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# Mean-Variance Cointegration and the Expectations Hypothesis 

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# Mean-Variance Cointegration and the Expectations Hypothesis ${ }^{1}$ 

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#### Abstract

The present work provides an economic explanation of a well-known (seeming) violation of the expectations hypothesis of the term structure (EHT) - the frequent finding of unit roots in interest rate spreads. We derive from EHT that the nonstationarity stems from the holding premium, which is hence cointegrated with the spread. We model the premium as being proportional to the integrated variance of excess returns and further propose a cointegration test. Simulating the distribution of the test statistic we actually find cointegration relations between premia and spreads in US data. The EHT appears to perform better than previously thought.


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[^1]
## 1 Introduction

The expectations hypothesis of the term structure (EHT) remains both one of the most examined and rejected theories. Comprehensive surveys covering early work are provided by Melino (1988), Shiller (1990) and Campbell and Shiller (1991). The present work focuses on the common implication of EHT that interest rate spreads should be stationary and explains why this property is so often not found in empirical studies. We show that this notorious lack of evidence can be attributed to a nonstationary premium in line with economic theory and verified by econometric results from unit root and cointegration analysis.

For interest rates integrated of order one $(I(1))$ and under the assumption of a constant term premium, Campbell and Shiller (1987) show that EHT implies cointegration between two arbitrary interest rates of different maturities with vector $(1,-1)$. However, Shea (1992), Zhang (1993), Wolters $(1995,1998)$ and Carstensen (2003), for instance, find that stationary spreads are often not reflected in the data. The larger the difference in maturity the more often this outcome occurs. Some of these studies conduct multivariate cointegration analysis building on the fact that stationary spreads imply cointegrating rank $n-1$ in a system of $n$ interest rates (see Hall et al. 1992). Yet, the cointegrating rank mostly turns out to be smaller than $n-1$, so that interest rates are driven by more than one stochastic trend.

Nonstationarity of spreads has far-reaching consequences for empirically testing the EHT. As an example consider the popular single equation regression where the interest rate spread explains future short rate changes. Such an error correction model (ECM) can be derived directly from economic theory. Theoretically, the relevant adjustment coefficient equals the ratio of long and short rate maturity (see, inter alia, Mankiw and Miron 1986, Kugler 1988 or Caporale and Caporale 2008). Yet, commonly values considerably smaller than those implied by EHT are found (see also Campbell and Shiller 1991). More importantly, the adjustment coefficient often proves to be even insignificant - indicating no predictive power of spreads for short rate changes. Such zero adjustment would occur if the equation were unbalanced and an ECM would simply not exist.

A number of authors argue that the assumption of a constant term premium may be inappropriate. Evidence for a time-varying premium is provided by Mankiw and Summers (1984), Mankiw and Miron (1986), Bollerslev et al. 1988, Engle et al. (1990) and Evans and Lewis (1994), to name just a few. A related and relatively recent series of papers quotes a (monetary) policy regime explanation for the adjustment coefficient to be different from its theoretically expected value (e.g. Kugler 2002, Caporale and Caporale 2008, Weber and Wolters 2009). Another influential strand of literature (e.g. Dai and Singleton 2000, Duffee 2002, Ang and Piazzesi 2003 and Balfoussia and Wickens 2007) describes the yield curve and risk premia as affine functions of observable and latent factors.

However, even if we allow for a time-varying but stationary premium, the implication that spreads must be $I(0)$ holds. Again, this is crucial since the mean-reverting property is a necessary precondition for spreads having forecasting power for short rate changes. Therefore, studying the stationarity properties implied by EHT can provide insights into whether this economic theory is at least remotely valid.

In the present paper we identify and estimate the source of nonstationarity of US interest rate spreads. We provide an economic explanation for integrated spreads by showing that they come along with nonstationary premia. The latter are described as the product of risk and its market price, equalling the expected excess return. Thereby, the conditional second moment of excess returns serves as risk measure ${ }^{4}$. We apply the standard definition of excess returns as the sum of interest rate spread and capital gain. Since capital gains prove to be clearly stationary, balancedness requires cointegration between spread and premium, i.e. mean-variance cointegration. The premium is estimated based on a generalized autoregressive conditional heteroskedasticity (GARCH) model (Engle 1982, Bollerslev 1986). We then show that the null of integrated conditional variance cannot be rejected. This result survives the inclusion of structural breaks under the alternative hypothesis. Finally, we propose a cointegration test and simulate the appropriate distribution of the test statistic. Empirically, we actually find cointegration relations between premia and spreads in US interest rate data. This explains the seeming violation

[^2]of the necessary condition for the EHT - the frequent finding of nonstationary spreads.

The paper proceeds as follows. The next section briefly reviews some economic theory. In section 3 we introduce econometric methodology and propose a procedure to test for cointegration. This is followed by the presentation of the empirical results and several robustness checks. The final section summarizes and contains concluding remarks.

## 2 Economic Motivation

Consider to buy a bond sold at par at time $t$ that matures at time $T$ and to sell it one period later. Let $\tilde{y}_{t+1}$ indicate the corresponding return at $t+1$ from holding the $T-t$ period bond for one period. This return can be written as $\tilde{y}_{t+1}=c_{t+1}+R_{t}$, where $c_{t+1}$ refers to the capital gain accruing through a change in the bond price and $R_{t}$ is defined as the one period interest rate (equal to the coupon payment ${ }^{5}$ ) of a $T-t$ period bond. On the other hand, regard the interest rate at $t$ of a one period bond, $r_{t}$, that is known today and assumed riskless.

Given rational expectations and absence of arbitrage, the pure EHT states that $\mathrm{E}\left[\tilde{y}_{t+1} \mid I_{t}\right]$ and $r_{t}$ must be equal. Here, E is the expectations operator and $I_{t}$ denotes the information set containing all information available up to time $t$. However, since it is not sure at $t$ how the price of the long term bond evolves (i.e., $c_{t+1}$ is uncertain at $t$ ), risk-averse investors demand a premium. According to the common EHT, this one period holding premium ${ }^{6}$ (the expected one period excess return) follows as:

$$
\begin{equation*}
\phi_{t+1}=\mathrm{E}\left[\tilde{y}_{t+1} \mid I_{t}\right]-r_{t} . \tag{1}
\end{equation*}
$$

Substituting for $\tilde{y}_{t+1}$ in (1) we get

$$
\begin{equation*}
\phi_{t+1}=\mathrm{E}\left[c_{t+1} \mid I_{t}\right]+s_{t} . \tag{2}
\end{equation*}
$$

[^3]Relation (2) shows that the holding premium consists of the expected capital gain and the interest rate spread, denoted by $s_{t}$. The latter is contained in the information set $I_{t}$. From now on assume that $\mathrm{E}\left[c_{t+1} \mid I_{t}\right]$ is $I(0)^{7}$. Then, (2) implies spread and holding premium either to be both stationary or both nonstationary ${ }^{8}$. Otherwise (2) would not be balanced. As noted in the introduction, there is evidence in the literature for a time-varying premium that might resolve some discrepancies between theory and empirical results. However, a time-varying but stationary premium cannot explain the frequent finding that $s_{t}$ is $I(1)$. By contrast, if nonstationary spreads were associated with integrated holding premia and, moreover, spreads and premia were cointegrated, this could explain the poor performance of the EHT documented in numerous studies. According to (2), the cointegrating vector would be $(1,-1)$.

