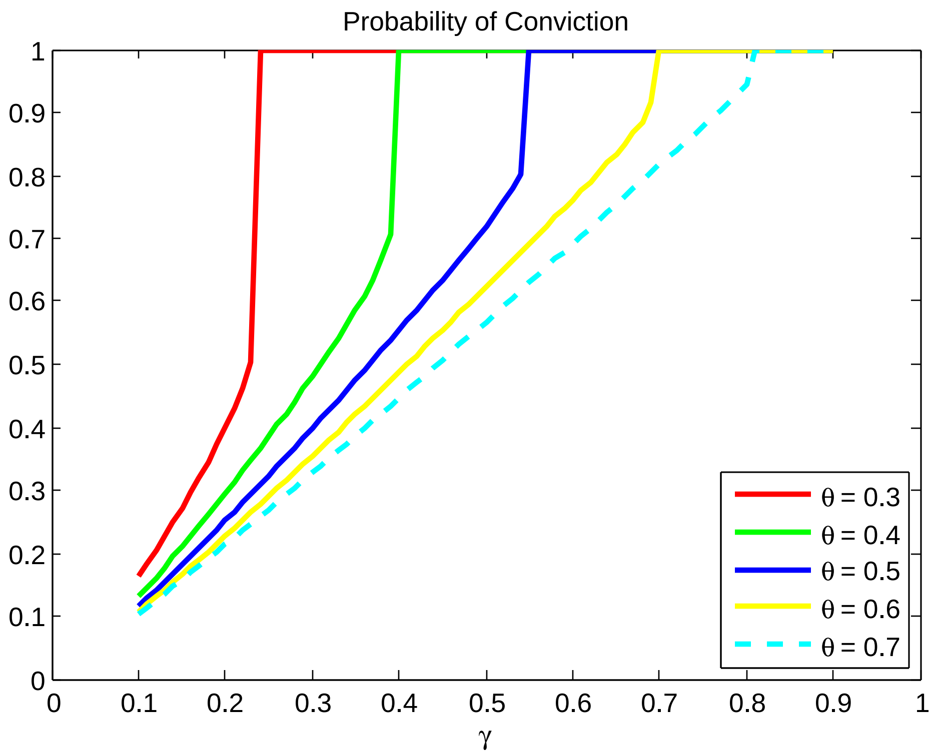

Figure 1, $\rho = (0, 1)$

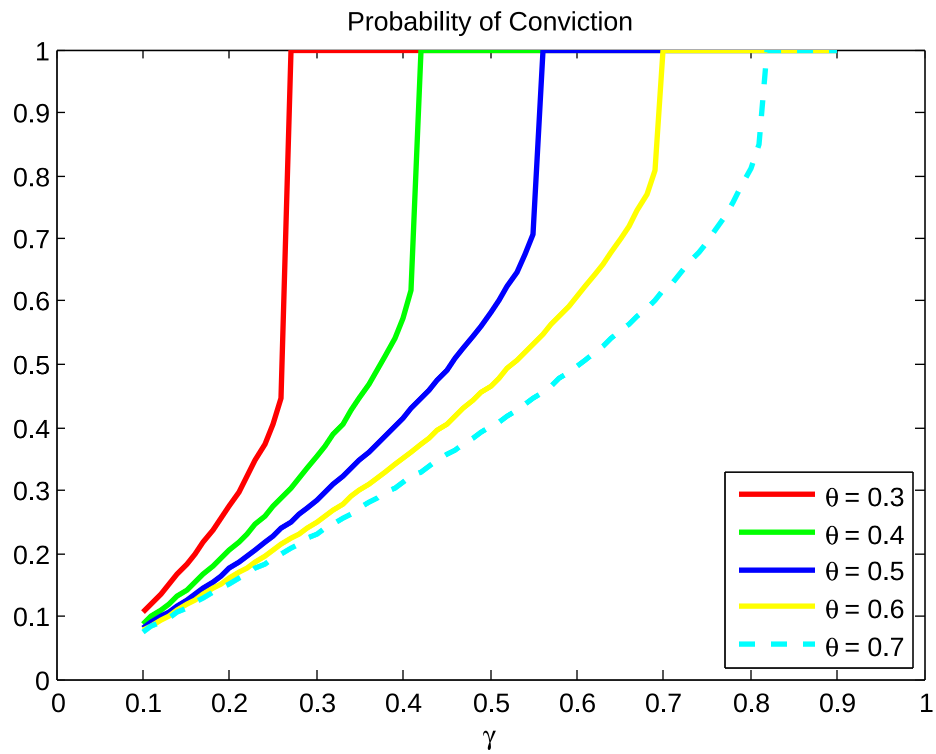

Figure 2, ρ = (0, 0.75)

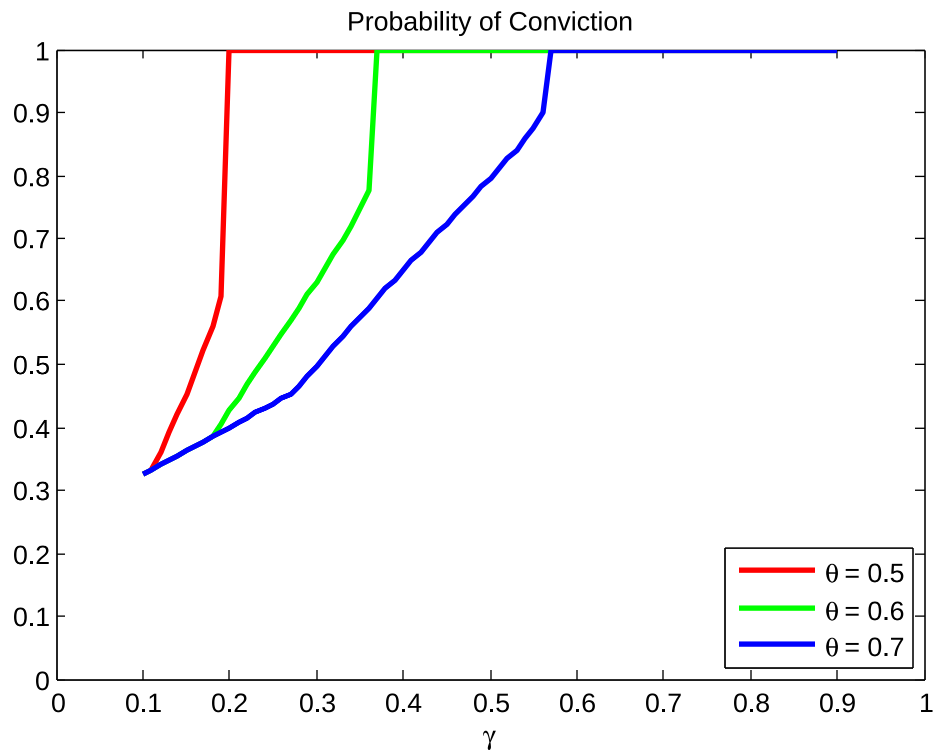

Figure 3, $\rho = (0.25, 1)$