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Aggregation of exponential smoothing processes with an application to portfolio risk evaluation

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Abstract

In this paper we propose a unified framework to analyse contemporaneous and temporal aggregation of exponential smoothing (EWMA) models. Focusing on a vector IMA(1,1) model, we obtain a closed form representation for the parameters of the contemporaneously and temporally aggregated process as a function of the parameters of the original one. In the framework of EWMA estimates of volatility, we present an application dealing with Value-at-Risk (VaR) prediction at different sampling frequencies for an equally weighted portfolio composed of multiple indices. We apply the aggregation results by inferring the decay factor in the portfolio volatility equation from the estimated vector IMA(1,1) model of squared returns. Empirical results show that VaR predictions delivered using this suggested approach are at least as accurate as those obtained by applying the standard univariate RiskMetrics™ methodology.

Keywords: contemporaneous and temporal aggregation, EWMA, volatility, Value-at-Risk.
JEL Classification: C10, C32, C43

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1 Introduction

The aim of this work is to focus on the effect of temporal and contemporaneous aggregation in the framework of exponential smoothing methods.[1] As well known, linear Exponentially Weighted Moving Average (EWMA) methods have equivalent representations based on ARIMA models. By "equivalent", as pointed out by Gardner and McKenzie (1985), we mean that the forecasts produced by the smoothing techniques are minimum mean squared error (MSE) forecasts for the corresponding ARIMA models.

According to a number of existing results in the econometric literature, the parameters and the orders of the aggregated models can be derived by establishing the relationship between the autocovariance structures of the disaggregated and aggregated models. In particular, to recover the aggregated parameters, in most of the cases it is necessary to solve nonlinear systems of equations, as detailed by Granger and Morris (1976), Stram and Wei (1986), Wei (1990), Marcellino (1999), Hafner (2008).[2]

Our main contribution is to derive algebraic solutions for the parameters of the contemporaneously and temporally aggregated model. Assuming that the data generating process is a vector integrated moving average of order one (IMA(1,1)), we provide closed form expressions for the autocovariances of the aggregated model. Those allow to derive exact functions linking the unknown aggregate parameters with the data generating process. Although the focus is on a specific class of moving average processes, results are valid for any aggregation frequency and for every level of contemporaneous aggregation.

The IMA(1,1) model is important in time series analysis since its predictions take the form of exponential smoothing. Its simplicity is also responsible for its popularity. In fact, the forecast function depends on a single smoothing parameter which expresses the weight by which past observations are discounted.

The importance of the EWMA process has been widely recognized in economics. The optimal properties of the simple exponential smoothing were first discussed in Muth (1960). In particular, the EWMA became the cornerstone process at the origin of the rational expectations theory as in Muth (1961). Since then, a large number of authors decided to employ this model, which is nowadays one of the most recommended method in forecasting macro time series.[3]

The EWMA is not only employed in economics, but also in finance. In particular, this model is widely used by practitioners to produce forecasts of volatilities of financial data. It has been popularized by RiskMetrics^{TM}, a risk management methodology for measuring market risk developed by J.P. Morgan based on the Value-at-Risk (VaR) concept. See Zaffaroni (2008) for a recent contribution on the asymptotic properties of the estimation approach suggested by RiskMetrics^{TM}.

[1] We refer to Gardner (1985, 2006) for a comprehensive review and thorough discussion of these models.

[2] Recently, Petkovic and Veredas (2009) study the impact of individual and temporal aggregation in linear models for panel data in terms of specification and efficiency of the estimated parameters.

[3] For example, Nelson and Schwert (1977) and subsequently Barsky (1987) claim that the EWMA is the best forecasting process, outperforming other univariate processes, in modeling inflation rate. Also Pearce (1979) supports the use of EWMA in forecasting price levels. Similar results have been achieved more recently by Rossana and Seater (1995), who argue that many economic time series sampled at relatively low frequencies can often be approximated by an IMA (1,1) model, and by Stock and Watson (2007).

VaR is one of the standard measures used to quantify market risk in the financial industry. Broadly speaking, it represents the tail of the distribution of a portfolio's loss over a certain time horizon. Conceptually, it is easy to interpret since it reduces the market risk associated with any portfolio to just one number (Manganelli and Engle, 2001). VaR is the basis of risk measurement and has a variety of applications, essentially in risk management and for regulatory requirements. For instance, Basel II Capital Accord imposes to financial institutions to meet capital requirements based on VaR estimates at a confidence level of 5 or 1 percent.

VaR evaluation (and prediction) is also the focus of our empirical application, which is an additional contribution of the paper. In the application, we illustrate how the results on temporal and contemporaneous aggregation can be applied in practice analysing the problem of volatility prediction and VaR calculation for a portfolio of multiple indices (contemporaneous aggregation) at several sampling frequencies (temporal aggregation). A method is proposed for deriving the MA coefficient and thus the decay factor in the EWMA volatility equation of the portfolio, for different levels of aggregation frequency of the returns. This requires estimating a multivariate IMA(1,1) model by approximate methods, and inferring the smoothing parameter as a function of the MA matrix coefficients. As far as estimation is concerned, we employ several benchmarks, such as: (a) the RiskMetrics^{TM} procedure (in its two versions, namely, when the decay factor is set to 0.94 on the basis of prior calibration and when it is estimated minimizing the mean squared forecast error); (b) the Student-t EWMA maximum likelihood estimator (MLE), which is obtained by maximizing a Student-t log-likelihood function in which the past squared returns are weighted exponentially; (c) a Student-t GARCH(1,1) MLE.

Our results show that VaR predictions based on the aggregation of the vector IMA(1,1) model are at least as accurate as those delivered by the Student-t EWMA MLE, the Student-t GARCH(1,1) MLE and the RiskMetrics^{TM} approach.

The rest of this paper is structured as follows. In Section 2 we set up the econometric framework, which is based on Lütkepohl (1984a, 1987). In Section 3 we derive algebraic solutions for the parameters of the contemporaneously aggregated process. In Section 4 we derive algebraic solutions for the parameters of the temporally aggregated process. In Section 5 we present the empirical application and discuss the results. Section 6 concludes. The proofs are relegated to an appendix.

2 The econometric framework: the vector IMA(1,1) model for the original data

A framework to deal with contemporaneous and temporal aggregation of vector ARMA (VARMA) type of models has been suggested and formalized by Lütkepohl (1984a, 1984b, 1987). In particular, this author shows that contemporaneous and temporal aggregation of a VARMA model corresponds to a linear transformation of a "macro process", which constitutes a different representation of the original VARMA. Therefore, the theory of linear transformations of VARMA processes can be applied to derive results on simultaneous temporal and contemporaneous aggregation.[4]

[4]Furthermore, it is possible to prove the closeness of VARMA models after aggregation, namely, the fact that the aggregated model is still in the VARMA class and keeps the same structure.

To cover the aggregation issue in full generality, the econometric framework is based on the commonly named "macro processes" above mentioned, introduced by Lütkepohl (1987). As it will become clear in the sequel, this formulation allows to consider contemporaneous and temporal aggregation in a *joint framework*.

We assume that the data generating process is a vector integrated moving average process of order one, that is, a vector IMA(1,1). In particular, we use a representation similar to the one proposed by Lütkepohl (2007, p. 441):

$$\underset{(Nk\times Nk)}{\begin{pmatrix} \boldsymbol{I}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ -\boldsymbol{I}_N & \boldsymbol{I}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \boldsymbol{I}_N \end{pmatrix}} \underset{(Nk\times 1)}{\begin{pmatrix} \boldsymbol{y}_{k(t-1)+1} \\ \boldsymbol{y}_{k(t-1)+2} \\ \vdots \\ \boldsymbol{y}_{k(t-1)+k} \end{pmatrix}} = \begin{pmatrix} \mathbf{O}_N & \mathbf{O}_N & \dots & \boldsymbol{I}_N \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{k(t-2)+1} \\ \boldsymbol{y}_{k(t-2)+2} \\ \vdots \\ \boldsymbol{y}_{k(t-2)+k} \end{pmatrix}$$
$$+ \begin{pmatrix} \boldsymbol{I}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ \boldsymbol{\theta} & \boldsymbol{I}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \boldsymbol{I}_N \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_{k(t-1)+1} \\ \boldsymbol{\varepsilon}_{k(t-1)+2} \\ \vdots \\ \boldsymbol{\varepsilon}_{k(t-1)+k} \end{pmatrix} + \begin{pmatrix} \mathbf{O}_N & \mathbf{O}_N & \dots & \boldsymbol{\theta} \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_{k(t-2)+1} \\ \boldsymbol{\varepsilon}_{k(t-2)+2} \\ \vdots \\ \boldsymbol{\varepsilon}_{k(t-2)+k} \end{pmatrix} \quad t \in \mathbb{N}. \tag{1}$$

A few words of clarification are at order. The model in (1) is a vector IMA(1,1) for the disaggregate data. In (1), each $\boldsymbol{y}_{k(t-i)+j}$, $i = 1, 2$, $j = 1, 2, \dots, k$, is an $(N \times 1)$ vector and each $\boldsymbol{\varepsilon}_{k(t-i)+j}$, $i = 1, 2$, $j = 1, 2, \dots, k$, is an $(N \times 1)$ vector. Furthermore, $\boldsymbol{\theta}$ is an $(N \times N)$ matrix. Note that k represents the temporal aggregation frequency. Similarly, N is the order of contemporaneous aggregation.[5]

In what follows we explain how to derive algebraic solutions to recover the parameters of the model for the contemporaneously and temporally aggregated series. For the sake of clarity, we discuss contemporaneous aggregation in Section 3 and temporal aggregation in Section 4, separately. However, we remark that our focus is on *simultaneous* temporal and contemporaneous aggregation: in other words, these two forms of aggregation should be thought as being acting at the same time on the original data.