Notice that dropping the expectation operator from the right hand side of (1) would yield the realized (or ex-post) excess return at $t+1$ which - on average should equal the rational expectations of investors at $t$. Excess returns are generally known to exhibit only very slight autocorrelation. However, as follows from Evans and LEWIS (1994), this empirical result may also occur if spreads are nonstationary since the variation of capital gains is usually so high relative to $s_{t}$ that the persistence of $c_{t+1}$ and $c_{t+1}+s_{t}$ is statistically indistinguishable. Put differently, due to a very small signal-to-noise ratio, statistical tests fail to detect the true order of integration of excess returns in case of $s_{t}$ being $I(1)$. This fact may also underlie the general difficulties of empirical finance to find a significant risk-return tradeoff when trying to explain (statistically) strongly mean reverting excess returns by highly persistent second moments. However, the theoretical result remains - the orders of integration of $s_{t}$ and $\phi_{t+1}$ must be equal.

The conclusion that spreads should be stationary can also be (and often was) drawn from the representation of the spread as a weighted average of expected future short rate changes (see Campbell and Shiller 1991),

[^4]\[

$$
\begin{equation*}
s_{t}=\theta_{t}+\sum_{i=1}^{m-1}\left(1-\frac{i}{m}\right) \mathrm{E}\left[\Delta r_{t+i} \mid I_{t}\right] \tag{3}
\end{equation*}
$$

\]

where $m=T-t$. Given interest rate series integrated of order one, the expectation terms should be $I(0)$. If furthermore the rollover premium $\theta_{t}$ is stationary, the same holds for the spread. It should be noted that by definition $\theta_{t}$ can be written as a sum of successive holding premia, where the first summand equals $\phi_{t+1}$ from (2) (see Shiller 1990). Therefore, theoretically, the orders of integration of $\boldsymbol{\phi}_{t+1}$ and $\theta_{t}$ must be equal.

As any other term premium, the holding premium in (2) is unobservable. We therefore model the holding premium as being proportional to the conditional second moment of excess returns ${ }^{9}$ (e.g., Engle et al. 1987 or Tzavalis and Wickens 1995). The tradeoff between risk and return as the fundamental idea of many theories has a long tradition in financial economics. First of all, this concerns (conditional) asset pricing models that trace back to the Sharpe-Lintner CAPM (Sharpe 1964, Lintner 1965). However, modeling the risk-return trade-off as a mean-variance relation is a common feature of many theoretical approaches developed since then and often supported by empirical evidence (see, e.g., Bansal and Lundblad 2002, Ghysels et al. 2005 and Lundblad 2007). In the basic CAPM expected returns are determined by the covariance of the asset with the market return. If we consider a stylized world that comprises only two assets, i.e. one risky asset (offering the long rate) and the riskless asset (offering the short rate), the conditional covariance with the market portfolio becomes the variance of the asset itself. Evidently, this is a very direct way to motivate a mean-variance relation for returns.

Formally, let $y_{t+1}=\tilde{y}_{t+1}-r_{t}$ denote the excess return series so that $\mathrm{E}\left[y_{t+1} \mid I_{t}\right]=\phi_{t+1}$. For the conditionally expected variance of excess returns $s_{t}$ plays no role since it is included in $I_{t}$. Consequently, the relation between mean and variance follows as

[^5]\[

$$
\begin{aligned}
\mathrm{E}\left[y_{t+1} \mid I_{t}\right] & =b \operatorname{Var}\left[y_{t+1} \mid I_{t}\right] \\
& =b \operatorname{Var}\left[c_{t+1}+s_{t} \mid I_{t}\right] \\
& =b \operatorname{Var}\left[c_{t+1} \mid I_{t}\right],
\end{aligned}
$$
\]

and therefore:

$$
\begin{equation*}
\phi_{t+1}=b \operatorname{Var}\left[c_{t+1} \mid I_{t}\right] . \tag{4}
\end{equation*}
$$

In a two-asset world, $b$ may be interpreted as the market price of risk. Plugging (4) into (2) reveals that given $\mathrm{E}\left[c_{t+1} \mid I_{t}\right] \sim I(0)$ and $s_{t} \sim I(1)$, the spread is expected to be cointegrated with the conditional variance of the capital gain series with vector $(1,-b)$. This is what we label mean-variance cointegration. Theoretical considerations thus lead us to examine the EHT in three steps:
(i) First, we determine the order of integration of interest rate spreads to obtain evidence of whether assuming stationary premia is appropriate.
(ii) If this is not the case, testing for integrated premia (respectively, variances) will follow.
(iii) Finally, if premia are actually found to be nonstationary, we will test for cointegration with the spreads and estimate the proportionality coefficient as well as the adjustment speed.

## 3 Econometric Modeling

The methodology to be introduced follows the three steps just mentioned. At first, we discuss how to test for unit roots in the conditional mean of a time series (the spread) that potentially exhibits heteroskedasticity. Secondly, the same will be done with respect to nonstationarity of the conditional variance of a time series (the capital gain). At last, we propose a test for cointegration between mean and variance.

### 3.1 Testing for Unit Roots

As explained in the previous section, given stationarity of capital gains and holding premia, the EHT implies that interest rate spreads are $I(0)$. This implication can be checked by unit root tests (see step (i), section 2). Since the present data exhibits heteroskedastic innovations, a usual property of financial time series, we review some recent developments in the field of unit root tests under GARCH. With regard to the impact of neglected GARCH on the (augmented) Dickey-Fuller (DF, Dickey and Fuller 1979) test see, e.g., Kim and Schmidt (1993), Haldrup (1994), Ling and Li $(1998,2003)$ and Ling et al. (2003). Due to the invariance principle, the DF test proves to be asymptotically robust to covariance stationary GARCH errors. However, small-sample properties were conjectured to be affected in case of very persistent variance processes. Hence, SEO (1999), for instance, proposes a more powerful test that uses the information arising from conditional heteroskedasticity by means of joint maximum likelihood estimation (MLE) of the autoregressive and the GARCH parameters (see also Cook 2008a, 2008b). Yet, Charles and Darné (2008) find that for many practically relevant GARCH parameter values ${ }^{10}$ the DF test performs better than the Seo test with respect to power and size. Recent work from Kourogenis and Pittis (2008) explicitly analyzes integrated GARCH (IGARCH) innovations in the context of standard unit root tests. ${ }^{11}$ In their Monte Carlo simulations the DF test is included as the special case of uncorrelated innovations. Accordingly, the standard DF test appears to perform surprisingly well in the IGARCH case. Nevertheless, we will later double-check the standard least squares ADF test results by the Seo test. The distribution in the Seo test depends on a nuisance parameter, the relative weight $0 \leq \tau \leq 1$, and is bounded between the DF distribution $(\tau=1)$ and the standard normal $(\tau=0)$. The well-known ADF test equation (respectively the mean equation in the Seo test) is given by

$$
\begin{equation*}
\Delta x_{t+1}=\delta+\psi x_{t}+\sum_{i=1}^{q} \delta_{i} \Delta x_{t+1-i}+u_{t+1} \tag{5}
\end{equation*}
$$

[^6]where $q$ denotes the lag length and $u_{t+1}$ is (possibly heteroskedastic) white noise.

