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USING SPLINE FUNCTIONS FOR THE SUBSTANTIATION OF TAX POLICIES BY LOCAL AUTHORITIES

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The paper aims to approach innovative financial instruments for the management of public resources. In the category of these innovative tools have been included polynomial spline functions used for budgetary sizing in the substantiating of fiscal and budgetary policies. In order to use polynomial spline functions there have been made a number of steps consisted in the establishment of nodes, the calculation of specific coefficients corresponding to the spline functions, development and determination of errors of approximation. Also in this paper was done extrapolation of series of property tax data using polynomial spline functions of order I. For spline impelementation were taken two series of data, one reffering to property tax as a resultative variable and the second one reffering to building tax, resulting a correlation indicator R=0,95. Moreover the calculation of spline functions are easy to solve and due to small errors of approximation have a great power of predictibility, much better than using ordinary least squares method.
In order to realise the research there have been used as methods of research several steps, namely observation, series of data construction and processing the data with spline functions. The data construction is a daily series gathered from the budget account, reffering to building tax and property tax. The added value of this paper is given by the possibility of avoiding deficits by using spline functions as innovative instruments in the publlic finance, the original contribution is made by the average of splines resulted from the series of data.
The research results lead to conclusion that the polynomial spline functions are recommended to form the elaboration of fiscal and budgetary policies, due to relatively small errors obtained in the extrapolation of economic processes and phenomena.
Future research directions are taking in consideration to study the polynomial spline functions of second-order, third-order, Hermite spline and cubic splines of class C2 .

Keywords: fiscal policy, budget deficits, spline functions, budget justification, debt crisis

JEL codification: C29, H72

I.Introduction

After about two years from the financial crisis debut, local governments from everywhere are preoccupied to ensure the financial resources needed to finance budget deficits and to ensure quality public services provided for the interests of beneficiaries.
The financial crisis has left deep scars in local communities. Financial pandemic has spread rapidly throughout the economic system. Budget revenues have fallen dramatically, budget deficits have become chronic and in front of these realities the local authorities had to identify solutions to avoid the financial collapse .
Often local authorities have had to turn to central government to finance budget deficits or to support throughout the budget transfers some categories of public services. The dependence of

central government proved to be mostly harmful. This is because local authorities could not avoid political influence in their own public policies. In this way there was created a financial dependency on central government, wich had had an impact on the quality of public services provided. The explanation for this financial dependence is not justified only by the economic crisis. There is another reason that explains this phenomenon resulting from the manner which it was understood the financial autonomy of local governments written in the European Charter on Local Autonomy in Strasbourg. The common denominator of financial autonomy is that local governments have the property tax as a revenue source for their own budgets. But this proved to be insufficient compared to the funding needs on public services that local governments have to ensure . The solution identified to introduce a new type of tax in the configuration of financial resources available to local governments has proved to be a useful solution yet insufficient . It is about shared taxes wich are included in own revenues of local governments in Romania, does not fully cover financing needs of their local communities. Therefore the needs for financing, most often over the financing capacity of local governments are the result of transferring competences, duties and responsibilities regarding public services. It is about public services such as preuniversity education, health and social protection, local police, evidence of the population and lately the public health services.

In this way budget transfers were maintained as a last minute solution for decentralized financing of public services wich are useful but only turns local authorities into simple financial resources cashiers who runs the operations of receipts and payments. Ther has been tried budgetary balance solution by means of balancing allowances contained sharing of income tax (currently 22%) and also for balancing amounts allocated by the central government to local governments, but the criteria allocating these amounts are subject to dispute between the Ministry of Finance and local governments interested in obtaining an opportunity to exploit the maximum amount in the allocation on these financial resources.

Therefore the central government in the process of budget elaboration divides financial resources into two major categories namely: own revenues and transfers from the budget. Also own revenues include income tax on property taxes, income tax shared revenues from concessions and assets of local governments wich are public property. Most of the times own income does not exceed 50% of the total revenues available to local governments, which confirms the theory that local governments are financially dependent on central governments and has a poor financial capacity wich is relatively low compared to the funding needs that local governments generally have.

Budgetary transfers even though they are significant in amount, they have most cases fixed destination so that local governments operates receipts and payments transactions which runs between the central government and local governments.

There is a natural question and in the same time legitimate: are there interested local governments to ensure a prudent and efficient management of own financial resources ? And the answer, especially in times of financial crisis, can only be yes.

