Event Risk Premia and Bond Market Incentives for Corporate Leverage

The growth of highly leveraged transactions during the 1980s had a profound effect on corporate shareholders and bondholders. Studies exploring the effects of the restructurings, mergers, leveraged buyouts, and recapitalizations of the last decade have typically focused on the welfare of firms and agents directly involved in the transactions Evidence suggests, however, that the repercussions of these activities have extended to a much broader circle of corporations and individuals

This article introduces and explores the hypothesis that the threat of leveraging from such events has worked to raise the risk premia paid on the debt of all but the most leveraged firms. In addition, it argues that the increase in the cost of debt has been greater for less leveraged firms The penalty for low leverage creates incentives for debt financing, thereby compounding existing distortions created by tax laws and limited shareholder liability In effect, the growth of an active market for corporate control has embedded in bond prices the possibility that firms will either leverage themselves or be leveraged in a change of ownership As a result, the marginal incentive to leverage increases. A related implication is that it no longer appears as feasible as it once was for a firm to hold down its marginal cost of debt capital by keeping a very large equity cushion

The article begins with a more precise exposition of the hypothesis. Subsequent sections provide empirical support and estimate the extent to which the leverage threat affects the schedule of borrowing rates faced by U.S. firms. The relevance of the hypothesis to the observed increase in U.S corporations' dependence on debt is also considered. In the closing sections, the article reviews the implications of the empirical results for the cost of capital to U.S corporations in the 1980s and the relationship of these findings to policy measures aimed at limiting corporate debt.

Effects of event risk premia

Our hypothesis posits that during the 1980s the threat of unanticipated leverage increased the risk premia paid by U S corporations on their debt and that this increase was greater for the debt of less leveraged firms. The hypothesis is based on two observations. First, a firm's existing bondholders face substantial losses from unanticipated increases in the firm's leverage Second, the risk is greater for less leveraged firms because they are better candidates for increased debt and because their bondholders face greater potential losses from the increase in debt A fuller understanding of this hypothesis requires familiarity with the concepts of the risk premium curve and the event risk premium

The risk premium curve

The risk premium is the sum a corporation must pay on its debt beyond the riskless interest rate in order to compensate bondholders for the possibility of default The most obvious determinant of the firm's risk premium is its leverage, although factors such as volatility of cash flow, access to credit markets, asset liquidity, firm size, and diversity of revenue sources also play a role.

The risk premium increases with leverage The lower curve in Chart 1 depicts the risk premium as a function of a measure of leverage — the debt ratio. The debt ratio is defined as the market value of a firm's debt divided by the total value (market value of equity plus market value of debt) of the firm The chart shows the risk premium curve as rising more than proportionally with the debt ratio. This representation of the curve is in the literature and, as demonstrated below, empirically verifiable

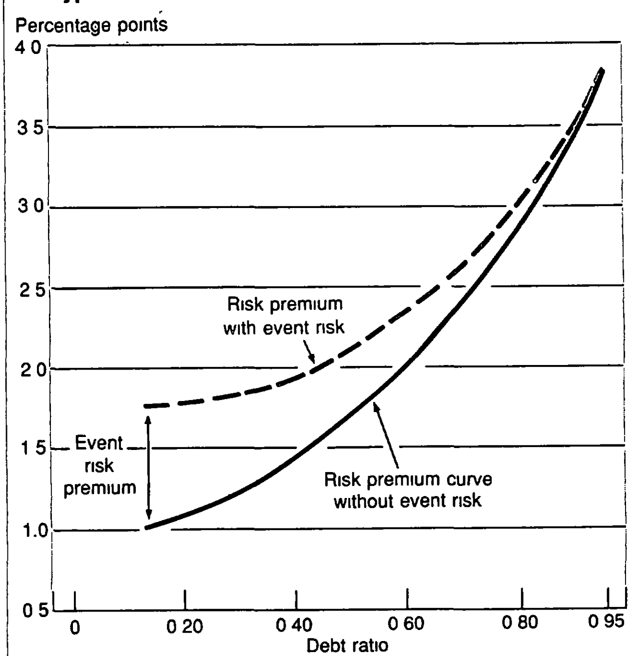
Event risk

Event risk will be defined as the risk of any significant unanticipated increase in the debt share of a firm. Such increases may result from leveraged buyouts, share repurchases, extraordinary dividends, and other leveraging tactics. The tactics include leverage both by outsiders who gain control and by current managers who adopt defensive measures.

The existence of event risk will prompt bondholders to require a risk premium in excess of that which would be demanded of a firm without such risk The size of this additional premium will largely be a function of the debt ratio, although factors such as industry cyclicality and cash flow volatility may also play a part. As noted earlier, a firm with a low debt ratio will tend to pay a higher event risk premium because it is a more likely candidate for additional leverage and because the bondholders have more to lose from the additional leveraging. The event risk premium thus acts as a penalty that decreases with leverage, as shown by the two curves in Chart 1 This chart illustrates the two basic effects of event risk. First, event risk raises the risk premium schedule, ratcheting up borrowing costs over a wide range of leverage. This effect may be termed "raising the intercept of the risk premium curve." Second, event risk flattens the risk premium curve This lowers the marginal cost of additional leveraging: for a given increase in leverage, the firm is facing a smaller increase in the risk premium In effect, bond buyers respond to event risk by making firms pay for leverage whether they are leveraged or not, thus making it desirable for firms to leverage up and reap the benefits of additional debt Even firms that are not otherwise inclined to take on new debt may be encouraged, if not compelled, to do so by event risk premia

Chart 1

Hypothetical Risk Premium Curves



Role of bond covenants

Event risk premia may not capture the total event risk penalty paid by firms since they do not reflect the implicit costs of bond covenants By issuing debt with strong covenants, a firm may eliminate a great deal of event risk and its accompanying premia But while the firm is no longer paying an explicit event risk premium, it nevertheless may bear other costs in honoring the covenant. Because covenants typically entail substantial indirect costs,[1] including loss of flexibility in decision making, they may not provide a way to reduce overall costs

Empirical estimation of event risk

This section assesses the extent to which event risk premia have grown. The hypothesis that leverage risk premia were smaller before the mid-1980s implies that differences in risk premia between high-leverage and low-leverage firms should have narrowed over the course of the decade

Chart 2 provides casual empirical evidence for the hypothesis. Note that since 1984 (roughly the start of the takeover boom), the spread between Aaa corporate and Treasury rates has grown relative to the spread between Baa corporate and Treasury rates This result suggests a change in the perception of Aaa corporate debt: once regarded as a near substitute for riskless debt, Aaa corporate debt is now an intermediate between Baa and riskless debt. This shift is consistent with the flattening of the risk premium curve caused by a rise in the risk premium on high-grade debt

It should be remembered that both rate spreads tend to widen during downturns, as Chart 2 suggests for the

[1]Clifford W Smith, Jr, and Jerold B Warner, "On Financial Contracting An Analysis of Bond Covenants," *Journal of Financial Economics*, June 1979

period corresponding to the 1981-82 recession. During the prolonged recovery period, however, the Aaa/Treasury spread has remained high, generally exceeding 1982 levels. In contrast, during the 1975-79 expansion, Aaa/Treasury spreads were smaller, absolutely as well as relative to Baa/Treasury spreads. The persistence of the increased Aaa/Treasury spread during the 1980s makes it unlikely that this change reflects fluctuations in market liquidity conditions. Indeed, the massive supply increase of Treasury debt over this period, combined with weak issuance of Aaa corporate debt,[2] makes the growing disparity especially impressive. These considerations further support the contention that since the mid-1980s, event risk premia have risen for high-grade debt in particular.

