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Higher tax morale implies a higher optimal income tax rate

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Abstract

We analyze the impact of (exogenous) tax morale on the optimal design of progressive income taxation. In our model, only universal basic income (transfer) is financed from a linear income tax and the financing of public goods is neglected. Each individual supplies labor and (un)declares earning, depending on his labor disutility and tax morale, respectively. Limiting the utilitarianism to the poorer parts of the population (defined by the *welfare share*), the optimal tax rate is an increasing function of the tax morale and a decreasing function of the welfare share.

Keywords: tax morale, progressive income tax, undeclared earning, labor supply, income redistribution

JEL Classification: H21, H26, H41, D58

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Jobb adómorál – nagyobb optimális adókulcs

Simonovits András

Összefoglaló

Hogyan hat az exogén adómorál a progresszív jövedelemadó optimális kulcsára? Ezt elemezzük egy olyan modellben, amelyben a lineáris jövedelemadó csak univerzális alapjövedelmet finanszíroz, és a közjavak pénzügyi fedezetét elhanyagoljuk. Minden egyén munkaáldozatától és adómoráljától függően vállal munkát, illetve titkolja el keresete egy részét. Általánosított Rawls-féle társadalmi jóléti függvény esetén igazoljuk, hogy a társadalmi jólétet maximalizáló egykulcsos adó mértéke az adómorál növekvő függvénye.

Tárgyszavak: adómorál, progresszív személyi jövedelemadó, eltitkolt kereset, munkakínálat, jövedelemújraelosztás

JEL kódok: H21, H26, H41, D58

1. Introduction

Between 1930 and 1970 the ratio of government tax and social security revenues to the GDP had risen sharply and has remained at a high level since 1970s in the developed world. It is a commonplace that rising wage-income (and other) tax rates may diminish labor supply and increase tax evasion. But comparing different countries, it becomes evident that the impact of taxation on economic activity also depends on the so-called "tax morale" (or morality). This concept refers to the propensity to pay taxes or capturing "the readiness with which individuals leave the official economy and enter the illegitimate (untaxed) hidden economy" (Frey and Weck-Hannemann (1984); see also Lago–Penas and Lago–Penas (2010) for its determinants). We should distinguish between *exogenous* and *endogenous* individual tax morales: the former is a given parameter of the utility function, the latter depends on the exogenous tax morale as well as on the observed behavior of the individual's neighborhood.

In the present paper, I try to analyze the impact of the exogenous tax morale on the socially optimal tax rate in a very simple *static* model, where a *flat-rate income tax* finances a universal basic income (transfer), neglecting the fiscal demand of providing public goods. In this model, the existence of tax morale makes monitoring and punishing tax evasion superfluous. Another factor determining the optimal tax rate is the *progressivity* of the social welfare function, meaning how much it favors redistribution. (With the usual CRRA social welfare function, the lower the power index, the more progressive it is.) Then *the optimal flat tax rate is an increasing function of the tax morale and of the progressivity of the social welfare function.*

If one works with traditional, strictly concave utility functions (from Mirrlees (1971) to Simonovits (2010)), then in this type of models, the calculations soon become excessively complex, requiring numerical illustrations. However, in a recent paper, Doerrenberg et al. (2011) introduced a very simple linear–quadratic utility function, where the optimal labor supply and undeclared earning are simple linear functions, namely the former is proportional to the net of tax rate and the latter is equal to the ratio of the wage rate to the tax morale.

Adopting this trick, the present paper works with a model which is more amenable to analytical investigation than my previous model was. To make sense of income redistribution, however, the purely utilitarian social welfare function used there had to be replaced by a generalized utilitarian social welfare function. We define a *truncated (or generalized Rawlsian) social welfare function* as the average of the first J lowest utilities out of I utilities, $J < I$, J being the *cutoff index*, and $\nu = J/I$ being the *welfare share*.

