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Social rationality, separability, and equity under uncertainty

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Abstract

Harsanyi (1955) proved that, in the context of uncertainty, social ratio- nality and the Pareto principle impose severe constraints on the degree of priority for the worst-off that can be adopted in the social evaluation. Since then, the literature has hesitated between an ex ante approach that relaxes rationality (Diamond (1967)) and an ex post approach that fails the Pareto principle (Hammond (1983), Broome (1991)). The Hammond-Broome ex post approach conveniently retains the separable form of utilitarianism but does not make it explicit how to give priority to the worst-off, and how much disrespect of individual preferences this implies. Fleurbaey (2008) studies how to incorporate a priority for the worst-off in an explicit formulation, but leaves aside the issue of ex ante equity in lotteries, retaining a restrictive form of consequentialism. We extend the analysis to a framework allowing for ex ante equity considerations to play a role in the ex post approach, and find a richer configuration of possible criteria. But the general outlook of the Harsanyian dilemma is confirmed in this more general setting.

Keywords: risk, inequality, social welfare, ex ante, ex post, fairness, Harsanyi theorem.

JEL Classification: D63, D71, D81

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1 Introduction

Harsanyi (1955) has presented a theorem that has attracted a lot of attention. The theorem says that in the context of lotteries, if the individuals and the social observer are expected utility maximizers, the Pareto principle applied to individual expected utilities over lotteries implies that the von Neumann-Morgenstern (VNM) utility of the social observer is affine with respect to the vector of individual VNM utilities. While Harsanyi viewed this result as a key argument in a justification of utilitarianism, many commentators understood it as a negative result. Namely, the combination of respect for individual preferences (Pareto) and social rationality (expected utility on behalf of the social observer) imposes severe constraints on the degree of priority for the worst-off. The social observers who want to be more egalitarian than allowed by Harsanyi's theorem have to choose between irrationality and paternalism, two great evils in mainstream welfare economics. The former occurs when one adopts the "ex ante" approach in which an inequality-averse social criterion is applied to individual expected utilities, while the latter haunts the "ex post" approach in which one computes the expected value of an inequality-averse social welfare function.

Hammond (1983) and Broome (1991) have proposed a version of the ex post approach in which the form of Harsanyi's utilitarianism is retained, but individual utilities are reinterpreted as the contribution to social welfare brought by individual situations. This approach is most elegant, but these authors have not explored in detail how to incorporate a priority for the worst-off in the measurement of individual utilities, and how much divorce with individual preferences over lotteries this would imply. As a consequence, their theories remain too abstract and implicit for concrete applications.

In a famous critique often invoked by the advocates of the ex ante approach, Diamond (1967) questioned the idea of requiring a social observer to maximize expected social welfare. Randomizing the allocation of a prize may not change the inequality in the final distribution significantly but may bring greater ex ante fairness between individuals. While Diamond focused on a critique of the independence axiom of the expected utility theorem, it was later noted (Machina (1989), Grant (1995)) that following Diamond on his prize example actually implies greater violations of rationality, and in particular a violation of dynamic consistency and stochastic dominance. While a fix can be proposed for dynamic consistency (Epstein and Segal (1992)), the issue of dominance is more serious. A natural step, in this respect, is to adopt a richer description of the consequences, so that one can make the difference among final consequences obtained with or without a fair lottery. With such a richer description of consequences, Diamond's critique seems powerless against the ex post approach which is then able to combine rationality and a concern for ex ante fairness.[1]

Hammond (1981) observed that when individual beliefs on probabilities are not trustworthy, respecting their ex ante preferences is not as compelling as in the case of full information. Fleurbaey (2008) argued that this is actually the general case, as probabilistic beliefs are generally different from actual probabilities. Truly enough, the social observer's own beliefs may not be much more reliable in general. However, Fleurbaey noted that in situations of randomized prizes as in Diamond's example, there is an interesting difference between a social observer who is sure of the final distribution of utilities and individuals who do not know their own final utility. Such situations are not risky for the observer and this may justify disrespecting individual preferences: preventing individuals from taking some risk is in the interest of the future losers, who are bound to exist and are ex ante ignorant of their true interests.

Dropping the Pareto principle, however, does not fully eliminate the difficulty. The argument of the previous paragraph only applies in cases of sure inequalities and the Pareto principle remains compelling when equality is preserved in all possible consequences. Fleurbaey shows that retaining the Pareto principle in cases of perfect equality and combining it with dominance singles out a social criterion: maximizing the expected value of the equally-distributed equivalent utility (Atkinson (1970)). This criterion is nice in several respects but it is strongly non-separable across subpopulations, as the equally-distributed equivalent utility in a state of nature will typically depend on the whole vector of utilities in that state. Therefore, the bulk of Harsanyi's theorem is preserved if one adds a requirement of separability across subpopulations.

Some separability is required in particular if, in practical applications, one wants to be able to make decisions for future risks independently of the utility of those who have lived in the distant past. This requirement of "Independence of the Utility of the Dead", introduced in Blackorby, Bossert, and Donaldson (2005), seems attractive if only for practical convenience. In summary, the dilemma for an inequality-averse social observer seems to involve three evils rather than just two: irrationality, paternalism, non-separability.

Fleurbaey's analysis shares with the Hammond-Broome theory the unpalatable feature that it is not fully explicit. While it is explicit about inequality aversion, it leaves it implicit how to incorporate a concern for ex ante fairness in the mea-

[1]See Adler and Sanchirico (2006) for a rich discussion of these issues and an endorsement of the ex post approach.

surement of final utilities. Formally, it retains a narrow form of consequentialism in which the evaluation of ex post consequences in a particular state of nature only involves the utilities obtained in this state of nature. The interplay between ex ante fairness and ex post inequality aversion is therefore left unexplored. In this paper, we set out to analyze the form of the dilemma when the evaluation of ex post consequences may involve the counterfactual utilities of other states of nature. Formally, this means that the requirement of dominance becomes much less constraining.

We also extend the analysis in another direction. Unlike many papers pursuing Harsanyi's work,[2] we will not assume that the evaluation of ex ante individual prospects, as referred to in the Pareto principle, is based on expected utility. In this way the analysis gains in generality and the negative results, if any, become even more problematic. Our results are not totally negative but they show that the essence of the dilemma remains. More precisely, we show that the combination of rationality and separability imposes such constraints on the social criterion that the dilemma between paternalism and priority for the worst-off is unescapable.

The structure of the paper is straightforward. In Section 2 the framework is presented, followed in Section 3 by the axioms that embody the requirements we want to impose on the social criterion. The results are stated in Section 4 and discussed in Section 5. Section 6 concludes. All the proofs are gathered in Section 7.

2 Setup

The framework involves state-contingent alternatives,[3] with a finite set of states of nature $\mathscr{S} = \{1, ..., s\}$. The population is a finite set $\mathscr{N} = \{1, ..., n\}$.

The objects of evaluation are prospects (u, z), in which $u \in \mathscr{U} = \mathbb{R}^{ns}$ is a utility matrix such that $u_{i\sigma}$ is the utility obtained by $i \in \mathscr{N}$ in state $\sigma \in \mathscr{S}$, and $z \in \mathscr{Z}$ denotes any ex ante non-utility information about states of nature (such as probabilities of occurrence) that may be relevant for the evaluation. The set $\mathscr{Z}$ is assumed to be a separable metric space. Let $u_i = (u_{i\sigma})_{\sigma \in \mathscr{S}}$ and $u_\sigma = (u_{i\sigma})_{i \in \mathscr{N}}$. Let $u_{-i} = (u_j)_{j \in \mathscr{N} \setminus \{i\}}$, and for $M \subseteq \mathscr{N}$, $u_M = (u_i)_{i \in M}$.

The subset of sure prospects, i.e., of prospects u such that $u_\sigma = u_\tau$ for all $\sigma, \tau \in \mathscr{S}$, is denoted $\mathscr{U}^c$. The subset of egalitarian prospects, i.e., of prospects u such that $u_i = u_j$ for all $i, j \in \mathscr{N}$, is denoted $\mathscr{U}^e$.

[2]Notable exceptions are Blackorby, Donaldson, and Mongin (2004), Gajdos, Tallon, and Vergnaud (2008).

[3]For the adaptation of Harsanyi's theorem to such a framework, see Blackorby, Donaldson, and Weymark (1999).

The utility figure $u_{i\sigma}$ must be interpreted as measuring the utility obtained by individual i in state σ, without consideration of inequality in society or fairness in the lottery. The goal of this paper is to define how to incorporate such considerations explicitly in the social evaluation. In contrast, the value of $u_{i\sigma}$ may include everything that is relevant in i's personal ex post situation, including the utility consequences of bearing risk in the ex ante situation. For instance, if i has taken a great risk and suffered anxiety, this may yield a low $u_{i\sigma}$ even in a lucky state of nature. We do not explicitly model individual preferences under uncertainty and the underlying economic allocations. We work directly with utility consequences.