Analysis of individual firms

Chart 2 gives only partial evidence of the flattening of the risk premium curve since it ignores changes in the composition of the firms in each rating category. A regression analysis of individual firms will provide more complete evidence of this development

To this end, a sample of forty-seven firms (Table 1) is selected to cover all industry classifications except banking, financial services, and certain utilities (see Table A1 in Appendix A for characteristics of sample firms) Electronics, vehicles, oil, and chemicals are overrepresented in the sample because of data availability. All firms in the sample pool have been in continuous operation since at least 1976, and most have a large share of publicly traded debt, particularly multiple issues. Firms whose bonds contain call provisions that are especially difficult to price are eliminated from the sample. To be comparable, bond yields should be adjusted to remove the effects of such call options, but the valuation can be problematic (see Appendix A).

Although representative size distribution is sought in the sample, the selection criteria bias the sample heavily toward large firms. If event risk premia are smaller for very large firms because such firms make more difficult takeover targets, then the analysis actually underestimates the effects of event risk premia

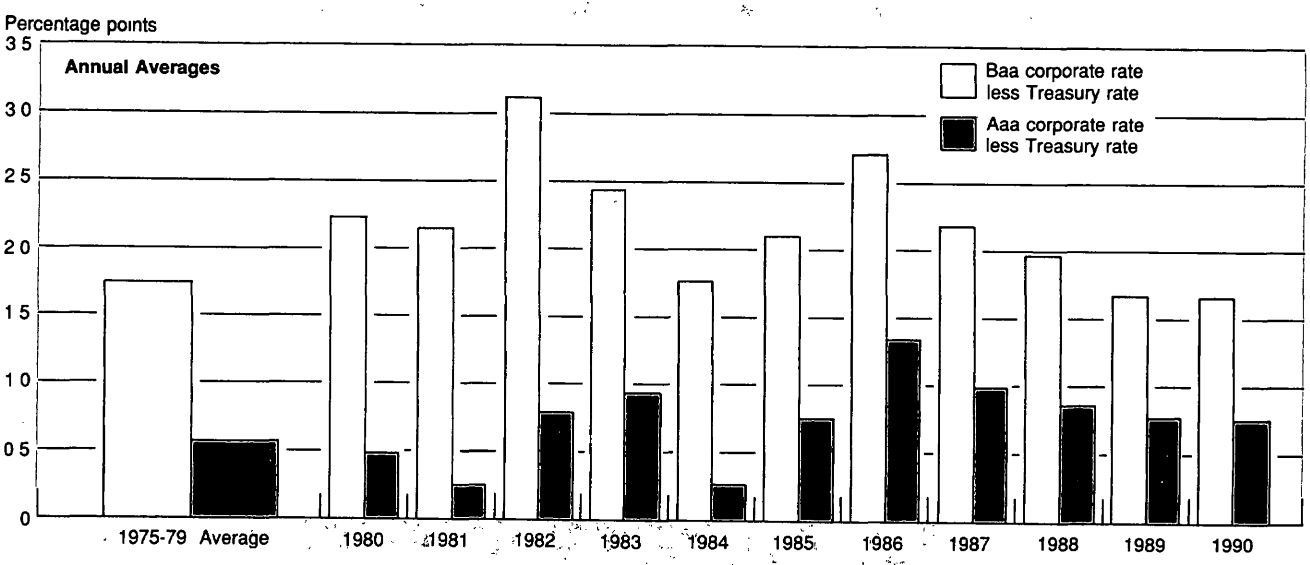
The sample dates chosen for the analysis are the last business day in June for the years 1980, 1988, and 1989.[3] The dates have the advantage of spanning the leverage boom and of representing periods in which

[2]Edward I Altman, "Measuring Corporate Bond Mortality and Performance," working paper, February 1988, Table 6, p 16

[3]Data from 1982 gave results similar to those from 1980 data, while data from 1984 gave results that were between 1980 and 1988 results The data from 1982 and 1984 may not be directly comparable with those from other years, however, because of significantly higher nominal interest rates in these periods

Chart 2

Credit Quality Spreads



Source: Moody's Bond Survey

Note. Chart shows yield to maturity on seasoned corporate and Treasury debt with ten years to maturity The 1990 yield is a first quarter average nominal interest rates were roughly equal (see Appendix A). In addition, interest rate volatility around the observation periods was low, a condition which ensures more reliable estimates of risk premia

The first step in the analysis is to compare the leverage of sample firms with the observed spread of their bond yields over Treasury yields. The measure of leverage used is market value of debt as a share of total firm market value—the debt ratio defined above To calculate the market value of debt, the firm's publicly traded debt is repriced at the prevailing market price All other debt is left at book value The total value of the firm is the sum of this market value of debt and the market value of outstanding equity.[4]

The risk premium used is the average of the differences in yield to maturity between the firm's publicly traded bonds and riskless debt of comparable maturity and coupon characteristics.[5] One problem with this measure is the timing of the yield quotes [6] The prevailing yield on a corporate bond reflects the most recent transaction, which may have occurred several days earlier. Although such lags do not systematically bias the slope of the estimated curve relating risk premia to leverage, they can systematically bias the intercept.[7] Nevertheless, steady interest rates in the period immediately preceding the sample dates make substantial bias unlikely, as do the active markets in high-grade bonds, the debt most affected by event risk premia

Other potential problems associated with the measure include intertemporal biases induced by systematic changes in the maturity structure of outstanding corporate debt or by changes in the relation of the maturity and seniority of traded and nontraded debt. These problems do not appear serious, however (see Appendix A)

Table 1

Firms in Sample by Industry Classification

Firm	Industry Classification†
Aluminum Company of America	Metal refining
American Telephone and Telegraph	Communications
Amoco	Petroleum refining
Armco	Steel refining
Ashland Oil	Petroleum refining
Brunswick Corporation	Miscellaneous manufacturing
Cabot Corporation	Chemicals and pharmaceuticals
CBS	Communications
Centel	Communications
Chevron	Petroleum refining
Chrysler Corporation	Vehicles
Coastal Corporation	Fuel exploration
Colt Industries	Vehicles
Combustion Engineering	Machinery
Contel	Communications
Corning	Glass and concrete
CSX Corporation	Transportation
Cummins Engine	Vehicles
Delta Air Lines	Transportation
Dow Chemical	Chemicals and pharmaceuticals
Dupont	Chemicals and pharmaceuticals
Exxon	Petroleum refining
Fairchild	Vehicles and electronics
Ford Motor Corporation	Vehicles
General Electric	Electronics
Goodyear Tire and Rubber	Rubber and plastics
GTE	Communications
Honeywell	Laboratory equipment
International Business Machines	Machinery
ITEL	Wholesale
Kroger	Retail
Martin Marietta	Vehicles
McDonnell Douglas	Vehicles
Minnesota Mining and Manufacturing	Miscellaneous manufacturing
Panhandle East	Natural gas and coal
PepsiCo Incorporated	Food and tobacco
Pitney Bowes	Machinery
Procter and Gamble	Chemicals and pharmaceuticals
Sara Lee	Food and tobacco
Teledyne	Vehicles
Texaco	Petroleum refining
Tosco Corporation	Petroleum refining
Trinova	Vehicles
United Telecom	Communications
Weyerhauser	Wood products
Whirlpool	Electronics
Williams Company	Fertilizer, energy, and materials

†Industry classifications are based on a scheme developed in William Lee, "Corporate Leverage and the Consequences of Macroeconomic Instability," Federal Reserve Bank of New York, unpublished working paper, 1989

[4]Book value of debt and market value of equity are taken from the COMPUSTAT data base Market price of outstanding bonds is taken from Standard and Poor's *Bond Guide* (July 1980, July 1988, and July 1989)