For truncated or exclusive social welfare functions (cf. Ravaillon, 1997), I was able to determine explicitly the socially optimal tax rate and show that it is an increasing function of the tax morale and a decreasing function of the welfare share (Theorems 1 and 2). In the simplest specification (when only the poorest's utility is considered: Ralwsian case), the optimal tax rate is given by $\tau[\mu] = 1/2 - 1/(2\mu)$, where τ denotes the tax rate and μ denotes the tax morale. I also investigated *numerically* this dependence for inclusive social welfare functions, and I invariably received qualitatively this schedule, similarly to Ravaillon (1997, p. 359)'s observation: "the theoretical distinction [between exclusive and inclusive social welfare functions] can sometimes be of very little practical consequence". I hope that the results remain valid in more general settings (cf. Simonovits (2010) with strictly concave utility functions and public expenditures). Note that the artificial specification of the utility functions and the exclusion of consumption- and social security taxes preclude any empirical verification. The present toy model is only able to illustrate certain causal relations.

In this framework, the intuition behind the major result is relatively simple. The government's objective is to maximize the social welfare function. Raising the tax rate increases the transfer share but diminishes the labor supply and thus the total output. For any given tax rate, the higher the tax morale, the lower is the undeclared earning, making room for more redistribution via raising the tax rate. But we need a model to make this intuitive argument watertight.

To relate the foregoing highly theoretical observation to the real world, I present a very stylized table, describing various combinations of tax morales (lower and higher) and tax shares (low, medium, high) defined as the ratio of tax (and pension) revenues to the GDP. These somewhat unreliable but still characteristic numbers refer to the pre-crisis era and contain many things directly *not* related to our problem (budget deficits, interest payments, different public pension systems etc). I tentatively interpret Table 1 as showing that a medium tax share may be *socially optimal* for a country with lower tax morale (e.g. the Czech Republic versus Slovakia or Hungary), while a high tax share may be optimal for a country with higher tax morale (like Sweden versus Japan or Germany).

Table 1. *Tax shares, tax morales and ranking of social welfare*

Tax share	Low cc. 30%		Medium cc. 40%		High cc. 50%
Lower morale	Slovakia	$<$	Czech Rep.	$>$	Hungary
Higher morale	Japan	$<$	Germany	$<$	Sweden

At this stage, I make a short review of the literature. In his pioneering paper, Mirrlees (1971) solved the theoretical problem of the socially optimal income taxation, when labor supply is flexible but productivity is private knowledge. Sheshinski (1972) simplified the analysis by confining his attention to linear taxes (cf. Feldstein, 1973). One limitation of these papers is that they did not consider tax evasion. Taking the opposite extreme position, Allingham and Sandmo (1972) analyzed income tax evasion, neglecting the flexibility of labor supply. In a sequel to that paper, Sandmo (1981) extended the research on tax evasion into the direction of social welfare maximization with flexible labor supply and raised a weaker form of the conjecture of the present paper (p. 279): "a natural question to ask is whether ... the marginal tax rate in some sense ought to be lower than otherwise have been because of the presence of tax evasion." Later on (p. 282) he gave alternative sufficient conditions, namely either "regular income is now a less reliable indicator of economic welfare" or "the numerical value of the compensated supply derivative in the regular market is increased" but did not commit himself to their validity.

From Feldstein (1999) to Chetty (2009) and Saez et al. (2009), a great number of papers studied tax avoidance and the deadweight loss of the income tax, putting the concept of *elasticity of taxable income* to the center of the analysis.

In their survey, Andreoni et al. (1998) extended the narrow neoclassical model and introduced soft but relevant concepts like moral sentiments and the satisfaction of the taxpayer with the provision of public goods and services. From our point of view, they made three important observations: (i) the morally more sensitive citizens declare a higher share of their true (pre-tax) incomes; (ii) the more unfair the tax-and-transfer system is in the eyes of citizens, the less income they declare; (iii) the less satisfied the taxpayers are with the provision of public goods and services, the less income they declare.

Traxler (2010) extended the analysis from exogenous to endogenous tax morales, where the individual tax morale depends on the observed degree of tax evasion. Combining the two approaches, Garay et al. (2011) and Méder et al. (2011) investigated the *dynamics* of the tax evasion process. Using the framework of Simonovits (2010), both papers neglected income redistribution and studied the dynamics of the declared (or taxable) incomes financing the provision of public goods.

In the paper already mentioned, Doerrenberg et al. (2011) considered differentiated taxation among different groups in different countries. Using econometric techniques, they found that typically in any country, "nice guys finish last: people with higher tax morale are taxed more heavily". Making the individual utilities dependent on others' utilities, Doerrenberg and Peichl (2010) discussed the opposite causality and found that greater tax progressivity implies higher tax morale.