[5]For instance, when $k = 3$, the original vector process is temporally aggregated over three subsequents periods. If $N = 4$, we are summing four processes along the cross-section dimension (think of gross domestic product, i.e. GDP, in one period, which is the sum of consumption, gross investment, government spending and net exports in that period). If $N = k = 4$, we are summing four processes across the cross-section and the data are also summed over four subsequent periods (e.g., sum of quarterly consumption, gross investment, government spending and net exports, and sum of the result over four periods. What we get is annual GDP).

3 Parameters of the contemporaneously aggregated model

Without loss of generality, let us focus on the first row of (1):[6]

$$\underset{(N\times 1)}{\boldsymbol{y}_{k(t-1)+1}} = \boldsymbol{y}_{k(t-1)} + \underset{(N\times 1)}{\boldsymbol{\varepsilon}_{k(t-1)+1}} + \underset{(N\times N)}{\boldsymbol{\theta}} \boldsymbol{\varepsilon}_{k(t-1)} \tag{2}$$

In (2), the size of the $\boldsymbol{y}$ and $\boldsymbol{\varepsilon}$ vectors and of the $\boldsymbol{\theta}$ matrix has been explicitly written out.

To deal with contemporaneous aggregation of the process in (2), it is useful to consider the following N-variate system representation:

$$\underset{(N\times 1)}{\begin{pmatrix} (1-L)y_{1,k(t-1)+1} \\ (1-L)y_{2,k(t-1)+1} \\ \vdots \\ (1-L)y_{N,k(t-1)+1} \end{pmatrix}} = \begin{pmatrix} (1+\theta_{11}L) & \theta_{12}L & \dots & \theta_{1N}L \\ \theta_{21}L & (1+\theta_{22}L) & \dots & \theta_{2N}L \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{N1}L & \theta_{N2}L & \dots & (1+\theta_{NN}L) \end{pmatrix} \begin{pmatrix} \varepsilon_{1,k(t-1)+1} \\ \varepsilon_{2,k(t-1)+1} \\ \vdots \\ \varepsilon_{N,k(t-1)+1} \end{pmatrix} \qquad t \in \mathbb{N}, \tag{3}$$

where L is the usual lag operator, such that $Lx_t = x_{t-1}$. In addition, by assumption, $\boldsymbol{\varepsilon}'_{k(t-1)+1} = (\varepsilon_{1,k(t-1)+1}, \varepsilon_{2,k(t-1)+1}, \dots, \varepsilon_{N,k(t-1)+1})$ is a vector of white noise innovations such that $\mathrm{E}(\boldsymbol{\varepsilon}_s) = \mathbf{0}$ and $\mathrm{E}(\boldsymbol{\varepsilon}_s\boldsymbol{\varepsilon}'_s) = \underset{(N\times N)}{\boldsymbol{\Sigma}}$. More specifically:

$$\begin{aligned} \mathrm{E}(\boldsymbol{\varepsilon}_{k(t-1)+1}\boldsymbol{\varepsilon}'_s) &= \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_N^2 \end{pmatrix} \qquad && if \quad s = k(t-1)+1 \\ \mathrm{E}(\boldsymbol{\varepsilon}_{k(t-1)+1}\boldsymbol{\varepsilon}'_s) &= \mathbf{O}_N && \text{otherwise.} \end{aligned} \tag{4}$$

We present the following preparatory lemma which establishes the order of the contemporaneously aggregated process, which is an IMA(1,1) for any level of aggregation.

Lemma 1 *Focusing on system (3), it is possible to show that the process generated after contemporaneous aggregation is an MA(1), that is:*

$$y_{k(t-1)+1} = \boldsymbol{F}((1-L)\boldsymbol{y}_{k(t-1)+1}) = (1+\theta L)a_{k(t-1)+1},$$

where $\boldsymbol{F}$ is an $(1 \times N)$ generic aggregation vector.[7]

In what follows we assume that $\boldsymbol{F} = (\omega_1, \omega_2, ..., \omega_N)$, with $\Sigma_{i=1}^N \omega_i = 1$.

[6] We could focus on any row of (1). Results in Section 3 would not be affected.

[7] The result stems from the fact that, in general, summing up across i moving average processes of order q_i leads to an MA(q*) where $q^* \leq max(q_i)$. A detailed proof can be found in Lütkepohl (2007, page 436).

Proposition 1 *Assuming a weighted aggregation scheme, the innovation variance of the contemporaneously aggregated process σ_a^2 and the MA parameter θ can be recovered as:*

$$\sigma_a^2 = 2^{-1}\left(\gamma(0) + \sqrt{\gamma(0)^2 - 4\gamma(1)^2}\right)$$

$$\theta = \frac{2\gamma(1)}{\gamma(0) + \sqrt{\gamma(0)^2 - 4\gamma(1)^2}}. \tag{5}$$

with

$$\gamma(0) = \mathrm{E}(y_{k(t-1)+1}^2) = \sum_{i=1}^{N}\sigma_i^2\omega_i^2 + \sum_{i=1}^{N}\alpha_i^2\sigma_i^2\omega_i^2 + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\sigma_{ij}\omega_i\omega_j + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\alpha_i\alpha_j\sigma_{ij}\omega_i\omega_j$$

and

$$\gamma(1) = \mathrm{E}(y_{k(t-1)+1}\ y_{k(t-1)}) = \sum_{i=1}^{N}\alpha_i\sigma_i^2\omega_i^2 + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\alpha_j\sigma_{ij}\omega_i\omega_j,$$

where $\alpha_N = \sum_{i=1}^{N}\theta_{iN}$.

In fact, considering that $\theta = \frac{\gamma(1)}{\sigma_a^2}$, it can be easily shown that σ_a^2 can be obtained by solving $\gamma(0) = \sigma_a^2\left(1 + \left(\frac{\gamma(1)}{\sigma_a^2}\right)^2\right)$ with respect to σ_a^2. The equation yields two different solutions, yet, the only solution leading to an invertible MA(1) process (i.e. $|\theta| < 1$) is the one as above.

Finally, it can also be shown that the aggregate error is a white noise process. Without loss of generality, we focus on the simple aggregation scheme (i.e. $\omega_1 = \omega_2 = ... = \omega_N = 1$). Such that the aggregate error can be expressed as: $a_{k(t-1)+1} = \sum_{i=1}^{N}\varepsilon_{i,k(t-1)+1} + \sum_{i=1}^{N}\alpha_i\varepsilon_{i,k(t-1)} - \theta a_{k(t-1)}$.

It can be easily seen that the first-order autocovariance is:

$$\mathrm{E}(a_{k(t-1)+1}a_{k(t-1)}) = \frac{\sum_{i=1}^{N}\alpha_i\sigma_i^2 + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\alpha_j\sigma_{ij} - \theta\,\mathrm{E}(a_{k(t-1)}^2)}{1-\theta^2} = 0.$$

Higher-order autocovariances are equal to zero, as well.

