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Underreported earnings and age-specific income redistribution in post-socialist economies

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Abstract

We analyze underreported earnings and age-specific income redistribution in post-socialist economies. Pensions, other transfers and public expenditures are financed from contributions and wage taxes, respectively. We derive the reported earnings and savings from individual utility maximization, when workers overly discount the future, obtain partial satisfaction from reporting earnings, cannot be excluded from the use of public services. The government maximizes a utilitarian social welfare function, corrected for discounting and taking into account the utility of public services. An optimal proportional pension system – complemented by appropriate means-testing – provides higher welfare than any system containing a significant flat component.

Keywords: reporting earnings, proportional (contributional) pension, flat pension, progressive wage tax, redistribution

JEL: H55, D91

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Keresetbevallás és korspecifikus jövedelem-újraelosztás posztszocialista gazdaságokban

Simonovits András

Összefoglaló

A keresetbevallást és a korspecifikus jövedelem-újraelosztást elemezzük a posztszocialista gazdaságokban. A nyugdíjakat, más transzfereket és a közkiadásokat járulékokból és adókból fedezik. Egyéni hasznosságmaximalizálásból származtatjuk a keresetbevallást és a megtakarítást, amelynek során a dolgozók túlzottan leszámítolják a jövőt, és részleges kielégülést nyernek a keresetbevallásból, valamint nem lehet őket kizárni a közszolgáltatások igénybevételéből. A kormány olyan utilitarista társadalmi jóléti függvényt maximalizál, amely eltekint a leszámítolástól, és figyelembe veszi a közszolgálatok hasznát. Egy megfelelő rászorultsági nyugdíjjal kiegészített optimális keresetarányos nyugdíj nagyobb jólétet nyújt, mint bármely olyan rendszer, amelyben jelentős mértékű alapnyugdíjat kap minden nyugdíjas.

Tárgyszavak: keresetbevallás, keresetarányos nyugdíjak, alapnyugdíj, progresszív jövedelemadó, újraelosztás

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December 2, 2009

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1. Introduction

In the classical socialist (centrally planned) economies, the heavy profits and modest contributions of the government-owned firms financed the welfare systems. In the transition economies (like any market economy), however, the welfare systems are financed from taxes and contributions. While the privatization of the whole economy made the economic system more efficient and compatible with democracy, it also opened up wide possibilities for employees and employers to avoid taxes and contributions, making the financing of the public sector, including the welfare system difficult. As a result of the structural pension reforms, started after 1998 in a number of post-socialist countries, the progressivity of the new pensions have been slowly or quickly phased out. These trends have strengthened the incentives to report earnings more fully but have reduced and will reduce the newly rewarded public pensions of those who reported and will report low lifetime earnings (with short reported lengths of employment or low reported earnings), regardless whether the report was correct or not. The voluntary old-age savings help myopic workers little, moreover, only the better paid evaders save at all. Similar problems arise in Latin American countries and on a smaller scale, in the developed world.

We speak of a *progressive* pension system if the ratio of benefit to earning is a decreasing function of the earning. Usually but not always the benefit increases with the earning. Representing a progressive system as a linear *combination* of a universal flat benefit plus a proportional benefit can be a good approximation (Disney, 2004). Some countries (e.g. the Czech Republic) finance proportional and flat pensions from contributions proportional to reported earnings, while other countries (e.g. Denmark) finance the flat pension from general taxes.

A lively debate has recently started in Hungary whether the phasing-out of the redistributive elements in the pension system since 1998 was an error which needs a quick correction or not. According to the critics, any future pension should be explicitly a sum of flat (or basic, or Beveridgian) and proportional (or contributional or Bismarckian) components. Other experts (including me) contend this view, defend the proportional system and propose a much more modest and cheaper remedy, namely means-testing, i.e. topping up only the low pensions, and the government may even restrict this addition to those with genuine low earnings, monitoring savings or assets. There is only a consensus in the following: the development of the pension system should be calculable and the subsequent governments should follow credible pension strategies.