Romer (1975) also obtained interesting results concerning the majority voting on linear income taxes—an alternative to welfare analysis. In such a political economy framework, Meltzer and Richard (1981) proved an interesting intuitive result: the greater the pre-tax income inequality, the greater redistribution will be chosen by the median voter. (By the way, Theorem 1* of the present paper reproduces this result, also preserving the influence of tax morale.)

Alesina and Angelitos (2005), however, found an apparent anomaly: compare two countries, say the US and Sweden. Though the US inequality of the pre-tax incomes is greater than the Swedish, the US personal income tax is less progressive than the Swedish. They created a model with country-specific beliefs on the role of luck in the determination of individual pre-tax earnings. Their major result was as follows: the stronger the presumed role of luck, the greater income redistribution is selected. In contrast, in our social welfare maximization framework, this anomaly can be explained by the difference between the countries' social welfare functions: the impact of the lower welfare share overrules the impact of the lower pre-tax income inequality, implying higher optimal tax rate.

The structure of the remainder of the paper is as follows: Section 2 presents the model and Section 3 displays the illustrations. Section 4 concludes.

2. The model

Following Doerrenberg et al. (2011), we shall use a very simple model. There are $I > 1$ types in the population, indexed as $i = 1, \dots, n$. Type i's *labor supply* is l_i, $0 < l_i \leq 1$, his pre-tax *wage rate* (independently of the tax system) is $w_i > 0$, both reals, thus his *earning* is $l_i w_i$. To achieve income redistribution, the government operates a linear tax with a *flat (marginal) tax rate* τ, $0 \leq \tau \leq 1$ and a *basic income* $\beta \geq 0$. Type i *undeclares* $e_i \geq 0$ from his earning, i.e. he evades tax τe_i, therefore his *net tax* is equal to $\tau(l_i w_i - e_i) - \beta = \tau y_i - \beta$, where $y_i = l_i w_i - e_i$ denotes the *declared (or taxable) earning*. Consequently, his *consumption* is given by $c_i = (1-\tau)l_i w_i + \tau e_i + \beta$. Note that for any positive basic income $\beta > 0$, even the flat-rate tax is progressive in the sense that the average net tax rate $t_i = (\tau y_i - \beta)/y_i$ is increasing in the declared earning y_i.

To derive the dual choice of labor supply and undeclared earning from individual utility maximization, we must assume an individual objective function. To obtain explicit formulas, we rely on a linear–quadratic utility function. In addition to a usual linear *consumption utility* $2c_i$ and a quadratic *labor disutility* function $-\alpha w_i l_i^2$, ($\alpha > 0$ being the coefficient of labor disutility), we introduce a quadratic *moral disutility* function of tax evasion $-\mu(\tau w_i)^{-1}(\tau e_i)^2$, ($\mu > 0$ being the coefficient of tax morale, for short, the tax morale). (We shall see that the factors w_i and $(\tau w_i)^{-1}$ make the optimal labor supply independent of and the optimal undeclared earning proportional to the type-specific wage rate, respectively. Note that we imitate Yitzaki (1974), who made the penalty to be proportional to the evaded tax rather than the undeclared earning. Finally, by doubling the consumption in the utility function, the fraction 1/2 is avoided in further formulas.) In sum, type i's utility function is

$$U_i = 2c_i - \alpha w_i l_i^2 - \mu\tau w_i^{-1} e_i^2.$$

Inserting formula for c_i into formula for U_i, we receive a *reduced* utility function

$$u_i(l_i, e_i) = 2l_i w_i(1-\tau) + 2e_i\tau + 2\beta - \alpha w_i l_i^2 - \mu\tau w_i^{-1} e_i^2.$$