Ex ante, the social planner faces a prospect $(u, z) \in \mathscr{U} \times \mathscr{Z}$. Ex post, the social planner faces a situation $(u, z, \sigma) \in \mathscr{U} \times \mathscr{Z} \times \mathscr{S}$. We are interested in three preference orderings, which are all supposed to belong to the same ethical observer who seeks to make a coherent assessment of ex ante prospects and ex post consequences.

- Ex post preferences on individual situations, denoted R, bearing on $(u_i, z, \sigma) \in \mathbb{R}^s \times \mathscr{Z} \times \mathscr{S}$.

  Such preferences do not bear only on $u_{i\sigma}$ because what could have happened in other (non-realized) states of nature may be important in order to assess whether the individual has been fairly treated.

- Ex post preferences on social situations, denoted R^p, bearing on $(u, z, \sigma) \in \mathscr{U} \times \mathscr{Z} \times \mathscr{S}$.

  Again, such preferences do not only bear on (u_σ, z, σ), because utility in counterfactual states may carry relevant information.

- Ex ante preferences on social prospects, denoted R^a, bearing on $(u, z) \in U \times Z$.

Let P and I denote the strict preference and indifference relations, respectively, corresponding to R. The relations P^p, I^p, P^a, and I^a are defined similarly.

3 Axioms

The axioms we want to impose on this triple of orderings fall under three headings: social rationality, individualism and separability. We do not introduce specific axioms that would capture the ideals of priority for the worst-off and ex ante fairness. The results we obtain make it clear how such ideals can be satisfied in combination with the axioms studied in this paper. This will be discussed in Section 5.

3.1 Social rationality

Harsanyi (1955) requires the social criterion to take the form of expected welfare. We also make rationality assumptions, but in a more general form that encompasses non-expected utility criteria and turns out, as we shall see, to be formally weak enough to accommodate the ex ante approach. First, the relations under consideration should be complete and continuous preorders.

Axiom 1 (Ordering)**.** The three relations R, R^p, R^a are transitive, reflexive, complete, and continuous.[4]

The key rationality axioms are dominance and independence. Dominance means that an improvement in all possible consequences for the different states of nature must yield a global improvement. This is really the minimal requirement of social rationality.

Axiom 2 (Dominance)**.** For all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$[\forall \sigma \in \mathscr{S},\ (u, z, \sigma) R^p (u', z', \sigma)] \Rightarrow (u, z) R^a (u', z'),$$

and

$$\left.\begin{array}{l} \forall \sigma \in \mathscr{S},\ (u, z, \sigma) R^p (u', z', \sigma) \\ \exists \hat{\sigma} \in \mathscr{S},\ (u, z, \hat{\sigma}) P^p (u', z', \hat{\sigma}) \end{array}\right\} \Rightarrow (u, z) P^a (u', z').$$

Independence is formulated here in a way that remains compatible with many non-expected utility approaches, because it is applied in a way that takes account of the whole matrix u in the evaluation of ex post consequences. This is therefore a rather weak axiom.

Axiom 3 (Independence)**.** For all $u, v, u', v' \in \mathscr{U}$, $y, z, y', z' \in Z$ and all $T \subseteq \mathscr{S}$,

$$\left.\begin{array}{l} \forall \sigma \in T,\ (u, y, \sigma) I^p (v, z, \sigma) \\ \forall \sigma \in T,\ (u', y', \sigma) I^p (v', z', \sigma) \\ \forall \sigma \in \mathscr{S} \setminus T,\ (u, y, \sigma) I^p (u', y', \sigma) \\ \forall \sigma \in \mathscr{S} \setminus T,\ (v, z, \sigma) I^p (v', z', \sigma) \end{array}\right\} \Rightarrow [(u, y) R^a (v, z) \Leftrightarrow (u', y') R^a (v', z').]$$

The next axiom is meant to rule out degenerate criteria for which the evaluation of ex post consequences is only based on ex ante information.[5] Introduced by Skiadas (1997), it requires sufficient richness in the possible evaluation of the ex post consequences of a given prospect.

[4] A relation $\tilde{R}$ on X is said to be continuous if, for all $x \in X$, the sets $\{y \in X \mid x\tilde{R}y\}$ and $\{y \in X \mid y\tilde{R}x\}$ are closed.

[5] In fact, such criteria are not completely excluded by this axiom. See footnote 9.

Axiom 4 (Solvability)**.** For all $((u^1, z^1), \cdots, (u^s, z^s)) \in (\mathscr{U} \times \mathscr{Z})^s$, there exists $(u, z) \in \mathscr{U} \times \mathscr{Z}$ such that for all $\sigma \in \mathscr{S}$, $(u, z, \sigma) I^p (u^\sigma, z^\sigma, \sigma)$.

We also introduce axioms that require a certain form of simplicity in the evaluation of final situations in different states. First, we require the role of states to be symmetric in the ex ante evaluation, as observed for instance in expected utilities which are sums of terms representing the contribution of each state to the expected value, each term being the product of the probability of the state by the utility attained in the state.

Axiom 5 (State Neutrality)**.** For all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, if there exists a permutation $\pi : \mathscr{S} \to \mathscr{S}$ such that $(u, z, \sigma) I^p (u', z', \pi(\sigma))$ for all $\sigma \in \mathscr{S}$, then $(u, z) I^a (u', z')$.

The next axiom requires that for riskless prospects, the contribution of each state to ex ante evaluation is equal for some informational configurations (e.g., equiprobable states in the case of expected utility).

Axiom 6 (State Equivalence)**.** There exists $\mathscr{Z}^c \subset \mathscr{Z}$ such that for all $u \in \mathscr{U}^c$, $z \in \mathscr{Z}^c$, $\sigma, \sigma' \in \mathscr{S}$, $i \in \mathscr{N}$

$$(u_i, z, \sigma) I (u_i, z, \sigma').$$

3.2 Individualism

In Harsanyi (1955), individualism is embodied in the Pareto principle applied to ex ante prospects. As noted in Hammond (1981) and emphasized in Fleurbaey (2008), the Pareto principle is not compelling when applied to uncertain prospects because unanimity among future winners and losers may be obtained only because they ignore their ultimate interests. We therefore limit the application of this principle to ex post consequences, in which full information prevails.

Axiom 7 (Ex Post Pareto)**.** For all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$ and $\sigma, \sigma' \in \mathscr{S}$,

$$[\forall i \in \mathscr{N},\ (u_i, z, \sigma) R (u'_i, z', \sigma')] \Rightarrow (u, z, \sigma) R^p (u', z', \sigma'),$$

and

$$\left.\begin{array}{l} \forall i \in \mathscr{N},\ (u_i, z, \sigma) R (u'_i, z', \sigma') \\ \exists i \in \mathscr{N},\ (u_i, z, \sigma) P (u'_i, z', \sigma') \end{array}\right\} \Rightarrow (u, z, \sigma) P^p (u', z', \sigma').$$

We also introduce a monotonicity axiom made of two parts. The first one is standard, and requires that the evaluation of individual situations is increasing in the components of the prospects.[6] The second part of the axiom essentially requires that

[6]We therefore implicitly assume that for all z in Z, there is no null state.

differences in information can always be compensated in the evaluation of individual situations by increasing or decreasing the prospect itself. Formally, this axiom is stated as follows.[7]

Axiom 8 (Monotonicity)**.**

1. For all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma \in \mathscr{S}$, $i \in \mathscr{N}$,

$$\begin{aligned} u_i &\geq u'_i \Rightarrow (u_i, z, \sigma) R (u'_i, z, \sigma), \\ u_i &> u'_i \Rightarrow (u_i, z, \sigma) P (u'_i, z, \sigma), \end{aligned}$$

2. There exists $z^* \in \mathscr{Z}^c$ such that, for all $u_i \in \mathbb{R}^s$, $\sigma \in \mathscr{S}$ and $z \in Z$, there exists two sure vectors $\underline{u}_i$ and $\bar{u}_i$ for which

$$(\bar{u}_i, z^*, \sigma) R (u, z, \sigma) R (\underline{u}_i, z^*, \sigma).$$

Finally, we require individuals to be treated equally.

Axiom 9 (Anonymity)**.** For all $u, u' \in \mathscr{U}$, $z \in \mathscr{Z}$, $\sigma \in \mathscr{S}$, if there exists a permutation $\pi : \mathscr{N} \to \mathscr{N}$ such that for all $i \in \mathscr{N}$, $u'_i = u_{\pi(i)}$,

$$(u, z, \sigma) I^p (u', z, \sigma).$$

3.3 Separability

Harsanyi (1955) derives a separable (indeed, additive) social ordering from the combination of social rationality and ex ante Pareto. With the axioms introduced so far very little separability is obtained, and it appears interesting to study a quite attractive principle of separability. This principle says that individuals who are not concerned and bear no risk should not influence the social evaluation. Individuals are not concerned when their personal situation is the same in the two prospects under consideration. This principle is inspired by the observation that in its absence, for practical applications of the criteria studied here, one should either take account of the utility of dead people in the evaluation of prospects,[8] or ignore it and violate dynamic consistency.