But managing their own financial resources can not be ensured without a budget process governed by modern tools and techniques that allow the decision-makers (local governments) to measure the level of these financial resources, to identify new alternative tax bases and then through the mechanisms of public funds to ensure financing of public services.(1)

The key element in the budget process is the budget formulation stage. Either it is known that the planned financial resources is equal to the financial resources consumed. The simple rule of budgetary balance. What did the science of public finances so far? has provided series of structured budgeting methods into two categories namely: traditional methods and modern methods budgeting. A brief recap of their category indicates wich in classical methods included direct method of increase / decrease and modern methods which include zero base approach or rationalize expenditures budgeting method.

These methods(2) had the disadvantage that are relatively unpredictable because are based throughout the series of data from the previous period of time and do not take into account the various correlations that may occur between different variables within in portfolio of any local budget revenues . Moreover, in the process of budget forecasting for time periods of three or five years, these methods have proved to be highly unnecessary as there had long-term low predictive power.(3)
The major disadvantages have been the motivation of developing this work, which was necessary to identify other ways to approach the predictions in the planning of budgetary resources to prove his worth. For this was an innovative area was identified in the science of public finances namely the use of spline functions.

II. What are spline functions and how they can be applied in the substantiating of fiscal and budgetary policies of local governments?

II.1. Brief history of spline functions. State of knowledge in the field of spline functions.

The notion "spline"comes from ancient times, from a mechanical device used by draftsmen in draw smooth curves. The term spline function in mathematics was first used by Schoenberg in 1946.(4)
In 1946, mathematicians began to study the shape of splines and derivatives of polynomial formula known as the term spline curve or the term spline function.(5)
This has led to widespread use such computer aided design functions, particularly in the areas vehicle designs. IJ Schoenberg had given the name of spline function after its resemblance to the mechanical used by draftsmen.
The word "spline"comes from term used by shipbuilders, referring to pieces of thin wood. In the past 40 years, functions have become very popular in computer graphics, computer animation and computer aided design field. It is difficult to find any industrial product, no spline substance . Also, they are widely used in studies of mathematics such as interpolation approximation. Spline gives another way to represent the class of functions. For decades, mathematicians had used for the polynomial calculus. In the early 20th century, with advances in approximation theory, spline functions had started to appear. The basic idea is simple. In essence, they consist of portions of polynomials with local support.
In the late 1950, two French mathematicians, Pierre Bézier from Renault and Casteljau Paul from Citroen, had independently developed spline-based computer aided design systems for cars.(6)
In last decades a very important place in the scientific field is taken by spline functions, wich its crowd forms a linear space. They present the major advantage to be easy programable leading to final numerical solutions. Also it important the fact that polinomyal spline functions have the property to minimize a seminorma of spline interpolation function odd degree, the using Hilbert space techniques can be defined as a unique spline element solution of a variational problem. This allows to obtain important properties of all multitude linear spline functions such as spline functions of polynomial, trigonometric, exponential, L-spline functions, etc. Chebyshev type. One can thus study the convergence of spline interpolation functions to the function which interpolates in usual rules Chebyshev and L2, and the order of convergence.(7)
Spline functions are characterized by their shapes subintervals between two nodes (which can be various known functions) and certain conditions for connecting the nodes.
Spline functions in recent years had found wide application, the main purpose being the interpolation. For spline interpolation was not deducted an expression of the error of approximation, a straightforward manner, as polynomial interpolation. It was retained only the statement that continuous function can be approximated as well all the interval $[x_1, x_N]$ when the number divisions increases, ie, spline interpolation is always convergent.. Because the derivative

of order m, $s(m)$, of a polinomial spline function is a step function (constant on intervals) while step function as well approximates continuous function splines on the interval when the number of divisions increases, we may give error assessment based on the maximum deviation between the derivatives $f_{(m)}(x)$ and $S_{(m)}(x)$ suppose that $f_{(m)}(x)$ exists and is continuos.(8)

Spline functions are used mostly in the industrial world. There are very few areas of production and engineering where these interesting functions are not used, and their use continues to grow at a rapid pace.

Robin Forrest describes "lofting", as a tehnique used in airplane industry during the World War II to build splines from thin wood for airplanes. Later the „lofting conic" was discovered, wich replaced so called splines in the early 1960 due to JC Ferguson`s work at Boeing.