[5]The risk premium is mathematically defined as follows

$$\sum_{i=1}^{k} w_i * (r_i^c - r_i^f),$$

where k = number of publicly traded bond issues
w_i = par value bond i / total par value of publicly traded debt
r_i^c = yield to maturity of corporate bond i, adjusted for callability
r_i^f = yield to maturity on riskless debt corresponding to bond i

[6]Corporate bond yields are taken from Standard and Poor's *Bond Guide* (July 1980, July 1988, and July 1989) and represent closing quotes from the last business day in June Government bond yields are from the *Wall Street Journal* for corresponding days

[7]Consider the following example Interest rates are rising in a market where the riskless debt is more heavily traded than the corporate debt Since the price quotes on the corporate debt are necessarily older, their yields will be biased downward relative to the riskless debt The risk premia on the corporate debt will not be biased relative to each other, however, although there will be more noise around the "true" risk premia

In calculations of risk premia, an effort is made to minimize the quote lag problem by taking quotes during weeks in which there was little interest rate movement and by relying on the more heavily traded corporate bonds

The next step is to calculate the volatility of total cash flow (pretax profit plus depreciation plus interest paid) over book value for each of the firms.[8] All other things equal, the firm with lower cash flow volatility will be less likely to default and should therefore pay a lower risk premium. Cash flow volatility will serve as an additional independent variable in some of the regressions performed below.

The risk premium is then modeled as a function of the debt ratio. As noted earlier, risk premia increase with leverage, although the exact relation is an empirical question. Risk premia are first modeled as a linear function of leverage: risk premium = $a + b * $ debt ratio. This relation works, but better results are achieved with nonlinear models (see Appendix B for a complete list of test results). The best overall results are obtained with the following relation: risk premium = $a + b *$ (debt ratio)$^{2.6}$.

The first part of Table 2 shows the results obtained by regressing risk premia solely on debt ratios. Note that the zero-leverage intercept has risen in each of the periods while the slope of the line has fallen. Statistical tests on the regressions indicate that the slope of the line in 1980 is significantly different from the slopes of the lines in 1988 and 1989, particularly when cash flow volatility is included as an independent variable in the regression.[9]

The flattening can be seen more clearly in Chart 3, which plots the curves generated by the regression results from Table 2. The risk premium curves for 1988 and 1989 are actually below the 1980 curve for high debt ratios. There are three possible explanations for this finding. First, as explained in greater detail below, changes in general liquidity conditions in fixed income markets contributed to an upward bias in the 1980 risk premium curve. Second, because of the low number of observations for debt ratios above 0.80, the 1988 and 1989 curves are probably estimated as being more linear than they actually are.[10] The true curve for these

[8] Cash flow volatility is measured as a twelve-quarter rolling variance of quarterly cash flow rates

$$\text{Volatility} = \sum_{i=t-11}^{t} (C_i - \overline{C}_t)^2 / 12,$$

where

$$C_i = \frac{\text{pretax profits + depreciation + interest paid in quarter } i}{\text{book value of firm at end of quarter } i}$$

$$\overline{C}_t = 1/12 * \sum_{i=t-11}^{t} C_i$$

[9] F-tests are performed to test the significance of the difference in the coefficient on the debt ratios between the 1980 and 1988 regressions and the 1980 and 1989 regressions. The results are as follows

Regression PRE = $a + b *$ LEV

Comparison	F-Statistic	Degrees of Freedom Numerator	Degrees of Freedom Denominator	Significance
1980 vs. 1988	4.76	1	77	0.95+
1980 vs. 1989	3.83	1	77	0.95−

Regression PRE = $a + b_1 *$ LEV $+ b_2 *$ VOL

Comparison	F-Statistic	Degrees of Freedom Numerator	Degrees of Freedom Denominator	Significance
1980 vs. 1988	9.10	1	76	0.995+
1980 vs. 1989	7.00	1	76	0.99+

where
PRE = required risk premium
LEV = debt ratio
VOL = variance of cash earnings (see footnote 8)

[10] Firms with debt ratios over 0.90 are excluded from the sample, partially because the exponential curve underestimates these values

Table 2

Required Risk Premium as a Function of Leverage

Regression Equation PRE = $a_0 + b * LEV^{2.6}$

	a_0	b	Standard Error of b	Adjusted R^2	Degrees of Freedom
1980	0.032	5.046	0.996	0.428	32
1988	0.366	2.710	0.566	0.323	45
1989	0.579	2.288	0.650	0.198	45

Required Risk Premium as Function of Leverage and Cash Flow Volatility

Regression Equation PRE = $a_0 + b_1 * LEV^{2.6} + b_2 * VOL$

	a_0	b_1	b_2	Standard Error of b_1	Standard Error of b_2	Adjusted R^2	Degrees of Freedom
1980	−0.23	4.65	31.30	1.01	21.27	0.448	31
1988	0.13	2.26	18.01	0.53	5.41	0.447	44
1989	0.17	2.17	22.99	0.61	8.11	0.306	44

Notes: PRE = risk premium over riskless rate for debt issues of individual corporations, LEV = debt ratio = market value of debt over total (debt plus equity) firm market value, and VOL = variance of earnings before interest and tax payments years should probably be flatter for debt ratios below 0.70 and steeper for debt ratios beyond that. Third, as markets for lower quality debt have improved in the 1980s, the yield on debt of more leveraged firms has actually fallen (although recent experience indicates that this trend may be reversing).

The second part of Table 2 presents the regression results obtained with the addition of cash flow volatility as an independent variable. In each case the regression fit is improved without a substantial change in the coefficient on leverage.

The coefficient on cash flow volatility falls sharply between the 1980 sample and the 1988 and 1989 samples, dropping from 31.3 in 1980 to 18.0 in 1988 and 23.0 in 1989. This effect is consistent with the results for the coefficient on leverage. low cash flow volatility no longer guarantees the firm cheap debt financing; the market assumes that firms with low cash flow volatility will be leveraged up.

Liquidity conditions and the risk premium curve

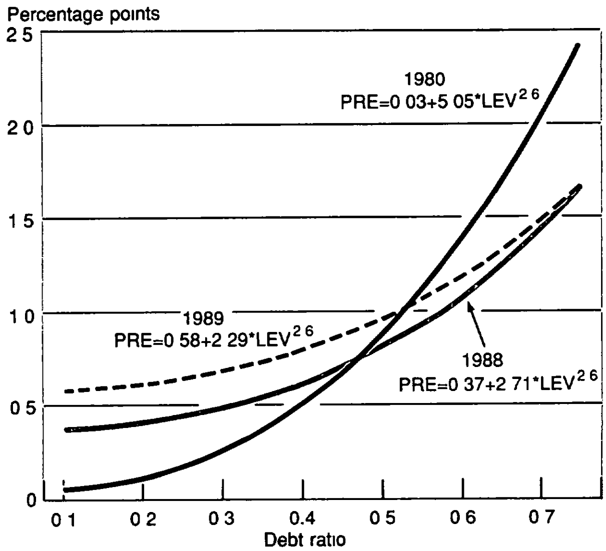
The difference between corporate and Treasury yields reflects not only the risklessness of government bonds, but also their higher liquidity Evidence suggests that the liquidity premium paid by corporate issuers over

Footnote 10 continued

and partially because the illiquidity of high-yield debt — particularly in 1980 — makes comparisons unreliable

Chart 3

Required Risk Premia on Corporate Bonds as a Function of Debt Ratio



Note PRE=required risk premium; LEV=debt ratio.

Treasury debt was higher in 1980 than in 1988 or 1989, biasing the 1980 risk premium curve upward relative to the 1988 and 1989 curves.

To obtain this finding, a proxy for the fixed-income liquidity premium in each of the years was estimated by comparing Treasury debt yields with yields on comparable World Bank debt and Federal National Mortgage Association debt (the procedure is described in Appendix A). The liquidity premium paid over Treasury debt by such issuers was about 25 basis points higher in 1980 than in 1988 and 1989.

