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A Theory of Industry Life Cycle

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This paper derives the equilibrium timing of entries and exits as well as the equilibrium output levels over the industry life cycle. This paper also examines the effects of the increase in entry costs. It turns out that the first entry may occur earlier when the entry costs increase. In an extended model with 3 potential entrants, it is shown that the first entry may be delayed with the third firm, and that the less efficient firm may be the first entrant in some exceptional cases.

I. Introduction

Competition among firms in growing industries has been one of the important issues in industrial organization. Gilbert and Harris (1984) and Mills (1988) showed that total profits from each additional investment (or production capacity) must be equal to zero because of the incentive to preempt the market in growing industries. These results are similar to the contestable market theory (Baumol, Panzar and Willig (1982)) in that the monopolist receives zero profits not by the real competition, but by the potential competition.

Firms may also compete to survive in a declining industry. Exit behavior of firms in declining industries is well explained by Ghemawat and Nalebuff (1985, 1987). They showed that a smaller firm outlasts a bigger one in declining industries. Yoo (1990), on the other hand, showed that it is possible for a larger firm to outlast a smaller one by relaxing their assumption that the output level must be equal to the production capacity.

Londregan (1990) put these two phases together, the phase of growth and the phase of decline. He explained the pattern of entries and exits over the industry life cycle, focusing on who preempts and who outlasts. He assumed, however, that production capacity is exogenously given and that all or nothing operation does exist. In his model, each firm decides only "in" or "out".

This paper assumes a situation similar to Londregan, a situation in which the industry evolves and subsequently devolves. Unlike Londregan, however, this paper assumes that reentry does not occur over the industry life cycle and that entry costs are the same for all firms. Instead, this paper endogenizes the choice on output levels (along with the choice on entries and exits). In this sense, this paper generalizes his model by relaxing his assumption that all or nothing operation does exist given the production capacity. By doing so, this paper examines what are the timing of entries and exits over the industry life cycle and what are the

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equilibrium output levels at each instant?

It is shown that the first entrant may have to enter the market when its net instant monopoly profit is still negative. Because of the incentive to preempt the market, the total profits during the monopoly period in a growing phase turns out to be close to zero when the cost difference is negligible. These results are similar to Gilbert and Harris (1984) and Mills (1990). In a declining phase, the less efficient firm exits as soon as its instant duopoly profit decreases to zero and the more efficient firm stays active until its instant monopoly profit decreases to zero. This pattern of exits is consistent with Ghemawat and Nalebuff (1985, 1987) and Yoo (1990). This paper also shows, in a two-firm model, that the early entrant (the late survivor) produces the monopoly output level of each instant during the monopoly period, and that each firm produces the Cournot output levels of each instant during the duopoly period. This result justifies the use of the monopoly output and the Cournot outputs over the industry life cycle where the market evolves and subsequently devolves.

This paper examines also the following important questions: What are the effects of the increase in entry costs on entry? What are the effects of the third potential entrant on entry? It is shown that the first entrant may have to enter the market earlier when the entry costs increase. It is also shown that the entry of the first entry may be delayed by the existence of the third potential entrant, and that the less efficient firm may be the first entrant in some exceptional cases. These results are interesting not only because they are new but also because they are contrary to our prior expectation.

This paper is organized as follows. Section II describes the model in which two firms compete over the industry life cycle. Based on this model, Section III derives the subgame perfect equilibrium of the whole game. Section IV examines the effect of the change in entry cost on the timing of entries. Section V extends the model by allowing the third potential entrant. Finally, Section VI concludes the paper.

II. Model

Consider a homogeneous market that evolves and subsequently devolves. Two potential entrants compete in the market. Let $\pi_i^m(t)$ and $\pi_i^d(t)$ be the profit-maximizing instant monopoly profit and the instant Cournot duopoly profit of firm i respectively. The term ‘instant’ denotes the stage game at time t.

Assume that $\pi_i^m(t)$ and $\pi_i^d(t)$ are twice continuously differentiable functions of time t. Assume further that $\pi_i^m(t=0)<0$, $\pi_i^d(t=0)<0$ for i = 1, 2; there exists T_{ai}^m and T_{ai}^d (i =1,2) such that

$$\partial\pi_i^m(t)/\partial t>0 \quad \forall t\in[0,T_{ai}^m) \text{ and } \partial\pi_i^m(t)/\partial t<0 \quad \forall t\in(T_{ai}^m,\infty)$$

$$\partial\pi_i^d(t)/\partial t>0 \quad \forall t\in[0,T_{ai}^d) \text{ and } \partial\pi_i^d(t)/\partial t<0 \quad \forall t\in(T_{ai}^d,\infty);$$

