Evidence on Stock Market Speculative Bubbles: Japan, the United States, and Great Britain

The sudden worldwide collapse of stock prices in October 1987 has puzzled observers of financial market developments. President Reagan commented that the fall had nothing to do with the economy. Market analysts described the event as "absurd" or as "mindless herd movement." Indeed, it is hard to justify such a large drop in stock prices. Economists typically attribute large swings in stock prices to the impact of important economic news on financial markets. New information can cause investors to make drastic reassessments of the size of future cash flows or the future discount rates at which these cash flows are capitalized. But the adverse economic news that preceded the fall in stock prices on Monday, October 19, 1987 does not appear dramatic enough to have caused the unusual drop of 23 percent.[1]

The view that the stock price collapse cannot be explained by economic fundamentals leads to the question, Did the collapse represent an abrupt downward correction of an overvalued market or did the market become grossly undervalued after prices fell? This article focuses specifically on the possibility of overvaluation before the October crash in the three major national stock markets of the United States, Japan, and Great Britain. It proposes a new method of detecting market overvaluation and finds evidence consistent with this phenomenon.

[1]The adverse news included the disappointing U S trade deficit figures announced the previous Friday and reports of a possible tax law that would negatively affect mergers and acquisitions

Overview

Persistent market overvaluation followed by market collapse is often referred to as a speculative bubble. Such bubbles may be triggered by an extraneous event that is unrelated to fundamental economic conditions; one group of investors buys with the expectation of a large capital gain, and others follow suit, without paying proper attention to economic factors such as future dividends or interest rates. If such behavior persists, it may feed on itself as consecutive waves of buying increase prices. Speculative bubbles may subsequently burst very suddenly; an overvalued market is fragile and a relatively unimportant piece of "bad" news may easily create pessimism and set off a selling wave.[2]

The traditional method of searching for market overvaluation or speculative bubbles counts the number of unusually high returns during the suspected bubble period and assesses the likelihood that the total number of these high returns could have arisen from chance.[3] An unusually high return (or a positive "abnormal" return) is a return higher than the risk-free rate plus the usual risk premium necessary to compensate

[2]Note that although the general description of a speculative bubble assumes some sort of collective market irrationality, irrationality is not a necessary characteristic of a speculative bubble In a special case described later, agents know the market is overvalued but they remain in the market because they expect to be compensated for staying in an overvalued market

[3]Olivier J Blanchard and Mark W Watson, "Bubbles, Rational Expectations, and Financial Markets," in Paul Wachtel, ed , *Crises in the Economic and Financial Structure* (Lexington, Massachusetts Lexington Books, 1982), chap 11, pp 295-315

risk-averse stockholders for the uncertainty associated with their security returns. In the absence of a speculative bubble, a very large number of unusually high returns would normally occur by chance only with a small probability. Hence a large number of unusually high returns constitutes evidence consistent with the presence of speculative bubbles. Unfortunately, although simple, the traditional test has low statistical power to detect speculative bubbles: Stock prices are very volatile and their swings generate both large positive and large negative returns. The latter tend to mask any existing bubble evidence.

In order to construct a more powerful test for bubbles, it is necessary to formulate a more precise economic account of the development of the bubble. One can imagine many different scenarios of market overvaluation, but this analysis restricts the possible scenarios to those in which investors *know* that the market is overvalued yet show no special desire to liquidate their positions and continue to buy or sell as they would in the absence of bubbles. This is a realistic working assumption for the period before October 1987. Robert Shiller provides survey evidence indicating that before October 1987, 71.7 percent of individual investors and 84.3 percent of institutional investors thought that the market was overvalued at the time[4]

Explaining why investors did not get out of an overvalued market is more difficult. One could argue that the presence of highly liquid futures markets and associated trading strategies such as portfolio insurance led investors to the false belief that they could enjoy large positive returns in an upward market yet still avoid suffering a large loss if the market took a big plunge.[5] This article, however, pursues an explanation that does not depend on some sort of collective irrationality. Academic economists call it the "rational speculative bubble" hypothesis.

[4]The survey results are described in Robert J Shiller, "Investor Behavior in the October 1987 Stock Market Crash," National Bureau of Economic Research, Working Paper no 2446, November 1987 The survey was conducted after the crash, so there is reason to suspect that the respondents' answers were influenced by the crash Shiller also reports consistent answers to a similar question only 36 1 percent of individual investors and 22 2 percent of institutional investors described themselves as bullish and optimistic relative to other investors before October 1987

[5]The expression portfolio insurance is a misnomer for a dynamic portfolio allocation strategy designed to guarantee a specified minimum return The strategy assumes a liquid market where one can sell stocks whenever the need arises Portfolio insurance can work well when the number of insurers is small and they cannot influence the market But when the same technique is employed by many insurers, liquidity in the market is destroyed This is particularly true when noninsurers act in anticipation of what portfolio insurers will do Thus portfolio insurers who assume the presence of a liquid market suffer from irrationality The October 1987 stock market collapse provided an example of portfolio insurance failure exactly at the time the insurance was needed most

In the case of a rational speculative bubble, investors know that the bubble may crash and that they will not be able to get out once the crash starts, but they remain in the market because they believe—for whatever reason—there is good probability that the bubble will continue to grow, bringing them large positive returns. These returns are expected to be higher than the risk-free rate plus the usual risk premium in the absence of bubbles, and large enough to compensate them *exactly* for the probability of a bubble crash and a large onetime negative return. Hence it is rational for investors to stay in the market. The expected extra return when no bubble crash occurs can be called the "bubble premium." The theory implies that the bubble premium is not only positive, but also increases during the lifetime of the bubble. The time trend in the bubble premium derives from the explosive nature of the bubble component of the stock price. As time goes on, the bubble component of the stock price grows larger and larger relative to the fundamental component. This growth implies that with the passage of time, the expected drop in the stock price in the case of a bubble crash grows larger too, necessitating a larger and larger bubble premium.

The evidence points to a positive and rising bubble premium for approximately a year and a half before October 1987 in the national stock markets of the United States and Japan. A positive and rising bubble premium is also present in the national stock market of Great Britain, but it appears much later, in mid-1987. Overall, the evidence is consistent with the hypothesis of rational speculative bubbles.

The nature of rational speculative bubbles

This section provides an intuitive description of a rational speculative bubble. A more detailed mathematical example is presented in the accompanying Box. Recall that a rational speculative bubble is a special case of a speculative bubble. The characteristic that makes a speculative bubble rational is the particular size of its expected rate of growth. The expected rate of growth of a rational speculative bubble is such that investors have no incentive to get out of the market, although they know the market is overvalued.