4 Parameters of the temporally aggregated model

In the previous section we have given the parameters of the contemporaneously aggregated process. After contemporaneous aggregation, the macro process in (1) simplifies to:

$$\begin{pmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} y_{k(t-1)+1} \\ y_{k(t-1)+2} \\ \vdots \\ y_{k(t-1)+k} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} y_{k(t-2)+1} \\ y_{k(t-2)+2} \\ \vdots \\ y_{k(t-2)+k} \end{pmatrix} + \begin{pmatrix} 1 & 0 & \dots & 0 \\ \theta & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} a_{k(t-1)+1} \\ a_{k(t-1)+2} \\ \vdots \\ a_{k(t-1)+k} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \dots & \theta \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} a_{k(t-2)+1} \\ a_{k(t-2)+2} \\ \vdots \\ a_{k(t-2)+k} \end{pmatrix} \quad t \in \mathbb{N}. \tag{6}$$

In (6), each row corresponds to an IMA(1,1) process. Without loss of generality, let us focus on the last row:[8]

$$(1 - L)y_{kt} = (1 + \theta L)a_{kt}, \tag{7}$$

where $|\theta| < 1$ is the contemporaneously aggregated MA parameter in (5) and a_{kt} is the white noise innovation term of the contemporaneously aggregated process, such that $a_{kt} \sim (0, \sigma_a^2)$. Note that the random variable y_{kt} is observed at the disaggregate frequency t.

We define the temporally aggregated process as:

$$\underset{(1\times1)}{y_\tau} = \sum_{j=0}^{k-1} y_{kt-j}. \tag{8}$$

Consequently, $\{y_\tau\}_{\tau=0}^{\infty}$ evolves in aggregate time units τ according to the aggregation scheme in (8). After temporal aggregation, sample information for the random variable at the aggregate frequency is available only every k^{th} period $(k, 2k, 3k, \dots)$, where k, an integer value larger that one, is the aggregation frequency or the order of temporal aggregation. It is interesting to note that the aggregation scheme in (8) corresponds to a linear transformation applied to the macro process in (6), that is, multiplying (6) by an aggregation vector of ones of size $(1 \times k)$.[9]

Following Weiss (1984), Stram and Wei (1986), Wei (1990) and Marcellino (1999), the Lemma below can be easily proved.

[8]We could focus on any row of (6). Results in Section 4 would not be affected.

[9]Besides (8), other important aggregation schemes are possible: namely, systematic sampling, weighted averaging and phase averaging sampling. We refer to Silvestrini and Veredas (2008), among others, for a discussion.

Lemma 2 *The temporal aggregation of the IMA(1,1) process in (6) is an IMA(1,1) in aggregate time units, whatever the aggregation frequency k.*

As a consequence, the temporal aggregation in (8) corresponds to an IMA(1,1), which may be represented as:

$$(1 - L^k)y_\tau = (1 + \Theta L^k)a_\tau, \tag{9}$$

where y_τ and a_τ are the temporally aggregated time series and innovation sequence, respectively. Moreover, Θ is the MA parameter and $\sigma^2_{a^*}$ is the innovation variance of the temporally aggregated process in (9).

At this stage, we need the first two autocovariances of the temporally aggregated process in order to derive the parameters of the temporally aggregated model.

Proposition 2 *The parameters of the temporally aggregated model in (9) may be obtained as in (5), considering the following autocovariances:*

$$\begin{cases} \Gamma(0) = (1+\Theta^2)\sigma^2_{a^*} = \left(\sum_{j=0}^{k-1} ((j+1)+j\theta)^2 + \sum_{j=k}^{2k-1} ((2k-j-1)+(2k-j)\theta)^2\right)\sigma^2_a \\ \Gamma(1) = \Theta\sigma^2_{a^*} = \left(\sum_{j=k}^{2k-1}((2k-j-1)+(2k-j)\theta)((j+1-k)+(j-k)\theta)\right)\sigma^2_a \end{cases} \tag{10}$$

The autocovariances of the temporally aggregated model as in (10) are not trivial to derive, and therefore they represent another contribution of this paper.

Proof. The derivation of $\Gamma(0)$ and $\Gamma(1)$ is provided in the Appendix. □

5 An empirical application to risk management

5.1 Data and preliminary analysis

We now present an empirical application dealing with Value-at-Risk (VaR) calculation, prediction and backtesting for a portfolio composed of multiple indices. As well known, in financial risk management, portfolio's VaR represents the α-th quantile of the portfolio's return conditional distribution, namely

$$\Pr[r_t < -VaR_t^{(\alpha)} | \mathcal{F}_{t-1}] = \alpha,$$

where $\mathcal{F}_{t-1}$ is the information set at time t-1.

The exercise is applied to an equally weighted portfolio composed by NASDAQ and Standard & Poor's 500 (S&P500) stock indices. Daily adjusted closing prices data, provided by Datastream, have been employed from July 09, 1999 to February 17, 2010 (2769 observations).[10] In the empirical application, log-returns series of NASDAQ (NAS_t) and S&P500 (SP_t) are considered such that an

[10] A similar data set has been recently analysed by Rombouts and Verbeek (2009).

equally weighted portfolio of both indices is constructed. The log-returns of the equally weighted portfolio (PTF_t) may be approximated as

$$PTF_t \approx 0.5NAS_t + 0.5SP_t. \tag{11}$$

Equal weighting is a common index weighting scheme. It is used in this application to provide an illustration of contemporaneous aggregation.

A plot of the daily continuously compounded returns (i.e., $r_t = \log P_t - \log P_{t-1}$, $t = 2, \ldots, 2769$, where P_t is the original price at time t) of NASDAQ and S&P500 is presented in Figure 1.

[FIGURE 1 ABOUT HERE]

The two time series display a few jumps, which are clearly visible along the whole sample and are particularly large for NASDAQ. Moreover, the NASDAQ and S&P500 financial log-returns exhibit the well known property of volatility clustering.

Descriptive statistics of log-returns are displayed in Table 1.

[TABLE 1 ABOUT HERE]

The two indices share similar features: both average returns are close to zero, although NASDAQ is more dispersed than S&P500. Both unconditional distributions have ticker tails than the Gaussian benchmark, while NASDAQ has lower kurtosis than S&P500. Furthermore, they are not symmetric: for NASDAQ, skewness is positive but close to zero; for S&P500, on the other hand, skewness is negative (i.e., the data are spread out more to the left of the mean) and the asymmetry is more pronounced. Consistently, the Jarque-Bera test, which is a test of distributional Gaussianity, rejects the null of normality for both indices at 5 percent level.

Figure 2 shows the sample autocorrelation functions (ACF) of daily log-returns of S&P500 and NASDAQ (panels a, b) and the ACF of daily squared log-returns (panels c, d). In each plot, the maximum time lag is 50. The starred lines represent the confidence bands of the sample autocorrelation. As expected, the ACF of daily log-returns is similar to that of a white noise: except in a very few cases, it is within the two standard error limits. Conversely, the plot of the ACF of the squared log-returns displays a strong form of serial correlation for both indices. Moreover, it exhibits a very slow decay.

[FIGURE 2 ABOUT HERE]

5.2 Estimation of volatility models

The aim of the exercise is to estimate 1-period horizon VaR for the portfolio in (11), by employing the EWMA model for conditional volatility introduced by RiskMetrics$^{\text{TM}}$ (1996). Of course, other approaches can be followed for volatility forecasting. In this exercise we focus on RiskMetrics$^{\text{TM}}$ because it incorporates the EWMA model for estimating the conditional volatility. Moreover, it is easy to estimate and widely used for risk management purposes.

In the original RiskMetrics^{TM} (1996) methodology, the log prices are assumed to follow the process[11]

$$\log P_t = \log P_{t-1} + \sigma_{t|t-1}\varepsilon_t, \quad \varepsilon_t \sim i.i.d.\ N(0,1), \tag{12}$$

where ε_t is an independent and identically distributed (i.i.d.) Gaussian sequence. As a consequence of (12), the compounded log-returns r_t are conditionally Gaussian and uncorrelated:

$$r_t|\mathcal{F}_{t-1} \sim i.i.d.\ N(0, \sigma^2_{t|t-1}).$$

The conditional volatility $\sigma^2_{t|t-1}$ is assumed to evolve according to the following recursion

$$\sigma^2_{t+1|t} = \lambda\sigma^2_{t|t-1} + (1-\lambda)r_t^2, \tag{13}$$

where λ $(0 < \lambda < 1)$, defined as the "decay factor", is the smoothing parameter of the EWMA scheme. The recursion in (13) is exactly equivalent to an integrated GARCH(1,1) model without constant term for the conditional volatility of the log-returns. Furthermore, it can be easily shown that (13) can be re-parameterized as an IMA(1,1) model for the squared returns,

$$(1-L)r_t^2 = \eta_t - \lambda\eta_{t-1}, \tag{14}$$

where $\eta_t := r_t^2 - \sigma^2_{t|t-1}$ is a martingale difference sequence disturbance term.