there also exists T_{bi}^m and T_{bi}^d (i =1,2) such that

$$p_i^m(T_{bi}^m) < 0 \ \text{ for } \ T_{ai}^m < T_{bi}^m < \infty$$

$$p_i^m(T_{bi}^d) < 0 \ \text{ for } \ T_{ai}^d < T_{bi}^d < \infty \, .$$

These assumptions imply an industry lfe cycle: monopoly(or duopoly) profit is negative at time 0; it increases until the peak time for the market demand; and then it decreases down to negative levels again. The existence of T_{bi}^m makes the game finite-horizon, eventually ruling out the existence of multiple equilibria usually found in infinite-horizon games. The equilibrium concept adopted in this paper is the subgame perfect equilibrium, and the series of the equilibrium of the stage game consist of the equilibrium of the whole game by the backward induction. For simplicity, this paper assumes large enough differences between T_{ai}^m and T_{bi}^m and between T_{ai}^d and T_{bi}^d. This assumption separates the entry and the exit phase. Assume further that firm 1 is more efficient than firm 2. That is, $p_1^m(t) > p_2^m(t),\ \ p_1^d(t) > p_2^d(t)$ for all $t > 0$.

The game is set in continuous time, but firms make decisions only at discrete time intervals, $t = \Delta,\ 2\Delta,\ 3\Delta,\ 4\Delta$ .... . Here, Δ is assumed to be close to zero ($\Delta \rightarrow 0$). By doing so, I avoid complications arising from the strategy space with uncountably many dimensions. In this sense, the equilibrium in this paper is the limit equilibrium of the limit game by Fudenberg and Levine (1986).

Firms decide their output levels at each instant (that is, at each stage game). Firms exhibit Cournot behavior. There exists a unique Cournot equilibrium output level at each instant. Each firm enters the market by choosing a positive output level and exits by choosing zero output level. A strategy, s_i is a sequence of functions s_{it} which maps from histories of the game onto the set of output levels [0, ∞). Unlike Londregan (1990) and Gilbert and Harris (1984), firms are allowed to increase or decrease the output level at any instant.[1] For simplicity, this paper assumes that the production capacity is flexible and costless. The operating costs of the firms are $C_i(Q,\ t)$. There is no barriers to entry or exit, but there exists entry (or reentry) cost of $e^{-rt}S$ for both firms discounted to time $t = 0$.[2] The game is under perfect and complete information, which implies that market demands as well as costs are common knowledge.

Define

1. Gilbert and Harris (1984) ignored the post-entry strategic reactions of the incumbent firm. In their model, the output level is not adjustable downward due to the assumption that the output level must be equal to the production capacity. Thus, the incumbent firm cannot reduce the output level below the existing capacity even after the entry of a new entrant and there is no strategic interaction in terms of the output level. The only choices open to the players are whether to increase the capacity at each instant of decision making and how much in such cases.
2. The assumption on the entry cost can be replaced with the assumption of exit cost without loss of generality.

$$t_{mi} \equiv \inf\left\{ t \,\middle|\, p_i^m(t) - rS > 0 \right\} \tag{1}$$

$$t_{di} \equiv \inf\left\{ t \,\middle|\, p_i^d(t) - rS > 0 \right\} \tag{2}$$

$$t_i^m \equiv \sup\left\{ t \,\middle|\, p_i^m(t) > 0 \right\} \tag{3}$$

$$t_i^d \equiv \sup\left\{ t \,\middle|\, p_i^d(t) > 0 \right\}. \tag{4}$$

t_{mi} and t_{di} denote the earliest instant at which the net instant monopoly and the net instant duopoly profit of the firm i become positive respectively. The term 'net' instant profit means the instant (monopoly or duopoly) profit minus the cost of entering the market one instant (Δ) earlier. It is the 'net' instant profit, not the instant profit that is relevant in deciding whether to enter or not, since firms have to take into account the cost (opportunity cost) of entering the market one instant earlier. t_i^m and t_i^d, on the other hand, denote the last instant at which the instant monopoly and the instant duopoly profits of the firm i remain positive respectively. Since there is no exit cost, the term 'net' is not necessary here.

Since the instant monopoly profit is always greater than the instant duopoly profits, $t_{mi} < t_{di}$ and $t_i^m > t_i^d$ must hold for i = 1, 2. It means that the earliest time at which the net instant monopoly profit (in a growing phase) becomes positive comes earlier than the earliest time at which the net instant duopoly profit becomes positive. It also means that the latest time at which the instant monopoly profit remains positive (in a declining phase) comes later than the latest time at which the instant duopoly profit remains positive. Figure 1 represents t_{mi}, t_{di}, t_i^m and t_i^d graphically.

