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Endogenous longevity, biological deterioration and economic growth*

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Abstract: The identification of the types of bidirectional interactions that take place between longevity and economic growth in the long-run is carried out by means of the integration of human capital accumulation, innovation in medical technology, a health goods sector, and individual decisions on health and longevity in a dynamic general equilibrium set-up. In this context, in which individual agents decide not only on their "quality" of life but also on its "quantity", the mere process of biological deterioration, that is to say, the continuous loss of health goods effectiveness in maintaining a given level of health as individuals age, provides the reason for an additional, and new, engine of growth.

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1.- Introduction

Much has been learnt since the first works that, in the second half of 1980's, renewed our interest in economic growth and provided the origin for Endogenous Growth Theory. One outstanding conclusion of this theory is that a limited number of parameters play a key role in the value of the long-run growth rate. These parameters are essentially three in number: the intertemporal discount rate of the utility, the productivity parameter of the human capital accumulation technology and the productivity parameter of the R&D sector technology. Those economies with high intertemporal discount rates of the utility will tend to grow more slowly than those with lower rates, ceteris paribus, while high productivities in the accumulation technologies of human capital and productive knowledge will result, by contrast, in high growth rates in correspondence with the engine of growth character of both activities. The very concept of engine of growth has been a key contribution of the theory of endogenous growth: an engine of growth is a phenomenon that ultimately lies behind growth, in that it is able to generate incentives that cause agents to take deliberate decisions that will, in turn, give rise to productivity growth.

Against this general background, one of the objectives of this paper is to draw attention to the existence of an additional and new engine of growth that, in our view, is worth considering. We will try to meet this objective by illustrating that, when it is properly considered, a new parameter typical of this additional engine of growth is determinant in the long-run growth rate. The importance of the three earlier-mentioned parameters is derived from situations where individuals must decide on the quality of a given time horizon that is fixed or infinite. When agents have to decide not only on their quality of life but also on its quantity, the introduction of aspects related to health and longevity become unavoidable. In what follows, we present a model which makes clear that, under some particular conditions, a new parameter in the field of endogenous growth, that is to say, the biological deterioration rate, acquires a leading role, with this parameter being understood as the rate at which the effectiveness of health goods in maintaining a given level of health decreases as individuals grow older. Somewhat unexpectedly, we find that in our model none of the three earlier mentioned parameters appears as relevant in the long-run growth rate, which is now determined only by the biological deterioration rate.

What, then, is the new engine of growth which lies behind this result? We have deliberately chosen not to introduce a permanent accumulation mechanism of human capital through education. Instead, what we introduce is a mechanism for the accumulation of medical knowledge aimed at fighting biological deterioration. Thus, the new engine of growth takes the form of innovation in the fight against biological deterioration: in the same way that human capital and technological innovation are engines of growth because they help to fight against technical restrictions to production, medical innovation is also an engine, in that it helps in the fight against biological restrictions. In this way, there is a complete symmetry between the three engines, both in their nature as fighters against restrictions and in their character as knowledge generators.

In fact, the general consensus admits that economic development goes hand-in-hand with important improvements in life expectancy. Thus, Sen (1998) points to mortality as a proven indicator of economic success and failure, emphasizing that the connection between both elements is not direct, since countries like South Korea and Japan have been able to increase their levels of life expectancy as fast as their economic growth, while others, like Brazil, have maintained smooth longevity increases despite their high rates of economic growth. At the other extreme, countries such as Sri Lanka, Costa Rica and the Hindu state of Kerala have experienced dramatic falls in their mortality rates without enjoying important rates of economic growth. Such differences are explained because the link between longevity and economic growth takes place through third elements, such as health and education. An identical argument is presented by Barro and Sala-i-Martín (1995), for whom life expectancy is an important factor of economic growth because longevity acts as a proxy for other variables related to social welfare in addition to health. In fact, health is not the only element connecting longevity and economic growth even though, as Bloom, Janison, Malaney and Ruger (1998) point out, health improvements always become productivity improvements. In this sense, the level of health depends on the innovation that takes place in the goods and services which provide medical care. Human capital derived from education plays also an important role. Insofar as human capital is inherently tied to individuals, the expansion of life expectancy allows for returns to be obtained over a longer period of time, which encourages its accumulation and, as a consequence, economic growth. Likewise, the degree of patience, measured through the intertemporal discount rate of the utility, is another factor to be taken into account, because individuals with a longer life expectancy are more patient, generate higher levels of savings and, as a consequence, more growth.

All these arguments are present in the theoretical literature. Most of the works concentrate on human capital as a link between longevity and growth. Meltzer (1992) offers a detailed study of the positive effect of longevity on human capital investment, showing that falls in mortality promote sustained economic growth. Ehrlich and Lui (1991 and 1994) also find that a fall in the mortality rate will have a positive influence on human capital investment and a negative one on fertility, provided that children materially ensure their parents' old age. By contrast, Zhang, Zhang and Lee (2001) show that the effect of a life expectancy extension on long-run economic growth will have a different sign, depending on the utility parameters and the social security system. Without social security or with funded social security, an increase in life expectancy will reduce fertility, encouraging human capital investment and growth only if parents value the number of their children to a greater extent than their welfare. With a pay-as-you-go social security system, an increase in longevity will accelerate growth only if parents prefer "quality" to "quantity" with respect to their children. De la Croix and Licandro (1999) also show that the effect of a longevity increase on growth can be negative because the extension of the educational period delays incorporation into production, thereby causing a fall in the activity rate. Using numerical simulations, these authors obtain that, for high longevity levels, the negative effect is dominant. Kalemli-Ozcan, Ryder and Weil (2000) find, as in the perpetual youth model of Blanchard (1985), that a reduction in mortality increases the stationary levels of physical capital and consumption. However, they do not find a longevity effect on the stationary growth rate. Reinhart (1999) maintains that the connection between longevity and economic growth is positive, because individuals with a higher life expectancy are more patient, and this characteristic leads to higher levels of savings and investment. Zhang and Zhang (2001) combine both channels and show that an increase in life expectancy leads to both an increase in the savings rate over the length of an individual's life, as well as to an increase in the investment in human capital. Van Zon and Muysken (2001), using a general equilibrium set-up, relate longevity and economic growth by way of health. Thus, health appears as a necessary condition for the provision of the human capital services that have been acquired from education. The steady state analysis leads to a positive relationship between longevity, health and economic growth. However, high discount rates would lead to high longevity levels, a result that is at odds with the rest of the literature.

Two conclusions immediately arise from this scenario. First, we can note that there is a clear lack of unanimity about whether the extension of the life horizon has a positive or negative effect on the rhythm of long-run growth, or simply does not affect it at all. This latter possibility would seem likely, given the continuous advances observed in longevity simultaneously with sustained growth at a relatively stable rate in the developed economies. In fact, Meltzer(1992) showed that theoretically the steady state growth rate can be independent of mortality under some assumptions, and that empirically declines in mortality can lead to trivial changes in the return to education once mortality among young adults reaches low levels. Secondly, we can also appreciate that the links between development and improvements in life expectancy take place in two directions: economic growth affects longevity, and longevity, in turn, has an influence on the allocation of resources that lead to growth. However, this bidirectionality has still not been sufficiently considered in a theoretical context of general equilibrium. Save for one example, that of Van Zon and Muysken (2001), all the papers cited earlier consider the fall in mortality as an exogenous event to economic development. However, life expectancy improvements are due not only to exogenous factors, but also to mechanisms that are endogenous to the growth process itself. Some contributions in which mortality is an endogenous variable have appeared since 2002 (Kalemly-Ozcam (2002), Blackburn and Cipriani (2002), Lagerlof (2003), and Chakraborty(2004), for example). However, these papers consider longevity as depending on other endogenous variables or, in an OLG context with a maximum life span, admit a variable mortality rate. In contrast with this approaches, we are interested in explaining the role played by longevity as an isolated variable, representing the duration of the life as a given number of time periods.

