

Università degli Studi di Salerno  
Centro di Economia del Lavoro e di Politica Economica

Lisa Crosato\*, Sergio Destefanis\*\*, Piero Ganugi\*  
\*DIPARTIMENTO DI SCIENZE ECONOMICHE E SOCIALI,  
UNIVERSITÀ CATTOLICA DEL SACRO CUORE DI PIACENZA  
\*\*CELPE AND CSEF, UNIVERSITÀ DI SALERNO

Technology and Firm Size Distribution:  
Evidence from Italian Manufacturing

Corresponding author:  
lisa.crosato@unicatt.it;

Discussion Paper 102

**Scientific Committee:**

Adalgiso Amendola, Floro Ernesto Caroleo, Cesare Imbriani, Pasquale Persico, Salvatore Vinci

C.E.L.P.E.

Centro di Ricerca Interdipartimentale di Economia del Lavoro e di Politica Economica

Università degli Studi di Salerno

Via Ponte Don Mellillo, 84084 Fisciano, I - Italy

<http://www.celpe.unisa.it>

E-mail: [celpe@unisa.it](mailto:celpe@unisa.it)

## **Indice**

|   |    |
|---|----|
| <i>Abstract</i> .....   | 4  |
| <i>Introduction</i> .....   | 5  |
| <i>1 The Dataset</i> .....  | 6  |
| <i>2 Technology, Efficiency, Scale Elasticity: The FDH Approach</i> ..... | 8  |
| <i>3 Technical Efficiency: The Empirical Evidence</i> .....               | 10 |
| <i>4 Scale Elasticity: The Empirical Evidence</i> .....                   | 14 |
| <i>5 The Zipf Plot of the Pareto IV Distribution</i> .....                | 19 |
| <i>6 Zipf Plot Concavity/Linearity and Returns to Scale</i> .....         | 20 |
| <i>Concluding Remarks</i> .....   | 25 |
| <i>References</i> .....   | 26 |

# Technology and Firm Size Distribution: Evidence from Italian Manufacturing<sup>1</sup>

Lisa Crosato\*, Sergio Destefanis\*\*, Piero Ganugi\*  
\*DIPARTIMENTO DI SCIENZE ECONOMICHE E SOCIALI,  
UNIVERSITÀ CATTOLICA DEL SACRO CUORE DI PIACENZA  
\*\*CELPE AND CSEF, UNIVERSITÀ DI SALERNO

## Abstract

This paper explores the relationship between firm size distribution and technology. Similarly to Crosato and Ganugi (2006), we focus on six industries from the Micro1 survey by the Italian Statistical National Office (ISTAT). Firm technology is analysed across selected industries by means of a non-parametric production analysis, the Free Disposal Hull approach (Deprins et al., 1984; Kerstens and Vanden Eeckaut, 1999). The existence of a link between technical efficiency and size on the one hand, and between scale elasticity and size on the other is investigated. Graphical analyses show the absence of a clear-cut relation in the first case, while an inverse relation is found in the second one. Building on this relation, we inquire whether the shape of the firm size-distribution is related to a particular pattern of returns to scale. This problem is studied through the Zipf Plot (Stanley et al., 1995) of the Pareto IV distribution, which is concave for firms up to a given threshold, and then becomes linear. Results show that firms in the concave part of the plot experience increasing returns to scale. On the contrary, firms in the linear part are mainly characterised by constant returns to scale.

**JEL classification:** L11, L6, D20, C14.

**Keywords:** Italian manufacturing, Free Disposal Hull, Pareto distributions, returns to scale.

---

<sup>1</sup> The present work is part of a more general research project: "Industry evolution: innovation, profitability and firm's growth", conducted within the Department of Economic and Social Sciences of the Università Cattolica del Sacro Cuore (UCSC), Piacenza, coordinated by Professor Maurizio Baussola in cooperation with ISTAT (Italian Statistical Office, regional office for Lombardy). Destefanis thanks the MIUR for financial support. We thank Francesco Addesa for skilful research assistance and Marco Vivarelli for perceptive comments on a previous draft. The usual disclaimer applies.

## Introduction

The basic goal of the present paper is to provide empirical evidence on the relationship between the shape of firm size distribution and firms' returns to scale. Similarly to Crosato and Ganugi (2006), we focus on six industries from the Micro1 survey by the Italian Statistical National Office (ISTAT), but we move a step forward to analyse firm technology across industries by means of a non-parametric production analysis. More precisely, this paper is based on the Free Disposal Hull approach first proposed by Deprins et al. (1984), which imposes very little a priori structure on the pattern of the returns to scale.

Ever since the seminal works of Pareto (1897) and Gibrat (1931) the existence of a recurrent model of firm size distribution has been investigated in the statistical as well as in the economic literature (for an exhaustive survey see Kleiber and Kotz, 2003). In early as much as in recent literature firm size has often been modelled by means of the Lognormal and Pareto I distributions (Hart and Prais, 1956; Steindl, 1965; Quandt 1966; Simon et al., 1955, 1977; Stanley et al., 1995; Okuyama et al., 1999; Axtell, 2001, Ganugi et al. 2003, 2005). Both distributions were shown to be the outcome of stochastic models of growth based on the Law of Proportionate Effect, postulating no effect of size on percentage growth rates (Gibrat, 1931). Nonetheless, the empirical fit of these distributions is not always so satisfactory as to warrant inference about firm-size growth dynamics from firm-size distributional properties (e.g. inferring Gibrat's law from Lognormality). Indeed the Lognormal generally fails to account for the right tail of firm sizes (Stanley et al., 1995; Hart and Oulton, 1997), while the Pareto I has problems in fitting their left tail.

Departures from the Pareto I and Lognormal distributions have been interpreted as deviations from Gibrat's law and therefore as instances of different regimes of growth. Crosato and Ganugi (2006) model these departures through a very flexible model, the Pareto IV distribution (Arnold, 1983), taking the hint from the literature on income distribution (Singh and Maddala, 1975; Dagum, 1977; Stoppa, 1990). The favourable evidence for the Pareto IV both at the aggregate and at the sectoral level in Italian manufacturing sheds doubt on the relevance of the law of proportionate effect in this ambit. Only the size distribution of larger firms can be successfully fitted through a Pareto I distribution, as the Pareto IV right tail conforms to the Pareto I's. This aspect is made particularly clear by the Zipf Plot (see Stanley et al., 1995, for an introduction to this tool), a double log scale plot providing an estimate of any firm's size under the null that the size is distributed according to a specific distribution. The Pareto I is characterised by a linear Zipf Plot, while only the right-hand tail of the Pareto IV is a straight line, providing evidence on the convergence of the two distributions.

Generally speaking, it is expected in the literature that firms lying in the linear part of the Zipf Plot experience constant returns to scale, while no clear-cut opinion is provided on the nature of returns to scale characterising firms in non-linear parts of the plot (Simon and Bonini, 1958; Ijiri and Simon, 1974; Vining, 1976; Lucas, 1978). Hence, we investigate here if large firms experience favourable conditions to grow in a proportionate fashion, discussing this way

Gibrat's law, and at the same time we explore the possibility that non-linear parts of the Zipf plot are systematically characterised by non-constant returns to scale.

The paper is divided in five sections. In section 1 we briefly present the main features of our dataset. In section 2 we describe our non-parametric approach to the measurement of technical efficiency and scale elasticity. In section 3, we investigate the empirical relationship between firm size and technical efficiency, while in section 4 we deal with scale elasticity. We find that elasticities of scale are sizably lower for large firms in all industries. In section 5 we illustrate the Zipf Plot of the Pareto IV distribution. In section 6 we determine the threshold above which firms conform to a Pareto I distribution, and find that firms display different returns to scale regimes below and above this cut-off point. We finally provide some final remarks and describe further directions of research.

## 1. The Dataset

In this paper we focus on six industries selected from the Micro1 survey, assembled by ISTAT through the matching of the Structural Business Statistics Survey (SCI) and the Community Innovation Survey (CIS<sup>2</sup>). The original dataset is composed of 5445 firms, followed from 1989 to 1997, with at least 20 employees and was extensively analysed in Crosato and Ganugi (2006), who also discuss the characteristics of the dataset in relation to Gibrat's law (about Italian manufacturing in the same period, see also Bottazzi et al., 2003, 2005).

Micro1 is a closed panel and hence exposed in principle to survivor bias (Mansfield, 1962; Jovanovic, 1982). According to the literature (Evans, 1987; Hall, 1987; Dunne and Hughes 1994), this bias should be considered and corrected in order to yield to a correct evaluation of the Law's validity. The survivor bias, regarding in particular the left-hand tail of the distribution, where entry-exit dynamics are more intense, can affect the average growth rates of small firms upwards, leading to unfounded conclusions against Gibrat's law. Small firms with low growth rates are more likely to exit the industry with respect to large firms, which can reduce in size but still survive, for a longer time period at least. Our analysis, however, should not suffer heavily from this bias, because the dataset excludes a priori firms with less than 20 employees. Besides, the main aim of the paper is not to test whether Gibrat's law applies to the whole set of firms. Rather, we want to ascertain whether firms situated in the size range conforming to a Pareto I distribution, and thus yielding evidence in favour of the Law from the distributional point of view, also provide evidence consistent with the Law from the standpoint of (constant) returns to scale. As will be apparent from the evidence shown below, this size range is situated well above the 20-employee threshold. Subsequently, it is unlikely that our conclusions are influenced by survivor bias, which is likely to hit more the left tail of the size distribution.

An important, and relatively uncommon, feature of this dataset is that it includes data not only on the number of employees, but also on the total number of work hours. This is likely to reduce an important source of bias in the

---

<sup>2</sup> In this paper we do not use the Innovation Survey.

measurement of returns to scale, as work hours track more closely than employees the cyclical evolution of value added. In any case, the dataset also allows consideration of an alternative specification of the labour input, splitting it into blue and white collars.

The industries we have chosen to analyse will be often denoted according to their 2-digit ATECO '91 (ISIC Rev.3) code. They are:

- Food products and beverages (DA 15);
- Textile (DB 17);
- Chemicals and chemical products (DG 24);
- Non-metallic mineral products (DI 26);
- Fabricated metal products except machinery and equipment (DJ 28);
- Machinery and equipment (DK 29).

| year | sectors |      |      |      |      |      |      |      |      |      |      |
|------|---------|------|------|------|------|------|------|------|------|------|------|
|      | DA15    | DB17 | DB18 | DG24 | DG25 | DI26 | DJ27 | DJ28 | DK29 | DL31 | DN36 |
| 1989 | 389     | 537  | 221  | 275  | 337  | 429  | 224  | 632  | 727  | 218  | 413  |
| 1990 | 390     | 538  | 220  | 275  | 344  | 430  | 219  | 631  | 729  | 216  | 410  |
| 1991 | 389     | 532  | 223  | 275  | 341  | 428  | 223  | 635  | 731  | 215  | 414  |
| 1992 | 391     | 522  | 228  | 259  | 317  | 429  | 215  | 615  | 742  | 214  | 421  |
| 1993 | 391     | 523  | 223  | 256  | 326  | 429  | 219  | 613  | 741  | 214  | 425  |
| 1994 | 391     | 523  | 225  | 256  | 321  | 428  | 215  | 611  | 749  | 214  | 425  |
| 1995 | 391     | 525  | 224  | 259  | 318  | 427  | 216  | 603  | 754  | 211  | 427  |
| 1996 | 391     | 526  | 224  | 262  | 314  | 428  | 215  | 605  | 754  | 212  | 426  |
| 1997 | 391     | 526  | 226  | 263  | 317  | 429  | 215  | 601  | 752  | 213  | 426  |

*Table 1: Number of firms by sector*

These sectors have been chosen through two different steps. First (Table 1) we have singled out eleven sectors with at least 200 firms in each year to assure reliable statistical estimates (see § 3).