### 3.2 Testing for IGARCH

In the following, the conditional means of spread and capital gain series are both specified as $\operatorname{AR}(p)$ processes. Thereby we already imposed a unit in $s_{t}$ which is found later during the empirical analysis. With $\operatorname{GARCH}(1,1)$ errors $\boldsymbol{\varepsilon}_{c, t+1}$ and $\varepsilon_{s, t+1}$ we get

$$
\begin{align*}
c_{t+1} & =a_{c}+\sum_{i=1}^{p_{c}} a_{c, i} c_{t+1-i}+\varepsilon_{c, t+1},  \tag{6}\\
h_{c, t+1} & =\omega_{c}+\alpha_{c} \varepsilon_{c, t}^{2}+\beta_{c} h_{c, t},
\end{align*}
$$

and

$$
\begin{align*}
\Delta s_{t+1} & =\sum_{i=1}^{p_{s}} a_{s, i} \Delta s_{t+1-i}+\varepsilon_{s, t+1}  \tag{7}\\
h_{s, t+1} & =\omega_{s}+\alpha_{s} \varepsilon_{s, t}^{2}+\beta_{s} h_{s, t}
\end{align*}
$$

where $\operatorname{Var}\left[c_{t+1} \mid I_{t}\right]=\mathrm{E}\left[\varepsilon_{c, t+1}^{2} \mid I_{t}\right] \equiv h_{c, t+1}$ and $h_{s, t+1}$ accordingly. The necessary condition for the unit root in autoregression (7) to be consistent with the EHT (see step (ii), section 2) would be met if the conditional variance of the capital gains in (6) was integrated ( $\operatorname{IGARCH}(1,1)$, see Engle and Bollerslev 1986). However, note that whether the variance process in (7) turns out to be covariance stationary or not is irrelevant for evaluating the validity of the EHT.
The parsimonious (I)GARCH $(1,1)$ specification is known to capture variance dynamics of most financial time series fairly well. This is also true for the present data. The $\operatorname{IGARCH}(1,1)$ hypothesis, meaning that the slope coefficients in the conditional variance equation sum up to one, is often checked by likelihood ratio (LR) tests (see, inter alia, Chou 1988, Tzavalis and Wickens 1995 and Fang and Miller 2009). Yet, Lumsdaine (1995) shows within a Monte Carlo investigation that the LR test is quite oversized in small samples. Busch (2005) proposes a robust LR test based on quasi MLE (QMLE). His test statistic proves to be well behaved in small samples. The correction term $k=0.5\left(\mathrm{E}\left[\xi_{t}^{4}\right]-1\right)$ with $\xi_{t}=\varepsilon_{t} / \sqrt{h_{t}}$ is calculated under the alternative of a covariance stationary GARCH process. In
that case the test statistic

$$
\begin{equation*}
\lambda=-\frac{2}{k}\left(l\left(\hat{\boldsymbol{\theta}}_{r}\right)-l\left(\hat{\boldsymbol{\theta}}_{u}\right)\right) \tag{8}
\end{equation*}
$$

has actual size close to nominal size, even for skewed disturbances. ${ }^{12}$ In (8), $\hat{\boldsymbol{\theta}}_{r}$ and $\hat{\boldsymbol{\theta}}_{u}$ are the restricted and the unrestricted QMLEs for the parameter vector $\boldsymbol{\theta}$. However, since persistence in variance is a central question in the present work, we have additionally conducted a small-sample simulation experiment. That is, we have simulated the respective distribution of $\boldsymbol{\lambda}$ under the null of a DGP according to (6) with parameter vector $\hat{\boldsymbol{\theta}}_{r}\left(\alpha_{c}+\beta_{c}=1\right)$. The resulting critical values will be applied in addition to the $\chi^{2}$ quantiles. Moreover, we will provide evidence from test variants allowing for structural breaks under the alternative hypothesis.

### 3.3 Testing for Mean-Variance Cointegration

We continue by discussing the methodological approach to examine step (iii) (see section 2). Thus, a test for cointegration between mean of the spread series and variance of the capital gain series is presented. For that purpose, we initially augment the above ADF test equation (5) for $s_{t}$ by the integrated variance series $h_{c, t+1}$ from (6) (i.e., under the restriction that $\alpha_{c}+\beta_{c}=1$ ):

$$
\begin{equation*}
\Delta s_{t+1}=a+\rho s_{t}+\gamma h_{c, t+1}+\sum_{i=1}^{p} a_{i} \Delta s_{t+1-i}+\varepsilon_{t+1} \tag{9}
\end{equation*}
$$

Additionally, we explicitly address the presence of (I)GARCH effects in $\boldsymbol{\varepsilon}_{t+1}$. Hence, (9) and the process for $\varepsilon_{t+1}^{2}$ should be estimated simultaneously by (Q)ML. Relation (9) describes an ECM for the interest rate spread. Note that the capital gain variance in (9) is conditional on the information available at $t$. In view of (2) and (4), that is exactly what follows from economic theory - cointegration between $s_{t}$ and $h_{c, t+1}$. For simplicity, lagged differences of $h_{c, t+1}$ are not included since they turn out to be insignificant. The established reasoning when testing for cointegration in an error correction framework applies (Stock 1987): In case of cointegration, the common nonstationary factor of the two variables cancels out

[^7]so that the linear combination $z_{t} \equiv(1, \gamma / \rho)\left(s_{t}, h_{c, t+1}\right)^{\prime}$ represents a stationary time series. If so, the relation in levels should significantly contribute to the explanation of $\Delta s_{t+1}$. On the contrary, under the null $z_{t}$ is nonstationary and thus $\rho$ is zero. Note that under the null of no cointegration the ECM in (9) becomes the first difference autoregression in (7).

In contrast to usual cointegration testing in an error correction framework, the proposed test equation (9) contains a latent regressor, the IGARCH series (the capital gain variance) estimated in a preceding step. Although $h_{c, t+1}$ is a martingale, it is known that the properties of such a series deviate in many aspects from those of a random walk (see Nelson 1990). Besides, the innovations in (9) themselves are IGARCH $^{13}$, as it turned out in LR tests. To the best of our knowledge, a theory on testing for cointegration between a conditional mean and an estimated conditional variance series has not yet been developed. In order to fill this gap we derive the distribution of the test statistic (the $t$-value of $\hat{\rho}$ ) via simulation. The subsequent steps sketch the procedure:

Step 1. Set initial values $h_{c, 0}$ and $h_{s, 0}$ in (6) and (7) equal to the mean of squares of $\hat{\boldsymbol{\varepsilon}}_{c, t+1}$ and $\hat{\boldsymbol{\varepsilon}}_{s, t+1}$, respectively ${ }^{14}$.
Step 2. Draw two random samples of size $N=783$ (equal to the number of observations in the present analysis) from a standard normal distribution. These random shocks are denoted by $\xi_{c, t+1}$ and $\xi_{s, t+1}$.
Step 3. Generate data recursively according to (6) and (7) with $\boldsymbol{\varepsilon}_{c, t+1}=\xi_{c, t+1} \sqrt{h_{c, t+1}}$ and $\varepsilon_{s, t+1}=\xi_{s, t+1} \sqrt{h_{s, t+1}}$.
Step 4. Estimate model (6) via ML (BHHH algorithm) and save $\hat{h}_{c, t+1}$.
Step 5. Estimate model (9) via ML using the generated spread series from Step 3 and the estimated capital gain variance from Step 4 and save the $t$-value of $\hat{\rho}$ based on robust standard errors following Bollerslev and Wooldridge (1992).
Step 6. Repeat Step 1 to Step 5 100,000 times.