We introduce two axioms capturing this idea. The first literally embodies the separability principle as just stated.

[7] For two vectors x, y, $x > y$ means that $x \geq y$ and $x \neq y$.

[8] The principle of "independence of the utility of the dead"has been introduced by Blackorby, Bossert, and Donaldson (2005). It is also invoked in Bommier and Zuber (2008).

Axiom 10 (Independence of the Utility of the Sure)**.** For all $u, u' \in \mathscr{U}$, $v, v' \in \mathscr{U}^c$, $z, z' \in \mathscr{Z}$, $M \subset \mathscr{N}$,

$$((u_M, v_{\mathscr{N}\setminus M}), z) R^a ((u'_M, v_{\mathscr{N}\setminus M}), z') \Leftrightarrow ((u_M, v'_{\mathscr{N}\setminus M}), z) R^a ((u'_M, v'_{\mathscr{N}\setminus M}), z').$$

The second says that when the subgroup that takes risks and is concerned is perfectly egalitarian in all possible states of nature, then the evaluation should proceed as if the whole society was doing the same. This means that, in this special case, the mere presence of unconcerned and risk-free individuals has no influence on the evaluation, a property that is not guaranteed under the previous axiom.

Axiom 11 (Restricted Independence of the Sure)**.** For all $u, u' \in \mathscr{U}^e$, $v \in \mathscr{U}^c$, $z, z' \in \mathscr{Z}$, $M \subset \mathscr{N}$, $M \neq \varnothing$,

$$((u_M, v_{\mathscr{N}\setminus M}), z) R^a ((u'_M, v_{\mathscr{N}\setminus M}), z') \Leftrightarrow (u, z) R^a (u', z').$$

4 Two families of social criteria

In the standard consequentialist framework, Axiom 10 (Independence of the Utility of the Sure) implies the following strong form of ex post separability (see Blackorby, Bossert, and Donaldson (2005) or Bommier and Zuber (2008)), which can also be justified normatively (see Broome (1991)).

Axiom 12 (Ex Post Separability)**.** For all $u, v, u', v' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $s, s' \in \mathscr{S}$, and $M \subset \mathscr{N}$,

$$\left.\begin{array}{l} \forall i \in M,\, u_i = v_i \\ \forall i \in M,\, u'_i = v'_i \\ \forall i \in \mathscr{N} \setminus M,\, (u_i, z, \sigma) I (u'_i, z', \sigma') \\ \forall i \in \mathscr{N} \setminus M,\, (v_i, z, \sigma) I (v'_i, z', \sigma') \end{array}\right\} \Rightarrow \left[(u, z, \sigma) R^p\, (u', z', \sigma') \Leftrightarrow (v, z, \sigma)\, R^p (v', z', \sigma')\right].$$

A similar implication can be obtained in our extended framework, as shown by the following lemma.

Lemma 1. Axioms 1 (Ordering), 2 (Dominance), 6 (State Equivalence), 7 (Ex Post Pareto), 8 (Monotonicity) and 10 (Independence of the Utility of the Sure) imply Axiom 12 (Ex Post Separability).

We are now ready to state our first main result. The principles introduced in Section 3 make it possible to single out two broad families of social criteria.

Proposition 1. If Axioms 1 (Ordering), 2 (Dominance), 3 (Independence), 4 (Solvability), 5 (State Neutrality), 6 (State Equivalence), 7 (Ex Post Pareto), 8 (Monotonicity) and 10 (Independence of the Utility of the Sure) are satisfied, then:

1. there exists a continuous function $\varphi : \mathbb{R}^s \times \mathscr{Z} \times \mathscr{S} \to \mathbb{R}$, increasing in its first s arguments and satisfying $\varphi(u_i, z, \sigma) = \varphi(u_i, z, \sigma')$ for all $\sigma, \sigma' \in \mathscr{S}$, $u \in \mathscr{U}^c$ and $z \in \mathscr{Z}^c$, such that for all $u_i, u_i' \in \mathbb{R}^s$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,
$$(u_i, z, \sigma) \, R \, (u_i', z', \sigma') \Leftrightarrow \varphi \left(u_i, z, \sigma\right) \geq \varphi \left(u_i', z', \sigma'\right);$$

2. there exist n continuous increasing functions $\varphi_i : \mathbb{R} \to \mathbb{R}$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,
$$(u, z, \sigma) R^p (u', z', \sigma') \Leftrightarrow \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \geq \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i', z', \sigma');$$

3. (a) either for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,
$$(u, z) R^a (u', z') \Leftrightarrow \sum_{\sigma \in \mathscr{S}} \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \geq \sum_{\sigma \in \mathscr{S}} \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i', z', \sigma')$$
and for all $i \in \mathscr{N}$ there exists a continuous increasing function ψ_i and a continuous function ξ_i such that for all $u \in \mathscr{U}^c$, $\sum_{\sigma \in \mathscr{S}} \varphi_i \circ \varphi(u_i, z, \sigma) = \psi_i(u_{i1}) + \xi_i(z)$.

   (b) or, there exists $\alpha \neq 0$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,
$$(u, z) R^a (u', z')$$
$$\Leftrightarrow \sum_{\sigma \in \mathscr{S}} \alpha \exp \left(\alpha \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \right) \geq \sum_{\sigma \in \mathscr{S}} \alpha \exp \left(\alpha \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i', z', \sigma') \right).$$
and for all $i \in \mathscr{N}$ there exists a continuous increasing function ϑ_i and a continuous function η_i such that for all $u \in \mathscr{U}^c$, $\varphi_i \circ \varphi(u_i, z, \sigma) = \vartheta_i(u_{i1}) + \eta_i(z, \sigma)$.

Note that we do not provide necessary and sufficient conditions for all the axioms to be satisfied. This is because Axiom 4 (Solvability) and the second part of Axiom 8 (Monotonicity) impose conditions that are not specially interesting, and a little heavy to write. However, we have the following result.

Proposition 2. Assume R, R^p and R^a are defined as in Proposition 1. Then Axioms 1 (Ordering), 2 (Dominance), 3 (Independence), 5 (State Neutrality), 6

(State Equivalence), 7 (Ex Post Pareto), 10 (Independence of the Utility of the Sure), and the first part of Axiom 8 (Monotonicity) are satisfied.

While the additive family (a) in Proposition 1 is reminiscent of Harsanyi's result, the exponential one (b) looks more singular. However, when Axioms 9 (Anonymity) and 11 (Restricted Independence of the Sure) are added, only the additive family remains.

Proposition 3. If Axioms 1 (Ordering), 2 (Dominance), 3 (Independence), 4 (Solvability), 5 (State Neutrality), 6 (State Equivalence), 7 (Ex Post Pareto), 8 (Monotonicity), 9 (Anonymity), 10 (Independence of the Utility of the Sure) and 11 (Restricted Independence of the Sure) are satisfied then there exists a continuous function $\varphi : \mathbb{R}^s \times \mathscr{Z} \times \mathscr{S} \to \mathbb{R}$ that is increasing in its first s arguments and satisfies $\varphi(u_i, z, \sigma) = \varphi(u_i, z, \sigma')$ for all $\sigma, \sigma' \in \mathscr{S}$, $u \in \mathscr{U}^c$ and $z \in \mathscr{Z}^c$ and $\sum_{\sigma \in \mathscr{S}} \varphi(u_i, z, \sigma) = \psi(u_i^1) + \xi(z)$ for all $u \in \mathscr{U}^c$, where ψ is continuous and increasing and ξ is continuous, and such that:

1. for all $u_i, u_i' \in \mathbb{R}^s$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u_i, z, \sigma)\, R\, (u_i', z', \sigma') \Leftrightarrow \varphi\,(u_i, z, \sigma) \geq \varphi\,(u_i', z', \sigma')\,,$$

2. for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u, z, \sigma)\, R^p\, (u', z', \sigma') \Leftrightarrow \sum_{i \in \mathscr{N}} \varphi\,(u_i, z, \sigma) \geq \sum_{i \in \mathscr{N}} \varphi\,(u_i', z', \sigma')\,,$$

3. For all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$(u, z) R^a (u', z') \Leftrightarrow \sum_{i \in \mathscr{N}} \sum_{\sigma \in \mathscr{S}} \varphi\,(u_i, z, \sigma) \geq \sum_{i \in \mathscr{N}} \sum_{\sigma \in \mathscr{S}} \varphi\,(u_i', z', \sigma)\,.$$

5 Discussion

Let us review the various possible social criteria that are singled out or ruled out by our results. In order to do so, it is useful to specify a little more the notion of individual ex ante utility. We will assume that it can be denoted $E_z u_i$, without implying that this must be an *expected* utility.