Of course, the value of the basic income β depends on the decisions of every worker (see below). Since the impact of any single worker on β can be neglected, our workers neglect it. Therefore type i maximizes the *curtailed* utility function $u_i(l_i, e_i) - 2\beta$. Taking the partial derivatives of this concave function with respect to l_i and e_i and equating them to zero, his optimal decisions are respectively

$$l_i^* = \alpha^{-1}(1-\tau) \quad \text{and} \quad e_i^* = \mu^{-1} w_i.$$

Note the simple meaning of these rules: the optimal labor supply l_i^* is proportional to 1 minus the tax rate (the so-called *net of tax rate*) $1-\tau$, where the proportionality coefficient is the reciprocal of α; the optimal undeclared earning e_i^* is proportional to the wage rate w_i, where the proportionality coefficient is the reciprocal of μ. In a *white economy* (studied by Mirrlees), $\mu = \infty$ and $e_i^* = 0$. Unfortunately, our optimal undeclared earning e_i^* is unrelated to the true earning $l_i^* w_i$ or the tax rate τ. Furthermore, a more complicated suitable utility function would imply a more general and realistic labor supply–tax rate function $l_i^* = \alpha^{-1} - \gamma_i \tau$ with $\alpha^{-1} > \gamma_i$ but we forsake this option.

To obtain feasible labor supply for any tax rate, it is appropriate to assume $\alpha \geq 1$, in the limit: $\alpha = 1$. It is also logical to assume that at the optimum, the undeclared earning is less than or equal to the true earning, i.e. $\mu^{-1} \leq \alpha^{-1}(1-\tau)$, i.e. $\tau \leq 1-\alpha/\mu$, implying $\alpha \leq \mu$. We can make the following observation: the higher the tax morale μ, the higher is the upper limit $1 - \alpha/\mu$ on the feasible tax rate!

Turning from individual to aggregate behavior, we assume that the weight of type i in the population is uniform, i.e. $1/I$. We shall need the average wage rate, to be normalized to unity:

$$W = \frac{1}{I}\sum_{i=1}^{I} w_i = 1.$$

Three more averages are introduced: average labor supply, average earning and average undeclared earning, respectively:

$$L = \frac{1}{I}\sum_{i=1}^{I} l_i, \quad Y = \frac{1}{I}\sum_{i=1}^{I} l_i w_i \quad \text{and} \quad E = \frac{1}{I}\sum_{i=1}^{I} e_i.$$

At the optimum, they are equal to $L^* = (1-\tau)\alpha^{-1} = Y^*$ and $E^* = \mu^{-1}$, respectively.

For the sake of simplicity, we assume that the total (or average) net tax is zero (neglecting the provision of public goods), i.e. we end up with the following budget constraint taken at the optimum:

$$\beta^* = \tau(Y^* - E^*) = \tau[\alpha^{-1}(1-\tau) - \mu^{-1}].$$

To find the socially optimal tax rate and the corresponding basic income, it is worth expressing the optimal reduced utilities as *indirect* utility functions:

$$u_i^* = w_i[\alpha^{-1}(1-\tau)^2 + \mu^{-1}\tau] + \tau[\alpha^{-1}(1-\tau) - \mu^{-1}].$$

Moving to social welfare maximization, note that contrary to Simonovits (2010), we cannot use a *purely utilitarian* social welfare function, because the individual utility is a linear function of the individual consumption—making any income redistribution not only useless but counterproductive. Rather we look for a family of generalized social welfare function which keep the simplicity of the purely utilitarian one but does not exclude redistribution. We shall introduce *truncated utilitarian (or generalized Rawlsian or exclusive)* social welfare functions, defined as the average of the J lowest utilities, J being the cutoff index. (Later on we shall work with the relative index $\nu = J/I$, to be called *welfare share*.) Note that in the present model, these indirect utilities are increasing linear functions of the wage rates. If we index the latter as $w_1 < w_2 < \cdots < w_{I-1} < w_I$, then $u_1^* < u_2^* < \cdots < w_{I-1}^* < w_I^*$. Hence the definition of the truncated social welfare function is simple:

$$V_J = \frac{1}{J}\sum_{i=1}^{J} u_i^*, \qquad J = 1, 2, \ldots, I.$$

The higher the cutoff index J, the more indifferent is the social planner to the utility differences. We display the two limit cases.
The purely utilitarian case:

$$V_I = \frac{1}{I}\sum_{i=1}^{I} u_i^*.$$

The Rawlsian case:

$$V_1 = u_1^*.$$

Note that the social welfare functions $V_1, \ldots, V_{I-1}$ fail to depend on all the utilities but they are simple and approximate well the much more complex CRRA social welfare functions, therefore we rely on them.

