Dipartimento di Scienze Economiche
Università degli Studi di Firenze

Working Paper Series

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P. Balduzzi, C. Graziano and A. Luporini

Working Paper N. 01/2011
January 2011

Dipartimento di Scienze Economiche, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze, Italia
www.dse.unifi.it

Stampato in proprio in Firenze dal Dipartimento Scienze Economiche
(Via delle Pandette 9, 50127 Firenze) nel mese di Gennaio 2011,
Esemplare Fuori Commercio Per il Deposito Legale
agli effetti della Legge 15 Aprile 2004, N.106

Voting in Small Committees*

Paolo Balduzzi†
Catholic University of Milan
IEF and CIFREL

Clara Graziano‡
University of Udine and CESifo

Annalisa Luporini§
University of Florence and CESifo

January 13, 2011

Abstract

We analyze the voting behavior of a small committee that has to approve or reject a proposal whose return is uncertain. Members have heterogenous preferences: some members want to maximize the expected value while other members have a bias toward project approval and ignore their private information. We analyze different voting games when information is costless and communication is not possible, and we provide insights on the optimal composition of these committees. Our main result is that the presence of biased members can improve the voting outcome by simplifying the strategies of unbiased members. Thus, committees with heterogeneous members can function at least as well as homogeneous committees and in some cases they perform better. In particular, when value-maximizing members hold 51% of votes, the socially optimal equilibrium becomes unique.

Key words: Voting, Small committees.

JEL classification: D71, D72.

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*We thank participants from the ASSET Conference 2008, the CEPET Workshop 2009, the EALE Conference 2009, CESifo Seminar in applied Micro 2010 and from seminars held at Stony Brook University, Catholic University of Milan, University of Milan-Bicocca for helpful suggestions. We thank Patrick Leyens and Roland Kirstein for valuable discussion and comments. All mistakes are the authors' own responsibility. Financial help from IEF (Catholic University) and PRIN 2008 is gratefully acknowledge.

†Email: paolo.balduzzi@unicatt.it; Skype: paolo.balduzzi.

‡Email: clara.graziano@uniud.it.

§Email: annalisa.luporini@unifi.it.

1 Introduction

In this paper, we focus on the most efficient use of scarce information in small decisional bodies, such as committees, where members have different preferences. The most straightforward example is given by boards of directors, whose objective, in principle, is to maximize a firm's profit. A peculiar feature of boards is that directors represent different stakes (majority and minority shareholders, investors, workers, etc.). As a result, members of the board may have different preferences which in turn reflect in different behaviors. Other examples are provided by special juries (e.g.: Supreme or Constitutional Courts) and by technical committees, where politicians, bureaucrats and experts meet to provide advice.

We look at a generic small committee and address two issues. First, we analyze the effect of heterogeneity by studying the voting behavior of a committee with two kinds of players: expected value maximizers and biased members. In this heterogeneous committee a further problem arises. Biased members find it optimal to destroy their information. So, why should they be allowed in these committees? Surprisingly, their presence ensures uniqueness *and* optimality of the equilibrium strategy profile. The intuition for this result is that the bias provides certainty about these members' strategies simplifying the response of the other members and therefore reduces the number of (otherwise) multiple equilibria. Second, we explore the behavior of uninformed members when they are forced to vote (e.g., they are not allowed to abstain). If these members want to maximize the probability that the committee takes the correct decision, they face the question of how to avoid influencing the committee decision and to let informed members determine it. Finally, our model provides some suggestions about the optimal composition of such committees.

The paper is organized as follows. In section 2 we present the basic model. In Section 3 we examine the voting game in a committee composed only of value-maximizing members. In Section 4, we introduce heterogeneity and we show how results change (or not) when members have different objectives. Then, in Section 5 we review the main literature and in Section 6 we conclude. All proofs are collected in the appendix.

2 The model

A committee is composed of $2n + 1$ members and its goal is to decide whether to approve a project (voting "yes") or reject it (voting "no") by majority vote. If the proposal is rejected, a value of 0 is realized. If accepted, the project can take one of two values, $v : \{-1, 1\}$, according to the specific state of nature. In particular, if the state of the world is low (L), $v = -1$; if the state of the world is high (H), $v = 1$. Each state, and thus each value, has the same prior (i.e., $\frac{1}{2}$). Voting correctly means achieving a maximum expected value of $\frac{1}{2}$ (rejecting the project in L and approving it in H). Note that a single uninformed decision maker would always achieve an expected value of 0.

A member of the committee learns the true state with probability $\alpha \in [\frac{1}{2}, 1)$ and with probability $1 - \alpha$ he learns nothing. As a consequence, the information set of a generic member of the board m is simply $\Omega_m = \{\omega_m\}$, with $\omega_m \in \{H; L\}$, when m is informed. On the contrary, the information set of an uninformed member is $\Omega_m = \{H, L\}$, as he does not know the true state of the world.

We assume that members cannot communicate and abstention is not possible. The first assumption is generally not made without loss of generality, and we consider that in a companion paper (Balduzzi, Graziano and Luporini [2011]). As to the lack of abstention, this assumption may seem restrictive and is worth an additional comment. Allowing abstention would bring other issues and restrictions into the picture, which would drive the attention away from the scope of the paper. First of all, it is not clear what would happen when everyone abstains. Second, it is neither clear nor straightforward what the decision rule should be: simple majority of members or simple majority of actual votes?[1].

As a consequence of the no abstention assumption, the action set for each player is simple: vote "yes" to accept the project or vote "no" to reject it. A strategy s_m is a member's voting behavior, conditional on his information set. A mixed strategy is defined as the probability that a member votes "yes".

