The Price Risk of Options Positions: Measurement and Capital Requirements

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Global markets for option products, both exchange-traded and over-the-counter, have expanded rapidly in recent years. The Bank for International Settlements (1994) reports that outstanding options now exist on notional principal amounts totaling at least $3.3 trillion. In the last four years, the outstanding interest rate, commodity, and equity-related options of U.S commercial banks have grown more than 40 percent annually and the banks' foreign exchange options more than 16 percent per year

The expansion of options markets underscores the need for supervisors to develop sound methods of monitoring the risks associated with these markets. This article assesses different supervisory approaches to the measurement and capital treatment of the market risk of options—the risk that an options contract will decline in value with changes in market prices or rates. The methods for measuring the market risk of options positions examined in the article fall into two broad categories. simple strategy methods and value-at-risk (or price sensitivity) methods. We compare the performance of the different methods in providing capital coverage for potential losses on a series of option portfolios.

We find that the simple strategy methods provide only a rough measure of potential losses Moreover, for market participants with large option positions, the simple strategy approach could lead to an excessive reporting burden. The value-at-risk methods, which are based on option pricing models, tend to provide better estimates of the market risk inherent in a position. The accuracy of the value-at-risk approach is also found to be significantly enhanced by adjustments for gamma risk, the risk that an option's price changes in a nonlinear fashion as a result of large movements in the price of the underlying instrument.[1]

Background and Methodology

The Unique Risks of Options

Like most other instruments, options contracts entail both price (or market) risk and credit risk Market risk arises when the value of the intermediary's portfolio is sensitive to changes in market prices or rates. On some occasions, intermediaries will attempt to eliminate such risk by engaging in offsetting transactions, that is, they attempt to "hedge" the market risk away On other occasions, however, intermediaries may attempt to earn a risk premium for bearing the market risk. Credit risk arises because a financial asset, such as a purchased option or a business loan, could become worthless if the counterparty to the asset does not make good on its obligations. This article focuses on market risk.

The form that market risk takes in options markets can be quite different from its form in other markets This is true in part because the values of options contracts can change extremely rapidly—often far more rapidly as a percentage of their value than do the assets that underlie options contracts In addition, the price sensitivity and volatility of a position can themselves change quickly, further complicating the risk management of options positions. An intermediary must constantly track the changes in the volatility of its portfolio that result from market movements. This task can be particularly difficult in periods of great market stress and lowered liquidity, such as that experienced in the market for European currency options in September 1992

Alert to these difficulties, supervisors must consider carefully how supervisory capital can best reduce the detrimental impact of options risks in the financial markets

Considerations in Setting Supervisory Capital Requirements

Determining appropriate supervisory capital standards for options positions involves several choices A sufficient level of prudence could probably be achieved by simply setting very high standards without regard to how the risks change in response to changing market conditions This approach could, however, require far more capital than is actually needed. The costs of this excessive safety would then translate into a slowdown in potentially beneficial options trading or perhaps a relocation of this trading to jurisdictions not imposing such onerous standards

More accurate measures of the risks of an options portfolio require more information about the composition of the portfolio and more calculations. As the analysis below makes clear, there is a definite trade-off between the efficiency of the capital charge and the resources required to compute the charge. The most efficient charges would, for example, require substantial data about the composition of the options portfolio and the risk factors affecting the portfolio's value, as well as estimates of the sensitivities of the portfolio's value to movements in the risk factors. To process such information in a timely fashion, an institution must be willing to commit significant resources.

The complexity of the capital calculation itself is also an important consideration. Complex supervisory charges may be difficult to implement uniformly Moreover, the more specific the rule, the greater the opportunities for finding exceptions or exclusions that were not intended Nevertheless, complex calculations could lead to more accurate and efficient charges, thereby lessening the regulatory burden on the options market.

A final consideration that is particularly important in setting supervisory capital for options is the use of option pricing models in the calculation. The development of these models has been critical to the growth of the options market, and

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their use in the market is pervasive However, the markets do not always conform to the assumptions of the models In particular, options pricing models typically assume continuous price changes and liquid markets. The risk of systemic problems, however, is very likely to go hand-in-hand with a loss of market liquidity and discontinuous price movements Therefore, in setting supervisory capital requirements, regulators may not wish to rely solely on pricing models.

Current and Proposed Supervisory Approaches

At the international level, there are a number of different existing and proposed supervisory capital treatments for the market risk of options The European Community's Capital Adequacy Directive (CAD), which takes effect in January

1996, includes a simple capital requirement for options that applies to all commercial banks and securities firms licensed in the Community The Basle Committee on Banking Supervision's April 1993 market risk proposal (BIS proposal) includes capital requirements for options similar to those of the CAD Unlike the CAD, however, the BIS proposal also considers allowing banks to apply a more sophisticated scenario-based methodology to calculate their capital requirement for options positions.

The rules of both the United Kingdom's Securities and Futures Authority (SFA) and the U.S Securities and Exchange Commission (SEC) already include capital requirements for the market risk of options. The SFA's requirements range from simple rules for small options players to a scenario-based approach for more sophisticated institutions. The SEC's capital requirements are based on a series of options trading strategies that are commonly employed by financial institutions.

Methodology of Evaluation

Conceptually, measuring the market risk of options for capital purposes consists of two basic steps. The first entails making assumptions about the potential movement of risk factors that could affect the value of an options portfolio over a defined holding period and at a certain level of confidence. This step may also account for correlations among the different risk factors The second step consists of measuring the sensitivity of options positions to the assumed movements in the underlying risk factors to arrive at an estimate of the portfolio's potential gain or loss. Each step can be carried out with various degrees of sophistication. This article focuses primarily on the second step, examining the performance of a number of capital rules in approximating potential portfolio losses resulting from given movements in the underlying risk factors.

The following section describes various methods of setting regulatory capital and the rationale that underlies each. Although in several cases the methods overlap with those of specific regulatory entities, our intent is not to assess the particular rules of different regulatory bodies, but to evaluate the generic approaches that might be taken towards regulatory capital [2]

Capital Rules for Options

Capital rules for the price risk of options fall into two categories: value-at-risk (or price sensitivity) rules and strategy-based rules The value-at-risk approach uses option pricing models to estimate a portfolio's potential losses. The strategy-based rules, by contrast, apply various formulas to the market price of an option and its underlying asset

While strategy-based rules estimate potential losses very roughly, the option pricing models of the value-at-risk approach in principle provide a much more accurate measure of potential changes in the value of options. Strategy-based rules have the additional drawback of requiring a complex parsing of a portfolio to rearrange its components into the particular set of strategies recognized for capital purposes

The value-at-risk approach uses a simple aggregation of all positions relating to a given underlying asset (for example, a particular equity or exchange rate). This aggregation of positions of the value-at-risk approach more reliably accounts for offsetting within a portfolio than does the strategy-based approach, which recognizes offsetting in only a piecemeal fashion—and only to the extent that positions fit into the trading strategies recognized by supervisors. Note, however, that this conclusion applies only to aggregation across instruments sensitive to a given underlying asset The appropriate approach to aggregation of positions across multiple underlyings is a separate problem.

Value-at-Risk Approaches

A portfolio's value at risk is a measure of its potential losses, where the losses are expressed in terms of some confidence level (for example, a loss of such magnitude that it is likely to occur in only one month out of a hundred) This risk measurement approach lends itself quite readily to the construction of a capital requirement. Specifically, the potential losses a portfolio might suffer are estimated and then used to determine an amount of capital sufficient to cover these losses with the desired level of confidence.

