The Pricing and Hedging of Market Index Deposits

Commercial banks and other financial institutions are currently devising new ways of tailoring their debt instruments to the portfolio needs of their customers A recent innovation has been deposits with returns linked not to market interest rates but to the performance of indexes or prices in markets other than those for fixed-income instruments. These market index deposits (MIDs) offer a specified portion of any gain in a market index and guarantee a minimum return. Two U.S. banks have recently begun to issue MIDs: Chase Manhattan has deposits linked to the S&P 500 Index and Wells Fargo has deposits linked to the price of gold. This article explains the pricing of such deposits and the means by which risks to the issuing banks can be controlled by hedging.

The new deposits are similar to index and gold warrants issued in the international bond market. These warrants, however, have been available only to large investors. A few mutual funds also provide returns linked to gains in the S&P 500 and offer a floor on the value of shares. In addition, a major investment bank offers a menu of short-term instruments, called PIPs,[1] allowing investors to choose from various combinations of minimum returns and links to foreign currencies and interest rates, equity indexes, and commodities prices. MIDs differ from these other instruments in that they are both available in retail denominations and carry FDIC insurance.[2] While the focus of this paper is on banks issuing MIDs, the analysis also applies to other institutions offering instruments with characteristics of options.

Our analysis suggests that, barring a severe market downturn, banks can offer MIDs without exposing themselves to excessive risk In principle, they can price and hedge these deposits so that they cost no more than conventional deposits regardless of market performance. To hedge, banks need only purchase the appropriate options, which provide a payoff matching that of an MID When the right options arê unavailable, as is frequently the case in practice, banks may construct synthetic ones. The strategy of hedging with synthetic options, however, has yet to be tested by an extreme market move.

We first outline the hedging of a prototypical MID and the pattern of returns that can be generated by such a deposit A central choice that the bank must make is to determine the proportion of increases in the index that it will pass on to depositors We show that the issuing bank faces a trade-off between the maturity, guaranteed minimum return, and proportion of index gain that it can offer at any given cost. We illustrate this trade-off for the case of a deposit linked to the S&P 500 This trade-off will not remain constant over time, and we next demonstrate the extent to which market fluctuations would have affected the terms of a hypothetical MID between July 1983 and June 1987.

We also show the return that a depositor would have

[1]These are described in *Performance-Indexed Paper An Indexed Money Market Instrument*, by Rajiv Nanda and James Callahan, Salomon Brothers, July 1987

[2]The minimum for the Chase index deposit is $1,000 and for the Wells Fargo gold deposit, $2,500 Even the "retail" currency

Footnote 2 continued
warrants that Citicorp has recently issued can be exercised only in lots of about $7,000 each, compared to the $55 to $80 value of the option embedded in a one-year $1,000 MID that guarantees only principal

received over that period from an efficiently priced MID linked to the S&P 500. We contrast this return with the yield that would have been obtained from an investment in an index fund (i.e one whose performance equals that of the S&P 500) and with an investment in a conventional bank certificate of deposit (CD). Our analysis demonstrates that investors in an MID would have realized returns that were generally between those obtained on a conventional CD and those obtained in the stock market, and with risk characteristics that also fell between these two alternative investments.

We then discuss in greater detail appropriate hedging strategies for the issuing bank. Because the strategies proposed do not perfectly hedge the bank's exposure, we assess the risks to the hedging strategy that might result from unanticipated jumps in the market index. We suggest that the risks to any bank that offers such a deposit can be attenuated *provided the bank is able to execute its hedging strategy.*

Pricing the deposit

An MID will pay at maturity either (1) a guaranteed minimum return, or (2) a fixed proportion of any gain in the market index over the life of the deposit, whichever is greater. The guaranteed return may be positive, zero, or even negative.[3] We call the fixed proportion of market gain the "upside capture."[4] If the MID is efficiently priced and hedged, it will offer terms that will cost the bank neither more nor less than alternative sources of funds of the same maturity.[5]

If a bank were to offer such a deposit without hedging it, then the cost of the deposit would be unknown at the time the bank issued it. Only when the deposit matured, and the value of the market index became known, would the bank know its cost. If the market were to remain constant or fall, the cost of the deposit would be simply the guaranteed minimum return. If the index were to rise, however, the cost could exceed the guaranteed return and, in principle, rise without limit A bank could reduce the uncertainty about its cost of funds by hedging its exposure to such risk.

The bank could protect itself against a large increase in the index over the life of the deposit by purchasing call options on the index that expire at the same time as the deposit.[6] The calls would have a strike price corresponding to the guaranteed return on the MID. In the case of an MID that guarantees only principal, for example, the strike price would be equal to the initial index value.[7] The payoff on an index option depends only on the value of the index when the option is exercised and is independent of the previous movements of the index Since the return on an MID similarly depends only on the value of the index at maturity, an option is the ideal hedge instrument When the appropriate options are difficult to obtain, it may be cheaper for the bank to construct synthetic options by using other financial instruments, such as futures, sometimes in combination with purchased options We discuss this technique, known as "dynamic hedging," below and in Appendix 1.[8] The price of an option or the cost of a synthetic one will be determined primarily by market volatility, short-term interest rates, and the option's expiration date

For the deposit to be efficiently priced, the cost of calls purchased must be equal in present value to the potential interest payments saved—specifically, the differential between the conventional CD rate and the guaranteed minimum return on the MID The size of this differential therefore determines the amount that the bank can profitably invest in options to hedge its deposit The number of options the bank can purchase in turn determines the amount of upside capture it can profitably offer depositors. The upside capture will therefore depend negatively on both the guaranteed return and the price of a call option [9]

Suppose a bank wishes to raise $1000 in one-year funds by issuing either a 6 percent one-year CD or a one-year MID with a guarantee of principal only, that is, with zero guaranteed interest. The present value of the interest payment on the conventional CD would be $56.60 ($60/(1.06)). For the same cost, the bank could offer an MID and hedge by purchasing $56.60 worth of calls with a strike price equal to the current index value. If the price of one such "at-the-money" call were $113.20 per $1,000 of the underlying asset, the

[3]By guaranteeing a floor on the return that the depositor will receive, the bank is effectively offering its depositors "portfolio insurance" Portfolio insurance is discussed in Mark Rubinstein, "Alternative Paths to Portfolio Insurance," *Financial Analysts Journal*, July/August 1985, pp 42-55

[4]Upside capture is sometimes called the "participation rate"