[^8]Step 7. Calculate the 1.00, 5.00 and 10.00 percentiles from the distribution of the $t$-value of $\hat{\rho}$.

## 4 Empirical Results

### 4.1 Data

The subsequent analysis is based on weekly yields from 1/03/1992 to 12/29/2006 provided by the US Federal Reserve Statistical Release. The 15 years of four US interest rate series should ensure a sufficient number of observations (783). All series are taken from the Treasury Constant Maturity data which allows to directly compare these rates. ${ }^{15}$ The sample period includes the timespan after the early 1990's recession and cuts off the ongoing financial crisis. We choose this period to reduce the probability of breaks in conditional first and second moments often leading to artificial persistence as shown, for instance, in Lamoureux and Lastrapes (1990) or Tzavalis and Wickens (1995). Excess holding returns and capital gains are calculated as, e.g., in Jones et al. (1998) or Christiansen (2000). The calculation method is also described in Ibbotson and Associates (1994). For the case of effectively infinitely-lived bonds see Engle et al. (1987). As mentioned earlier, the excess return is defined as the return on holding a longer term bond over one period in excess of the one period (riskless) spot rate. Longer term bonds have maturities of 5, 7 and 10 years. The riskless spot rate is assumed to equal the standard 3 -month Treasury rate, which is from now on referred to as the short rate. This is a common assumption (see, e.g., Bollerslev 1988, Nelson 1991, Jones et al. 1998, Christiansen 2000) and considered to be the best alternative against using a one week money market rate, which would imply, amongst other issues, discontinuities or outliers on settlement days.

As usual, mean excess holding returns increase with maturity of the long term bond ( $0.032,0.044$ and 0.055 ). The same is true for the empirical standard deviations ( $0.488,0.628$ and 0.786 ). Capital gains as part of the excess returns are denoted

[^9]by $C 5, C 7$ and $C 10$. They equal the change in the present value (price) from one week to another (see Figure 1).


Abbildung 1: Capital Gains (Losses)

Since we use weekly observations of annualized interest rates, spreads are calculated as the fraction $(1 / 52)$ of the difference between the respective long rate and the short rate, thus corresponding to a holding period of one week. Spread series are labeled S5, S7 and S10 and are depicted in annualized form in Figure 2.


Abbildung 2: Annualized Interest Rate Spreads

### 4.2 Unit Root Tests

In order to determine the integration order of interest rate spreads, the ADF test is applied. Note that as far as levels are concerned the test equation includes a constant whereas for the first differences there is no deterministic part. A linear trend would not be meaningful for interest rate spreads and is also not supported by the data. The number of lagged differences has been chosen according to the Schwarz information criterion (SIC). HECQ (1996) shows that even in the IGARCH case standard information criteria can be applied. The SIC performs best compared to the Hannan Quinn criterion (HQC) and the final prediction error (FPE). Yet, since ADF test results are known to be sensitive to the number of lagged differences in the test equation we double-checked our results by using HQC and FPE. Table 1 summarizes the unit root test results.

Tabelle 1: ADF Tests - Interest Rate Spreads

|  | Levels | First differences |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Variable | $\hat{t}$ | $q$ | $\hat{t}$ | $q$ |
| $S 5$ | $-1.651(0.456)$ | 1 | $-22.916(0.000)$ | 0 |
| $S 7$ | $-1.466(0.551)$ | 1 | $-22.430(0.000)$ | 0 |
| $S 10$ | $-1.275(0.643)$ | 1 | $-22.021(0.000)$ | 0 |

Note: Test statistics are denoted by $\hat{t} . q$ refers to the number of lagged differences and $p$-values are given in parentheses.

It can be seen that nonstationarity is far from being rejected. Thus, all three spread series should be considered as integrated of order one ${ }^{16}$. When we apply HQC and FPE, both criteria suggest including more lags but do not lead to different test decisions as test statistics do barely change. Moreover, the null of a unit root also cannot be rejected when performing Seo tests.

As the stationarity properties of interest rate spreads are crucial for the followi-

[^10]ng analysis, we also conducted the KPSS test (Bartlett kernel and Newey-West bandwidth selection) with the null hypothesis of stationarity. This is to assure that non-rejections of nonstationarity are not simply due to the possible power problem of ADF-type unit root tests. Yet, for the present data this seems not to be the case since the KPSS test clearly rejects stationarity ${ }^{17}$.

### 4.3 IGARCH Tests

In the previous section it turned out that all spreads should be considered as $I(1)$. Therefore, the EHT can only be valid in presence of nonstationary holding premia, see equation (2). ${ }^{18}$ As pointed out earlier, the unobservable premium linearly depends on the conditional variance of the capital gains. To the latter series we fit models like (6) under the restriction that $\alpha_{c}+\beta_{c}=1$. In all three cases we choose $p_{c}=1$ following the SIC. Thereafter, we test these models against the alternative of $\mathrm{AR}(1)$ processes with covariance stationary variance series, that is against AR(1)-GARCH(1,1) models. Table 2 summarizes the test results.

Tabelle 2: Testing for Integrated Variances

| Variable | $p_{c}$ | $\hat{\alpha}_{c}$ | $\hat{\beta}_{c}$ | $l\left(\hat{\boldsymbol{\theta}}_{u}\right)-l\left(\hat{\boldsymbol{\theta}}_{r}\right)$ | $\hat{k}$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C 5$ | 1 | 0.035 | 0.951 | 2.116 | 1.598 | 2.648 |
|  |  | $[0.014]$ | $[0.022]$ |  |  |  |
| $C 7$ | 1 | 0.037 | 0.948 | 1.898 | 1.406 | 2.699 |
|  |  | $[0.016]$ | $[0.025]$ |  |  |  |
| $C 10$ | 1 | 0.033 | 0.953 | 1.342 | 1.241 | 2.163 |
|  |  | $[0.015]$ | $[0.024]$ |  |  |  |

Note: $p_{c}$ refers to the lag length in (6). $\hat{\alpha}_{c}$ and $\hat{\beta}_{c}$ refer to the coefficients in the respective conditional variance equation obtained through unrestricted estimation where standard errors are given in brackets. $\hat{k}$ and $\lambda$ denote the correction term and corresponding test statistics to test the restriction $\alpha_{c}+\beta_{c}=1$.

The null of integrated variances cannot be rejected even at the $10 \%$ level since

[^11]$\chi_{0.90}^{2}(1) \approx 2.706$. This is true for all series. We additionally conducted a further experiment, where we simulated the distribution of $\lambda$ for $C 5, C 7$ and $C 10$ with conditional normal distribution and 100,000 replications. The $90 \%$ quantiles turned out to be 3.418, 3.470 and 3.448 . Thus, these results strengthened the test decision not to reject the null of IGARCH. Yet, it is well known that spurious persistence can be caused by structural breaks neglected in the model specification (Lamoureux and Lastrapes 1990). Therefore, in our robustness section below we account for possible breaks in the unconditional variance, too.