At Boeing, these functions are used not only to represent geometrical models, for shape and curve modelling but also at the engineering analysis.(9)

Engineering analyisis and simulation is very important especially for a big engineering company such as Boeing. The examples are various, from sophisticated simulations for fluid dynamic calculation to simple codes of linear analysis wich uses simple physics models. Some codes are used to calibrate the calculations, while others use B-spline, wich forms a base for any kind of space given by the spline function as finite elements. In the finite element case, engineering codes creates spline approximation to de ordinary diferential equations, equations with partial derivatives, integral equations and algebrical, beeing a popular approach done by Boor and Swarts in 1973.(10)

II.2.Spline functions algorithm

To clarify the spline functions algorithm we start from several features considered as work hypotheses to explain this rational mechanism with the possibility of applying in economic sciences, namely:

a) we consider a series of data with the values $(x_1, x_2, ...x_{n-1}, x_n)$ wich are in the same time a division $\Delta \cdot a = x_1 \prec x_2 \prec ...... \prec x_{n-1} \prec x_n = b$ of the interval $[a,b]$;

b) the space of spline function is considered as:

$$S_m^k(\Delta) = \{s, s \in C^k[a,b], s/_{|x,x_{i-1}|} \in P_m, i = 1,2,.....,n-1\} \quad \text{with} \quad m \geq 0, k \in N \cup \{-1\}$$

where m represents the spline degree and k the smoothness class of spline function`s space .

If $k = m$ then function $s \in S_m^m(\Delta)$ are polynomials.

For $m = 1$ and $k = 0$ we obtain linear splines.

The division points of the interval $[a,b]$ are called nods, and in each nod we know the value of the function $f(x_i)$ form $y_i = f(x_i)$;

If we note $s(x_i)$ the spline function with wich approximate the function $f(x_i)$ on the interval $[a,b]$ then we have to identify the form of the function $s(x_i) = f(x_i)$ wich take values $(x_1, x_2, ...x_{n-1}, x_n)$.

In the interpolation approximation we suppose that (x_i, y_i) are known. If due to various reasons the nods are affected by errors then the interpolation criterion is inadequate.(11)

With other words spline function $s(x_i)$ approximate the function $f(x_i)$ on the $[x_i, x_{i-1}]$ interval with the following conditions:

1. the restrictions of spline functions on the subintervals $[x_i, x_{i-1}]$ are polynomial of m degree, $s| \, [x_i, x_{i-1}] = p_{m,i}$;

2. $s(x_i)$ is derivable (m-1) times on the interval $[a,b]$;

3. When connecting the nods, each nod is conditioned by:

$p_{m,i}^{(k)}(xi+1) = p_{m,i+1}^{(k)}(xi+1), k = 0, 1, \ldots, m-1,$(1)

wich means that on the x_{i+1} frontier between the two subintervals, the polynomial from left side of $p_{m,i}$ and its first *m*-1 derivatives must have the same values with the polynomial from the right side of, $p_{m,i+1}$. Outside the interval [a,b] the function $s(x_i)$ it can extend throughout polynomial of degree$\leq m$.(12)

The three cumulative conditions defines the spline function $s:[a,b] \to R$ wich is known in the scientific literature as polynomial spline function of m degree.(13)

In these conditions the polynomial spline functions can substantial contribute in the economic field in interpolating problems but also in approximation of functions wich describes the economic processes and phenomena with direct influence in economic variables predictibility and identifying the financial risk helping the managers to administrate the resources in the public sector.(14)

In the scientific literature there are three categories of polynomial spline functions known, 1st degree, 2nd degree and 3rd degree.

The **polynomial spline functions of first degree** have the following expression.

$$S(x) = S_i(x) = s_{i,0} + s_{i,1}(x - x_i), x \in [x_{i-1}, x_i] \ (2)$$

with the coefficients $s_{i,k}$ wich are determined throughout the condition:

$$S(x_i) = y_i, i = \overline{0,n}; \ (3) \ \square\text{i} \ S_i(x_i) = S_{i+1}(x_{i+1}), i = \overline{1,n-2}; (4)$$

The spline functions of first degree, also applied in this paper approximate with remarcable results, series of data with linear distribution and increasing evolution.