Our paper compares the voting outcome in two different committees. First, we study the voting behavior and the performance of a committee composed uniquely by members who wish to maximize the probability that the committee makes the correct decision (i.e., voting "yes" if the state of the world is H and "no" otherwise). Then, we study what happens when some members have a bias and always want to approve the proposal, independently of the true state of the world. We assume that members are risk neutral and that their types are common knowledge. Let O denote members maximizing expected value and I members with a bias. Then, we call o and i the probability of voting "yes" when uninformed for a member of type O and I respectively.

The utility function of an O type positively depends on the expected value of the project; in particular, we assume that it actually corresponds to its expected value: $u_O(E(v)) = E(v)$.

[1] In many committees abstention is explicitly or implicitly ruled out (the Italian Constitutional Court and the European Courts of Human Rights are two examples).

Given this utility function, an O member will choose the strategy that maximizes $E(v)$. To this end, he will condition his strategy on being pivotal, because that is the only case where he can actually influence the outcome of the voting process and therefore his own utility. Since any strategy is optimal when the player is not pivotal, without loss of generality, we concentrate on weakly dominant strategies.

Notice that, given the values the project can take, maximizing $E(v)$ is equivalent to maximizing the probability that the committee takes the correct decision. Indeed, the latter is given by the weighted sum of the probability that "yes" wins when the actual value of the alternative is 1 and that "no" wins when the actual value of the alternative is -1:

$$\frac{1}{2}\left\{Y(\cdot \mid v=1)+\frac{1}{2}[1-Y(\cdot \mid v=-1)]\right\}, \tag{1}$$

where the function $Y(\cdot)$ is the probability that the committee as a whole votes "yes". The expected value of the project, $E(v)$, is:

$$E(v)=\frac{1}{2}[E(\pi|v=1)+E(\pi|v=-1)]=\frac{1}{2}[Y(\cdot|v=1)-Y(\cdot|v=-1)] \tag{2}$$

and it can be immediately noticed that expressions (1) and (2) are strategically equivalent.

The utility function of an I type positively depends on the approval of the project[2]. He always supports the proposal, regardless of the value which is *ex post* realized. His utility u_I is therefore a function of the final decision of the committee, D, where $D=Y$ when the proposal is approved ("yes" wins) and $D=N$ when the project is rejected ("no" wins). Accordingly, $u_I(D)$ assumes the following two values: $u_I(D)=1$ if if $D=Y$, and $u_I(D)=1$ if $D=N$. This clearly implies that always voting "yes" is a dominant strategy for I. For simplicity, we abstract from additional problems, such as member I's reputation when his proposal creates a loss or is rejected.

Compensating Strategy

Before analyzing the voting behavior of the committee, we introduce the definition of compensating strategy that will be useful in the following sections.

Definition 1 (Compensating strategy) *Two players are compensating for each other when the following conditions are satisfied: i) they are both uninformed; ii) they play "yes" with probabilities whose sum is equal to 1. When these probabilities take extreme values (0 or 1), we have compensation in pure strategies.*

[2] Alternatively we can interpret the behavior of this member type as the consequence of his overconfidence in the validity of the proposal. This however implies that the prior he attaches to $v=1$ is higher than $\frac{1}{2}$.

We consider we concentrate on equilibria where directors of the same type follow the same type of strategy, i. e. either they all play pure strategies or they all play mixed strategies.

3 The social optimum

We define socially optimal a committee where each member has the object to maximize the expected value; i.e., a committee composed only of O members. Notice that this composition maximizes the probability that the committee is correct.

Suppose that $\alpha = \frac{1}{2}$ and $n = 2$, so that there are five members O_i, $i = 1, 2, 3, 4, 5$. It can be easily shown that there are only two types of equilibria differing as to the behavior of uninformed members. The first is an asymmetric equilibrium, where four members compensate in pure strategies when uninformed; the second is a symmetric equilibrium, where four members compensate in mixed strategies when uninformed. Whenever informed, instead members vote according to their information. As the identity of members who vote "yes" and members who vote "no" in the asymmetric equilibrium is interchangeable, there exist in fact a multiplicity of such equilibria all of which yield the same expected value.

Consider an asymmetric equilibrium where each member plays a pure strategy. In order to understand its features, suppose that, contrary to the equilibrium strategies, 4 members vote "yes" when uninformed. The remaining member is pivotal only if two members vote "no". But this happens only if these two members are informed. The remaining member therefore should vote "no". The same argument holds to show that there cannot exist a symmetric equilibrium in pure strategies where everyone votes "no". So, an equilibrium in pure strategies can only be an asymmetric one. Moreover, by applying the same argument as above, it can be shown that a situation where 4 members vote "yes" (or "no") when uninformed cannot be an equilibrium: voting "yes" ("no") is not the best response for an uninformed individual when there are already 3 members following such a strategy. Intuitively, any uninformed member has an incentive to leave the final decision to others, who may be informed. As that member cannot abstain, it is optimal for him to vote the opposite of another uninformed member.