Before announcing our main theorem, as a counterpart to V_J, we shall define the average wage rate of the J lowest types (for short, J-minimum average wage rate):

$$W_J = \frac{1}{J}\sum_{i=1}^{J} w_i. \quad j = 1, 2, \ldots, I.$$

Because w_is are increasing, so do W_Js: $w_1 = W_1 < W_2 < \cdots < W_{I-1} < W_I = W = 1$.

Here is our major result.

Theorem 1. *Let us choose a cutoff index $J < I$ and assume that the J-minimum average wage rate is lower than the critical value:*

$$W_J < \bar{w} = \frac{2(1-\alpha/\mu)}{2-\alpha/\mu} \le 1.$$

Then for the J-truncated social welfare function, the J-optimal tax rate is positive and is given by

$$\tau_J(\alpha,\mu) = \frac{2(1-\alpha/\mu) - (2-\alpha/\mu)W_J}{2(2-W_J)}.$$

Remarks. 1. For a fixed cutoff index J and for any sufficiently low fixed labor disutility coefficient $\alpha \ge 1$, the tax rate–tax morale function $\tau_J[\mu]$ is increasing in the tax morale μ and its upper limit is achieved in the white economy ($\mu = \infty$):

$$\tau_J[\infty] = \frac{1-W_J}{2-W_J} \le \frac{1}{2}.$$

2. The special Rawlsian optimal tax rate is given by

$$\tau_1(\alpha,\mu) = \frac{2(1-\alpha/\mu) - (2-\alpha/\mu)w_1}{2(2-w_J)} > 0 \quad \text{if} \quad w_1 < \bar{w}.$$

If there are workers with zero wage rates: $w_1 = 0$, then the Rawlsian tax rate is $\tau_1 = (1-\alpha/\mu)/2$.

3. The literature has concentrated on the dependence of the tax rate on the labor disutility, to be denoted here by $\tau_J\{\alpha\}$. Of course, this function is decreasing in labor disutility coefficient α until reaching zero.

4. Note that in our model, for every cutoff index J, the optimal tax rate only depends on the ratio of the tax morale to the labor disutility parameter, α/μ rather than on α and μ. Probably, this is only an implication of our specification of the utility functions.

Proof. The J-truncated social welfare function is the average of the J lowest indirect utility functions and each of them is a linear function of the corresponding wage rate, i.e.

$$V_J(\tau) = W_J[\alpha^{-1}(1-\tau)^2 + \mu^{-1}\tau] + 2\tau[\alpha^{-1}(1-\tau) - \mu^{-1}].$$

Taking the derivative of $V_J(\tau)$ and equating it with zero, yields $\tau_J(\alpha, \mu)$. Since $V_J'(\tau)$ turns from positive into negative at the stationary point, it is a local (and global) maximum. $W_J < \bar{w}$ is equivalent to $\tau_J(\alpha, \mu) > 0$. The qualitative behavior of $\tau_J[\mu]$ is obvious. ∎

If we give up the uniformity of the tax morales and the labor disutilities, then the ordering of the indirect utilities becomes cumbersome.

We turn now to the dependence of the optimal tax rate on the cutoff index J, measuring the extent of exclusion of the richer groups. Intuitively, we expect that the lower the cutoff index, the higher is the optimal tax rate. Indeed, this is the case.

Theorem 2. *Let $w_1 < \bar{w}$ and let K be an integer such that $W_K < \bar{w} \le W_{K+1}$. Then the socially optimal positive tax rates are decreasing in the cutoff index J: $\tau_1 > \tau_2 > \cdots > \tau_K > \tau_{K+1} = \cdots = \tau_I = 0$.*

Proof. Since $w_1 < \bar{w} < 1 = W$, there exists a K appearing in Theorem 2. By Theorem 1, τ_J is an decreasing function of W_J, which in turn is increasing in J. ∎

As is usual in welfare economics, for any J, it is worth calculating the *degree of the suboptimality* of the presumed tax morale-specific optimum $\tau_J[\hat{\mu}]$ in an economy with a tax morale μ: $\hat{\mu} \ne \mu$. Fixing the ratios of pre-tax wage rates, we look for that average wage rate $\tilde{W}$, for which $\tau_J[\mu, \hat{\mu}]$ yields the same welfare as the original unit average wage rate and $\tau_J[\mu]$ do.

To simplify the formulas, we only consider a very simple case.