In an earlier paper (Simonovits, 2008), I studied the interrelation between reported earnings and redistribution in the pension system with the help of an elementary model. There it was assumed that the workers report either their full earnings or the statutory minimum wage, for short, minimum wage. Both the universal flat pension and the means-tested pension are financed from value added taxes, whose payment cannot be avoided. Assuming that among people reporting minimum earnings, the evaders and the low paid can be told apart, it has been shown that the means-tested system is more attractive than the universal flat system.

The present paper "complicates" the elementary model by endogenizing reporting wages and savings. Following the methodology of neoclassical economics, it derives the reported earnings and savings from individual utility maximization, overly discounting the future, partially enjoying to report earnings, while free-riding on public goods and services. A worker only reports his earnings to increase his resulting earnings-related benefit and moral satisfaction or decrease his transaction costs. A sensible model of the interaction between underreporting wages and income redistribution cannot neglect the existence of wage taxes either, because anyone who avoids paying contribution also avoids paying wage tax. A progressive wage tax, in turn, may redistribute between the working poor and the working rich, while a progressive pension system can do the same among the pensioners. We must also consider public goods financed from taxes, to be used freely by everybody (or alternatively, assume a balanced tax system, where public goods are paid from individual incomes).

The government has a social welfare function, built up from paternalistic, objective utility functions (without excessive discounting and with due respect to public goods and services) and the parameter values of the tax- and pension system are obtained from social welfare maximization. The major result of the present paper is as follows: if the workers have common reporting preferences, then there is a critical preference value, for which an optimal proportional pension system and an optimal progressive pension system (containing a flat pension component) achieve the same social welfare. (We shall see that if positive savings occur, then the social optimum is indeterminate.) Below that preference value (in a dark grey economy), the proportional system achieves higher welfare than the progressive system, above that preference value (in a light grey economy), the reverse holds. In the limiting case, when the workers report their full earnings (white economy), and the labor supply is still exogeneously given, it is worthwhile to introduce fully redistributive earning tax and pure flat pension.

We also add another idealization: there is only a wage tax but no value added tax. We investigate the impact of the structure of the tax- and pension system on the reported earnings and the savings of individuals with different earnings and reporting preferences in a general equilibrium framework.

As a technical simplification, we assume that the population and the economy, including the pension system, are stationary, moreover, there is full employment. We also assume that the workers fully understand the mechanism of the system and the government determines its tax- and pension strategy as a benevolent dictator.

In reality, population aging increases the burden on the pension systems. Full employment is only present in few countries during exceptionally good periods, furthermore in the short run, population aging may mitigate unemployment. Labor supply is also sensitive to the tax wedge (Diamond, 2003). Obviously, a large part of the workers does not understand the basic rules of the pension system. Note also that governments frequently concentrate on short-term political interests rather than on long-term social welfare. The resulting unpredictable government behavior undermines the workers' trust in the system.

The stability of the tax- and pension system is also an important factor of satisfactory reporting of earnings. This stability, however, varies from country to country. For example, in the US, the public pension system is basically stable, while in the otherwise similar Great Britain, the pension system's basic structure has often been changed in recent decades. Mentioning a closer example: the Czech pension system has hardly been changing, while the erstwhile twin, Slovakia radically transformed its pension system in 2004, from a flat system to a proportional system. The U-turns of the Hungarian pension policy would require a separate study (Gál et al., eds. 2008).

Having outlined the general model, we analyze the issue with a simple example, where the utility functions are logarithmic (though one function also contains a linear term). To be able to derive analytical expressions, further simplifications are needed. Even then we were only able to derive the conditional individual decisions, without solving the general equilibrium problem in a closed form. The only exception is the case of the proportional tax- and pension system or the case of the white economy. In the latter case, where every worker reports his full wage, even the socially optimal tax and contribution rates can be analytically determined. Interesting results can also be obtained for the black economy, where the workers only report the minimum wage, regardless of their true wages.