### 4.4 Mean-Variance Cointegration Tests

So far, our results from section 4.2 have shown that interest rate spreads are $I(1)$. The last section has demonstrated that the three capital gain variances are nonstationary, too. Unit roots in spreads are at odds with the usual interpretation of the expectations hypothesis. However, we have demonstrated that integrated spreads can very well be consistent with the EHT if the nonstationarity originates from integrated holding premia. If so, we have further pointed out in chapter 2 that some linear combination of the interest rate spread and the corresponding capital gain variance series should be stationary, i.e. mean-variance cointegration. Recall that the holding premium is assumed to be a multiple of the capital gain variance series. Applying the cointegration test procedure introduced in section 4, the following three ECMs have been estimated:

$$
\begin{align*}
& \Delta S 5_{t+1}=\underset{[-3.347]}{-0.0007}-\underset{[-3.967]}{0.022} S 5_{t}+\underset{[4.047]}{0.005} \hat{h}_{c, t+1}+\underset{[5.685]}{0.192 \Delta S 5_{t}+\hat{\varepsilon}_{t+1}},  \tag{10}\\
& \hat{h}_{t+1}=\underset{[3.893]}{0.029} \hat{\varepsilon}_{t}^{2}+\underset{[132.517]}{0.971} \hat{h}_{t}, \\
& \Delta S 7_{t+1}=\underset{[-2.821]}{-0.0006}-\underset{[-3.455]}{0.016} S 7_{t}+\underset{[3.430]}{0.003} \hat{h}_{c, t+1}+\underset{[6.001]}{0.216 \Delta} \Delta S 7_{t}+\hat{\varepsilon}_{t+1},  \tag{11}\\
& \hat{h}_{t+1}=\underset{[3.626]}{0.028} \hat{\varepsilon}_{t}^{2}+\underset{[126.516]}{0.972} \hat{h}_{t}, \\
& \Delta S 10_{t+1}=\underset{[-3.898]}{-0.0008}-\underset{[-4.116]}{0.017} S 10_{t}+\underset{[4.435]}{0.002} \hat{h}_{c, t+1}+\underset{[6.025]}{0.222} \Delta S 10_{t}+\hat{\varepsilon}_{t+1},  \tag{12}\\
& \hat{h}_{t+1}=\underset{[3.680]}{0.028} \hat{\varepsilon}_{t}^{2}+\underset{[128.945]}{0.972} \hat{h}_{t} .
\end{align*}
$$

Compared to the test statistics from ADF tests in section 4.2, $t$-values of the lagged level $s_{t}$ considerably increased in (10), (11) and (12). Again, the lag length is chosen according to the $\mathrm{SIC}^{19}$ and supported by specification tests for no residual autocorrelation. Furthermore, IGARCH for the ECM residuals has not been rejected. Q statistics of standardized (squared) residuals as well as LM tests for remaining GARCH show that the parsimonious $\operatorname{IGARCH}(1,1)$ specification proves to be reasonable.

We compare the $t$ statistics to simulated critical values. Some experiments made clear that these depend on the conditional variance parameters of the DGPs (6) and (7). Figure (3) illustrates the dependence on the variance process. It contains two Epanechnikov kernel density estimates ${ }^{20}$ of the distributions of the test statistics for $\alpha_{c}=0.033$ and $\alpha_{c}=0.3\left(\beta_{c}=1-\alpha_{c}\right)$ with all other parameters unchanged and equal to our estimates for $C 10$ and $\Delta S 10$ in (6) and (7). It can be seen that the increase in $\alpha_{c}$ shifts the distribution to the right. The mean (variance) changes from $-1.841(0.791)$ to -1.771 ( 0.765 ). Both distributions are slightly skewed ( 0.244 and 0.226 ) and exhibit kurtosis of 3.404 and 3.359. For $\alpha_{c} \rightarrow 0$ the distribution moves leftwards but critical values change only at the second decimal place.

Hence, we have simulated three different distributions of the test statistic in (10), (11) and (12) with parameters in (6) and (7) according to our empirical estimates. Table 3 includes individual $1 \%, 5 \%$ and $10 \%$ quantiles for each of the three models. The critical values vary just slightly as the DGPs are very similar. Test results are clear-cut: In models (10) and (12) the null of no cointegration can be rejected at the $1 \%$ level. In (11) we reject at the $5 \%$ level. This is considered as strong evidence in favor of the existence of a cointegration relation. Economically, this means that there is actually a long-run equilibrium between US interest rate spreads and the corresponding one period holding premium, as implied by EHT.

Moreover, following our discussion in section 2, the coefficient $b$ in the respective attractor can be interpreted as the market price of risk (PoR). Recall that we do not estimate the PoR via the standard GARCH-M model on the basis of excess return

[^12]

Abbildung 3: Kernel Density Estimates of the Small-Sample Distribution of the Cointegration Test Statistic
Note: The solid line shows the density of the test statistic in model (12) with $\alpha_{c}=0.033$. The dotted line describes the density for $\alpha_{c}=0.3$ with all other parameters unchanged. Changing $\alpha_{c}$ moves the $5 \%$ quantile from -3.247 to -3.149 .
data but through a cointegration relation between the nonstationary component of the excess return - the spread - and the integrated variance of the capital gain - the component of excess returns not included in the information set. Thereby, consistency of the estimator is guaranteed by the existence of cointegration. In (10), (11) and (12) we have estimated coefficients of $0.228,0.164$ and 0.127 . Compared to other findings these coefficients are relatively, but not implausibly small (see, e.g., Tzavalis and Wickens 1994, Bali and Engle 2010). For example, investors holding 7 -year Treasury bonds expect at $t$ that, on average, the excess return they will realize at $t+1$ equals about one sixth of its variance.

Tabelle 3: Critical Values - Cointegration Test

| Model | $\hat{t}$ | $1 \%$ | $5 \%$ | $10 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta S 5$ | -3.967 | -3.842 | -3.247 | -2.946 |
| $\Delta S 7$ | -3.455 | -3.838 | -3.249 | -2.939 |
| $\Delta S 10$ | -4.116 | -3.830 | -3.247 | -2.949 |

Note: $\hat{t}$ refers to the coefficient of the lagged level $s_{t}$ in the mean equation of (9).

Regarding the above ECMs, adjustment coefficients may at first glance appear quite small. If, for instance, the spread $S 10_{t}$ exceeds its equilibrium value by one unit (one percentage point), then the spread decreases one week later by 0.017 units (percentage points). However, after 13 weeks (one quarter) the initial equilibrium error has reduced to 0.771 and the half-life of the shock implied by system (12) equals just 33 weeks. Figure 4 illustrates the annualized attractor, i.e. the long-run relation $s=\frac{a}{\rho}+\frac{\gamma}{\rho} h_{c}$, in the respective ECMs.