Scenario-based and Simulation Methods with Full Revaluation

The most precise method for estimating value at risk entails calculating a portfolio's gains and losses by using option pricing models to explicitly revalue the portfolio over a set of postulated price changes. The resulting capital charge is simply the largest loss over the set of postulated price changes, measured at a certain level of confidence

The postulated price changes can be obtained using either a scenario approach or a simulation method The scenario approach revalues the portfolio at several distinct values of the underlying asset within a given interval—for example, within the interval defined by the current underlying price plus or minus three standard deviations of monthly moves. Alternatively, changes in the price of the underlying asset may be simulated, using either historical price changes or Monte Carlo methods In either type of simulation, the entire portfolio is revalued at each point generated by the simulation. Either the largest loss or some conservative percentile of the losses, depending on the desired degree of confidence, can then be selected as the value at risk. The simulation approaches allow for sampling of portfolio value changes over a more continuous range of price changes in the underlying asset than does the scenario approach, which focuses on a more limited number of specific price movements

The scenario approach is included in the SFA capital rules as a preferred alternative. It is also similar to the SPAN system of margin requirements used by a number of derivatives exchanges

Methods Based on Price Sensitivities—First- and Second-Order Approximations

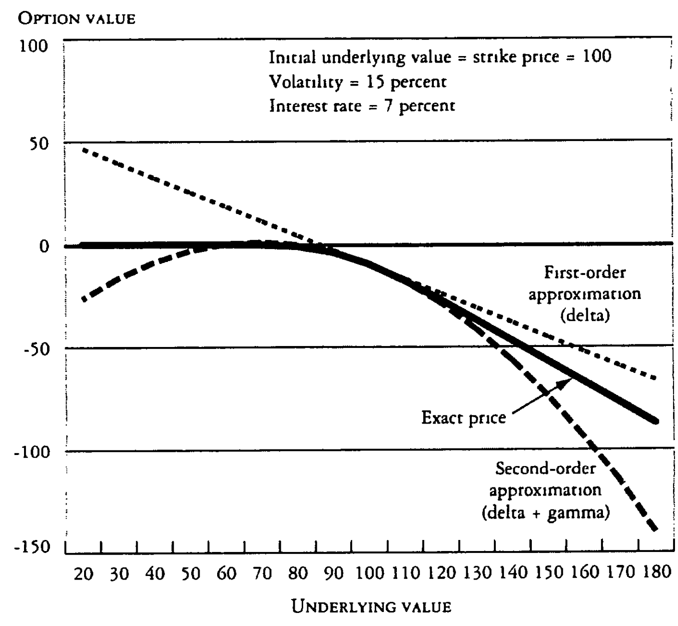
Another class of value-at-risk rules relies on a simple approximation of an option's price sensitivity to changes in the price of the underlying asset (see Appendix I) All of these rules use various combinations of an option's basic price sensitivity measures—the option's delta and gamma values An option's delta value is the change in the price of an option resulting from a small change in the price of the underlying asset. An option's gamma value measures the change in delta with respect to movements in the price of the underlying asset. The gamma value can be used not only to improve upon the approximation obtained from the delta value but also to evaluate the quality of hedges—a delta-hedged options position with a large negative gamma is vulnerable to large changes in the price of the underlying asset. The delta-equivalent rule, the principal approach of the European Community's CAD, is based only on the option's delta value, while other rules use both the delta and gamma values Chart 1 illustrates the results of using first- and second-order approximations in the case of a written call option

Note that the second-order approximation is not necessarily more accurate than the first-order approximation, particularly if the changes in the underlying value are large Thus, for applications that rely on accurate measures of portfolio changes resulting from large market movements (for example, "stress testing"), models involving approximations should be used with caution.

An advantage of the price sensitivity approach is its simplicity. It requires only the most basic options-related data used by standard risk management systems. the standard deviation (volatility) of the underlying asset, the portfolio's delta value, and the portfolio's gamma value. Additionally, a convenient feature of deltas and gammas is their additivity across instruments written on the same underlying asset, allowing offsetting exposures within a portfolio to be properly netted and reflected in the capital charge.

Chart 1

First- and Second-Order Approximations of a Written Call Option



Nevertheless, price sensitivity approximations, even those that incorporate gamma values, may underestimate potential losses because the price sensitivity at current prices may not be representative of an option's behavior at other prices. Option prices do not move in a linear fashion. Some options portfolios can have a very small gamma at the current price of the underlying asset but possess a much more negative gamma at different prices (for example, a written out-of-the money put). In this case, price sensitivity approaches can underestimate the fall in the portfolio's value resulting from a large decline in the price of the underlying asset.

Both the scenario and simulation approaches can also be combined with price sensitivity approximations. In this case, the portfolio is not explicitly revalued at each simulated price change, but instead the changes in portfolio value are approximated for each simulated price change Although the approximations are based on option pricing models, their use will inevitably lead to a loss of accuracy relative to the explicit use of the models themselves.

INCORPORATION OF VOLATILITY RISK

The discussion to this point has addressed only potential losses resulting from changes in the price of the underlying asset. Unlike other instruments, however, options are also exposed to changes in price volatility. A capital adjustment for volatility risk can be incorporated in the price sensitivity approximation approach and the simulation or scenario-based methods.

STRATEGY-BASED RULES

The capital requirement under strategy-based rules is derived from a series of defined options trading strategies commonly employed by financial institutions. The strategy-based approach may recognize offsetting for certain types of trades, but in general, it does not easily accommodate the netting of opposite positions Since strategy-based rules are collections of formulas that apply to various specific positions, it is impossible to define a generic form for these rules Thus, to test this type of rule, it is convenient to use its implementation by a particular regulatory authority, the SEC.

The SEC's Net Capital Rule (15(c)3-1) for options contains two strategy-based rules: Appendix A, which applies to securities firms holding options for proprietary trading, and Paragraph (c)(2)(x), which applies to market-making firms. Both rules apply a lower capital charge for certain offsetting options strategies, but Paragraph (c)(2)(x) recognizes more offsetting strategies than does Appendix A.[3]

The SEC has released for public comment (Federal Register 1994) a proposal that would permit broker-dealers to calculate their capital requirements for listed options using a scenario-based approach similar to the one described earlier in this section.[4] According to the SEC's proposal, "haircuts for options and related positions, when computed using this model, would more accurately reflect the risk inherent in broker-dealers' option positions." SEC requirements for over-the-counter options would continue to be calculated using the strategy-based rules outlined in this paper.

The strategy-based capital rules do not closely parallel most firms' risk control and/or trading systems because the capital charges may not reflect portfolio-wide risks. For instance, the risk of a delta-hedged short call strategy is equal to the risk of a delta-hedged covered call strategy with the same set of parameters. However, the capital charges could be different depending on the strategy chosen to compute them

QUANTITATIVE ANALYSIS OF PRICE RISK RULES

This section assesses the performance of the two classes of rules discussed in the previous section. Under the value-at-risk approach, we consider three different methods for constructing a capital charge: (1) the delta-equivalent rule, which bases the capital charge on the delta-equivalent amount (that is, delta multiplied by the value of the underlying position); (2) the Taylor series rule, which supplements the delta-equivalent rule with an adjustment for gamma (positive or negative), and (3) the gamma rule, which supplements the delta-equivalent rule with an adjustment for negative gamma. Under the strategy-based approach, we consider the SEC's Appendix A rule and C2X rule

We evaluate the accuracy with which these rules measure the price risk of a variety of options positions and the degree to which they provide adequate capital levels for the portfolios' potential losses. Broadly, the analysis performed in this exercise reveals that the approximate value-at-risk rules tend to estimate portfolio risk more accurately and thus produce more efficient levels of capital than the simple strategy-based rules. Within the value-at-risk category, the gamma and Taylor series rules provide a more accurate measure of potential losses than does the delta-equivalent rule, highlighting the importance of an adjustment for gamma risk

METHODOLOGY

The riskiness of an options portfolio is measured as the maximum potential loss that would occur over a range of changes in the underlying asset price. Using a scenario-based methodology, we calculate gains and losses by revaluing each portfolio over a set of postulated price and volatility changes to find the largest loss (price and volatility assumptions are listed in Table 1) Specifically, the underlying price is varied within a

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range of ±3 standard deviations of one-month price changes in discrete increments of 5 percent of the initial price. The volatility of the underlying price is varied within a range of ±5 percentage points of the initial volatility in increments of one percentage point.[5] The parameters used in this analysis provide a consistent benchmark for comparing the different capital rules Although the parameters are intended to be realistic, they are not intended to represent a prescribed absolute level of confidence.