[5]More precisely, the bank should equate marginal costs across all funding sources of any given maturity For example, the marginal cost of 6-month funds from an MID should be the same as that of 6 month funds from any other source

[6]A call option confers on its purchaser the right, but not the obligation, to purchase at the strike price the underlying index Settlement of an index option consists of a cash payment representing the difference between the current index value and the strike price Options, including index options, are discussed in Laurie S Goodman, "New Option Markets," this *Quarterly Review*, Autumn 1983, pp 35-47

[7]The precise relationship between the guaranteed return and the strike price is given in Appendix 2

[8]However, the realized cost and payoff of a synthetic option no longer has the desirable property of being independent of the path that the index follows before expiration

[9]The exact formula for the upside capture is provided in Appendix 2

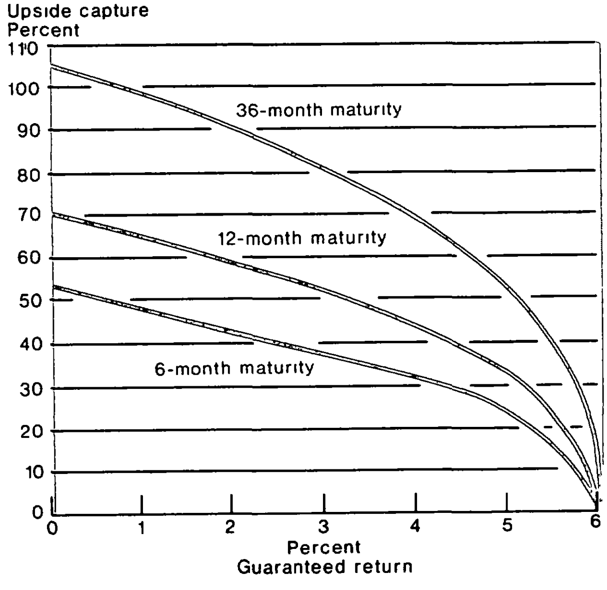
bank could purchase half a call. Because it could thus hedge only half the deposit, the bank would offer an upside capture of only one-half. If the index then rose over the year, the bank would exercise its half of a call to receive a payoff just sufficient to pay the MID depositor. If the index fell, the call would expire worthless while the depositor would be owed no interest. In either case, the MID would cost the bank the same as the conventional CD.

In principle, given MIDs that are efficiently priced, the relationship among upside capture, guaranteed return, and maturity is precise. Chart 1 shows the various amounts of upside capture that could be offered on an MID that would cost the bank the same as a conventional CD paying a 6 percent annual return. The three curves show the combinations of guaranteed annual return and upside capture for maturities of 6, 12, and 36 months[10] On any point on any of the curves, the bank's cost for the MID is the same 6 percent annual rate The lower the guaranteed return or the longer the maturity, the higher the upside capture.

[10]The example assumes that the CD rate and dividend yield are 6 percent and 3 5 percent, respectively, and that market volatility on the S&P 500 is 17 percent These assumptions are consistent with actual market conditions in the first half of 1987

Chart 1

Upside Capture, Guaranteed Return, and Maturity



Note The results are computed as described in Appendix 2 using the Black-Scholes model We assume a dividend yield of 3 5 percent and a marginal cost of funds of 6 percent

For each curve, upside capture falls to zero as the guaranteed return approaches 6 percent. If the bank guarantees a return of 6 percent—its assumed marginal cost of funds—it cannot offer any additional return linked to the index. For any given maturity, a lower guaranteed return implies a larger interest differential relative to a conventional CD and a larger sum to allocate to the purchase of options, and thereby greater upside capture. Given the guaranteed return, a longer maturity also allows greater upside capture, since the savings from the interest differential rise faster with time to expiration than do call prices.[11]

The top curve shows that if maturity is long enough, an MID can offer more than 100 percent of the gain in the stock index and still guarantee no capital loss. Such an MID, however, is not a way to beat the market. Since stock index gains represent only capital appreciation, the MID holder does not receive the dividends from the stocks underlying the index. Hence, the expected return to the depositor will be lower than that on an index fund. Chase Manhattan, for example, introduced an MID for large investors that initially offered 115 percent of the S&P 500 over three years with a guaranteed minimum return of zero.[12] Given a dividend yield of 3.5 percent on the S&P 500, the index would have to rise more than 30 percent a year for the Chase account to outperform an index fund. Since the index is unlikely to do so well for three years, investors in the account are sacrificing some expected return to protect the value of their principal.

Deposit behavior over time

To get a sense of the effect of actual market conditions on MIDs, we can observe how these deposits would have behaved over time had they been issued in the past. Such a demonstration serves to illustrate the risk and return characteristics of an MID in comparison with those of alternative instruments. In this section, we track the behavior of hypothetical 90-day MIDs with zero guaranteed return, assuming they had been issued each month from July 1983 through June 1987. First we illustrate the upside capture a bank could have offered. We then compare the performance of the MID with that of an index fund and a conventional CD.

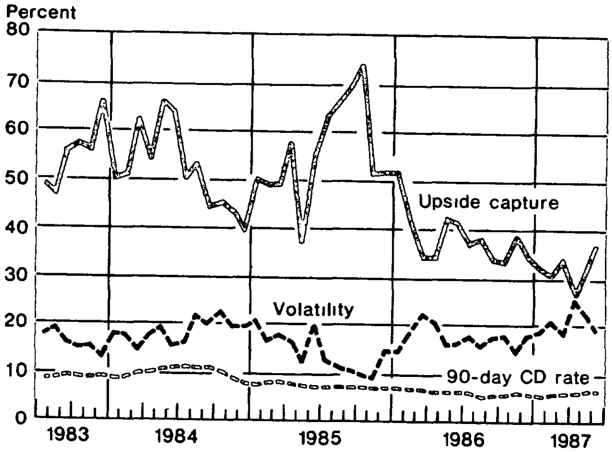
Upside capture—Chart 2 plots the monthly upside capture that a bank could have offered on the 90-day

[11]This property can be shown from the formula in Appendix 2 See also Chapter 5, "An Exact Pricing Formula," in John C Cox and Mark Rubinstein, *Options Markets* (Prentice-Hall, 1985)

[12]Salomon Brothers has issued SPINs offering the greater of a guaranteed return of 2 percent and 100 percent of the percentage increase in the S&P 500 over four years

Chart 2

Upside Capture and Its Determinants



Note The chart shows the upside capture of a 90 day MID with guaranteed return of principal, and its two key determinants the annual volatility implied by traded call option prices and the interest rate on conventional CDs

MID with zero guaranteed return. The feasible upside capture is chosen so as to cost the bank the same amount as a conventional 90-day CD issued in the same month.[13] The chart shows two principal determinants of upside capture: the volatility of stock returns and the interest rate on the conventional CD. The CD rate and upside capture are both generally high at the beginning of the sample period and low towards the end. This pattern reflects the fact that when CD rates are high, the bank's interest savings allow the purchase of more call options. Month-to-month variation in the upside capture, however, is dominated by movements in volatility,[14] which has a major influence on the price of the relevant call option. The chart also shows that a bank offering efficiently priced MIDs would be expected to change the terms of the deposit—at least one term among the guaranteed return, upside capture, and maturity—quite sharply over time.