Adjusting for the change in liquidity premia would shift the 1988 and 1989 curves in Chart 3 up by 25 basis points relative to the 1980 curve. Such an adjustment would suggest, for example, that the required premium on a firm with a 40 percent debt ratio is about 45 basis points higher in 1988 or 1989 than in 1980 owing to event risk.

Quantifying the debt costs of event risk premia

The long-run increase in debt costs from event risk can be quantified. If the upshift in the risk premium curve is about 15 basis points on average,[11] then U.S. corporations eventually stand to pay an additional $1.33 billion (15 basis points on $885 billion corporate bonds outstanding in 1988[12]) annually on their notes and bonds alone Initially, bond investors take the loss, but corporations must pay as maturing bonds are refinanced.

Most studies of event risk proceed by measuring the losses incurred by bondholders of buyout targets and weighing the losses against the gains realized by the target's stockholders. One of the largest estimates of lost bondholder wealth has been advanced by Asquith and Wizman, who calculate that, on average, bondholders lose 2.5 percent of bond value from a buyout, with greater losses for bondholders lacking covenant protection.[13] Asquith and Wizman note that even if the losses are applied to all target firms' debt, bondholder

[11]The model gives this figure as the direct increase to an average (debt ratio = 0 50) corporate bond issuer over the period 1980 to 1989 without an adjustment for liquidity Such an adjustment would add another 25 basis points

[12]Board of Governors of the Federal Reserve System, *Flow of Funds Accounts Financial Assets and Liabilities, Year-End, 1965-1988*, September 1989

[13]Paul Asquith and Thierry A Wizman, "Event Risk, Wealth Redistribution and the Return to Existing Bondholders in Corporate Buyouts," *Journal of Financial Economics*, forthcoming The losses reported represent abnormal returns over the period from month-end two months before the determination of buyout until month-end two months after the announcement of outcome The sample is taken from sixty-five large completed buyouts

Asquith and Wizman calculate losses of 5 4 percent for bondholders with no covenant protection and losses of 2 8 percent for bondholders with weak covenant protection, while bondholders with strong covenant protection experience gains of 2 3 percent

Box: Event Risk Premia and Additional Incentives for Leveraging

The reduction in leverage disincentives brought about by event risk can be measured more precisely by calculating the marginal increase in debt cost resulting from additional leverage Specifically, one can calculate the increase in total risk premiums paid as a result of a 1 percentage point increase in the debt ratio (total risk premiums paid at [x + 1] percent debt ratio less total risk premiums paid at x percent debt ratio). Mathematically, this can be expressed as follows

$$TRP = LEV * PRE = LEV * \{ a + b * (LEV^{2.6})\},$$

where TRP = total risk premium as percentage of firm value

LEV = debt ratio

PRE = required risk premium

The relation can be expressed as follows.

$$\text{Marginal cost of leveraging} = \frac{dTRP}{d\,LEV} = a + (3.6b * LEV^{2.6})$$

The marginal increase in total risk premia associated with a percentage point increase in the debt ratio is shown as the marginal cost curve in the chart Note that this marginal increase is on average about 40 percent lower for 1988 than for 1980 (the 1989 curve is omitted for readability) Put another way, the marginal penalty for leveraging exacted by the bond market has, on average, fallen by about 40 percent because of event risk.

Let us now compare the marginal risk premia cost of leverage with the marginal tax benefits of leverage. The marginal tax benefit of leverage is defined as the increase in the total tax shield resulting from a 1 percent increase in the debt ratio.

The total tax shield can be expressed as

$$t_c * \{ LEV * [r_f + PRE]\} = t_c * \{ LEV * [r_f + (a + b * LEV^{2.6}]\},$$

where t_c = top bracket corporate tax rate

r_f = risk-free interest rate

The marginal change in the total tax shield with respect to leverage is then:

$$t_c * \{ r_f + a + (3.6b * LEV^{2.6})\}.$$

If there were no risk premia, then the marginal tax benefit function would be a flat line equal to the riskless rate multiplied by the corporate tax rate Since risk premia are increasing, however, marginal tax benefits are an increasing function of leverage

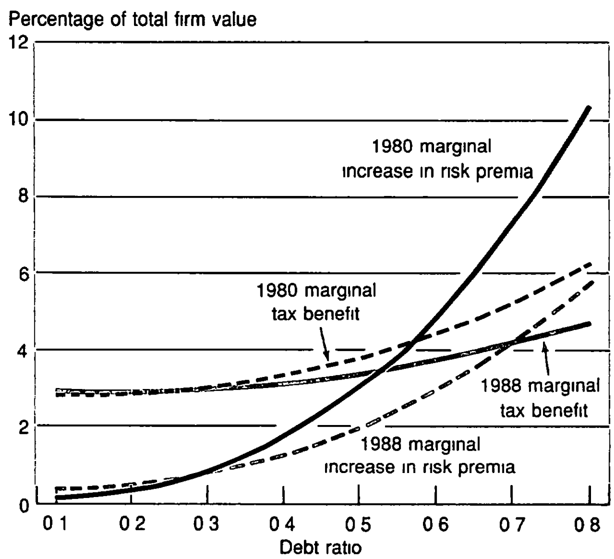
The marginal tax benefit curve is calculated using 1989 tax rates (34 percent, state and local rates not considered) in each of the years This step is taken in order to isolate the event risk premia effects from tax effects that occurred over the same period, since a shift in corporate tax rates can move the marginal benefit curve in a way that amplifies or dampens the effect of the event risk–induced shift of the marginal cost curve.

The chart shows the marginal risk premia cost curves against marginal tax benefit curves for the 1980 and 1988 samples Note that the marginal tax benefits of leveraging are slightly higher in the 1980 case because of the steeper risk premium schedule For the comparison we use a riskless interest rate of 8 percent, roughly the rate prevailing in the three time periods

The "optimal" debt ratio is reached at the point where the marginal cost and benefit curves intersect, although factors such as strategic incentives for debt are not reflected in the ratio. It is still worth noting, however, that the "equilibrium" debt ratio suggested by the model rises from 0 56 in 1980 to 0 69 in 1988, and to 0.72 in 1989 (not shown on chart). The 1980 value is a reference point, not an actual historical estimate, because it is calculated using 1989 tax rates.

Note that the optimal debt ratio also depends on the interaction of equity costs and the debt ratio Appendix B develops a model of this relation that suggests that the optimal debt ratio is almost completely determined by the equation of the marginal costs and benefits of debt.

Marginal Tax Benefits versus Marginal Increases in Total Risk Premia



Note Calculations assume an 8 percent riskless interest rate losses are less than 8 percent of stockholder gains.

Event studies, however, measure only part of the bondholder loss from buyouts since they only consider the impact upon directly affected bondholders. Costs of event risk that follow the upshift in the risk premium curve are incurred by firms not involved in buyouts—to the extent that they have to refinance at higher interest rates—and by the bondholders of these firms—to the extent that the value of their longer term debt holdings is eroded by unexpected rises in event risk. These losses should be added to the losses of bondholders of the involved firms. Our conservative estimate of a 15 basis point upshift in the risk premium curve represents (at a 10 percent capitalization rate) a 1.5 percent discount in the value of bonds issued by uninvolved firms. Again, this loss applies to all bond-issuing firms, not just those involved in buyouts. It appears, therefore, that indirect bondholder and issuer losses from event risk are far greater than the direct losses borne by bondholders of buyout targets.

Implications of event risk premia

The advent of event risk premia has had several implications for corporations. First, firms now find it much less attractive to finance themselves largely through equity. Thus financial market developments which would appear to have increased the range of options for corporate treasurers by making high leverage more feasible may have actually narrowed the range by making the choice of low to moderate leverage more expensive. Second, the increase in the cost of debt for firms of low to moderate leverage raises the possibility that the corporate cost of capital has risen. Third, event risk premia tend to amplify the effects of tax measures aimed at decreasing corporate leverage. Each of these implications is examined in greater detail below.