We measure potential losses for a set of options portfolios with 30 days to maturity and for otherwise equivalent portfolios with 180 days to maturity Analyzing the effect of an option's time to maturity on the accuracy of a rule is important because of its influence on an option's time value and gamma. Gamma is substantially larger for options nearing expiration—especially around the money—than for options with longer time to maturity. The effect of gamma risk is generally greatest for portfolios consisting of a net short position, where gamma is negative.

Each sample set contains thirty-five portfolios constructed from different combinations of common options positions. Among the thirty-five portfolios are several that correspond to strategies recognized directly by the SEC rules. Other portfolios are then added by building on the SEC strategies Portfolios include naked positions, simple strategies, and various combinations of SEC-recognized strategies. Delta-hedged portfolios are also included to generate results that more closely parallel gains and losses from realistic trading practices. The composition of each portfolio and the parameters used to compute options prices and risks are listed in Table 1.[6] Portfolio sizes are normalized to ensure that differences in potential losses across portfolios result from the portfolios' inherent riskiness rather than their size.[7]

COMPARISON OF CAPITAL RULES

EFFICIENCY OF CAPITAL RULES

To evaluate the efficiency of individual rules, we perform a linear regression of the capital charge produced by a given rule on the largest loss as measured by the scenario approach discussed in the previous section. There are thus thirty-five data points in each regression, corresponding to the thirty-five portfolios considered. The R^2 coefficient, which is one measure of correlation between two variables, reflects the strength of the linear relationship between portfolio losses and capital requirements Rules with higher R^2 coefficients tend to assign capital levels that are more closely correlated to potential losses R^2 coefficients for the rules examined are summarized in Table 2.

The Taylor series and gamma charge rules display a stronger relationship between capital charges and losses than does the delta-equivalent rule. The reason is that the capital charge of the delta-equivalent rule is based only on a portfolio's delta value at a given point in time and therefore does not capture the nonlinearity of an option's price changes. In contrast, both the Taylor series and gamma charge rules approximate the option's nonlinear price behavior by reflecting the gamma values in the capital computation.

Overall, both strategy-based rules—Appendix A and C2X—exhibit significantly lower R^2 coefficients than the value-at-risk rules, suggesting that capital charges de-

Table 1
OPTIONS PORTFOLIOS

	Portfolio	Position	Strike Price
* 1	Short call–in the money	Short 1 call	90
* 2	Short call–out of the money	Short 1 call	110
* 3	Short call–at the money	Short 1 call	100
4	Delta-hedged/short call–in the money	Portfolio #1, delta-hedged	
5	Delta-hedged/short call–out of the money	Portfolio #2, delta-hedged	
6	Delta-hedged/short call–at the money	Portfolio #3, delta-hedged	
* 7	Covered short call–in the money	Long 1 underlying Short 1 call	 90
* 8	Covered short call–out of the money	Long 1 underlying Short 1 call	 110
* 9	Covered short call–at the money	Long 1 underlying Short 1 call	 100
10	Synthetic long futures with split strike Strikes of put and call are different	Long 1 call Short 1 put	110 90
* 11	Bull call spread Strike of short is higher than strike of long	Long 1 call Short 1 call	95 115
12	Delta-hedged/bull call spread	Portfolio #11, delta-hedged	
* 13	Bear call spread Strike of short is lower than strike of long	Long 1 call Short 1 call	115 95
14	Delta-hedged/bear call spread	Portfolio #13, delta-hedged	
* 15	Bear call spread/symmetric to #15 Symmetrically in the money and out of the money, as in #15	Long 1 call Short 1 call	105 85
16	Delta-hedged/bear call/symmetric	Portfolio #15, delta-hedged	
* 17	Short straddle	Short 1 call	100
18	Delta-hedged/short straddle	Portfolio #17, delta-hedged	
* 19	Protected call–in the money (synthetic put)	Short 1 Underlying Long 1 call	 90
* 20	Protected call–out of the money (synthetic put)	Short 1 Underlying Long 1 call	 110
* 21	Protected call–at the money (synthetic put)	Short 1 Underlying Long 1 call	 100
22	Delta-hedged/protected call–in the money	Portfolio #19, delta-hedged	
23	Delta-hedged/protected call–out of the money	Portfolio #20, delta-hedged	
24	Delta-hedged/protected call–at the money	Portfolio #21, delta-hedged	
* 25	Short put–in the money	Short 1 put	110
* 26	Short put–out of the money	Short 1 put	90
* 27	Short put–at the money	Short 1 put	100
* 28	Long put–in the money	Long 1 put	110
* 29	Long put–out of the money	Long 1 put	90
* 30	Long put–at the money	Long 1 put	100
31	Ratio call spread 2 1 ratio Long a call with a lower strike and short 2 otherwise identical calls with a higher strike	Long 1 call Short 2 calls	100 110
32	Delta-hedged/ ratio call	Portfolio #31 delta-hedged	
33	Ratio call backspread 2 1 ratio Short a call with a lower strike and long 2 otherwise identical calls with a higher strike	Long 2 calls Short 1 call	100 90
34	Delta-hedged/ratio call backspread	Portfolio #33 delta-hedged	
35	Vertical box Combination of a bull spread and a bear spread Long call at higher strike and short call at lower strike Long put at higher strike and short put at lower strike	Long 1 call Short 1 call Long 1 put Short 1 put	115 95 115 95

Note The parameters used in the portfolio simulation program are as follows
Underlying price = 100
Annual dividend rate = 0 percent
Annual volatility = 30 percent
Interest rate = 3 5 percent
Time to expiration = 180 days (set 1)
30 days (set 2)

*Option strategies that are officially recognized under the SEC's Appendix A and C2X capital rules rived under the simple strategy-based approach are not closely correlated with the potential losses of options portfolios The low R^2 could be attributed to the fact that strategy-based rules provide only a rough measure of the portfolios' risk, largely because they do not account for internal risk management practices or changing market conditions.