Once primary needs are covered, individuals direct their consumption towards other types of goods and services, especially those related to health. This dynamic of individual behavior towards a growing interest in health care, together with the use of medical technology, are key components to understand the evolution of longevity. Van Zon and

Muysken (2001) assume that longevity is proportional to the health level of the economy, and this level is determined in their model. Unfortunately, they do not relate this health level to agents' deliberate decisions aimed at achieving health objectives, a relation that has been successfully characterized by Grossman (1972) and Ehrlich and Chuma (1990). However, this two latter works adopt a partial equilibrium approach from which it is not possible to derive a general perspective with bidirectional interactions between longevity and growth. It seems, then, that there is a case for trying to establish a theoretical set-up that is able to endogenously integrate growth, longevity as an isolated variable and those elements which, according to the economic literature, could play an important role in their connection. This represents the second of the objectives of this paper which, from an endogenous growth point-of-view, integrates human capital accumulation, medical technology innovation, health and longevity. In particular, with respect to human capital and medical technology innovation, we follow the classical approaches of Lucas (1988) and Romer (1990), respectively. Furthermore, some aspects taken from Grossman (1972) and Ehrlich and Chuma (1990) are particularly relevant in order to introduce the need to allocate economic resources to health goals so as to produce opportunities for a longer life. These last two papers justify individual health expenditure by virtue of the need to maintain the health level above a certain minimum. Given that the resources directed to medical care slow down the deterioration in health resulting from ageing, the time of death is delayed because it takes place when the health level falls below this minimum. Finally, the endogenous character of longevity requires some assumptions on population dynamics, preferences and wealth transmissions between generations.

The resulting framework allows for a clear identification of which and what type of bidirectional interactions take place between longevity and economic growth in the long-run. Our context must be understood as appropriate only for developed economies. Essentially, we indicate what are the factors causing level effects, and which are those with a growth effect. The process of biological deterioration appears as the central axis of the long-run dynamics. The need to offset this biological deterioration encourages medical research, which takes place at a rhythm that fits exactly with the rate of this deterioration. Medical science provides new goods to combat deterioration which, at the same time that have the effect of mitigating it, thereby improve the health conditions of the new generations. As a result, individuals' productive capacity increases and economic growth takes place. On the one hand, this economic growth provides the necessary resources to finance medical research and health expenditure. On the other, the rhythms of medical innovation and economic growth lead to a constant stationary level of longevity which determines the time devoted to education.

We are referring here to the steady state behaviour. In the transition, if investment in health technology increases life expectancy in a meaningful way only for some older generations, there will be little or no effect on their returns to education. Moreover, and for the shake of simplicity, we do not consider a period of retirement, so living individuals are always economically active.

Although the economy maintains a positive growth rate, the long-run longevity level is constant. The model does not admit the possibility of continuous life expectancy growth, not precisely because of the presence of biological deterioration, but as a consequence of the preferences. However, improvements in long-run longevity are possible as a consequence of changes in individuals' preferences related to the importance of an externality in the own consumption, the interest in the descendants, or the degree of patience. An increase in any of these elements would increase the long-run life expectancy level.

Faced with a perspective of greater longevity, individuals would choose to make greater investment in human capital and, hence, the production level of the economy would increase. In this way, longevity has a level effect on aggregate output, but not on growth. However, the biological deterioration rate, which coincides with the long-run growth rate of the economy, does have a positive effect on longevity. In summary, we have been able to determine the long-run bidirectional interactions between medical innovation, economic growth, standard of living and longevity.

The rest of the paper is organized as follows. In Section 2 we present the proposed model and consider the characteristics of the problem faced by an individual over the length of his life. Individual behavior and market equilibria are determined in Section 3. Finally, Section 4 closes the paper with a review of the main conclusions.

2.- The model

2.1. Population dynamics

Given that the goal of our analysis turns on the phenomenon of longevity, we assume that the number of births is constant during each period of time. This is a non-trivial assumption, because we know the clear association between mortality decline and fertility decline during the demographic transition. From this association we recognise that we can deduce an endogenous link; however, in our model we do not give any consideration to it. Three are the reasons that support our assumption: 1) we only are interested in the steady state and, in such a situation, fertility must be constant; 2) for high levels of development, longevity improvements do not generally coincide with declines in fertility[1]; 3) our main focus is on a new way to consider endogenously longevity.

Let us assume that *A* individuals are born at time *t*, in such a way that they form cohort *t*. Let z_t be the birth date of the oldest cohort in t (cohort z_t). Each cohort has its own life expectancy with T_j being the life span of every member of cohort *j*. We assume $T_j \leq T_{j+\varepsilon}$ for all the living cohorts in *t*, with ε being an arbitrary positive number, which guarantees that all the individuals born between z_t and *t* are alive. Under these conditions, the population in *t*, that we call L_t, can be represented as:

$$L_t = \int_{z_t}^{t} A dj = AE_t \,, \qquad (1)$$

where $E_t = t - z_t$ is the age of the oldest cohort. E_t will not change if the life expectancy of all the cohorts is the same at all times. However, if longevity continuously increases, then both the maximum age of the cohorts and the population will be greater and greater.

The population structure we have described is a generalization of that proposed by Stokey (1991), which assumes a constant longevity for all individuals, fixed and exogenous, in such a way that the age of the oldest cohort remains the same over time. This

[1] Zhang et al. (2003) make the same assumption using the following reasoning: *"we abstract from....fertility choices since in most developed countries… fertility rates are near or below the replacement level"* page 85.

characterization would exclude the possibility of studying the determinants of longevity. However, if changes in the age of the oldest cohort are possible, then we have a general set-up where changes in longevity can occur as a consequence of individual decisions.

2.2. Preferences

The second differential feature of the model is related to the specification of agents preferences. The utility function of a member of cohort j can be represented as:

$$U(j) = \int_{j}^{j+T_j} \frac{1}{\alpha} c_{j,t}^{\alpha} c_t^{1-\alpha} e^{-\rho(t-j)} dt + e^{-\beta(j+T_j-j)} \int_{j+T_j}^{\infty} c_{j,j+T_j} e^{(g_{j+T_j}-\rho)[t-(j+T_j)]} dt, \qquad 0< \alpha <1 \quad (2)$$

where $c_{j,t}$ is the final goods consumption in *t* of an individual born in *j*, c_t is the average final goods consumption of the economy in *t*, α measures the own consumption elasticity of the instantaneous utility, ρ is the discount rate of the utility derived from the final goods consumption, β is the discount rate of the utility obtained from the welfare of a representative descendant (we assume $\beta>\rho$), T_j is the longevity (life expectancy) of all the members of cohort *j*, $c_{j,j+T_j}$ is the level of final goods consumption of every individual in cohort *j* at the end of his life, and g_{j+T_j} is the growth rate of the economy at this final instant. This expression presents two clearly differentiated parts: the first represents the utility during the individual's life until $j+T_j$, whilst the second represents the utility obtained from the expected welfare of a representative descendant since $j+T_j$.

The functional form of an individual's utility during his life is similar to those used by Abel (1990) and Galí (1994), that is to say, of the "Catching up with the Joneses" and "Keeping up with the Joneses" type, respectively, where the inclusion of the average consumption allows for the influence of the average living standard of the economy on the individual's utility[2]. In our case, a greater average consumption makes the individual better

[2] The incorporation of externalities in consumption is a typical feature in different fields of the economic literature. Thus, Abel (1990), Galí (1994) and Campbell and Cochrane (1999) introduce consumption externalities in models of asset valuation and portfolio decisions. Crês, Ghiglino and Tvedes (1997) consider this same idea to represent external effects between generations. Even Duesenberry(1952) pointed out that

off, reflecting a preference for an environment which is as good as possible. In any event, we would have the case in which the externality is irrelevant when α tends to one[3].

On the other hand, the utility obtained from the expected consumption of a representative descendant is valued taking as reference the agent's own level of consumption at the moment of death, together with the expectation of a growth rate identical to the rate observed in the economy at that moment. Such an expectation has its basis in the continuity of the consumption behavior across successive generations[4]. The individual believes that the average standard of living of his descendants will be near to his own level, although he perceives that it will grow at a rate which he approximates with the observed rate of growth. We assume that ρ is always greater than g_{j+T_j}. The literature contains other examples of utility functions that also incorporate expectations on descendants' welfare. A characteristic example is the utility function proposed by Barro and Becker (1988), widely used in the study of the interactions between fertility and economic growth, that integrates expectations derived from assuming perfect foresight and an identical level of utility for all the descendants[5]. However, the concept of descendant we use here is not exactly the concept of offspring. If this were the case, then the introduction of bequests would be justified. Rather, we speak of the representative descendant as any survivor of the cohort born at the moment of the decision-maker's death. The reason for the presence of this term in the utility is altruism, reflected in the fact that individuals value the utility that a consumption identical to theirs in their last period of life, together with the expectation that this consumption will grow at the same rate as that observed in such a period, could be obtained by a representative descendant, provided that a given level of utility is more expensive for society the older its individual members are as a consequence of biological deterioration. In fact, while the discount rate applied to the utility provided by the expected consumption of the representative descendant is greater than ρ, the value of α

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consumer behavior does not depend only on his absolute level of income, but also on the level of the social group to which he belongs.