In the analysis, indeed, we have relied on substantial requirements of social rationality and separability, but left a secondary role for individualism and the respect of individual preferences. However, it is easy to see what happens if such notions are taken on board. Consider the criterion based on $\sum_{i \in \mathscr{N}} \sum_{\sigma \in \mathscr{S}} \varphi\,(u_i, z, \sigma)\,.$

If $u \in \mathscr{U}^e$, i.e., if u is perfectly egalitarian in all states of nature, the criterion boils down to maximizing $\sum_{\sigma\in\mathscr{S}} \varphi(u_i, z, \sigma)$. If this formula is congruent with individual ex ante utility (which appears reasonable when there is no inequality), there is an increasing function G such that $\sum_{\sigma\in\mathscr{S}} \varphi(u_i, z, \sigma) \equiv G(E_z u_i)$. This implies that for all $u \in \mathscr{U}$, not just egalitarian prospects, the social criterion is based on $\sum_{i\in\mathscr{N}} G(E_z u_i)$. It is clear that this criterion takes no account whatsoever of ex post inequalities and focuses at most on ex ante inequalities. Therefore Prop. 3 implies that there is a clear dilemma between respecting ex ante individual utility in absence of inequalities and giving priority to the worst-off in every state of nature.

Therefore, in comparison with the more restrictive analysis in Fleurbaey (2008), allowing for a richer evaluation of final consequences that takes account of counterfactual states makes it possible here to combine ex ante inequality aversion with ex post rationality, separability, and some respect of ex ante utility. The introduction of ex post inequality aversion remains problematic.

However, even ex ante inequality aversion raises some difficulty in our setting. Let us look again at the identity $\sum_{\sigma\in\mathscr{S}} \varphi(u_i, z, \sigma) \equiv G(E_z u_i)$. If $E_z u_i$ is linear in the vector u_i (as in expected utility or rank-dependent expected utility), and if $\varphi(u_i, z, \sigma)$ is more sensitive to $u_{i\sigma}$ than to the other components of u_i, we almost have a Pexider equation — this is exactly a Pexider equation if $\varphi(u_i, z, \sigma)$ depends only on the component $u_{i\sigma}$, as in the strict form of consequentialism adopted in Fleurbaey (2008).

It is therefore difficult for G to be strictly concave. Let us illustrate this with an example. Suppose that $\varphi(u_i, z, \sigma) = f_\sigma^z(\lambda u_{i\sigma} + (1-\lambda) E_z u_i)$. It is easy to make sense of such a form, the evaluation of a final situation depending jointly on ex post utility and ex ante utility. The quasi Pexider equation becomes:

$$\sum_{\sigma\in\mathscr{S}} f_\sigma^z(\lambda u_{i\sigma} + (1-\lambda) E_z u_i) = G(E_z u_i).$$

If $E_z u_i$ is linear in the vector u_i, it is also linear in $(\lambda u_{i\sigma} + (1-\lambda) E_z u_i)_{s\in S}$ and we then really have a Pexider equation, unless $\lambda = 0$. Therefore, if $E_z u_i$ is linear in the vector u_i the dilemma is the following: either G is affine, so that there is no ex ante inequality aversion in R^a, or $\lambda = 0$, meaning that the ex post evaluation $\varphi(u_i, z, \sigma)$ is based solely on the ex ante information contained in $E_z u_i$ — a rather absurd kind of ex post evaluation according to which "bad luck does not hurt".[9]

[9] Solvability is meant to rule out this degenerate kind of ex post evaluation, but does not exclude it completely. Letting $P(z, \sigma)$ be the probability of state $\sigma \in \mathscr{S}$ when the information is $z \in \mathscr{Z}$, suppose that $\varphi(u_i, z, \sigma) = P(z, \sigma) G(E_z u_i)$ and that the range of $G(E_z u_i)$ for $u_i \in \mathbb{R}^s$ is $\mathbb{R}_+$. Then Solvability is satisfied by adjusting z jointly with u_i.

Therefore, even though a seemingly ex ante criterion such as $\sum_{i\in\mathscr{N}} G\left(E_z u_i\right)$ can be surprisingly reconciled with our ex post approach because our Dominance and Independence axioms are weak enough, this is obtained at the cost of making the ex post criteria R^p and R unreasonable.

We illustrate these dilemmas in Fig. 1 which gives examples of criteria R^a and R^p for the various possibilities left under the conditions of Prop. 3.

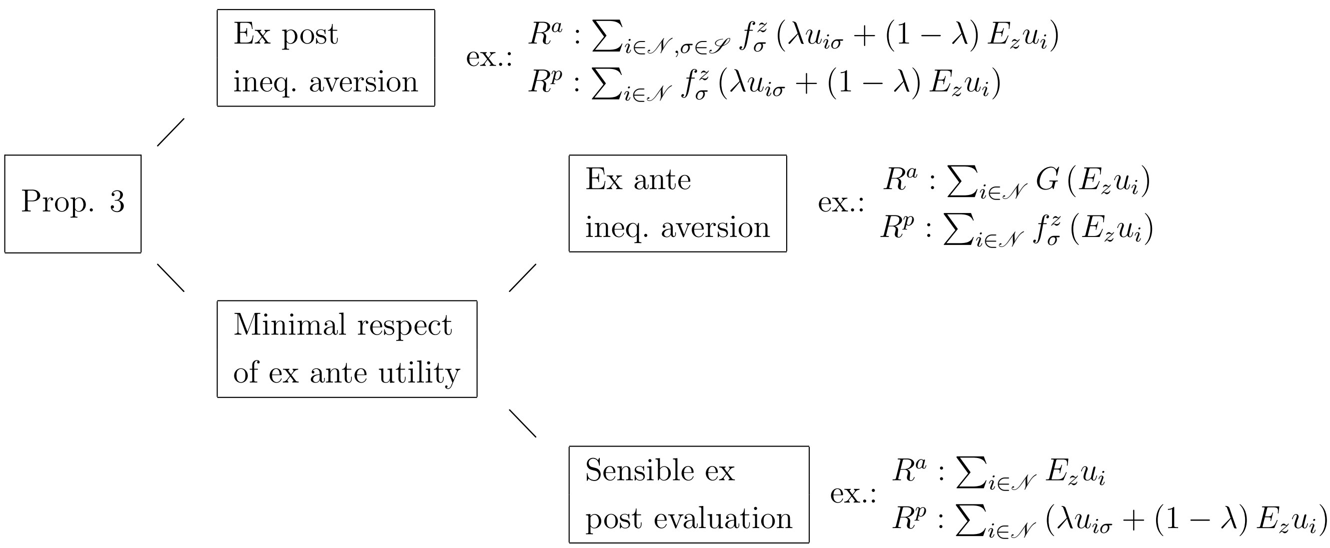

Figure 1

Let us now move backward and examine another possibility left open in Prop. 1 and excluded in Prop. 3, namely, the case in which R^a is represented by

$$\alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in\mathscr{N}} \varphi\left(u_i, z, \sigma\right)\right).$$

To compare with the above discussion and to ensure Independence of the Utility of the Sure (see Prop. 1), assume that $\varphi\left(u_i, z, \sigma\right) = \frac{1}{n}\Big(\eta(z,\sigma) + f(\lambda u_{i\sigma} + (1-\lambda)E_z u_i)\Big)$. For instance, one can take $\eta(z,\sigma) = \frac{\ln P(z,\sigma)}{\alpha}$, where $P(z,\sigma)$ is the probability of state $\sigma \in \mathscr{S}$ when the information is $z \in \mathscr{Z}$, to obtain the expression:

$$\alpha \sum_{\sigma\in\mathscr{S}} P(z,\sigma) \exp\left(\alpha \sum_{i\in\mathscr{N}} \frac{1}{n} f\Big(\lambda u_{i\sigma} + (1-\lambda)E_z u_i\Big)\right).$$

This criterion embodies a concern for ex ante and ex post inequality as well as for ex ante fairness. The cost is of course to lose Restricted Independence of the Sure. The criterion can also be made consistent with the respect of ex ante individual utility in absence of inequality. We can indeed have

$$\alpha \sum_{\sigma \in \mathscr{S}} P(z,\sigma) \exp\left(\alpha f\left(\lambda u_{i\sigma} + (1-\lambda) E_z u_i\right)\right) = G(E_z U_i)$$

provided $f(y) = \frac{\ln(y+\gamma)}{\alpha}$. One problem with this criterion however is that it is not well defined when $\lambda u_{i\sigma} + (1-\lambda)E_z u_i < -\gamma$. Hence the dilemmas exposed above cannot be solved satisfactorily on the whole space $\mathscr{U}$ even when relaxing Restricted Independence of the Sure.