A graphical analysis supports the statistical results: The general impression is quite a strong co-movement between the spreads and the corresponding one period holding premia. From the beginning of the sample till the New Economy boom the average level of all spreads decreases along with the volatilities. We see three noticeable peaks during that period, in 1994, between 1996 and 1997 as well as around the turn of the millennium. Whereas the first one results from long rates rising faster than the short rate, the second and third ones arise from increasing long rates when the short rate remained roughly constant. In view of equation (2), rising spreads are associated with growing holding premia and hence with rising capital gain variances. Indeed, we clearly see that the variance movement features similar peaks. However, most eye-catching is the period after 2001 when the short rate fell steeply for several years and, accordingly, spreads went up. In support of the cointegration test results, this timespan is also characterized by high volatility.


Abbildung 4: Attractor - Spread and Corresponding One Period Holding Premium

## 5 Robustness Checks

Finally, we conducted several robustness checks. Among other things we were concerned with initial values issues, with the conditional normal distribution assumption and with spurious persistence in the conditional variance due to neglected structural breaks.

As mentioned before, the choice of initial values has no impact on our simulation results. However, the shape of the estimated capital gain variance series varies
slightly - particularly during the first year (about 52 observations) when we initialize GARCH models using, for instance, backcast exponential smoothing (where $\left.h_{0}=\kappa^{N} / N \sum_{t=0}^{N} \hat{\varepsilon}_{t}^{2}+(1-\kappa) \sum_{j=0}^{N} \kappa^{N-j-1} \hat{\varepsilon}_{N-j}^{2}, 0<\kappa \leq 1\right)$ instead of simply the mean of squares of residuals. Since after about one year the initial value impact has essentially vanished, we re-estimated the ECMs starting the sample at the beginning of 1993 using 731 observations. ${ }^{21}$ Compared to the estimates (10), (11) and (12), test statistics decreased (i.e. increased in absolute value) to $-4.512,-4.476$ and -4.402. The lower number of observations affects critical values only at the second decimal place. We therefore reject the null at the $1 \%$ level in all ECMs.

Furthermore, we have investigated how the distributional assumption affects our test and simulation results. First of all, we have drawn the random samples $\boldsymbol{\xi}_{c, t+1}=$ $\varepsilon_{c, t+1} / \sqrt{h_{c, t+1}}$ and $\xi_{s, t+1}=\varepsilon_{s, t+1} / \sqrt{h_{s, t+1}}$ from Student t-distributions in both simulation experiments - the LR test for IGARCH (section 3.2) and the cointegration test (section 3.3). Degrees of freedom are set equal to estimated values under the null and lie between 8 and 18. Most of the estimated values are clearly bigger than 10 indicating that the initial assumption of normality is fairly appropriate for the present data. Since sample excess kurtosis of all series is relatively small (between 0.8 and 1.5), this is not surprising. So as to analyze the effect of possibly incorrectly specified innovations our estimates become actually QMLEs in the sense that Gaussian likelihoods are maximized even though we have generated t-innovations. As expected, the smaller the number of degrees of freedom the more the distribution of $\lambda$ shifts to the right. Hence, the test decision not to reject the null of integrated variances in section 4.3 is strengthened. Similarly, the distribution of the $t$-value of $\hat{\rho}$ also shifts to the right so that we can reject the null of no cointegration in section 4.4 at an even higher significance level (since the $t$-values are negative).

At last, we would like to stress the point that persistence in conditional second moments can be an artifact of a neglected structural change in the variance. LAmoureux and Lastrapes (1990) provide examples of that phenomenon. A specific example is also found in Tzavalis and Wickens (1995) who show that the persistence in volatility of US holding premia between 1970 and 1986 is the result

[^13]of a structural shift during a period of exceptionally high variances (October 1979 - September 1982). In general, the timing of structural breaks is quite difficult. Regarding the present data, one could suspect a level shift in the variance during the period of high volatility between the end of 2001 and the end of 2004. On $11 / 16 / 2001$, for instance, the conditional variance roughly equals its mean and increases by around $50 \%$ till next week on $11 / 23 / 2001$. After having reached its sample maximum on $11 / 08 / 2002$ the variance returns to its mean on 12/10/2004. We therefore augmented the unrestricted conditional variance equation by a dummy that takes the value 1 between $11 / 23 / 2001$ and $12 / 03 / 2004$ and 0 elsewhere. Both the persistence $\left(\alpha_{c}+\beta_{c}\right)$ and the value of the log likelihood hardly changed. This is true for all series under consideration. Besides, the dummy turns out to be insignificant in the variance equations.

In order to avoid the arbitrariness of choosing break dates exogenously, we conducted an endogenous break search. In detail, we selected the date where the shift dummy has the highest $t$-value. As shown in the unit root literature (e.g. Zivot and Andrews 1992), the additional step of estimating the break date affects the distribution of the test statistic. Therefore, we simulated the distribution allowing for a break in the GARCH constant under the alternative hypothesis. The results show that for none of the three capital gain series the null of IGARCH can be rejected at the $10 \%$ level. We also allowed for two level shifts with endogenous break dates. This improved the likelihood under the alternative hypothesis only very little so that nonstationarity was not rejected in this case, too.

## 6 Concluding Summary

The present paper empirically examines a well-known implication of the expectations hypothesis of the term structure: interest rate spreads should be stationary. This implication has been the pivotal element in many studies, especially concerning cointegration (e.g. Cuthbertson et al. 1996, Wolters 1998 and Carstensen 2003). For a system of $n$ interest rates, stationary spreads imply a cointegrating rank of $n-1$. Stationarity of spreads is also the crucial precondition when analyzing their predictive power for short rate changes or other macroecono-
mic variables like inflation and GDP growth (see, inter alia, Mankiw and Miron 1986, Kugler 1988 or Caporale and Caporale 2008). The consequences of theoretically implied stationarity properties of interest rate spreads are obviously far-reaching. We are therefore concerned with the question why they are so often not met and argue that an explanation consistent with the expectations hypothesis is provided by a nonstationary term premium.

The theoretical starting point is the one period holding premium defined as the sum of interest rate spread and expected capital gain. Given stationary capital gains, spread and holding premium must exhibit the same order of integration. We make the usual assumption that holding premia are proportional to the conditional variance of excess returns. If the spread is $I(1)$ so has to be the premium and hence the conditional variance of excess returns should be integrated. Finally, economic theory implies cointegration between spread and premium. A mean-variance cointegration test in an error correction framework is proposed and the small-sample distribution of the test statistic is derived through simulation. Our approach may be seen as a quasi IGARCH-M cointegration test as the variance that enters the mean equation is estimated in a preceding step and is driven by the squared residuals from a different mean equation.

The empirical analysis is based on weekly observations of US Treasury Constant Maturity data. We examine three different spreads between the short rate (3-month Treasury rate) and long rates with maturities of 5, 7 and 10 years. Following the ADF test results all spreads prove to be nonstationary. Further tests unanimously confirm nonstationarity of the spreads, what is a common result in the empirical literature. Subsequently, estimating conditional variances of excess returns, it turns out that the null hypothesis of IGARCH cannot be rejected. Additionally, this result holds when incorporating endogenous structural breaks. Hence, we conclude that premia are also integrated. The last step is to test for cointegration between premia and spreads. As the main result of the present work, we actually find highly significant long-run relations between all spreads and corresponding premia (respectively, capital gain variances).