Incentives for leveraging

Having learned the hard lesson that the strong can abruptly become the weak, investors in the bond market tend to discount the bonds of financially strong firms As a result, firms of low and moderate leverage find themselves already paying for a portion of any increase in leverage they may be contemplating. In other words, the change in the market environment has moderated the increase in the interest rate associated with an increase in leverage.

The Box calculates a measure of the incentives to shift to debt finance created by event risk premia. The estimated marginal costs of leverage resulting from increased risk premia are compared to an estimate of the marginal tax benefits of leverage resulting from the deductibility of interest The calculation in the example implies that, when all other factors are held constant, the debt ratio balancing tax benefits and escalating debt costs rises by 16 percentage points over the period 1980-89. Although the impact of this change must be considered in conjunction with other influences on capital structure, these calculations suggest that event risk premia may have been responsible for inducing sizable increases in debt during the 1980s.

Event risk premia and the cost of capital

The risk that well-capitalized firms might be leveraged up raises the cost of debt by increasing the risk premium paid by all but the most leveraged borrowers. In turn, the rising cost of debt could conceivably entail an increase in the overall cost of capital.[14]

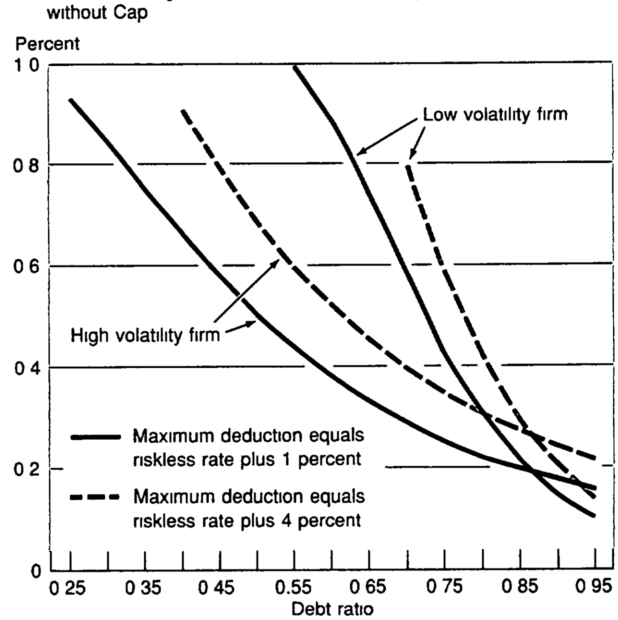
One can argue to the contrary that if share prices rise to reflect the potential for substantial gains to shareholders from leveraging transactions, the higher cost of debt might be offset But while this argument is attractive in the abstract, practical considerations

[14]Albert Ando and Alan L Auerbach, "The Cost of Capital in the U S and Japan A Comparison," *Journal of Japanese and International Economics*, vol 2 (1988), pp 134-58, and Robert N McCauley and Steven A Zimmer, "Explaining International Differences in the Cost of Capital," this *Quarterly Review*, Summer 1989, pp 7-28

Chart 4

Incentives for Debt Financing under an Interest Deduction Cap

Marginal Change in Total Interest Deduction
Ratio of Change with Interest Deduction Cap to Change without Cap



weigh against it. Consider the position of a treasurer in a well-capitalized firm who watched in the 1980s as debt-financed takeovers drove up the yields on prime corporate debt relative to Treasury yields. Would that treasurer be easily persuaded that the measurable costs of event risk premia could be offset by the much less tangible benefits of lower equity costs?

Tax policy

Tax policies to limit corporate leverage are more effective in the presence of event risk premia. Any weakening in the tax incentives for debt financing as opposed to equity financing will tend to mitigate event risk Therefore, in addition to its direct effect on leverage incentives, a reduced tax incentive for debt finance will work to steepen the risk premium schedule. Even policies aimed at reducing extreme leverage will raise the marginal cost of debt for firms with low or moderate leverage. As an example, consider the elimination of deductibility for corporate interest payments paid at a rate above some specified spread over Treasury yields. A cap on interest deductibility reduces the incentive for leverage over a surprisingly wide range of leverage. The effect of such a cap may be measured as the ratio of the marginal increase in the value of the interest deduction with the cap to the value of the interest deduction without the cap (Chart 4, Appendix 2 provides the complete derivation). For instance, even a 4 percent cap reduces the marginal tax benefit by over one-half for a low-volatility firm with a debt ratio of 80 percent A cap becomes effective at lower leverage for a firm with a more volatile cash flow because bondholders demand compensation for the greater risk of debt service difficulties entailed by such a cash flow

By sharply reducing the worst risks to bondholders and thereby steepening the risk premium schedule, even caps set quite high can work indirectly to discourage additional debt over the entire range of leverage. This observation does not constitute a case for such a policy, which would affect firms that are risky for reasons other than high leverage and which might pose administrative challenges. But factoring individual changes in risk premia into the assessment of tax policy can show it to have greater potency than has been acknowledged in the past.

Conclusion

Analysis of U.S. corporate bond yields demonstrates that investors learned to demand a higher premium over U.S. Treasury yields in the course of the 1980s. This rise in the risk premium schedule resulted from mergers and acquisitions activity that made maintaining a given firm's credit standing more uncertain. Debt-financed changes in corporate control changed the bond market environment for less leveraged firms and made it more difficult for those firms to keep debt costs low by maintaining a comfortable equity cushion.

The least leveraged firms experienced the largest increase in borrowing costs relative to Treasury yields. Consequently, the marginal cost of leverage declined enough to induce firms, according to one calculation, to increase the debt fraction of total financing by 16 percentage points.

The rise in the cost of debt has been significant. The fear of debt-financed takeover activity has caused a percentage reduction in the value of all U.S. corporate bonds that is comparable to the percentage reduction in the value of bonds immediately affected by takeover events. The effect of event risk premia on the cost of capital, however, is ambiguous. The demonstrated rise in debt costs for less leveraged firms must be balanced against the effect of takeover premia in cheapening the cost of equity.

The analysis suggests that tax policy can exert a particularly powerful effect in the presence of event risk premia. Any reduction in the tax benefits of leveraging, even narrowly drawn reductions, can indirectly discourage leveraging across the spectrum of firms.

Steven A. Zimmer

Appendix A: Characteristics of Sample Firms

Callable bonds

The yields of the corporate bonds in the sample are not fully comparable with Treasury yields because most of them reflect some type of call provision To make the yields on different corporate bonds comparable, it is necessary to make price adjustments for the callability of certain issues

The call option on corporate bonds can be priced using contingent claims theory if certain assumptions are made about adjustment to a "natural" interest rate We elect not to use this method, however, because it generally yields poor results.† The method's basic weaknesses are compounded by heterogeneous perceptions of "natural" interest rates and debt issuance costs, which drive a wedge between holder and issuer call valuation

To get around the call valuation problem, we use non-callable debt whenever possible. If forced to value a call option, we try to use debt with call valuations that are deeply out of the money, although in some cases we use yield to call on debt that is clearly going to be called

In valuing calls, we follow a conservative application of the call value techniques of Kim, Ramaswamy, and Sundaresan and of Gastineau.‡

†The best attempt to value debt through a contingent claims analysis is probably E Phillip Jones, Scott P Mason, and Eric Rosenfeld, "Contingent Claims Valuation of Corporate Liabilities Theory and Empirical Tests," National Bureau of Economic Research, Working Paper no 1143, June 1983

The debt pricing in this study returns an average absolute error of 6 05 percent on market value of debt, although the estimates with an average prediction error of 0 64 percent, are relatively unbiased

‡Joon Kim, Krishna Ramaswamy, and Suresh Sundaresan, "Valuation of Corporate Fixed Income Securities," working paper, December 1986, and Gary L Gastineau, *The Options Manual* (New York McGraw Hill, 1988), pp 347-64

Maturity of bonds in sample

The regression results are potentially sensitive to the maturity of the bonds used to determine the risk premium In particular, the maturity of the bonds in the sample can affect the slope of the curve relating risk premia to leverage.