Moving backward again, it appears that the exponential criterion belongs to a broader family. One may indeed question Independence of Utility of the Sure and examine what can be obtained in this way. It is then easy to refine the criterion proposed by Fleurbaey (2008) in order to take account of ex ante fairness. For instance, let $\varphi(u_i, z, \sigma) = P(z,\sigma) f(\lambda u_{i\sigma} + (1-\lambda) E_z u_i)$ with f a strictly concave, increasing and continuous function. The social ordering R^p may be based on the equally-distributed equivalent

$$P(z,\sigma) f^{-1}\left(\frac{1}{n} \sum_{i \in \mathscr{N}} f(\lambda u_{is} + (1-\lambda) E_z u_i)\right)$$

and R^a may be based on maximizing the expected value of this expression:

$$\sum_{\sigma \in \mathscr{S}} P(z,\sigma) f^{-1}\left(\frac{1}{n} \sum_{i \in \mathscr{N}} f(\lambda u_{is} + (1-\lambda) E_z u_i)\right).$$

This expression is very similar to the one obtained for the exponential case. Like the exponential criterion, it embodies a concern for ex ante and ex post inequality as well as for ex ante fairness. When $u \in \mathscr{U}^e$, it boils down to $E_z u_i$ and therefore respects individual utility in absence of inequality. It satisfies all the axioms of our list except the separability axioms and Ex Post Pareto, and allows for any finite degree of inequality aversion.

6 Conclusion

The general outlook of our results is negative. In a framework that allows for an explicit incorporation of ex ante and ex post inequality aversion and ex ante fairness in the social evaluation of risky prospects, a list of reasonable axioms that capture basic notions of social rationality, individualism and separability impose such constraints on the social ordering that no fully satisfactory candidate is singled out.

Inequality aversion enters in conflict with a minimal respect of individual ex ante utility, and this occurs whether one is interested in ex post or even simply ex ante inequality aversion. The conflict is alleviated only if separability is relaxed.

A more positive conclusion is that we have found general conditions that single out an additively separable criterion of the form $\sum_{i\in\mathscr{N}}\sum_{\sigma\in\mathscr{S}}\varphi\left(u_i,z,\sigma\right)$ without assuming a strict form of consequentialism. Although we have directly assumed separability across states of nature via the Independence axiom, separability across subpopulation has been introduced only through weak separability axioms involving the subpopulations who take no risk. Another positive result is the exponential criterion which, even though it fails some attractive axioms, illustrates how a specific moderate degree of inequality aversion may be imposed by the analysis.

We believe our axioms to be all reasonable, but the list is long and we are not able to show that each of them is needed in our results. This is the cost to pay for a very flexible framework with very weak rationality conditions. Our conditions are so weak that ex ante criteria of the form $\sum_{i\in\mathscr{N}}G\left(E_z u_i\right)$ can fulfill them, but in a rather implausible way as far as ex post evaluation is concerned. The dilemmas identified in this paper therefore deserve to be further explored in order to pin down the key principles of rationality, individualism and separability which are responsible for the difficulty. In particular, one may think that Continuity, Solvability, and State Neutrality are mostly technical requirements and it would be very interesting to see what happens to our two main results without them. This is left for future research.

7 Proofs

7.1 Proof of Lemma 1

Proof. Let $u,v,u',v'\in\mathscr{U}$, $z,z'\in\mathscr{Z}$, $\sigma,\sigma'\in\mathscr{S}$, and $M\subset\mathscr{N}$ be such that

$$\begin{aligned}
&\forall i\in M,\ u_i=v_i\\
&\forall i\in M,\ u_i'=v_i'\\
&\forall i\in\mathscr{N}\setminus M,\ (u_i,z,\sigma)I(u_i',z',\sigma')\\
&\forall i\in\mathscr{N}\setminus M,\ (v_i,z,\sigma)I(v_i',z',\sigma').
\end{aligned}$$

Let $z^*\in\mathscr{Z}^c$ be as defined in Axiom 8 (Monotonicity). By Axioms 1 (Ordering) and 8 (Monotonicity), for all $i\in\mathscr{N}$ there is a sure $\bar{u}_i$ such that $(u_i,z,\sigma)\,I\,(\bar{u}_i,z^*,\sigma)$ and a sure $\bar{u}_i'$ such that $(u_i',z',\sigma')\,I\,(\bar{u}_i',z^*,\sigma')$. Similarly there is a sure $\bar{v}_i$ such that $(v_i,z,\sigma)\,I\,(\bar{v}_i,z^*,\sigma)$ and a sure $\bar{v}_i'$ such that $(v_i',z',\sigma')\,I\,(\bar{v}_i',z^*,\sigma')$. For all $i\in M$, $\bar{u}_i=\bar{v}_i$ and $\bar{u}_i'=\bar{v}_i'$. For all $i\in\mathscr{N}\setminus M$, $(\bar{u}_i,z^*,\sigma)\,I\,(\bar{u}_i',z^*,\sigma')$ and $(\bar{v}_i,z^*,\sigma)\,I\,(\bar{v}_i',z^*,\sigma')$, which, by Axiom 6 (State Equivalence), implies $\bar{u}_i=\bar{u}_i'$ and $\bar{v}_i=\bar{v}_i'$.

By Axiom 7 (Ex Post Pareto),

$$\begin{aligned}
&(u, z, \sigma)\, I^p\, (\bar{u}, z^*, \sigma)\,, \\
&(u', z', \sigma')\, I^p\, (\bar{u}', z^*, \sigma')\,, \\
&(v, z, \sigma)\, I^p\, (\bar{v}, z^*, \sigma)\,, \\
&(v', z, \sigma)\, I^p\, (\bar{v}', z^*, \sigma)\,.
\end{aligned}$$

Suppose that $(u, z, \sigma) R^p\, (u', z', \sigma')$, which, by transitivity, is equivalent to $(\bar{u}, z^*, \sigma)\, R^p\, (\bar{u}', z^*, \sigma')$. By Axiom 6 (State Equivalence), $(\bar{u}, z^*, \tau)\, R^p\, (\bar{u}', z^*, \tau)$ for all $\tau \in \mathscr{S}$. By Axiom 2 (Dominance), $(\bar{u}, z^*)\, R^a\, (\bar{u}', z^*)$.

By Axiom 10 (Independence of Utility of the Sure),

$$\left(\left(\bar{u}_M, \bar{u}_{\mathscr{N}\setminus M}\right), z^*\right) R^a \left(\left(\bar{u}'_M, \bar{u}_{\mathscr{N}\setminus M}\right), z^*\right) \Leftrightarrow \left(\left(\bar{u}_M, \bar{u}'_{\mathscr{N}\setminus M}\right), z^*\right) R^a \left(\left(\bar{u}'_M, \bar{u}'_{\mathscr{N}\setminus M}\right), z^*\right).$$

This also reads

$$(\bar{u}, z^*)\, R^a\, (\bar{u}', z^*) \Leftrightarrow (\bar{v}, z^*)\, R^a\, (\bar{v}', z^*)\,.$$

Suppose one had $(\bar{v}', z^*, \sigma')\, P^p\, (\bar{v}, z^*, \sigma)$. Then, by Axioms 6 (State Equivalence) and 2 (Dominance), one would have $(\bar{v}', z^*)\, P^a\, (\bar{v}, z^*)$, a contradiction. Therefore, $(\bar{v}, z^*, \sigma)\, R^p\, (\bar{v}', z^*, \sigma')$. By transitivity, $(v, z, \sigma)\, R^p(v', z', \sigma')$.

We have proved that

$$(u, z, \sigma) R^p\, (u', z', \sigma') \Rightarrow (v, z, \sigma)\, R^p (v', z', \sigma').$$

By symmetry, the converse holds. □

7.2 Proof of Proposition 1

Claim 1. There exists a continuous function $\varphi : \mathbb{R}^s \times \mathscr{Z} \times \mathscr{S} \to \mathbb{R}$, increasing in its first s arguments and satisfying $\varphi(u_i, z, \sigma) = \varphi(u_i, z, \sigma')$ for all $\sigma, \sigma' \in \mathscr{S}$, $u \in \mathscr{U}^c$ and $z \in \mathscr{Z}^c$, such that for all $u_i, u'_i \in \mathbb{R}^s$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u_i, z, \sigma)\, R\, (u'_i, z', \sigma') \Leftrightarrow \varphi\, (u_i, z, \sigma) \geq \varphi\, (u'_i, z', \sigma')\,.$$

Proof. By Axiom 1 (Ordering), there is a real-valued continuous function φ that represents R. By Axiom 8 (Monotonicity), it is increasing in each component of u_i. By Axiom 6 (State Equivalence), it must be the case that $\varphi(u_i, z, \sigma) = \varphi(u_i, z, \sigma')$ for all $\sigma, \sigma' \in \mathscr{S}$, $u \in \mathscr{U}^c$ and $z \in \mathscr{Z}^c$. □