Rational bubbles and investor behavior

To understand how a rational speculative bubble works, let us consider a concrete example: Suppose that investors require a rate of return of 10 percent in order to invest in the stock market. This required rate of return of 10 percent equals the risk-free rate that they could get by investing, say, in Treasury bills or eurodollars, plus an extra return that represents a compensation for the risk they assume when investing in stocks,

Box: An Example of a Rational Speculative Bubble

For expositional simplicity, assume that the sum of the risk-free rate and the risk premium is constant over time and equals r. The presence of a time-varying risk-free rate or risk premium does not affect the main point of the example. Rationality of behavior and of expectations, together with market clearing, implies that the expected rate of return on a stock equals the required rate r:

(B1) $E_t R_{t+1} = r, \quad R_{t+1} \equiv (p_{t+1} - p_t + d_{t+1}) / p_t,$

where p denotes the stock price, d the dividend, and E_t the expectations operator at time t Rearranging the expected arbitrage condition, B1, yields.

(B2) $p_t = (1/(1+r)) E_t p_{t+1} + (1/(1+r)) E_t d_{t+1}.$

Substituting for p_{t+1} in equation B2 and continuing recursively, one can derive the familiar present value model, which states that the price equals the infinite sum of expected future dividends discounted at the required rate of return r:†

(B3) $p^*_t = \sum_{i=1}^{\infty} [1/(1+r)]^i E_t d_{t+i}$

Here p^*_t is the "market fundamental value" of the stock. Note, however, that p^*_t is not the only solution to B2 Any p_t of the following form is a solution as well:

(B4) $p_t = p^*_t + b_t, \quad \text{with } E_t b_{t+1} = (1+r)\, b_t.$

To see that equation B4 is a solution, observe that according to B4, $p_{t+1} = p^*_{t+1} + b_{t+1}$. Then, substitute p_t and p_{t+1} in B2 and check that B2 is satisfied. Equation B4 says that the market price can deviate from the market fundamental value by b_t, a bubble component, without violating the expected arbitrage condition B2 or B1. The intuitive reason why a bubble component can exist is straightforward: arbitrage conditions in financial markets are expressed in terms of rates of return, not in terms of price levels Therefore, even if an asset is, say, overvalued by an amount b_t, it is still "rational" for an investor to buy it, if the degree of overvaluation is expected to grow every period at the rate r

Observe that the characteristic that makes a bubble rational is its expected rate of growth and not necessarily its actual rate of growth. Therefore, theory cannot determine with precision the actual form of the bubble process. A multiplicity of bubble processes may exist An obvious candidate is the following:

(B5) $b_{t+1} = (1+r)\, b_t + v_{t+1}, \quad E_t v_{t+1} = 0.$

The bubble process B5 satisfies the condition $E_t b_{t+1} = (1+r)\, b_t$ but explodes to infinity with the passage of time. Since the stock price cannot be infinite, the bubble process B5 is not realistic.

The following example, advanced by Blanchard, is a realistic scenario of a bubble that grows for some time but eventually bursts:‡

(B6) $b_{t+1} = (1/q)\,(1+r)\, b_t + v_{t+1}$ with probability q

v_{t+1} with probability 1-q

with $E_t v_{t+1} = 0.$

In each period the bubble continues with probability q or crashes with probability 1-q. It is straightforward to check that the bubble process B6 satisfies the condition $E_t b_{t+1} = (1+r)\, b_t$, that is, the bubble is expected to grow at the rate r. However, on the condition that the bubble does not crash, the bubble is expected to grow at a rate higher than r: $E_t b_{t+1} | NC = (1/q)(1+r) b_t$. This implies that conditional on the event of no bubble crash, markets expect to receive a rate of return higher than r. This extra expected return is the bubble premium §

The bubble premium is positive and increasing with time To clarify these two properties of the bubble premium, let us continue the example by assuming that expected dividend payments are constant:

(B7) $d_{t+i} = d + u_{t+i}, \text{ with } E_{t+i-1} u_{t+i} = 0, \quad i = 1, 2, \ldots$

In this case the fundamental component p^*_t is constant over time and equals d/r, and the realized abnormal rate of return during the lifetime of the bubble is.

(B8) $R_{t+1} - r = [(1+r)((1\text{-}q)/q)\, b_t + u_{t+1} + v_{t+1}] / p_t.$

The expected abnormal rate of return conditional on no bubble crash taking place, that is, the bubble premium, is:

(B9) $E_t(R_{t+1} - r \mid NC) = [(1+r)(1\text{-}q)/q]\, b_t/p_t.$

Equation B9 shows that the bubble premium is positive and increasing with time. The term b_t/p_t is an increasing function of b_t. Since b_t is itself rising as the bubble unfolds, $E_t(R_{t+1}\text{-}r \mid NC)$ is increasing with time. Our test exploits these two properties of the bubble premium. In contrast, the Blanchard-Watson test exploits the fact that the realized abnormal return in equation B8 is on average positive One of the reasons the Blanchard-Watson test has low statistical power is the presence of the noise terms u_{t+1} and v_{t+1}, which are absent from the bubble premium of equation B9.

†It is assumed that the transversality condition holds, that is, $\lim_{i\to\infty} E_t[1/(1+r)^i]P_{t+i} = 0$

‡Olivier J Blanchard, "Speculative Bubbles, Crashes and Rational Expectations," *Economics Letters*, vol 3 (1979), pp 387-89

§In "Rational Inflationary Bubbles," *Journal of Monetary Economics*, vol 21 (January 1988), pp 35-46, Behzad T Diba and Herschel I Grossman argue that once a rational bubble bursts, it cannot restart However, they also show that a rational bubble can periodically shrink to a very small positive number For the purposes of this article, the distinction between shrinking and bursting bubbles does not matter the risk premium. Suppose also that investors expect to receive a constant dividend equal to $5.0 each year. Then, according to the present value model of stock prices, the fundamental or bubble-free price of the stock is $5.0/0.10 = $50.0. The price will stay at $50.0 as long as the required rate of return (the discount rate) and the expected dividend remain constant. The investors' expected rate of return over their holding period equals the expected dividend of $5.0 plus the zero expected capital appreciation or depreciation, divided by $50.0. Thus the expected rate of return is 10 percent, the same as the required rate of return, and investors are satisfied.