Figure 1 Profit Functions

III. The Subgame Perfect Equilibrium of the Game

Define

$$t_1^* \equiv \inf\{\, t \mid \int_t^{t_{d2}} e^{-rt}[\pi_1^m(t) - rS]dt - \int_{t_{d1}}^{t_{d2}} e^{-rt}[\pi_1^d(t) - rS]\,dt \; > 0\} \tag{5}$$

$$t_2^* \equiv \inf\{\, t \mid \int_t^{t_{d1}} e^{-rt}[\pi_2^m(t) - rS]dt + \int_{t_{d1}}^{t_{d2}} e^{-rt}[\pi_2^d(t) - rS]dt \; > 0\}\,. \tag{6}$$

t_i^* denotes the earliest time when firm *i*' s profits from entering the market as the first entrant are bigger than its profits from entering later at t_{di} as the second entrant. It is under the condition that the rival (firm j) enters at time t_{dj}. It is the earliest time firm i would ever want to preempt the market if firm j enters the market at t_{dj} as the second entrant. Later, it will be shown that the second entrant (let us say firm *j*) enters the market at t_{dj} in the subgame perfect equilibrium. Lemma III-1 is derived from the fact that firm 1 is more efficient than firm 2. The proof is straightforward and hence is omitted.

Lemma III-1:
$t_1^* < t_2^*$, $t_{m1} < t_{m2}$, $t_{d1} < t_{d2}$, $t_1^d > t_2^d$, $t_1^m > t_2^m$.

The following lemma shows that market preemption may occur before the net instant monopoly profit becomes positive.

Lemma III-2:
$t_i^* < t_{mi}$ for i =1, 2 if its rival (firm *j*) enters the market at t_{dj}.

Proof:
Firm i earns positive net instant monopoly profits at least during the period [t_{mi}, t_{dj}] if firm i enters the market as the first entrant and if the rival (firm *j*) enters the market at t_{dj} as the second entrant. Therefore, $t_i^* < t_{mi}$. **Q.E.D.**

In order to simplify the analysis, this paper makes two additional assumptions. First, total duopoly profits are enough to cover the entry cost when either firm enters the market at t_{di} as the second entrant (i=1,2). Second, both firms are better off if they remain active in a growing phase once entered at t_i^* or after, instead of exiting and reentering at t_{di}. That is,

$$\int_{t_{di}}^{t_i^d} e^{-rt}\pi_i^d(t)dt > e^{-rt_{di}}S \quad \text{for } i=1,\; 2 \tag{7}$$

$$\int_{t_i^*}^{t_{di}} e^{-rt}\pi_i^d(t)dt > -e^{-rt_{di}}S \quad \text{for } i=1,\ 2 \tag{8}$$

The following lemma is also helpful in deriving the equilibrium of the whole game.

Lemma* Ⅲ*-3:

The late entrant (let us say firm j) must enter the market, in the subgame perfect equilibrium, at t_{dj} at which the net instant duopoly profit becomes positive.

Proof:

The proof is rather straightforward. Since it is better for both firms to remain active in a growing phase, once entered at t_i^* or after, it is better for the early entrant (let us say firm i) not to exit in a growing phase. Thus, the late entrant (firm j) must enter the market at t_{dj} at which the net instant duopoly profit becomes positive. **Q.E.D.**

Then, the subgame perfect equilibrium of the whole game is summarized in the following proposition.

Proposition* Ⅲ*-1:

The subgame perfect equilibrium of the whole game is:

1) Firm 1 enters the market at $t^* = \min(t_2^* - \Delta,\ t_{m1})$ $\min(t_2^*,\ t_{m1})$ and produces the monopoly output level (of each instant) until t_{d2}.

2) Firm 2 enters the market and produces monopoly output level (of each instant) at any instant during $[t_2^*,\ t_{d2}]$ if firm 1 did not enter the market before that instant. Firm 2 enters the market at t_{d2} no matter what.

3) Each firm produces its Cournot output level (of each instant) during the duopoly period, [t_{d2}, t_2^d].

4) Firm 2 exits at $t_2^d + \Delta$ t_2^d.

5) Firm 1 produces the monopoly output level (of each instant) during $(t_2^d,\ t_1^m]$ and exits at $t_1^m + \Delta$ t_1^m.

Proof:

Let us first consider a declining phase. Derivation of the equilibrium timing of exits is similar to Ghemawat and Nalebuff (1985, 1987) and Yoo (1990). Note that t_i^m is the instant when firm i has to exit even as a monopolist and $t_1^d > t_2^d$ and $t_1^m > t_2^m$. If both firms are active at t_2^m, firm 1 does not exit and produces monopoly output level at the instant $t_2^m + \Delta$. It is because firm 2 has to exit at $t_2^m + \Delta$ when its instant monopoly profit becomes negative regardless of firm 1' s decision. At time t_2^m, firm 1 would stay in order to become a monopolist at $t_2^m + \Delta$. This means that firm 2 has to exit at time

t_2^m. With the backward induction,[3] one can easily show that firm 2 must exit in the subgame perfect equilibrium at time t_2^d, that is, when firm 2 begins to receive negative instant duopoly profit. Firm 1 would exit at $t_1^m + \Delta$, that is, when firm 1 begins to receive negative instant monopoly profit. Again with the backward induction, firm 1 produces, in the subgame perfect equilibrium, the monopoly output level (of each instant) during the monopoly period.