Second, we concentrated on six out of these eleven sectors according to their importance, in terms of value added, with respect to the whole of Manufacturing. As can be seen from Table 2, sectors DA15, DB17, DG24, DI 26 and DK29 are on top of the value added ranking over the whole period. The three sectors which occupied alternatively the sixth place were DJ27, DJ28 and DL31. We retain DJ28 because it contains almost three times the firms of the other two.

Following Crosato and Ganugi (2006) we will consider total assets as proxy of firm size because of the lack of volatility of this variable through time.

| year | sectors |      |      |       |      |      |      |      |       |      |      |
|------|---------|------|------|-------|------|------|------|------|-------|------|------|
|      | DA15    | DB17 | DB18 | DG24  | DG25 | DI26 | DJ27 | DJ28 | DK29  | DL31 | DN36 |
| 1989 | 8.45    | 5.52 | 1.49 | 9.15  | 4.38 | 5.60 | 5.23 | 4.08 | 12.17 | 4.69 | 2.23 |
| 1990 | 8.90    | 5.70 | 1.45 | 10.13 | 4.29 | 5.65 | 4.76 | 4.21 | 11.87 | 5.06 | 2.28 |
| 1991 | 9.02    | 5.35 | 1.52 | 10.22 | 4.31 | 5.67 | 4.26 | 4.16 | 11.60 | 5.31 | 2.33 |
| 1992 | 9.73    | 4.99 | 1.91 | 10.54 | 4.09 | 5.90 | 4.08 | 4.14 | 12.29 | 5.21 | 2.40 |
| 1993 | 10.09   | 5.06 | 1.94 | 10.91 | 4.36 | 5.88 | 4.28 | 4.28 | 13.42 | 5.16 | 2.58 |
| 1994 | 9.11    | 5.22 | 1.82 | 10.96 | 4.36 | 5.79 | 4.58 | 4.30 | 13.76 | 4.67 | 2.50 |
| 1995 | 8.63    | 5.22 | 1.82 | 10.98 | 4.48 | 5.69 | 5.16 | 4.61 | 13.73 | 4.48 | 2.44 |
| 1996 | 9.39    | 5.06 | 1.82 | 11.45 | 4.66 | 5.69 | 4.68 | 4.88 | 14.44 | 4.23 | 2.49 |
| 1997 | 8.97    | 5.08 | 1.73 | 10.74 | 4.48 | 5.58 | 4.64 | 4.69 | 14.36 | 4.58 | 2.43 |

Table 2: Percentage share of value added by sector with respect to the whole Manufacturing

## 2. Technology, Efficiency, Scale Elasticity: The FDH Approach

Non-parametric methods provide estimates of the upper boundary of a production set (the so-called production frontier) without supposing the existence of a functional relationship between inputs and outputs (Farrell, 1957; Fried et al., 1993). The frontier is supported by some of the observed producers, which are defined efficient. Non-parametric methods are divided between those that impose upon the production set the hypothesis of convexity (usually gathered under the label of Data Envelopment Analysis, or DEA) and those that do not need this assumption (the Free Disposal Hull - FDH - approach proposed in Deprins, *et al.*, 1984, Tulkens, 1993). In the latter case, the only property imposed on the production set is strong input and output disposability, while in DEA the additional hypothesis of convexity is made. More formally, in FDH, for a given set of producers  $Y_0$ , the reference set  $Y(Y_0)$  is characterised, in terms of an observation  $i$ , by the following postulate:

$$(X^i, Y^i) \text{ observed, } (X^i + a, Y^i - b) \in Y(Y_0), \quad a, b \geq 0$$

where  $a$  and  $b$  are vectors of free disposal of input and output, respectively. In other words, due to the possibility of strong input and output disposability, the reference set includes all the producers which are using the same or more inputs and which are producing the same or less output in relation to observation  $i$ .

Let us take as an example Fig.1, where we consider a technology with one input ( $X$ ) and one output ( $Y$ ). The input-output pairs correspond to producers examined at a given point in time. Beginning with observation B, we define every



observation located at its right and/or below it (that is with more input and same output, or with less output and same input; or else with more input and less output, as F) as dominated by B.

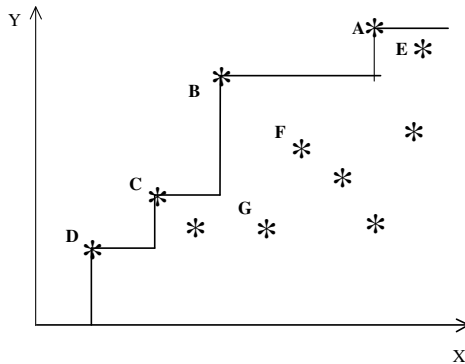


Figure 1: An FDH Production Frontier

In FDH, this comparison is carried out for every observation, and observations not dominated by any other observation are considered efficient producers, belonging to the frontier of the reference set: On the other hand, the observations that are dominated are considered inefficient. In DEA, on the other hand, the frontier of the overall reference set is found by constructing a convex envelope around the production set; this implies the assumption not only of free input and output disposal, but also of convexity. Hence, the DEA frontier must exhibit by construction non-decreasing returns to scale for relatively smaller observations, and non-increasing returns for relatively larger observations. This is not true in FDH and is of crucial importance for the present research.

One problem with FDH is that possibly many observations may be efficient because they are located in an area of the production set where there are no other observations with which they can be compared (*efficiency by default*). To circumvent it, we use a refinement of traditional FDH, the VP-FDH (variable-parameter FDH) proposed by Kerstens and Vanden-Eeckaut (1999), which decisively reduces the problem of efficiency by default. VP-FDH is defined as the intersection of FDH technologies that impose by assumption non-decreasing and non-increasing returns to scale. First, each observation is compared not only to any other observation but also to their smaller or larger proportional replicas; then, one selects for each given observation the assumption about returns to scale that yields the highest efficiency score. While still relaxing the hypothesis of convexity (meaning that the nature of returns to scale is not restricted a priori), VP-FDH imposes more structure on the production set than traditional FDH, greatly increasing the scope for comparisons between observations, and reducing correspondingly the problem of efficiency by default.

In all non-parametric methods the distance of a producer from the frontier provides its measure of technical inefficiency, or, for short, its efficiency score. Typically, the (output-oriented or input-oriented) measure of Debreu-Farrell is used. If the measure is output-oriented, technical inefficiency is equal to the complement to one of the maximum output expansion consistent with the

utilisation of a given input. A producer which is technically efficient (and which is therefore on the frontier of reference) will not be able to attain such an expansion of output, achieving an efficiency score equal to one. If the measure of Debreu-Farrell is input-oriented, it is given by the complement to one of the maximum input reduction which allows to keep up the production of a given output. If the technologies considered have more than one output or one input, then the two measures of Debreu-Farrell are equal to, respectively, the complement to one of the maximum equiproportional expansion of all the outputs consistent with using a given vector of inputs, and to the complement to one of the maximum equiproportional reduction of all inputs that allows to keep up the production of a given vector of outputs.

There are various methods in non-parametric frontier analysis to assess the nature of returns to scale on the frontier point relevant for any given producer (see the discussions in Førsund, 1996, or in Kerstens and Vanden Eeckaut, 1999). In a qualitative sense, one must ascertain whether the frontier point relevant for an inefficient producer according to the variable-returns-to-scale technology must be scaled up or down to obtain the frontier point relevant for an inefficient producer according to the constant-returns-to-scale technology. In the first case, the frontier exhibits increasing returns to scale, while the contrary holds true in the opposite case. If the two frontier points coincide, the frontier exhibits constant returns to scale. There exist also some procedures that allow the derivation of quantitative measure of returns to scale from non-parametric frontier analysis. A simple and attractive measure, derived from Frisch's Beam variation equations, is the ratio between the natural log of the output-increasing efficiency score and the natural log of the input-decreasing efficiency score (Førsund and Hjalmarsson, 1979; Førsund, 1996). This ratio is only an average measure and is determined by the (generally non-measurable) magnitude of returns to scale in the two frontier points relevant for the producer taken into consideration. Hence it exists only for given data intervals (not for given points), and only for inefficient producers.

It should be kept in mind that a major problem of small-sample bias arises when non-parametric frontier approaches are used (Kneip et al., 1998; Gijbels et al., 1999; Kittelsen, 1999). As reported in Kittelsen (1999) these approaches begin to be characterised by substantial biases for sample sizes around 100 to 150 observations. This suggested to restrict empirical analysis to sectors well exceeding the 100 to 150 observations (per year) mark.

### 3. Technical Efficiency: The Empirical Evidence

We now proceed to calculate technical efficiency scores, through both traditional and VP-FDH, separately for each industry and year. The scores are subsequently used to compute the measure of scale elasticity proposed in Førsund and Hjalmarsson (1979). We rely on two different production sets. In the first one the output is value added, while number of work hours and the book value of fixed assets are the inputs. Value added is the output also in the second production set, but inputs include, along with the book value of fixed assets, the number of blue-collar employees and the number of white-collar employees.

Indeed the empirical literature on Italian manufacturing firms suggests that employees should be divided between blue-collar and white-collar, in order to take more satisfactory account of the quality of labour inputs. In such a case, however, it is not possible to take into account the hours worked by each category of employees.

Below we present our main results and provide some comments about them. Some remarks about the notation adopted. Capital letters H and W stand for the results obtained respectively with the first (work hours) and the second (blue- and white collars) production set. Capital letters I and O stand for input- and output-oriented technical efficiency scores. Finally, the VP-FDH results are denoted by VP.

Some summary statistics of the technical efficiency scores are provided in Table 3. For the sake of brevity, we pool together all years in presenting these results. We obtain efficiency scores with reasonably high mean and median values, and acceptably low dispersion. These results point to a satisfactory specification of the production set and lend robustness to our subsequent analysis.