When we turn on the computer, and fill the model with illustrative numbers, we are able to take into account the balance conditions, too. The most important results are contained in Table 6: in the dark grey economy, the progressive system is inferior to the proportional system, while in the light grey economy, the reverse holds. We hope that even in the latter case, the completion of the proportional system with means-testing or pension credit is superior to the progressive one.

The technical problems of modeling required utmost simplifications. We hope that notwithstanding its abstractness, our toy model contributes to the better understanding of the interaction among reporting of earnings, saving, taxation and the public pension system. Anyway, we must consider the conclusions with utmost care.

Having presented the model, now we outline the related literature. The issue of universal versus means-tested pension was already discussed by Friedman and Cohen (1972): they suggested the replacement of the former by the latter, thus reducing the size of the welfare program. Akerlof (1978) investigated the issue in a more general framework and rejected the idea of a negative wage tax (similar to a flat pension) on the ground that it puts a too heavy marginal burden on the shoulders of a typical taxpayer. Modeling the choice between flat and means-tested pensions, Feldstein (1987) introduced individual utility functions and a utilitarian social welfare function built from paternalistic utility functions, without discounting (see also Docquier, 2002). He only considered homogeneous wage rates, fixed labor supply and flat benefits. Though he proved that the means-tested system is generally superior to a progressive one, he found the opposite atypically when (i) the former "may induce some utility-maximizing workers to save nothing" or (ii) to avoid problem (i), the means-tested benefit is set lower than socially optimal. In a more modern approach, the time-inconsistent behavior of excessive discounting was modeled by the hyperbolic discounting in Diamond and Kőszegi (2003).

The recent history of the British pension system provides ample evidence on these issues. Indeed, our questions have already been discussed in the context of the ongoing British reforms: the universal flat benefit has been topped-up since 1997 (means-testing), which in turn was relaxed by the introduction of a pension credit in 2003 (Clark and Emmerson, 2003). In a very sophisticated model, Sefton at al. (2008) demonstrated that the introduction of the pension credit raises the savings of the lower-paid but reduces the others, only modestly improving the overall situation. Similar issues have surfaced in the reform of Chile's means-tested pension system (Barr and Diamond, (2008), Chapter 13.2). Note that a Special Pension Study (2006) discussed the issue of minimum wage provision and the adequacy of pensioners' consumption.

Following this line of thought, Fehr et al. (2008a) created a computable general equilibrium model with individual risk of career paths and life span and determined numerically the key parameters of the socially optimal German "progressive pension arrangement". Their results are shocking: the quite generous but strongly earning-related pension system should be replaced by a progressive system, plus uniformly relieving the earnings from pension contribution up to 30% of the average earnings. They acknowledge the hike in the marginal pension contribution rate, the resulting reduction in the originally higher pensions, the generated heavy loss of labor supply but appreciate the simultaneous increase in social insurance so high that accept it. In a companion paper (Fehr et al. 2008b), they evaluate the German public pension system "with rational and hyperbolic consumers" and show the advantage of the system over the private one in ensuring life annuity, especially for workers with weak will-power. Barr and Diamond (2008, Chapter 7) also strongly emphasize the insurance (neglected here) provided by the progressive pension system when the future earnings are uncertain at the start. For an early model, see Varian (1980).

In her pioneering study, Augusztinovics (2005) showed that—due to fractured labor careers—a large part of the Hungarian cohorts retiring in the next decades would have quite low pensions. (For a newer treatment, see Augusztinovics and Köllő, 2008.) To avoid mass old-age poverty, she proposed a significant reduction of the present proportional benefits and the corresponding contribution rate, while introducing a universal flat benefit, to be financed from new value added taxes. An additional advantage of such a system is that however small contribution during however short period worth be paid, because the accruing pension is added to the flat benefit. Nevertheless, such a system weakens the incentives to report wages above a quite low threshold.