Second, let us consider a growing phase. As lemma III-3 shows, the late entrant, firm *j*, must enter the market at t_{dj}, that is, when the net instant duopoly profit becomes positive. In the growing phase, however, both firms are expected to preempt the market as long as it is profitable to do so. Note that t_i^* is the earliest time firm *i* would ever want to preempt the market under if the other firm enters the market at t_{dj}. Since $t_1^* < t_2^*$ as in Lemma III-1, it is better for firm 1 to enter the market at $t_2^* - \Delta$ t_2^* as the first entrant. Otherwise, firm 2 would enter the market at t_2^* as the first entrant. Of course, this is only when $t_2^* \le t_{m1}$. If $t_{m1} < t_2^*$, firm 1 would enter the market at t_{m1} any way since its net instant monopoly profit is already positive. In fact, t_2^* is the latest time at which firm 1 must enter to become the first entrant. Therefore, the subgame perfect equilibrium strategy of firm 1 is to enter the market at $t^* = \min(t_2^* - \Delta,\ t_{m1})$ $\min(t_2^*,\ t_{m1})$ and to produce the monopoly output level (of each instant) until t_{d2}. Firm 2 enters the market and produces monopoly output level (of each instant) during $[t_2^*,\ t_{d2}]$ if firm 1 did not enter the market before that instant. Firm 2 enters the market at t_{d2} no matter what, since its duopoly profit is positive. Again with the backward induction, each firm produces its Cournot output level (of each instant) during the duopoly period, $[\,t_{d2},\ t_2^d\,]$ in the subgame perfect equilibrium.

Q.E.D.

The above proposition is consistent with Gilbert and Harris (1984) and Mills (1988). The existence of the potential entrant may cause the first entrant to enter early. The first entrant, firm 1 may have to enter the market when its net instant monopoly profit is still negative. Because of the incentive to preempt the market, the total profits during the monopoly period in a growing phase turn out to be close to zero when the cost difference is negligible and thus t_1^* t_2^*. Consumers are better off, on the other hand, with the early entry of the leader. The proposition 1 is also consistent with

3. If the two firms are identical, which implies $t^* \equiv t_1^* = t_2^*$, $t_m \equiv t_{m1} = t_{m2}$, $t_d \equiv t_{d1} = t_{d2}$, $t^d \equiv t_1^d = t_2^d$, and $t^m \equiv t_1^m = t_2^m$, there are three types of subgame perfect equilibria in exit phase of the game: First, firm 1 exits at t^d and firm 2 exits at t^m; second, firm 2 exits at t^d and firm 1 exits at t^m; third, fully-mixed strategy equilibrium in which both firms are indifferent between staying and exiting in every instant from t^d to t^m. In this case, backward induction cannot be applied to derive the subgame perfect equilibrium of the whole game. But this case is ruled out by the assumption.

Ghemawat and Nalebuff (1985, 1987) and Yoo (1990). The less efficient firm exits as soon as its instant duopoly profit becomes negative in a declining phase while the more efficient firm stays active until its instant monopoly profit remains positive.

The proposition 1 also shows that the early entrant (who is actually the late survivor) produces the monopoly output level (of each instant) during the monopoly period, and that each firm produces its Cournot output level (of each instant) during the duopoly period. It means that the monopoly output level and the Cournot output levels of each instant consist of the equilibrium strategy of the whole game over the industry life cycle.

IV. Effects of the Change in Entry Costs on Entry

The above results are similar to Londregan (1990), except the fact the output level is endogenized. Because of its simplicity, however, the model in this paper can be used to answer many interesting questions. One example is the effects of the increase in entry costs on the timing of entries. One may argue that the entry must be delayed with the increase in entry costs. It turns out, however, that the first entry may occur earlier even with the increase in entry costs. In other words, when entry costs increase, the total monopoly profits of the first entrant may increase because of the delay in the second entry.

Before entry costs change, firm 1 (the first entrant) enters the market at t_2^* and firm 2 (the second entrant) enters the market at t_{d2}. t_2^* is derived so as to make the area *cde* in Figure 2 equal to the area *abc* + the area *efg*. The area *cde* shows the gains from the preemption and the area *abc* and the area *efg* show the costs of the preemption. t_2^* is the earliest time firm 2 may want to enter the market as the first entrant.

Figure 2 Equilibrium Timing of Entries before the Change in Entry Costs

When entry costs increase from $e^{-rt}S$ to $e^{-r't}S$, the entry of the second entrant are delayed. Of course, it is firm 2 that enters as the second entrant in the subgame perfect equilibrium. The equilibrium timing of entry of firm 1, that is, t_2^*' is derived so as to make the area $c'd'e'$ in Figure 3 equal to the area $a'b'c'$ + the area $e'f'g'$. It can be easily shown that t_2^*' may be either bigger or smaller than t_2^*.