Results of the present investigation suggest that the expectations hypothesis of the term structure possibly performs better than previously thought. Two extensions
might be interesting: First, our approach could be generalized to non-diversifiable risk (e.g. Bollerslev et al. 1988 or Balfoussia and Wickens 2007): The frequent failure of EHT would be explained by integrated covariance series that could be obtained from multivariate GARCH models (see BAUwEns et al. 2006 for a survey). Second, since appropriate modeling of the persistence of the premium has proven to be crucial, a further possible extension would be to allow for fractional integration in interest rates (Connolly et al. 2007) and conditional variances (Baillie et al. 1996). If, for example, the order of integration of spreads and premia is equal but appears to be less than one, cointegration tests may be carried out in a fractionally integrated framework.

## Literatur

[1] Ang, A. and M. Piazzesi. A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables. Journal of Monetary Economics, 50(4):745-787, 2003.
[2] Baillie, R.T., T. Bollerslev, and H.O. Mikkelsen. Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 74(1):3-30, 1996.
[3] Balfoussia, H. and M.R. Wickens. Macroeconomic Sources of Risk in the Term Structure. Journal of Money, Credit and Banking, 39(1):205-236, 2007.
[4] Bali, T.G. and R.F. Engle. The Intertemporal Capital Asset Pricing Model with Dynamic Conditional Correlations. Journal of Monetary Economics, 57(4):377-390, 2010.
[5] Bansal, R. and C. Lundblad. Market Efficiency, Asset Returns, and the Size of the Risk Premium in Global Equity Markets. Journal of Econometrics, 109(2):195-237, 2002.
[6] Bauwens, L., S. Laurent, and J.V.K. Rombouts. Multivariate Garch Models: A Survey. Journal of Applied Econometrics, 21(1):79-109, 2006.
[7] Bollerslev, T. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31(3):307-327, 1986.
[8] Bollerslev, T. and J. Wooldridge. Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances. Econometric Reviews, 11(2):143-172, 1992.
[9] Bollerslev, T., R.F. Engle, and J.M. Wooldridge. A Capital Asset Pricing Model with Time-Varying Covariances. Journal of Political Economy, 96(1):116-131, 1988.
[10] Busch, T. A Robust LR Test for the GARCH Model. Economics Letters, 88(3):358-364, 2005.
[11] Campbell, J.Y. and R.J. Shiller. Cointegration and Tests of Present Value Models. Journal of Political Economy, 95(5):1062-1088, 1987.
[12] Campbell, J.Y. and R.J. Shiller. Yield Spreads and Interest Rate Movements: A Bird's Eye View. Review of Economic Studies, 58(3):495-514, 1991.
[13] Caporale, B. and T. Caporale. Political Risk and the Expectations Hypothesis. Economics Letters, 100(2):178-180, 2008.
[14] Carstensen, K. Nonstationary Term Premia and Cointegration of the Term Structure. Economics Letters, 80(3):409-413, 2003.
[15] Cavaliere, G. and A.M.R. Taylor. Testing for Unit Roots in Time Series Models with Non-Stationary Volatility. Journal of Econometrics, 140(2):919947, 2007.
[16] Charles, A. and O. Darné. A Note on Unit Root Tests and GARCH Errors: A Simulation Experiment. Communications in Statistics - Simulation and Computation, 37(2):314-319, 2008.
[17] Chou, R.Y. Volatility Persistence and Stock Valuations: Some Empirical Evidence Using GARCH. Journal of Applied Econometrics, 3(4):279-294, 1988.
[18] Christiansen, C. Macroeconomic Announcement Effects on the Covariance Structure of Government Bond Returns. Journal of Empirical Finance, 7(5):479-507, 2000.
[19] Connolly, R.A., Z.N. Güner, and K.N. Hightower. Evidence on the Extent and Potential Sources of Long Memory in U.S. Treasury Security Returns and Yields. Journal of Money, Credit and Banking, 39(2-3):689-702, 2007.
[20] Cook, S. Maximum Likelihood Unit Root Testing in the Presence of GARCH: A New Test with Increased Power. Communications in Statistics Simulation and Computation, 37(4):756-765, 2008a.
[21] Cook, S. Joint Maximum Likelihood Estimation of Unit Root Testing Equations and GARCH Processes: Some Finite-Sample Issues. Mathematics and Computers in Simulation, 77(1):109-116, 2008b.
[22] Cuthbertson, K., S. Hayes, and D. Nitzsche. The Behaviour of Certificate of Deposit Rates in the UK. Oxford Economic Papers, 48(3):397-414, 1996.
[23] Dai, Q. and K.J. Singleton. Specification Analysis of Affine Term Structure Models. The Journal of Finance, 55(5):1943-1978, 2000.
[24] Dickey, D.A. and W.A. Fuller. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. Journal of the American Statistical Association, 74(366):427-431, 1979.
[25] Duffee, G.R. Term Premia and Interest Rate Forecasts in Affine Models. The Journal of Finance, 57(1):405-443, 2002.
[26] Engle, R.F. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50(4):987-1007, 1982.
[27] Engle, R.F. and T. Bollerslev. Modelling the Persistence of Conditional Variances. Econometric Reviews, 5(1):1-50, 1986.
[28] Engle, R.F., D.M. Lilien, and R.P. Robins. Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model. Econometrica, 55(2):391-407, 1987.
[29] Engle, R.F., V.K. Ng, and M. Rothschild. Asset Pricing with a FactorARCH Covariance Structure: Empirical Estimates for Treasury Bills. Journal of Econometrics, 45(1-2):213-237, 1990.
[30] Evans, M.D.D. and K.K. Lewis. Do Stationary Risk Premia Explain It All?: Evidence from the Term Structure. Journal of Monetary Economics, 33(2):285-318, 1994.
[31] Fang, W. and S.M. Miller. Modeling the Volatility of Real GDP Growth: The Case of Japan Revisited. Japan and the World Economy, 21(3):312-324, 2009.
[32] Ghysels, E., P. Santa-Clara, and R. Valkanov. There is a Risk-Return Trade-Off After All. Journal of Financial Economics, 76(3):509-548, 2005.
[33] Haldrup, N. Heteroscedasticity in Non-Stationary Time Series, Some Monte Carlo Evidence. Statistical Papers, 35(1):287-307, 1994.
[34] Hall, A.D., H.M. Anderson, and C.W.J. Granger. A Cointegration Analysis of Treasury Bill Yields. The Review of Economics and Statistics, 74(1):116-126, 1992.
[35] Hecq, A. IGARCH Effect on Autoregressive Lag Length Selection and Causality Tests. Applied Economics Letters, 3(5):317-23, 1996.
[36] Ibbotson and Associates. Stocks, Bonds, Bills, and Inflation Yearbook. Ibbotson Associates, Chicago, 1994.
[37] Jones, C.M., O. Lamont, and R.L. Lumsdaine. Macroeconomic News and Bond Market Volatility. Journal of Financial Economics, 47(3):315-337, 1998.
[38] Kim, K. and P. Schmidt. Unit Root Tests with Conditional Heteroskedasticity. Journal of Econometrics, 59(3):287-300, 1993.
[39] Kourogenis, N. and N. Pittis. Testing for a Unit Root Under Errors with Just Barely Infinite Variance. Journal of Time Series Analysis, 29(6):10661087, 2008.
[40] Kugler, P. An Empirical Note on the Term Structure and Interest Rate Stabilization Policies. The Quarterly Journal of Economics, 103(4):789-792, 1988.
[41] Kugler, P. The Term Premium, Time Varying Interest Rate Volatility and Central Bank Policy Reaction. Economics Letters, 76(3):311-316, 2002.
[42] Lamoureux, C.G. and W.D. Lastrapes. Persistence in Variance, Structural Change, and the GARCH Model. Journal of Business \& Economic Statistics, 8(2):225-234, 1990.
[43] Li, W.K., S. Ling, and H. Wong. Estimation for Partially Nonstationary Multivariate Autoregressive Models with Conditional Heteroscedasticity. Biometrika, 88(4):1135-1152, 2001.
[44] Ling, S. and W.K. Li. Limiting Distributions of Maximum Likelihood Estimators for Unstable Autoregressive Moving-Average Time Series with General Autoregressive Heteroscedastic Errors. The Annals of Statistics, 26(1):84-125, 1998.
[45] Ling, S. and W.K. Li. Asymptotic Inference For Unit Root Processes With Garch(1,1) Errors. Econometric Theory, 19(04):541-564, 2003.
[46] W. K. Li Ling, S. and M. McAleer. Estimation and Testing for Unit Root Processes with GARCH $(1,1)$ Errors: Theory and Monte Carlo Evidence. Econometric Reviews, 22(2):179-202, 2003.
[47] Lintner, J. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. The Review of Economics and Statistics, 47(1):13-37, 1965.
[48] Lumsdaine, R.L. Finite-Sample Properties of the Maximum Likelihood Estimator in $\operatorname{GARCH}(1,1)$ and $\operatorname{IGARCH}(1,1)$ Models: A Monte Carlo Investigation. Journal of Business \& Economic Statistics, 13(1):1-10, 1995.
[49] Lundblad, C. The Risk Return Tradeoff in the Long Run: 1836-2003. Journal of Financial Economics, 85(1):123-150, 2007.
[50] Mankiw, N.G. and J.A. Miron. The Changing Behavior of the Term Structure of Interest Rates. The Quarterly Journal of Economics, 101(2):211228, 1986.
[51] Mankiw, N.G. and L.H. Summers. Do Long-Term Interest Rates Overreact to Short-Term Interest Rates? Brookings Papers on Economic Activity, 15(1):223-248, 1984.
[52] Melino, A. The Term Structure of Interest Rates: Evidence and Theory. Journal of Economic Surveys, 2(4):335-366, 1988.
[53] Nelson, D.B. Stationarity and Persistence in the GARCH(1,1) Model. Econometric Theory, 6(3):318-334, 1990.
[54] Nelson, D.B. Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59(2):347-370, 1991.
[55] Seo, B. Distribution Theory for Unit Root Tests with Conditional Heteroskedasticity. Journal of Econometrics, 91(1):113-144, 1999.
[56] Seo, B. Asymptotic Distribution of the Cointegrating Vector Estimator in Error Correction Models with Conditional Heteroskedasticity. Journal of Econometrics, 137(1):68-111, 2007.
[57] Sharpe, W.F. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. The Journal of Finance, 19(3):425-442, 1964.
[58] Shea, G.S. Benchmarking the Expectations Hypothesis of the Interest-Rate Term Structure: An Analysis of Cointegration Vectors. Journal of Business E Economic Statistics, 10(3):347-366, 1992.
[59] Shiller, R.J. The Term Structure of Interest Rates. In B.M. Friedman and F.H. Hahn, editors, Handbook of Monetary Economics, pages 627-722. North-Holland, Amsterdam, 1990.
[60] Silverman, B.W. Density Estimation for Statistics and Data Analysis. Chapman and Hall, London, UK, 1986.
[61] Stock, J.H. Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors. Econometrica, 55(5):1035-1056, 1987.
[62] Tzavalis, E. and M.R. Wickens. The Persistence in Volatility of the US Term Premium 1970-1986. Economics Letters, 49(4):381-389, 1995.
[63] Weber, E. and Wolters, J. The US Term Structure and Central Bank Policy. Working paper 436, University of Regensburg, 2009.
[64] Wolters, J. On the Term Structure of Interest Rates - Empirical Results for Germany. Statistical Papers, 36(1):193-214, 1995.
[65] Wolters, J. Cointegration and German Bond Yields. Applied Economics Letters, 5(8):497-502, 1998.
[66] Wong, H., W.K. Li, and S. Ling. Joint Modeling of Cointegration and Conditional Heteroscedasticity with Applications. Annals of the Institute of Statistical Mathematics, 57(1):83-103, 2005.
[67] Zhang, H. Treasury Yield Curves and Cointegration. Applied Economics, 25(3):361-67, 1993.
[68] Zivot, E. and W.K. Andrews. Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. Journal of Business $\mathcal{E}$ Economic Statistics, 20(1):25-44, 1992.