Now suppose there is a bubble component on the stock price equal to $4.0, so that the market price is $54.0.[6] For simplicity, let the holding period horizon of investors be one year and assume that the probability that the bubble will crash to zero during the year is, say, 1/10, and the probability that it will continue after the end of the year is 9/10. Normally, if investors know the stock price is overvalued by $4.0, they will attempt to sell the stock, driving its price down to the fundamental level of $50.0. In the case of a general speculative bubble, investors stay in the market because they do not pay sufficient attention to fundamentals and do not know that the market is overvalued. However, in the case of a rational bubble, investors know the market is overvalued but have no incentive to sell because they expect that the bubble component will grow and compensate them appropriately.[7] Specifically, suppose that if the bubble continues, it will reach the level of $4.89 at the end of the year. Then the expected level of the bubble at the end of the holding period equals the probability of a bubble crash times the value of zero[8] plus the probability of no bubble crash times $4.89, that is, 1/10 $0.00 + 9/10 $4.89 = $4.40. The expected bubble level of $4.40 is exactly 10 percent higher than the original level of $4.0 and implies an expected capital gain of $0.40. It also implies that the expected rate of return from the stock is $(5.0 + 0.4)/$54.0 = 10 percent, which is the rate of return required to satisfy investors. Thus, when the expected value of the bubble at the end of the year is 10 percent above its level at the beginning of the year, investors have no incentive to get out of the market.

Positive bubble premium

Next, let us see how an unfolding rational bubble is consistent with the presence of a positive bubble premium. Recall that a bubble premium is the extra compensation that investors expect to receive while the bubble continues to grow. In the previous example, the rate of return investors expect to receive if the bubble does not crash is $(5.0 + 4.89)/$54.0 = 10.91 percent, and thus the bubble premium is 0.91 percent. The bubble premium is positive because it compensates investors for the negative excess return in case the bubble crashes. In the previous example, the expected return in case of a bubble crash is $(5.0 − 4.0)/$54.0 = 1.85 percent, which implies a negative expected excess return of −8.15 percent. Observe that 9/10 (0.91 percent) + 1/10 (−8.15 percent) = 0. This is the sense in which the bubble premium exactly compensates investors for the probability of a bubble crash and a negative onetime excess return.[9]

Increasing bubble premium

The bubble premium is not only positive but also grows progressively larger as long as the bubble continues. This growth occurs because the bubble component gets larger and larger relative to the fundamental component of the stock price. A higher bubble component implies a larger loss in case of a bubble crash, an outcome which necessitates a larger bubble premium. To clarify this point, let us continue the previous example. Assume that at the end of the year the bubble reaches the level of $4.89, which implies a new stock price of $54.89. If the bubble crashes at the end of the second year, investors expect to make a return of $(5.0 − 4.89)/$54.89 = 0.20 percent, which implies a negative excess return of −9.80 percent. This expected loss, larger in absolute terms than the corresponding excess return of −8.15 percent of the first year, necessitates a larger bubble premium. The new bubble premium

[6]For a discussion of how a bubble can start, see Diba and Grossman, "Rational Inflationary Bubbles."

[7]It is perhaps difficult to understand why investors expect the bubble to grow in the state of no crash. However, the point of the example is not to explain how such expectations are formed, but to show that once these expectations are formed, they can be consistent with an equilibrium in which the required rate of return is 10 percent.

[8]The bubble value of zero, that is, a return of the stock price to the fundamental value of $50.0, is assumed only for purposes of simplicity. It is possible that when a bubble crashes, the stock price overshoots or undershoots its fundamental value. The example in the Box allows for such possibilities by adding an error term to the bubble component.

[9]Clearly, the greater the probability of a bubble crash, the larger the bubble premium. To understand this point, let the probability of a bubble crash be 1/3 instead of 1/10. If investors are to stay in the market, they must expect that so long as no crash occurs, the bubble will grow from $4.0 to $6.6, or that the stock price will increase from $54.0 to $56.6. The overall expected bubble level is (2/3) $6.6 + (1/3) $0.0 = $4.4, which is 10 percent larger than the current bubble of $4.0, as is the case in the example of the text. As in the text, the expected rate of return from the stock (which equals the sum of the dividend of $5.0 plus the expected capital gain of $0.4, divided by the current price of $54.0) is 10 percent, thus, investors have no incentive to get out of the market. However, the bubble premium is larger. The bubble premium can be found from the expected rate of return in the state of no crash, which is equal to $(5.0 + 2.6)/$54.0 = 14.07 percent. Thus the bubble premium is 4.07 percent and is larger than the bubble premium of 0.91 percent in the text.

should be such that 9/10 times the bubble premium, bp, equals 1/10 times 9.80 percent, that is, (9/10) bp = (1/10) 9.80 percent. This equality implies a bubble premium of (10/9)(1/10) 9.80 percent = 1.09 percent, which is larger than the previous bubble premium of 0.91 percent.

To see the rising bubble premium in an alternative way, recall that investors will have no incentive to liquidate their positions if during the second period they expect to receive their required rate of return of 10 percent. Since the stock price at the beginning of the second year is $54.89, investors are satisfied if in addition to the $5.0 in dividends, they also expect the price on the average to rise by $0.49 to $55.38, so that the expected rate of return is ($5 0 + $0.49)/$54.89 = 10 percent. Now recall that the example assumes two possible states, one with a bubble crash and one without. In the state of a bubble crash, the bubble component, currently at $4.89, will drop to zero, causing the price to drop to $50.0. If investors are satisfied with an overall expected price level of $55.38, they must expect that in the state of no bubble crash, the price will rise by $1.09 to the level of $55.98, so that the expected price is (9/10) $55.98 + (1/10) $50.0 = $55.38. Put differently, investors expect that in the absence of a bubble crash they will make a return of ($5.0 + $1.09)/$54.89 = 11.09 percent. Thus the bubble premium is 1.09 percent and is higher than the bubble premium of the first year.

Realized abnormal return versus bubble premium

If at the end of the second year the bubble does not crash, the example suggests that the price will rise to $55.98 and investors will receive exactly their bubble premium of 1.09 percent. In the example, the realized abnormal return at the end of a year does not differ from the bubble premium, a subjective expected abnormal return at the beginning of the year; thus, realized abnormal returns are also positive and growing over time. In practice, however, realized abnormal returns are positive and growing over time only in an average sense. For example, if the bubble does not crash, there is no guarantee that at the end of the second year the stock price will be $55.98. Many unforeseen events occur that may affect the stock price, driving it either above or below the level of $55.98. Thus, during the lifetime of the bubble, realized abnormal returns will fluctuate considerably, and this volatility may mask their upward trend. Empirically, this volatility is particularly problematic when the lifetime of the bubble is short. In contrast, the bubble premium is not affected directly by the volatility of the stock market. For this reason, the empirical methodology focuses on the bubble premium.