The coefficients of polynomial functions are determined from the two specific conditions of spline functions namely that value of the spline function in nods is equal with the value of the function $y_i = f(x_i)$ and from the continuity condition of spline function in each ond of the $[x_i, x_{i-1}]$ interval;

The **second degree polynomial spline** function use also in problems of interpolation and approximation have the expression:

$$S(x) = S_i(x) = s_{i,0} + s_{i,1}(x - x_i) + s_{i,2}(x - x_i)^2, x \in [x_{i-1}, x_i] \ (5)$$

with the coefficients determined from the conditions:

$$S(x_i) = y_i, i = \overline{0,n};$$

$$S_i(x_i) = S_{i+1}(x_i), i = \overline{1,n-1}; (6)$$

$$S_i^I(x_i) = S_{i+1}^I)x_i, i = \overline{1,n-1}; (7) \ ; \ S_0^I(x_0) = 0; (8) \ (15)$$

The polynomial functions of second degree is introducing two suplimentary conditions in order to find the coefficients namely, the condition for the first derivatives in the continuity points of spline functions and the local extremum condition for one of the extremities of second degree spline wich usually is chosen in x_0.

The **polynomial spline function of third order** has the expression:

$$S(x) = S(x_i) = s_{i,o} + s_{i,1}(x - x_i) + s_{i,2}(x - x_i)^2 + s_{i,3}(x - x_i)^3, x \in [x_i, x_{i+1}] (9)$$

with the following conditions needed to determine the coefficients:

$$S(x_i) = y_i, i = \overline{0,n};$$

$$S_i(x_i) = S_{i+1}(x_i), i = \overline{1, n-1};$$
$$S_i^I(x_i) = S_{i+1}^I(x_i), i = \overline{1, n-1};$$
$$S_i^{II}(x_i) = S_{i+1}^{II}, i = \overline{1, n-1}; (10)$$
$$S_i^{II}(x_0) = S_{n-1}^{II}(x_{n-1}) = 0; \text{ or } S_1^I(x_0) = y_0^I, S_{n-1}^I(x_{n-1}) = y_{n-1}^I$$

Spline functions of third-order polynomial best fit the functions whose graphical representation is sine wave.

Additional conditions also refers to local conditions for two extreme ends of the second degree polynomial spline function.

The specialized literature has established and developed series of Spline functions along with the polynomial, as outlined above, with different applications especially in technical sciences. In the category of spline functions used for interpolation we know the cubic splines, the spline functions with Hermite interpolation on intervals, cublic splines of class C^2 etc.(16)

II.3. Spline functions mechanism in the public finance

The mechanism resumes to present the algorithm of application of polynomial spline functions in public finance. This mechanism was applied on series of data at a local budget level, in comparison with other tehniques studied in the econometric field such as linear regression.

These steps are presented below for the polynomial spline function of first order.

Step I: Establishing the nods $x_i; i = \overline{1, n}$; Usually the nods are established randomly, equidistant, for every determined interval $[x_i; x_{i+1}]$ spline functions to achieve accurate approximation;
Step II: For each interval $[x_i; x_{i+1}]$ we establish the form of the splin function $S_i(x_i)$ usin the expressions described above for the polynomial spline functions of first order;
Step III: Determination of coefficients $s_{i,k}$ from the spline function conditions, to take value in the delimitated nods in Step 1and from the continuity conditions on the interval $[x_i; x_{i+1}]$;
Step IV: Establishing the form of the spline functions from regarding the coefficients determined for each interval and verifying the error of approximation using the relation:
$e_x = f(x_i) - f(s_i)$ (16)
Step V: Establishing the spline function used for prediction by calculating the average of the spline functions for each interval $[x_i; x_{i+1}]$ wich they can estimate the values of function $f(x_i) = y_i$ when x_i is known for the future period.

Tabel no.1 – The algorithm of applying spline functions for series of data registered in economic process and phenomena analysis

The steps mentioned above are not limitative. They describe first-order polynomial spline functions adapted to the needs of economic researchers to estimate and study the predictability of economic phenomena and processes.

II.4. Using possibilities of polynomial spline functions of first-order in tax policies substantiation of local authorities

Nowadays tax and budgetary policies are very important for local governments wich are aware of the importance of financial resources.