Suppose that two members vote no when uninformed, e.g. $o_1 = o_2 = 0; o_3 = o_4 = o_5 = 1$. It can be easily shown that

$$E(v) = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2}\left[1 - \frac{1}{8}\right] = \frac{7}{16} < \frac{1}{2}$$

In the spirit of Condorcet, the expected value of the project, as defined in (2), is much bigger than the value obtained by a single decision maker. Still, this optimal committee does not provide full aggregation of information. In other words, it does not necessarily *always* take the correct decision. Nonetheless, no alternative asymmetric equilibrium in pure strategies can achieve a higher expected value. To make this point more clear, we provide also the remaining possible asymmetric pure strategies profiles, with the respective outcomes (we ignore cases which are simple permutation of others):

Profile	$E(v)$
$o_1 = o_2 = 0; o_3 = o_4 = o_5 = 1$	$\frac{1}{2}\left[1 - \frac{1}{8}\right] = \frac{7}{16}$
$o_1 = o_2 = o_5 = 0; o_3 = o_4 = 1$	$\frac{1}{2}\left[\frac{7}{8}\right] = \frac{7}{16}$
$o_1 = o_2 = o_3 = o_4 = 0; o_5 = 1$	$\frac{1}{2}\left[\frac{11}{16}\right] = \frac{11}{32}$
$o_1 = 0; o_2 = o_3 = o_4 = o_5 = 1$	$\frac{1}{2}\left[1 - \frac{5}{16}\right] = \frac{11}{32}$

It can be immediately seen that in the last two profiles, given other members' strategies, any member has an incentive to change his strategy in order to increase $E(v)$. Thus, the only possible equilibria are the ones where four members compensate in pure strategies.

These, however, are not the only equilibria of the game. There also exists an additional equilibrium where all members compensate in symmetric mixed strategies:

Profile	$E(v)$
$o_1 = o_2 = o_3 = o_4 = o_5 = \frac{1}{2}$	$\frac{203}{512} < \frac{7}{16}$

Intuitively, any member can randomize as long as, given other members' strategies, he is pivotal in both states of the world with the same probability. But this is true for any member only if all members are pivotal in each state of the world with the same probability. The only profile which is compatible with this logic is the one described above. This logic rules out any other possible equilibria in mixed strategies: whenever a member is not indifferent among voting "yes" and voting "no" (because the probability of being pivotal in a state is higher than the probability of being pivotal in another state), he plays a pure strategy. But then, other members will have an incentive to deviate from any mixed strategy to compensate for another pure strategy. Proposition 1 generalizes our findings.

Proposition 1 (Benchmark) *In a voting game with $2n + 1$ members of type O, informed members always play according to their information. As to the behavior of uninformed players there are two types of equilibria: i) n players vote "no", n players vote "yes", and one player chooses $o \in \{0, 1\}$; ii) $2n + 1$ players randomize with probability $\frac{1}{2}$. Equilibria of type i) yield expected value $E(v) = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$. Equilibria of type ii) yield lower expected value.*

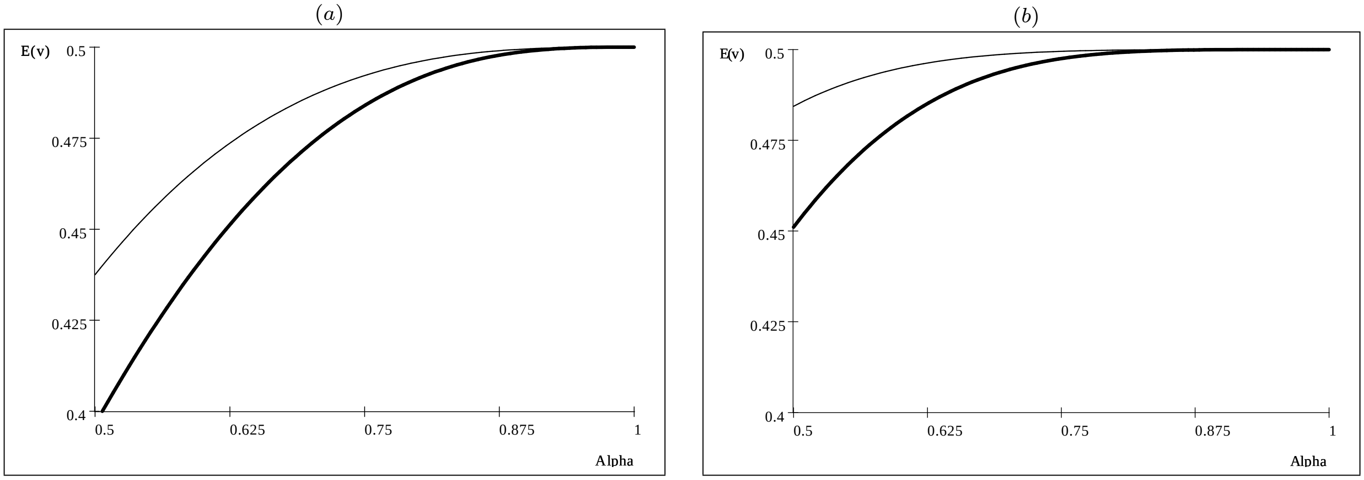
The intuition for this result is the same as in the five members case: uninformed members do not want to influence the outcome of the voting process, so they compensate each other and leave the final decision to informed members. This is also what happens in the second kind of equilibria where uninformed players compensate in mixed strategies. Compensation is more effective when played in pure strategies, as it is realized with probability one: $E(v)$ is higher in the asymmetric equilibria in pure strategies than in symmetric equilibrium in mixed strategies. Then, in what follows we take this level of $E(v)$ as our benchmark.

Definition 2 *An equilibrium is defined optimal if it yields* $E(v) = \frac{1}{2}[1 - (1-\alpha)^{n+1}]$, *i.e. if it yields the same expected value given by equilibria of type i).*

Just for illustration, in Graph 1 we draw the relationship between the $E(v)$ and the probability of having informed members (α), in committees with 5 and 9 members. In both graphs, we compare the optimal equilibrium outcome (pure strategies, thin line) with the symmetric mixed strategies equilibrium outcome (thick line). The relationship between $E(v)$ and α is clearly positive. Indeed, when all members are informed, there is no difference between the two equilibria and $E(v)$ are the same.