Valdés-Prieto (2009) analyzed a model, where workers choose the *density of contribution*, i.e. "the proportion of active life worked in fully covered jobs (density) rather than hours worked." Instead of diminishing the worker's utility as underreporting does in our model, he explicitly models the reduction of productivity in the informal sector and compares the efficiency of various insurance systems, including pension credit.

In most models, it is the labor supply rather than the reported earning, that plays the main role. Cremer et al. (2007) and Koethenbuerger et al. (2008) considered the following question: what type and what size of a pension system do voters vote for? (In addition to pension contributions, the second paper also considered wage taxes which finance public goods and services.) The political economy approach supplements our welfare approach and yields a similar result: *the more redistributive a tax- and pension system, the smaller is its size.* Cremer et al. (2008) are even closer to the present model, emphasizing the distinction between subjective and objective utility functions and the role of the credit constraint in the forced saving.

Since the present paper also models the tax system even if in a rudimentary form, we mention a few models on reporting wages for taxation purposes. For the case of flexible labor supply, the unique socially optimal linear tax rate can easily be determined (Atkinson and Stiglitz, 1980). The classical paper of tax evasion is Allingham and Sandmo (1972), which determined the optimal reported wage under given wage, tax rate, audit probability and fine. Pestieau and Possen (1991) modeled the connection between tax evasion behavior and the choice of work place, anticipating a most serious problem of any transition economy. Feldstein (1999) and Chetty (2009) analyzed tax avoidance and the deadweight loss of the income tax, including underreported incomes into the generalized labor supply. In their survey, Andreoni et al. (1998) extended the narrow neoclassical model and introduced soft but relevant concepts like moral sentiments and the satisfaction of the taxpayer with public goods and services. From our point of view, they made three important claims: (i) the morally more sensitive citizens declare a higher share of their true wages; (ii) the more unfair the tax system is in the eyes of the citizens, the less wage they declare; (iii) the less satisfied the taxpayers are with the public goods and services, the less wage they declare. In a related study, Lackó (2007) demonstrated that the preference to report is closely related to the citizens' opinion of the utility of public goods and services.

Mc Gillivrey (2001) and Christie and Holzner (2006) extended the field from tax evasion to contributions evasion. In their empirical work, the latter considered both old and new EU members and obtained interesting estimates on the degree of dual evasion. For example, according to their Table 1.19, in Hungary, 2002, individuals reported 69.8 percent of their personal income tax base, while only 63.6 percent of their gross earnings base. In contrast, their Table 1.10 contains a pair 76.6 and 67.5 percents on the Czech Republic, 2003. Similarly, Janky (2007) considered the public attitudes toward tax (and contribution) evasion in Hungary and he cited remarkable data from CEORG Omnibus survey 1999 (Table 2). To the question "Is it improper for a tax-payer to provide a false income statement in order to avoid taxes?", 53% of the Hungarians and 77% of the Czechs answered with disapproval, while 42% of the Hungarians and 18% of the Czechs found it excusable. While I doubt if such a sharp difference exists in real life, it is consistent with the prevalence of contributive pensions in Hungary and of redistributive pensions in the Czech Republic. Nevertheless, the redistributive pension system pushes the self-employed in the direction of underreporting in both countries. Bakos et al. (2008) discussed the tax evasion behavior of the Hungarian employees.

OECD (2002) gave a wide survey on the 'non-observed' economy. For a comparative analysis of the hidden economies of transitional countries, see also Lackó (2000). Tonin (2007) and Elek et al. (2009) analyzed the related interaction of minimum wage and tax evasion. Studying the effects of Russian flat tax reform of 2001, Gorodnichenko et al. (2009, p. 505), concluded that "(i) the tax evasion response to the tax change was large, but that (ii) the productivity response was relatively week ...." This justifies our concentration on underreported earnings rather than restrained labor supply.