If the risk premium is a function of the time to maturity, then any systematic relationship between time to maturity and leverage will bias the slope of the risk premium curve. For example, consider a situation in which low-debt firms in one sample year tend to have shorter maturity debt than their more leveraged counterparts If risk premia are an increasing function of time to maturity, then the risk premia of the less leveraged firms will be biased downward relative to the risk premia of the more leveraged firms, steepening the apparent slope of the risk premium schedule.

Ideally, the risk premia in our sample should be independent of time to maturity Table A1 shows the results obtained by regressing time to maturity on leverage in each of the sample years The coefficient on time to maturity is negative in each of the regressions. The coefficients for the 1988 and 1989 cases are of much greater magnitude than the coefficient for the 1980 case, which is not significantly different from 0.

The importance of these results depends upon the relationship between time to maturity and the required risk premium Theoretical work by Merton and by Pitts and Selby concludes that for investment grade bonds, the required risk premium rises with time to maturity out to one year and then declines very slowly afterward.§ The required risk premium on lower grade debt declines asymptotically with time to maturity. Later work by Sarig and Warga provides strong empirical support for these

§Robert C Merton, "On the Pricing of Corporate Debt The Risk Structure of Interest Rates," *Journal of Finance*, vol 29 (1974)

Table A1

Characteristics of Sample Groups

	1980	1988	1989
Average debt ratio	0 540	0 529	0 562
Average risk premium	1 220	0 983	1 190
Average earnings rate variance	0 012	0 019	0 019
Average time to maturity of debt	14 25	11 36	9 62

Table A2

Regressions of Leverage on Time to Maturity

Debt ratio = $a + b \cdot$ (time to maturity of debt)

	1980	1988	1989
a	0 5551	0 6556	0 6307
b	−0 00003	−0 01113	−0 00714
error of *b*	0 00004	0 00456	0 00481
R^2	0 0267	0 1171	0 0466

Appendix A: Characteristics of Sample Firms *(continued)*

theories about the time pattern of the premia.||

Since the average time to maturity of bonds in our sample is roughly ten years (Table A2), we can treat the risk premium as a downward sloping function of time to maturity. Since the coefficient on time to maturity is negative in each sample period, we know from our earlier argument that the slope of the risk premia curves in each of the sample periods is biased upward Further, this bias is stronger for the 1988 and 1989 samples, suggesting that we underestimate the actual extent of the flattening of the risk premium curve.

Timing of rate quotations

As noted in the article, shifts in interest rates around the sampling period can bias the estimated intercept of the risk premium curve This occurs because Treasury debt is more heavily traded than corporate debt, with the result that the observed yields on corporate securities are less current than those on government securities.

Examination of interest rate movements immediately before the observation period helps to determine the direction of the bias — if interest rates are falling in the period before the observation, then the intercept will be biased upward. The patterns of interest rate movements in the period preceding the sample date have additional significance because the spread errors induced by them will depend upon how recently the issue was traded and will therefore vary across debt issues. This does not bias the slope of the risk premia curve, but it does increase the variance of the individual observations around the curve and is a potential source of estimation error

Chart A1 traces the path of changes in the yield to maturity of the benchmark five-year government bond (noncallable, five years to maturity, closest to par coupon) over the week ending in the observation date Note that in 1980, 1988, and 1989 the Treasury yields are similar and are relatively steady for the week preceding the sample period The rate rises somewhat in 1980, suggesting an underestimation of the risk intercept, and falls in 1989, suggesting an overestimation. The rate is steady in 1988. As a rough estimation of the intercept bias introduced by the rate movement, we take the average of the rate observations prevailing in the week before the observation date and compare it to the rate prevailing on the observation date

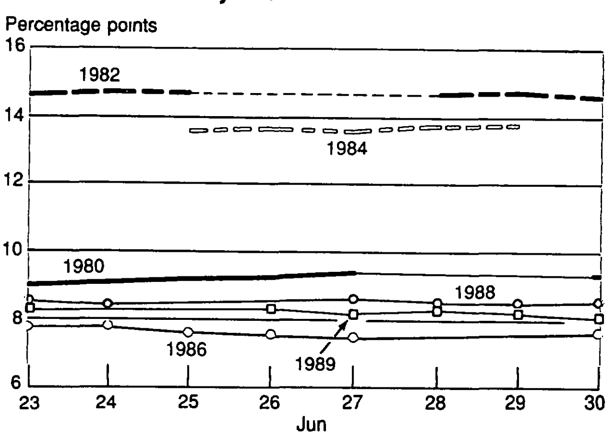
||C G C Pitts and M J P Selby, "The Pricing of Corporate Debt A Further Note," *Journal of Finance*, vol 38 (1983), and Oded Sarig and Arthur Warga, "Some Empirical Estimates of the Risk Structure of Interest Rates," *Journal of Finance*, December 1989

	Five-Year Rate	
Year	Last Week Average	Observation Date
1980	9 19	9 32
1988	8 47	8.46
1989	8 24	8 13

The difference between the average of the week's observations and the final observation may be taken as a rough measure of the intercept bias on the risk premium curve. The 1980 curve is therefore biased downward roughly 14 basis points relative to the 1988 curve, and 24 basis points relative to the 1989 curve

Chart A1

Five-Year Treasury Rates



Note Rates are yield to maturity on par coupon Treasury debt with five years to maturity

Liquidity premia

The yield difference between Treasury debt and certain government agency debt can be used as a measure of liquidity since some agency debt is essentially riskless, but less liquid than Treasury debt The best agency for this calculation is probably the Federal National Mortgage Association (FNMA), which has large issues of nearly riskless straight debt with coupon values similar to Treasury debt. FNMA debt does pose two problems, however. First, its credit quality is thought to have improved slightly between 1980 and 1988, biasing the observed liquidity spread down slightly Second, FNMA debt, unlike Treasury debt, is not deductible at the state

Appendix A: Characteristics of Sample Firms *(continued)*

and local level Because federal personal income tax rates have fallen since the early 1980s, the effective tax on FNMA debt has risen relative to Treasury debt, biasing the observed liquidity premium upward # Nevertheless, since the two potential sources of error appear small and work in opposite directions, we will assume that they can be ignored

Chart A2 shows the FNMA/Treasury spread for comparable ten-year debt• over the last two weeks of June in each of the years. The observed liquidity premium is clearly higher for 1980 than for 1988 or 1989 Over the final week of June, the spread averaged 52 basis points in 1980 as against 33 basis points in 1988 and 26 basis points in 1989

The liquidity spreads on FNMA debt are consistent with our observations of liquidity spreads on World Bank debt. World Bank debt is comparable to FNMA debt but is characterized by lower issue volume and liquidity, particularly for longer maturities

At the end of June 1980, five-year World Bank bonds were yielding 81 basis points more than comparable Treasuries †† This difference fell to 47 basis points in 1988 and 63 basis points in 1989 This drop again represents a downward shift of about 25 basis points in the liquidity premium between the earlier and later periods

#The tax rate on FNMA debt exceeds the tax paid on Treasury debt by the effective state and local tax rates This difference varies with the tax status of the holder We can express the effective state and local rate for a given tax bracket as $t_s * (1 - t_f)$, where t_s is the combined state and local tax rate and t_f is the federal tax rate The subtraction of the federal rate reflects the deductibility of federal taxes from state and local taxes Since the top bracket personal federal tax rate