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[^2]:    ${ }^{4}$ Common examples for that approach are Engle et al. (1987) and Bollerslev et al. (1988).

[^3]:    ${ }^{5}$ Note that yield data are used in the subsequent analysis.
    ${ }^{6}$ The literature sometimes confusingly uses "term premium" as an umbrella term for forward, holding and rollover premium. Since for the following work it is important to use exact definitions we apply those from the notes of Shiller (1990).

[^4]:    ${ }^{7}$ Theoretically, $c_{t}$ can be considered as a series of price changes. Since prices normally behave like random walks or slightly more general $I(1)$ processes, agents would expect $c_{t+1}$ to be $I(0)$. Indeed, as will be seen later, this property of capital gains is found in the data
    ${ }^{8}$ Note that throughout the present work the term stationarity always refers to covariance stationarity.

[^5]:    ${ }^{9}$ It is also common to use the square-root or the logarithm of the conditional variance.

[^6]:    ${ }^{10}$ That is for a sum of the ARCH and GARCH coefficients between 0.8 and 1 and for a GARCH parameter larger than the ARCH parameter.
    ${ }^{11}$ Cavaliere and Taylor (2007) discuss a general class of nonstationary variances.

[^7]:    ${ }^{12}$ For further details see BUSCH (2005).

[^8]:    ${ }^{13}$ For a discussion of potential consequences of GARCH effects on standard cointegration tests see, e.g., Li et al. (2001), Wong et al. (2005) and Seo (2007).
    ${ }^{14}$ Dependence on the initial values turned out to be negligible. The initial values of $s_{t}, t=$ $0, \ldots, p_{s}$ are arbitrarily chosen in the sense that they are realizations of two standard normally distributed random variables.

[^9]:    ${ }^{15}$ All yields represent bond equivalent yields for securities that pay semiannual interest.

[^10]:    ${ }^{16}$ The integration order of the interest rate series has been checked, too. According to ADF test results there is very strong evidence that all interest rate series can be considered as integrated of order one.

[^11]:    ${ }^{17}$ All test results being not reported can be obtained upon request
    ${ }^{18}$ All capital gain series proved to be clearly $I(0)$ with ADF test statistics of $-21.727,-13.363$ and -13.290 for $C 5, C 7$ and $C 10$.

[^12]:    ${ }^{19}$ Using different information criteria, all suggesting more lags, does not change cointegration test results.
    ${ }^{20}$ We use data-based bandwidth selection according to Silverman (1986).

[^13]:    ${ }^{21}$ Persistence in variances proves not to be affected by initial conditions.