Theorem 3. *Assume that $\alpha = 1$ and the poorest workers' wage rate is zero: $w_1 = 0$. Then the degree of suboptimality of applying a Rawlsian tax rate $\tau_1[\hat{\mu}]$ in an economy with true tax morale μ is*

$$\tilde{W}(\mu, \hat{\mu}) = \frac{(1-\mu^{-1})^2}{(1-\hat{\mu}^{-1})(1+\hat{\mu}^{-1} - 2\mu^{-1})} \ge 1.$$

Remark. The most important case is when the government presumes a white economy with $\hat{\mu} = \infty$. Then the welfare loss is equal to

$$\tilde{W}(\mu, \infty) - 1 = \frac{\mu^{-2}}{1 - 2\mu^{-1}} \quad \text{for} \quad \mu > 2.$$

Proof. For $w_1 = 0$, the perceived utility function is $u_1^*(\mu, \hat{\mu}, \tau) = 2\tau_1(1 - \tau - \hat{\mu}^{-1})$. Hence $\tau_1[\hat{\mu}] = (1 - \hat{\mu}^{-1})/2$. Substituting it into u_1^* yields

$$u_1^*[\mu, \hat{\mu}] = u_1^*(\mu, \hat{\mu}, \tau_1[\hat{\mu}]) = (1 - \hat{\mu}^{-1})[(1/2)(1 + \hat{\mu}^{-1}) - \mu^{-1}].$$

At correct estimation, $u_1^*[\mu, \mu] = (1-\mu^{-1})^2/2$. Using the definition $\tilde{W}u_1^*[\mu, \hat{\mu}] = u_1^*[\mu, \mu]$, yields the result. ∎

At this point we make a short detour into the realm of political economy. Assuming that $I = 2M-1$, denote the median wage rate by w_M. Then the *median* worker's indirect utility satisfies the single-peaked condition and the optimal tax rate corresponds to that of Theorem 1, only the J-minimal wage rate is replaced by the median one.

Theorem 1.* *Assume that the median wage rate is less than the critical value (of Theorem 1):*

$$w_M < \bar{w} = \frac{2(1-\alpha/\mu)}{2-\alpha/\mu} \leq 1.$$

Then the median voter's preferred tax rate is positive and is given by

$$\tau^*(\alpha,\mu) = \frac{2(1-\alpha/\mu)-(2-\alpha/\mu)w_M}{2(2-w_M)}.$$

Remarks. 1. In accordance with Meltzer and Richard (1981), in our model the greater the pre-tax earning inequality, here measured by the difference between the average and the median wage rates $1-w_M$, the greater redistribution will be chosen by the median voter.

2. Since the M-minimal average wage rate is generally lower than the median wage rate: $W_M < w_M$, therefore $0 < \tau^*(\alpha,\mu) < \tau_M(\alpha,\mu)$, i.e. the M-optimum tax rate is higher than the median voter's. Also, the same upper bound on W_M and on w_M means much stronger restriction in the political economy model than in the welfare maximization model.

3. Numerical illustrations

To give a feeling of the magnitudes and display the difference between our truncated and the usual CRRA social welfare functions, we rely on numerical illustrations. We shall work with the following extremely simple specifications. We shall use a stylized version of US income distribution with quintiles ($I = 5$), taken as a wage rate distribution, and normalized its expected value to 1 (see the first column of Table 2 below). We work with the minimal labor disutility parameter, namely $\alpha = 1$.

To make our presentation less dependent on the number of types, we shall work with the relative share of preferred workers in the population to be called *welfare share*: $\nu = J/I$ rather than their absolute numbers or the cutoff index J. We start the illustrations with index $J = 2$ or rather with welfare share $\nu = 0.4$. First we display a simple run with tax morale $\mu = 4$, i.e. workers undeclare 1/4 of their wage rates (or their potential earnings), regardless of their earnings, implying the numerically determined optimal tax rate, being equal to 0.287. (Note that due to less than maximal labor supply, the ratio of undeclared earning to actual earning is higher than 1/4.) Table 2 produces sensible results and presumably can be used for further calculations. The redistribution is quite spectacular: the signed net transfers paid by the workers being equal to $T_i^* = \tau(w_i l_i^* - e_i^*) - \beta^*$, the poorest quintile receives about half its potential earnings and the richest quintile pays about 8% of its potential earnings.