Budget deficits, public loans, public services, all of them are depending on tax policies. Unfortunately the tax policy is based almost entirely on the property tax wich proved to be insufficient to satisfy the public needs of local communities. Although apparently were part of fiscal policies, shared taxes, are not in direct "coordination" of local authorities. This is because the shared taxes are collected by the central authorities after which the shares are allocated to local communities.

Based on these consideration we tried to realise a group of fiscal policies based on their role in stimulating the economic development. Thes attempts led to dividing the fiscal policies into the following category, namely:

Agressive fiscal policies: wich are characterized by the fact that the tax rate and tax base are established at extreme values, imposed by law so they don`t have a stimulative effect for the bussines environment neither to economic development.

Stimulative fiscal policies: are policies wich are characterized by the fact that the tax rate and tax base plays a great role in the bussines environment stimulation. In this sense the local authorities provides tax incentives depending on the investment values and number of jobs created in order to stimulate the investments in the region.

Neutral fiscal policies: are those policies wich have on their base the property tax but having values that doesn`t change the taxpayer`s behaviour.

Also budgetary policies includes the expenditure policy administration wich are based on cutting the operational expenditures or based on supporting the long-term investments wich leads to the local economic development.

Regardless of the theory of fiscal policy budget how spline functions are used in the substantiation of the fiscal policy? The answer is not simple. It can be formulated on two different directions, namely: by using the $r_i \times B_i$ method or predicting the budgetary revenues collected on the entire budget.

The first way, which is used at substantiation of property tax revenue it has to be known the base for a period of time, minimum one year in order to establish the future collected tax.

Polynomial spline functions of first-order will have the expression:

$S(B_i) = s_{1,0} + s_{1,1}(B_i - B_{ii}), B_i \in [B_i, B_{i+1}]$ with the conditions imposed for the coefficients $(s_{i,k})$ of spline functions regarding the equality of the values in nods and the continuity condition through nods $S_i(B_{ii}) = S_{i+1}(B_{ii})$.(17)

This way of using spline functions determinates the substantiation of fiscal policies depending on tax base. That is way local authorities have to identify a benchmark for the collected revenues from the property tax , and have to identify ways to increase the tax base at the a certain level proposed to realise.

The second direction , within fiscal policy of local authorities aims to predict the collected taxes from property when there is registrated certain values of building tax. In this way using a polynomial spline function of first-order allows to establish certain property tax when building tax projection is known.

Regardless of the way spline functions are used, they permit approximating the evolution of tax property or other categories of revenues with a greater accuracy than other means known in the economic field (ie. econometry).

III. The research results obtained by applying functions in the substantiation of fiscal-budgetary policies

III.1. Research methodology

In order to realise the research there have been used as methods of research several steps, namely observation, series of data construction and processing the data with spline functions. The data construction is a daily series gathered from the budget execution, reffering to building tax and property tax, and in the end the process of data was made according to the methodology described at **2.3**.

III.2. Research results obtained through series of data processing using polynomial splin functions of first-order

To study the use of spline functions were considered two types sets of data from the budget execution for the year 2009 of Oradea Municipality (due to its easy acces at the database), respectively the **property tax** and **building tax indicators** registered in **February** 2009 and verified the appliance of spline functions in order to size the revenues for next month (**March**). The reason why past series were chosen is to verify the accuracy of using spline functions in extrapolations facinging them with the real data registred in March 2009 for property tax.

From the given series of February were determined the polynomial spline functions of first-order wich is relevant for the evolution of tax property depending on building tax.

There were identified four nods, established at a range of seven days and were determined four polynomial spline functions of first-order for each week of February. Each spline is related to a week, taking equidistant range between the nods. Also were established the function wich describes the series throug the **ordinary least squares (OLS**) method in order to compare the results of research using the extrapolation for OLS and for polynomial spline function of first-order.

In order to verify the accuracy of extrapolation using spline functions were considered three cases to reflect de difference between the function determined with OLS and polynomial spline functions of first-order.

Case I

Last spline function on February has been considered $S_4(x_i)$.

Y= 1.29872X+123675

Case II

We consider the average of four spline functions resulted for each week

Y=1,316065X+10685

Case III

The function resulted from linear regression has been determined (Ordinary Least Square method)

Y=1,28617X+154722

In order to underline the efficiency of these 3 methods, throughout Gretl software, we have compared the extrapolated values for the property tax with the real values of property tax for March.