The example discussed in this section has made a number of simplifying assumptions. These include a constant risk-free rate, a constant risk premium, and a constant probability that the bubble will collapse even as the bubble grows. These assumptions were made for expository purposes only. They are not required for the empirical analysis that follows.

Measurement of the bubble premium

The previous section showed that in the presence of rational speculative bubbles, investors expect to receive a positive bubble premium that increases over time as the bubble unfolds. This section describes the empirical measurement of the bubble premium. Recall that the bubble premium is an extra return investors expect to receive over their required rate of return, as long as the bubble does not burst.

Methodology

Let R denote the realized rate of return of a stock, which consists of the dividend payment during the period plus the realized capital gain or loss at the end of the period. During the lifetime of the bubble, R can be decomposed into the following four components:

(1) $R = rf + rp + bp + e$,

where rf denotes the risk-free rate, say, the eurodollar deposit rate, an observable variable at the beginning of the period; rp and bp denote the risk premium and the bubble premium respectively, variables known subjectively to market participants at the beginning of the period but not directly observable; and e denotes an unanticipated random disturbance arising from unforeseen events. The sum of the risk-free rate and the risk premium represents the required rate of return, or the discount rate. The sum bp + e is the realized abnormal return during the bubble period. The bubble premium, bp, is zero during periods with no bubbles.

The sum of the bubble premium and risk premium, bp + rp, represents the excess return over the risk-free rate that market participants *expect* to receive provided that the bubble does not crash. The realized excess return at the end of the period, R − rf, is what market participants *actually* receive. The difference between the actual and the expected excess return is the disturbance e. If investors' expectations are rational, then the disturbance e cannot be predicted at the beginning of the period and has an expected value of zero. Put differently, the assumption of rational expectations implies that the investors' expected compensation for assuming risk and investing in a bubble period, rp + bp, is *on the average* equal to the actual compensation, R − rf Thus the observable R − rf can be used in conjunction with the assumption of rational expectations in order to estimate the unobservable sum of the risk premium and bubble premium

The sum of the risk premium and bubble premium can be estimated by regressing R − rf on variables known to market participants at the beginning of the period The regression equation decomposes R − rf into a predictable component—an estimate of bp + rp—and an unpredictable random component representing news that develops after the beginning of the holding period. The regression equation is as follows.

$$(2)\quad R - rf = [a + b_1 x_1 + \ldots + b_I x_I] + e,$$

where $x_1, \ldots, x_I$ are variables known to market participants at the beginning of the period. When this regression is run over the sample period before the crash, the regression fit—the estimated item in the brackets—represents the excess return expected if no bubble crash takes place and is a proxy for the sum of the risk premium and bubble premium.

The information variables $x_1, \ldots, x_I$ of the above regression equation were chosen to maximize explanatory power over the entire sample period (September 1977 through December 1987). They are financial variables such as volatility measures and interest rate spreads within or across countries. Volatility measures are obvious empirical proxies for the risk premium, but interest rate spreads are also good proxies for risk and bubble premia To understand this point, recall that financial variables are the aggregate outcomes of investors' actions in financial markets. These actions are motivated not only by investors' expectations of future profits but also by their willingness to assume risk and their knowledge of a possible underlying bubble Thus, in equilibrium, financial variables provide information about the risk premium and the bubble premium. For example, the spread between the Japanese 10-year government and industrial bond yields represents a proxy for corporate risk; the spread between the 12-month and 3-month eurodollar deposit rates represents a proxy for the risk of a change in interest rates 3 months hence.[10]

The holding period over which returns are calculated is assumed to be either 3 or 12 months. Shorter holding periods are perhaps more representative of the horizons of active investors but are less useful for

[10]For more information on the regression variables and the statistical techniques that were employed, see the technical version of this paper, entitled "Evidence on Stock Market Speculative Bubbles Japan, United States, and Great Britain," Federal Reserve Bank of New York, Research Paper no 8810, February 1988

Table 1

Realized Excess Stock Returns

(Percent Return in Domestic Currency)

	Three-Month Holding Period					
	December 1977 to March 1985			April 1985 to December 1987		
	Japan	United States	United Kingdom	Japan	United States	United Kingdom
Mean	10 1	3 1	8 4	22 6	9 7	7 6
Standard deviation	25 0	29 7	31 4	44 9	44 0	49 7
Correlation with						
Japan	1 00			1 00		
United States	0 24	1 00		0 58	1 00	
United Kingdom	0 39	0 52	1 00	0 64	0 88	1 00
	Twelve-Month Holding Period					
	September 1978 to March 1985			April 1985 to December 1987		
	Japan	United States	United Kingdom	Japan	United States	United Kingdom
Mean	9 8	2 9	9 8	33 1	17 1	19 3
Standard deviation	11 6	15 7	12 2	20 6	10 3	12 6
Correlation with						
Japan	1 00			1 00		
United States	0 30	1 00		0 37	1 00	
United Kingdom	0 32	0 61	1 00	0 29	0 75	1 00

Note Excess stock returns are realized total returns, including dividends, minus the 3-month (12-month) eurodeposit rate of 3 (12) months earlier They correspond to R-rf of equation 2 in the text All returns are annualized Note that in the period March-April 1985, the dollar began a downward slide uncovering speculative bubbles because returns over short periods have a large variance and are not easily predictable.[11] The large swings in stock prices over short horizons would mask any evidence of a positive and rising bubble premium.[12] Note, however, that the holding period assumption is only a practical tool and does not affect the conclusions regarding the presence of speculative bubbles.

Preliminary data analysis

Before turning to the estimation of the bubble premium and the risk premium, it is instructive to perform a preliminary data analysis. Table 1 presents summary statistics of the realized excess rate of return R − rf. Consistent with the previous discussion, excess rates of return are less volatile in the 12-month horizon than in the 3-month horizon. Also note that after March 1985, excess rates of return increased, a necessary development if bubbles are to be found.[13]

Table 2 presents summary information from prelimi-

[11]This technique of searching for bubbles depends critically on the predictability of stock returns because it utilizes the bubble premium, which is an expected as opposed to a realized abnormal return Financial economists have recently shown that contrary to the traditional random walk hypothesis of stock prices, stock returns are indeed predictable, but over longer horizons See Eugene F Fama and Kenneth R French, "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, vol 96 (April 1988), pp 246-73, and Gikas A Hardouvelis, "Margin Requirements, Volatility, and the Transitory Component of Stock Prices," Federal Reserve Bank of New York, Research Paper no 8818, July 1988