One notable exception to this overall finding occurs when options are nearer to maturity (30-day series) In this case the correlation between capital charges and potential losses is stronger under Appendix A than under the delta-equivalent rule The consistently high capital charge of the Appendix A rule appears to mesh well with the high gamma risk often associated with short-dated options[8] In contrast, the delta-equivalent rule performs less efficiently in this case because it does not capture gamma risk

Coverage of Capital Rules

Capital rules should provide not only a high correlation between capital requirements and losses but also adequate coverage of portfolio losses One way to evaluate coverage is to compare the slope coefficients of regression lines produced by each rule A slope coefficient gives the marginal amount of capital that corresponds to a $1 difference in the risk exposure of a portfolio For example, a slope coefficient of 1 indicates that an incremental loss of $1 is associated with $1 more of capital The slope coefficients are also summarized in Table 2

The differences between slope coefficients for the Taylor series and gamma charge rules are slight In contrast, the delta-equivalent rule produces materially lower slope coefficients than do the gamma and Taylor series rules The Appendix A slope coefficient is significantly above one for the 30-day options, and below one (although not significantly so) for 180-day options The C2X coefficients are significantly lower than those of the other rules and are closer to zero than to one.

The capital coverage of the various rules is captured graphically in Charts 2 through 6 The "45 degree" lines in the charts depict capital-to-loss ratios of 1 1. The points above the 45 degree line in charts 2 through 6 represent portfolios for which the capital requirements exceed portfolio losses. The points falling below the 45 degree line represent portfolios for which the capital requirements are insufficient to cover portfolio losses From the charts, it is clear that the gamma and Taylor measures produce the patterns most similar to the 45 degree line itself, while the patterns for strategy-based rules are the least similar

Another approach to the evaluation of the methods is to examine the "errors" produced by each method. In Charts 2 through 6, capital deficits and surpluses can be measured by the vertical distance, expressed in absolute dollar amounts, of a portfolio from the 45 degree line. To compare the aggregate magnitude of the capital shortages and excesses resulting from each rule, we summed the portfolio deficits and surpluses separately (Table 2).

From a supervisory perspective, the gamma charge rule appears slightly more attractive than the Taylor series rule The total deficit is higher under the Taylor series rule than under the gamma charge rule As Appendix I explains in greater detail, the Taylor series rule gives capital credit whenever gamma is positive, while the gamma rule does not pro-

Table 2

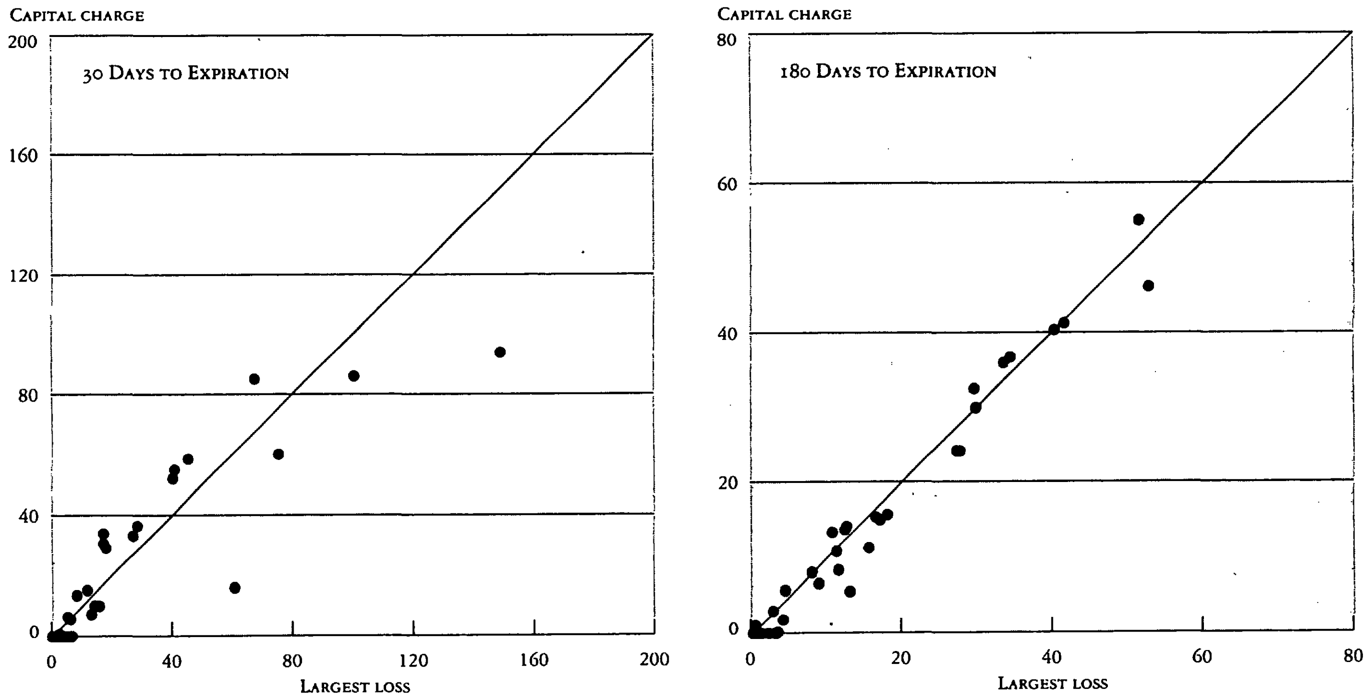
Comparison of Value-at-Risk and Strategy-Based Rules

Statistic	Taylor	Gamma	Delta	Appendix A	C2X
30-Day Options					
R^2 coefficient	0.842	0.828	0.381	0.555	0.026
Slope coefficient	0.96*	0.91*	0.52	1.54	0.19
Total capital deficit	188	158	497	131	634
Total capital surplus	173	274	204	1176	580
180-Day Options					
R^2 coefficient	0.974	0.927	0.667	0.112	0.010
Slope coefficient	1.02*	1.02*	0.69	0.67*	0.12
Total capital deficit	60	33	187	112	253
Total capital surplus	16	74	62	695	325

*The hypothesis that the slope coefficient equals 1 cannot be rejected at the 5 percent level