We outline the structure of the present paper. Section 2 develops the model and then Section 3 presents some analytical results. Sections 4 displays numerical illustrations and Section 5 concludes.

2. The model

This Section starts with the determination of that pair of earnings report and saving which maximize the lifetime utility function of a given worker. Then we outline the macro framework and the social welfare function to be maximized by the government.

The utility maximizing worker

We shall make the following extreme, nevertheless meaningful assumptions. The population and the economy are stationary. Every young person works and every old person is retired. The worker is employed for a unit time period and the pensioner enjoys his retirement for a period of length μ, $0 < \mu < 1$. (In practice, the more one earns, the longer he lives on average but here we neglect this relation.) Most existing systems superfluously differentiate between employer's and employee's contributions, but we assume a unified contribution. Contrary to practice, we work with the total wage cost w rather than gross wages (their difference is the employer's contribution) and we calculate on its basis. Every worker files in a report v on his earnings w, $v \in [w_m, w]$ and pays a positive pension contribution $\tau v > 0$, where $w_m > 0$ denotes the *minimum wage*. The pension system is in balance. In addition, the worker pays a linear wage tax θv with a tax rate $\theta > 0$ and receives a lump-sum return $\varepsilon > 0$. The net tax is $\theta v - \varepsilon$. (This is the simplest formulation of progressive wage taxation.) The net tax finances public goods and services including health care. (It will be clear that without considering taxes, the model is not only unrealistic but mathematically confined to an unnecessarily narrow domain in the model space.)

In the present model, every worker has two characteristics, namely his *wage* w and his *preference to report* on his earning w, p. Assume that the two characteristics have a joint discrete probability distribution with relative frequencies f_i, perhaps index $i = (j, k)$ has two dimensions. We assume that a worker with (w, p) reports an earning $v \in [w_m, w]$ and saves $s \geq 0$. Due to a saving technology, the pensioner will have s/μ to dissave per period. Note that this means that the efficiency of the private saving is equal to that of the pension system. This is an assumption frequently used (e.g. Cremer et al., 2008). Others (like Feldstein, 1987) insist on the superiority of private savings over public savings, consequently they introduce an interest factor ρ, which is higher than $1/\mu$. Taking into account the high cost of providing indexed life annuities, one can also argue for $\rho < 1/\mu$.

The life annuity b consists of two parts: a universal flat benefit $\alpha \geq 0$ and a labor pension proportional to earnings report, βv, where $\beta \geq 0$ is the marginal accrual rate, therefore $b = \alpha + \beta v$.

The intensities of the worker's and the pensioner's consumption are respectively

$$c = w - (\tau + \theta)v + \varepsilon - s \quad \text{and} \quad d = \alpha + \beta v + \mu^{-1}s.$$

(Note that both consumptions are to be positive and the total consumption of a pensioner is μd.) Let us introduce the total burden rate $t = \tau + \theta$, its complement is called the *tax wedge*: $1 - t > 0$. Hence $c = w - tv + \varepsilon - s$. In our linear wage tax system, the individual and the per-capita net average taxes (projected to the total lifetime) are given respectively by

$$a = \frac{\theta v - \varepsilon}{1 + \mu}, \qquad \bar{a} = \frac{\theta \bar{v} - \varepsilon}{1 + \mu}.$$

If one consider the involvement of public goods to the model, (s)he can get rid of it by adding a balance condition, namely $\bar{a} = 0$ or equivalently, $\varepsilon = \theta\bar{v}$.

The *subjective* lifetime utility function of every worker consists of four parts: (i) the worker's utility $u(c)$, (ii) the pensioner's utility $\delta\mu u(d)$, where $0 < \delta < 1$ is the excessive discount factor, (iii) the utility of the public good $q(\bar{a})$ financed by the average tax $\bar{a}$ and (iv) the utility of reporting wage v when one earns w: $z(w, p, v)$. (This corresponds to the moral sentiments in Andreoni et al. (1998) plus the utility loss due to the cost of underreporting in Feldstein (1999).) In sum,

$$\hat{Z}(w, p, c, d, v) = u(c) + \delta\mu u(d) + (1 + \mu)q(\bar{a}) + z(w, p, v). \tag{1}$$

(The hat distinguishes this function from the one to be used below.) As is usual, $u(\cdot)$ and $z(w, p, \cdot)$ are smooth, strictly increasing and strictly concave functions.