[12]Stock price volatility is more pronounced at the daily level Thus it is, not surprising that Santoni was unable to find evidence consistent with the presence of speculative bubbles when he examined daily stock returns in the U S stock market before October 1987 See Gary S Santoni, "The Great Bull Markets 1924-29 and 1982-87 Speculative Bubbles or Economic Fundamentals?" Federal Reserve Bank of St Louis *Review*, November 1987, pp 16-30

[13]Data on eurodeposit rates represent London midmorning rates (after October 1986 they are closing rates) during the last trading day of the month and were provided by DRI Stock returns are based on national stock market indexes on the last trading day of the month For Japan and Great Britain, the data come from the Morgan Stanley Capital International Indices data bank For the United States, the data reflect the S&P 500 and come from the Citibase data bank and the *Wall Street Journal*

Table 2

The Predictability of Excess Stock Returns

$$R - rf = a + b_1 x_1 + \ldots + b_j x_j + e$$

Sample December 1977 to December 1987, 121 observations

			Significance Level			
				Chow Tests		
Three-Month Holding Period	$\bar{R}^2$	**SEE**	Test of $b_1 = \ldots = b_j = 0$	Periods 1 vs (2+3)	Periods 1 vs 2	Periods 2 vs 3
Japan	07	30 9 percent	060	002	190	001
United States	13	31 9	000	880	020	044
United Kingdom	10	35 1	001	260	000	007
Twelve-Month Holding Period						
Japan	47	13 3 percent	000	000	041	000
United States	16	14 3	427	000	001	000
United Kingdom	32	10 5	000	000	000	003

Notes $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom, SEE is the regression standard error Significance levels lower than 050 constitute evidence for rejecting the null hypothesis R is the annualized total gross return, and rf is the risk-free rate, that is, the 3- or 12-month eurodeposit rate for each country and holding period The estimation uses all overlapping observations with the necessary adjustments, see Lars P Hansen, "Large Sample Properties of Generalized Methods of Moments Estimators," *Econometrica*, vol 50 (July 1982), pp 1029-54 Period 1 runs from December 1977 through July 1982, period 2 from August 1982 through March 1985, and period 3 from April 1985 through December 1987 In the 12-month horizon of Japan, the sample begins in September 1978 The information variables x_i are as follows (1) for Japan, in the 3-month horizon spread between 10-year Japanese government bond yield and 3-month euroyen rate, spread between 3-month and 1-month euroyen rates, spread between 3-month and 1-month eurodollar rates, spread between 10-year Japanese government and industrial bond yields, and spread between 3-month eurodollar and euroyen rates, in the 12-month horizon spread between Japanese government 10-year bond yield and 12-month euroyen rate, spread between 12-month and 3-month eurodollar rates, spread between 12-month eurodollar and euroyen rates, and yen/dollar exchange rate volatility, (2) for the United States, in the 3-month horizon spread between 3-month and 1-month eurodollar rates, and spread between 3-month eurodollar and euroyen rates; in the 12-month horizon lagged dependent variable, spread between 12-month eurodollar and europound rates, spread between 12-month and 3-month eurodollar rates, and spread between 30-year and 5-year U S government yields, (3) for the United Kingdom, in the 3-month horizon lagged dependent variable, spread between 12-month eurodollar and europound rates, and volatility of U K stock prices, in the 12-month horizon lagged dependent variable, europound rate, and spread between the 20-year and 5-year British government bond yields nary regressions utilizing the whole sample, including postcrash data Observe that the explanatory power of the information variables, measured by the R^2 statistic, is much higher in the 12-month holding period This result is consistent with the finding of Fama and French and others that stock returns are more predictable over longer horizons The table shows the results of testing the hypothesis that all slope coefficents $b_1, \ldots, b_j$ are jointly zero $b_1 = \ldots = b_j = 0$ Zero slope coefficients would imply that the sum of the risk premium and bubble premium is constant over time In five of the six cases the hypothesis is, however, rejected, an outcome that shows that excess stock returns are partially predictable and that, in the absence of bubbles, risk premia are time-varying

Table 2 also presents tests of structural change of the parameters a, $b_1, \ldots, b_j$, with the break points occurring in July 1982, the time when a bull market began around the globe, and in March 1985, the time when the dollar began a downward slide Although coefficient instability can be caused by the inability of the information variables $x_1, \ldots, x_j$ to capture the variability in the risk premium adequately, it can also be caused by the presence of speculative bubbles The tests reveal considerable instability, particularly at the March 1985 break point, indicating that speculative bubbles could be present after March 1985 [14]