Claim 2. There exist n continuous increasing functions $\varphi_i : \mathbb{R} \to \mathbb{R}$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u, z, \sigma) R^p (u', z', \sigma') \Leftrightarrow \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \geq \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u'_i, z', \sigma').$$

Proof. By Axiom 1 (Ordering) and 7 (Ex Post Pareto), there is a continuous and increasing function $\Gamma : (\operatorname{rge} \varphi)^n \to \mathbb{R}$[10] such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u, z, \sigma) R^p (u', z', \sigma') \Leftrightarrow \Gamma((\varphi(u_i, z, \sigma))_{i \in \mathscr{N}}) \geq \Gamma((\varphi(u'_i, z', \sigma'))_{i \in \mathscr{N}}).$$

By Lemma 1, Axiom 12 (Ex Post Separability) holds, so that the ordering over $(\operatorname{rge} \varphi)^n$ represented by Γ is separable. Therefore, there exist n continuous functions $\varphi_i : \mathbb{R} \to \mathbb{R}$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u, z, \sigma) R^p (u', z', \sigma') \Leftrightarrow \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \geq \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u'_i, z', \sigma').$$

By Axiom 7 (Ex Post Pareto), each φ_i is increasing. □

Claim 3. There exists a continuous and increasing function $\Psi : \mathbb{R} \to \mathbb{R}$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$(u, z) R^a (u', z') \Leftrightarrow \sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \right) \geq \sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u'_i, z', \sigma') \right),$$

Proof. Define $\psi(u, z, \sigma) = \sum_{i \in \mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma)$. The function ψ is continuous and increasing in each component of u.

By Axiom 4 (Solvability),

$$\{(\psi(u, z, \sigma))_{\sigma \in \mathscr{S}} | (u, z) \in \mathscr{U} \times \mathscr{Z}\} = \prod_{\sigma \in \mathscr{S}} \operatorname{rge} \psi(\cdot, \cdot, \sigma).$$

Let $\mathscr{D} = \prod_{s \in \mathscr{S}} \operatorname{rge} \psi(\cdot, \cdot, \sigma)$. Define $\succeq$ on $\mathscr{D}$ as follows: $a \succeq b$ iff there exist $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$\left. \begin{array}{l} \forall \sigma \in \mathscr{S},\ \psi(u, z, \sigma) = a_\sigma \\ \forall \sigma \in \mathscr{S},\ \psi(u', z', \sigma) = b_\sigma \end{array} \right\} \text{ and } (u, z) R^a (u', z').$$

By Axioms 1 (Ordering), 2 (Dominance) and 4 (Solvability), $\succeq$ is a well-defined, complete and continuous ordering, as we now show.

[10] For any function f, $\operatorname{rge} f$ denotes the range of f.

First, observe that $\succeq$ is well-defined. Indeed, let $u, v, u', v' \in \mathscr{U}$ and $y, z, y', z' \in \mathscr{Z}$ be such that for all $\sigma \in \mathscr{S}$, $\psi(u, z, \sigma) = \psi(v, y, \sigma) = a_\sigma$ and $\psi(u', z', \sigma) = \psi(v', y', \sigma) = b_\sigma$. This implies that for all $\sigma \in \mathscr{S}$, $(u, z, \sigma)I^p(v, y, \sigma)$ and $(u', z', \sigma)I^p(v', y', \sigma)$. By Axiom 2 (Dominance), $(u, z)I^a(v, y)$ and $(u', z')I^a(v', y')$. Therefore,

$$(u, z)R^a(u', z') \Leftrightarrow (v, y)R^a(v', y').$$

Since $\mathscr{D} = \{(\psi(u, z, \sigma))_{\sigma\in\mathscr{S}} | (u, z) \in \mathscr{U} \times \mathscr{Z}\}$ and R^a is complete, $\succeq$ is also complete.

Finally, we show that $\succeq$ is continuous. The function $\bar{\psi} : \mathscr{U} \times \mathscr{Z} \to \mathscr{D}$ defined by $\bar{\psi}(u, z) = (\psi(u, z, \sigma))_{\sigma\in\mathscr{S}}$ is continuous. Consider any $b \in \mathscr{D}$ and the set $\mathscr{A} = \{a \in \mathscr{D} | a \succeq b\}$. Let v, y be such that $\bar{\psi}(v, y) = b$. The set $\mathscr{A}$ is the image by $\bar{\psi}$ of the set

$$\{(u, z) \in \mathscr{U} \times \mathscr{Z} | (u, z)R^a(v, y)\}.$$

As R^a is continuous, this set is closed. Since $\bar{\psi}$ is continuous, $\mathscr{A}$ is also closed. A similar argument shows that the set $\{a \in \mathscr{D} | b \succeq a\}$ is closed as well, and therefore $\succeq$ is continuous.

By Axiom 2 (Dominance), $\succeq$ is strictly monotonic in each component. By Axiom 3 (Independence), it is separable. By Axiom 5 (State Neutrality), it is symmetric, i.e., indifferent to permutations of components.

Therefore there exists a continuous and increasing function $\Psi : \mathbb{R} \to \mathbb{R}$ such that for all $a, b \in \mathscr{D}$,

$$a \succeq b \Leftrightarrow \sum_{\sigma\in\mathscr{S}} \Psi(a_\sigma) \geq \sum_{\sigma\in\mathscr{S}} \Psi(b_\sigma).$$

□

Claim 4. One can restrict attention either to $\Psi(x) = x$ or to $\Psi(x) = \alpha e^{\alpha x}$ for some $\alpha \in \mathbb{R} \setminus \{0\}$.

Proof. Let $\{C, R\}$ be a partition of $\mathscr{N}$, with $|C| \geq 2$, and let $z^* \in \mathscr{Z}^c$. Let $\mathscr{U}_C^c \subset U$ be the subset of matrices u such that u_C is risk-free. Finally, let $r = |R|$.

For $i \in \mathscr{N}$ and $x \in \mathbb{R}$, let $\phi_i(x) = \varphi_i \circ \varphi((x, ..., x), z^*, \sigma)$. Note that by Axiom 6 (State Equivalence) this value does not depend on σ. Each function ϕ_i is continuous and increasing. Without loss of generality, we can impose $\phi_i(0) = 0$.

By Axiom 10 (Independence of the Utility of the Sure), for R^a the subset $R \cup A$ is separable for all $A \subsetneq C$ (including $A = \emptyset$). Therefore, by Theorem 1 in Gorman (1968), every subset of C, including C itself, is also separable. By Corollary of Theorem 1 in Gorman (1968), there exist continuous functions $h : \mathbb{R}^{rs} \to \mathbb{R}$ and

$\hat{\phi}_i : \mathbb{R} \to \mathbb{R}$, $i \in C$, such that for all $u, v \in \mathscr{U}_C^c$,

$$(u, z^*) R^a (v, z^*) \Leftrightarrow h(u_R) + \sum_{i \in C} \hat{\phi}_i(u_{i1}) \geq h(v_R) + \sum_{i \in C} \hat{\phi}_i(v_{i1}).$$

Therefore, there exists an increasing function $\bar{f}$ such that for all $u \in \mathscr{U}_C^c$,

$$\sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in R} \varphi_i \circ \varphi(u_i, z^*, \sigma) + \sum_{i \in C} \phi_i(u_{i1}) \right) = \bar{f} \left(h(u_R) + \sum_{i \in C} \hat{\phi}_i(u_{i1}) \right). \quad (1)$$

Fixing u_R, one sees that this implies that there is an increasing function $\bar{g}$ such that for all u_C such that $(u_R, u_C) \in \mathscr{U}_C^c$,

$$\sum_{i \in C} \hat{\phi}_i(u_{i1}) = \bar{g} \left(\sum_{i \in C} \phi_i(u_{i1}) \right).$$

Moreover, letting $\phi_i^* = \hat{\phi}_i \circ \phi_i^{-1}$, this reads

$$\sum_{i \in C} \phi_i^*(\phi_i(u_{i1})) = \bar{g} \left(\sum_{i \in C} \phi_i(u_{i1}) \right).$$

This is a variant of the Pexider equation, implying that $\bar{g}$ and ϕ_i^* must be affine (Rado and Baker (1987)), so that there exist γ, δ such that $\phi_i(u_{i1}) = \gamma \hat{\phi}_i(u_{i1}) + \delta_i$.