Graph 1: $E(v)$ and α

(a) (b)



5 members 9 members

From visual comparisons of the pictures, it is also clear that $E(v)$ is growing in n. This relation is formalized by Corollary 1 which immediately follows from $E(v) = \frac{1}{2}[1-(1-\alpha)^{n+1}]$

Corollary 1 *When an optimal equilibrium is played, an increase in n or in α induces an increase in $E(v)$.*

4 Heterogeneous preferences

Having defined the social optimum, we can compare this benchmark to the outcome of a committee composed of members with heterogeneous preferences. Consider again a committee with $n = 2$ (five members), and $\alpha = \frac{1}{2}$ but let now members be either of type O or of type I. If members of type I hold a strict majority, the committee always approves the proposal and remaining members are never pivotal. Then, there are only two interesting cases: the committee is composed of O_1, O_2, O_3, O_4 and I_5 and the committee is composed of O_1, O_2, O_3, and I_4 and I_5.

We start from the latter case where it can be easily shown that there exists a unique type of equilibrium. Members of type I always approve the proposal, independently of their information. Members of type O, whenever informed, vote according to their information; when they are not informed, they reject the proposal. Given the strategies of I members, this rejection is optimal, as the probability that an uninformed member is pivotal is obviously higher when the state of the world is L. In this case, we can easily compute:

$$E(v) = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2}\left[\frac{7}{8}\right] = \frac{7}{16}$$

Quite surprisingly, the performance of this committee is the same as the performance of the committee with only value-maximizing members. In addition, this type of equilibrium is unique. To understand the importance of this finding, recall that in the committee composed only of O members, the equilibrium in (symmetric) mixed strategies was suboptimal. Thus, we can say that, provided members of type O hold the strict majority, the heterogeneous committee ensures the socially optimal outcome.

As to the case in which the committee is composed of O_1, O_2, O_3, O_4 and I_5, two types of equilibria emerge differing as to the behavior of uninformed O members. Whenever informed in fact O members vote according to their information and the unique I member always approves the project in both types of equilibria. In the equilibria of the first type, two O members compensate in pure strategies when uninformed and the remaining O members reject the proposal. In the equilibrium of the second type, all the four members of type O play the same mixed strategy when uninformed.

Equilibria of the first type are optimal:

$$E(v) = \frac{1}{2}[Y(\cdot|v = 1) - Y(\cdot|v = -1)] = \frac{1}{2}\left[\frac{7}{8}\right] = \frac{7}{16}$$

The equilibrium of the second type is suboptimal. Solving for the equilibrium under the constraint that all of the O members choose the same mixed strategy when uninformed, we find the following solution: $o_1 = o_2 = o_3 = o_4 = \frac{5-\sqrt{13}}{6}$, and

$$E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \simeq \frac{197}{512} < \frac{7}{16}$$

Intuitively, when O members are more than 51%, they have two possible choices. Either one single member offsets the bias of I_5 in pure strategies and the remaining members compensate in pure strategies, or all of the O members play the same mixed strategy with the aim to collectively compensate the bias of I_5. Hence, with heterogenous members, the symmetric mixed strategy of O members is biased towards rejection.

Proposition 2 generalizes these results.

Proposition 2 *Consider a committee with $2n+1$ members where members of type I always approve the project and informed members always vote according to their information. We can distinguish two cases:*

i) if there are n members of type I and $n+1$ members of type O the game has a a unique equilibrium in which all members of type O vote "no" when uninformed. This equilibrium coincides with the social optimum;

ii) if there are $n-k$ members of type I $(n > k > 0)$ and $n+1+k$ members of type O, the voting game may have multiple equilibria, one of which coincides with the social optimum. In the optimal equilibrium $2k$ members of type O compensate in pure strategies and the remaining $n-k+1$ members O vote "no" when uninformed.

From the above proposition, Corollary 2 immediately follows.

Corollary 2 *$E(v)$ is not increased by increasing the proportion of outside members above $(n+1)\,/\,(2n+1)$.*

For a given size of the committee, increasing the proportion of O members is not profitable, provided that they already hold the majority. By increasing the proportion of O members, socially optimal equilibria can still be obtained but there may also exist other kind of equilibria. Thus, if the proportion of biased members reaches 49% of the committee, the situation is greatly simplified with respect to our benchmark case, because the socially optimal equilibria are unique.

Increasing the size of the committee (increasing n), on the contrary, increases $E(v)$. Recall that

$$E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)];$$

then the following corollary holds.

Corollary 3 *If the socially optimal equilibrium is played, an increase in n induces an increase in $E(v)$.*

Provided that a strict majority of O members is maintained, the probability of approving the project when the state of nature is unfavorable $Y(\cdot|v = -1)$ is not increased by an increase in n because the effect of additional biased members is compensated for by the O members rejecting the proposal when uninformed. On the contrary, the probability of approving the project when it is profitable, $Y(\cdot|v = 1)$, increases with n, because it is equal to the probability that at least one of those $n + 1$ members of type O who choose $o = 0$, is informed. Hence adding new members (including biased ones) is profitable[3].

5 Related literature

Since Condorcet's seminal contribution, namely his Jury Theorem, the literature about voting has been constantly growing. A lot of papers have generalized the Jury Theorem[4], and many others have extended voting games to include both naive and strategic voting[5].

In the voting literature, preferences are usually assumed to be homogeneous, although a few papers have addressed the issue of conflict of interest. The main contributions are Feddersen and Pesendorfer [1996] and [1997] who analyze how well simultaneous voting in large elections can aggregate private information. They show that the probability of electing the "wrong" candidate asymptotically goes to zero. Conflict of interest is more problematic in small committees where information aggregation may be severely limited by strategic voting induced by divergent interests. In a standard Condorcet Jury Theorem framework, this problem may be partially offset by providing minority members with optimal incentives to participate in voting, thus not wasting their information (Chwe [1999]).