Risk and return—Although an efficiently priced MID will cost the bank the same as a conventional CD, the yield that a depositor actually receives will not, in general, be expected to equal the CD rate. Indeed the MID would be expected to yield a return that is higher on average than that on a CD, but one that is more variable. Chart 3 plots the annualized returns on. (1) the S&P 500 with dividends reinvested,[15] (2) the 90-day MID (with the same upside capture as in Chart 2), and (3) a 90-day CD. The returns are computed ex-post, that is, they represent the yield that an investor would have realized over 90 days, rather than the yield the investor had expected at the time of the investment.

The chart illustrates vividly the differences between returns on the alternative investments. The role of the MID in putting a floor on the returns is apparent from the fact that the MID line never dips beneath zero. The cost of this floor protection is shown clearly by the fact that the MID investor would receive only around one-half of the positive yields that a stock market investor would obtain. It should be stressed, however, that had the market fallen over the period, rather than risen, the MID investor would have benefited more frequently from the floor protection provided by the MID. Between July 1983 and March 1987, the mean ex-post annualized yield on the efficiently priced MID was 9.3 percent, compared to 19.1 percent on the S&P 500, and 8.5 percent on the 90-day CD. The standard deviation for

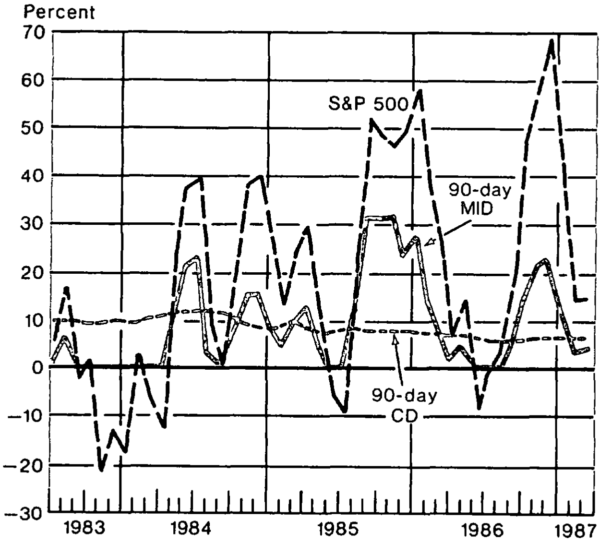
[13]The feasible upside capture is computed iteratively using the principle described in the previous section (and specified precisely in Appendix 2), together with the Black-Scholes formula

[14]The volatility measure is an average for the month of the volatility implied by current call option prices and the Black-Scholes formula

[15]Equivalently, the chart plots the yield that would be obtained by investing in an S&P 500 index fund

Chart 3

Ex-Post Return on the S&P 500, an MID, and a CD



Note The figure shows ex-post 90-day returns on the S&P 500 including dividends, a conventional CD, and an MID with no guaranteed interest rate the MID return was 10.1 percent, less than half that for the S&P 500, which was 23.0 percent [16]

Choice of hedging instrument

A bank issuing a market index deposit could hedge its risk exposure with a number of alternative instruments, used separately or in combination with each other. To hedge a deposit linked to a stock index, the bank could use the stocks comprising the index, index options, index futures, or options on index futures. To hedge a deposit linked to a commodity price, the bank could use the actual commodity, commodity futures, or options on commodity futures. We discuss below the advantages and disadvantages of these alternative hedging instruments.

Listed options—The hedging instruments that are the simplest conceptually would be exchange-traded European options[17] with maturities and strike prices matching exactly the terms of the deposit. If such options were available, the bank using them would be exposed to no hedging risk other than the credit risk of the exchange. The Chase and Wells Fargo deposits, for example, are issued weekly and should be hedged with European calls expiring each Tuesday, the day on which the deposits mature. While some listed index options are European, they expire at most once a month and are not available for maturities longer than five months, with most of the trading concentrated in the nearest months.[18] Listed options on index futures are American, expire only quarterly and are not available for longer than three quarters. Strike prices for either type of option on the S&P 500 are available only in increments of five index points The mismatch in maturities or strike prices would make the use of listed options more expensive than is necessary for the hedging bank's purposes. Hedging with an option expiring after the deposit matures, for example, would mean paying for some of the remaining time value of the option.

[16]These numbers can also be compared to an ex-post mean annual yield of 9 1 percent on a 90-day MID with a guaranteed minimum return of 1 percent In this case, the standard deviation was 9 3 percent

[17]A European option permits exercise only at the expiration date By contrast, an American option may be exercised on or before expiration

[18]Since American options allow early exercise, they may be more expensive than European options with otherwise identical terms For purposes of hedging an MID, however, early exercise would be necessary only if options with the same maturity as the MID were unavailable Hence, if both types of options had the right maturity, the bank would prefer the European Of course, if maturities cannot be matched precisely, American options would be preferable to European options, since the former can be exercised as deposits mature European options include those on the Institutional Index (on the American Stock Exchange) and the S&P 500 (on the Chicago Board Options Exchange)

Over-the-counter options—The bank could hedge with over-the-counter European options, which might be tailored to have exactly the right maturities and strike prices. This approach, however, is likely to be infeasible because of the lack of liquidity in this market. The bank would have to find suitable option writers each time it issued a deposit

Stocks or commodities—In the absence of appropriate options, the bank could create synthetic options by holding a portfolio of stocks or commodities and adjusting its position in response to price movements Using a method known as "dynamic hedging," the bank could, in principle, replicate the risk-return profile of any option by taking varying positions in the underlying asset and cash In the case of an index option, the underlying asset would be the portfolio of stocks making up the index In the case of a commodity option, the asset would be the physical commodity. We illustrate the operation of dynamic hedging in Appendix 1.