Figure 3 Equilibrium Timing of Entries after the Increase in Entry Costs

The above result implies that the first entrant may enter the market earlier when entry costs increase. An intuitive explanation is as follows. When entry costs increase, first entrant may enjoy longer monopoly periods. It means that firm 1 has to enter the market earlier in order to preempt the market. In such cases, the direct effect of increase in entry costs (that is, the disincentive to enter the market early) is dominated by the indirect effect of increase in entry costs via extension of the monopoly period (that is, incentive to enter the market early).

V. An Extension of the Model: The Case of 3 Potential Entrants

What happens if there are three potential entrants competing in a growing industry? Does the third firm cause the entry to occur earlier? Is the order of the entry unaffected by the third firm?

Since the pattern of exits in a declining phase is quite similar to the case of two potential entrants,[4] this section focuses on entry in a growing phase. As is in section III, let us assume, for simplicity, that firm 1 is the most efficient while firm 3 is the least. That is, $\pi_1^m(t) > \pi_2^m(t) > \pi_3^m(t)$, $\pi_1^d(t) > \pi_2^d(t) > \pi_3^d(t)$, $\pi_1^t(t) > \pi_2^t(t) > \pi_3^t(t)$ for all $t > 0$. Here, $\pi_i^t(t)$ denotes the instant trigopoly profit of firm i at time t. Assume that there exists t_i^t such that

$$t_{ti} \equiv \inf \{t \,|\, \pi_i^t(t) - rS > 0\} \tag{9}$$

t_{ti} denotes the earliest instant at which the net instant trigopoly profit becomes positive.

Define

$$t_i^s(j) \equiv \inf\{ t \,|\, \int_t^{t_{tj}} e^{-rt}[\pi_i^d(t) - rS]dt - \int_{t_{ti}}^{t_{tj}} e^{-rt}[\pi_i^t(t) - rS]dt > 0\} \text{ if } i \le j \tag{10}$$

$$t_i^s(j) \equiv \inf\{ t \,|\, \int_t^{t_{tj}} e^{-rt}[\pi_i^d(t) - rS]dt + \int_{t_{ti}}^{t_{tj}} e^{-rt}[\pi_i^t(t) - rS]dt > 0\} \text{ if } i > j \tag{11}$$

Here, $t_i^s(j)$ denotes the earliest time when firm i' s profits from entering as the second entrant are bigger than its profits from entering later at t_{ti} as the third entrant, given that one firm (let us say firm k) already entered the market. It is the earliest time firm i would ever want to enter the market as a second entrant (rather than entering the market at t_{ti} as the last entrant), given the condition that the last entrant (let us say firm j)enters the market at t_{tj}. Again, one can easily show that the last entrant (let us say firm j), in the subgame perfect equilibrium, actually enters the market at t_{tj}, that is, when its net instant trigopoly profit becomes positive.[5]

This section makes the following assumptions. First, total trigopoly profits of the last entrant, whatever it may be, are enough to cover the entry cost when it enters the market when its net instant trigopoly profit becomes positive. Second, it is better for any firm (firm i, i=1,2,3) to remain active in a growing phase rather than to exit and reenter at t_{ti} once entered at $t_i^s(k)$ or later. Lemma V-1 follows from these assumptions.

4. In a declining phase, the subgame perfect equilibrium is characterized as follows. Firm 3 (the least efficient firm) exits first when its instant trigopoly profit becomes negative, firm 2 exits next when its instant duopoly profit becomes negative, and then firm 1(the most efficient firm) exits last when its instant monopoly profit becomes negative.

5. The proof is similar to Lemma III-4.

***Lemma V-1*:**

i) $t_{m1} < t_{m2} < t_{m3}$, $t_{d1} < t_{d2} < t_{d3}$, $t_{t1} < t_{t2} < t_{t3}$,

ii) $t_i^s(j) < t_i^d$ for *i*, *j*=1, 2, 3 and $i \neq j$

***Proof*:** Similar to those of lemma III-1, III-2.

i) of lemma V-1 follows from the assumption that firm 1 is the most efficient while firm 3 is the least. ii) follows from the fact that firm *i* is able to earn positive net instant duopoly profits during the period [t_{di}, t_{ti}).

***Lemma V-2*:**

$t_1^s(2) < t_2^s(1) < t_3^s(1)$, $t_1^s(3) < t_2^s(3)$ and $t_1^s(2) < t_3^s(2)$.

Proof:

First, let us consider the subgame in which two firms (firm *i* and *j*) compete to be the second entrant after one firm (let us say firm *k*) already entered. The earliest time at which the more efficient firm (let us say firm *i*) would ever want to enter the market as the second entrant (rather than entering the market at t_{ti} as the last entrant) comes earlier than the less efficient firm.[6] Therefore, $t_i^s(j) < t_j^s(i)$ for all $i < j$, under the condition that the last entrant enters the market when the instant trigopoly profit becomes positive. Second, the earliest time at which the more efficient firm (firm *i*) would want to enter the market as the second entrant comes earlier than that of the less efficient firm (firm *j*), when faced with the same firm (firm *k*). This is because the less efficient firm enters the market later as the last entrant, and thus the second entrant can enjoy longer duopoly periods when his competitor is less efficient.[7] Therefore, $t_i^s(k) < t_j^s(k)$ for all $i < j$ and for all $k \neq i, j$. **Q.E.D.**

Assume further that the profits of firm *k* are greater when its competitor is less efficient. Then, the earliest time at which firm *k* would want to enter the market as the second entrant comes earlier when its competitor is less efficient. That is, $t_k^s(j) < t_k^s(i)$ for all $i < j$ and for all $k \neq i, j$.