Trough Figures 2 and 3, we do not find any kind of systematic relationship between the efficiency scores and the firm size. This is what one expects from the empirical literature, and is consistent with the model proposed in Jovanovic (1982), where some firms are more efficient than others at all levels of output. Inefficient firms decline and abandon the industry while efficient firms grow and survive. At the same time efficiency levels can vary throughout the output range in absence of any specific trend. Note that we should not expect particularly low efficiency levels in our dataset, composed by firms having survived for nine years which hence belong, at least for the period in analysis, to the efficient and able-to-grow bunch.

|      | min      | 1 <sup>st</sup> qu | med    | mean   | 3 <sup>rd</sup> qu | max | min      | 1 <sup>st</sup> qu | med   | mean  | 3 <sup>rd</sup> qu | max |
|------|----------|--------------------|--------|--------|--------------------|-----|----------|--------------------|-------|-------|--------------------|-----|
|      | TE-H-I   |                    |        |        |                    |     | TE-H-O   |                    |       |       |                    |     |
| DA15 | 0,085    | 0,500              | 0,655  | 0,662  | 0,823              | 1   | 0,017    | 0,438              | 0,588 | 0,608 | 0,766              | 1   |
| DB17 | 0,100    | 0,450              | 0,586  | 0,606  | 0,746              | 1   | 0,031    | 0,403              | 0,529 | 0,559 | 0,691              | 1   |
| DG24 | 0,100    | 0,599              | 0,750  | 0,740  | 0,911              | 1   | 0,030    | 0,550              | 0,719 | 0,711 | 0,905              | 1   |
| DI26 | 0,124    | 0,596              | 0,733  | 0,730  | 0,875              | 1   | 0,022    | 0,522              | 0,675 | 0,675 | 0,844              | 1   |
| DJ28 | 0,114    | 0,527              | 0,653  | 0,663  | 0,792              | 1   | 0,083    | 0,470              | 0,608 | 0,621 | 0,767              | 1   |
| DK29 | 0,056    | 0,473              | 0,605  | 0,624  | 0,762              | 1   | 0,026    | 0,433              | 0,566 | 0,590 | 0,734              | 1   |
|      | TEVP-H-I |                    |        |        |                    |     | TEVP-H-O |                    |       |       |                    |     |
| DA15 | 0,085    | 0,414              | 0,541  | 0,554  | 0,688              | 1   | 0,013    | 0,332              | 0,436 | 0,463 | 0,571              | 1   |
| DB17 | 0,091    | 0,388              | 0,507  | 0,532  | 0,667              | 1   | 0,019    | 0,328              | 0,433 | 0,464 | 0,567              | 1   |
| DG24 | 0,100    | 0,481              | 0,592  | 0,603  | 0,731              | 1   | 0,028    | 0,412              | 0,530 | 0,545 | 0,668              | 1   |
| DI26 | 0,124    | 0,521              | 0,644  | 0,645  | 0,774              | 1   | 0,017    | 0,429              | 0,554 | 0,564 | 0,691              | 1   |
| DJ28 | 0,114    | 0,471              | 0,578  | 0,590  | 0,705              | 1   | 0,072    | 0,410              | 0,514 | 0,530 | 0,637              | 1   |
| DK29 | 0,056    | 0,397              | 0,498  | 0,525  | 0,634              | 1   | 0,023    | 0,339              | 0,426 | 0,453 | 0,543              | 1   |
|      | TE-W-I   |                    |        |        |                    |     | TE-W-O   |                    |       |       |                    |     |
| DA15 | 0,09177  | 0,6129             | 0,7869 | 0,754  | 0,924              | 1   | 0,017    | 0,499              | 0,678 | 0,680 | 0,889              | 1   |
| DB17 | 0,1213   | 0,6142             | 0,75   | 0,744  | 0,8947             | 1   | 0,042    | 0,524              | 0,681 | 0,680 | 0,859              | 1   |
| DG24 | 0,1366   | 0,7143             | 0,8684 | 0,8279 | 1                  | 1   | 0,053    | 0,677              | 0,842 | 0,799 | 1                  | 1   |
| DI26 | 0,104    | 0,700              | 0,838  | 0,813  | 1,000              | 1   | 0,033    | 0,598              | 0,767 | 0,748 | 0,960              | 1   |
| DJ28 | 0,1389   | 0,6364             | 0,7764 | 0,7587 | 0,9048             | 1   | 0,107    | 0,532              | 0,695 | 0,690 | 0,858              | 1   |
| DK29 | 0,09223  | 0,6259             | 0,7692 | 0,7553 | 0,9139             | 1   | 0,031    | 0,553              | 0,708 | 0,705 | 0,881              | 1   |
|      | TEVP-W-I |                    |        |        |                    |     | TEVP-W-O |                    |       |       |                    |     |
| DA15 | 0,092    | 0,513              | 0,667  | 0,657  | 0,813              | 1   | 0,013    | 0,406              | 0,537 | 0,550 | 0,689              | 1   |
| DB17 | 0,121    | 0,536              | 0,667  | 0,666  | 0,800              | 1   | 0,032    | 0,450              | 0,572 | 0,582 | 0,715              | 1   |
| DG24 | 0,068    | 0,588              | 0,706  | 0,694  | 0,824              | 1   | 0,038    | 0,522              | 0,651 | 0,642 | 0,778              | 1   |
| DI26 | 0,104    | 0,591              | 0,720  | 0,706  | 0,836              | 1   | 0,031    | 0,485              | 0,627 | 0,619 | 0,764              | 1   |
| DJ28 | 0,130    | 0,571              | 0,692  | 0,687  | 0,808              | 1   | 0,075    | 0,473              | 0,592 | 0,599 | 0,723              | 1   |
| DK29 | 0,081    | 0,537              | 0,657  | 0,657  | 0,789              | 1   | 0,029    | 0,462              | 0,570 | 0,582 | 0,705              | 1   |

Table 3: Summary statistics of technical efficiency scores. annual scores are pooled

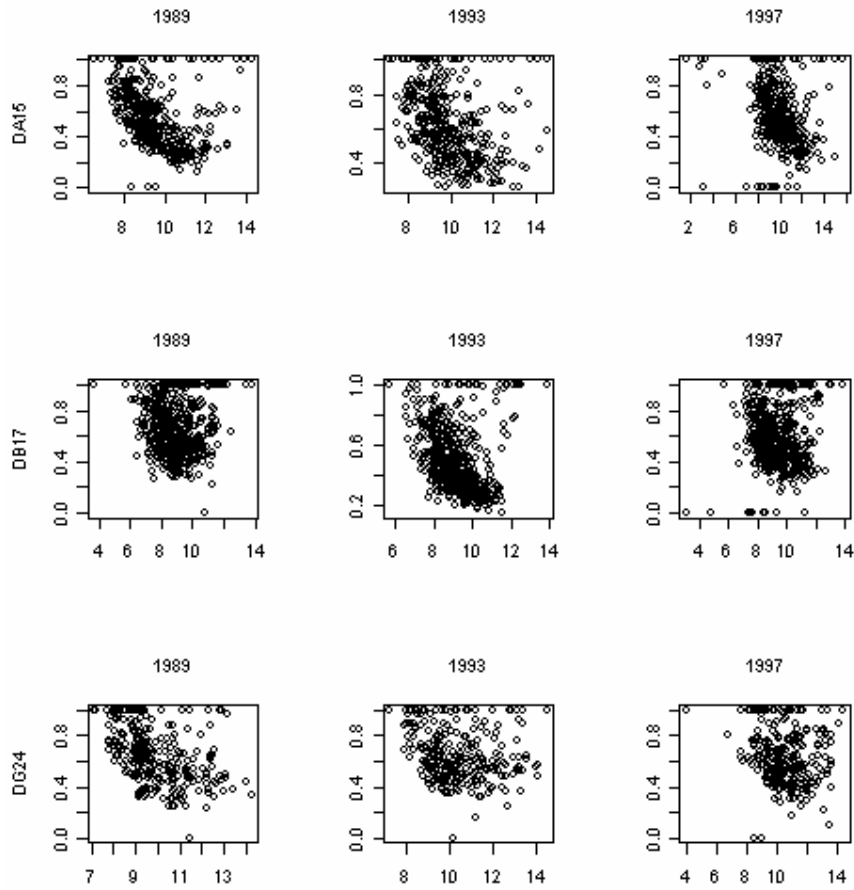


Figure 2: The efficiency-size relationship. Technical efficiency scores on the vertical axis (TEVP-H-I), logarithm of Total Assets on the horizontal axis (Sectors DA15, DB17 and DG 24)

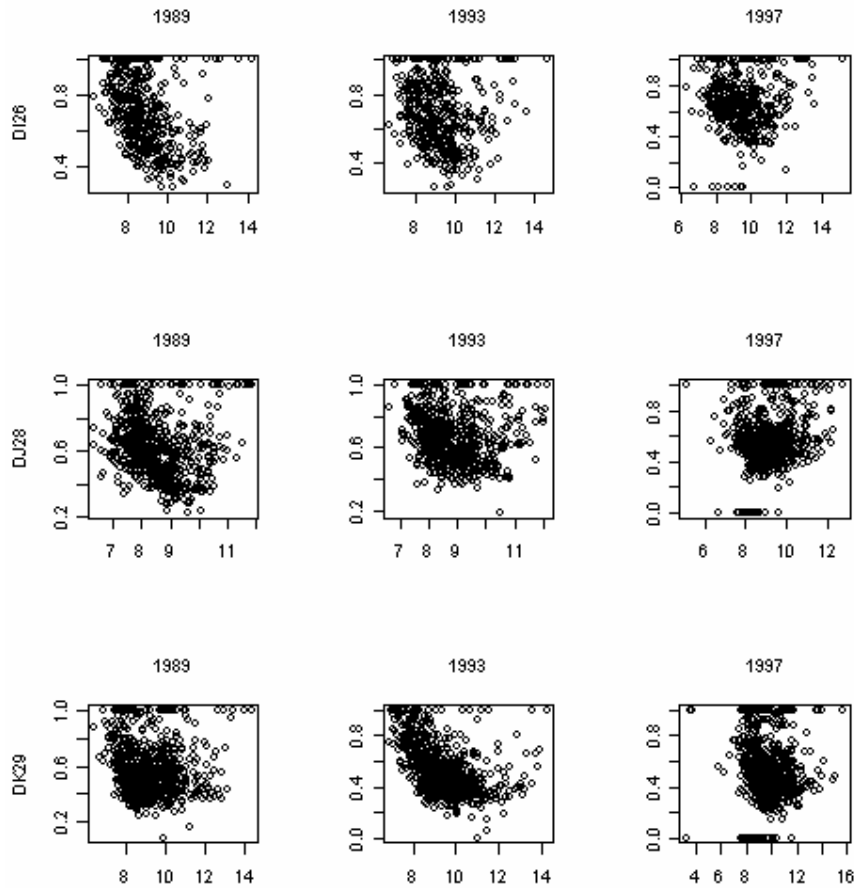


Figure 3: The efficiency-size relationship. Technical efficiency scores on the vertical axis (TEVP-H-I), logarithm of Total Assets on the horizontal axis (Sectors DI26, DJ28 and DK 29).

#### 4. Scale Elasticity: The Empirical Evidence

In order to better understand the characteristics of our scale elasticity measures, it is useful to consider Figure 4, which provides the rates of growth of Italian GDP in the years included in the analysis. Clearly, 1989 was the year closer to a cycle peak, while 1993 is the cycle trough. Toward the end of the sample the cycle picks up again.