[3] Galí (1991) introduces two types of consumption externalities: positive externalities to formalize the notion of keeping up with the Joneses, and negative externalities because the good of the economy could show some "public good" features. Note that the latter is not our case.

[4] In some models children' consumption appears closely influenced by the consumption level of their parents. See, for example, Becker (1992) and De La Croix and Michel (1999).

[5] See Nerlove and Raut (1997).

in that case is one because then the decision-maker is only concerned with the interest of the descendant.

It would be possible to wonder about the reasons for which we need both altruism and externalities in the utility function. The answer is that both assumptions provide generality to the model. On the one hand, altruism plays a crucial role in determining the lifespan chosen by each cohort. In particular, when β tends to infinite (there is not altruism at all) the solution in the steady state for the longevity is infinite. It is clear that this value will never be attained and we should develop the transitory dynamics towards that value, along which the longevity would be constantly increasing. But this result would be very particular. The reason why we obtain a much more general range of results is precisely the presence of altruism in the utility function. Moreover, we are able to prove that the very reason for the existence of a finite longevity in the steady state is altruism, at least in our set-up. This is one of our main results.

This result implies nothing for the actual behaviour of the longevity, because this must be necessarily finite, in that whatever the level of the finite value of the steady state longevity, there will also be a finite restriction on it given the available resources. This is a question related to the transitory dynamics which lies outside the scope of this paper.

On the other hand, the externality in the instantaneous utility provides a more general result in the behaviour of the consumption along the lifespan of each individual. In our case, we can have a constant, increasing or decreasing path for the consumption along the life. In the absence of externality, the only possibility would be a decreasing path. Moreover, we are able to determine that the own consumption elasticity has an influence on the determination of the value of the longevity.

As regards to the expression of the representative descendant, this is a nominal terminology, in that with it we avoid the reference to the number of descendants that would make necessary the introduction of the fertility problem. In fact, we follow the traditional approach in the overlapping generation models. From this approach, the second part of the utility can be obtained from the expectations about future generations' consumption and both the assumptions that longevity of future generations is the same as that of the agent and that intergenerational discount rate between them is not β but ρ. Assuming j=0, and denoting the successive representative descendants as 1, 2, …., we can write:

$$
\begin{aligned}
U(0) &= \int_0^{T_0} e^{-\rho t}\,\frac{1}{\alpha} c_{0,t}^{\alpha} c_t^{1-\alpha}\,dt + e^{-\beta T_0} U(1)\\
U(1) &= \int_{T_0}^{2T_0} e^{-\rho(t-T_0)} c_{0,T_0} e^{g_{T_0}(t-T_0)}\,dt + e^{-\beta T_0} U(2)\\
U(2) &= \int_{2T_0}^{3T_0} e^{-\rho(t-2T_0)} c_{0,T_0} e^{g_{T_0}(t-T_0)}\,dt + e^{-\beta T_0} U(3)\\
U(3) &= \int_{3T_0}^{4T_0} e^{-\rho(t-3T_0)} c_{0,T_0} e^{g_{T_0}(t-T_0)}\,dt + e^{-\beta T_0} U(4)\\
&.\\
&.
\end{aligned}
$$

By substituting the corresponding values, we arrive at the following expression:

$$
\begin{aligned}
U(0) = &\int_0^{T_0} e^{-\rho t}\,\frac{1}{\alpha} c_{0,t}^{\alpha} c_t^{1-\alpha}\,dt +\\
&+ e^{-\beta T_0} c_{0,T_0}\left[\int_{T_0}^{2T_0} e^{(g_{T_0}-\rho)(t-T_0)}\,dt + e^{(\rho-\beta)T_0}\int_{2T_0}^{3T_0} e^{(g_{T_0}-\rho)(t-T_0)}\,dt + e^{(\rho-\beta)2T_0}\int_{3T_0}^{4T_0} e^{(g_{T_0}-\rho)(t-T_0)}\,dt + ...\right]
\end{aligned}
$$

And now, putting *j* instead of *0* and $\beta=\rho$ inside the square brackets, we directly obtain expression (2).

2.3. Health care

Each individual decides his final goods consumption level $c_{j,t}$, that will provide him with utility. Furthermore, the individual acquires the “health goods” that will allow him to reach the desired life expectancy and the required level of productivity at work. The term “health goods” includes medication, surgical treatment, diagnosis and any other good or service that is able to offset the biological deterioration caused by illness or merely by ageing. Let $x_{m,j,t}$ be the amount of the variety of health good *m* acquired by an individual of cohort *j* at time *t*. The reason why individuals devote part of their resources to health care is due to the logical need to achieve an average level of health required in order to be employed and to obtain the desired life expectancy. This requirement is analytically reflected in the physiological restriction represented by the expression:

$$h_{s_j} T_j \leq \int_j^{j+T_j} \left[\int_0^{M_t} x_{m,j,t}^{\gamma} dm \, e^{-\delta(t-j)} \right] dt, \qquad 0<\delta<\rho, \qquad 0<\gamma<1. \tag{3}$$

The physiological restriction, an inherent condition in the longevity problem, has already been specified in the literature. In fact, our characterization is an adaptation of the restriction formulated by Grossman (1972) and incorporated later by other partial equilibrium models that have considered the longevity problem[6]. These works explain individual health investment as a requirement to maintain the level of health above a certain minimum. They assume that resources devoted to health care reduce the deterioration caused by ageing, in this way delaying the moment of death, which takes place when the average health level is below a minimum threshold. Although their approach is not entirely suitable to our case[7], we maintain its main features.

First of all, given that the basic reason why individuals devote part of their resources to health care is precisely to achieve a level of health that allows them to reach the standard health level required in the labor market and to attain the desired life span, a simple way to analytically represent the health requirements of every individual of cohort j is $h_{s\,j} T_j$, where $h_{s\,j}$ denotes the level of health of every individual of cohort j and T_j his life expectancy. This requirement is fulfilled precisely through the acquisition of health goods $x_{m,j,t}$. When a member of cohort j decides to acquire an amount $x_{m,j,t}$ of each of the M_t varieties of health goods that medical research makes available, the amount of health obtained at each moment of time t is $\int_0^{M_t} x_{m,j,t}^{\gamma} dm e^{-\delta(t-j)}$, with the parameter γ measuring the

[6] See Ehrlich and Chuma (1990), Grossman (1998) and Ried (1998), amongst others.

[7] In particular, we omit any reference to the concept of health capital, an element that is not present in our model.

efficacy of each variety of health good. For the sake of simplicity, we consider that the efficacy of each variety is the same. Furthermore, we assume that γ takes a value between 0 and 1, that is to say, the medical technology is characterized by decreasing returns[8]: whilst the resources allocated to health care have a positive effect on the health level, the marginal increase obtained decreases with the amount of health good demanded.

Moreover, these decreasing returns are reinforced by ageing. Longevity and ageing are two coincidental phenomena. Although greater longevity has a clearly positive connotation, ageing is a significant cause for concern in developed economies, given that the efficacy of the resources devoted to health decreases with this ageing, in such a way that the expenditure required to maintaining a given level of health increases. To reflect this property in the physiological restriction, we consider that the efficacy of the resources devoted to health decreases exponentially at a rate δ depending on the biological age. This parameter was already present in Grossman (1972). While it is difficult to give a unique empirical approximation and future research should be carried out on the issue, it is possible to make some references to the type of phenomenon we are referring to. For example, the proportion of Adverse Drugs Reactions (ADR) grows with age, a proof of the defiance that longevity represents for the natural mechanisms of the human body. Also, the capacity of cellular regeneration falls with age and the level of testosterone, being at its maximum at the age of twenty, only reach its half at the age of forty and its quarter at sixty. It is obvious that we are simplifying because in this process we are not drawing a distinction between childhood and adulthood. The process we are representing is describing only the second part of the life.