A shortcoming of dynamic hedging is that the method works imperfectly in practice. As we explain in the next section, the method is subject to tracking error and execution risk, which allow only an approximate replication of options. How close the approximation is depends on the skill of the hedger and the state of the market. Moreover, positions are revised so frequently in dynamic hedging that transactions costs are significant One practitioner has estimated that the transactions costs involved in replicating a one-year option on the S&P 500 with stocks would amount to 56 basis points [19] In commodity markets, transactions costs are likely to be even higher

Futures—Synthetic options could be created out of futures instead of the underlying asset Futures usually have the advantage of liquidity, the markets in the S&P 500 index futures and in gold futures are both highly liquid. As a result, it is relatively cheap to transact trades In the case of the S&P 500, for example, dynamic hedging implemented with futures is estimated to entail transactions costs only one-third those of hedging with stocks.[20] Futures, however, are one market removed from the market on which the relevant index or commodity price is based Since arbitrage between markets is imperfect, some mispricing of futures relative to the underlying asset is to be expected. Moreover, this mispricing—or "cash-futures basis"—works against the dynamic hedger. Despite this limitation, however, futures tend to be the preferred instrument for dynamic hedging because of advantages in liquidity and transactions costs

[19]See Rubinstein, "Alternative Paths"

[20]See Rubinstein, "Alternative Paths"

Risks of dynamic hedging

Dynamic hedging has become a familiar technique in portfolio insurance, and there is now some experience with its performance under moderate market conditions. The technique entails buying more of futures or the underlying asset as prices rise, and selling off as prices fall. Buying high and selling low in this way necessarily implies capital losses. In the absence of transactions costs, these losses should equal in present value the price of the option being replicated. However, two specific types of problems arise with option-replicating strategies: (1) tracking error and (2) execution risk.[21] We discuss these two problems in terms of hedging with futures for MIDs

Tracking error—Tracking error occurs when the hedger fails to hold the right positions in futures that will replicate the call option The exact positions are calculated from an options-pricing model. We consider below three conceptually distinct reasons why the hedger might fail to hold the correct positions: (1) inadequacies of the model used, (2) variations in the cash-futures basis, and (3) incorrect forecasts of index volatility or short-term interest rates. One practitioner estimates the cost of tracking error from the three sources together for hedging with index futures to be typically about 20 basis points.[22] This cost is in addition to transactions costs, which, as we have pointed out, are also significant in dynamic hedging

First, the assumptions underlying the options pricing model may be violated, in which case the model may fail to simulate the actual market precisely. Second, the cash-futures basis is such that futures generally trade at a premium when prices are rising and at a discount when prices are falling. Since the dynamic hedger buys as prices rise and sells as prices fall, this systematic mispricing adds to the cost of the strategy

The third source of error is incorrect estimation of volatility or interest rates. If volatility is underestimated at the time the MID is priced, the deposit will end up costing more than anticipated. The hedger will be buying at high prices and selling at low prices more frequently than anticipated, thereby incurring greater capital losses than expected On the other hand, if volatility is overestimated, the hedger will make larger trades than are warranted by price movements, and the MID will still end up costing more than necessary. In Chart 4, we show the losses the bank would incur if it incorrectly estimated volatility in hedging a 90-day deposit linked to the S&P 500 with zero guaranteed minimum return. The relationship is not quite linear even for an at-the-money option. However, over a wide range of volatilities, the bank would lose 20 basis points for each 1 percent error in the volatility estimate.[23]

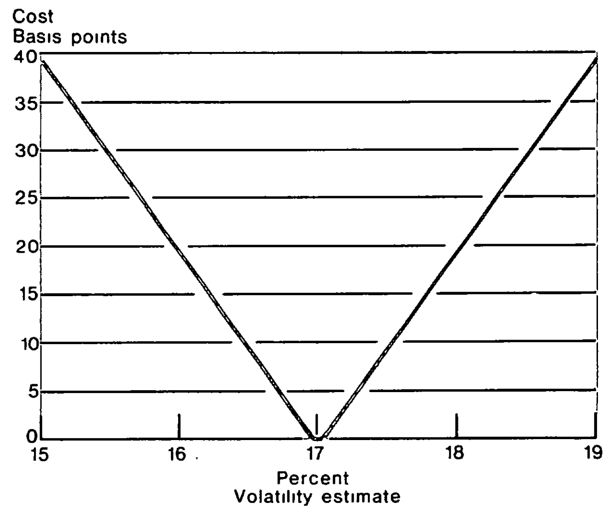
Similarly, if short-term interest rates are initially underestimated, the deposit will end up costing more than anticipated. Unlike volatility changes, however, interest rate changes are easily observed, so the hedging program can in principle be revised to replicate the appropriate option at the lowest cost possible

Execution risk—Execution fails when the hedging bank is unable to adjust its portfolio sufficiently rapidly in response to price movements. This risk is illustrated in the final section of Appendix 1. Clearly the problem is most severe when the hedger is obliged to make large transactions at the same time that the market price is changing rapidly. When the option is deep-in- or deep-out-of-the-money, or when the expiration date of the option is distant, the hedge adjustments required by the synthetic option are generally small. However, when the asset is close to maturity and near-the-money, the logic of dynamic hedging will imply large portfolio changes.

Chart 4

The Cost of Incorrectly Estimating Volatility

When the true value is 17 percent



[21]Appendix 1 provides an illustration For an introductory exposition, see Mark Rubinstein and Hayne Leland, "Replicating Options with Positions in Stock and Cash," *Financial Analysts Journal*, July/August 1981, pp 63-72 Since option-replicating strategies involve considerable amounts of trading, the hedger may also be exposed to problems that might arise in existing settlement and payments systems

[22]This estimate assumes the hedging program is executed well See Richard Bookstaber, "Does Execution Matter?" Morgan Stanley Fixed Income Research Special Report, 1987

[23]The costs portrayed in Charts 4 and 5 were computed using the Black-Scholes option-pricing formula