Chart 2

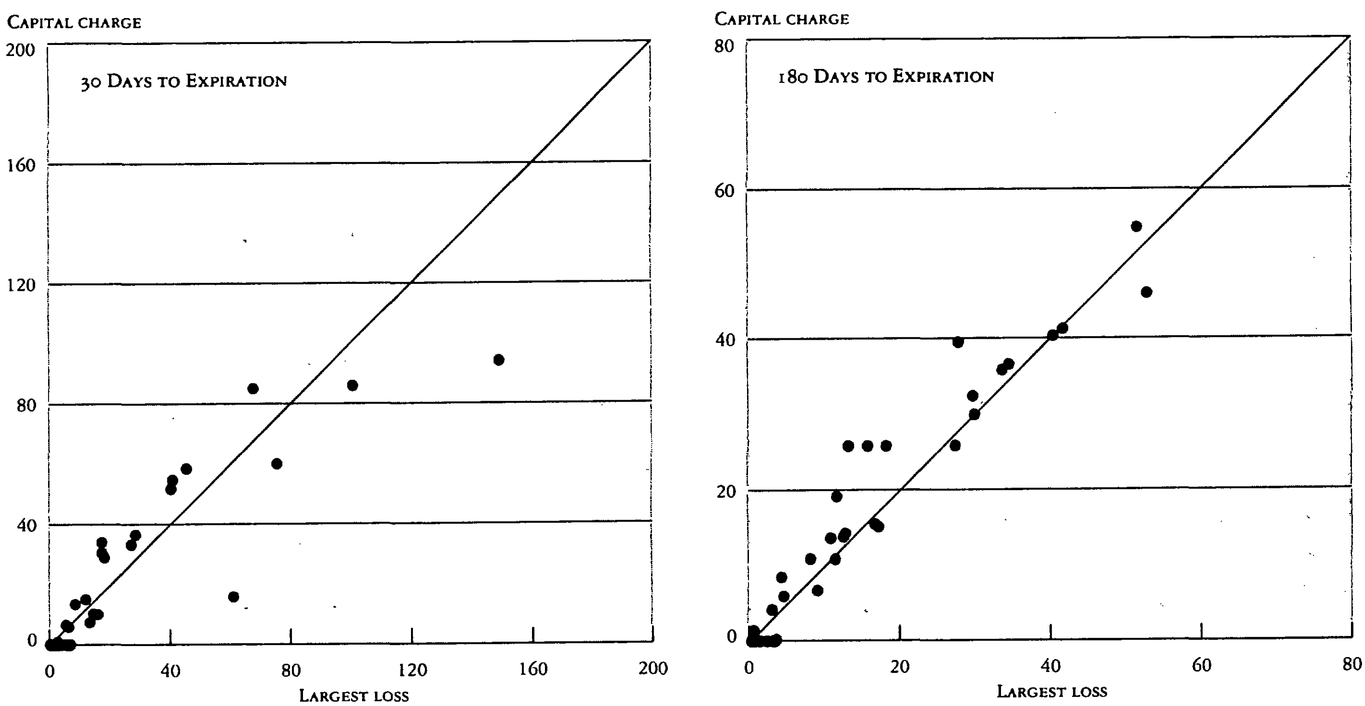
TAYLOR SERIES RULE



Note To maintain comparability of scale across charts, the coordinate pair (161, 205) is not shown in the left panel

Chart 3

GAMMA CHARGE RULE



Note To maintain comparability of scale across charts, the coordinate pair (161, 205) is not shown in the left panel

Chart 4

DELTA EQUIVALENT RULE

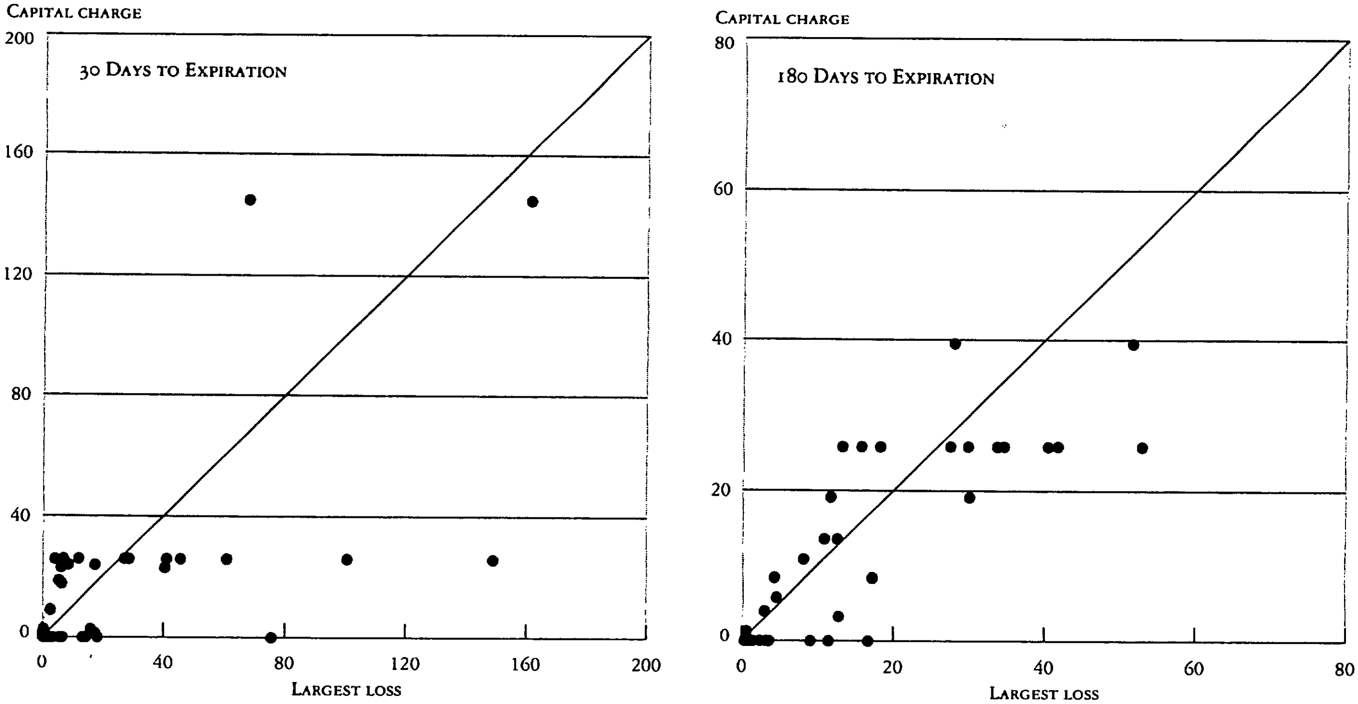
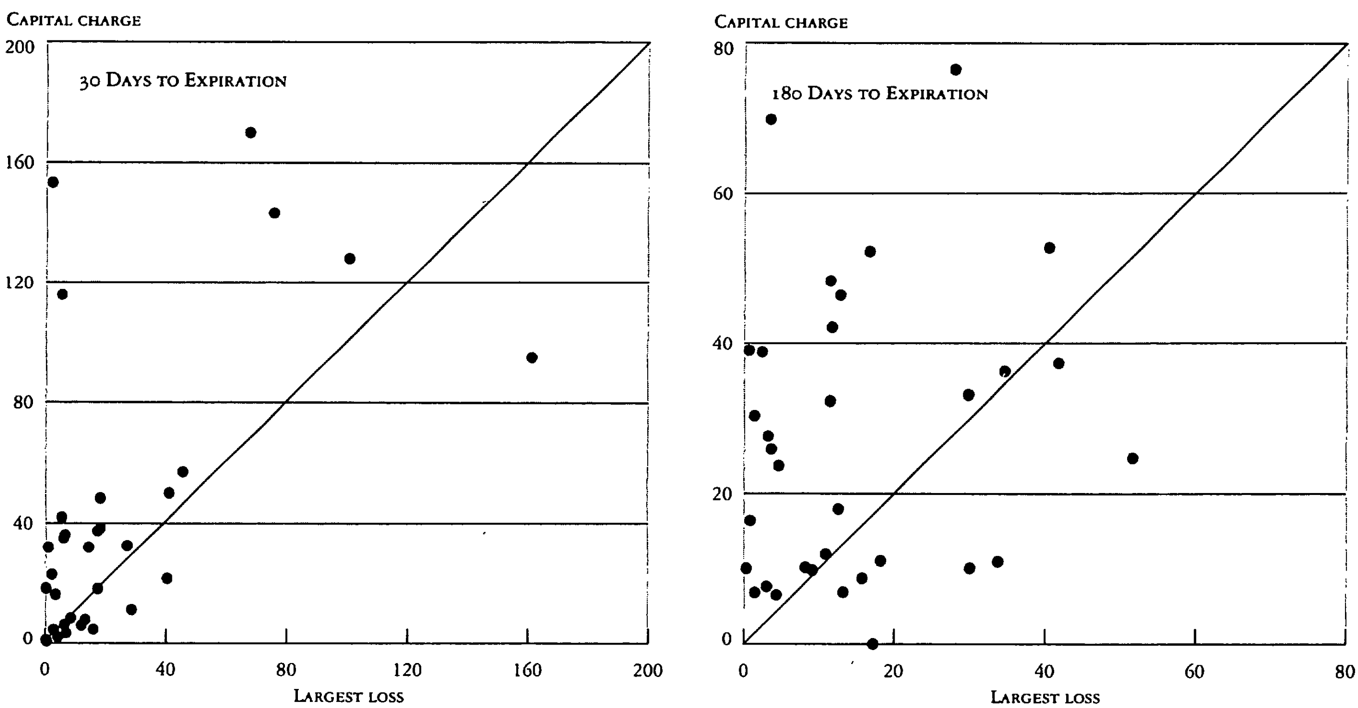


Chart 5

SEC APPENDIX A RULE



Note To maintain comparability of scale across charts, the following coordinate pairs are not shown (100,264) (149,408) in the left panel and (2,99) (53,149) in the right panel vide for such credit Under the delta-equivalent rule, the magnitude of uncovered losses is much greater than under either of the foregoing methods. In Chart 4, the capital charges computed for several portfolios under the delta rule diverge significantly from the 45 degree line.