Figure 4 shows it graphically by taking an example of $t_3^s(2)$ and $t_3^s(1)$. $t_3^s(2)$ makes area *abe* + area *jkl* equal to area *eij* and $t_3^s(1)$ makes area *cde* + area *ghl* equal to area *efg*.

6. For example, $t_1(2) < t_2(1)$ implies that the earliest time firm 1 would ever want to enter the market as a second entrant (rather than entering the market at t_{t1} as the last entrant) comes earlier than the firm 2 when firms 1 and 2 are competing to be the second entrant, because firm 1 is more efficient than firm 2.

7. For example, $t_1(2) < t_1(3)$ implies that the less efficient his opponent is, the earlier firm 1 would ever want to enter the market as a second entrant (rather than entering the market at t_{t1} as the last entrant), because the less efficient firm enters the market later as the last entrant.

Figure 4 Profit Functions

***Lemma V-3*:**

In the subgame where one firm, let us say firm k, already entered, firm i must enter the market at $\min(t_j^s(i) - \Delta,\ t_{di})$ $\min(t_j^s(i),\ t_{di})$ and firm j must enter the market at t_{tj} if firm i is the more efficient between the two (that is, if $i < j$).

Proof:

Lemma V-3 is similar to 1) of Proposition III-1. Let us assume that firm k already entered the market. Then, more efficient firm i is able to become the second winner (that is, the second entrant) if he enters the market at $t_j^s(i) - \Delta$, since $t_i^s(j) < t_j^s(i)$. $t_j^s(i) - \Delta$ is the latest time at which less efficient firm j would never want to enter the market as the second entrant. Therefore, firm i has to enter the market at $t_j^s(i) - \Delta$ to become the second entrant if $t_j^s(i) \le t_{di}$. If $t_j^s(i) > t_{di}$, firm i enters the market as soon as his duopoly profit becomes positive (at t_{di}), since it is enough to be the second winner by doing so in that case. **Q.E.D.**

Lemma V-4:

Firm 3 cannot be the first entrant in the subgame perfect equilibrium.

Proof:

Let us assume that firm 3 enters the market as the first entrant at t' in a subgame perfect equilibrium. Then, the following conditions must be satisfied. First, firm 3 must prefer entering at t' as the first entrant to entering at t_{t3} as the last entrant.

Second, firm 1 must enter the market at min($t_2^s(1)$, t_{d1}) as a second entrant and firm 2 must enter the market at t_{t2} as the last entrant. Now, firm 2 knows that firm 1 would enter the market at min($t_3^s(1)$, t_{d1}) as a second entrant, and that firm 3 would enter the market at t_3^t if firm 2 deviates and enter at $t'-\Delta$. Then, firm 2 has an incentive to deviate and to enter the market at $t'-\Delta$ as the first entrant. Note that i) the second entrant (firm 1) enters the market later when firm 2 deviates, since min($t_3^s(1)$, t_{d1}) is bigger or equal to min($t_2^s(1)$, t_{d1}), ii) the last entrant (firm 3) enters the market later when firm 2 deviates, since t_{t3} is bigger than t_{t2}, and iii) the firm 2's profit is bigger than that of firm 3. Therefore, firm 2 must prefer entering at $t'-\Delta$ as the first entrant to entering at t_{t2} as the last entrant if firm 3 prefers entering at t' as the first entrant to entering at t_{t3} as the last entrant. Since this is contradictory, firm 3 cannot be the first entrant in subgame perfect equilibrium. **Q.E.D.**

Corollary V-1:
Firm 3 must be the last entrant in the subgame perfect equilibrium.

Proof:
If firm 3 cannot be the first entrant, it must be the last entrant, since $t_1^s(3) < t_1^s(2) < t_2^s(1) < t_3^s(1)$ as shown in lemma V-2. **Q.E.D.**

Now, let us investigate which firm (firms 1 or 2) enters first and when, in order to answer the following questions on the effect of the third potential entrant: 1) Does the entry of the first entrant occur earlier when there exists the third firm? 2) Is the order of the entry unaffected by the third firm?

Since $t_{d1} < t_{d2}$ and $t_3^s(2) < t_3^s(1)$, there are five possible cases: i) $t_{d1} < t_{d2} \le t_3^s(2) < t_3^s(1)$, ii) $t_{d1} \le t_3^s(2) < t_{d2} \le t_3^s(1)$, iii) $t_{d1} \le t_3^s(2) < t_3^s(1) < t_{d2}$, iv) $t_3^s(2) < t_{d1} \le t_3^s(1) < t_{d2}$, v) $t_3^s(2) < t_3^s(1) < t_{d1} < t_{d2}$. The five cases can be regrouped into three.