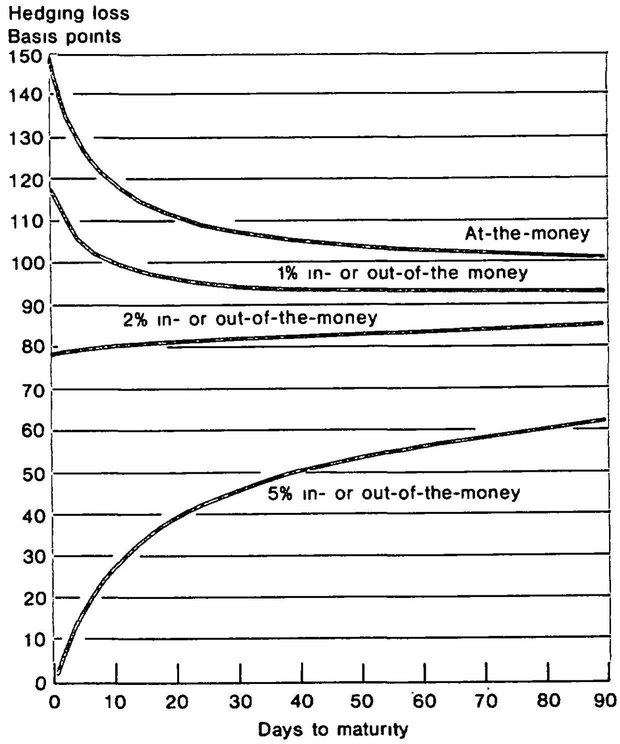
Intuitively, these large portfolio changes result from the requirement of the technique that, just before the expiration of the option, the investor be either fully in cash or fully in the index future The investor should be fully in cash if the option is out-of-the-money, and fully in the index future if the option is in-the-money. If the option is at-the-money immediately before expiration, the portfolio will be 50 percent invested in cash. The bank will then be faced with trading half of its portfolio in a very short period of time

We can calculate the potential costs of execution risk in a "worst case" scenario We assume that the hedger is completely unable to execute any portfolio transactions at a time when there is an adverse 4 percent shock to the S&P 500 index.[24] Chart 5 shows how the cost varies with time to maturity and "closeness-to-the-money" for a 90-day deposit under current market volatility and interest rates.[25] As we explained above, the largest costs are incurred on an at-the-money option that is close to maturity, and these can be as high as 150 basis points.[26] Options that are "away-from-the-money" suffer lower costs at all maturities. The chart demonstrates that an option that is 5 percent away-from-the-money suffers very little from an unexpected price change that occurs near maturity, because the change makes little difference to the probability that the option will expire in-the-money. The chart suggests that a bank dynamically hedging $100 million of 90-day MIDs outstanding would expect to lose between $400,000 and $1 million in this scenario. Having said this, we should emphasize that the performance of synthetic options has not been tested in market conditions more adverse than those assumed above. It is possible that actual trading losses could exceed the above amounts.

Chart 5

Execution Risk and Time to Maturity

Cost of an unhedged 4-percent price change



Conclusion

Commercial banks have recently begun to attract funds by offering accounts that return to depositors the greater of a guaranteed minimum return and a proportion of any increase in a market index Such a market index deposit potentially exposes the issuing bank to considerable risk. We have shown, however, the ways in which the issuing bank can minimize its risk when liquid instruments are available for hedging.

Clearly, investors with access to options or futures markets could to some degree replicate the investment characteristics of MIDs Such replication, however, would be impractical for small investors. Existing MIDs may appeal to the latter group because they offer straightforward access to options with low transactions costs and FDIC insurance, and because they require minimal effort on the part of the investor.

For MIDs to appeal to larger investors, banks will have to offer more than simple convenience, they will have to provide value added. MIDs could be attractive to some investors, particularly institutions that are not large enough to replicate at reasonable cost the risk-return profile that the deposit offers. Deposits linked to an index on which there are no publicly traded options with

[24]This was the magnitude of the decline in the S&P 500 on September 11, 1986, the third largest single-day decline in history William Brodsky, Chairman of the Chicago Mercantile Exchange, stated that few portfolio insurers were practicing intraday adjustment at that time He further noted that few portfolio insurers were able to execute on January 23, 1987, during the 70 minutes when the S&P 500 dropped 6 percent, producing a record trading range for a day (See *International Financing Review*, March 21, 1987, pp 1001-1005)

Footnote 24 continued

[25]The costs are calculated as the difference between the theoretical price of a call option immediately before and immediately after the 4 percent index shock

[26]Execution risk in dynamic hedging can be reduced by combining futures with purchased options in a technique called "delta-gamma hedging" See *Recent Innovations in International Banking*, Bank for International Settlements, Basel, April 1986, pp 101-120

the maturity that the bank offers would therefore represent potentially viable products. In such a case, the bank would be providing portfolio insurance by using dynamic hedging techniques that would be uneconomic to the individual investor. It remains to be seen if this type of wholesale deposit will prove to be more successful than competing financial instruments.

Stephen R. King
Eli M. Remolona

Appendix 1

Hedging the Market Index Deposit: A Simple Numerical Example

With this numerical example, we first show various ways to hedge a market index deposit (an MID) using options, stocks, and index futures. Then we illustrate the execution risk of a hedge that relies on a synthetic option of either stocks or futures.

Pricing the MID

An MID that is properly priced and hedged must cost the issuing bank neither more nor less than alternative sources of funds regardless of index performance.

Consider an MID linked to the S&P 500 Index. Suppose the current value of the index is 300. Divide the time until the deposit matures into two periods. Over the first period, the index can move with equal probability either up to 336 or down to 267. If it moves up to 336, then over the second period it can move to either 376 or 300. If it moves down to 267, then it can move to either 300 or 238. These index movements result in three possible outcomes, as shown in the top panel of Figure A-1. Outcome I represents a gain of slightly more than 25 percent over two periods, in outcome II there is no gain or loss, and in outcome III there is a loss of about 21 percent.