Besides volatility, VaR calculation depends on a tail quantile of the portfolio's log-return conditional distribution. Figure 3 shows the left tail of the histogram of the portfolio's log-returns in (11). The daily returns have been normalized by dividing for the square root of the volatility in (13).[12] For illustrative purposes, a Student-t distribution with twelve degrees of freedom (dotted line), rescaled to have a unit variance, and a standard Normal (solid line) are displayed.[13] The Student-t density seems to fit the data reasonably well. It is often well suited to deal with the fat-tailed and leptokurtic features. Indeed, consistently with residual properties, the graph suggests that the Student-t distribution provides a better description of the tails than the Gaussian distribution.

[FIGURE 3 ABOUT HERE]

In the empirical application, we calculate 1-period horizon VaR predictions for the portfolio in (11), working with stock price data sampled at five different time frequencies (1/2/3/4/5 business days). In this way we provide an illustration of how the temporal aggregation scheme works at relevant sampling frequencies. The EWMA recursion in (13) is used to deliver 1-step-ahead volatility forecasts.

Concerning the estimation of the EWMA model, RiskMetrics^{TM} (1996) suggests to select the decay factor in (13) by searching for the smallest root mean squared forecast error (MSFE) over

[11] See RiskMetrics^{TM} (Technical Document, 1996, p. 85).

[12] Note that, to draw the normalized log-returns in Figure 3, the λ parameter has been fixed to 0.94 over the whole sample.

[13] We are aware that different distributional assumptions can be made: yet, for simplicity, we focus only on the Student-t and on the Gaussian.

different values of λ (the resulting λ is termed "optimal decay factor").[14] Formally:

$$\lambda^{opt} = \underset{\lambda \in [\lambda_{min}, \lambda_{max}]}{\operatorname{argmin}} \sqrt{\frac{1}{T} \sum_{t=2}^{T} (r_t^2 - \sigma_{t|t-1}^2(\lambda))^2}, \tag{15}$$

where $[\lambda_{min}, \lambda_{max}]$ is a compact set over which the optimization takes place (i.e., [0,1]). Or simply to fix λ at a certain value (0.94 for daily volatility).

However, Zaffaroni (2008) points out that the estimation method in (15) is non-consistent for λ. In addition, the resulting estimator lacks of the usual asymptotic statistical properties. As an alternative, he suggests the pseudo maximum likelihood estimator (PMLE), which requires to maximize a Gaussian pseudo log-likelihood function with respect to the unknown decay factor.

As far as EWMA estimation is concerned, different procedures are examined in the application: we refer to "Riskmetrics EWMA" if the decay factor is chosen as in (15), by a simple grid search; we also consider the standard Riskmetrics^{TM} approach, in which the decay factor is set to 0.94 ("Riskmetrics EWMA 0.94"). Furthermore, "Student-t EWMA MLE" is the usual ML estimator. Since we make an explicit Student-t assumption for the normalized log-returns, we can estimate the decay factor by maximizing a Student-t log-likelihood.[15] The log-likelihood function has two unknown parameters: the decay factor λ and the Student's degrees of freedom parameter, which is estimated jointly with λ.[16] Finally, we also consider a Student-t GARCH(1,1) model, whose estimation is carried out by ML as well.

We shall call "Aggregation vector IMA(1,1)" our proposed approach for the determination of λ. As a matter of fact, equation (14) allows the theoretical framework presented in this paper to be employed to infer the decay factor at several sampling frequencies and levels of contemporaneous aggregation. Given that the portfolio in (11) is the weighted sum of two indices at daily frequency, we propose to estimate a trivariate IMA(1,1) model for the system composed by the two conditional variances and the autocovariance, namely,

$$\operatorname{vech}(\boldsymbol{r}_t \boldsymbol{r}_t') = \operatorname{vech}(\boldsymbol{r}_{t-1} \boldsymbol{r}_{t-1}') + \operatorname{vech}(\boldsymbol{\eta}_t) - \boldsymbol{\Lambda} \operatorname{vech}(\boldsymbol{\eta}_{t-1}),$$

which is the multivariate version of (14) and where $\boldsymbol{\Lambda}$ is a 3×3 moving average coefficient matrix. Once obtained the estimates for the MA matrix of coefficients, the decay factor is analytically computed for 1-day-ahead portfolio's volatility forecasts (i.e., θ), applying the results on contemporaneous aggregation of the IMA(1,1) model. Similarly, results on the temporal aggregation of the IMA(1,1) can be applied if interested in a decay factor for portfolio's volatility predictions at a different time frequency (i.e., Θ).

Regarding vector IMA(1,1) estimation, there are several existing vector autoregressive (VAR)-based procedures for the estimation of vector MA models. We employ the method proposed by Galbraith, Ullah and Zinde-Walsh (2002), which relies on a VAR approximation to the vector MA process. This procedure consists of basically two steps. In the first step, a VAR model is

[14] We refer to See RiskMetrics^{TM} (Technical Document, 1996, Section 5.3) for further details.

[15] Note, therefore, that "Student-t EWMA MLE" refers to the ML estimator, not to the PMLE suggested by Zaffaroni (2008).

[16] The MATLAB function fmincon.m is used for ML estimation. The λ parameter has a lower bound of 0.005 and an upper bound equal to 0.995.

estimated by the Yule-Walker estimator (Lütkepohl, 2007, Section 3.3.4) with a lag length ($p > 1$) that allows to recover the vector of innovations whose properties are close to the vector MA process. The maximum VAR lag length is 50. We use the Yule-Walker estimator since it delivers estimates of the VAR matrix coefficients which are always in the stability region.[17] In the second step, the vector MA coefficient matrices are obtained by using some known relations between the coefficient matrices of an infinite-order VAR model and their vector MA counterparts (see, for instance, Lütkepohl, 2007, Section 2.1.2).[18]

Somehow, it can be seen that the estimation of the decay factor based on the contemporaneously and temporally aggregated vector IMA(1,1) model can be considered as a variant of the RiskMetrics^{TM} approach and of the standard MLE, bearing in mind that all these procedures aim to calculate in different ways the decay factor, which is subsequently used as an input for the EWMA estimator of the portfolio's conditional volatility in (13).

5.3 VaR diagnostic test results

In order to assess the quality of the estimation methods and the resulting VaR predictions, we use standard backtests of VaR models. Backtesting is a statistical procedure consisting of calculating the percentage of times that the actual portfolio returns fall outside the VaR estimate, and comparing it to the confidence level used.

To implement the backtesting exercise, we carry out an out-of-sample analysis. We consider 1-step ahead forecasts using a fixed rolling window scheme.[19] The size of the window is set to 1000 daily observations (approximately 4 years), i.e., to generate VaR predictions, a training period of 1000 observations of past log-returns is used. The use of a fixed rolling window scheme is very common in the financial econometrics literature and is standard practice in RiskMetrics^{TM}.

We split the sample in 2 parts: from t=1 to t=1000; from t=1001 to t=2768. The first part of the sample is used for model estimation. The second part of the sample as out-of-sample validation period. The five examined estimation methods are implemented in order to calculate 1-step-ahead volatility forecasts, based on (13), and 1-period horizon VaR predictions. For all estimation methods under consideration, the decay factor is chosen over a range of values comprised between $\lambda_{min} = 0.005$ and $\lambda_{max} = 0.995$. The models are estimated and VaR predictions are calculated using observations from 1 to 1000. Then the sample is rolled forward, the models are re-estimated using data from 2 to 1001, VaR predictions are updated, and so on. Using daily data with a rolling estimation window of 1000 observations and starting the out-of-sample forecasting at t=1001, we have a total of 1767 estimates of the aggregated MA parameter and of the decay factor.

When a different time frequency is adopted for VaR prediction, in implementing "Riskmetrics EWMA", "Riskmetrics EWMA 0.94", "Student-t EWMA MLE" and "Student-t GARCH(1,1) MLE", the decay factors are estimated starting directly from the aggregate time series. For "Aggregation vector IMA(1,1)", on the other hand, the decay factor is systematically sampled at

[17] We are grateful to Helmut Lütkepohl who pointed out this property.

[18] Due to the type of exercise, in selecting the VAR(p) lag length, we adopt the criterion in (15), as for the RiskMetrics^{TM} approach. That is, the choice of the lag length of the VAR is made in order to analytically obtain a decay factor that minimizes the RMSE over different values.

[19] The developed MATLAB code is available from the authors upon request.

the corresponding desired frequency, such as the sequence,

$$\{\lambda_{tk}\}_{t=1}^{\lfloor 1767/k \rfloor}, \quad k = 1, 2, 3, 4, 5, \tag{16}$$

where $\lfloor b \rfloor$ indicates the integer part of a real number b.