 # continued
has fallen from 70 percent to 34 percent in the period between the early and late sample periods, we may assume that federal rates have fallen more than statutory state and local rates and that the effective state and local rate has therefore risen

•The yield to maturity is for debt with similar coupons that matures in ten years from the observation date

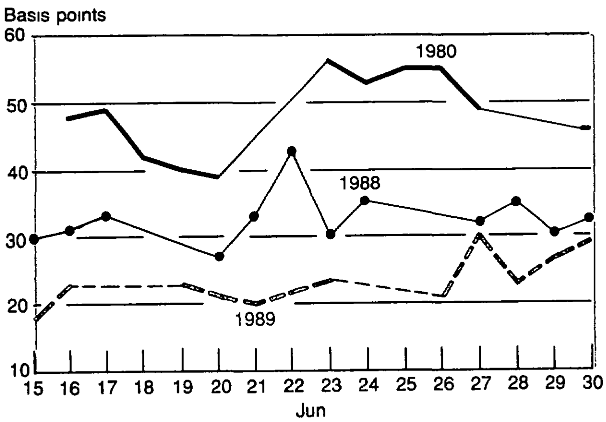
††Rate comparisons use yields on World Bank debt for the last day in June on which the issue traded The Treasury rate is taken as the yield to maturity on comparable (five years to maturity, similar coupon characteristics) Treasury debt using the average of several observations from the day of World Bank debt observation

Date	Treasury Yield	World Bank Yield	Spread
6-24-80	8 91	9 72	0 81
6-29-88	8 44	8 91	0 47
6-28-89	8 25	8 88	0 63

Chart A2

Liquidity Premia

Yield on Ten-Year Federal National Mortgage Association Debt less Yield on Ten-Year Treasury Debt



Note Rates are yields on comparable bonds with ten years to maturity

Appendix B: The Risk Premium Curve and the Optimal Debt Ratio

The cost of equity and the optimal debt ratio

We define the cost of capital as the weighted average of debt and equity costs The firm determines the optimal debt ratio as the debt ratio that minimizes CK, the cost of capital (more properly, the "cost of funds").

$$CK = (LEV * D) + \{(1 - LEV) * E\},$$

where LEV is the debt ratio, D is the real after-tax cost of debt, and E is the average cost of equity

The firm solves

$$\frac{d\,CK}{d\,LEV} = (D - E) + \left(LEV * \frac{dD}{d\,LEV}\right) + \left((1 - LEV) * \frac{dE}{d\,LEV}\right) = 0.$$

The first term represents the simple substitution of a unit of equity for a unit of debt. The second term represents the resulting change in average debt costs multi-

Appendix B: The Risk Premium Curve and the Optimal Debt Ratio *(continued)*

plied by the size of the debt share (the term $\frac{dD}{d\,\mathrm{LEV}}$ is simply the slope of the risk premium curve on an after-tax basis), and the third term represents the resulting change in average equity costs multiplied by the size of the equity share.

We can prove that the equilibrium debt ratio used in the text satisfies the above equation if we make certain assumptions about a CAPM model The first step is to rewrite D, the real after-tax cost of debt, as $D = r_r - (t_c * i_r)$, where r_r is the real interest rate facing the firm, t_c is the corporate tax rate, and i_r is the nominal interest rate facing the firm. We can then rewrite our optimal leverage equation

$$\frac{d\,\mathrm{CK}}{d\,\mathrm{LEV}} = (r_r - E) + \left(-(t_c * i_r) + \mathrm{LEV} * \frac{dD}{d\,\mathrm{LEV}}\right) + \left((1 - \mathrm{LEV}) * \frac{dE}{d\,\mathrm{LEV}}\right) = 0$$

The second term in the equation is now simply the difference between the marginal cost of leveraging and the marginal tax benefit. At the equilibrium point calculated in the text, these two values are equal, and consequently the second term disappears. We now have to show that the first and third terms are equal.

If we accept CAPM, we can write $E = r_f + \beta * (E_m - r_f)$, where r_f is the risk-free rate of interest and E_m is the

Table B1

Required Risk Premium as a Function of Leverage

Regression Equation	a_0	b	Standard Error of b	Adjusted R^2	Degrees of Freedom
PRE = a_0 + b•LEV					
1980	−1 281	4 700	1 193	0 306	32
1988	−0 448	2 701	0 580	0 311	45
1989	−0 325	2 700	0 785	0 190	45
PRE = a_0 + b•LEV$^{1\,5}$					
1980	−0 585	4 478	1 045	0 345	32
1988	−0 025	2 528	0 528	0 322	45
1989	0 149	2 389	0 682	0 197	45
PRE = a_0 + b•LEV$^{2\,0}$					
1980	−0 227	4 635	1 000	0 383	32
1988	0 199	2 561	0 530	0 327	45
1989	0 397	2 299	0 651	0 200	45
PRE = a_0 + b•LEV$^{2\,2}$					
1980	−1 126	4 752	0 994	0 398	32
1988	0 263	2 601	0 539	0 326	45
1989	0 468	2 288	0 648	0 199	45
PRE = a_0 + b•LEV$^{2\,4}$					
1980	−0 041	4 890	0 994	0 413	32
1988	0 318	2 652	0 551	0 325	45
1989	0 527	2 285	0 648	0 199	45
PRE = a_0 + b•LEV$^{2\,6}$					
1980	0 032	5 046	0 996	0 428	32
1988	0 366	2 710	0 566	0 323	45
1989	0 579	2 288	0 650	0 198	45
log(PRE) = a_0 + b•log(LEV)					
1980	1 326	2 482	0 506	0 411	32
1988	0 625	1 262	0 225	0 399	45
1989	0 459	0 895	0 310	0 137	45

Notes PRE = risk premium over riskless rate for debt issues of individual corporations, LEV = debt ratio = market value of debt over total (debt plus equity) firm market value

Appendix B: The Risk Premium Curve and the Optimal Debt Ratio *(continued)*

market equity return The cost of equity then varies with leverage through the beta coefficient. The beta coefficient varies with the debt ratio as follows $\frac{d\beta}{d\,LEV} = \frac{1}{1-LEV}$. Here we assume that the firm has systematic asset risk that is completely absorbed by the equity, hence the equity beta varies with the inverse of the equity ratio In reality, we would expect the beta coefficient for a firm's debt to increase with leverage, accounting for some of the systematic asset risk, but bond betas tend to be very low and can be ignored for our purposes

We can now write $\frac{dE}{d\,LEV} = \frac{(E_m - r_f)}{1-LEV}$ The third term of the optimum debt equation then becomes $(E_m - r_f)$ Setting the second term of the optimum leverage equation equal to zero, we obtain $\frac{d\,CK}{d\,LEV} = (E_m - E) + (r_r - r_f)$ For a market representation of firms we have $E_m = E$ The value then becomes $(r_r - r_f)$, a positive but very small number because it is taken over a differential change in leverage. Recall that our equation for $\frac{d\beta}{d\,LEV}$ overlooked the fact that not all of the systematic asset volatility would remain in equity as we leveraged Adjusting for this omission would lower the value of the third term and move the sum of the first and third terms toward zero In addition, the extent to which systematic asset volatility is not absorbed by equity is a positive function of leverage, just as $(r_f - r_m)$ is. This suggests that the relation between the first and third terms is