If we turn from the value-at-risk approaches to the strategy-based rules, we find that Appendix A and C2X tend either to underestimate risk significantly, yielding large capital deficits, or to overcompensate for risk, producing large capital surpluses. The C2X rule, which is designed for market makers on the trading floor, could result in especially large uncovered losses. It produces the highest deficits among all of the capital rules considered.

The Appendix A rule appears to perform well when options with 30 days to maturity are considered However, Table 2 shows that Appendix A also leads to significant capital surpluses. By incorporating only simple trading strategies, Appendix A does not entirely allow for hedging or offsetting portfolio effects, causing substantial excess capital burdens for a number of portfolios.

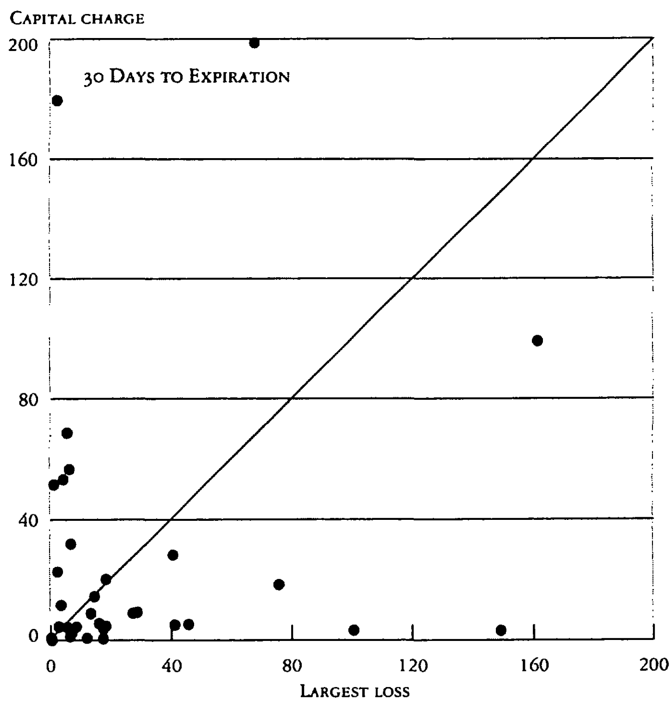
The large capital surpluses produced by the strategy-based rules may be partly attributable to their intended coverage of nonprice risks. When the results of the strategy-based rules are examined on a portfolio-by-portfolio basis, however, a systematic pattern appears For portfolios representing SEC-recognized strategies, the strategy-based rules tend to underestimate the potential loss, generating very small capital requirements. For portfolios representing strategies not recognized by the SEC, the rules tend to overestimate the potential loss, generating correspondingly large capital requirements[9]

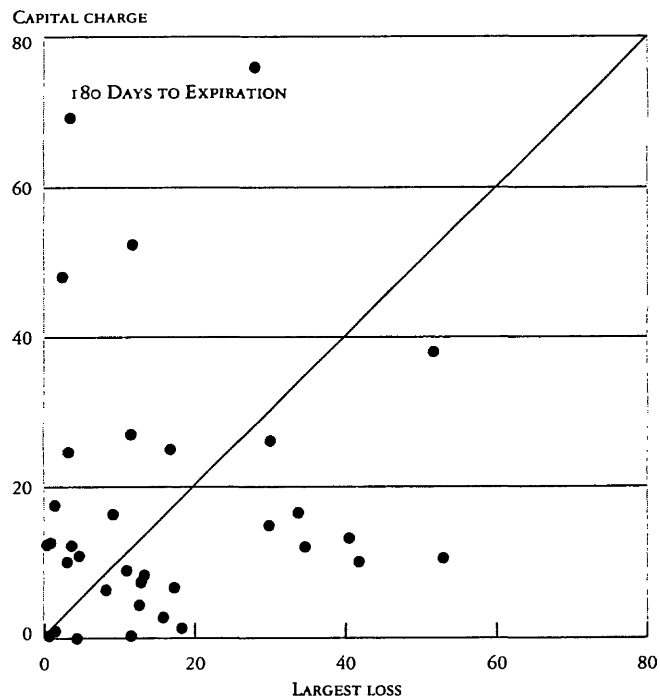
Finally, all of the rules provide a better estimate of potential losses for the set of portfolios with 180 days to maturity than for the set with 30-day options. As noted above, this result can be attributed to the high level of gamma risk associated with short-dated options.

In summary, our evaluation of the efficiency and coverage of the capital rules indicates that the approximate value-at-risk rules tend to perform better than the simple strategy-based rules. Within the category of value-at-risk rules, the gamma and Taylor series rules provide relatively similar results. Both outperform the delta-equivalent rule,

Chart 6

SEC C2X Rule





and the gamma rule tends to be a bit more conservative than the Taylor series rule.

Volatility Risk Add-ons

To investigate the potential benefits of a volatility risk add-on, we supplement the Taylor series rule with a charge equal to the absolute value of the vega of each position multiplied by a 5 percentage point change in volatility.[10] Since vega rises with the tenor of the option, the volatility add-ons are substantially larger for the options with remaining maturity of 180 days than for those with remaining maturity of 30 days. Across all of the positions, the add-on specification increases total required capital by 4.2 percent for options with remaining maturity of 30 days and 14.4 percent for options with remaining maturity of 180 days.

For the 30-day positions the vega add-on has little tangible effect on measures of capital efficiency. The R^2 increases from 0.842 to 0.844, while the slope increases from 0.96 to 0.97. With respect to coverage, the total capital deficit decreases from 188 to 168, while the surplus increases from 173 to 189. The effect of the volatility risk add-on is larger for the 180-day option positions. The R^2 of the regression of capital on largest loss rises from 0.974 to 0.978, while the slope rises from 1.02 to 1.06. The total capital deficit declines from 60 to 21, while the surplus rises from 16 to 31.

In summary, a volatility risk add-on certainly does not reduce the efficiency of the capital charge, but neither does it markedly increase it relative to the Taylor series rule. The volatility risk add-on does increase coverage of the largest losses as measured by the total capital deficit.

Other Considerations

Simplicity and Reporting Burden

In general, the introduction of specific supervisory capital requirements for the market risks arising from banks' options positions requires a certain level of sophistication on the part of both banks and supervisors. Nevertheless, the proposals examined in this paper vary significantly in their complexity and therefore in their potential reporting burden.

Strategy-based methods can be easily applied by even the least sophisticated options players because they do not require the use of complicated options pricing models. Consequently, these methods do not create excessive reporting burdens for banks that rely on only a few simple options strategies, primarily for hedging purposes. But while the capital requirement of simple strategy-based methods may be relatively easy to calculate, these methods can lead to substantial reporting burdens for more sophisticated institutions that carry out a wide range of trading strategies, since each type of position would require a different capital requirement. In addition, capital requirements that rely exclusively on the strategy-based method may compel regulators to revise their capital requirements repeatedly as financial institutions develop new options instruments and trading strategies.