Working with daily data, we have a total of 1767 tests for VaR at each confidence level (α=0.10, 0.05, 0.025, 0.01). For backtesting, we count the proportion of observations where the actual portfolio loss exceeded the estimated VaR for α=0.10, 0.05, 0.025, 0.01. This can be done by means of the exception indicator (also known as violation, or failure indicator), a binary variable which takes the value 1 if the log-returns exceed in absolute value the predicted VaR and 0 otherwise:

$$I_t(\alpha) = \mathbf{1}_{\left\{VaR_t^{(\alpha)}|\mathcal{F}_t < r_t\right\}}, \quad t = 1001, 1002, \ldots, 2768.$$

Similarly at the other time frequencies. If the VaR model is correctly specified and the confidence level α is, say, 95%, we expect portfolio log-returns to exceed the VaR predictions on about 5% of the cases. Formally:

$$\Pr(I_t(\alpha) = 1) = \alpha. \tag{17}$$

Equation (17) represents the "correct unconditional coverage" hypothesis, which can be statistically tested. If the number of exceptions is significantly lower or higher than α, the VaR model contributes to an overestimation or underestimation of risk.

To test whether the hypothesis of "correct unconditional coverage" holds, we use the likelihood ratio test statistic proposed by Kupiec (1995); the likelihood ratio test for "independence of VaR violations" (Christoffersen, 1998), which assesses whether or not exceptions are clustered in time, is also performed. Furthermore, "correct unconditional coverage" and "independence of VaR violations" are jointly tested by means of the "correct conditional coverage" test (Christoffersen, 1998), which examines both the number and the timing of violations.

[TABLES from 2 to 6 ABOUT HERE]

The empirical backtesting results are summarized in Tables 2-6. At all frequencies and for all the estimation methods, we adopt quantiles of a standardized Student's t-distribution (with twelve degrees of freedom) to provide VaR predictions. The degrees of freedom have been fixed to twelve since this value was close to what we got, on average, when estimating the decay factor (and the Student's degrees of freedom) with maximum likelihood. For a given significance level α, from top to down, each cell in the table displays the out-of-sample empirical coverage (i.e., proportion of exceptions) and, for VaR prediction, the p-values of the corresponding unconditional coverage test (Kupiec, 1995), independence of VaR violations test and conditional coverage test (Christoffersen, 1998). A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

The five methods perform similarly and reasonably well for $T = 1767, 883, 589, 441, 353$ (i.e., 1/2/3/4/5-days sampling frequencies). Overall, evaluation results indicate that the examined estimation methods pass the Kupiec and Christoffersen tests most of the times, with few exceptions: at 1-day frequency ($T = 1767$) and $\alpha = 0.05$, the observed failure rate for "Student-t GARCH(1,1) MLE" is significantly lower than 0.05 (risk is overestimated); moreover, at 2-days frequency ($T = 883$) and $\alpha = 0.10$, the proportion of violations for "Student-t GARCH(1,1) MLE" is 0.0805, far behind 0.10; similarly, at 4-days frequency ($T = 441$) and $\alpha = 0.10$, the proportion of violations is 0.0773 for "Student-t GARCH(1,1) MLE", and therefore the null hypothesis of the unconditional coverage test is not accepted. At 3-days frequency ($T = 589$) and $\alpha = 0.01$, the failure rate for "Riskmetrics EWMA" is significantly higher (0.0255) than the theoretical value at the specified confidence level (risk is underestimated); also at 5-days frequency ($T = 353$) and $\alpha = 0.01$, both for "Riskmetrics EWMA" and for "Aggregation vector IMA(1,1)", the observed failure rates are significantly higher (0.0284) than the theoretical values.

In summary, up to 5-days frequency, it comes out that "Student-t GARCH(1,1) MLE" and "Riskmetrics EWMA" perform worse than the other competing techniques, which allow to estimate VaR more accurately. In particular, the "Aggregation vector IMA(1,1)" approach and the standard "Riskmetrics EWMA 0.94" are the most successful techniques in evaluating risk. Therefore, "Aggregation vector IMA(1,1)" provides a rule to infer estimates of the decay factor and volatility predictions up to 5-days forecast horizon which works satisfactorily well.

In general, even when the null hypothesis is not rejected, it is worth noting that "Aggregation vector IMA(1,1)" reports higher (or equal) p-values than "Riskmetrics EWMA" most of the times. This seems suggesting the use of "Aggregation vector IMA(1,1)" when moving from high to low frequencies, at least up to 5-days. The standard "Riskmetrics EWMA 0.94", in which the decay factor is not estimated but simply suggested, delivers a very good performance (this probably explains why it is still one of the most widely used methodology for measuring market risk in financial industry). These results have to be evaluated with caution since they are based on a simple exercise and on a single data set. Further empirical research is needed to explore the performance of the aggregate Vector IMA(1,1) model in VaR analysis.

6 Concluding remarks

In this paper we deal with temporal and contemporaneous aggregation of IMA(1,1) processes, which constitute the equivalent ARIMA representation of the simple exponential smoothing. Relying on the closeness of VARMA processes with respect to linear transformations, we derive algebraic solutions for the unknown parameters of the aggregated model.

We provide evidence that no complicated numerical algorithms need to be implemented to infer the parameters of the aggregated model. Indeed, we derive algebraic solutions to recover them. In this way, we show that it is possible to establish a direct mapping between the parameters of the original model and those of the model for the corresponding contemporaneously and temporally aggregated data. The mapping fully relies on autocovariances of the aggregated model and can be easily obtained by applying the moment-based procedure discussed in the paper.

To illustrate the practical relevance of aggregation results for the vector IMA(1,1) model, we have presented an application of the techniques discussed to the calculation and prediction of VaR for a portfolio composed by NASDAQ and S&P500 stock indices, at different sampling frequencies. An empirical comparison shows that VaR predictions based on the vector IMA(1,1) model are at least as accurate as those based on the optimization procedure in (15) suggested by RiskMetrics$^{\text{TM}}$ or on maximum likelihood.

7 APPENDIX

For the proof of Proposition 2, we need this preparatory Lemma.

Lemma 3 *It is possible to express the square of the polynomial $1 + L + L^2 + \ldots + L^{k-1}$, whatever k, as:*

$$\left(\sum_{j=0}^{k-1} L^j\right)^2 = \sum_{j=0}^{k-1}(j+1)L^j + \sum_{j=k}^{2k-1}(2k-j-1)L^j. \tag{18}$$

Proof. We use induction to prove Lemma 3.

- Step 1. Check. For $k = 1$:

$$\left(\sum_{j=0}^{0} L^j\right)^2 = 1^2 = \sum_{j=0}^{0}(j+1)L^j + \sum_{j=1}^{1}(2-j-1)L^j = 1.$$

- Step 2. Induction. We assume that the result holds true for $k = n$:

$$\left(\sum_{j=0}^{n-1} L^j\right)^2 = \sum_{j=0}^{n-1}(j+1)L^j + \sum_{j=n}^{2n-1}(2n-j-1)L^j.$$

We need to show that the result follows for $k = n + 1$, under the hypothesis it works for $k = n$, i.e.,

$$\left(\sum_{j=0}^{n} L^j\right)^2 = \sum_{j=0}^{n}(j+1)L^j + \sum_{j=n+1}^{2n+1}(2n+1-j)L^j. \tag{19}$$

Note that we can express a sum over $n + 1$ terms as a sum over the first n terms plus the final term. For $k = n + 1$, therefore, the square of the sum over $n + 1$ terms may be developed as:

$$\begin{aligned}
\left(\sum_{j=0}^{n} L^j\right)^2 &= \left(\sum_{j=0}^{n-1} L^j + L^n\right)^2 \\
&= \left(\sum_{j=0}^{n-1} L^j\right)^2 + L^{2n} + 2\left(\sum_{j=0}^{n-1} L^j\right)L^n \\
&= \sum_{j=0}^{n-1}(j+1)L^j + \sum_{j=n}^{2n-1}(2n-j-1)L^j + L^{2n} + 2\left(\frac{1-L^n}{1-L}\right)L^n \quad \text{(from the assumption)} \\
&= \sum_{j=0}^{n-1}(j+1)L^j + \sum_{j=n}^{2n-1}(2n-j-1)L^j + \frac{L^n}{1-L}(2 - L^{n+1} - L^n).
\end{aligned}$$