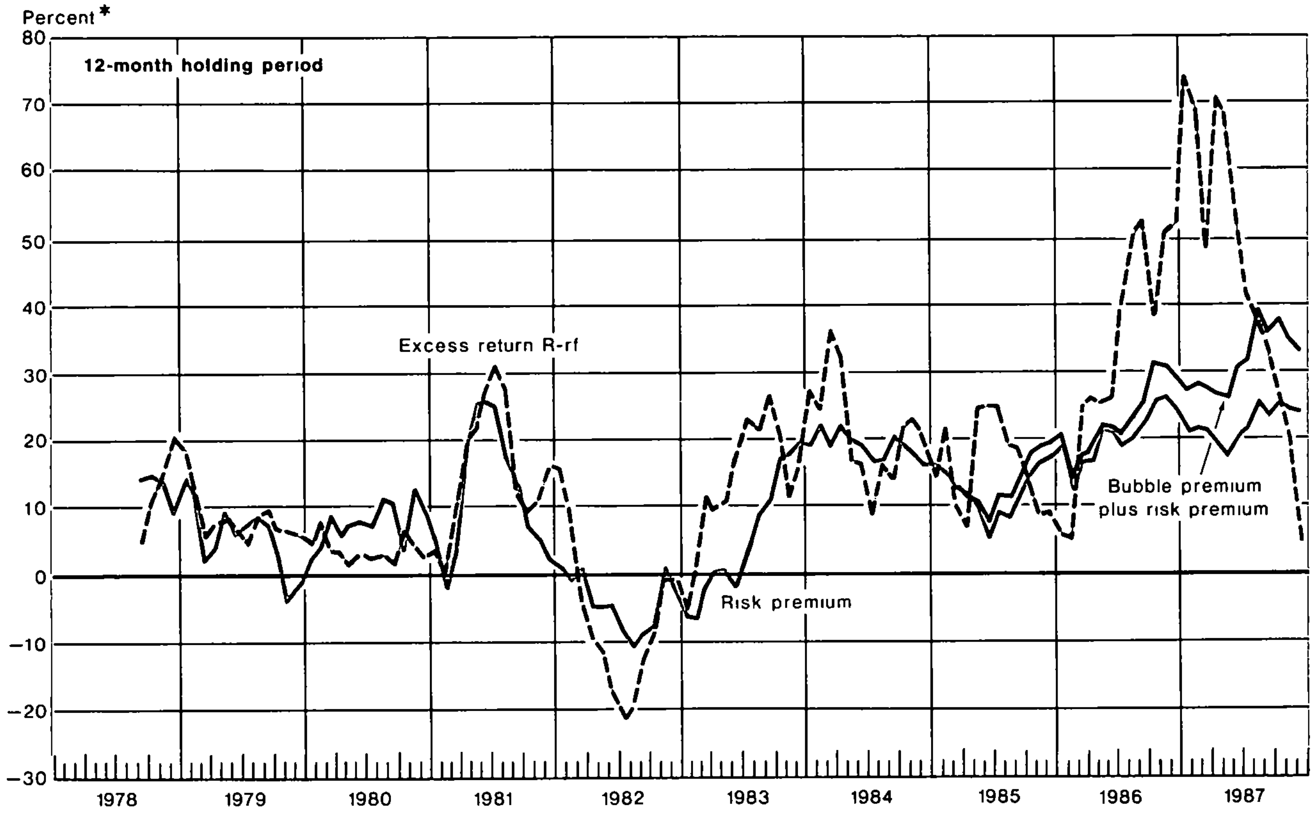
Estimation

Let us turn now to the actual estimation of the risk premium and the bubble premium Regression equation 2 can be used to estimate the sum of the risk premium

[14] The presence of bubbles can cause instability, but the reverse is not true the presence of instability does not necessarily imply the presence of speculative bubbles Coefficient instability can be caused by many other factors

Chart 1

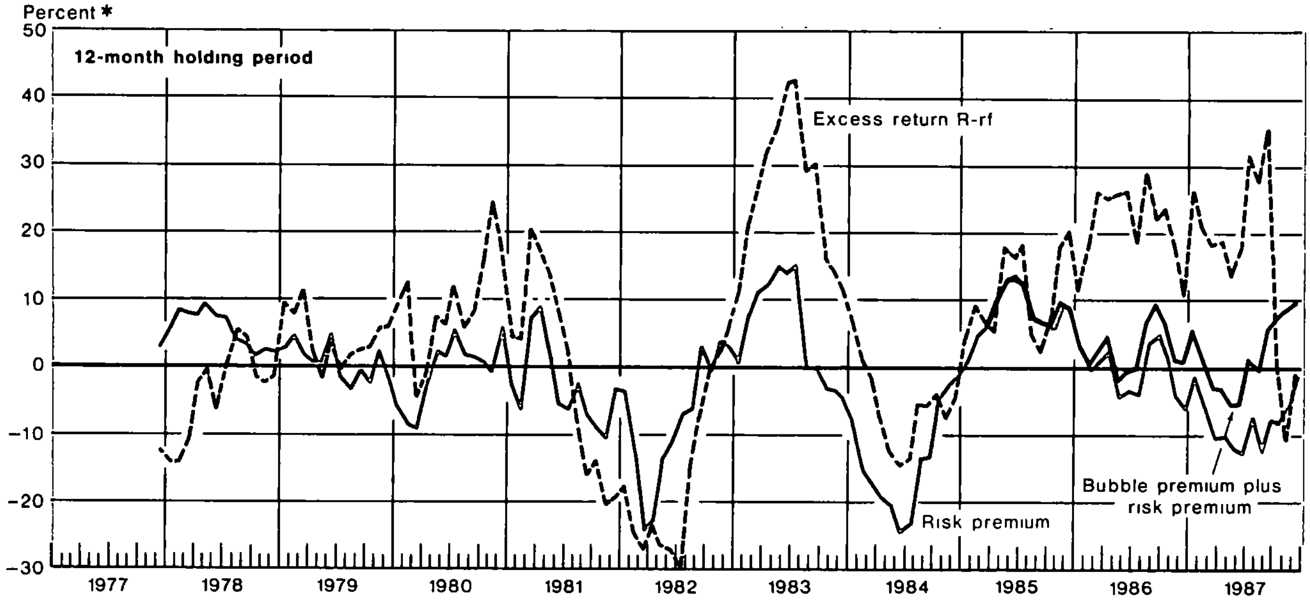
Japanese Stock Returns



*Annualized rates of return

Chart 2

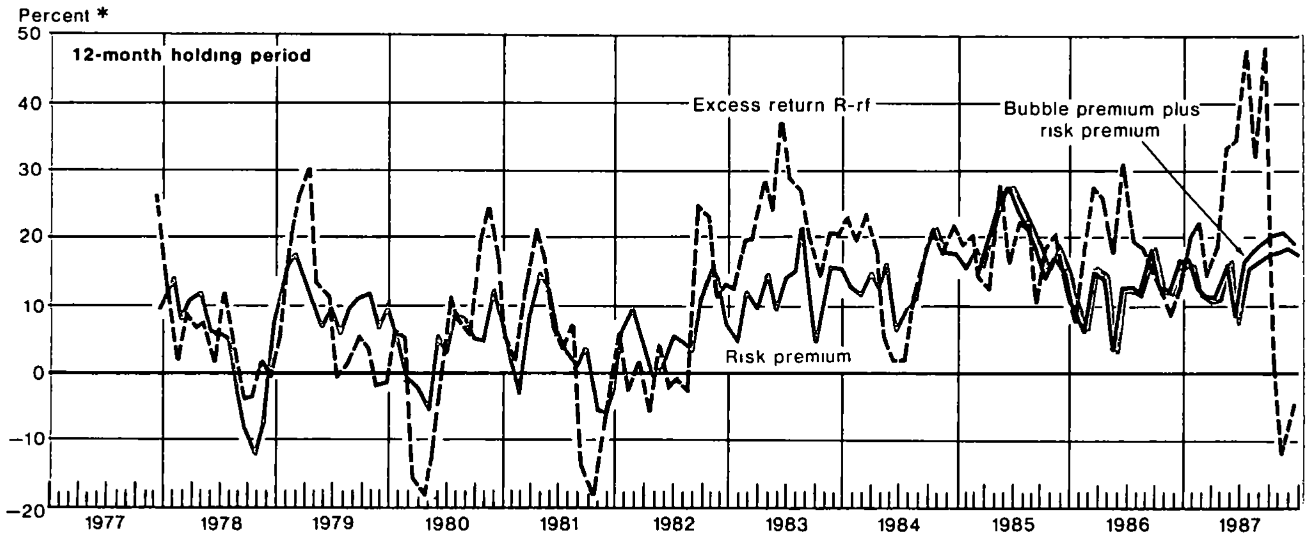
U.S. Stock Returns



*Annualized rates of return

Chart 3

U.K. Stock Returns



*Annualized rates of return

and bubble premium. In order to partition the estimated sum into its two separate components, it is necessary to make two reasonable assumptions first, it is assumed that during the earlier part of the sample there were no speculative bubbles This assumption makes it possible to use the data of the earlier period, specifically from September 1977 through March 1985, in order to estimate the parameters a, b_1, , b_j of equation 2 that characterize the evolution of the risk premium [15] Second, it is assumed that the parameters a, b_1, . , b_j that characterize the risk premium over the subperiod from September 1977 through March 1985 remain the same during the subperiod from April 1985 through September 1987 and that any observed changes are caused by the presence of a bubble