Things become more complicated when the relevant issue is not to find optimal voting rules but rather an optimal way to aggregate "useful (or correct)" information. Li, Rosen and Suen [2001] examine a two-person committee where each member receives a private signal and reports his information. Since members have conflicting interests, strategic considerations

[3] In a companion paper (Balduzzi, Graziano and Luporini [2011]), we consider positive costs of information; this allows us to provide insights on the optimal size of such committees.

[4] See, for instance, Duggan and Martinelli [2001], Myerson [1998] and McLennan [1998]. See also Piketty [1999] for a brief review of recent contributions about the information-aggregation role of political institutions.

[5] See Austen-Smith and Banks [1996] and an experiment on the use of strategic voting by Eckel and Holt [1989].

induce information misreporting and there is no truth-telling equilibrium. Conflict of interest prevents full information aggregation also in the model by Maug and Yilmaz [2002]. The authors suggest to group voters into separate classes because when voters have sufficiently strong conflict of interests and different information, a two-class voting mechanism may alleviate the incentive to withhold information and is better than one-class voting in the sense that voting decisions become more informative. Wolinsky [2002] suggests to solve the problem in a similar way, by partitioning experts in different groups. This may enhance their incentive to reveal information, even if their preferences differ from those of the principal, at least when they think they are pivotal. Cai [2009] develops a model of committee size, where information gathering is costly and preferences are heterogeneous. In his model, experts learn their preferences by gathering information. On the contrary, in our model preferences are known and are strategically used exactly to maximize the expected value of the project. Another difference between his model and ours is that our main focus is on the composition of the committee rather than on its size.

Finally, our paper is close to the literature on social learning (Banerjee [1992]; Bikhchandani, Hirshleifer and Welch [1992; 1998]; Gale [1996]; Palley [1995]; Scharfstein and Stein [1990]) and, in particular, on the role of voting in social learning (Ali and Kartik [2008]; Dekel and Piccione [2000]; Goeree, Palfrey and Rogers [2006]; Palley [1995]; Ottaviani and Sørensen [2001]; Smith and Sørensen [2000]). We share with most of these papers features like the role of voting as an aggregating information device, binary vote, impossibility of abstention[6] and of communication[7].

6 Conclusions

We have analyzed the voting behavior of a small committee that has to approve or reject a proposal whose return is uncertain. Members have heterogenous preferences: some members want to maximize expected value but some other members have a bias toward project approval and ignore their private information. We have shown that committees with heterogeneous members can function at least as well as committees with homogeneous members and in some cases they can perform better. In particular, when value-maximizing members are just 51%, the presence of biased members can improve the voting outcome by simplifying the voting strategies of unbiased members. Thus, the socially optimal equilibrium becomes unique. Furthermore, increasing the number of value-maximizing members above 51% does not increase the expected value.

[6] Feddersen and Pesendorfer [1997] is an exception.

[7] The relevance of debate and communication is emphasized by several other authors, such as Austen-Smith [1990] and Doraszelski *et al.* [2001].

7 Appendix

7.1 Proof of Proposition 1

First, in equilibrium, each informed member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. Second, recall that value-maximizing members choose their strategies as if they were pivotal, as what they do when they are not pivotal is irrelevant for the voting outcome. Thus, we concentrate on equilibria in weakly dominant strategies. Third, we focus only on two types of equilibria: equilibria where all members play pure strategies and equilibria where all members play mixed strategies. So in principle we can have the following four types of equilibria:

i) asymmetric equilibria in pure strategies (AP)

ii) symmetric equilibria in mixed strategies (SM);

iii) symmetric equilibria in pure strategies (SP);

iv) asymmetric equilibria in mixed strategies (AM).

We prove that, when the committee is composed of $2n + 1$ members of type O, there only exist equilibria of type i) and ii). In particular, in the equilibria of type i) n members choose $o_j = 0$, n members choose $o_j = 1$, and one member, denoted by O_i $i \neq j$, chooses $o_i \in \{0, 1\}$; in the unique equilibrium of type ii), all the $2n+1$ members choose $o_j = \frac{1}{2}$. Finally, we prove that equilibria of type b) maximize expected value.

Equilibria of type i): Asymmetric equilibria with compensation in pure strategies.

The proof of the existence of these equilibria is divided in four steps. First we prove that, given the strategies of the other $2n$ players , player i is voting optimally; then we prove that the remaining $2n$ are voting optimally as well (steps 2 and 3). Finally, we prove that this is the only asymmetric equilibria in pure strategies of the game.

1. If n members choose $o_j = 1$, n members choose $o_j = 0$, the best response of O_i, $i \neq j$, is to choose $o_i \in \{0, 1\}$.

When n members are voting "yes" and n members are voting "no" O_i is pivotal in both states of the world with the same probability. Indeed, when $v = 1$, O_i is pivotal when everybody else is uninformed or when the only informed members are those who would vote "yes" even when uninformed (thus not changing their votes whether informed or not). As the former case (everybody is uninformed) is a subcase of the latter, the probability that O_i is pivotal is

$$(1-\alpha)^n \left[\sum_{j=0}^{n} \frac{n!}{j!(n-j)!} \alpha^{n-j} (1-\alpha)^j \right] = (1-\alpha)^n.$$

Ceteris paribus, when $v = -1$, O_i is pivotal when everybody else is uninformed or when the only informed members are those who would vote "no" even when uninformed. Then the probability that O_i is pivotal is still $(1-\alpha)^n$. Hence, O_i is indifferent between the two possible values of $o_i \in \{0, 1\}$.