Table B2

Required Risk Premium as a Function of Leverage and Cash Flow Volatility

Regression Equation	a_0	b_1	b_2	Standard Error of b_1	Standard Error of b_2	Adjusted R^2	Degrees of Freedom
$PRE = a_0 + b_1 \cdot LEV + b_2 \cdot VOL$							
1980	−1 47	4 24	38 00	1 19	23 05	0 341	31
1988	−0 52	2 22	17 86	0 54	5 50	0 431	44
1989	−0 63	2 49	21 99	0 74	8 25	0 287	44
$PRE = a_0 + b_1 \cdot LEV^{1\ 5} + b_2 \cdot VOL$							
1980	−0 83	4 07	35 85	1 05	22 51	0 375	31
1988	−0 17	2 09	17 71	0 50	5 46	0 441	44
1989	−0 20	2 22	22 21	0 64	8 19	0 296	44
$PRE = a_0 + b_1 \cdot LEV^{2\ 0} + b_2 \cdot VOL$							
1980	−0 49	4 24	33 75	1 01	21 96	0 408	31
1988	0 00	2 12	17 75	0 50	5 42	0 446	44
1989	0 01	2 16	22 53	0 61	8 14	0 303	44
$PRE = a_0 + b_1 \cdot LEV^{2\ 2} + b_2 \cdot VOL$							
1980	−0 39	4 36	32 92	1 00	21 73	0 422	31
1988	0 05	2 16	17 82	0 51	5 42	0 447	44
1989	0 08	2 16	22 67	0 61	8 13	0 304	44
$PRE = a_0 + b_1 \cdot LEV^{2\ 4} + b_2 \cdot VOL$							
1980	−0 31	4 50	32 11	1 01	21 50	0 435	31
1988	0 10	2 20	17 90	0 52	5 41	0 447	44
1989	0 13	2 16	22 83	0 60	8 12	0 306	44
$PRE = a_0 + b_1 \cdot LEV^{2\ 6} + b_2 \cdot VOL$							
1980	−0 23	4 65	31 30	1 01	21 27	0 448	31
1988	0 13	2 26	18 01	0 53	5 41	0 447	44
1989	0 17	2 17	22 99	0 61	8 11	0 306	44
$\log(PRE) = a_0 + b_1 \cdot \log(LEV) + b_2 \cdot VOL$							
1980	2 31	2 48	0 21	0 51	0 24	0 407	31
1988	1 70	1 13	0 28	0 22	0 11	0 462	44
1989	1 47	0 86	0 25	0 31	0 15	0 166	44

Note VOL = Variance of earnings before interest and tax payments

Appendix B: The Risk Premium Curve and the Optimal Debt Ratio *(continued)*

stable.

Even if one does not accept all of the assumptions of the CAPM model, it should be clear that the methods employed yield a fairly consistent estimate of the optimal debt ratio Nevertheless, modelling equity costs as a function of leverage is a subject requiring further empirical work

Generating a risk premium schedule

A schedule of risk premia is generated for help in determining the correct functional form to use in the risk premia regressions and for use in evaluating the effectiveness of an interest deduction cap

The risk premium curve is calculated for a high-volatility firm (annualized standard deviation of total firm value = 0.310), a low-volatility firm (annualized standard deviation of total firm value = 0 220) and a very low (takeover target) volatility firm (annualized standard deviation of total firm value = 0 175) †

The calculations assume the following 1) the standard deviation of total firm market value (debt plus equity) is constant over the life of the debt; 2) the riskless interest rate is 7 percent, and 3) the debt ratio is determined after solving for the required risk premium ‡

The model uses seven-year zero coupon debt to approximate the duration of ten-year coupon debt The risk premia on the medium-term debt for the high- and low-volatility firms form risk premia curves similar to those estimated in the text

Estimating the functional form of the risk premium curve

We perform the following regressions on the generated points: log (risk premium) = $a + b \cdot$ log (debt ratio) Some results are given below:

Volatility	Time to Maturity	a	b	R^2
0.175	3 years	−33.04	7 718	0.9954
0 220	7 years	−14.79	3 711	0 9991
0.310	7 years	− 8 80	2 490	0.9993

We then perform additional regressions on the generated points risk premium = $a + b \cdot$ (debt ratio)^k, letting k take on different values. Some results for a firm with low volatility (0 220) and seven-year zero-coupon debt follow

k	a	b	b error	R^2
1	−5 33	12 21	0.684	0 8886
1 5	−3 00	10 46	0 467	0 9263
2	−1.82	9.86	0 340	0 9546
2 4	−1 21	9.74	0.265	0 9712
2 6	−0 97	9 75	0 232	0 9780

The values generated by the Black-Scholes model are considered only over the debt ratio interval of 0 10 to 0 90. The exponential relationship with the generated points begins to break down with debt ratios greater than 0.90

The results obtained from regressions on curves generated from the Black-Scholes model serve as a guide for regressions on the sample data The shape of the risk premium curve is empirically estimated using various functional forms. Table B1 presents the results obtained by regressing the risk premium solely on the debt ratio, and Table B2 gives the results obtained by regressing the risk premium on the debt ratio and cash flow variance

Estimating the effects of an interest deduction cap

To determine the effect of an interest deduction cap on the marginal tax incentive to leverage, we begin by estimating the risk premium schedule facing firms Although a schedule of this kind was developed in the text, the estimation of the risk premium curves excluded the use of firms with debt ratios in excess of 90 percent For this reason, and because the exponential form in the text tends to underestimate risk premia for debt ratios in excess of 90 percent, we will use the risk premium curve generated by a log/log regression on a curve generated by a Black-Scholes model, as calculated earlier in this appendix

Next, we define the benchmark rate (i_b) as the highest coupon rate that the firm is allowed to deduct for interest expense The nominal interest rate paid by the firm is i_r The deductible interest rate (i_d) of a firm under the cap can be defined as

$$i_d = i_r \quad \text{if } i_r < i_b$$
$$i_d = i_b \quad \text{if } i_r > i_b$$

†Estimates of volatility of total firm value may be found in E Phillip Jones, Scott P Mason, and Eric Rosenfeld, "Contingent Claims Valuation of Corporate Liabilities Theory and Empirical Tests," National Bureau of Economic Research, Working Paper no 1143, June 1983

‡Arbitrage pricing is done under the assumption that the current value of the firm equals 100 The value of equity is calculated as a call on total firm value, the value of debt is calculated as a residual Different debt ratios are obtained by varying nominal redemption prices of debt The risk premium is calculated as the yield required on the market value of debt in order for debt to reach redemption price, less the riskless rate Mathematically, the risk premium must satisfy

$$(1 + i_f + \text{PRE})^t \cdot \text{LEV} = K,$$

where i_f is the riskless interest rate, PRE is the required risk premium, t is the life of debt, and K is the face value redemption price of debt

Solving for PRE in continuous time, we obtain

$$\text{PRE} = t^{-1} \cdot (\log K - \log \text{LEV}) - i_f$$

Appendix B: The Risk Premium Curve and the Optimal Debt Ratio *(continued)*

Without the cap, we always have $i_d = i_r$.

The annual tax savings from the debt shield is simply the deductible rate times the debt ratio of the firm, LEV:

$$i_d * \text{LEV} * t_c.$$

The marginal change in the total interest deduction with respect to a change in the debt ratio is:

$$\text{(B-1)} \qquad \left\{\left(\frac{d\, i_d}{d\,\text{LEV}} * \text{LEV}\right) + i_d\right\} * t_c$$

Without the interest cap (the unrestricted case) we can substitute i_r for i_d, and $\frac{d\, i_d}{d\,\text{LEV}}$ in (B-1) will always be positive. With the interest cap (the restricted case), the value of (B-1) will equal that of the unrestricted case when $i_r < i_b$. If $i_r > i_b$, then in the restricted case the value for (B-1) becomes $(i_b * t_c)$. The values shown in Chart 4 in the text are simply the ratio of $(i_b * t_c)$ to the value calculated in (B-1) for the unrestricted case over the range for which $i_r > i_b$.