As a consequence:

$$\left(\sum_{j=0}^{n} L^j\right)^2 = \sum_{j=0}^{n-1}(j+1)L^j + \sum_{j=n}^{2n-1}(2n-j-1)L^j + \frac{L^n}{1-L}(2 - L^{n+1} - L^n). \tag{20}$$

To prove that this is the desired result for $k = n + 1$, from (19) and (20), it is enough to show that:

$$\left(\sum_{j=0}^{n}(j+1)L^j + \sum_{j=n+1}^{2n+1}(2n+1-j)L^j\right) - \left(\sum_{j=0}^{n-1}(j+1)L^j + \sum_{j=n}^{2n-1}(2n-j-1)L^j\right) \stackrel{?}{=} \frac{L^n}{1-L}(2 - L^{n+1} - L^n). \tag{21}$$

To show that the equation above is true, we develop (21) as:

$$\begin{aligned}
& \left(\sum_{j=0}^{n}(j+1)L^j - \sum_{j=0}^{n-1}(j+1)L^j\right) + \left(\sum_{j=n+1}^{2n+1}(2n+1-j)L^j - \sum_{j=n}^{2n-1}(2n-j-1)L^j\right) \\
= \quad & (n+1)L^n + \left(\sum_{j=n+1}^{2n+1}(2n+1-j)L^j - \sum_{j=n}^{2n-1}(2n-j-1)L^j\right) \\
= \quad & (n+1)L^n + \left(\sum_{j=n+1}^{2n+1}(2n-j)L^j + \sum_{j=n+1}^{2n+1}L^j - \sum_{j=n}^{2n-1}(2n-j)L^j + \sum_{j=n}^{2n-1}L^j\right) \\
= \quad & 2L^n + L^{2n} + 2\sum_{j=n+1}^{2n-1}L^j = 2L^n + L^{2n} + 2L^n\frac{L-L^n}{1-L} = \left(\frac{L-L^n}{1-L}+1\right)2L^n + L^{2n} \\
= \quad & \left(\frac{1-L^n}{1-L}\right)2L^n + L^{2n} = \frac{L^n}{1-L}(2-L^{n+1}-L^n).
\end{aligned} \tag{22}$$

Therefore (22) confirms (21). This completes Step 2. We have proven Lemma 3. □

Proof of Proposition 2. The original model at the disaggregate frequency is an IMA(1,1), say,

$$(1-L)y_{kt} = (1+\theta L)\,a_{kt}. \tag{23}$$

Bearing in mind the temporal aggregation scheme in (8), that we list here below for the sake of the exposition,

$$y_\tau = \sum_{j=0}^{k-1} y_{kt-j} = \left(\frac{1-L^k}{1-L}\right)y_{kt},$$

and applying it to both sides of (23), we get:

$$(1-L)y_\tau = (1+\theta L)\left(\frac{1-L^k}{1-L}\right)a_{kt}.$$

Since the temporally aggregated model is an IMA(1,1) and sample information for y_τ is available only every k^{th} period, the equation here above needs to be transformed into:

$$\begin{aligned}
(1-L^k)y_\tau \quad &= \quad (1+\theta L)\left(\frac{1-L^k}{1-L}\right)^2 a_{kt} \\
&= \quad (1+\theta L)\left(\sum_{j=0}^{k-1}L^j\right)^2 a_{kt}.
\end{aligned} \tag{24}$$

Let us focus on the first row of (10). The autocovariance of order zero of the temporally aggregated model defined in (24) is:

$$\begin{aligned}
\Gamma(0) \quad &= \quad \mathrm{E}\left[(1-L^k)y_\tau \times (1-L^k)y_\tau\right] \\
&= \quad \mathrm{E}[(1+\theta L)(1+L+\ldots+L^{k-1})^2 a_{kt} \times (1+\theta L)(1+L+\ldots+L^{k-1})^2 a_{kt}].
\end{aligned}$$

From (18) we know that:

$$\left(\sum_{j=0}^{k-1}L^j\right)^2 = \sum_{j=0}^{k-1}(j+1)L^j + \sum_{j=k}^{2k-1}(2k-j-1)L^j. \tag{25}$$

Therefore, based on this, we can express $(1-L^k)y_\tau = (1+\theta L)(1+L+\ldots+L^{k-1})^2 a_{kt}$ in (24) as:

$$(1-L^k)y_\tau = (1+\theta L)\left(\sum_{j=0}^{k-1}(j+1)L^j + \sum_{j=k}^{2k-1}(2k-j-1)L^j\right)a_{kt}.$$

The expression here above can be written out as:

$$
\begin{aligned}
(1-L^k)y_\tau &= \left(\sum_{j=0}^{k-1}(j+1)L^j + \sum_{j=k}^{2k-1}(2k-j-1)L^j + \sum_{j=0}^{k-1}(j+1)\theta L^{j+1} + \sum_{j=k}^{2k-1}(2k-j-1)\theta L^{j+1}\right) a_{kt} \\
&= \left(\sum_{j=0}^{k-1}(j+1)L^j + (j+1)\theta L^{j+1}\right) a_{kt} + \left(\sum_{j=k}^{2k-1}(2k-j-1)L^j + (2k-j-1)\theta L^{j+1}\right) a_{kt} \\
&= \left(\sum_{j=0}^{k-1}(j+1)L^j + \sum_{j=1}^{k} j\theta L^j + \sum_{j=k}^{2k-1}(2k-j-1)L^j + \sum_{j=k+1}^{2k}(2k-j)\theta L^j\right) a_{kt} \\
&= \left(1+\sum_{j=1}^{k-1}(j+1)L^j + \sum_{j=1}^{k-1} j\theta L^j + ((k-1)+k\theta)L^k + \sum_{j=k+1}^{2k-1}(2k-j-1)L^j + \sum_{j=k+1}^{2k-1}(2k-j)\theta L^j\right) a_{kt} \\
&= \left(1+\sum_{j=1}^{k-1}((j+1)+j\theta)L^j + ((k-1)+k\theta)L^k + \sum_{j=k+1}^{2k-1}((2k-j-1)+(2k-j)\theta)L^j\right) a_{kt}.
\end{aligned}
\tag{26}
$$

Thus, the autocovariance of order zero is:

$$
\begin{aligned}
\Gamma(0) &= \left(1+\sum_{j=1}^{k-1}((j+1)+j\theta)^2 + ((k-1)+k\theta)^2 + \sum_{j=k+1}^{2k-1}((2k-j-1)+(2k-j)\theta)^2\right)\sigma_a^2 \\
&= \left(\sum_{j=0}^{k-1}((j+1)+j\theta)^2 + \sum_{j=k}^{2k-1}((2k-j-1)+(2k-j)\theta)^2\right)\sigma_a^2,
\end{aligned}
\tag{27}
$$

as we aimed to prove.

Let us switch to the second row of (10). We need to calculate the first-order autocovariance of the temporally aggregated model in (24), that is:

$$
\begin{aligned}
\Gamma(1) &= \mathrm{E}\left[(1-L^k)y_\tau \times (1-L^k)y_{\tau-1}\right] \\
&= \mathrm{E}[(1+\theta L)(1+L+\ldots+L^{k-1})^2 a_{kt} \times (1+\theta L)(1+L+\ldots+L^{k-1})^2 a_{k(t-1)}].
\end{aligned}
$$

From (26), we already know that:

$$
\begin{aligned}
&(1-L^k)y_\tau \\
&= \left(1+\sum_{j=1}^{k-1}((j+1)+j\theta)L^j + ((k-1)+k\theta)L^k + \sum_{j=k+1}^{2k-1}((2k-j-1)+(2k-j)\theta)L^j\right) a_{kt}.
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
&(1-L^k)y_{\tau-1} \\
&= \left(L^k + \sum_{j=k+1}^{2k-1}((j+1-k)+(j-k)\theta)L^j + ((k-1)+k\theta)L^{2k} + \sum_{j=2k+1}^{3k-1}((3k-j-1)+(3k-j)\theta)L^j\right) a_{kt}.
\end{aligned}
$$

Consequently:

$$\begin{aligned}
\Gamma(1) &= \mathrm{E}\left[(1-L^k)y_\tau \times (1-L^k)y_{\tau-1}\right] \\
&= \left(((k-1)+k\theta)+\sum_{j=k+1}^{2k-1}((2k-j-1)+(2k-j)\theta)((j+1-k)+(j-k)\theta)\right)\sigma_a^2 \\
&= \left(\sum_{j=k}^{2k-1}((2k-j-1)+(2k-j)\theta)((j+1-k)+(j-k)\theta)\right)\sigma_a^2.
\end{aligned}$$

The second part of Proposition 2 follows from the properties of second degree equations. The proof is now complete. □