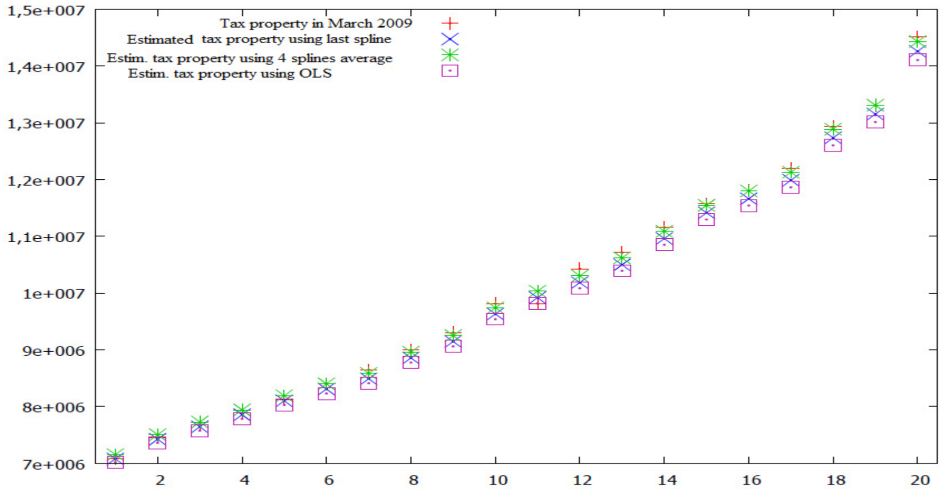


Fig.no.1 Comparison between the expected values fo March 2009 and real values,
Source: Budgetary Account Oradea 2009

We can notice that approximating the revenues from property tax depending on building tax for March 2009 was realised with the greates accuracy by the average of 4 spline functions resulted from the February 2009 series.
Also the function resulted using OLS method proved to be less efficient to describe the trend for the next period of time.
To underline the efficiency of these three methods were calculated the errors.

$$\varepsilon Y = Y - \hat{Y} \text{ , (11)}$$

where: Y – real property tax, ε – error, $\hat{Y}$ - estimated property tax;

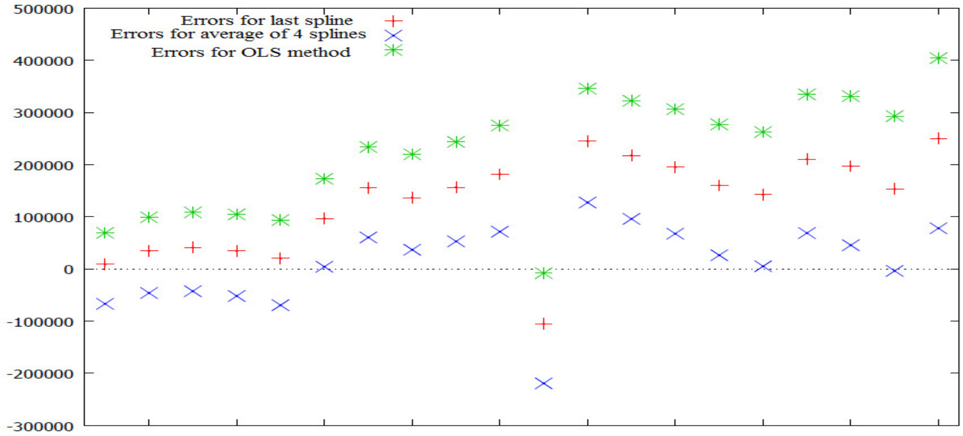
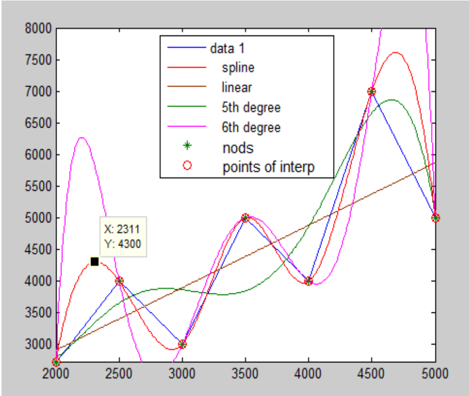


Fig.no.2 Resulted errors from extrapolation
Source: Budgetary Account Oradea 2009