2. If n members choose $o_j = 1$, $n-1$ members choose $o_j = 0$, and member O_i, $i \neq j$, chooses $o_i \in \{0,1\}$ the best response of the remaining member (denoted by k, $k \neq i, j$) is to choose $o_k = 0$.

If O_i chooses $o_i = 0$, we are back to point 1. So the optimal response of the remaining O_k is $o_k \in \{0, 1\}$. On the contrary, if $o_i = 1$, then O_k is pivotal only if $v = -1$; hence, choosing $o_k = 0$ is a weakly dominant strategy.

3. If $n-1$ members choose $o_j = 1$, n members choose $o_j = 0$, and member O_i, $i \neq j$, chooses $o_i \in \{0,1\}$ the best response of the remaining member (denoted by k, $k \neq i, j$) is to choose $o_k = 1$.

The argument is symmetric to the one used at point 2.

4. This is the only possible asymmetric equilibria in pure strategies.

Suppose $n+i$ members ($i \in \{2, 3, ..., n\}$) are choosing $o_j = 1$. Then every remaining member knows that he is pivotal with a higher probability when $v = -1$. Hence, the remaining $n - i - 1$ members choose $o_j = 0$. However, this cannot be an equilibrium. Also members voting $o_j = 1$ know that they are pivotal with a higher probability when $v = -1$. Hence, they have an incentive to change their strategy and play $o_j = 0$. This is true as long as $i \neq 0$.

Equilibria of type ii): Symmetric equilibrium with compensation in mixed strategies.

The proof of the existence of this equilibrium is divided in two steps.

1. If $2n$ members choose $o_j = \frac{1}{2}$, the best response of the remaining member (denoted by i, $i \neq j$) is to choose $o_i = \frac{1}{2}$.

O_i is pivotal when a) everybody is uninformed, n members vote "yes" and the others n members vote "no", or b) only up to n members are informed and uninformed members vote in such a way that results in n members voting "yes" and n members voting "no". Both states of the world are equally possible, so that O_i is pivotal with the same probability in both states. Consequently he is indifferent among any $o_i \in [0, 1]$. Given that the same holds true for every member, it immediately follows that $o = \frac{1}{2}$ for all members sustains an equilibrium of the game. The probability that a member is pivotal in this case is given by

$$(1-\alpha)^n \left(\frac{1}{2}\right)^n \left[\sum_{h=0}^{n} \frac{n!}{h!(n-h)!} \alpha^{n-h}(1-\alpha)^h \left(\frac{1}{2}\right)^h\right].$$

2. If at least one member chooses $o_i \neq \frac{1}{2}$, the best response of a generic agent $k \neq i$ is to choose a pure strategy.

If O_i were to choose $o_i > \frac{1}{2}$, the best response of a generic agent k, $k \neq i$, would be $o_k = 0$, because O_k would be pivotal with higher probability in $v = -1$ than in $v = 1$. With $2n - 1$ members choosing $o_j = \frac{1}{2}$ and O_k choosing $o_k = 0$, however the best response of O_i becomes $o_i = 1$, because O_i would be pivotal with higher probability in $v = 1$ than in $v = -1$.

Symmetrically if O_i were to choose $o_i < \frac{1}{2}$, the best response of a generic agent $k \neq i$, would be $o_k = 1$, because O_k would be pivotal with higher probability in $v = -1$ than in $v = 1$. With $2n - 1$ members choosing $o_j = \frac{1}{2}$ and O_k choosing $o_k = 1$, however the best response of O_i becomes $o_i = 0$, because O_i would be pivotal with higher probability in $v = -1$ than in $v = 1$.

Inexistence of other equilibria

There is no symmetric equilibrium in pure strategies (type iii)). Suppose in fact that $2n$ uninformed members always vote "yes". Then, the remaining member is pivotal only if some members voted "no". But this happens only if those who voted "no" were informed. The remaining member therefore should vote "no". The same arguments holds to prove that there does not exist an equilibrium of type iii) where everyone votes "no".

There is no asymmetric equilibrium in mixed strategies (type iv)). This indirectly follows from the fact the we proved the existence of a symmetric mixed strategies equilibrium with $o_j = \frac{1}{2}$. As shown in that proof, as soon as a member j deviates and plays $o_j \neq \frac{1}{2}$, any other member z, $z \neq j$, has an incentive to play a pure strategy (either $o_z = 0$ or $o_z = 1$).

Optimal equilibria

Equilibria with compensation in pure strategies (equilibria of type i), are optimal, i. e. they are associated to the highest possible $E(v)$.

In equilibria of type i), $E(v)$ is equal to

$$\begin{aligned} E(v)_{PS} &= \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \\ &= \frac{1}{2}\left[1-(1-\alpha)^{n+1}\right]. \end{aligned}$$

In the equilibrium of type ii), $E(v)$ is equal to

$$\begin{aligned} E(v)_{MS} &= \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \\ &= \frac{1}{2}\left[1 - Y(\cdot|v=-1) - (1 - Y(\cdot|v=1)\right]. \end{aligned}$$

We can focus on the probability of taking the wrong decision

$$Y(\cdot|v=-1) + (1 - Y(\cdot|v=1).$$

This is equal to

$$(1-\alpha)^{n+1}\left(\frac{1}{2}\right)^{n}\sum_{i=0}^{n}\frac{(2n+1)!}{i!(2n+1-i)!}\left(\alpha+\frac{(1-\alpha)}{2}\right)^{i}\left(\frac{(1-\alpha)}{2}\right)^{n-i}.$$