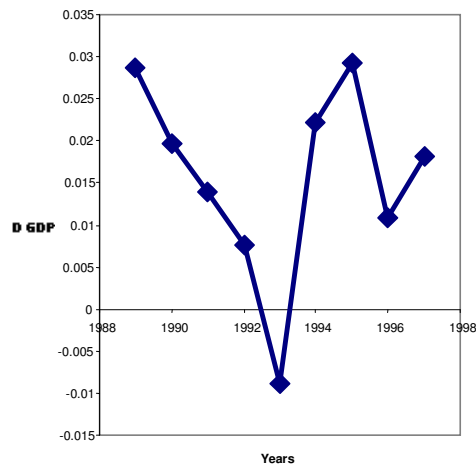


Figure 4: Growth rates of the Italian GDP: 1989-97

If the inputs included in the production set do not closely track the cyclical fluctuation of the output, our measures of scale elasticity could be cyclically biased. Hence it is important to compare their values in different periods. For the sake of brevity we carry out this comparison for 1989 (peak), 1993 (trough) and 1997 (last year, with the GDP growth rate very close to its sample mean). As can be seen from Table 4, there is no evidence of a cyclical bias in our scale elasticities.

In particular, the 1993 values are not distinctly higher than the other ones, as one would expect to be the case in the presence of slack inputs.

Table 4 also makes it clear that the measurement of scale elasticities through FDH (and, to some extent, also through VP-FDH) produces some highly anomalous values. These values occur when firms are close to a very high or a very wide “step” of the FDH frontier. In the following analysis the lowest and the highest 2.5% of the elasticity scores will be trimmed out of the sample. In any case, it generally turns out that scale elasticity is inversely related to firm size. This inverse relationship is depicted in Figures 5 and 6 for the trimmed sample.

|      | sc. el. H    |                    |       |       |                    |       | sc. el. W    |                    |        |       |                    |        |
|------|--------------|--------------------|-------|-------|--------------------|-------|--------------|--------------------|--------|-------|--------------------|--------|
|      | Min          | 1 <sup>st</sup> Qu | Med   | Mean  | 3 <sup>rd</sup> Qu | Max   | Min          | 1 <sup>st</sup> Qu | Med    | Mean  | 3 <sup>rd</sup> Qu | Max    |
| 1989 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.127        | 0.951              | 1.387 | 1.499 | 1.910              | 5.654 | 0.156        | 0.835              | 1.584  | 2.249 | 2.876              | 11.500 |
| DB17 | 0.235        | 0.840              | 1.217 | 1.459 | 1.698              | 6.552 | 0.102        | 0.722              | 1.260  | 1.616 | 2.045              | 8.033  |
| DG24 | 0.162        | 0.679              | 1.149 | 1.506 | 1.910              | 8.858 | 0.059        | 0.494              | 0.908  | 1.650 | 1.810              | 10.110 |
| DI26 | 0.854        | 1                  | 1.254 | 1.482 | 1.688              | 4.063 | 1.002        | 1.240              | 1.525  | 1.899 | 2.090              | 7.168  |
| DJ28 | 0.195        | 0.766              | 1.299 | 1.488 | 1.877              | 6.045 | 0.119        | 0.843              | 1.420  | 1.778 | 2.349              | 6.920  |
| DK29 | 0.223        | 0.776              | 0.995 | 1.219 | 1.419              | 4.497 | 0.545        | 1                  | 1      | 1.342 | 1.484              | 4.502  |
| 1993 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.118        | 0.816              | 1.307 | 1.685 | 2.149              | 9.016 | 0.103        | 0.819              | 1.410  | 1.806 | 2.240              | 8.429  |
| DB17 | 0.168        | 0.787              | 1.259 | 1.589 | 2.063              | 6.436 | 0.065        | 0.796              | 1.332  | 1.683 | 2.157              | 6.436  |
| DG24 | 0.140        | 0.623              | 1.071 | 1.272 | 1.663              | 4.496 | 0.054        | 0.502              | 0.6785 | 1.511 | 1.957              | 6.705  |
| DI26 | 0.396        | 1                  | 1.275 | 1.505 | 1.655              | 5.507 | 0.396        | 1                  | 1.275  | 1.505 | 1.655              | 5.507  |
| DJ28 | 0.167        | 0.865              | 1.182 | 1.360 | 1.644              | 5.245 | 0.120        | 0.806              | 1.307  | 1.664 | 1.953              | 7.305  |
| DK29 | 0.157        | 0.699              | 1.147 | 1.423 | 1.751              | 5.239 | 0.739        | 1                  | 1123   | 1.440 | 1.500              | 4.386  |
| 1997 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.155        | 0.841              | 1.284 | 1.681 | 1.963              | 10.13 | 0.136        | 0.884              | 1.608  | 2.485 | 2.909              | 13.450 |
| DB17 | 0.170        | 0.876              | 1.165 | 1.470 | 1.665              | 8.005 | 0.061        | 0.677              | 1.198  | 1.575 | 1.963              | 6.444  |
| DG24 | 0.094        | 0.628              | 0.952 | 1.361 | 1.655              | 6.983 | 0.083        | 0.607              | 0.970  | 1.751 | 1.827              | 10.230 |
| DI26 | 0.555        | 1                  | 1.227 | 1.342 | 1.451              | 3.668 | 0.651        | 1                  | 1.336  | 1.735 | 1.930              | 5.581  |
| DJ28 | 0.221        | 0.622              | 1     | 1.114 | 1.403              | 4.551 | 0.147        | 0.655              | 1.155  | 1.609 | 2.167              | 6.916  |
| DK29 | 0.194        | 0.708              | 1.004 | 1.277 | 1.583              | 4.850 | 0.583        | 1                  | 1125   | 1.331 | 1.420              | 4.268  |
|      | sc. el. H-VP |                    |       |       |                    |       | sc. el. W-VP |                    |        |       |                    |        |
|      | Min          | 1 <sup>st</sup> Qu | Med   | Mean  | 3 <sup>rd</sup> Qu | Max   | Min          | 1 <sup>st</sup> Qu | Med    | Mean  | 3 <sup>rd</sup> Qu | Max    |
| 1989 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.540        | 1                  | 1.436 | 1.477 | 1.812              | 2.757 | 0.384        | 1                  | 1.519  | 2.056 | 2.765              | 8.691  |
| DB17 | 0.488        | 1.114              | 1.114 | 1.346 | 1.560              | 4.376 | 0.431        | 1                  | 1.084  | 1.520 | 1.667              | 6.648  |
| DG24 | 0.811        | 1                  | 1.012 | 1.407 | 1.497              | 4.606 | 0.868        | 1                  | 1      | 1.241 | 1.227              | 3.508  |
| DI26 | 0.934        | 1                  | 1.179 | 1.418 | 1.595              | 4.063 | 1.002        | 1.244              | 1.556  | 2.050 | 2.166              | 9.119  |
| DJ28 | 0.392        | 1                  | 1.228 | 1.270 | 1.539              | 2.898 | 0.415        | 1                  | 1.202  | 1.604 | 1.960              | 5.957  |
| DK29 | 0.544        | 0.972              | 1     | 1.150 | 1.112              | 3.209 | 0.121        | 0.782              | 1212   | 1.501 | 1.899              | 6.522  |
| 1993 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.811        | 1                  | 1.256 | 1.570 | 1.950              | 3.986 | 0.676        | 1                  | 1.092  | 1.603 | 1.869              | 6.054  |
| DB17 | 0.275        | 1                  | 1.440 | 1.641 | 2.089              | 4.747 | 0.517        | 1                  | 1.368  | 1.561 | 1.867              | 4.457  |
| DG24 | 0.602        | 1                  | 1     | 1.160 | 1.087              | 3.787 | 0.566        | 1                  | 1      | 1.134 | 1.117              | 3.628  |
| DI26 | 0.445        | 1                  | 1.217 | 1.449 | 1.589              | 5.315 | 0.445        | 1                  | 1.217  | 1.449 | 1.589              | 5.315  |
| DJ28 | 0.588        | 1                  | 1.205 | 1.343 | 1.554              | 3.460 | 0.610        | 1                  | 1.154  | 1.566 | 1.749              | 6.690  |
| DK29 | 0.666        | 1                  | 1.179 | 1.430 | 1.465              | 4.363 | 0.068        | 0.765              | 1243   | 1.640 | 2.074              | 6.660  |
| 1997 |              |                    |       |       |                    |       |              |                    |        |       |                    |        |
| DA15 | 0.521        | 1                  | 1.213 | 1.473 | 1.792              | 3.415 | 0.540        | 1                  | 1.319  | 1.956 | 2.179              | 8.395  |
| DB17 | 0.425        | 1                  | 1.103 | 1.345 | 1.560              | 3.978 | 0.535        | 1                  | 1.098  | 1.522 | 1.763              | 5.837  |
| DG24 | 0.818        | 1                  | 1     | 1.148 | 1.188              | 2.475 | 0.492        | 1                  | 1      | 1.308 | 1.328              | 5.831  |
| DI26 | 0.557        | 1                  | 1.189 | 1.297 | 1.410              | 3.628 | 0.715        | 1                  | 1.105  | 1.513 | 1.636              | 5.480  |
| DJ28 | 0.484        | 0.898              | 1     | 1.097 | 1.079              | 2.874 | 0.623        | 1                  | 1      | 1.457 | 1.549              | 5.145  |
| DK29 | 0.630        | 0.961              | 1     | 1.217 | 1.327              | 3.649 | 0.139        | 0.721              | 1284   | 1.604 | 2.104              | 6.107  |

Table 4: Summary statistics of the scale elasticities. 1989. 1993. 1997



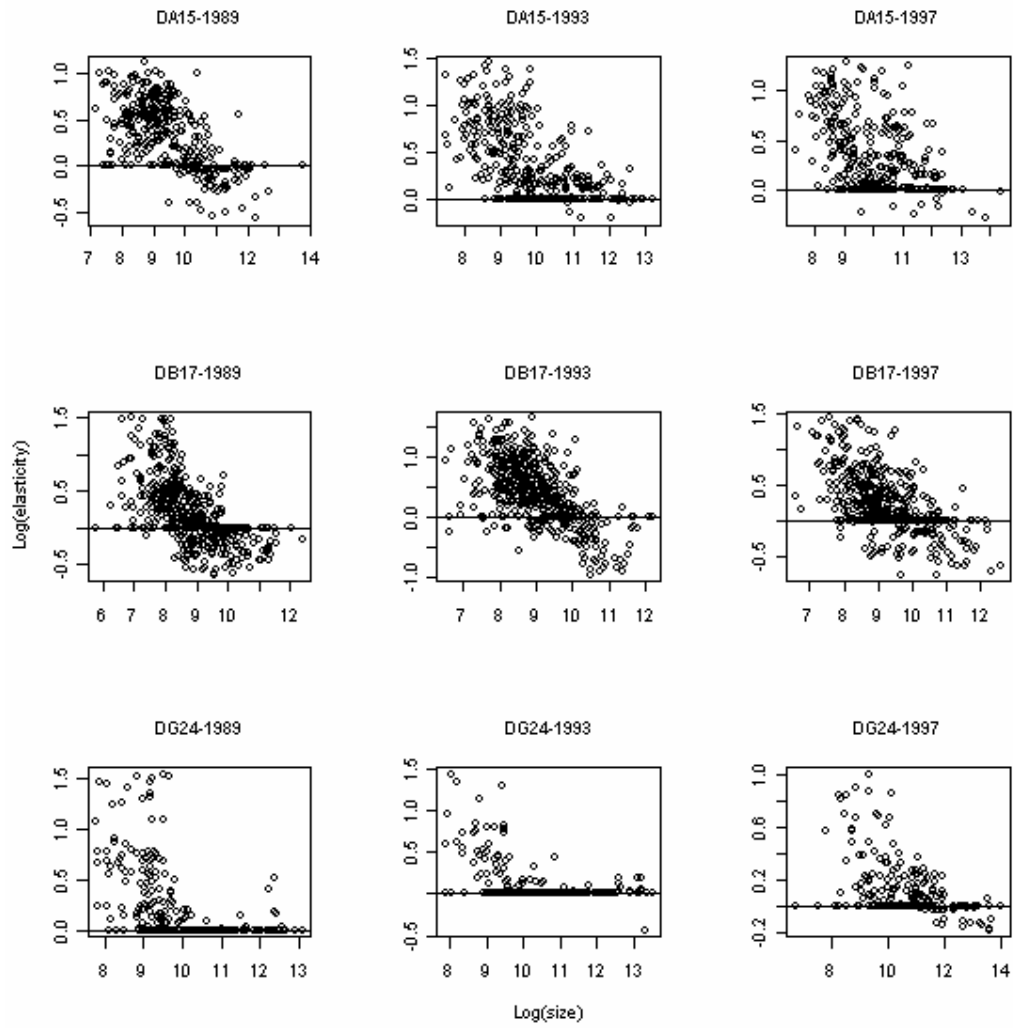


Figure 5: The scale elasticity-size relationship ( elasticity scores are calculated through FDH-VP, with respect to the first production set). Sectors DA15, DB17, DG24