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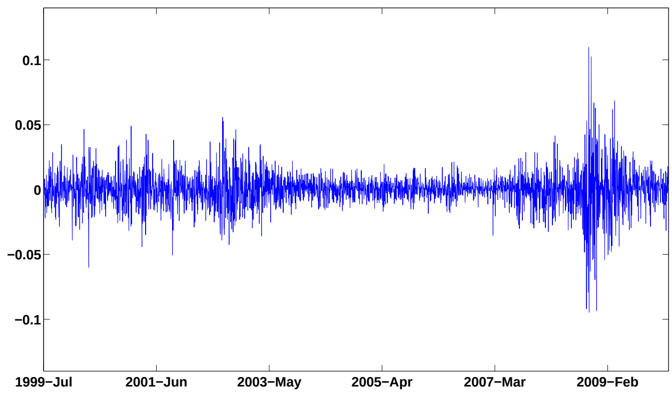
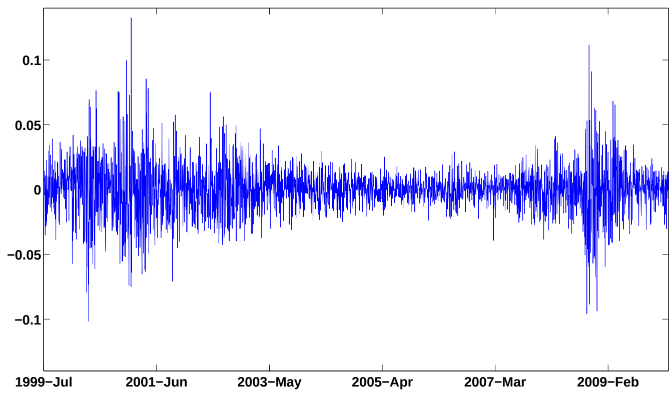
Table 1: NASDAQ and S&P500 log-returns summary statistics

Statistics	**NASDAQ**	**S&P500**
Number of observations	2768	2768
Maximum	0.132546	0.109572
Minimum	-0.101684	-0.094695
Mean	-8.193816e-005	-8.813140e-005
Median	2.046986e-004	2.636947e-005
Range	0.234231	0.204267
St.Dev.	0.018709	0.013596
Skewness	0.091602	-0.104871
Kurtosis	7.344184	10.965030
Jarque-Bera	2.180435e+003	7.322017e+003
	(pval=1.0000e-003)	(pval=1.0000e-003)

Descriptive statistics for NASDAQ and S&P500 daily log-returns. Closing prices data from July 09, 1999 to February 17, 2010.

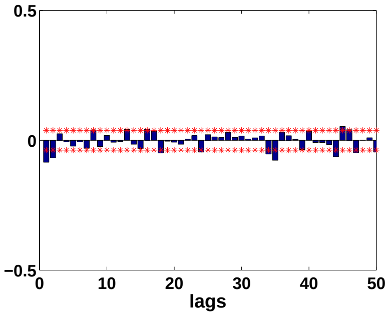


(a) S&P500

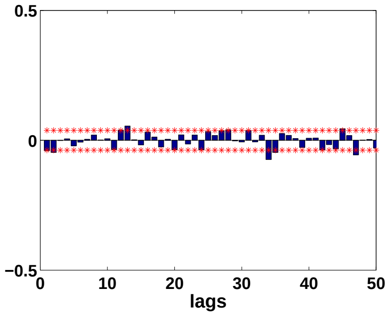


(b) NASDAQ

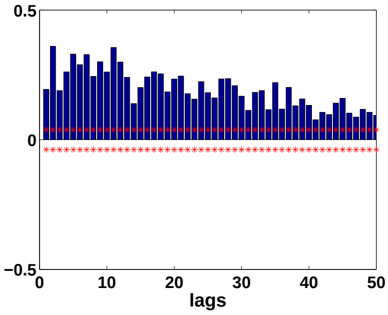
Figure 1: Daily log-returns series of the S&P500 and NASDAQ indices from July 09, 1999 to February 17, 2010.



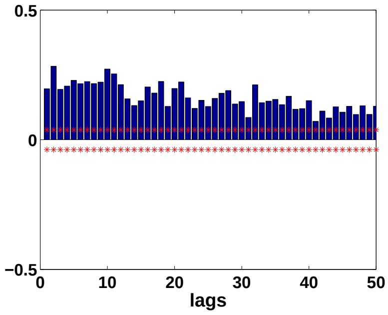
(a) S&P500 log-returns



(b) NASDAQ log-returns



(c) S&P500 squared log-returns



(d) NASDAQ squared log-returns

Figure 2: Autocorrelation function of S&P500 and NASDAQ log-returns and squared log-returns.

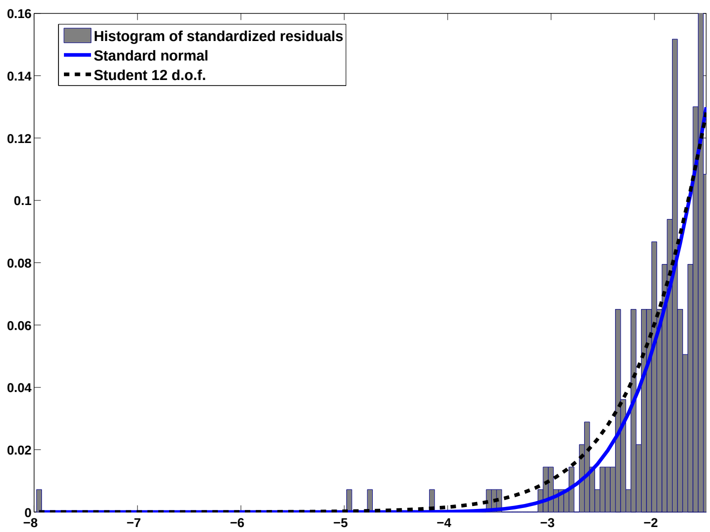


Figure 3: Histogram (left tail) of the daily normalized log-returns of the portfolio in (11).

Table 2: VaR evaluation tests (T=1767)

T=1767 BACKTESTING DATA (sampling frequency: 1 day)	Riskmetrics EWMA (min MSFE)	Riskmetrics EWMA 0.94 (λ set at 0.94)	Aggregation vector IMA(1,1)	Student-t EWMA MLE	Student-t GARCH(1,1) MLE
Confidence level (α)	0.1000	0.1000	0.1000	0.1000	0.1000
Proportion of exceptions	0.1076	0.1053	0.1059	0.1076	0.0900
P-value Unconditional coverage test	0.2931	0.4594	0.4134	0.2931	0.1563
P-value Independence test	0.1602	0.2296	0.2105	**0.0939**	0.1885
P-value Conditional coverage test	0.2147	0.3696	0.3268	0.1414	0.1542
Confidence level (α)	0.0500	0.0500	0.0500	0.0500	0.0500
Proportion of exceptions	0.0555	0.0555	0.0555	0.0532	0.0391
P-value Unconditional coverage test	0.2976	0.2976	0.2976	0.5378	**0.0287**
P-value Independence test	0.8393	0.8393	0.8393	0.9977	0.2209
P-value Conditional coverage test	0.5695	0.5695	0.5695	0.8271	**0.0432**
Confidence level (α)	0.0250	0.0250	0.0250	0.0250	0.0250
Proportion of exceptions	0.0255	0.0266	0.0255	0.0255	0.0187
P-value Unconditional coverage test	0.8972	0.6672	0.8972	0.8972	0.0754
P-value Independence test	0.1249	0.1088	0.1249	0.1249	0.2621
P-value Conditional coverage test	0.3055	0.2520	0.3055	0.3055	0.1098
Confidence level (α)	0.0100	0.0100	0.0100	0.0100	0.0100
Proportion of exceptions	0.0102	0.0096	0.0091	0.0102	0.0085
P-value Unconditional coverage test	0.9354	0.8738	0.6866	0.9354	0.5137
P-value Independence test	0.5425	0.5653	0.5885	0.5425	0.6121
P-value Conditional coverage test	0.8280	0.8370	0.7963	0.8280	0.7105

At all frequencies, a Student-t distribution with 12 degrees of freedom is used to provide a VaR estimate. A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

Table 3: VaR evaluation tests (T=883)