Proposition V-1:
i) If $t_{d1} < t_{d2} \le t_3^s(2) < t_3^s(1)$, the existence of firm 3 does not affect the entry of firm 1 and 2,
ii) If $t_{d1} \le t_3^s(2) < t_{d2} \le t_3^s(1)$ or if $t_{d1} \le t_3^s(2) < t_3^s(1) < t_{d2}$, the existence of firm 3 makes firm 1 as well as firm 2 to enter earlier (or at least at the same time) without the change in the order of entry,
iii) If $t_3^s(2) < t_{d1} \le t_3^s(1) < t_{d2}$ or if $t_3^s(2) < t_3^s(1) < t_{d1} < t_{d2}$, the existence of firm 3 may delay the entry of firm 1. It may even change the order of entry in this case.

Proof:
Case i): $t_{d1} < t_{d2} \le t_3^s(2) < t_3^s(1)$

Firm 1 enters at t_{d1} in the subgame when firm 2 already entered, and firm 2 enters at t_{d2} in the subgame when firm 1 already entered. Thus, firm 1 enters the market at t_2^* and firm 2 enters the market at t_{d2}. Thus, the existence of firm 3 does not affect the entry of firms 1 and 2.

Case ii): $t_{d1} \le t_3^s(2) < t_{d2} \le t_3^s(1)$ or $t_{d1} \le t_3^s(2) < t_3^s(1) < t_{d2}$

Firm 1 enters at t_{d1} in the subgame when firm 2 already entered, but firm 2 enters at $t_3^s(2)(< t_2^d)$ in the subgame when firm 1 already entered. This means that firm 2 enters the market as a second entrant before his duopoly profit becomes positive if firm 3 exists. This increases the incentive of firm 2 to be the first entrant and, thus, firm 1 must preempt the market earlier.

Let us define t_i^{f*} to be the earliest time at which firm *i* would want to enter the market as the first entrant instead of entering as a second entrant. Then, t_1^{f*} and t_2^{f*} can be represented as follows since $t_{d1} < t_3^s(2)$.

$$t_1^{f*} \equiv \inf\{\, t \mid \int_t^{t_3^s(2)} e^{-rt}[\boldsymbol{p}_1^m(t) - rS\,]dt - \int_{t_{d1}}^{t_3^2(2)} e^{-rt}[\boldsymbol{p}_1^d(t) - rS\,]dt > 0\} \tag{12}$$

$$t_2^{f*} \equiv \inf\{\, t \mid \int_t^{t_1^d} e^{-rt}[\boldsymbol{p}_2^m(t) - rS\,]dt + \int_{t_1^d}^{t_3^2(2)} e^{-rt}[\boldsymbol{p}_2^d(t) - rS\,]dt > 0\} \tag{13}$$

Figure 5 Effects of the Existence of Firm 3 on the Entry of Firm 1

As shown in Section III, firm 1 enters the market at min($t_2^* - \Delta$, t_{m1}) as the first entrant when there are only two firms. t_2^* is defined so as to make area *cde* + area *ghk* equal to area *efg* in Figure 5. When there are three firms, firm 1 enters the market at min($t_2^{f*} - \Delta$, t_{m1}) as the first entrant. t_2^{f*} is defined so as to make area *abe* + area *ghji* equal to area *efg*. Since t_2^{f*} < t_2^*, it that firm 1 enters the market earlier when there exists firm 3.[8]

Case iii): $t_3^s(2) < t_{d1} \le t_3^s(1) < t_{d2}$ or $t_3^s(2) < t_3^s(1) < t_{d1} < t_{d2}$,

Firm 1 enters at t_{d1} in the subgame when firm 2 already entered, but firm 2 enters at $t_3^s(2)(< t_{d1})$ in the subgame when firm 1 already entered. Firm 2 enters earlier than firm 1 as a second entrant. Faced with the same firm (firm 3), firm 2 has to enter the market earlier than firm 1 as a second entrant, since firm 3 has more incentive to be the second entrant when competing with firm 2 rather than firm 1.