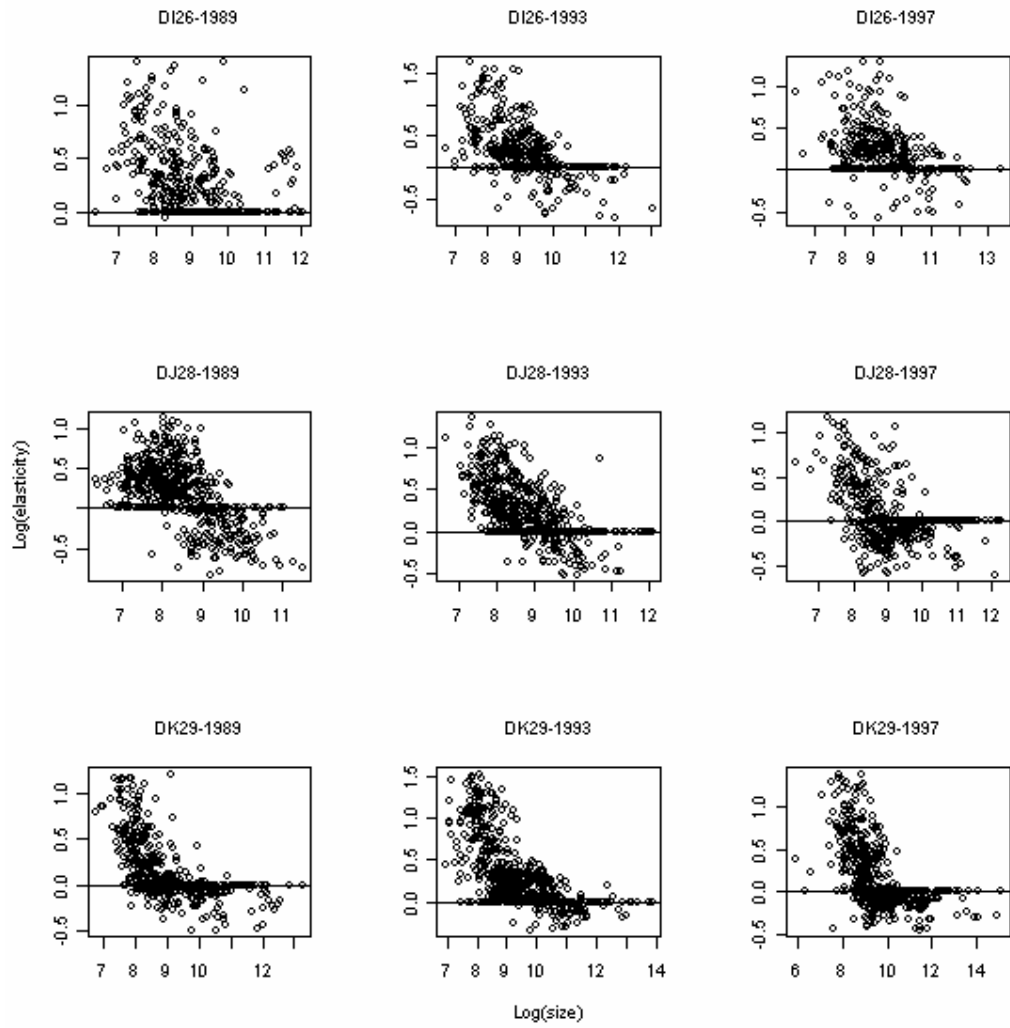


Figure 6: The scale elasticity-size relationship ( elasticity scores are calculated through FDH-VP, with respect to the first production set). Sectors DI26, DJ28, DK29

## 5. The Zipf Plot of the Pareto IV Distribution

The Zipf Plot (see Stanley et al., 1995), is a graph of the log of the rank versus the log of the variable being analysed, in this case firm size. Let  $X$  be the random variable size,  $(x_1, x_2, \dots, x_N)$  be the vector of its realizations on a set of  $N$  firms and  $F_X(x)$  its cumulative distribution function. Next, suppose that the observations are ordered from the largest to the smallest so that the index  $i$  is the rank of the  $i$ -th firm. The sample Zipf plot is the graph of  $\ln(x_i)$  against  $\ln(i)$ . Further, because of the ranking,

$$i/N = 1 - F_X(x_i), \text{ so that } \ln(i) = \ln[1 - F_X(x_i)] + \ln(N), \text{ and}$$

$$\ln(x_i) = \ln[F_X^{-1}(1 - i/N)]$$

Therefore, if a distribution function  $\hat{F}_X(x)$  has been satisfactorily fitted on the random variable  $X$ , it is possible to provide for each firm an estimate of its size, starting from its rank. This estimate is achieved by means of the inverse of the same distribution function, i.e. the quantile function

$$\hat{x}_i = \hat{F}_X^{-1}(1 - i/N)$$

The passage from the cumulative distribution function to the rank permits to estimate the size of sector biggest firm, second biggest firm and so on, when firm size in that sector is reasonably proved to follow a particular probability law. Accordingly, systematic differences between effective and estimated size reveal regimes of growth different from those assumed by the growth process, if any, associated with the probability law.

The quantile function of the Pareto IV distribution is defined as:

$$Q_{P4}(x) = \left[ (1 - y)^{-1/\alpha} - 1 \right]^\gamma \sigma + \mu$$

The Pareto IV is a general Pareto model with four parameters, respectively  $\mu$  of location,  $\sigma$  of scale,  $\alpha$  and  $\gamma$  of shape and nests the Pareto I as a particular case, i.e. for particular specification of the parameters ( $\mu = \sigma$  and  $\gamma = 1$ ). More generally, the Pareto IV possesses a Pareto I right-hand tail, with coefficient  $\alpha/\gamma$  (Bell and Klonner, 2005): the Pareto I shape coefficient is approximated by the ratio of the Pareto IV shape coefficients. This is clearly highlighted by the Zipf Plot in that the Pareto I's Zipf Plot is a straight line (Ijiri and Simon, 1977) and the Pareto IV's one becomes a straight line beyond a given size threshold.

In Figure 7 we depict the Zipf plots for the six industries. In all cases the plot starkly differs from a Pareto I's straight line. Details about the Maximum Likelihood estimation of the plots are provided in Crosato and Ganugi (2006).

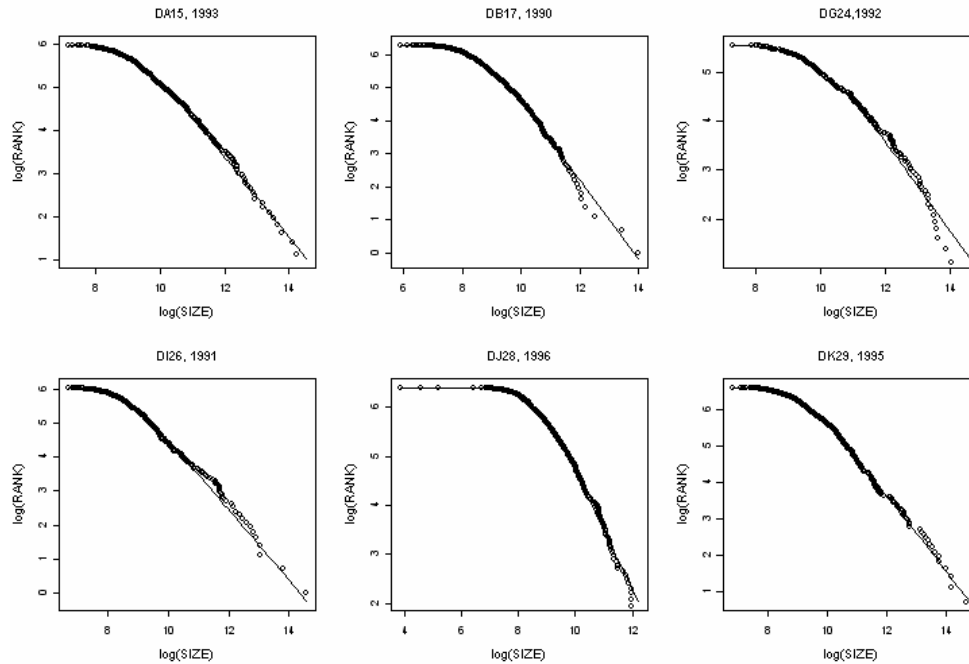


Figure 7: Zipf Plots of the six sectors. Dots represent the observed rank-size relationship, the solid line represents the relationship between the observed rank and the size estimated through the quantile function.

## 6. Zipf Plot Concavity/Linearity and Returns to Scale

The existence of a link between the shape of firm-size distribution and of returns to scale has been the object of some attention in the literature (Simon and Bonini, 1958; Ijiri and Simon, 1967, 1974, 1977; Vining, 1976; Lucas, 1978). Ijiri and Simon (1977) pointed out (see also Simon and Bonini, 1958) that Gibrat's law is consistent with a regime of constant returns to scale, at least if firms are able to reach the Minimum Efficient Scale. Later, Lucas (1978) drew on this point, building a model where firms characterised by constant returns to scale grow according to Gibrat's law. Matters are less clear-cut when considering Zipf plots characterised by non-linearities. Ijiri and Simon (1967, 1974, 1977) suggested that concave (downward) from the straight line in the Zipf plots can still be compatible with constant returns to scale in the presence of either strong autocorrelation of growth rates or mergers and acquisitions. However, Vining (1978) when simulating a Gibrat-like process, augmented by autocorrelation in growth rates, finds a convex curvature, rather than concave, on the Zipf plot. Hence, although the literature does not provide any clear conclusion, a concave curvature in the log-log chart may stand for «... something inherent in the very nature of size that causes a progressive decline in... [the growth rate of a firm] as it expands its activities » (Vining, 1976, 370). In other words, either decreasing returns to scale step in or increasing returns to scale phase out as firms grow larger.