We further assume that ρ is greater than δ. According to all the previous arguments, the effective resources devoted to health by a member of cohort j throughout the course of his life are measured by the expression $\int_{j}^{j+T_j}\left[\int_{0}^{M_t} x_{m,j,t}^{\gamma}\,dm\; e^{-\delta(t-j)}\right]dt$.

With these resources the individual satisfies his health needs in order to be alive with the required health level during his complete life span, being at least $h_{s\,j}T_j$. We

[8] See Ehrlich and Chuma (1990) and Van Zon and Muysken (2001).

assume that the average level of health of each cohort, $h_{s\,j}$, is derived from a health minimum, h_{min}, and from the level of medical technology in j, M_j, in such a way that $h_{s\,j}$ comes given by:

$$h_{s\,j} = h_{\min} M_j \, . \tag{4}$$

The term $h_{\min}$ represents the minimum average level of health required to be alive when M_j is one, where M_j reflects the state of medical science. Each generation will enjoy an average level of health that comes determined by the degree of development reached by medical technology at the time of its birth. The aim of this characterization is to reflect the fact that a greater diversity of medical techniques increases the average level of individuals' health over the length of their life. Medical advances allow for the possibility of living longer and in better conditions once individuals perceive and take into account the opportunities that science and technology offer[9]. In this regard, Chetty (1998) is particularly interesting, in that considers the interrelations between patients, physicians and the use of medical cover through a stochastic general equilibrium model. This author argues that while in the neoclassical models consumers maximize their preferences subject to budget constraints, the demand for medical services is mainly subject to restrictions of an exclusively technological character. Individuals pursue a given longevity in the best possible conditions and there is no doubt that, in health terms, such conditions are determined by the developments achieved in medical technology.

As M_j is not bounded, it could seem strange that from equations (4) and (3) the higher the state of medical science, the higher are the health expenditures needed to ensure a certain lifespan T_j. It would seem that a more highly developed medical science causes the physiological restriction (3) to be more binding. It is important to understand that the level of health required for each generation varies depending on the level of medical knowledge, due to both technological and sociological or cultural reasons simultaneously. And these last reasons come mainly from the labour market, where the requirements are higher as M_j grows, given that human capital productivity increases with health improvements, as does the rewards that individuals receive. It should be appreciated that

[9] See Fuchs (1982), Krugman (1997) and Van Zon and Muysken (2001).

the outcomes provided by the improvements in health result in more time alive or more production. This is the reason why the restriction is more binding as medical knowledge improves, and why the expenditures needed to ensure a lifespan must be higher[10].

Note here that the proposed model does not incorporate the health level as an argument in the utility function. Individuals do not obtain utility directly from their level of health; rather, they spend resources in order to achieve a certain level of health required to live a given number of periods and develop the activities, among them final goods consumption, that themselves provide this utility. Health is a requirement for living and carrying out activities and, as such, it mainly plays the role of a restriction.

2.4. Education and labor

Each individual decides his final goods consumption level $c_{j,t}$ and the demand for each of the available varieties of health goods, $x_{m,j,t}$, also taking into account which is his best decision with respect to his life expectancy T_j. He also decides the time to devote to education which, together with his health level, will determine the stock of human capital that he will hire during his working life. The time that an individual of cohort *j* devotes to education is denoted by f_j, with the rest of his life, $T_j - f_j$, being devoted to work. The stock of human capital acquired through education by a member of cohort *j* is determined by the expression:

$$h_{e\,j} = \left(f_j\right)^v, \qquad v>0, \tag{5}$$

so that a proportion $\left(\frac{f_j}{T_j}\right)$ of every instant of time is devoted to study and a proportion $\left(\frac{T_j - f_j}{T_j}\right)$ to work, in such a way that the only education cost is the opportunity cost [11].

[10] Cutler and McClellan (2001) point out that "…(medical) *technology often leads to more spending, but outcomes improve by even more*" . p 23.

[11] See Kalemli-Ozcan, Ryder and Weil (2000)

While the human capital derived from education is a broadly extended element in the literature on growth, the opinion that education and health are two closely aspects of human capital has recently gained a degree of acceptance[12]. In this paper we follow the formulation proposed by Van Zon and Muysken (2001), who develop a model of endogenous growth where health is a *sine-qua-non* condition for the provision of human capital services, in the sense that *"people can provide effective human capital services only if they are alive and healthy"* (page 170). Then, the overall human capital, or productive capacity of every member of cohort *j*, h_j takes the form:

$$h_j = h_{e\,j} h_{s\,j}\,, \tag{6}$$

where $h_{e\,j}$ is the stock of human capital that the individual acquires with his education and $h_{s\,j}$ is the level of health enjoyed during his life. An agent can work in both the final goods sector and the R&D sector. The time devoted to the final goods sector is remunerated at a rate $w_{Y\,t}$ for each human capital unit, whilst the remuneration in the R&D sector is $w_{M\,t}$ for each human capital unit. We denote as $u_{j,t}$ and $(1-u_{j,t})$ the time participations of any individual of generation *j* in the final goods sector and in the R&D sector, respectively. In summary, agents decide their levels of consumption, their demand for health goods, their education and their work time in each sector, with these decisions also determining their life expectancy.

2.5. Capital transfers and the budget constraint

We now introduce the role of a social institution devoted to managing capital transfers between generations without cost. This institution assigns an initial capital endowment to each new agent and, in order to obtain the necessary resources, applies a tax ϕ_t on the individual capital stock in *t*. Moreover, when a cohort dies, the same public institution takes charge of distributing the "legacy" among all the surviving cohorts in an

[12] See Ram and Schultz (1979), Fuchs (1982 and 1996), Meltzer (1992), De la Croix and Licandro (1999), Kalemli-Ozcan, Ryder and Weil (2000) and Van Zon and Muysken (2001).

egalitarian way. We denote as $\tilde{k}_{z_t,t}$ the legacy that each individual will receive in *t* provided that the oldest cohort, z_t, dies.

The budget constraint for a representative individual of cohort *j* at time *t* takes the following form:

$$\dot{k}_{j,t} = (r_t - \phi_t)k_{j,t} + \frac{\tilde{k}_{z_t,t}}{E_t} + \left(1 - \frac{f_j}{T_j}\right)\left[w_{Y_t} u_{j,t} + w_{M_t}(1 - u_{j,t})\right]h_j - c_{j,t} - p_t \int_0^{M_t} x_{m,j,t}\, dm, \tag{7}$$

$$\tilde{k}_{z_t,t} = \begin{cases} k_{z_t,t} \quad if \quad E_t = T_{z_t} \text{ and } \quad j \neq z_t \\ -k_{z_t,t} E_t \quad if \quad E_t = T_{z_t} \text{ and } \quad j = z_t \\ 0 \quad if \quad E_t < T_{z_t} \end{cases}$$

where $k_{j,t}$ is the capital stock owned by each member of cohort *j* at time *t* ($\dot{k}_{j,t} = \frac{dk_{j,t}}{dt}$), r_t is the interest rate, ϕ_t is the tax rate on the individual capital stock to provide the initial endowment to the new cohorts and p_t is the health goods average price.

Expression (7) shows the capital accumulation dynamics of an individual of cohort *j* over the length of his life. The public institution managing the capital transfers establishes a number of rules. First, the moment at which the capital transfer to the rest of society takes place coincides with the last instant of life, that is to say, an individual belonging to cohort *j* makes the capital transfer to the rest of society at time $j+T_j$. However, he will receive the interest returns on his capital corresponding to the last period, and thus the resources he receives in the last moment of his life are the wages and the capital returns on the stock accumulated until then.