By definition, the worker determines his reported wage $v(w, p)$ and saving $s(w, p)$ so as to maximize the subjective utility $\hat{Z}(w, p, c, d, v)$. By substituting the consumption equations into $\hat{Z}(w, p, c, d, v)$ and dropping hat and the individually uncontrollable $q(\cdot)$, yields

$$Z(w, p, v, s) = u(w - tv + \varepsilon - s) + \delta\mu u(\alpha + \beta v + \mu^{-1}s) + z(w, p, v),$$

and partial derivation by v and s yields the optimum. Except for the black economy, where $v = w_m$, we shall always assume that the reported earning is greater than the minimum wage: $v > w_m$.

Slack credit constraint (S)

First assume that the worker's saving is also an internal optimum: *slack* credit constraint. Note that each type consists of many similar and independent workers, therefore the impact of a single individual on the average tax is practically nil. At this point we call the reader's attention to the fact that we shall use corresponding letters S and B to differentiate between slack and binding credit constraints. Then the first-order necessary conditions are

$$Z'_v(w, p, v, s) = -tu'(c) + \delta\mu\beta u'(d) + z'_v(w, p, v) = 0 \tag{2S}$$

and

$$Z'_s(w, p, v, s) = -u'(c) + \delta u'(d) = 0. \tag{3S}$$

Under suitable regularity assumptions, this system of equations can be solved as follows: we express d from (3S) as a function of c: $d = d(c)$. Then we substitute it into (2S):

$$z'_v(w, p, v) = (t - \beta\mu)u'(c), \tag{4S}$$

whence v can be similarly expressed: $v = v(c)$. (If we neglected the tax system and the flat benefit, then the RHS of (4S) would disappear, and the reported earnings would become independent of the pension system. Moreover, every worker would fully report his earnings!)

To have c as a function of individual characteristics, we must eliminate s. By multiplying equation for d by μ and adding to equation for c, s cancels out and we arrive to the *lifetime individual budget constraint*:

$$c + \mu d(c) = \mu\alpha + w - (t - \beta\mu)v(c) + \varepsilon. \tag{5S}$$

From this equation, we can determine $c^* = c^S(w,p)$, then $d^* = d^S(w,p)$, $v^* = v^S(w,p)$ and finally, $s^* = s^S(w,p)$.

Binding credit constraint (B)

What happens if the above optimal saving is negative? Then it only remains a plan and the credit constraint becomes *binding*: $s = 0$. (If we accepted negative savings, then we should model that the worker should pay much higher interest after his credit than what he receives after his savings, i.e. the interest on loan should be positive.) Then (3S) is replaced by

$$Z'_s(w,p,v,0) = -u'(c) + \delta u'(d) < 0 \quad \text{for} \quad s = 0. \tag{3B}$$

Consequently, the other optimum condition, namely for v^B, becomes

$$Z'_v(w,p,v,0) = -tu'(w - tv + \varepsilon) + \delta\mu\beta u'(\alpha + \beta v) + z'_v(w,p,v) = 0, \tag{2B}$$

yielding directly $v = v^B(w,p)$. Finally, (3S) and (5S) are replaced by

$$c^B(w,p) = w - tv^B(w,p) + \varepsilon \quad \text{and} \quad d^B(w,p) = \alpha + \beta v^B(w,p). \tag{5B}$$

The positivity of c and d are guaranteed by the usual assumptions. One must also pay attention to the feasibility of the reported earnings but with suitable parameter values, $w_m \le v^B \le w$ holds.