In this paper we provide empirical evidence on this topic by linking explicitly the curvature of the Pareto IV distribution with the pattern of returns to scale of the production frontier, as appraised through the FDH approach. Following the above discussion we can put to test two alternative hypotheses. The first one envisages constant returns to scale regardless of their position in the Zipf plot, in which case some explanation should be found for the curvature of the latter. According to the second hypothesis, as firms grow larger, they tend to be characterised by increasing, constant and decreasing returns to scale. Considering the Zipf plots depicted in Figure 7, we expect constant returns to scale for firms lying in the linear part of the plot, while smaller firms experience increasing returns to scale. We can see here why a crucial advantage of FDH is that it imposes virtually no a priori structure on the pattern of returns to scale, while DEA exhibits by construction non-decreasing returns to scale for relatively smaller observations and non-increasing returns for relatively larger observations.

Turning now to the evidence, we recall that in § 4.2, an inverse relation was found between elasticity of scale and size. A careful look at Figures 5 and 6 suggests that, above a given size threshold, elasticities align themselves on the horizontal line demarking constant returns to scale. This tendency is observable in all sectors aside from the Textile one -DB17- where elasticities are first larger and then smaller than unity. On the other hand, in § 5.1 we stressed out that the Zipf plot of the six industries under scrutiny is concave for firms up to a given threshold, and then becomes linear.

Our research strategy is then to ascertain whether the level of Total Assets (TA) that divides the non-linear from the linear part of the Zipf plots can also be considered a threshold in terms of returns-to-scale regimes. To do this, we need to analyse the distribution of elasticities separately for firms on each side of the linearity threshold, which has to be fixed.

In order to determine this threshold, or cutoff point, we rely on the convergence of the Pareto IV to the Pareto I distribution through two steps. First, we estimate the Pareto I model moving from the left to the right of the distribution. We take one percentile in turn as the scale parameter of the model and estimate the shape parameter, through maximum likelihood estimates, for firms remaining on the right of that percentile. We then fix the linearity threshold at the percentile giving the smallest difference between the Pareto I shape parameter and the ratio of Pareto IV's  $\alpha$  and  $\gamma$ . This step is exemplified in Figure 8, where the sequence of such differences along the percentiles can be seen for the "Food products and beverage" industry.

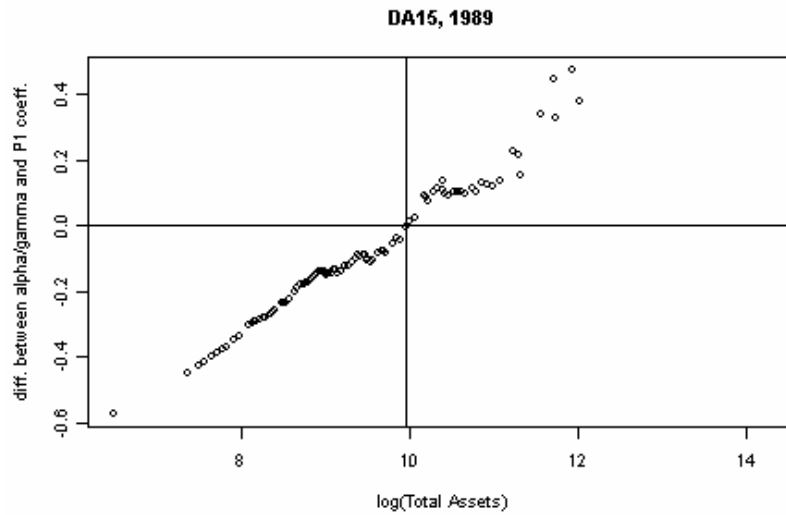


Figure 8: Fixing of the linearity threshold (vertical line). Food products and beverage (DA15).

In the second step, once determined the cutoff point, we check for acceptance of the Pareto I model, on its right, by a standard Chi-Squared test.

The outcomes of the procedure for the first, central and final years are summarized in Table 5. The second and third columns respectively contain the percentile from which the fitted Zipf Plot becomes linear and the number of firms belonging to the linear part. As can be seen, the linearity threshold shifts both between sectors and, within sectors, over time. A comparison between the fourth and the fifth column clearly highlights the proximity between the Pareto I shape parameter and the ratio of Pareto IV's  $\alpha$  and  $\gamma$ . Finally, the Chi-Squared test p-values reported in the last column validate a good fit of the Pareto I distribution for firms with TA exceeding the identified threshold. Overall, the obtained p-values widely exceed the 5% level of significance.

After the cutoff point has been determined for each year, firms can be straightforwardly classified as belonging either to the concave or the linear part of the plot. To analyse the type of returns to scale prevailing for firms on the left and on the right of the boundary, we divide the elasticities in three classes, precisely below 0.95 excluded, from 0.95 to 1.05 and above 1.05, representing decreasing, constant and increasing returns to scale, respectively. We report the results for this taxonomy in Table 6, by pooling all years together.

| year | percentile | n.of.firms | P1<br>shape<br>parameter | P4<br>$\alpha/\gamma$ | p.value (P1) |
|------|------------|------------|--------------------------|-----------------------|--------------|
| DA15 |            |            |                          |                       |              |
| 1989 | 66%        | 132        | 0.894                    | 0.897                 | 0.550        |
| 1993 | 79%        | 82         | 0.941                    | 0.937                 | 0.903        |
| 1997 | 85%        | 59         | 1.069                    | 1.070                 | 0.943        |
| DB17 |            |            |                          |                       |              |
| 1989 | 73%        | 145        | 1.155                    | 1.154                 | 0.245        |
| 1993 | 70%        | 157        | 1.202                    | 1.197                 | 0.450        |
| 1997 | 77%        | 121        | 1.182                    | 1.162                 | 0.096        |
| DG24 |            |            |                          |                       |              |
| 1989 | 65%        | 96         | 0.843                    | 0.860                 | 0.078        |
| 1993 | 68%        | 82         | 0.896                    | 0.896                 | 0.164        |
| 1997 | 83%        | 45         | 1.117                    | 1.109                 | 0.208        |
| DI26 |            |            |                          |                       |              |
| 1989 | 84%        | 69         | 1.000                    | 1.001                 | 0.653        |
| 1993 | 89%        | 48         | 1.009                    | 1.020                 | 0.908        |
| 1997 | 71%        | 125        | 0.962                    | 0.964                 | 0.849        |
| DJ28 |            |            |                          |                       |              |
| 1989 | 65%        | 221        | 1.200                    | 1.209                 | 0.197        |
| 1993 | 61%        | 239        | 1.104                    | 1.107                 | 0.209        |
| 1997 | 67%        | 198        | 1.171                    | 1.173                 | 0.537        |
| DK29 |            |            |                          |                       |              |
| 1989 | 65%        | 255        | 0.957                    | 0.948                 | 0.304        |
| 1993 | 87%        | 97         | 1.077                    | 1.077                 | 0.816        |
| 1997 | 85%        | 113        | 1.077                    | 1.077                 | 0.664        |

Table 5: Estimates of Pareto I model on the linear tail of the Pareto IV model. Linearity threshold in the Pareto 4 Zipf Plot and Chi-squared test p-values.

| Sectors | Zipf Plot linear part |               |        |      | Zipf Plot concave part |        |               |      |
|---------|-----------------------|---------------|--------|------|------------------------|--------|---------------|------|
|         | DRS                   | CRS           | IRS    | tot  | DRS                    | CRS    | IRS           | tot  |
| DA15    | 16.20%                | <u>44.20%</u> | 39.60% | 100% | 0.60%                  | 31.50% | <u>67.80%</u> | 100% |
| DB17    | <u>55.00%</u>         | 32.40%        | 12.60% | 100% | 16.90%                 | 17.10% | <u>66.10%</u> | 100% |
| DG24    | 4.70%                 | <u>82.10%</u> | 13.20% | 100% | 0.10%                  | 38.90% | <u>61.00%</u> | 100% |
| DI26    | 26.20%                | <u>57.53%</u> | 16.27% | 100% | 8.74%                  | 26.14% | <u>65.13%</u> | 100% |
| DL28    | 40.00%                | <u>42.90%</u> | 17.20% | 100% | 9.80%                  | 18.50% | <u>71.70%</u> | 100% |
| DK29    | 31.00%                | <u>63.70%</u> | 5.30%  | 100% | 10.30%                 | 26.40% | <u>63.30%</u> | 100% |

*Table 6: Percentage of firms in the linear and concave part according to scale elasticity*

As can be seen from Table 6, firms in the concave part clearly exhibit increasing returns to scale in all sectors. On the contrary, firms staying in the linear part are mostly concentrated in the interval  $[0.95, 1.05]$ , hence sharing a tendency to constant returns to scale. An exception is the Textile Sector (DB17), where most firms experience decreasing returns to scale.

Accordingly, our evidence favours the second hypothesis outlined above. The Pareto IV distribution is consistent with different returns-to-scale regimes: as firms grow larger, they tend to be characterised by increasing and then constant returns to scale. Most of firms in the concave part experience an IRS regime, while the linear tail overwhelmingly includes firms facing CRS in five out of six sectors.



## Concluding Remarks

In the literature on firm-size distribution, the relationship between technology and firm size has been treated mostly from a theoretical point of view (Simon and Bonini, 1958; Ijiri and Simon, 1967, 1974, 1977; Vining, 1976; Lucas, 1978). In this paper we provide empirical evidence on this topic analysing six industries from Italian manufacturing.

In a previous research (Crosato and Ganugi 2006) we have pointed out that the size distribution of firms in Italian manufacturing industries cannot be satisfactorily modelled by means of the Lognormal and Pareto I distributions. On the contrary, a good fit is achieved through the Pareto IV distribution, a more general Paretian model. As it can be easily shown through the Zipf plot, the Pareto IV model possesses a Pareto I right-hand tail, linear in double log-scale. In this paper we link explicitly the curvature of the Pareto IV distribution with the returns to scale of the production frontier, as appraised through the non-parametric Free Disposal Hull (FDH) approach, both in the traditional version from Deprins et al. (1984) and in the refined version by Kerstens and Vanden Eeckaut (1999). Utilisation of FDH is of crucial importance, as this approach imposes very little a priori structure on the pattern of returns to scale.

A first result of our research is the absence of a clear relationship between technical efficiency and size. This is consistent with the composition of our dataset, a closed panel which records firms from 20 employees onwards. According to Jovanovic (1982), in fact, firms with exceedingly low efficiency values should shrink and exit the industry and could not survive for nine years. Efficiency is therefore distributed neutrally with respect to size among surviving firms.

More crucially, we find an inverse relationship between scale elasticity and firm size. Building on this result, we investigate the connection between elasticities and the firm size distribution shape. We find that firms occupying the concave part of the Pareto IV clearly exhibit increasing returns to scale in all sectors. On the contrary, firms in the linear Pareto I-type part generally show constant returns to scale, with the exception of the Textile industry where returns to scale are mainly decreasing. The existence of two different technological regimes results is therefore consistent with the existence of different regimes of growth along the Pareto IV distribution. Furthermore, since both constant returns to scale and linearity of the Zipf plot are compatible with Gibrat's law, their joint presence in the right tail of the size distribution leaves room for a reappraisal of the law for larger firms of Italian manufacturing (Lotti et al., 2004).