2.6. Productive sectors of the economy

The economy has three productive sectors: a sector that produces final goods, an R&D sector devoted to health research, and a sector that produces health goods. Final goods sector firms are competitive. They produce a unique type of good, whose price is normalized to one, using as productive factors physical capital, K_t, and an amount H_{Y_t} of human capital:

$$Y_t = BK_t^{\sigma} H_{Y_t}^{1-\sigma}, \qquad 0<\sigma<1, \qquad B>0. \tag{8}$$

The aggregated physical capital is defined as:

$$K_t = \int_{z_t}^{t} Ak_{j,t}\, dj, \tag{9}$$

and, similarly, the human capital stock available for the final goods and R&D sectors comes given as:

$$H_t = \int_{z_t}^{t} A\left(1 - \frac{f_j}{T_j}\right) h_j\, dj. \tag{10}$$

A part of this stock, H_{Y_t}, is devoted to the final goods sector and the rest, H_{M_t}, is directed to the R&D sector in order to generate new medical knowledge[13]:

$$\dot{M}_t = \Delta H_{M_t}, \qquad \Delta > 0, \tag{11}$$

that will be put to commercial use by the health goods firms. This sector presents a monopolistically competitive market structure[14] with free entry. Firms gain access to a patent at a price P_{M_t}, monopolize the production of the corresponding variety of health good at a unit cost of η units of final good, and commercialize it.

[13] See Funke and Strulik (2000).
[14] Wong (1996) justifies empirically this type of structure for the health sector.

3.- Market equilibrium

Once we know the elements that characterize the economy, we are in a position to derive both the optimal behaviour of the individuals and that of the economy. The reader should bear in mind that some of the results reported here reflect special cases related to the functional forms chosen, especially the constant elasticity functional form for the instantaneous utility function and the production function.

3.1. Individual behavior.

All the elements presented in the previous section characterize the problem faced by an individual of cohort *j*:

$$\text{Max } U(j) = \int_j^{j+T_j} \frac{1}{\alpha} c_{j,t}^{\alpha} c_t^{1-\alpha} e^{-\rho(t-j)} dt + \frac{c_{j,j+T_j}}{\rho - g_{j+T_j}} e^{-\beta T_j}$$

s. t.:

$$\dot{k}_{j,t} = (r_t - \phi_t) k_{j,t} + \frac{\tilde{k}_{z_t,t}}{E_t} + \left(1 - \frac{f_j}{T_j}\right)\left[w_{Y_t} u_{j,t} + w_{M_t}\left(1 - u_{j,t}\right)\right] h_j -$$
$$- c_{j,t} - p_t \int_0^{M_t} x_{m,j,t} dm$$

$$h_j = h_{e\,j} h_{s\,j}$$

$$h_{s\,j} = h_{\min} M_j \qquad (12)$$

$$h_{e\,j} = \left(f_j\right)^{\nu}$$

$$\dot{\psi}_{j,t} = \int_0^{M_t} x_{m,j,t}^{\gamma} dm \, e^{-\delta(t-j)}$$

$$\psi_{j,j+T_j} \geq h_{s_j} T_j$$

$$k_{jt} \geq 0$$

$$k_{j,j} = \phi_j \frac{K_j}{A}.$$

We have simplified the objective function, including the result of the integral in the second term and introducing the variable ψ, given that we have a control problem subject to an integral restriction[15]. The Hamiltonian has the following expression:

$$\mathcal{H}_t = \frac{1}{\alpha} c_{j,t}^{\alpha} c_t^{1-\alpha} e^{-\rho(t-j)} +$$

$$+ \lambda_{j,t} \left\{ (r_t - \phi_t) k_{j,t} + \frac{\tilde{k}_{z_t,t}}{E_t} + \left(1 - \frac{f_j}{T_j}\right) \left[w_{Y_t} u_{j,t} + w_{M_t} (1 - u_{j,t}) \right] (f_j)^{\nu} h_{\min} M_j \right. \tag{13}$$

$$\left. - c_{j,t} - p_t \int_0^{M_t} x_{m,j,t} dm \right\} + \mu_{j,t} \int_0^{M_t} x_{m,j,t}^{\gamma} dm \, e^{-\delta(t-j)},$$

with $\lambda_{j,t}$ and $\mu_{j,t}$ being the costate variables corresponding to $k_{j,t}$ and $\psi_{j,t}$, respectively. As control variables we have the final goods consumption $c_{j,t}$, the demand for each of the health goods varieties $x_{m,j,t}$, the period of education f_j, and the participation in the final good sector $u_{j,t}$. The life expectancy (or longevity) T_j is also an endogenous variable, with its endogenous character being materialized in an additional condition corresponding to the optimal control problems when the terminal time is free[16]. The solution meets the following necessary conditions:

$$c_{j,t}^{\alpha-1} c_t^{1-\alpha} e^{-\rho(t-j)} = \lambda_{j,t}, \tag{14}$$

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[15] See Beavis and Dobbs (1990).

[16] In this problem with free terminal time, this additional condition is:

$$\mathcal{H}_{j+T_j} + \frac{\partial \left(\dfrac{c_{j,j+T_j} e^{-\beta T_j}}{\rho - g_{j,j+T_j}} \right)}{\partial (j + T_j)} = 0$$

See Chapters 2 and 3 of *Optimal Control Theory* by Sethi and Thompson (1981).

$$\gamma \mu_{j,t} x_{m,j,t}^{\gamma-1} e^{-\delta(t-j)} = p_t \lambda_{j,t} , \tag{15}$$

$$\left(1-\frac{f_j}{T_j}\right) v \left(f_j\right)^{v-1} = \frac{1}{T_j}\left(f_j\right)^{v} , \tag{16}$$

$$w_{Y_t} = w_{M_t} , \tag{17}$$

$$-\left(r_t - \phi_t\right)\lambda_{j,t} = \dot{\lambda}_{j,t} , \tag{18}$$

$$\dot{\mu}_{j,t} = 0 , \tag{19}$$

$$\begin{aligned} &\frac{1}{\alpha} c_{j,j+T_j}^{\alpha} c_{j+T_j}^{1-\alpha} e^{-\rho T_j} + \lambda_{j,j+T_j} \dot{k}_{j,j+T_j} + \mu_{j,j+T_j} \dot{\psi}_{j,j+T_j} \\ &- \frac{\beta}{\rho - g_{j,j+T_j}} c_{j,j+T_j} e^{-\beta T_j} = 0. \end{aligned} \tag{20}$$

The transversality conditions are the following:

$$\lambda_{j,j+T_j} = \frac{\partial\left(\frac{c_{j,j+T_j}}{\rho - g_{j,j+T_j}} e^{-\beta T_j}\right)}{\partial k_{j,j+T_j}} = 0 \tag{21a}$$

$$\begin{aligned} &\mu_{j,j+T_j} \geq 0 \\ &\psi_{j,j+T_j} \geq h_{sj} T_j \\ &(\psi_{j,j+T_j} - h_{sj} T_j)\mu_{j,j+T_j^-} = 0 \\ &(\psi_{j,j+T_j} - h_{sj} T_j)\mu_{j,j+T_j^+} = (\psi_{j,j+T_j} - h_{sj} T_j)\mu_{j,j+T_j} = 0 \end{aligned} \tag{21b}$$

$$\lambda_{j,j+T_j^-} - \lambda_{j,j+T_j^+} = \lambda_{j,j+T_j^-} - \lambda_{j,j+T} > 0$$

$$\mu_{j,j+T_j^-} - \mu_{j,j+T_j^+} = \mu_{j,j+T_j^-} - \mu_{j,j+T} > 0 \tag{21c}$$

With respect to this individual behavior, the first characteristic to which attention should be drawn is the close relation between the growth rates of individual consumption $g(c_{j,t})$ and the average consumption of the economy $g(c_t)$[17], reflecting the effect that the environment has on the individual's decision. From (14) and (18) we obtain that:

$$g(c_{j,t}) = \frac{1}{(1-\alpha)}(r_t - \phi_t - \rho) + g(c_t). \tag{22}$$

The growth rate of the final goods consumption of an individual of cohort *j* will be higher than, equal or lower than that corresponding to the average consumption if the net return of physical capital is higher than, equal or lower than the discount rate. Moreover, the lower the elasticity α, the lower the willingness of the individual to move away from the average path of the economy, given the gap between the net return on physical capital and the discount rate.