Suppose the marginal cost of bank funds, say the interest rate on a conventional CD, is 8 percent over two periods. Suppose also the MID pays a minimum return of zero (that is, only the principal is guaranteed). Then under outcomes II and III, the MID pays zero while the conventional CD pays 8 percent. For the MID to cost the same as the alternative source of funds under these outcomes, the spread between zero and 8 percent must be the cost of the hedge. Specifically, on a deposit size of D, the hedge must cost $0.08 \times D$ at maturity or $0.08 \times D/1.08$ at the start of the first period.*

One way to hedge would be to purchase call options on the index with the same maturity as the MID and a strike price equal to the initial value of the index.† Under outcome I, the call would be worth 76 index points at maturity (an index value of 376 minus a strike price of 300) and zero under outcomes II and III. The call would end "in-the-money" under outcome I, "at-the-money" under outcome II, and "out-of-the-money" under outcome III. The bottom panel of Figure A-1 shows how the price of this call would move with the index. At the start of the first period, the price of the call would be about 30 index points. If the index rose to 336 at the end of the first period, the price of the call would be about 48 index points, and if the index fell to 267, the price would be zero.‡

Suppose the deposit principal is \$3,000 and an index point is worth \$10. If the bank held one call against the MID, the proceeds from the call at maturity would allow the bank to pass on to depositors the entire gain in the index should it rise, and to guarantee the principal should the index fall. Under outcome I, the bank would exercise the call at maturity to receive \$760 (76 index points times \$10 per point), an amount that would be just sufficient to pay on the MID a return equal to the percentage change in the index. Under outcomes II and III, the bank would not exercise, thus losing no money on the index and thereby retaining its ability to guarantee the deposit principal. However, to purchase a call in the beginning would have cost the bank \$300 (30 index points times \$10 per point), while the spread between the cost of funds and the minimum return specifies a hedge costing only \$222 ($0.08 \times \$3000/1.08 \approx \$222$). If the bank held one call against the MID and the call ended out-of-the-money, the MID would turn out to be a more costly source of funds than a conventional CD.

The cost of the hedge requires the bank to purchase

*If r^G is the guaranteed minimum return and r is the cost of funds, the cost of the hedge must be $(r - r^G) \times D$ at maturity or $(r - r^G) \times D/(1 + r)$ at the start.

†If the guaranteed minimum return is r^G and the offered proportion of index gain is ϕ, the strike price should be $1 + r^G/\phi$ times the initial value of the index. Hence, in the case where $r^G>0$, the option price and proportion of index gain have to be solved for simultaneously.

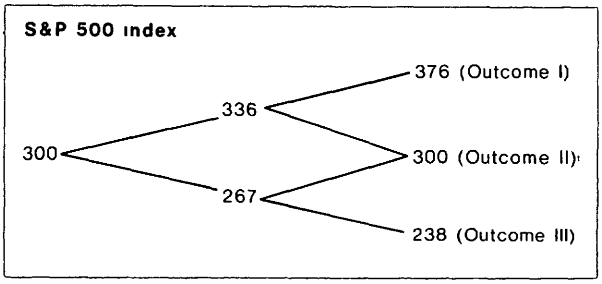
Footnote † continued

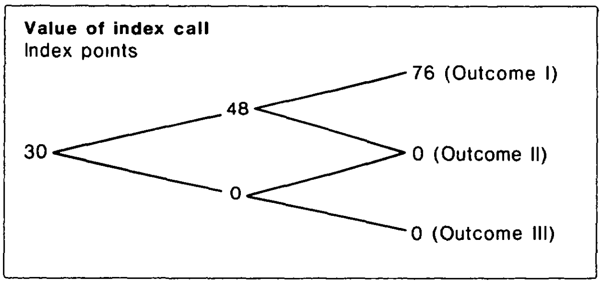
‡Option prices here assume that writers of the index call face the same 8 percent cost of funds and that the stocks underlying the index pay no dividends. The prices are derived using the binomial approach in Cox, Ross, and Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, Vol. 7 (1979), pp. 229-263.

Hedging the Market Index Deposit *(continued)*

Figure A-1

Index Movements and Value of Index Call





just three-quarters of a call ($222/$300 ≈ 3/4) and thus to offer to compensate the MID depositor for only 75 percent of the gain in the index. Hence the relationship between minimum return and upside capture (or proportion of index gain) can be expressed in terms of (1) the price of an appropriately specified option, and (2) the spread between the bank's marginal cost of funds and the minimum return it guarantees In Appendix 2, we present the general formula for this relationship.

Hedging with a purchased call

The table shows how the hedge would work on the $3,000 MID offering 75 percent of the gain in the index Under outcome I, the bank would pay the depositor a return of 19 percent (0.75 times the index gain) or $570. This payment would be fully covered by the exercise of the three-quarters of a call the bank would be holding (3/4 times $760 option value at maturity is $570). Under outcomes II and III, the bank would pay no interest on the MID, and the call would expire with zero value. Given the 8 percent cost of funds, the $222 starting price of three-quarters of a call would cost the bank $240 at maturity (1 08 × $222), an amount equal to the interest that the bank would pay if it took the $3,000 as a conventional CD. Hence, the MID would cost the bank neither more nor less than a conventional CD under any outcome.

Hedging with a stock portfolio

With dynamic hedging, the bank may also hedge by actively trading a portfolio of stocks replicating the S&P 500 Index § The idea is to hold a portfolio with the same risk profile as the index call—in effect, to construct a synthetic option At each point in time, the size of the stock position we need for the synthetic index call is given by the option's "hedge ratio" or "delta," usually computed from an options-pricing model such as Black-Scholes. In our example, the delta at the start of each period is calculated as the ratio of (1) the difference between the high and low values of the index call on the next move, and (2) the difference between the high and low values of the underlying index on the next move.|| Hence, at the start of the first period, (1) is 48 − 0 = 48, and (2) is 336 − 267 = 69, so the delta is about 0.7 (≈48/69). At the start of the second period, there are two possible deltas If the index rose to 336, the delta would be (76 − 0)/(376 − 300) = 1 If the index fell to 267, the delta would be (0 − 0)/(300 − 238) = 0

§One way to do this would be to trade shares in an index fund based on the S&P 500

||Again see Cox, Ross, and Rubinstein, "Option Pricing" In general, the delta depends on the interest rate, the current price of the underlying asset, the volatility of the asset's return, the strike price, and the time to maturity

Cash Flows for a Hedged MID and a Conventional CD

	Starting Date	Maturity Date: Outcome I	Maturity Date: Outcomes II and III
S&P 500 MID			
Take MID	3,000	−3,570	−3,000
Borrow at 8%	222	−240	−240
Buy 3/4 call	−222	570	0
Total	3,000	−3,240	−3,240
Conventional CD			
Take deposit	3,000	−3,240	−3,240

Assumptions The MID pays 0 75 of the gain in the S&P 500 Index and guarantees the principal The outcomes refer to Figure A-1 with each index point assumed to be worth $10

Hedging the Market Index Deposit *(continued)*

Figure A-2 shows how the bank would construct the synthetic option needed to hedge the $3,000 MID offering 75 percent of the market gain. At the start of the first period, a delta of 0.70 applied to three-quarters of a call tells the bank to buy an S&P 500 stock portfolio worth $1,575 (0.70 × (3/4) × $3,000), financing its purchase with funds costing 8 percent

If the index then rose to 336, the value of the portfolio underlying one index call would be $3,360 (336 index points times $10 a point). A delta of one for three-quarters of a call would then tell the bank to hold a stock portfolio worth $2,520. Since the stock portfolio carried over from the first period would now be worth about $1,760,¶ the bank would buy $760 more of the portfolio, again borrowing the amount for the purchase.