In future research, we intend to carry on a systematic study of the convergence of the Pareto IV to the Pareto I distribution, analysing why the linearity threshold changes across sectors and time. Possible hypotheses to be considered in this ambit are the influence of the age of the firm on its growth regime (Lotti et al., 2003), of company groups or conglomerates on the pattern of returns to scale, and of GDP fluctuations and business cycles. Finally, it would be interesting to investigate whether the cross-industry variability of the linearity threshold is related to differences in the Minimum Efficient Scale (MES) across sectors.

## References

- Arnold B (1983) Pareto Distributions. Fairland, MD: International Co-operative Publishing House
- Axtell R (2001) Zipf distribution of U.S. firm sizes. *Science*, 293: 1818-1820
- Bell C, Klonner S (2005) Output, prices and the distribution of consumption in rural India, *Agricultural Economics*, 33: 29-40.
- Bottazzi G, Cefis E, Dosi G, Secchi A (2003) Invariances and Diversities in the Evolution of Manufacturing Industries, Koopmans Research Institute, Discussion Paper Series 03-17, Utrecht University. (Forthcoming on *Small Business Economics*).
- Bottazzi G, Grazzi M, Secchi A (2005) Characterizing the Production Process: a Disaggregated Analysis of Italian Manufacturing Firms, *Rivista di Politica Economica*, I-II: 243-270.
- Crosato L, Ganugi P (2006), Statistical Regularity of Firm Size Distribution: the Pareto IV and Truncated Yule for Italian SCI Manufacturing, forthcoming in *Statistical Methods and Applications* (available on <http://dx.doi.org/10.1007/s10260-006-0023-7>).
- Dagum C (1977) A new model of personal income distribution: Specification and estimation, *Economie Appliquée*, 30: 413-437.
- Deprins D., Simar L., Tulkens H. (1984) Measuring labor-efficiency in post offices, in Marchand M., Pestieau P., Tulkens H. (eds.), *The performance of public enterprises: concepts and measurement*, Amsterdam: North Holland.
- Dunne P, Hughes A (1994) Age, size, growth and survival: UK companies in the 1980s. *Journal of Industrial Economics*, 42(2): 115-140.
- Evans DS (1987) Tests of Alternative Theories of Firm Growth, *Journal of Political Economy*, 95: 657-674.
- Farrell MJ (1957) The Measurement of Productive Efficiency, *Journal of the Royal Statistical Society, Series A, Part III*, 70: 253-281.
- Førsund F (1996) On the Calculation of Scale Elasticity in DEA Models, *Journal of Productivity Analysis*, 7: 283-302.
- Førsund F, Hjalmarsson L (1979) Generalised Farrell Measures of Efficiency: An Application to Milk Processing in Swedish Dairy Plants, *Economic Journal*, 89: 294-315.
- Fried H, Lovell CAK, Schmidt SS (eds.) (1993), *The Measurement of Productive Efficiency: Techniques and Applications*, Oxford University Press, London.
- Ganugi P, Grossi L, Crosato L (2003) Firm size distribution and stochastic growth models: a comparison between ICT and Mechanical Italian Companies, *Statistical Methods and Applications*, 12: 391-414
- Ganugi P, Grossi L, Gozzi G (2005) Testing Gibrat's law in Italian macro-regions: analysis on a panel of Mechanical companies, *Statistical Methods and Applications*, 14(1).
- Gibrat R (1931) *Les Inégalités Économiques*, Paris: Librairie du Recueil Sirey.
- Gijbels I, Mammen E, Park BU, Simar L (1999) On estimation of monotone and concave frontier functions, *Journal of the American Statistical Association*, 94: 220-228.
- Hall BH (1987) The relationship between firm size and growth in the U.S. Manufacturing sector, *Journal of Industrial Economics*, 35: 583-606.
- Hart PE, Oulton N (1997) Zipf and the size distribution of firms, *Applied Economics Letters*, 4(4): 205-206.
- Ijiri Y, Simon HA (1967) A Model of Business Firm Growth, *Econometrica*, 35: 348-355
- Ijiri Y, Simon HA (1974) Interpretations of departures from the Pareto curve firm-size distributions. *Journal of Political Economy*, 82: 315-331
- Ijiri Y, Simon HA (1977) *Skew Distributions and the Sizes of Business Firms*. Amsterdam: North Holland.
- Jovanovic B (1982) Selection and evolution of industry, *Econometrica*, 50: 649-670.

- Kerstens K, Vanden Eeckaut P (1999) Estimating returns to scale using non-parametric deterministic technologies: a new method based on goodness-of-fit, *European Journal of Operational Research*, n. 113: 206-214.
- Kittelsen SAC (1999) Monte Carlo simulations of DEA efficiency measures and hypothesis tests, Memorandum N. 9, Department of Economics, University of Oslo.
- Kleiber C, Kotz S (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, New York: Wiley.
- Kneip A, Park BU, Simar L (1998) A note on the convergence of nonparametric DEA estimators for production efficiency scores, *Econometric Theory*, 14: 783-793.
- Lotti F, Santarelli E, Vivarelli M (2003) Does Gibrat's Law hold among young, small firms?, *Journal of Evolutionary Economics*, 13: 213-235.
- Lotti F, Santarelli E, Vivarelli M (2004), Gibrat's Law and market selection, *Discussion Papers on Entrepreneurship, Growth and Public Policy*, Max Planck Institute of Economics n. 2004-28.
- Lucas RE (1978) On the size distribution of business firms, *Bell Journal of Economics*, 9: 508-523.
- Mansfield E (1962) Entry, Gibrat's Law, Innovation, and the Growth of Firms, *American Economic Review*, 52: 1023-1051.
- Okuyama K, Takayasu M, Takayasu H (1999) Zipf's law in income distribution of companies, *Physica*, A269: 125-131
- Pareto V (1897), *Cours d'Économie Politique*, professé à l'Université de Lausanne, Lausanne, Paris: Rouge-Pichon.
- Quandt RE (1966) On the size distribution of firms, *American Economic Review*, 3: 416-432
- Simon HA, Bonini CP (1958) The Size Distribution of Business Firms, *American Economic Review*, 48: 607-617
- Singh SK, Maddala GS (1975) A Function for Size Distribution of Incomes. *Econometrica*, 44: 963-970
- Stanley MHR, Buldyrev SV, Havlin S, Mantegna RN, Salinger MA, Stanley HE (1995) Zipf Plots and the Size Distribution of Firms, *Economics Letters*, 49: 453-457.
- Stoppa G (1990) A new model for income size distributions, in Dagum C, Zenga M (eds.), *Income and Wealth Distribution, Inequality and Poverty - Proceedings of the 2nd international conference on income distribution by size: generation, distribution, measurement and applications*. Berlin: Springer, pp 33-41.
- Tulkens H (1993) On FDH efficiency analysis: some methodological issues and applications to retail banking, courts and urban transit, *Journal of Productivity Analysis*, 4: 183-210.
- Vining DR (1976) Autocorrelated growth rates and the Pareto Law: A further analysis, *Journal of Political Economy*, 84: 369-380.

## CELPE's Discussion Papers

- 2007, 101      Guglielmo Maria CAPORALE, Alexandros KONTONIKAS  
*The Euro and Inflation Uncertainty in the European Monetary Union*
- 2006, 100      Francesco Paolo VOLPE  
*Principio democratico e giustizia nell'amministrazione*
- 2006, 99        Niall O'HIGGINS  
*Still With Us After All of These Years: Trends in Youth Labour Market Entry, Home-Leaving And Human Capital Accumulation in Italy 1993-2003*
- 2005, 98        Floro Ernesto CAROLEO, Gianluigi COPPOLA  
*The Impact of the Institutions on Regional Unemployment Disparities*
- 2005, 97        Carlo ALTAVILLA, Antonio GAROFALO, Concetto Paolo VINCI  
*Is the Discouraged Worker Effect Time-Varying?*
- 2005, 96        F. BUSATO, B. CHIARINI, P. DE ANGELIS, E. MARZANO  
*Capital Subsidies and Underground Production*
- 2005, 9Z        Lucio Valerio SPAGNOLO, Mario CERRATO  
*No euro please, We're British!*
- 2005, 94        Roberto BASILE, Mauro COSTANTINI, Sergio DESTEFANIS  
*Unit root and cointegration tests for cross-sectionally correlated panels. Estimating regional production functions*
- 2005, 93        Sergio DESTEFANIS, Raquel FONSECA  
*Matching Efficiency and Labour Market Reform in Italy. A Macroeconometric Assessment*
- 2005, 92        Cesare IMBRIANI, Antonio LOPES  
*Banking System Efficiency and the Dualistic Development of the Italian Economy in the Nineties*
- 2005, 91        Carlo ALTAVILLA, Antonio GAROFALO, Concetto Paolo VINCI  
*Designing the Optimal Length of Working Time*
- 2005, 90        Marco MANACORDA, Barbara PETRONGOLO  
*Regional Mismatch and Unemployment: Theory and Evidence from Italy, 1977-1998*

- 2004, 89 Roberta TROISI  
*Teoria dell'impresa e responsabilità parapenale: le implicazioni organizzativo-gestionali.*
- 2004, 88 Roberta TROISI  
*Enti non profit: tipologie ed opzioni organizzative*
- 2004, 87 Lavinia PARISI  
*La povertà: una rassegna sul confronto tra due approcci. Capability vs. Unidimensionalità*
- 2004, 86 Giuseppe CELI  
*Quality Differentiation, Vertical Disintegration and the Labour Market Effects of Intra-Industry Trade*
- 2004, 85 Niall O'HIGGINS  
*Recent Trends in Youth Labour Markets and Employment Policy in Europe and Central Asia*
- 2004, 84 Carlo ALTAVILLA, Floro Ernesto CAROLEO  
*Evaluating Asymmetries in Active Labour Policies: The Case of Italy*
- 2004, 83 Floro Ernesto CAROLEO, Francesco PASTORE  
*La disoccupazione giovanile in Italia. La riforma dei sistemi d'istruzione e di formazione professionale come alternativa alla flessibilità numerica per accrescere l'occupabilità*
- 2004, 82 Francesco PASTORE, Izabela MARCINKOWSKA  
*The Gender Wage Gap among Young People in Italy*
- 2004, 81 Elisabetta MARZANO  
*Dual Labour Market Theories And Irregular Jobs: Is There a Dualism Even in The Irregular Sector?*
- 2004, 80 Corrado ANDINI  
*Unemployment and Welfare Participation in a Structural VAR: Rethinking the 1990s in the United States*
- 2004, 79 Floro Ernesto CAROLEO  
*Fondamenti teorici della rigidità salariale nell'ambito dei "Non Market clearing Models"*
- 2004, 78 Adalgiso AMENDOLA, Floro Ernesto CAROLEO, Gianluigi COPPOLA  
*Regional Disparities in Europe*
- 2003, 77 Fernanda MAZZOTTA  
*Flessibilità, povertà e istruzione: un approccio Sen – istituzionale*
- 2003, 76 Adalgiso AMENDOLA, Annamaria NESE  
*Mobilità intergenerale nel livello d'istruzione nella società femminile italiana ed endogenità de titolo di studio in un modello di partecipazione alla Forza Lavoro.*