Table 2. *The individual optimal outcomes for 2-quintile optimum*

Wage rate w_i	Undeclared earning e_i^*	Transfer paid T_i^*	Consumption c_i^*
0.2	0.050	–0.106	0.249
0.4	0.100	–0.080	0.365
0.7	0.175	–0.040	0.539
1.1	0.275	0.013	0.771
2.6	0.650	0.213	1.642

Remarks. $\nu = 0.4$, $\mu = 4$, $\tau^{0.4} = 0.287$, $L^* = 0.713$. We use notation τ^ν rather than τ_ν, to distinguish the utilitarian optimum τ^1 from the Rawlsian optimum τ_1.

As a detour, we present the political economy equilibrium. Distinguishing the third quintile from the three lowest quintiles, we shall use subindex [0.5] rather than 0.6. The political economy equilibrium is much lower than the 2-quintile optimum: $\tau^{[0.5]} = 0.106 < 0.237 = \tau^{0.6}$ (see Table 5 below). Correspondingly, the redistribution is also lower, but the labor supply is higher: $L^* = 0.894$.

Table 3. *The individual optimal outcomes in political economy*

Wage rate w_i	Undeclared earning e_i^*	Transfer paid T_i^*	Consumption c_i^*
0.2	0.050	–0.055	0.233
0.4	0.100	–0.041	0.399
0.7	0.175	–0.020	0.646
1.1	0.275	0.007	0.977
2.6	0.650	0.109	2.216

Remark. $\mu = 4$, $\tau^{[0.5]} = 0.106$ and $L^* = 0.894$.

Next we move to studying the impact of tax morale on optimal average outcomes, and usually drop the adjective *average*. In Table 4, the tax morale runs from 2 to 12 to infinity (white economy) and see the quantitative side of Theorem 1: the optimal tax rate rises from 0.162 to 0.370 to 0.407. Note that even in the white economy, there remains a huge discrepancy between the highest and the lowest consumption; their ratio drops from 9.9 to 3.8, but not to 1.

Table 4. *The impact of tax morale on optimal outcomes for 2-quintiles*

Tax morale μ	Lowest rate $\tau^{0.4}[\mu]$	Lowest consumption c_1	Consumption ratio c_5/c_1
2	0.162	0.211	9.9
4	0.287	0.249	6.6
6	0.328	0.267	5.5
8	0.349	0.277	5.0
10	0.362	0.283	4.8
12	0.370	0.288	4.6
...			...
∞	0.407	0.308	3.8

Remark. $\nu = 0.4$.

Next, we illustrate the impact of the welfare share ν on the social optimum. We return to the tax morale of Table 2, $\mu = 4$. In harmony with our Theorem 2, as the welfare share drops from 1 (pure utilitarianism) to 0.2 (Rawls), the optimal tax rate rises from 0 to 0.319. However, the labor supply falls correspondingly. As a result, the absolute value of the lowest quintile's consumption hardly rises after leaving out the richest quintile from the social welfare function, only the ratio of the highest quintile's consumption to the lowest's, c_5/c_1 drops from 13 to 6.3 but does not approach 1.

Table 5. *The impact of welfare share on optimal outcomes*

Welfare share ν	Tax rate $\tau^{\nu}[\mu]$	Lowest consumption c_1	Consumption ratio c_5/c_1
1.0	0.000	0.200	13.0
0.8	0.161	0.244	8.3
0.6	0.237	0.250	7.2
0.4	0.287	0.249	6.6
0.2	0.319	0.246	6.3

Remark. $\mu = 4$.

To study the sensitivity of the result to the specification of the social welfare function, for a while, we replace our truncated version with the more accepted CRRA social welfare function, having an *indifference index* ρ, dropping from 1 to $-\infty$.

$$V_\rho = \rho^{-1} \sum_{i=1}^{I} f_i u_i^{*\rho}.$$

To determine the socially optimal tax rate, now we must rely on numerical calculations. Table 6 shows a similar picture as Table 5. Here are the results.

Table 6. *The impact of the indifference index on optimal outcomes, CRRA SWF*

Indifference index ρ	Tax rate $\tau^{\rho}[4]$	Lowest consumption c_1	Consumption ratio c_5/c_1
1	0.000	0.200	13.0
–1	0.236	0.250	7.2
–3	0.286	0.249	6.6
–5	0.303	0.248	6.4
–7	0.311	0.247	6.4
...			...
∞	0.407	0.308	3.8

Remark. $\mu = 4$, CRRA social welfare function.