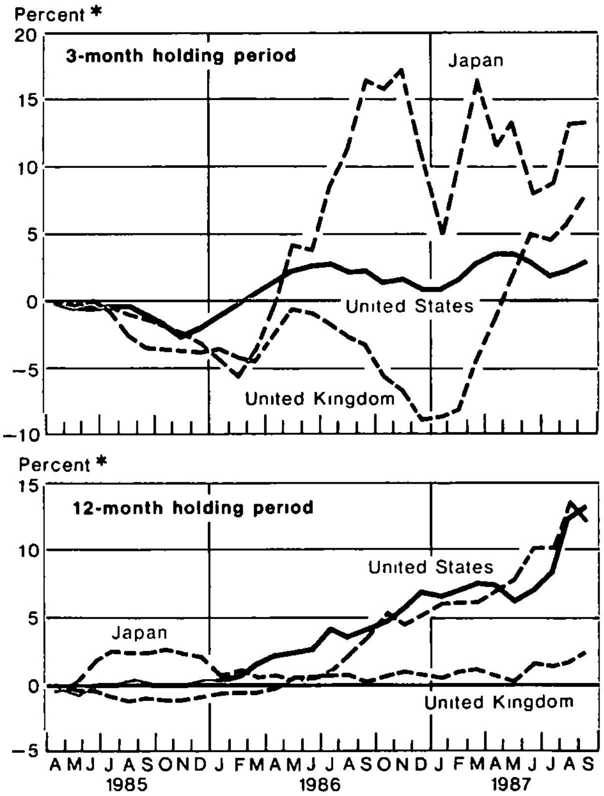
These assumptions make it straightforward to estimate the risk premium over the period from April 1985 through September 1987. One simply utilizes the estimated parameters a, b_1, ., b_j from the September 1977-March 1985 sample together with the information variables of the April 1985-September 1987 sample

Constructing the bubble premium involves estimating a new set of parameters, a′, b_1', b_2', . , b_j', over the period from April 1985 through September 1987 The bubble premium is calculated using the difference between the new estimates a′, b_1', . , b_j' and the old estimates a, b_1, . , b_j, together with the information variables of the later period Specifically, the estimation method allows for possible instability in the regression coefficients throughout the April 1985 to September 1987 sample period through the use of a rolling regression beginning in April 1985, the coefficients of equation 2 are reestimated every month, with a new month added to the sample each time.[16] Thus every month in the post-March 1985 sample has an associated set of regression coefficients These coefficients, together with the information variables of each month, provide an empirical proxy for the sum of the risk premium plus the bubble premium Since the risk premium is already estimated, one can promptly deduce the size of the bubble premium by simple subtraction.[17]

[15]The assumption that no bubbles were present before April 1985 simplifies the exposition but does not invalidate the conclusion on the presence of a bubble after April 1985 If a bubble were present during the earlier part of the sample, then the results of the article would simply be interpreted as evidence that the bubble became stronger after April 1985

Chart 4

Bubble Premium



*Annualized rates of return

Empirical evidence on rational speculative bubbles

Charts 1, 2, and 3 present plots of the realized excess return R − rf (dashed black line), the risk premium (solid colored line), and the post-March 1985 sum of the risk premium and the bubble premium (solid black line) for the national stock markets of Japan, the United States, and Great Britain A 12-month holding period horizon is assumed in each case Observe that

[16]A single regression over the April 1985 to September 1987 period allows for a more abrupt change in the estimated coefficients from the earlier period but does not allow the coefficients to vary during the April 1985 to September 1987 period itself It turns out that the resulting bubble premium from a single regression is very similar to the one from the rolling regression

[17]As noted earlier, one of the assumptions underlying the methodology is that the parameters of the reduced form equation 2 that describe the time variability of the risk premium do not change during the post-March 1985 suspected bubble period However, the mere presence of bubbles should increase the riskiness of holding stocks In a rational bubble, investors expect the volatility of stocks to increase as the bubble unfolds because the size of the potential loss (and gain) increases with time Thus the constructed bubble premium is the sum of two components the expected abnormal return conditional on no crash taking place plus the extra risk premium due to the presence of bubbles The presence of this extra risk premium does not affect the interpretation of the results, however, because it cannot exist without the presence of rational bubbles, if it exists, it indicates the presence of a bubble

after 1985, realized excess returns are positive in all three countries Indeed, during the last three years investors received higher rates of return in the stock markets than in the eurocurrency markets, a finding that suggests but does not constitute firm evidence that speculative bubbles are present Evidence for the presence of speculative bubbles would be a positive and rising bubble premium Recall that the bubble premium is an expected excess return over and above the risk-free rate plus the risk premium and is present only during the lifetime of the bubble. In Charts 1, 2, and 3 the bubble premium is the gap between the two solid lines after March 1985; for clarity, it is plotted separately in Chart 4.

Chart 4 shows that, indeed, the bubble premia are positive and increasing with time Japan and the United States show the strongest bubble evidence, and in both countries the evidence is stronger when a 12-month holding period is utilized In Great Britain the evidence is mixed because in the 3-month holding period the bubble premium becomes positive only after mid-1987

To confirm the upward trend of the bubble premium, one can regress the bubble premium, bp, on a linear time trend:

(3) $bp = c + d\ TIME + u,$

and test the hypothesis that the slope coefficent, d, is positive Table 3 presents the regression results In all six cases the slope coefficients are positive and significantly different from zero and thus confirm the positive evidence of the charts on the existence of rational speculative bubbles.

Table 3 also presents results from regressions in which the dependent variable is the realized abnormal return, bp + e, instead of the expected abnormal return, bp These results offer similar evidence of an upward trend, but the evidence is relatively weak As noted earlier, the noise term e creates excessive volatility and tends to mask the upward trend.[18]

This method of detecting speculative bubbles differs significantly from the traditional method of counting the number of abnormal positive returns, bp + e The traditional test requires independent observations and thus, if an adequate number of observations is to

[18]The sample in Table 3 begins in January 1986 because earlier estimates of the bubble premia tend to be negative and thus result in an overestimate of the upward trend in the bubble premia

Table 3

Is There a Time Trend in the Bubble Premium?