- 2003, 74 Antonio LOPES  
*Innovazione nel Sistema Creditizio del Mezzogiorno negli Anni Novanta*
- 2003, 73 Sergio DESTEFANIS, Vania SENA  
*Public Capital and Total Factor Productivity New Evidence from the Italian Regions*
- 2003, 72 Giuseppina AUTIERO, Bruna BRUNO  
*Social Preferences in Wage Bargaining: a Neocorporatist Approach*
- 2003, 71 Gianluigi COPPOLA, Maria Rosaria GAROFALO, Fernanda MAZZOTTA  
*Industrial Localisation and Economic Development. A Case Study*
- 2002, 70 Francesco GIORDANO, Fernanda Mazzotta  
*Salario di Riserva, Probabilità di Occupazione ed Efficacia dell'Istruzione Universitaria: un'Analisi sugli Studenti dell'Università di Salerno*
- 2002, 69 Giuseppe RUSSO  
*Istituzioni del Mercato del Lavoro e Occupazione: dai Costi di Aggiustamento all'Appropriabilità*
- 2002, 68 Floro Ernesto CAROLEO, Francesco PASTORE  
*Training Policy for Youth Unemployed in a Sample of European Countries*
- 2002, 67 Maria Rosaria GAROFALO, Maria Rosaria SUPINO  
*Il Disegno Normativo del Welfare Municipale in Italia come Strumento per lo Sviluppo Economico e l'Allargamento delle Opportunità Occupazionali. Una Lettura Neoistituzionale della L. 328/00*
- 2002, 65 Pietro SENESI  
*Cyclical dynamics under continuous time equilibrium*
- 2001, 64 Marcello D'AMATO, Vincenzo GALASSO  
*E' la Riforma Dini Politicamente Sostenibile?*
- 2001, 63 Sergio DESTEFANIS, Ornella Wanda MAIETTA  
*"Assessing the Productive Efficiency of Non-Profit Organisations: a Comparative Analysis"*
- 2001, 62 Floro Ernesto CAROLEO, Francesco PASTORE  
*How fine targeted is ALMP to the youth long term unemployed in Italy?*
- 2001, 61 Paolo COCCORESE  
*Strategic Advertising for Entry Deterrence Purposes*
- 2001, 60 Alessandra AMENDOLA  
*Modelling Asymmetries in Unemployment Rate*
- 2001, 59 Sergio DESTEFANIS  
*Differenziali Territoriali di Produttività ed Efficienza negli Anni '90: i Livelli e l'Andamento*

- 2001, 58      Giuseppina AUTIERO, Fernanda MAZZOTTA  
*Job Search Methods: the Choice between the Public and the Private Sector*
- 2001, 57      Giuseppina AUTIERO, Bruna BRUNO, Fernanda MAZZOTTA  
*A Correspondence Analysis of Labour Market Institutions*
- 2000, 56      Giuseppina AUTIERO  
*Governmental Organized Learning and Coordination Problems: The case of Japan in 1950s*
- 2000, 55      Giuseppina AUTIERO, Fernanda MAZZOTTA  
*The Choice of Search Methods: Some Empirical Evidence from Italy*
- 2000, 54      Giuseppe CELI  
*The Impact of International Trade on Labour Markets. The Case of Outward Processing Traffic between the European Union and Central Eastern European Countries.*
- 2000, 53      Giuseppe RUSSO, David VEREDAS  
*Institutional Rigidities and Employment on the Italian Labour Market: the Dynamic of the Employment in the Large Industrial Firms.*
- 2000, 52      Floro Ernesto CAROLEO  
*Le Politiche per l'Occupazione in Europa: una Tassonomia Istituzionale*
- 2000, 51      Andrew NEWELL, Francesco PASTORE  
*Regional Unemployment and Industrial Restructuring in Poland*
- 1999, 50      Giuseppe CELI, Alasdair SMITH  
*Quality Differentiation and the Labour Market Effects of International Trade.*
- 1999, 49      Giuseppe CELI  
*Vertical and Horizontal Intra-Industry Trade: What is the Empirical Evidence for the UK?*
- 1999, 48      Cesare IMBRIANI, Filippo REGANATI  
*Productivity spillovers and regional differences: some evidence on the italian manufacturing sector.*
- 1999, 47      Adalgiso AMENDOLA, Annamaria NESE  
*L'impatto del background familiare sul livello d'istruzione dei figli.*
- 1998, 46      Adalgiso AMENDOLA, Annamaria NESE  
*Mobilità intergenerazionale nella società femminile italiana.*
- 1998, 45      Floro Ernesto CAROLEO, Fernanda MAZZOTTA  
*Youth Unemployment and youth employment policies in Italy.*
- 1997, 44      Annamaria NESE  
*Mobilità intergenerazionale in Italia*

- 1997, 43      Adriana BARONE, Concetto Paolo VINCI  
*Fairness: un concetto relativo nell'analisi del mercato del lavoro*
- 1997, 42      Adriana BARONE, Concetto Paolo VINCI  
*Wage differentials and factor intensity reversals*
- 1997, 41      Rosa CAPOLUPO  
*L'ipotesi di convergenza nel recente dibattito sulle teorie della crescita*
- 1997, 40      Rosa CAPOLUPO  
*Endogenous Vs exogenous models of growth: the convergenze debate*
- 1997, 39      Fernanda MAZZOTTA, Annamaria NESE  
*Transizioni "In and Out" il mercato del lavoro in Italia: un'analisi microeconomica*
- 1997, 38      Fernanda MAZZOTTA  
*Disoccupazione e probabilità di occupazione in Italia: un'analisi su microdati*
- 1997, 37      Maria Rosaria GAROFALO, Bruna BRUNO  
*Equivalenza istituzionale" dei modelli di contrattazione sul salario: oltre il dibattito tra centralizzazione e decentramento*
- 1997, 36      Adalgiso AMENDOLA, Floro Ernesto CAROLEO, Gianluigi COPPOLA  
*Differenziali territoriali nel mercato del lavoro e sviluppo in Italia*
- 1996, 35      Adalgiso AMENDOLA  
*Istituzioni e mercato del lavoro. Deregolazione, occupazione e paradigma istituzionale*
- 1996, 33      Annamaria NESE  
*Modelli microeconomici per l'analisi della domanda abitativa*
- 1996, 32      Annamaria NESE  
*Test semiparametrici per modelli parametrici*
- 1996, 31      Giuseppe CELI  
*Vertical intra-industry trade and skill intensity in Europe: a cross sector analysis*
- 1996, 30      Sergio DESTEFANIS  
*Nominal rigidities and real activity. A cross-industry analysis for Italy, 1951-93*
- 1996, 29      Cesare IMBRIANI, Filippo REGANATI  
*International technology transfer into the italian manufacturing sector*
- 1996, 28      Stefania di SERAFINO, Alberto GANDOLFI  
*The choice of the Government optimal procurement mechanism: the first-price sealed bid auction with one and/or two winners.*



- 1996, 27 Raul de LUZENBERGER  
*Redistribution, and public debt with liquidity constraints*
- 1996, 26 Bruno CHIARINI  
*Un modello VAR per la domanda di lavoro*
- 1995, 25 Maria Rosaria CARILLO, Alberto ZAZZARO  
*Innovazione tecnologica e distruzione di capitale umano in un modello neo-shumpeteriano di crescita.*
- 1995, 24 Raul de LUZENBERGER  
*Macroeconomia e politiche redistributive: il caso di vincoli di liquidità*
- 1995, 23 Annamaria NESE  
*Tenure choice and demand for housing in Italy*
- 1995, 22 Filippo REGANATI  
*La struttura delle preferenze per prodotti orizzontalmente differenziati.*
- 1995, 21 Sergio DESTEFANIS, Michele LA ROCCA, Cosimo VITALE  
*Forecasting train ticket sales with linear model-based approaches and with edats*
- 1995, 20 Stefania di SERAFINO  
*Bounded rationality and incomplete contracts*
- 1995, 19 Adalgiso AMENDOLA, Floro E. CAROLEO, Maria Rosaria GAROFALO  
*Istituzioni, mercato del lavoro e decentramento delle decisioni.*
- 1995, 18 Niall O'HIGGINS  
*Why did the youth training scheme reduce reduce the wages of young people? A story of human capital formation, reservation wages and job offers.*
- 1994, 17 Antonio CARDONE  
*Misure di efficienza: alcuni aspetti teorici*
- 1994, 16 Maria Rosaria GAROFALO, Luisa ZANCHI  
*Neo-corporativismo, centralizzazione e dispersione inter-settoriale dei salari.*
- 1994, 15 Sergio DESTEFANIS  
*Allowing for frequency of negotiation in the aggregate wage equation. A study for italian manufacturing, 1973/92.*
- 1994, 14 Marcello D'AMATO  
*Tariffs for a foreign industry with market power under incomplete information on demand*
- 1994, 13 Raul DE LUZENBERGER, Cesare IMBRIANI, Giancarlo MARINI  
*Sustainability Issues in the process of European Integration*

- 1994, 12 Riccardo MARSELLI, Antonio CARDONE  
*Interdipendenza tra regioni: un'analisi su dati di Panel*
- 1993, 11 Adriana BARONE, Concetto Paolo VINCI  
*Dilemma del prigioniero e persistenza della disoccupazione involontaria*
- 1993, 10 Maria Rosaria CARILLO  
*Mutamenti strutturali ed offerta di lavoro.*
- 1993, 9 Niall O'HIGGINS  
*The effectiveness of YTS in Britain: an analysis of sample selection in the determination of employment and earnings*
- 1993, 8 Giuseppe CELI  
*Politiche valutarie ed integrazione commerciale: l'esperienza dello SME negli anni '80*
- 1992, 7 Paolo COCCORESE  
*Attività innovativa e configurazione industriale*
- 1992, 6 Maria Rosaria GAROFALO, Gian Luigi CORINTO  
*La Razionalità dell'Allocazione del Tempo di Lavoro in Agricoltura. Con un'Applicazione Empirica ad un Sistema Locale attraverso la PL..*
- 1992, 5 Adalgiso AMENDOLA, Maria SCATTAGLIA  
*Disoccupazione e Tassi di Attività nel Mezzogiorno*
- 1992, 4 Floro Ernesto CAROLEO  
*La Disoccupazione Strutturale*
- 1991, 3 Giovanni URGA  
*Dynamic Models of Labour Demand in the Italian Industrial Sector: Theories and Evidence from Panel Data*
- 1991, 2 Adalgiso AMENDOLA  
*Teoria dei Contratti Impliciti. Rigidità del Salario e Disoccupazione*
- 1991, 1 Guido CELLA  
*Sulla Integrazione Produttiva Interregionale: il Caso del Mezzogiorno*

CELPE – Centro di Economia del Lavoro e di Politica Economica  
Università degli Studi di Salerno  
Depositato ai sensi di Legge.