Then

$$\begin{aligned} E(v)_{MS} &= \frac{1}{2}\left[1 - Y(\cdot|v=-1) - (1 - Y(\cdot|v=1)\right] \\ &= \frac{1}{2}\left[1-(1-\alpha)^{n+1}\left(\frac{1}{2}\right)^{n}\sum_{i=0}^{n}\frac{(2n+1)!}{i!(2n+1-i)!}\left(\alpha+\frac{(1-\alpha)}{2}\right)^{i}\left(\frac{(1-\alpha)}{2}\right)^{n-i}\right]. \end{aligned}$$

Considering that

$$\begin{aligned} &\left(\frac{1}{2}\right)^{n}\sum_{i=0}^{n}\frac{(2n+1)!}{i!(2n+1-i)!}\left(\alpha+\frac{(1-\alpha)}{2}\right)^{i}\left(\frac{(1-\alpha)}{2}\right)^{n-i} > \\ &> \left(\frac{1}{2}\right)^{n}\frac{(2n+1)!}{i!(2n+1-i)!}\left(\alpha+\frac{(1-\alpha)}{2}\right)^{n} > 1, \end{aligned}$$

it can be immediately verified that $E(v)_{MS} < E(v)_{PS}$.

7.2 Proof of Proposition 2

In equilibrium, each informed O member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. Recall that O members choose their strategies as if they were pivotal, as what they do when they are not pivotal is irrelevant for the voting outcome. Thus, we concentrate on equilibria in weakly dominant strategies. Third, we focus only on two types of equilibria: equilibria where all members of the same type play pure strategies and equilibria where all members of the same type play mixed strategies. We prove that:

i) when the committee is composed of $n+1$ value-maximizing members and n biased members, there exists a unique equilibrium where each O member votes "no" when uninformed;

ii$_1$) when the committee is composed of $n+1+k$ value-maximizing members and $n-k$ biased members, there always exist equilibria where $n-k$ value-maximizing members vote "no" when uninformed and the remaining $2k$ value-maximizing members compensate in pure strategies. These are the only pure strategy equilibria of the voting game.

ii$_2$) when the committee is composed of $n+1+k$ value-maximizing members and $n-k$ biased members, there may exist equilibria where all the $n+1+k$ value-maximizing members play the same mixed strategy;

iii) The equilibria in pure strategies (sub i) and sub ii$_1$)) are optimal.

Point i) To prove that the unique equilibrium is the one where all O members vote "no" whenever uninformed ($o_i = 0; i = 1, 2, ..., n+1$), consider outsider O_{n+1}.

When $v = 1$, O_{n+1} is pivotal only if all other O members are uninformed and vote "no", which happens with probability:

$$(1-\alpha)^n \prod_{j=1}^{n} (1 - o_j).$$

When $v = -1$, O_{n+1} is pivotal if:

- all other O members are uninformed and vote "no", which happens with probability

$$(1-\alpha)^n \prod_{j=1}^{n} (1 - o_j),$$

- all the other O members are informed, which happens with probability

$$\alpha^n,$$

- at least one (but not all) of the other O members is informed and the others vote "no" when uninformed, which happens with probability

$$\sum_{h=1}^{n} \frac{n!}{h!(n-h)!} \alpha^{n-h} (1-\alpha)^h \prod_{j=1}^{h} (1 - o_j).$$

where $\frac{n!}{h!(n-h)!}$ represents the number of combination with h uninformed value-maximizing members and $n-h$ informed value-maximizing members. It is straightforward that O_{n+1} is pivotal with a higher probability in the bad state. Hence O_{n+1} chooses $o_{n+1}=0$. As the same reasoning holds for any other outsider $j \neq n+1$, it follows that every O member will vote "no" when uninformed.

Finally, note that we have not restricted o_j, $j \neq n+1$, to any particular value, so the result also proves that this equilibrium is unique.

Point ii$_1$) In the case of $n-k$ insiders ($n > k > 0$) and $n+1+k$ value-maximizing members, there exist multiple equilibria with $n-k+1$ value-maximizing members voting against the project and $2k$ value-maximizing members compensating for each other.

We prove the existence of these equilibria in three steps. In the first step, we prove that when $n-k$ value-maximizing members vote against the project and $2k$ value-maximizing members compensate for each other, the remaining O member has still an incentive to vote against the project; in the second step, we prove that when $n-k$ value-maximizing members vote against the project to contrast the $n-k$ insiders, and a majority of the other value-maximizing members also vote against the project, the remaining O member has an incentive to compensate, voting "yes". Finally, we show that there are no other equilibria in pure strategies.

1. If n value-maximizing members choose $o_j = 0$, and k value-maximizing members choose $o_j = 1$, the best response of O_i, $i \neq j$, is to choose $o_i = 0$.

When $v = 1$, O_i is pivotal if all the value-maximizing members are uninformed or if at least one of those k value-maximizing members who choose $o_j = 1$ when uninformed, is in fact informed. Thus, O_i is pivotal with probability

$$(1-\alpha)^n \left[\sum_{j=0}^{k} \frac{k!}{j!(k-j)!} \alpha^{k-j} (1-\alpha)^j \right] = (1-\alpha)^n$$

where $\frac{k!}{j!(k-j)!}$ represents the number of combination with j uninformed value-maximizing members, $k-j$ informed O members and the term in bracket is equal to 1 from the binomial theorem. When $v = -1$, O_i is pivotal if all the O members are uninformed or if at least one of those n value-maximizing members who chooses $o_j = 0$ when uninformed, is in fact informed. Then O_i is pivotal with probability

$$(1-\alpha)^k \left[\sum_{j=0}^{n} \frac{n!}{j!(n-j)!} \alpha^{n-j} (1-\alpha)^j \right] = (1-\alpha)^k$$

Given that $(1-\alpha)^k > (1-\alpha)^n$, the probability that O_i is pivotal is higher when $v = -1$ than when $v = 1$. Hence O_i chooses $o_i = 0$.