Sample January 1986 to September 1987, 21 observations

$bp = c + d\ TIME + u$

$bp+e = c' + d'\ TIME + v$

	c	d	$\bar{R}^2$	SEE	c′	d′	$\bar{R}^2$	SEE
Three-month holding period								
Japan	−6 88	77*	43	5 4	56 14†	−1 32	−01	38 8
	(5 57)	(24)			(31 21)	(1 58)		
United States	−0 43	13*	36	1 0	10 30	21	−05	26 3
	(1 14)	(05)			(19 28)	(83)		
United Kingdom	−14 34*	39*	21	4 2	−92 71*	3 17*	23	33 2
	(5 45)	(19)			(46 23)	(1 35)		
Twelve-month holding period								
Japan	−7 39*	63*	91	1 2	12 74	1 72*	29	15 8
	(1 09)	(05)			(14 26)	(74)		
United States	−4 81*	52*	89	1 1	7 21	92*	47	5 9
	(56)	(03)			(4 48)	(22)		
United Kingdom	−2 64*	10*	70	0 4	−15 10	81†	18	9 7
	(0 39)	(01)			(13 75)	(43)		

*Significant at the 5 percent level

†Significant at the 10 percent level

Note bp is the bubble premium, the expected abnormal return conditional on no bubble crash taking place, and bp + e is the realized abnormal return (see equation 1 of the text) $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom SEE is the regression standard error Numbers in parentheses are corrected OLS standard errors, the correction is due to the overlapping intervals be obtained, can only be performed using the 3-month holding period. There is only a maximum of two non-overlapping observations in the case of a 12-month holding period. Table 4 tabulates the realized abnormal returns, bp + e, for the 3-month holding period. Take, for instance, the subperiod January 1986 to September 1987, which contains seven nonoverlapping observations. In Japan, the holding period sequence January-April-July-October shows four positive and three negative abnormal returns, while the other two holding period sequences, February-May-August-November and March-June-September-December, show five positive and two negative abnormal returns Clearly, one cannot reject the null hypothesis that these abnormal returns were generated by chance. For example, the case of five positive abnormal returns out of seven returns can arise from chance with probability 0 227 [19] In the United States, the number of positive abnormal returns is five for the first holding period sequence, four for the second, and six for the third In Great Britain, the evidence against the likelihood that the results could be due to chance is the weakest: four positive abnormal returns in the first sequence, three in the second, and four in the third. Overall, these findings show that the traditional runs test is unable to reject the null hypothesis of no bubbles. The procedure of relying on bubble premia clearly has more power.

Finally, it should be noted that the hypothesis of rational bubbles cannot predict how much a market would collapse once a bubble bursts. Although the example presented in the Box assumes that the market returns to its fundamental value after the bubble bursts, the market could, in reality, overshoot or undershoot its fundamental value. For example, in October 1987 the U K market that had earlier shown weak bubble evidence fell by about as much as the U.S. market, although the latter had shown strong bubble evidence.[20]

Table 4

Realized Abnormal Returns*

April 1985 to September 1987
Three-Month Holding Period
(In Percent)

Date	Japan	United States	United Kingdom
1985			
April	−4 80	−16 39	−6 76
May	−2 30	−4 55	2 88
June	−7 60	6 87	−35 27
July	11 78	4 86	−35 12
August	−17 96	−19 40	−16 64
September	−26 27	−38 86	−22 94
October	−4 99	−21 13	−2 28
November	−26 28	11 87	−7 84
December	−10 22	44 54	7 43
1986			
January	−15 98	18 91	−13 11
February	13 51	28 45	−3 27
March	74 07	37 17	42 27
April	71 29	34 27	18 24
May	67 69	22 02	−24 02
June	−2 25	8 00	−14 00
July	39 98	−11 80	−38 04
August	68 83	−4 42	−17 17
September	62 10	−41 02	−65 83
October	−4 83	−1 47	−28 52
November	−22.00	−19 03	−35 07
December	−8.20	2 34	−11 70
1987			
January	77 58	33 46	19 01
February	58 64	46 00	46 66
March	60 56	73 17	54 20
April	57 92	5 74	37 19
May	62 49	−5 15	53 31
June	9 90	2 32	68 84
July	−36 45	25 23	63 63
August	−15 99	41 02	10 61
September	5 36	8 59	19 20

*Realized abnormal returns refer to bp+e of equation 1 in the text They are annualized

Conclusion

Despite the difficulty of uncovering speculative bubbles from the data, this article isolated evidence consistent with the hypothesis of rational bubbles in the national stock markets of Japan and the United States before the October crash. In Great Britain the evidence is somewhat weaker. Evidence for the presence of rational bubbles is a positive and increasing bubble premium, which market participants require in order to invest during a bubble period. During the lifetime of a rational speculative bubble, market participants expect to receive positive abnormal returns (bubble premia) as compensation for the probability of a bubble crash and

[19]This is the probability of obtaining two or less negative tickets (five or more positive tickets) when drawing seven times with replacement from a box that contains two tickets, one positive and one negative There are 128 possible sequences of positives and negatives, out of which 1 sequence contains exactly zero negatives (seven positives), 7 sequences contain exactly one negative (six positives), and 21 sequences contain exactly two negatives (five positives) Thus (1 + 7 + 21) / 128 = 0 227

[20]Those who question the rational bubbles hypothesis as a general characteristic of stock market fluctuations typically argue that any evidence interpreted as a rational bubble can also be interpreted as arising from the econometrician's ignorance about unobservable market fundamentals For a review of such arguments, see James D Hamilton, "On Testing for Self-Fulfilling Speculative Bubbles," *International Economic Review*, vol 27 (October 1986), pp 545-52 Although this criticism of speculative bubbles is plausible in general, it cannot be easily applied to the specific evidence in the text It is very hard to construct a story based on fundamentals that can explain both the sudden collapse of stock prices in October 1987 and the previous upward trend This difficulty becomes immediately evident once one tries to use Hamilton's examples

a large onetime loss. The size of the bubble premium grows over time as the bubble unfolds because the degree of market overvaluation rises. As the magnitude of the potential loss during a crash increases, investors require progressively larger compensation. Indeed, the data show a positive and rising bubble premium for one and a half years before October 1987 in the national stock markets of Japan and the United States, and for half a year before October 1987 in the national stock market of Great Britain.

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