T=883 BACKTESTING DATA (sampling frequency: 2 days)	Riskmetrics EWMA (min MSFE)	Riskmetrics EWMA 0.94 (λ set at 0.94)	Aggregation vector IMA(1,1)	Student-t EWMA MLE	Student-t GARCH(1,1) MLE
Confidence level (α)	0.1000	0.1000	0.1000	0.1000	0.1000
Proportion of exceptions	0.0930	0.0918	0.0964	0.0964	0.0805
P-value Unconditional coverage test	0.4818	0.4132	0.7180	0.7180	**0.0464**
P-value Independence test	0.8840	0.3228	0.9379	0.1637	0.9000
P-value Conditional coverage test	0.7726	0.4389	0.9340	0.3552	0.1365
Confidence level (α)	0.0500	0.0500	0.0500	0.0500	0.0500
Proportion of exceptions	0.0544	0.0533	0.0522	0.0601	0.0431
P-value Unconditional coverage test	0.5522	0.6574	0.7706	0.1820	0.3349
P-value Independence test	0.6760	0.2579	0.2850	0.4505	0.5760
P-value Conditional coverage test	0.7680	0.4779	0.5412	0.3087	0.5372
Confidence level (α)	0.0250	0.0250	0.0250	0.0250	0.0250
Proportion of exceptions	0.0261	0.0204	0.0261	0.0295	0.0227
P-value Unconditional coverage test	0.8388	0.3672	0.8388	0.4072	0.6534
P-value Independence test	0.2668	0.3862	0.2668	0.7932	0.4715
P-value Conditional coverage test	0.5287	0.4575	0.5287	0.6853	0.6977
Confidence level (α)	0.0100	0.0100	0.0100	0.0100	0.0100
Proportion of exceptions	0.0102	0.0113	0.0125	0.0102	0.0068
P-value Unconditional coverage test	0.9516	0.6958	0.4775	0.9516	0.3111
P-value Independence test	0.6665	0.6318	0.5979	0.6665	0.7742
P-value Conditional coverage test	0.9096	0.8259	0.6761	0.9096	0.5746

At all frequencies, a Student-t distribution with 12 degrees of freedom is used to provide a VaR estimate. A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

Table 4: VaR evaluation tests (T=589)

T=589 BACKTESTING DATA (sampling frequency: 3 days)	Riskmetrics EWMA (min MSFE)	Riskmetrics EWMA 0.94 (λ set at 0.94)	Aggregation vector IMA(1,1)	Student-t EWMA MLE	Student-t GARCH(1,1) MLE
Confidence level (α)	0.1000	0.1000	0.1000	0.1000	0.1000
Proportion of exceptions	0.0935	0.0901	0.1003	0.0986	0.0867
P-value Unconditional coverage test	0.5978	0.4182	0.9781	0.9123	0.2737
P-value Independence test	0.5624	0.5542	0.1466	0.7302	0.4336
P-value Conditional coverage test	0.7357	0.6050	0.3485	0.9366	0.4043
Confidence level (α)	0.0500	0.0500	0.0500	0.0500	0.0500
Proportion of exceptions	0.0561	0.0476	0.0510	0.0527	0.0425
P-value Unconditional coverage test	0.5037	0.7895	0.9099	0.7640	0.3933
P-value Independence test	0.9113	0.1866	0.6300	0.7715	0.3939
P-value Conditional coverage test	0.7947	0.4035	0.8848	0.9165	0.4830
Confidence level (α)	0.0250	0.0250	0.0250	0.0250	0.0250
Proportion of exceptions	0.0391	0.0306	0.0340	0.0340	0.0221
P-value Unconditional coverage test	0.0426	0.3995	0.1841	0.1841	0.6470
P-value Independence test	0.2933	0.5740	0.2349	0.1730	0.2839
P-value Conditional coverage test	0.0736	0.5987	0.2044	0.1635	0.5071
Confidence level (α)	0.0100	0.0100	0.0100	0.0100	0.0100
Proportion of exceptions	0.0255	0.0170	0.0204	0.0221	0.0136
P-value Unconditional coverage test	**0.0016**	0.1206	0.0262	0.0109	0.4048
P-value Independence test	0.3903	0.1546	0.4791	0.2839	0.0908
P-value Conditional coverage test	**0.0047**	0.1088	0.0657	0.0221	0.1691

At all frequencies, a Student-t distribution with 12 degrees of freedom is used to provide a VaR estimate. A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

Table 5: VaR evaluation tests (T=441)

T=441 BACKTESTING DATA (sampling frequency: 4 days)	Riskmetrics EWMA (min MSFE)	Riskmetrics EWMA 0.94 (λ set at 0.94)	Aggregation vector IMA(1,1)	Student-t EWMA MLE	Student-t GARCH(1,1) MLE
Confidence level (α)	0.1000	0.1000	0.1000	0.1000	0.1000
Proportion of exceptions	0.0864	0.0864	0.0886	0.0977	0.0773
P-value Unconditional coverage test	0.3301	0.3301	0.4187	0.8733	**0.0992**
P-value Independence test	0.1339	0.3308	0.7800	0.3576	0.3906
P-value Conditional coverage test	0.2024	0.3879	0.6935	0.6467	0.1777
Confidence level (α)	0.0500	0.0500	0.0500	0.0500	0.0500
Proportion of exceptions	0.0568	0.0500	0.0591	0.0614	0.0409
P-value Unconditional coverage test	0.5203	1.0000	0.3944	0.2901	0.3669
P-value Independence test	0.6270	0.4139	0.6237	0.7849	0.2147
P-value Conditional coverage test	0.7227	0.7162	0.6169	0.5506	0.3083
Confidence level (α)	0.0250	0.0250	0.0250	0.0250	0.0250
Proportion of exceptions	0.0295	0.0250	0.0273	0.0295	0.0227
P-value Unconditional coverage test	0.5526	1.0000	0.7634	0.5526	0.7565
P-value Independence test	0.3893	0.2674	0.4115	0.3893	0.4947
P-value Conditional coverage test	0.5788	0.5406	0.6821	0.5788	0.7549
Confidence level (α)	0.0100	0.0100	0.0100	0.0100	0.0100
Proportion of exceptions	0.0136	0.0091	0.0159	0.0136	0.0136
P-value Unconditional coverage test	0.4676	0.8457	0.2513	0.4676	0.4676
P-value Independence test	0.6834	0.7862	0.6339	0.6834	0.6834
P-value Conditional coverage test	0.7068	0.9458	0.4624	0.7068	0.7068

At all frequencies, a Student-t distribution with 12 degrees of freedom is used to provide a VaR estimate. A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

Table 6: VaR evaluation tests (T=353)

T=353 BACKTESTING DATA (sampling frequency: 5 days)	Riskmetrics EWMA (min MSFE)	Riskmetrics EWMA 0.94 (λ set at 0.94)	Aggregation vector IMA(1,1)	Student-t EWMA MLE	Student-t GARCH(1,1) MLE
Confidence level (α)	0.1000	0.1000	0.1000	0.1000	0.1000
Proportion of exceptions	0.0767	0.0824	0.0881	0.0824	0.0739
P-value Unconditional coverage test	0.1303	0.2573	0.4472	0.2573	**0.0880**
P-value Independence test	0.9536	0.6803	0.8638	0.2947	0.9543
P-value Conditional coverage test	0.3178	0.4837	0.7381	0.3041	0.2329
Confidence level (α)	0.0500	0.0500	0.0500	0.0500	0.0500
Proportion of exceptions	0.0568	0.0483	0.0597	0.0625	0.0483
P-value Unconditional coverage test	0.5653	0.8827	0.4191	0.2995	0.8827
P-value Independence test	0.8876	0.8426	0.5116	0.7189	0.2407
P-value Conditional coverage test	0.8392	0.9699	0.5817	0.5472	0.4970
Confidence level (α)	0.0250	0.0250	0.0250	0.0250	0.0250
Proportion of exceptions	0.0341	0.0369	0.0455	0.0369	0.0199
P-value Unconditional coverage test	0.3001	0.1801	0.0271	0.1801	0.5242
P-value Independence test	0.4149	0.4934	0.2163	0.4934	0.1179
P-value Conditional coverage test	0.4193	0.3221	0.0405	0.3221	0.2405
Confidence level (α)	0.0100	0.0100	0.0100	0.0100	0.0100
Proportion of exceptions	0.0284	0.0256	0.0284	0.0227	0.0170
P-value Unconditional coverage test	**0.0046**	0.0141	**0.0046**	0.0396	0.2274
P-value Independence test	0.2752	0.2154	0.4437	0.1628	0.0807
P-value Conditional coverage test	**0.0099**	0.0228	0.0134	0.0455	0.1050

At all frequencies, a Student-t distribution with 12 degrees of freedom is used to provide a VaR estimate. A p-value smaller than α implies a rejection of the null hypothesis (bold numbers).

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