2. If $n+1$ value-maximizing members choose $o_j = 0$ and $k-1$ value-maximizing members choose $o_j = 1$, the best response of O_i, $i \neq j$ is to choose $o_i = 1$

When $v = 1$, O_i is pivotal if only one of the $n+1$ value-maximizing members choosing $o_j = 0$ is informed and votes "yes". This happens with probability

$$(n+1)(1-\alpha)^n \alpha.$$

On the contrary, O_i is never pivotal when $v = -1$. Hence, he chooses $o_i = 1$.

Finally, note that any O member can be in the position of O_i or of an O_j voting "yes", or also of an O_j voting "no". Thus, there is a multiplicity of equilibria such as the one we are considering.

3. There cannot exist other equilibria in pure strategies than those characterized at points 1 and 2.

We must now consider what happens if either a) more than n value-maximizing members vote "no" and the others vote "yes", or b) more than k value-maximizing members vote "yes" and the rest vote "no".

a) *If $n-h$ value-maximizing members choose $o_j = 0$, and $k+h$ value-maximizing members choose $o_j = 1$, $n \geq h > 0$, the best response of O_i, $i \neq j$, is to choose $o_i = 0$* because O_i is never pivotal when $v = 1$ while he may be pivotal when $v = -1$. This happens in the case where h of those $n+h$ value-maximizing members who choose $o_j = 1$ if uninformed, are in fact informed. As this is true for any $h > 0$, we are back to the case examined at point 1 above.

b) If $n+h$ value-maximizing members choose $o_j = 0$, and $k-h$ value-maximizing members choose $o_j = 1$, $k \geq h > 1$, the best response of O_i, $i \neq j$, is to choose $o_i = 1$ because O_i is never pivotal when $v = -1$ while he may be pivotal when $v = 1$. This happens in the case where h of those $n+h$ value-maximizing members who choose $o_j = 0$ if uninformed, are in fact informed. As this is true for any $h > 1$, we are back to the case examined at point 2 above.

Point ii$_2$) In the case of $n-k$ insiders $(n > k > 0)$ and $n+1+k$ value-maximizing members, there may exist equilibria where all the value-maximizing members choose the same mixed strategy.

In this case, each generic O member solves the following problem:

$$\begin{aligned}\max_{o_i} E(v) &= \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \\ s.t.\ o_i &\in (0,1) \\ o_i &= o_j, \forall j \neq i \\ & I_z \text{ always votes "yes" } \forall z\end{aligned}$$

where the first constraint refers to the fact that we are looking for a mixed strategy (i.e., an internal solution), the second constraint imposes the symmetry of this strategy, and the third constraint takes into account that any z member of type I follows his dominant strategy.

We have solved this problem for a committee of five members, one of which is of type I (hence, I_1 always votes "yes") and four of which are of type O. To make the maximization problem more explicit, we rewrite it as follows where $\bar{o} \equiv o_1 = o_2 = o_3 = o_4$:

$$\max_{\bar{o}\in(0,1)} \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)],$$

where

$$Y(\cdot|v=1) = \sum_{i=1}^{3} \frac{4!}{i!(5-i)!}[\alpha + (1-\alpha)\bar{o}]^{5-i}[(1-\alpha)(1-\bar{o})]^{i-1}$$

is the probability that the committee votes "yes" when the state of the world is H, member I always votes "yes" and all of the four O members plays the same mixed strategy $\bar{o}$. Analogously,

$$Y(\cdot|v=-1) = \sum_{i=1}^{3} \frac{4!}{i!(5-i)!}[(1-\alpha)\bar{o}]^{5-i}[\alpha + (1-\alpha)(1-\bar{o})]^{i-1}.$$

The solution of the problem, evaluated for $\alpha = \frac{1}{2}$, is:

$$o_1 = o_2 = o_3 = o_4 = \frac{5-\sqrt{13}}{6}.$$

We do not characterize the solution for different committee size or composition as it is sufficient for our purpose to show that such an equilibrium may exist.

Point iii) Optimal equilibria

Recall that

$$E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)].$$

In the unique equilibrium of the case with n biased members and $n+1$ value-maximizing members (point i), as well as in the pure strategy equilibrium of the case with $n-k$ biased members ($n>k>0$) and $n+1+k$ value-maximizing members (point ii$_1$), $Y(\cdot|v=-1)=0$ and $Y(\cdot|v=1)$ is equal to the probability that at least one of the $n+1$ members who choose $o_j=0$ is informed. Then in both cases $E(v)$ is equal to

$$\frac{1}{2}\sum_{j=0}^{n}\frac{(n+1)!}{j!(n+1-j)!}\alpha^{n+1-j}(1-\alpha)^j=\frac{1}{2}\left[1-(1-\alpha)^{n+1}\right]$$

implying that the pure-strategy equilibria are optimal.

7.3 Proof of Corollary 3

Given that there are at least $n+1$ members of type O, after an increase in n we still have $Y(\cdot|v=-1)=0$. On the other hand, $Y(\cdot|v=1)$ increases as n is increased, because in the equilibrium where $n+1$ members O choose $o_j=0$, and the other members O (if present) choose $o_j=1$, we have

$$Y(\cdot|v=1)=\sum_{j=0}^{n}\frac{(n+1)!}{j!(n+1-j)!}\alpha^{n+1-j}(1-\alpha)^j=1-(1-\alpha)^{n+1}.$$

So Corollary 3 follows.

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