Compared with the simple strategy methods, methods based on price sensitivity require a higher level of sophistication on the part of both banks and regulators. Nevertheless, price sensitivity methods rely on standard risk management techniques already employed by most major financial institutions to manage the risk of their options portfolios. Most larger banks use internal risk management systems that track on an ongoing basis the delta, gamma, and volatility risks of their options portfolios. As a result, in the case of larger options players, the price sensitivity approach should impose lower reporting burdens than the strategy-based approach because the methodology reflects banks' existing hedging practices and can be applied uniformly to all options positions. In addition, the price sensitivity approach can simplify the task of regulators: it is flexible

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enough to incorporate market innovations in options instruments and trading strategies without requiring additional capital standards.

Scenario methods also tend to be more complex than simple strategy-based methods. However, like the price sensitivity approach, these methods rely on variables that most larger banks already monitor through their standard risk management systems. Simulation methods are the most complex to implement and may lead to excessive reporting burdens for all but the most sophisticated banking institutions.

Conformity with Existing and Proposed Rules

The methods examined in this article also vary in the degree of their conformity with existing capital measurement approaches. By combining credit risk and market risk in one capital charge, the simple strategy method conflicts with the SFA, CAD, and proposed BIS market risk standards, which apply separate capital charges for the two types of risk. In addition, while the SFA, CAD, and BIS proposals allow for a limited number of simple strategy-based trades, none of these supervisory approaches permits an across-the-board application of strategy-based methods In contrast, the price sensitivity approach can easily be incorporated into the capital framework of the SFA, the CAD, and the BIS and SEC proposals, each of which allows for a price-sensitivity-based method.

The SFA allows a scenario method, while the BIS market risk proposal considers permitting scenario or simulation methods as alternatives to the price sensitivity approach However, because the scenario approach does not strictly employ the building block methodology as defined by the BIS market risk proposal—which distinguishes specific from general market risk—it might have to be "carved out" of the building block methodology of the BIS framework.

Summary of Findings

We have compared various supervisory approaches to the capital treatment of the market risk of options. Our comparison rested on three criteria. capital coverage of potential losses, simplicity and reporting burden, and conformity with existing and proposed supervisory approaches. The main findings of the analysis are summarized below.

Strategy-based Methods

The principal advantage of strategy-based capital requirements is their simplicity. Strategy-based methods do not require the use of option pricing models and are therefore most appropriate for banks that carry out only a limited amount of options business, primarily for the purpose of hedging. However, simple strategy-based methods can only roughly estimate the potential losses of an options portfolio.

A further drawback of the strategy-based method is that it may pose reporting problems for sophisticated options players Each trading strategy is subject to a different capital requirement, and the approach is generally unrelated to the internal risk management systems employed by banks. Moreover, the method lacks the flexibility to incorporate future market developments in options instruments and trading strategies. As a result, regulators must continuously upgrade and expand their capital requirements.

Value-at-Risk Methods

The value-at-risk methods examined above differ from each other primarily in the treatment of gamma risk. One advantage of these methods is that they can be incorporated in the framework of the SFA, the CAD, and the proposed BIS and SEC market risk capital requirements

Delta-Equivalent Method. The delta-equivalent rule provides banks with a relatively simple capital rule. Since most banks that are active in options activities generally measure delta risk as part of their overall risk management strategy, the delta-equivalent method would not present an excessive reporting burden for these banks. Further advantages of the delta-equivalent method are that it can be applied to all options positions uniformly, it may allow for portfolio effects, and it is flexible enough to incorporate market innovations

However, the delta-equivalent rule can seriously underestimate potential losses in the case of large price movements because it does not incorporate gamma risk. In addition, it cannot by itself account for changes in the volatility of the underlying instrument.

Delta-Equivalent Method with Gamma Adjustment The delta-equivalent method with gamma adjustment is similar to the delta-equivalent method except that it also modifies the capital requirement for the risk that delta will change as the price of the underlying moves. Our calculations show that adjusting for gamma risk (with either the Taylor or gamma rules) typically leads to a significantly improved approximation of an options portfolio's potential losses

DELTA-EQUIVALENT METHOD WITH GAMMA AND VOLATILITY ADJUSTMENTS. In addition to measuring delta and gamma risk, this approach also takes into account volatility risk, the risk that the variability of the underlying instrument's price will increase or decrease, affecting the price of the option. A volatility adjustment could play an important role, depending on the specific portfolios involved.[11]

SCENARIO-BASED AND SIMULATION METHODS WITH FULL REVALUATION. Scenario methods can vary in their sophistication but generally rely on options pricing models to measure the gains or losses on each option position in response to defined movements in the price or volatility of the underlying instruments. Scenario methods with full revaluation implicitly account for delta and gamma risk, and they can also explicitly account for volatility risk. As a result, these methods may provide a more exact measure of potential losses than the approximation-based price sensitivity methods. Simulation methods, using either real historical data or a Monte Carlo methodology, may facilitate the incorporation of other variables, such as an option's time value or its sensitivity to changes in interest rates.

APPENDIX I: DETAILED DESCRIPTION OF VALUE-AT-RISK RULES

BASIC FRAMEWORK

The change in the value (V) of an options portfolio associated with a given change in the price (u) of the underlying asset is given by:

$$(1) \qquad \Delta V = V(u + \Delta u) - V(u).$$

This change may be approximated using a second-order Taylor series expansion.

$$(2) \qquad \Delta V = \frac{\partial V}{\partial u} \Delta u + \frac{1}{2} \frac{\partial^2 V}{\partial u^2} \Delta u^2,$$

where ΔV and Δu are changes in the values of the portfolio and the underlying asset, and the partial derivatives $\partial V/\partial u$ and $\partial^2 V/\partial u^2$ are the portfolio's delta and gamma values.[12]

To determine the size of the capital requirement, the measure of price sensitivity is combined with a measure of the volatility of the underlying asset's price. In terms of equation 2, the change in the price of the underlying asset, Δu, is replaced by a measure of the asset's price volatility For example, with a multiple m of the standard deviation of the price change, equation 2 becomes:

$$(3) \qquad \Delta V = \frac{\partial V}{\partial u} (m\sigma) + \frac{1}{2} \frac{\partial^2 V}{\partial u^2} (m\sigma)^2$$

The multiple of the standard deviation determines the confidence level of the capital rule [13]

The magnitude of value at risk typically depends on both the portfolio's sensitivity to changes in the price of the underlying asset and the price volatility of the underlying asset. In terms of equation 3, if the portfolio were well hedged, then the price sensitivity terms $\partial V/\partial u$ and $\partial^2 V/\partial u^2$ would both be small because of the offsetting positions in the portfolio, thereby leading to a low capital requirement. Given the price sensitivity of the portfolio, however, the capital requirement would also be smaller if the underlying asset had a small price volatility: a smaller likelihood of large price changes implies that potential losses will be smaller.

METHODS BASED ON PRICE SENSITIVITIES— FIRST- AND SECOND-ORDER APPROXIMATIONS

Delta-Equivalent Rule. The delta-equivalent rule uses only the first term in equation 2, the delta value of the portfolio.

$$(4) \qquad K = \left| \frac{\partial V}{\partial u} m\sigma \right|,$$

where K is the capital charge and *m* is a multiple of the standard deviation that determines the degree of confidence in the capital rule. The delta-equivalent is the basic component of the CAD's rule and the proposed BIS rule.