At this point we want to obtain an estimation of the welfare loss due to using tax morale coefficient $\hat{\mu}$ rather than the true one, without assuming $w_1 = 0$ (as was done in Theorem 3). Table 7 displays the results for $\tau^{0.4}$ for $\mu = 4$ and $\hat{\mu} = 6$. The end result is disappointing: efficiency drops only about 2%. The details are much more interesting. The rise of tax rate from 0.287 to 0.362 reduces the labor supply from 0.713 to 0.638. On the one hand, it decreases labor disutility, i.e. increases the utility; on the other hand, it reduces the output, and diminishes or even eliminates the impact of the redistribution. Though the transfer to the poorest quintile rises from 0.106 to 0.112; their consumption drops from 0.249 to 0.240.

Table 7. *The individual suboptimal outcomes for 2-quintile optimum*

Wage rate w_i	Undeclared earning e_i^*	Transfer paid T_i^*	Consumption c_i^*
0.2	0.050	–0.112	0.240
0.4	0.100	–0.084	0.340
0.7	0.175	–0.042	0.489
1.1	0.275	0.014	0.688
2.6	0.650	0.225	1.435

Remarks. $\nu = 0.4$, $\mu = 4$, $\tau^{0.4}[10] = 0.362$, $L^* = 0.638$.

Finally, to explain the anomaly found by Alesina and Angelitos (2005) in our framework, we display two countries with two welfare shares: $\nu_L = 0.2$ (low) and $\nu_H = 0.6$ (high) and two pre-tax wage rate inequality setups. For the sake of simplicity, we keep the original wage rates of Table 2 for the high-inequality set up, and create a low-inequality set up by scaling down the deviations from the mean: $w_i(\omega) = 1 + \omega(w_i - 1)$ with $\omega = 0.8$. Since the (dis)utility parameters are uniform, the pre- and post-tax indicators are the same.) Comparing the economies represented by rows 2 and 3 of Table

8, lower welfare share 0.2 (vs. 0.6) overrules the impact of lower earning inequality and leads to a higher optimal tax rate 0.265 (vs. 0.237). Note that smaller differences in welfare shares may not overrule the impact of higher wage rate inequality. For example, if the higher inequality country reduces $\nu = 0.6$ to 0.4, then its optimal tax rate would rise to 0.287 (cf. Table 2), surpassing the other country's 0.2-optimum.

Table 8. *Lower welfare share may overrule lower earning inequality*

Earning inequality ω	Welfare share ν	Tax rate $\tau^{\nu}[\mu]$
0.8	0.6	0.187
1.0	0.6	0.237
0.8	0.2	0.265
1.0	0.2	0.319

Remark. $\mu = 4$.

We could continue the numerical exploration without any difficulty but for our purposes, this seems to be sufficient to show the basic idea of the model: in addition to the much studied elasticity of labor supply and welfare share (or the indifference index), tax morale also plays an important role in the design of optimal income taxation. This observation is also supported by my previous paper (Simonovits, 2010), where a distinctly different specification of the problem (with logarithmic utility functions, fixed labor supply and purely utilitarian social welfare function) gave qualitatively similar results.

4. Conclusions

In this very simple toy model with linear–quadratic utilities, we were able to study the impact of the exogenous tax morale on the socially optimal tax rate. Under certain assumptions (uniform linear-quadratic utilities, truncated (or exclusive) social welfare function), we proved analytically Theorems 1 and 2: higher morale and lower welfare share imply higher socially optimal tax rate. We can add a third observation: higher earning inequality implies a higher optimal tax rate, but this can be reversed by a higher welfare share. Incidentally, political economy considerations implied similar results: a higher tax morale implies a higher equilibrium tax rate. Further work should be done to check the robustness of our results, i.e. extend Theorems 1 and 2 to other utility functions, social welfare functions and heterogeneous tax morales. More importantly, the exogenous tax morales and simple labor disutility functions should be replaced by endogenous tax morales and sophisticated labor disutilities implying realistic labor supplies. Provision of public goods and its efficiency also require attention.

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