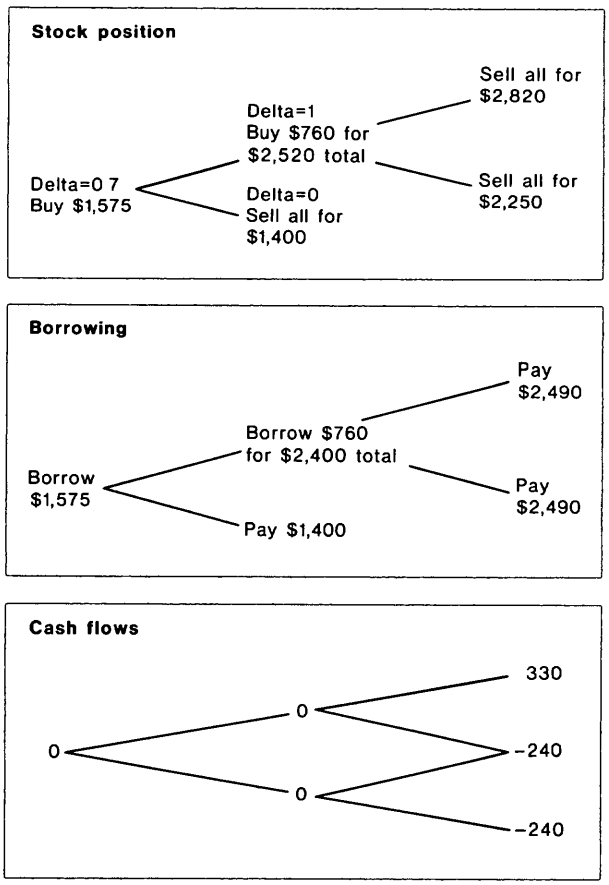
Under outcome I, the bank would end up with a stock position worth about $2,820. At the same time the bank would owe about $1,700 (1.08 × $1,575) from the first loan and about $790 from the second. After the bank liquidates the stocks and pays off the loans, it would have $330 left. Since the interest payment on the MID would be $570, the hedge and the MID would cost the bank $240 net, exactly the cost of a conventional CD Discounted to the start of the first period, this is also the price of a purchased option, as we saw in the last section.

Under outcome II, the stock portfolio would be worth $2,250. Paying off $2,490 in loans would again leave the bank $240 short. Since the MID would require no interest, the cost of the hedge would exactly match the cost of a conventional CD.

If the index fell to 267 at the end of the first period, the bank would be holding $1,400 in stocks. A delta of zero would then tell it to liquidate its stock position. After using the proceeds to pay its loan, the bank would still owe about $235 at the start of second period or $240 at maturity. Since only outcomes II and III would be possible, the MID would require no interest, and again the cost of the hedge would be the cost of a conventional CD.

Figure A-2

Dynamic Hedging With a Stock Portfolio



Hedging with index futures

The bank may also choose to form its synthetic option out of index futures. Index futures are priced to require no exchange of cash in the beginning, except for the initial margin.** This amounts to setting the price at a premium to the underlying index to reflect borrowing costs (less dividend yield if any). During the course of the contract, each price change is settled in cash.†† The side favored by the price change receives a payment while the other side pays. As shown in Figure A-3, when the index is 300 and the "cost-of-carry" is 8 percent over two periods, a futures contract expiring at the end of two periods would be priced at 324 points (1.08 × 300). If the index went to 336 at the end of the first period, the futures price would be 349 points (1.04 × 336), and the holder would receive the cash equivalent of 25 points (349 − 324). If the index fell to 267, the futures price would be 278 points, and the holder would pay 46 points

¶Recall that we assume no dividend payments

**The initial margin on an S&P 500 futures contract is $10,000 and each index point is worth $500

††In practice, this is done daily The cash settlement is called a "variation margin"

Hedging the Market Index Deposit *(continued)*

(324 − 278). At maturity, the futures price necessarily reverts to the value of the index. A final cash payment is made to reflect the last price change, and the contract expires.

Dynamic hedging with futures uses the same delta that is used for dynamic hedging with stocks. To create a synthetic index call, the bank would start with a delta of 0.70. At this point, a whole futures contract would be priced at $3,240 (at an assumed $10 per index point). However, replicating three-quarters of a call with index futures does not mean buying $1,700 worth of the contract (a delta of 0 70 times three-quarters times $3,240). Because of the cash settlement following each price change, we need to discount for the fact that the futures price reflects a two-period premium. As Figure A-3 shows, the correct amount to buy would be about $1,635 ($1,700/1.04), so that the bank, in effect, would pay only for one period of carrying costs

If the futures price then rose to 349 points at the end of the first period, the bank would be holding futures worth about $1,760 ($1,635 × 349/324), and it would receive $125 in cash ($1,760 − $1,635) from the price change. The new delta of one would tell the bank to buy about $855 more futures for a total of about $2,615 (three-quarters of a call times $3,490). Under outcome I, the bank's futures position would be about $2,815 at maturity. The bank would receive $200 in cash ($2,815 − $2615) as final settlement. At the same time, the earlier cash settlement would have grown to about $130 (1.04 × $125), so the bank would come out $330 ahead. It would then pay $570 on the MID for a net cost of $240, exactly what a conventional CD would cost. Under outcome II, the futures position would be about $2,250. The bank would pay a final cash settlement of roughly $370. This time the MID would require no interest payment. With the earlier cash settlement, the net cost of the MID would again be $240.

If the futures price fell to 278 points at the end of the first period, the bank's futures position would decline to about $1,405, requiring a cash payment of $230 ($1,635 − $1,405). The new delta of zero would tell the bank to sell all its futures. The sale would be carried out without any further exchange of cash. At the end of the second period, the cash payment made earlier would cost the bank about $240 (1.04 × $230) There would be no interest payment on the MID and no other gains or losses from the hedge. Again the net cost of the MID would be the same as that of a conventional CD.