> *Example.* Consider a portfolio consisting of a \$100 long position in the underlying asset and a written call option on \$100 of the asset. If the delta of the option equals .25, then the delta of this portfolio is equal to 75 (the written call has a negative delta of −.25 and the underlying has a delta of 1, where both are weighted equally since the underlying amount is the same for each position). If we assume that a three standard deviation confidence interval for the capital rule implies a \$20 change in the price of the underlying asset, then the capital charge will be \$15 (\$15 = (.75) x (\$20)).

The delta-equivalent can often underestimate risk because it is a linear approximation to the price sensitivity of options, which are inherently nonlinear. This approximation error can be reduced by incorporating an option's gamma value, a measure of the nonlinearity of an option's price sensitivity. The augmented delta-equivalent rules include such an adjustment.

Taylor Series Rule. One use of the gamma value to augment the delta-equivalent rule is the direct application of the Taylor series in equations 2 and 3 An option's price sensitivity can be different for increases and decreases in the price of the underlying security. Hence, to obtain the capital charge, a price increase as well as a price decrease (of magnitude $m\sigma$) is applied to the Taylor series equation, and the capital charge is the largest resulting loss:

$$(5) \quad K = \left| \min\left[\left(\frac{\partial V}{\partial u} m\sigma + \frac{1}{2}\frac{\partial^2 V}{\partial u^2}(m\sigma)^2\right), \left(\frac{\partial V}{\partial u}(-m\sigma) + \frac{1}{2}\frac{\partial^2 V}{\partial u^2}(m\sigma)^2\right), 0\right] \right|$$

For a book of written options (when options risk is most extreme), the Taylor series rule will always produce a larger capital requirement than the delta-equivalent However, for a book of purchased options, the Taylor series rule will require less capital than the delta-equivalent because the latter overestimates the price risk of purchased options.

Gamma Charge Rule. A more conservative application of gamma values is to use the sum of the absolute values of delta and gamma (when gamma is negative):

$$(6) \qquad K = \left| \frac{\partial V}{\partial u}(m\sigma) \right| + \left| \min\left(\frac{1}{2}\frac{\partial^2 V}{\partial u^2}(m\sigma)^2, 0\right) \right|.$$

This gamma charge capital requirement is one of the alternative rules in the SFA capital requirements.

The capital charge in this rule is at least as large as the Taylor series rule (equation 5) because of the way the delta and gamma interact in the Taylor series when they have different signs For a portfolio of written options (negative gamma), the Taylor series rule and the gamma charge rule produce identical capital charges. In a portfolio of purchased options, however, the delta term will overestimate potential losses, and in the Taylor series rule, the positive gamma (of the long options) will reduce the capital charge by moderating the effects of the delta term in equation 5. The gamma charge rule (equation 6), however, does not provide such "credit" for the delta's overestimate of risk. In this case (a book of purchased options), the delta-equivalent and the gamma charge rule produce the same capital charge.

> *Example.* Recall the earlier example of the delta-equivalent rule, and suppose that the option had a gamma value of −0.1. The gamma charge, in this

case, is \$20 = $|(-0.1)(\$20)^2/2|$, and the total capital requirement is \$35 = \$15 + \$20. (The \$20 term that is squared in the calculation is derived from a volatility of 20 percent weighted by the \$100 underlying asset; the \$35 term is the sum of the delta charge from the earlier example and the gamma charge.)

INCORPORATION OF VOLATILITY RISK

Capital coverage for volatility risk can be incorporated through an additional capital charge. This charge would be based on

$$V(\sigma + \Delta\sigma) - V(\sigma) \tag{7}$$

or, if an approximation is used,

$$\frac{\partial V}{\partial \sigma}\,\Delta\sigma, \tag{8}$$

where $\Delta\sigma$ denotes the change in volatility. In equation 8, the options' volatility sensitivity (vega or lambda) is weighted by the change in volatility, while in equation 7 the options are revalued at different volatilities. The SFA capital requirements incorporate such a volatility risk add-on, using a one percentage point change in volatility. Although equation 7 is a more exact estimate, an option's sensitivity to volatility changes tends to be a linear relationship, and the use of a volatility sensitivity measure as in equation 8 is a reasonable estimate of volatility risk.

APPENDIX II: EXAMPLES OF STRATEGY-BASED RULES

The following two examples of strategy-based rules are taken from the SEC's Appendix A rule, which applies to securities firms holding options for proprietary trading.

STRATEGY 1. Long call option

Capital charge: 50 percent of the market value of the option

Strategy 2: Long call option and short the underlying stock

Capital charge: If the call is out-of-the-money, the lesser of
- a) 30 percent of the underlying, or
- b) the out-of-the-money amount

If the call is in-the-money, no charge is required.

Endnotes

1 A more detailed discussion of options risk and supervisory capital rules is found in a related paper by the same authors "Options Positions Risk Measurement and Capital Requirements," Federal Reserve Bank of New York Research Paper

2 The paper cited in the preceding note provides a detailed account of specific methodologies actually used or proposed by regulatory authorities

3 For examples of specific strategy-based rules, see Appendix II

4 In the interim, the SEC's Division of Market Regulation issued a "no action letter" on March 15, 1994, that effectively allows broker-dealers to use the proposed method

5 The annual volatility of underlying price changes is assumed to be 30 percent Historical volatility for the U S S&P 500 index has typically ranged from 10 to 20 percent, but individual stocks can display much higher volatilities The average volatility for individual stocks in the Dow Jones Industrial Average is currently around 30 percent

6 Capital requirements under the SEC rules could differ depending on the sequence in which recognized strategies are pulled out from a portfolio Recognizing that a firm is likely to use the lowest cost interpretation of the capital rule, we chose the sequence that yielded the lowest level of capital requirement

7 The normalization method used is a delta-based normalization, that is, the larger of the sum of gross negative delta-equivalent values or the sum of gross positive delta-equivalent values of the options in a portfolio, *excluding positions in the underlying asset,* is normalized to $100.00 Further discussion of approaches to normalization is contained in the paper cited in note 1

8 Appendix A appears to overcompensate substantially for losses of a few portfolios, resulting in points with extreme capital levels. capital-to-loss ratios under the Appendix A rule are as high as 69 1, in contrast to 6 1 under the delta equivalent rule Charts 4 and 5 illustrate the differences in capital levels

9 Portfolios that have extreme capital-to-loss ratios are synthetic long futures, delta-hedged protected call, delta-hedged bull call, delta-hedged bear call, ratio call spread, protected call, delta-hedged ratio call spread, delta-hedged ratio call back spread, and box spread The portfolios that represent SEC-recognized strategies are so indicated in Table 1

10 Vega is defined as the change in the price of an option in response to a unit change in the volatility of the underlying asset

11 Volatility risk is largest for one-sided portfolios (all written or all purchased), for options with longer maturities, and for options that are close to the money

12 These delta and gamma values represent net portfolio values that are the arithmetic sum (taking account of the signs) of the deltas and gammas of all instruments and transactions in the portfolio

13 Although the degree of confidence provided by a specific multiple of the standard deviation depends on the precise underlying probability distribution, a rough idea of the degree of protection provided by, say, two or three standard deviations may be obtained Given a normal probability distribution, at three standard deviations, losses will exceed capital with a likelihood of 1 in 700, whereas at 2 33 standard deviations, losses will exceed capital with a likelihood of one in a hundred

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FEDERAL REGISTER 1994 Vol 59, No 54, March 21