We can notice from Fig.2 that the smallest errors are given by the average of 4 splines method, the evolution of errors beeing close to zero. Also for the first week from March the better extrapolation is done by the last spline resulted from the last week of February, though the average of four splines proves to be the most accurate method.
The linear regression using OLS proves to be less efficient, the errors achieving high values. In order to underline the utility of spline functions we have built 2 series of data. By using MATLAB programming language we have generated the function wich approximate the

distribution of variables fixing equidistant nods and points of interpolation in the nods for a better aprproximation.
We observe in Fig.no.3 the fact that spline functions is approximating the most efficient way in comparison with the regular polynomials. The polynomial of 6th order is decently approximating but in the end spline function is most accurate. The secret of splines is using polynomials of 3rd order on the intervals (2000-2500,2500-3000, … , 4500-5000) known as cubic splines.



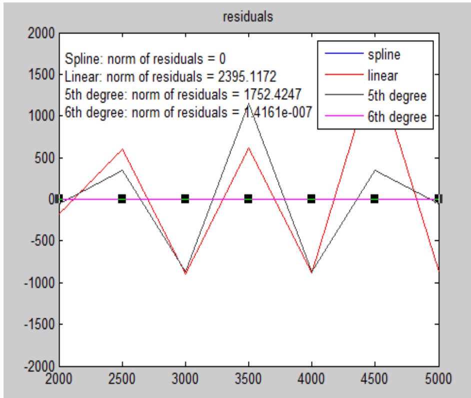


Fig.no.3 Comparison between spline functions and regular polynomials Source: author

Fig.no.4 Error comparison between splines and regular polynomials Source: author

Also in the fig.4 we can observe the residuals beeing insignifiant for the 6th order polynomial and even zero for spline functions.

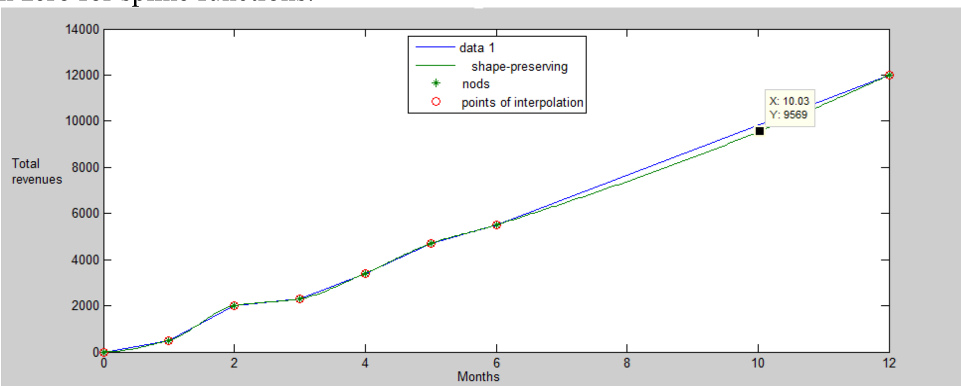


Fig.5 Monitoring the budget execution Source: author

As well, monitoring and controlling the budget execution can be made using polynomial spline functions by fixing a nod at the end of a year. We know the values of revenues collected in the first 6 months and we can compare the revenues in month10 with the collected revenues in order to see de difference between the estimated with the collected revenues. This method helps the public manager to size the expenditures for the next period in order to avoid budget deficits.

IV Conclusions

As a result of simulations made on real and theoretical data the opportunity of using polynomial spline functions as innovative instruments in sizing financial resources as well in other economic fields.
The accuracy of results obtained in the process of budget sizing, due to small errors of approximation, recommends spline functions as modern instruments in sizing the resources of budget.

Therefore, spline functions can be used for public **revenues predictibility**, for finding the **trend of expenditures** in accordance to the **Case II** presented earlier also **tracking deviations recorded in the budgetary execution** of the planned budget. The two approaches of using spline functions increase the efficiency of public management. Using spline functions can result in avoiding incontrollable budget deficits and treasury deficits wich have an impact on taxpayers who will support the cost of public loans.

Using splines in the process of budget elaboration may lead the managers to be more aware of budget deficits and can take decisions according to the situation.

Regarding the tax policies of local authorities, by using splines can be established the tax policy wich has to be adopted when are establishing the revenues from property tax.

The polynomial spline functions has the advantage of easy calculation, free of specific complex tests used in linear regression.

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