Figure A-3

Dynamic Hedging With Index Futures

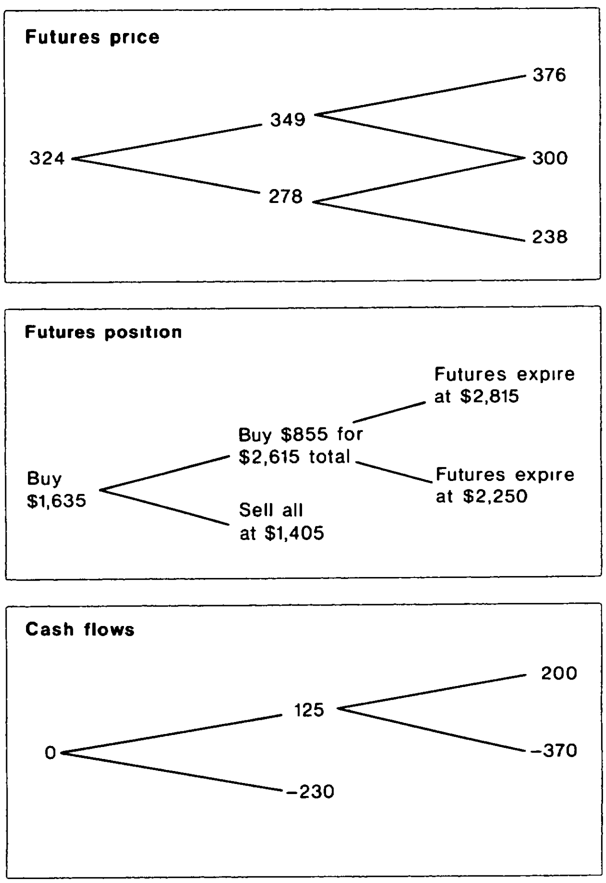
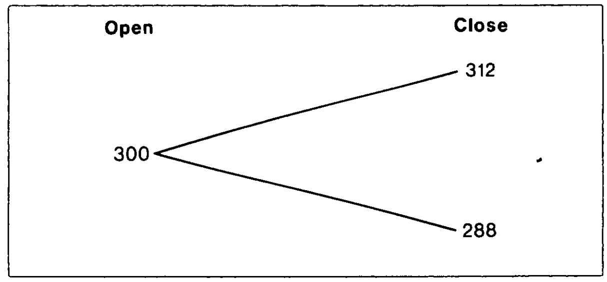


Figure A-4

Index Movement on Maturity Day



Hedging the Market Index Deposit *(continued)*

Execution risk

In existing markets, large changes in either stock or futures positions are hard to execute when prices are moving swiftly Thus a dynamic hedger faces execution risk when the delta happens to be very sensitive to prices during a period of unusually wide price swings In this situation, the delta would require a large hedge adjustment at the very time that prices are moving too fast to allow the hedger to execute a large trade. The delta is most sensitive to prices when the option to be replicated is both near maturity and at-the-money.

In order to illustrate execution risk when the delta is most sensitive to prices, let us suppose that we have the same $3,000 MID as before But this time, suppose that on the maturity date, as shown in Figure A-4, the index opens at 300 and can close at either 312 or 288 (with the MID linked to the closing value).§§ Hence, the trading day opens with the synthetic option being at-the-money. At this point the delta is one-half and the bank should hold (0.5) × (0.75) × $3,000 = $1,125 in stocks or futures for its dynamic hedge. Suppose that the bank is able to implement this investment position at the start of the day, but that during the day, prices move too quickly for the bank to execute any further adjustment.

If the index rose to 312, the bank would be holding a position worth $1,170 ($1,125 × 312/300), for a gain of $45 ($1,170 − $1,125). The MID, however, would require an interest payment of $90. The net loss to the bank would be $45 or 1.5 percent of the deposit principal. If the index fell to 288, the bank would be holding a position worth $1,080, for a loss of $45. Since the MID would require no interest payment, the net loss would still be 1.5 percent of principal. In theory, the accumulated cost of dynamic hedging|||| from the first day of the MID to the day of maturity would have already amounted to the cost of a purchased option. Hence, the loss on the last day would be an additional cost due to the failure of execution.

§§This is a rise or fall of 4 percent, somewhat smaller than the magnitude of the fall in the index on Thursday, September 11, 1986

||||As the previous sections show, this cost arises from the fact that the strategy of buying stocks or futures as prices rise and selling as prices fall amounts to buying "high" and selling "low" In the absence of transactions costs, these capital losses cause the cost of dynamic hedging to equal the cost of an equivalent option

Appendix 2

The General Pricing Formula for the MID

To hedge MIDs with maturity T years and guaranteed minimum annual return r^G, the bank must explicitly or implicitly purchase index calls with the same maturity and with a strike price corresponding to the current index value, guaranteed minimum return, and upside capture or proportion of index gain. The price of such an option will be a function of the current index value S, the strike price K, the annual volatility σ, the cost-of-carry r^C, and the time to maturity T* The function is increasing in all its arguments but the strike price. We can write the cost of one option per dollar of the deposit as

$$C(S,K,\sigma,r^C,T)/S$$

where C(.) is a standard call price formula for the case in which the strike price is

$$K = S(1 + ((1+r^G)^T - 1)/\phi)$$

*In the case of a stock index, r^C is equal to the short-term interest rate less the dividend yield For a commodity option, r^C is the sum of the short-term interest rate and the cost of physical storage

and ϕ is the upside capture to be offered.

The amount of options the bank can profitably hedge with will depend on the interest it would otherwise pay on funds with the same maturity (and denomination) as the MID. The MID will be priced efficiently relative to a conventional CD if the cost of the hedge in proportion to the deposit exactly equals in present value the difference between the interest cost of a conventional CD r^D and the guaranteed minimum return offered by the bank r^G. Hence the hedge expenditure per dollar of deposit should equal

$$h(r^G,r^D) = ((1+r^D)^T - (1+r^G)^T)/(1+r^D)^T.$$

The upside capture ϕ is therefore calculated as the ratio that solves

$$\phi = h(r^G,r^D)S/C(S,K,\sigma,r^C,T)$$

where K itself is a function of ϕ, S, r^G, and T. The solution can be written as $\phi(r^D,r^G,r^C,\sigma,T)$ and can be shown to be increasing in the deposit rate r^D and maturity T, decreasing in all its other arguments, and independent of the initial index value S.