Since firm 2 has to enter the market earlier as a second entrant than firm 1, firm 2' s cost disadvantage is an advantage in the race for the first entrant as well. Thus, the order of entry is determined by the relative importance of disadvantage in costs and advantage in entry timing as a second entrant. Note that the definitions of t_1^{f*} and t_2^{f*} are changed as follows since $t_3^s(2) < t_{d1}$.

$$t_1^{f*} \equiv \begin{cases} \inf\{\, t \mid \int_t^{t_3^s(2)} e^{-rt}[\boldsymbol{p}_1^m(t) - rS]dt + \int_{t_3^s(2)}^{t_{d1}} e^{-rt}[\boldsymbol{p}_1^d(t) - rS]\,dt > 0\} \;\; if \;\; t_{d1} \le t_3^s(1) \\ \inf\{\, t \mid \int_t^{t_3^s(2)} e^{-rt}[\boldsymbol{p}_1^m(t) - rS]dt + \int_{t_3^s(2)}^{t_3^s(1)} e^{-rt}[\boldsymbol{p}_1^d(t) - rS]\,dt > 0\} \;\; if \;\; t_{d1} > t_3^s(1)\,. \end{cases} \tag{15}$$

$$t_1^{f*} \equiv \begin{cases} \inf\{\, t \mid \int_t^{t_{d1}} e^{-rt}[\boldsymbol{p}_1^m(t) - rS]dt - \int_{t_3^s(2)}^{t_{d1}} e^{-rt}[\boldsymbol{p}_1^d(t) - rS]\,dt > 0\} \;\; if \;\; t_{d1} \le t_3^s(1) \\ \inf\{\, t \mid \int_t^{t_3^s(1)} e^{-rt}[\boldsymbol{p}_1^m(t) - rS]dt - \int_{t_3^s(2)}^{t_3^s(1)} e^{-rt}[\boldsymbol{p}_1^d(t) - rS]\,dt > 0\} \;\; if \;\; t_{d1} > t_3^s(1)\,. \end{cases} \tag{16}$$

It turns out that the entry of firm 1 (as the first entrant) is delayed by the existence of firm 3 in this case. Since t_2^{f*} is the earliest time at which firm 2 would want to enter the market as the first entrant, firm 1 is able to become the first entrant if it enters at $t_2^{f*} - \Delta$. Since t_2^* < t_2^{f*}, the entry of firm 1 is delayed by the existence of firm 3 even if firm 1 is the first entrant.

Furthermore, it is possible for firm 2 to be the first entrant in the subgame perfect equilibrium in this case. Figure 6 shows an example. Assume *i*) the profits of firm 1 and 2 are very similar up to t_{d1} and then diverge after t_{d1}, ii) t_{d1} t_{d2}, iii) $t_{d1} \le t_3^s(1)$, and iv) $t_3^s(2) < t_{d1}$. Then, $t_2^{f*} < t_1^{f*}$, since t_2^{f*} is defined so as to make area

8. Of course, it is firm 1 that enter the market first in the subgame perfect equilibrium since $t_1^{f*} < t_2^{f*}$.

abe equal to area *eij* and t_1^{f*} is defined so as to make area *cde* + area *ghj* equal to area *efg*. The earliest time firm 2 would ever want to enter the market as the first entrant comes earlier than firm 1 and, thus, firm 2 enters the market as the first entrant at t_1^{f*} in the subgame perfect equilibrium. In this case, disadvantage in costs is negligible in the early stage of the game, while the advantage in the timing of entry as the second entrant is not. The disadvantage in costs is actually dominated by the advantage in the timing of entry as the second entrant in this case. **Q.E.D.**

Figure 6 Example 1

VI. Conclusions

When an industry expands and subsequently shrinks, the timing of entries and exits as well as output levels are important issues not only for entrepreneurs but also for government policy makers. Entrepreneurs have to decide when to enter, when to exit and how much to produce while operating. Government policy makers should also be aware of the timing of entries and exits before they decide their policies.

In this paper, I derived the equilibrium timing of entries and exits as well as the equilibrium output levels over the industry life cycle. The first entrant may have to enter the market when its instant monopoly profit is still negative because of the incentive to preempt the market. In such circumstances, the total profits of the monopolist are reduced, but consumers are better off with the early introduction of goods or services. Also, each firm produces the monopoly output level (of each instant) during the monopoly period, and each firm produces its Cournot output level (of each instant) during the duopoly period. This result justifies the use of the monopoly output and the Cournot outputs over the industry life cycle.

This paper also examines the effects of the increase in entry costs on entry. Contrary to our prior expectation, it turns out that the entry of the first entrant may occur earlier. It is because the first entrant may enjoy the longer monopoly periods since the entry of the second entrant is delayed by the increase in entry costs. Thus, the net effect of the increase in entry costs depends on the relative magnitude between the direct effect of the increase in entry costs and the indirect effect arising from the delay of the second entry.

In an extended model with 3 potential entrants, it is shown that the entry of the first entrant may be delayed by the existence of the third potential entrant. It is because the second entrant enters the market sooner when faced with the competition from the third entrant, which, in turn, reduces the incentive to be the first entrant. In some exceptional cases, it is even possible that the less efficient firm may be the first entrant in the subgame perfect equilibrium. It occurs when the profit functions of the most efficient firm and the less efficient firm are very close up to a certain point and then diverge thereafter. In this case, the advantage in costs is dominated by the disadvantage in the timing of entry as a second entrant, since the less efficient firm may have to enter the market earlier as the second entrant than the most efficient firm.

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