As a result, one can simplify equation (1) and write that there exists an increasing function f such that for all $u \in \mathscr{U}_C^c$,

$$\sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in R} \varphi_i \circ \varphi(u_i, z^*, \sigma) + \sum_{i \in C} \phi_i(u_{i1}) \right) = f \left(h(u_R) + \sum_{i \in C} \phi_i(u_{i1}) \right). \quad (2)$$

For $u_C = 0$, this implies

$$h(u_R) = f^{-1} \left(\sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in R} \varphi_i \circ \varphi(u_i, z^*, \sigma) \right) \right).$$

Substituting in equation (2), we obtain

$$\sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in R} \varphi_i \circ \varphi(u_i, z^*, \sigma) + \sum_{i \in C} \phi_i(u_{i1}) \right) \\ = f \left(f^{-1} \left(\sum_{\sigma \in \mathscr{S}} \Psi \left(\sum_{i \in R} \varphi_i \circ \varphi(u_i, z^*, \sigma) \right) \right) + \sum_{i \in C} \phi_i(u_{i1}) \right).$$

Defining $x_\sigma = \Psi\left(\sum_{i\in R}\varphi_i\circ\varphi(u_i, z^*, \sigma)\right)$, this reads

$$\sum_{\sigma\in\mathscr{S}}\Psi\left(\Psi^{-1}(x_\sigma)+\sum_{i\in C}\phi_i(u_{i1})\right)=f\left(f^{-1}\left(\sum_{\sigma\in\mathscr{S}}x_\sigma\right)+\sum_{i\in C}\phi_i\left(u_{i1}\right)\right),$$

which, for a fixed u_C, is a Pexider equation defined on the range of $\left(\Psi\left(\sum_{i\in R}\varphi_i\circ\varphi(u_i, z^*, \sigma)\right)\right)_{\sigma\in\mathscr{S}}$ for $u_R\in\mathbb{R}^{rs}$. As the functions Ψ and $\varphi_i\circ\varphi$ are continuous in the relevant arguments, this set has a connected non-empty interior. Therefore the equation implies that both sides are affine in x. That is, letting $t=\sum_{i\in C}\phi_i(u_{i1})$, there exist $\gamma(t)>0$, $\delta(t)$ such that

$$\Psi\left(\Psi^{-1}\left(x_\sigma\right)+t\right)=\gamma(t)x_\sigma+\delta(t),$$

or equivalently, letting $y_\sigma=\Psi^{-1}(x_\sigma)=\sum_{i\in R}\varphi_i\circ\varphi(u_i, z^*, \sigma)$,

$$\Psi\left(y_\sigma+t\right)=\gamma(t)\Psi\left(y_\sigma\right)+\delta(t).$$

By Corollary 1 (p. 150-151) in Aczél (1966) this equation implies that $\Psi(x)$ is affine in x or affine in $e^{\alpha x}$ for some $\alpha\neq 0$. □

Claim 5. For all $u\in\mathscr{U}^c$ and $i\in\mathscr{N}$ it must be the case that

1. when $\Psi(x)=x$, $\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(u,z,\sigma)=\psi_i(u_{i1})+\xi_i(z)$ where ψ_i is continuous and increasing and ξ_i is continuous
2. when $\Psi(x)=\alpha e^{\alpha x}$, $\varphi_i\circ\varphi(u,z,\sigma)=\vartheta_i(u_{i1})+\eta_i(z,\sigma)$ where ϑ_i is continuous and increasing and η_i is continuous.

Proof. Case 1: $\Psi(x)=x$. Let $u,u'\in\mathscr{U}$, $v,v'\in\mathscr{U}^c$, $z\in\mathscr{Z}$ and $z_0\in\mathscr{Z}$ a reference informational content (for instance, equiprobable states of the world). By Axiom 10 (Independence of Utility of the Sure), it must be the case that:

$$((u_{-i},v_i),z)R^a((u'_{-i},v_i),z_0)\Leftrightarrow((u_{-i},v'_i),z)R^a((u'_{-i},v'_i),z_0)$$

Using the representation in Case 1, this means that:

$$\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z,\sigma)+\sum_{j\neq i}\sum_{\sigma\in\mathscr{S}}\varphi_j\circ\varphi(u_j,z,\sigma)\geq\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z_0,\sigma)+\sum_{j\neq i}\sum_{\sigma\in\mathscr{S}}\varphi_j\circ\varphi(u'_j,z_0,\sigma)$$
$$\Leftrightarrow\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v'_i,z,\sigma)+\sum_{j\neq i}\sum_{\sigma\in\mathscr{S}}\varphi_j\circ\varphi(u_j,z,\sigma)\geq\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v'_i,z_0,\sigma)+\sum_{j\neq i}\sum_{\sigma\in\mathscr{S}}\varphi_j\circ\varphi(u'_j,z_0,\sigma)$$

Hence the difference $\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z,\sigma)-\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z_0,\sigma)$ is independent of v_i: there exists a function ξ_i such that $\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z,\sigma)-\sum_{\sigma\in\mathscr{S}}\varphi_i\circ\varphi(v_i,z_0,\sigma)=$

$\xi_i(z)$. Denoting $\psi_i(v_{i1}) = \sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(v_i, z_0, \sigma)$ yields the result. Axioms 1 (Ordering) and 8 (Monotonicity) imply the properties of ψ_i and ξ_i.

Case 2: $\Psi(x) = \alpha e^{\alpha x}$. Let $u, u' \in \mathscr{U}$, $v \in \mathscr{U}^c$, $z \in \mathscr{Z}$. Let $\mathbf{0}$ denotes the sure prospect in $\mathbb{R}^s$ with all its components equal to 0. By Axiom 10 (Independence of Utility of the Sure), it must be the case that:

$$((u_{-i}, v_i), z)R^a((u'_{-i}, v_i), z) \Leftrightarrow ((u_{-i}, \mathbf{0}), z)R^a((u'_{-i}, \mathbf{0}), z)$$

Using the representation in Case 2, and assuming without loss of generality that $\alpha > 0$. this means that:

$$\begin{aligned}
&\sum_{\sigma\in\mathscr{S}} \exp\Big(\alpha\varphi_i \circ \varphi(v_i, z, \sigma)\Big) \exp\Big(\alpha \sum_{j\neq i} \varphi_j \circ \varphi(u_j, z, \sigma)\Big) \geq \\
&\quad \sum_{\sigma\in\mathscr{S}} \exp\Big(\alpha\varphi_i \circ \varphi(v_i, z, \sigma)\Big) \exp\Big(\alpha \sum_{j\neq i} \varphi_j \circ \varphi(u'_j, z, \sigma))\Big) \\
&\qquad \Leftrightarrow \sum_{\sigma\in\mathscr{S}} \exp\Big(\alpha\varphi_i \circ \varphi(\mathbf{0}, z, \sigma))\Big) \exp\Big(\alpha \sum_{j\neq i} \varphi_j \circ \varphi(u_j, z, \sigma))\Big) \geq \\
&\qquad\quad \sum_{\sigma\in\mathscr{S}} \exp\Big(\alpha\varphi_i \circ \varphi(\mathbf{0}, z, s))\Big) \exp\Big(\alpha \sum_{j\neq i} \varphi_j \circ \varphi(u'_j, z, \sigma))\Big)
\end{aligned}$$

Let $D_{iz} = \{(d_1, \cdots, d_s) \in \mathbb{R}^s : \exists u \in \mathscr{U}, \forall \sigma \in \mathscr{S}, d_\sigma = \sum_{j\neq i} \varphi_j \circ \varphi(u_j, z, \sigma)\}$. D_{iz} is connected and has a non empty interior. Let $f_\sigma(d_\sigma) = \exp\Big(\alpha\varphi_i \circ \varphi(v_i, z, \sigma)\Big)e^{\alpha d_\sigma}$ and $g_\sigma(d_\sigma) = \exp\Big(\alpha\varphi_i \circ \varphi(\mathbf{0}, z, \sigma)\Big)e^{\alpha d_\sigma}$. By the above result, the following two representations are ordinally equivalent: $\sum_{\sigma\in\mathscr{S}} f_\sigma(d_\sigma)$ and $\sum_{\sigma\in\mathscr{S}} g_\sigma(d_\sigma)$. By the unicity of additive representations up to an increasing affine transformation, there must exists $a > 0$ and scalars b_σ such that $f_\sigma = a g_\sigma + b_\sigma$. In view of the forms of functions f_σ and g_σ, we need $b_\sigma = 0$ for all $\sigma \in \mathscr{S}$.

To sum up, for every sure v_i and all $\sigma \in \mathscr{S}$, there exists $a(v_i)$ such that $\exp(\alpha\varphi_i \circ \varphi(v_i, z, \sigma)) = a(v_i)\exp(\alpha\varphi_i \circ \varphi(\mathbf{0}, z, \sigma))$. Denoting $\vartheta_i(v_{i1}) = \frac{\ln \circ a(v_i)}{\alpha}$ and $\eta_i(z, \sigma) = \varphi_i \circ \varphi(\mathbf{0}, z, \sigma)$, we obtain that $\varphi_i \circ \varphi(u, z, \sigma) = \vartheta_i(u_{i1}) + \eta_i(z, \sigma)$. The properties of ϑ_i and η_i follow from Axioms 1 (Ordering) and 8 (Monotonicity) . □

7.3 Proof of Proposition 2

Proof. It is immediate to see that the defined orderings satisfy Axioms 1 (Ordering), 2 (Dominance), 3 (Independence), 5 (State Neutrality), 7 (Ex Post Pareto), and the first part of 8 (Monotonicity).