Secondly, we verify that condition (15) holds for all of available varieties of health goods. Thus, at every moment in time the individual will demand the same amount of each of the different types of health goods:

$$x_{m,j,t} = x_{m',j,t} \qquad m \neq m'. \tag{23}$$

In order to simplify the notation, from now on we will denote as $\bar{x}_{j,t}$ the amount of each variety of health good demanded by an individual of cohort *j* at time *t*, with its growth rate, according to (15) and (18), being:

$$g(\bar{x}_{j,t}) = \frac{1}{(1-\gamma)}(r_t - \phi_t - g(p_t) - \delta). \tag{24}$$

An individual of cohort *j* will decide to increase (decrease) the demand for each variety of health good if the net return of physical capital exceeds (does not exceed) the

[17] We use g(q) to represent the growth rate of the variable q.

sum of the rate of change of the average price of the health goods and the rate of biological deterioration, with the response being greater when greater is the effectiveness of the variety measured by parameter γ.

Repeating the solution of the problem for the rest of the cohorts, we can verify that the growth rate of the demand for any good, whether final or health good, at time t, is the same for all the individuals of any cohort:

$$g\left(c_{j,t}\right) = g\left(c_{j+h,t}\right), \tag{25}$$

$$g\left(\bar{x}_{j,t}\right) = g\left(\bar{x}_{j+h,t}\right), \qquad h \neq 0. \tag{26}$$

However, this is not the only similarity in the behaviour of the individuals making up the different cohorts. Condition (16) implies that all the individuals decide to devote to education a time that is proportional to their life expectancy:

$$f_j = \frac{v}{1+v} T_j \ . \tag{27}$$

Thus, the greater the life expectancy, the longer the period of education, a result that has already been presented in Ram and Schultz (1979), Fuchs (1982 and 1996), Meltzer (1992), De la Croix and Licandro (1999) and Kalemli-Ozcan, Ryder and Weil (2000). The intrinsic association of human capital with the physical person implies that an extension of life allows for returns to be obtained over a longer period of time, a fact that logically encourages education investment. The proportion between longevity and education time remains fixed for all the generations, with such a proportion depending exclusively on the rate of return of the time devoted to education: when v is greater, the proportion is larger.

Related to the transversality conditions we conclude that there is a jump in both λ and μ in the moment $j+T_j$, in such a way that in $j+T_j^-$ are positive but in $j+T_j^+ = j+T_j$ are nought. The reason is that there is a change in the operative restrictions on the two state variables on the right of this point of time. $\lambda_{j,j+T_j}$ is null by (21a) and consequently $\mu_{j,j+T_j}$

by (15) provided that $x_{m,j,j+T_j}$ is different from zero for all *m*. Additionally, from (21c) we conclude that $\psi_{j,j+T_j} = h_{sj} T_j$.

We finish the characterization of the individual behavior with expression (20), which acquires a singular importance given that it allows for the obtention of life expectancy. Given that in $j+T_j$ the two costate variables are null, a marginal increase in longevity has a double influence on the present value of the individual utility: on the one hand, the agent enjoys an increase in the utility derived from his own consumption in that additional instant; on the other, he also experiences a fall in the utility provided by a representative descendant. Logically, the individual chooses that level of longevity in which both incentives, positive and negative, are compensated[18]:

$$e^{(\beta-\rho)T_j} = \frac{\alpha\beta}{\rho - g_{j,j+T_j}} \left(\frac{c_{j,j+T_j}}{c_{j+T_j}} \right)^{1-\alpha} . \tag{28}$$

3.2. The behavior of the economy.

The relationships described up to this point outline the behavior of each individual and the link with the average magnitudes of the economy. We now need a description of the aggregate behaviour. The path of the average demand for final goods is characterized by the following expression[19]:

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[18] $c_{j,j+T_j}$ and $x_{m,j,j+T_j}$ come from (14) and (16) respectively, in such a way that:

$$c^{\alpha}_{j,j+T_j} c^{1-\alpha}_{j+T_j} e^{-T_j} = \lambda_{j,j+T_j^-}$$

$$\gamma\mu_{j,j+T_j^-} x^{\gamma-1}_{m,j,j+T_j} e^{-T_j} = p_t \lambda_{j,j+T_j^-}$$

[19] Equation (29) is the result of taking logarithms and deriving with respect to *t* in the expression $c_t = \frac{\int_{z_t}^{t} c_{j,t} dj}{E_t}$.

The same could be done for $k_t = \frac{\int_{z_t}^{t} k_{jt} dj}{E_t}$.

$$g(c_t) = g(c_{j,t}) + \frac{1}{E_t}\frac{c_{t,t} - c_{z_t,t}}{c_t} + g(E_t)\left(\frac{c_{z_t,t}}{c_t} - 1\right), \tag{29}$$

where $c_{t,t}$ is the demand corresponding to the cohort born in *t* and $c_{z_t,t}$ is the final goods consumption of each member of the oldest cohort in *t*. An analogous expression can be obtained for the other average values of the economy, for example, for the average physical capital stock.

As the final goods sector firms are competitive, the interest rate and the wages coincide in equilibrium with the marginal product of physical and human capital, respectively. The wage per unit of human capital perceived in the final goods sector must coincide with the wage in the R&D sector in order to make possible the stability of the allocation of work time between sectors.

With respect to the health goods sector, the demand for each firm producing a variety in *t*, D_t, is the sum of the individual demands, $D_t = \int_{z_t}^{t} A\bar{x}_{j,t}\,dj$. The profit in t, taking into account condition (15) and the fact that the production unit cost is η units of final good, is:

$$\pi_t = (p_t - \eta)\int_{z_t}^{t} A\left[\frac{p_t \lambda_{j,t} e^{\delta(t-j)}}{\gamma \mu_{jt}}\right]^{\frac{1}{\gamma - 1}} dj\,. \tag{31}$$

Profit maximization leads each firm to fix a price:

$$p_t = \frac{\eta}{\gamma}\,. \tag{32}$$

The price of each variety of health good remains fixed in equilibrium, although any reduction in the production costs and/or an improvement in the efficacy of the health goods, which leads to a reduction in the mark-up will, cause this price to fall.

Taking into account all the previous relations, it is possible to synthesize the steady state behavior in the form of some propositions, as follows.

PROPOSITION 1. – *The growth rate of the aggregate endogenous variables of the economy is* δ *and the steady longevity is constant for all the cohorts.*

PROOF.- The rate of growth of the aggregate physical capital stock in the steady state is[20]:

$$g\left(K_t^{e.e}\right)=\frac{Y_t^{e.e}}{K_t^{e.e}}-\frac{C_t^{e.e}}{K_t^{e.e}}-\frac{M_t^{e.e}D_t^{e.e}}{K_t^{e.e}}. \tag{33}$$

In the steady state the rate of growth of the endogenous variables of the economy must be constant. From (33) the aggregate physical capital stock, final goods output, final goods consumption and health goods consumption all grow at the same rate:

$$g\left(K_t^{e.e}\right)=g\left(Y_t^{e.e}\right)=g\left(C_t^{e.e}\right)=g\left(M_t^{e.e}D_t^{e.e}\right). \tag{34}$$

Similarly, the human capital stock allocated in the steady state to the production of final goods, $H_{Y_t}^{e.e}$, grows at the same rate as final goods output, $Y_t^{e.e}$, according to (8). In the same way, from (11) the stock of human capital devoted to the R&D sector, $H_{M_t}^{e.e}$, grows at the same rate as the medical innovation $M_t^{e.e}$. Given that $H_t^{e.e}=H_{Y_t}^{e.e}+H_{M_t}^{e.e}$, it is clear that the aggregate variables of the economy grow at an identical rate:

$$g\left(K_t^{e.e}\right)=g\left(Y_t^{e.e}\right)=g\left(C_t^{e.e}\right)=g\left(M_t^{e.e}D_t^{e.e}\right)=g\left(H_{Y_t}^{e.e}\right)=g\left(H_{M_t}^{e.e}\right)=g\left(H_t^{e.e}\right)=g\left(M_t^{e.e}\right). \tag{35}$$

An immediate conclusion from (35) is that, although the total demand for health goods $M_t^{e.e}D_t^{e.e}$ can grow, the demand for each of the varieties, $D_t^{e.e}$, must be constant in the steady state.

The behavior of the individual agents determines that the total stock of available human capital for the productive sectors is:

$$H_t^{e.e}=\frac{v^v h_{\min}}{(1+v)^{v+1}}\int_{z_t}^{t}\left(T_j^{e.e}\right)^v M_j^{e.e}\,dj\,. \tag{36}$$

[20] The superindex e.e denotes the steady state values of the corresponding variables.