The condition $\varphi(u_i, z, \sigma) = \varphi(u_i, z, \sigma')$ for all $u \in \mathscr{U}^c$ and $z \in \mathscr{Z}^c$ guarantees that they satisfy Axiom 6 (State Equivalence).

For Axiom 10 (Independence of Utility of the Sure), consider $M \subset \mathscr{N}$ and $u, u' \in \mathscr{U}$, $v \in \mathscr{U}^c$, $z, z' \in \mathscr{Z}$. In the case, $\Psi(x) = x$, we obtain

$$\begin{aligned}((u_M, v_{\mathscr{N}\setminus M}), z) R^a ((u'_M, v_{\mathscr{N}\setminus M}), z') \;\Leftrightarrow\; & \sum_{i\in M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(u_i, z, \sigma) + \sum_{i\in\mathscr{N}\setminus M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(v_i, z, \sigma) \geq \\ & \quad \sum_{i\in M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(u'_i, z', \sigma) + \sum_{i\in\mathscr{N}\setminus M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(v_i, z', \sigma) \\ \Leftrightarrow\; & \sum_{i\in M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(u_i, z, \sigma) + \sum_{i\in\mathscr{N}\setminus M} \xi_i(z) \geq \\ & \quad \sum_{i\in M}\sum_{\sigma\in\mathscr{S}} \varphi_i \circ \varphi(u'_i, z', \sigma) + \sum_{i\in\mathscr{N}\setminus M} \xi_i(z')\end{aligned}$$

This is clearly independent of v, in accordance with Axiom 10 (Independence of Utility of the Sure).

In the case $\Psi(x) = \alpha \exp(\alpha x)$, and assuming without loss of generality that $\alpha > 0$, we obtain

$$\begin{aligned}&((u_M, v_{\mathscr{N}\setminus M}), z) R^a ((u'_M, v_{\mathscr{N}\setminus M}), z') \\ &\quad\Leftrightarrow \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi_i \circ \varphi(u_i, z, \sigma) + \sum_{i\in\mathscr{N}\setminus M} \varphi_i \circ \varphi(v_i, z, \sigma)\right) \geq \\ &\qquad \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi_i \circ \varphi(u'_i, z', \sigma) + \sum_{i\in\mathscr{N}\setminus M} \varphi_i \circ \varphi(v_i, z', \sigma)\right) \\ &\quad\Leftrightarrow \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi_i \circ \varphi(u_i, z, \sigma)\right) \exp\left(\alpha \sum_{i\in\mathscr{N}\setminus M} \eta_i(z, \sigma)\right) \geq \\ &\qquad \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi_i \circ \varphi(u'_i, z', \sigma)\right) \exp\left(\alpha \sum_{i\in\mathscr{N}\setminus M} \eta_i(z', \sigma)\right)\end{aligned}$$

This is also independent of v.

□

7.4 Proof of Proposition 3

By Proposition 1, we know that there exist functions φ, $(\varphi_i)_{i\in\mathscr{N}}$ such that for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$, $\sigma, \sigma' \in \mathscr{S}$,

$$(u, z, \sigma) R^p (u', z', \sigma') \Leftrightarrow \sum_{i\in\mathscr{N}} \varphi_i \circ \varphi(u_i, z, \sigma) \geq \sum_{i\in\mathscr{N}} \varphi_i \circ \varphi(u'_i, z', \sigma').$$

By Axiom 9 (Anonymity), one can take the $(\varphi_i)_{i\in\mathscr{N}}$ to be identical. Letting φ denote $\varphi_i \circ \varphi$, we then obtain the first two equivalences of this proposition.

By Proposition 1, we know that either for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$(u, z)R^a(u', z') \Leftrightarrow \sum_{\sigma\in\mathscr{S}}\sum_{i\in\mathscr{N}} \varphi(u_i, z, \sigma) \geq \sum_{\sigma\in\mathscr{S}}\sum_{i\in\mathscr{N}} \varphi(u'_i, z', \sigma),$$

or for some $\alpha \neq 0$, for all $u, u' \in \mathscr{U}$, $z, z' \in \mathscr{Z}$,

$$(u, z)R^a(u', z') \Leftrightarrow \alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in\mathscr{N}} \varphi(u_i, z, \sigma)\right) \geq \alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in\mathscr{N}} \varphi(u'_i, z', \sigma)\right). \tag{3}$$

Suppose the latter is true, and assume without loss of generality that $\alpha > 0$. Consider $u, u' \in \mathscr{U}^e$, $v \in \mathscr{U}^c$, $M \subseteq \mathscr{N}$ with $m = |M|$ and $z \in \mathscr{Z}$. By Axiom 11 (Restricted Independence of the Sure), for all $i \in M$,

$$\begin{aligned}((u_M, v_{\mathscr{N}\setminus M}), z)R^a((u'_M, v_{\mathscr{N}\setminus M}), z)& \\ \Leftrightarrow \alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha n\varphi(u_i, z, \sigma)\right) &\geq \alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha n\varphi(u'_i, z, \sigma)\right).\end{aligned}$$

But the left-hand side also reads, by Equation (3),

$$\begin{aligned}\alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi(u_i, z, \sigma) + \alpha \sum_{i\in\mathscr{N}\setminus M} \varphi(u_i, z, \sigma)\right)& \\ \geq \alpha \sum_{\sigma\in\mathscr{S}} \exp\left(\alpha \sum_{i\in M} \varphi(u'_i, z, \sigma) + \alpha \sum_{i\in\mathscr{N}\setminus M} \varphi(u'_i, z, \sigma)\right)&.\end{aligned}$$

By the condition on sure prospects for the exponential case in Proposition 1 , the inequality simplifies into

$$\sum_{\sigma\in\mathscr{S}} \exp\Big(\alpha(n-m)\eta(z, s)\Big) \exp(\alpha m\varphi(u_i, z, \sigma)) \geq \sum_{\sigma\in\mathscr{S}} \Big(\alpha(n-m)\eta(z, s)\Big) \exp(\alpha m\varphi(u'_i, z, \sigma)).$$

Let $X = \{(\exp(\alpha\varphi(u_i, z, \sigma)))_{\sigma\in\mathscr{S}} | u_i \in \mathbb{R}^s\}$, and $a_\sigma = \exp(\alpha\eta(z, s))$. We have obtained that for all $x, y \in X$, all $k \in \{1, \cdots, n\}$,

$$\sum_{\sigma\in\mathscr{S}} (a_\sigma)^{n-k}(x_\sigma)^k \geq \sum_{\sigma\in\mathscr{S}} (a_\sigma)^{n-k}(y_\sigma)^k \Leftrightarrow \sum_{\sigma\in\mathscr{S}} (x_\sigma)^n \geq \sum_{\sigma\in\mathscr{S}} (y_\sigma)^n.$$

This is possible only if there is $a \in \mathbb{R}^s_{++}$ such that for all $x \in X$, there is $\lambda \in \mathbb{R}_{++}$,

$x = \lambda a$. This argument applies to all $z \in \mathscr{Z}$. This implies that for all $(u_i, z) \in \mathbb{R}^s \times \mathscr{Z}$, there is $\gamma(u_i, z) \in \mathbb{R}$, $\varphi(u_i, z, \sigma) = \gamma(u_i, z) + \frac{\ln a_\sigma}{\alpha}$. The function γ must be increasing in u_i. Let $\beta_\sigma = \frac{\ln a_\sigma}{\alpha}$.

Define $\psi(u, z, \sigma) = \sum_{i \in \mathscr{N}} \varphi(u_i, z, \sigma)$. The above reasoning implies that $\psi(u, z, \sigma) = n\beta_\sigma + \sum_{i \in \mathscr{N}} \gamma(u_i, z)$. Let $\big((u^1, z^1), \cdots, (u^s, z^s)\big) \in (\mathscr{U} \times \mathscr{Z})^s$. One has, for all $\sigma \in \mathscr{S}$, $\psi(u^\sigma, z^\sigma, \sigma) = n\beta_\sigma + \sum_{i \in \mathscr{N}} \gamma(u_i^\sigma, z^\sigma)$. Axiom 4 (Solvability) requires that there is $(u, z) \in \mathscr{U} \times \mathscr{Z}$ such that for all $\sigma \in \mathscr{S}$,

$$\psi(u, z, \sigma) = n\beta_\sigma + \sum_{i \in \mathscr{N}} \gamma(u_i, z) = n\beta_\sigma + \sum_{i \in \mathscr{N}} \gamma(u_i^\sigma, z^\sigma).$$

This is possible in general only if γ is constant. This yields a contradiction.

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