Thus it can be derived that in the steady state we must have:

$$vg\left(T^{e.e}\right)\left\{1-e^{\left[vg\left(T^{e.e}\right)+g\left(M^{e.e}\right)\right]E_t}\right\}=\left[vg\left(T^{e.e}\right)+g\left(M^{e.e}\right)\right]\dot{E}_t^{e.e}, \tag{37}$$

that is only possible when $g\left(T^{e.e}\right)$ is zero, equivalent to $\dot{E}_t^{e.e}=0$. As a consequence, the individual's life expectancy remains constant in the steady state, that is to say, for all j:

$$T_j^{e.e}=T^{e.e}. \tag{38}$$

This result is a direct consequence of (35), in that H_t and M_t must grow at the same rate, which is only compatible with $g(T^{e.e})=0$.

The longevity of the individuals is constant and, given the population dynamics of the model, the population of the economy will also be constant. In a sense, the economy recovers in the steady state the population structure proposed by Stokey (1991), with the important difference that, in our case, the longevity is endogenous.

As the demand faced by each health good is time invariant and the population is constant, from (24) we can conclude that in the steady state each individual demands the same amount of each type of health good over the course of his life:

$$\bar{x}_{j,t}^{e.e}=\bar{x}_j^{e.e}, \tag{39}$$

The results obtained allow us to write the physiological restriction as:

$$\int_j^{j+T_j} M_t^{e.e}\left(\bar{x}_j^{e.e}\right)^{\gamma} e^{-\delta(t-j)}dt=\left(h_{\min}T^{e.e}\right)M_j^{e.e}. \tag{40}$$

Differentiating with respect to time:

$$M_{j+T^{e.e}}^{e.e}\left(\bar{x}_j^{e.e}\right)^{\gamma} e^{-\delta T^{e.e}}-M_j^{e.e}\left(\bar{x}_j^{e.e}\right)^{\gamma}=0, \tag{41}$$

which implies that $M_{j+T^{e.e}}^{e.e}=M_j^{e.e}e^{\delta T^{e.e}}$, from where it can be deduced that the rate of growth of the stock of medical knowledge in the steady state is δ and, as a consequence, from (35) the rest of the endogenous variables of the economy also grow at a rate δ (q. e. d.).

This result could seem strange because, at first glance, technological advance in health treatments should be accompanied by lower costs in the ensuring of a given lifespan. The reason why this is so is not that biological deterioration acts in a different way depending on each cohort, more particularly in a growing manner. In fact, the question is that this increasing expenditure improves the quality of life in terms of its potential outcomes. Where are these potential outcomes spent? This greater quality of life does not pay off in the utility function, because the level of health is not an argument in this function. The true reason is that the level of health' improvements are human capital investment. The ever increasing health expenditure is effective in terms of human capital productivity and individual remuneration, given the fact that human capital is increasing at a rate δ due to the growth in the health level. Perhaps the most striking point here is that, in the physiological restriction, the increase in medical technology (M_j) is not able to admit lower and lower expenditure to maintain a given lifespan[21]. This might be so in the absence of increasing labor requirements in terms of health. This greater health is spent in the productive activity and, as a consequence, there is a cost that has its reflection in the physiological restriction. With this explanation, we think the mechanisms at work in the steady state of the general equilibrium are clear in this model, in which we have found a fresh explanation for the steady state longevity that, we believe, contains many new aspects of interest.

PROPOSITION 2. - I*n the steady state the individual levels of saving and consumption of final and health goods at every stage of the life increase, cohort by cohort, at a rate δ. Thus, the individual paths of both saving and consumption of final and health goods shift upwards, cohort by cohort, in a parallel form in a proportion δ.*

PROOF.- From (40) it can be derived that the individual amount demanded of each type of health good is:

[21] According to Krugman (1997), *"….there is a consensus among health care experts that the main driving force behind rising costs is neither greed, nor inefficiency, not even the aging of our population, but technological progress. Medical expenditures used to be small, not because doctors were cheap or hospital were well managed, but because there was only so much medicine had to offer….We spend ever more on medicine because we keep on finding good new things that money can buy".*

$$\bar{x}^{e.e} = \left(h_{\min}\right)^{\frac{1}{\gamma}}. \tag{42}$$

Agents, whatever their cohort, demand in the steady state the same amount of each of the different existing varieties of health goods, with this amount increasing with the health minimum. Logically, the amount demanded for each variety decreases with its efficacy measured by γ. The total demand for health goods by each individual of cohort j, $X_{j,t}^{e.e}$, increases with the availability of new varieties:

$$X_{j,t}^{e.e} = \left(h_{\min}\right)^{\frac{1}{\gamma}} M_t^{e.e}. \tag{43}$$

From (28) and (29) we can deduce that the initial level of consumption of the individuals also grows cohort by cohort according to the rate of economic growth; that is to say, at a rate δ:

$$c_{j,j}^{e.e} = \left\{ e^{\frac{(\beta-\rho)T^{e.e}}{1-\alpha}} \left(\frac{\rho-\delta}{\alpha\beta}\right)^{\frac{1}{1-\alpha}} + \left[\delta - g\left(c_{jj}^{e.e}\right)\right] T^{e.e} \right\} c_j^{e.e}. \tag{44}$$

From (24), the return on physical capital net of taxes, $r_t^{e.e} - \phi^{e.e}$, takes a value δ. Taking into account this result, from (22) we can derive that the final goods consumption along the life of each individual can either grow, be constant, or decrease depending on the values of the parameters α, δ and ρ:

$$g\left(c_{jt}^{e.e}\right) = \delta - \frac{\rho-\delta}{1-\alpha}. \tag{45}$$

Both the individual and average total demand for health goods grow at a rate δ. The average final goods consumption grows at the same rate and agents initially choose a level of final goods consumption that also grows at a rate δ. The necessary resources to meet these initial growing needs are provided by the society transferring a larger and larger endowment to the newly born, financed by the steady capital tax $\phi^{e.e}$:

$$k_{j,j}^{e.e} = \phi^{e.e} k_j^{e.e} T^{e.e} = \left(\sigma \frac{Y^{e.e}}{K^{e.e}} - \delta \right) k_j^{e.e} T^{e.e}, \tag{46}$$

where $\left(\sigma \frac{Y^{e.e}}{K^{e.e}} \right)$ is the interest rate in the steady state. Additionally, at each point in time the agents receive the corresponding part of physical capital transferred by way of the earlier mentioned public institution from the oldest deceased cohort. In his last period of life each individual receives the remuneration from his work, $(1-\sigma) \frac{Y^{e.e}}{H_Y^{e.e}} h_j^{e.e}$, and the return on his physical capital stock accumulated until that time, $\left(\sigma \frac{Y^{e.e}}{K^{e.e}} \right) k_{j,j+T^{e.e}}^{e.e}$.

In summary, the individual stationary behaviour is characterized, first, by an initial demand for health goods that grows at rate δ cohort by cohort, with this growth rate being maintained throughout the individual's life span. Secondly, the final goods consumption of an individual at the beginning of his life increases cohort by cohort at a rate δ, following a path over the course of his life with a constant growth rate, which is lower than δ, and not necessarily positive. As a result of the individual decisions, together with the received capital endowment, from the budget constraint we can conclude that the saving of each agent also shifts upwards cohort by cohort at a rate δ (q. e. d.).

PROPOSITION 3. - *The steady state equilibrium value of the longevity comes given by the solution to the equation:*

$$e^{\frac{\beta-\delta}{1-\alpha} T^{e.e}} - e^{\frac{\beta-\rho}{1-\alpha} T^{e.e}} = \left(\frac{\alpha\beta}{\rho-\delta} \right)^{\frac{1}{1-\alpha}} \left(\frac{\rho-\delta}{1-\alpha} \right) T^{e.e},$$

provided that the condition $\alpha\beta > \rho - \delta$ *is fulfilled.*

PROOF.- Equation (44) fixes the individual's initial demand for final goods as a function of the average consumption of the economy at the time of birth. Multiplying both sides by the term $e^{\left(\delta - \frac{\rho-\delta}{1-\alpha}\right) T^{e.e}}$, we obtain:

$$c^{e.e}_{j,j+T^{e.e}} = \left[e^{\frac{(\beta-\rho)T^{e.e}}{1-\alpha}} \left(\frac{\rho-\delta}{\alpha\beta} \right)^{\frac{1}{1-\alpha}} + \frac{\rho-\delta}{1-\alpha} T^{e.e} \right] c^{e.e}_{j+T^{e.e}} e^{\left(-\frac{\rho-\delta}{1-\alpha}\right)T^{e.e}} . \tag{47}$$

By substituting condition (28) we can derive a function that allows us to implicitly determine the value of the longevity:

$$e^{\frac{\beta-\delta}{1-\alpha}T^{e.e}} - e^{\frac{\beta-\rho}{1-\alpha}T^{e.e}} = \left(\frac{\alpha\beta}{\rho-\delta} \right)^{\frac{1}{1-\alpha}} \left(\frac{\rho-\delta}{1-\alpha} \right) T^{e.e} . \tag{48}$$

If the left and right-hand terms of equation (48) with a generic T , instead of $T^{e.e}$,are denoted by z_1 and z_2, respectively, we can verify graphically in Figure 1 that this equation has a unique non-null solution provided that $\alpha\beta > \rho - \delta$. This condition guarantees a slope of z_2 when $T=0$ greater than the slope of z_1, which is a necessary condition for the existence of a solution finite and strictly positive (q.e.d.).

(Figure 1 about here)

The meaning of this inequality is important for the adequate understanding of the model. This condition fixes a minimum value for the discount rate of the expected utility from the representative descendant, or a maximum for the altruism. In other case $T^{e.e}$ would be zero.

The elements which determine the value of the steady state longevity are the elasticity α, the discount rate of the utility of future generations β, the discount rate of the utility ρ, and the growth rate of the economy δ. A study of comparative static shows a positive relation of $T^{e.e}$ with α and δ. The relation is positive with β provided that $\beta < \frac{1}{T^{e.e}}$ and it is always negative with ρ.

The expected relation with α is positive, as it is with β. The parameter α measures the weight (or the importance) of the own consumption related to the average consumption in the instantaneous utility of the individual. As it measures the importance of the own consumption in the utility obtained while being alive, it can be fairly expected that the greater the value of this parameter, the greater the longevity.

The parameter β represents a factor which is opposite to the degree of altruism; the greater its value, the lesser the degree of altruism, and the greater the longevity. However, we can observe that this direction in the relationship only takes place when $\beta < \frac{1}{T^{e.e}}$ [22]. In this context, if the increase in the degree of economies' development supposes, to some extent, an increment in β, representing a decrease in the degree of altruism, then the growth of the economy could be associated temporarily with increases in longevity.

Patient individuals (low value of ρ) are associated with greater longevity, in this way verifying a result that coincides with general opinion[23]. Thus, we conclude that individuals characterised by a concern for the future will enjoy a longer life expectancy. High discount rates mean impatience for consumption, and such impatience is not coherent with longevity. In our case, the model presents the expected result. Could it be possible that the discount rate decreases according to the pace of economic development? If this were so, then the coincidence of growth in both per-capita output and longevity would be feasible along the evolution from low to high income.

On the other hand, the analysis shows an unambiguous positive link between growth rate δ and life expectancy. This parameter appears in the second term of the preferences reflecting the valuation of the expected utility from a representative descendant; the greater the rate of growth, the shorter the period necessary to value this expectation, that is to say, the greater the longevity. Initially, this parameter is the rate of biological deterioration and the analysis shows an unambiguous positive link with the long-run life expectancy. This result might appear to be paradoxical, even contradictory, if we take into account its implication that longevity is greater when the rate of biological deterioration is greater. However, this is precisely one of the main results of our work, in that the rate of biological deterioration is the key element in the rhythm of both medical progress and economic growth and, as expression (48) shows, in the stationary level of longevity. Given a rate of biological deterioration δ, the economy reacts by offsetting its effects through increases in

[22] This result means that a greater discount rate β only leads to longer longevity when β is low compared to $T^{e.e}$. When it is relatively high, the effect of its increments will be a lower longevity, as a consequence of the sharp fall in the second part of the utility function.
[23] Fuchs (1982, 1996).

medical knowledge at the same rate. Medical innovation improves the health of the new generations, thereby enhancing their productive capacity and, as a result, sustaining economic growth. This economic growth, in turn, finances medical research and health expenditure, in this way establishing a feedback interrelation between economic growth and medical innovation. New generations incur more health expenditure because this increases both their quality of life and their human capital. The increase in human capital, cohort by cohort, at a rate δ generates resources that allow for growth in saving and in the consumption of final and health goods, cohort by cohort, at the same rate.

The rhythms of medical innovation and economic growth coexist in the steady state with a constant longevity. Whilst from our model we derive the conditions for the inexistence of long-run growth in the individuals' life expectancy, we nevertheless find that health expenditure grows steadily cohort by cohort. This growth implies an increase in the individuals' "quality" of life, in terms of health, rather than in their "quantity" of life. Medical innovation, that is to say, the growth in the variety of health goods, is closely connected in the steady state to the quality of life. However, the possibility of increments in steady state longevity is open, provided that there are changes in individuals' attitudes as regards the elasticity α, the discount rate of the expected utility from the descendants, or patience: an increase in any of these elements would extend life expectancy. Given the prospect of enjoying a longer life, individuals would choose to devote more time to education which, together with the corresponding increase in population, would raise the human capital stock of the economy and increase the output level[24]. In this way, we find that whilst long-run longevity has a positive effect on the level of production, it has no effect on growth. By contrast, the rate at which the innovation takes place in the fight against biological deterioration, that is to say, the rate of growth of the economy, does have a positive effect on longevity. Finally, if agents decide not only on the quality, but also on the quantity of their life, then innovation in the fight against biological deterioration represents an engine of growth, but it will not lead necessarily to an indefinite growth in longevity.

[24] It must be pointed out that we do not consider a period of retirement in the life horizon of the individuals.

4.- Conclusions

We have adopted a dynamic general equilibrium perspective in order to integrate the accumulation of human capital, innovation in medical technology, health and longevity, in such a way that we have been able to clearly identify which and what types of long-run bidirectional interactions take place between longevity and economic growth. When agents decide not only on their "quality" of life, but also on its "quantity", the mere process of biological deterioration, that is to say, the efficacy loss of health goods in maintaining a given level of health as individuals age, becomes an engine of growth in the long-run. The need to offset this biological deterioration encourages medical research, at a rate that fits exactly with the rate of deterioration. Medical science offers new health goods to combat this deterioration which, at the same time, have the effect of mitigating it, thereby improving the conditions of health of new generations. As a result, individuals' productive capacity improves and economic growth takes place, precisely at the rate of biological deterioration. This economic growth generates a sufficient amount of resources for the financing of medical research and health expenditure. At the same time, medical and economic progress determines a constant stationary longevity. The stationary longevity of individuals cannot be expanded indefinitely, not precisely because of the existence of biological deterioration, but as a consequence of the preferences; specifically due to the existence of altruism. However, long-run longevity improvements are possible with changes in individual preferences related to the own consumption elasticity of the utility function, the discount rate of the descendants' utility, or the degree of patience. An increase in any of these three elements would increase life expectancy, in such a way that individuals would choose to extend their education period, which would itself enhance the stock of human capital and, hence, the output level of the economy. In this way, we find that whilst longevity affects the production level, it has no effect on growth. However, we also find that the long-run growth rate of the economy does have a positive effect on longevity.

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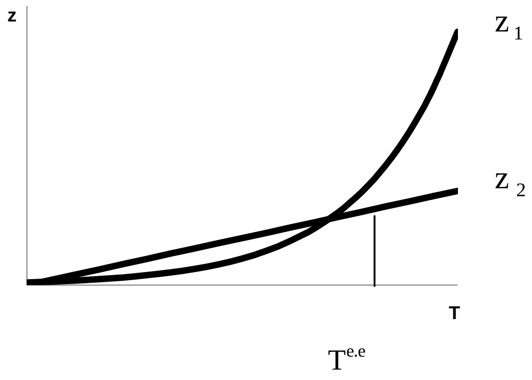
Figure 1.- Long-run longevity determination



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