Macro framework

Until now we have only considered a single, though arbitrary worker. In reality, there exists a lot of types, type i characterized by wage w_i and preference to report p_i, $i = 1, 2, \ldots, I > 1$. They could also differ by their discount factors and expected lengths in retirement, but we gloss over the latter complexities. (Note, however, that in Feldstein (1987) or Cremer et al. (2007), the heterogeneity of the discount factor played a crucial role.)

It is time to define the average reported earnings:

$$\bar{v} = \sum_{i=1}^{I} f_i v_i, \tag{6}$$

where the optimal v_i is either v_i^S or v_i^B and $\bar{v}$ depends on $(\alpha, \varepsilon, \theta, \tau, \beta)$. Note that we have suppressed the dependence of reported earnings on the preferences to report.

We also assume that the pension system is balanced, therefore for any given feasible nonnegative parameter vector $(\beta, \varepsilon, \theta, \tau)$, we are looking for a fifth parameter $\alpha \geq 0$, so that using (6),

$$\tau\bar{v} = \mu(\alpha + \beta\bar{v}). \tag{7}$$

If $\tau \geq \mu\beta$, then $\alpha > 0$ exists.

Finally, we outline the government's welfare maximizing task. As a starting point, we define a paternalistic *objective* utility function for type (w_i, p_i) as

$$U(c_i, d_i, \bar{a}, v_i) = u(c(w_i, p_i)) + \mu u(d(w_i, p_i)) + (1 + \mu)q(\bar{a}) + z(w_i, p_i, v_i), \tag{8}$$

where the myopic discounting disappears.

The social welfare function is defined as the average of the objective utilities taken at the subjective optima (starred):

$$V(\alpha, \varepsilon, \theta, \tau) = \sum_{i=1}^{I} f_i U(c_i^*, d_i^*, \bar{a}^*, v_i^*). \tag{9}$$

The government looks for a vector $(\alpha, \varepsilon, \theta, \tau, \beta)$ such that—in addition to satisfying (7) with definition (6)—maximizes the social welfare function (9).

Means-tested benefit (M)

Finally, we must also discuss the *means-tested* pension system, its symbol is M. The starting point is simple: the lower the share of the flat benefit in the total benefit, the greater is the danger that the lower paid's benefit is too low. This problem can be tackled if the average pension is raised. In fact, the lower is the share of the flat component, the larger is the size of the pension system (cf. Koethenbuerger et al., 2008). But in a system suffering just from underreporting of earnings, there are strong hurdles to this solution, and it may be more appropriate to introduce means-testing. This is, however, a two-edged weapon. On the one hand, it reduces the burden; on the other hand, it eliminates the connection between the contribution and the benefits of the low-paid, undermining their incentives to report earnings.

Let γ be the value of the means-tested benefit. Then the proportional benefit rule $b = \beta v$ is replaced by

$$b^{(M)} = \max[\beta v, \gamma].$$

Of course, the government must choose such a value for γ, which saves the low-paid from poverty. It should be at least as great as the flat benefit: $\gamma \geq \alpha$, but it should not be too attractive. For example, $\gamma = \alpha$ is a good compromise. Or, if this is awkward, then the free-riders should be excluded on a selection process based on external signals (Simonovits, 2008).

It is quite difficult to derive the optimum conditions of such a system, because in addition to the S–B distinction, we should also make a difference whether the type (w, p) receives a normal or a means-tested benefit. In the latter case, in the equation for the optimal report, $\beta = 0$, and in the equation of means testing, $\gamma - \beta v > 0$ is financed by others. (7) modifies as follows:

$$\tau\bar{v} = \mu(\alpha + \beta\bar{v}) + \mu \sum_{i=1}^{I} f_i[\gamma - \beta v_i]_+, \tag{7M}$$

where x_+ is the positive part of a real number x: $x_+ = x$ if $x \geq 0$, $x_+ = 0$ if $x < 0$. In reality, proportional systems with $\alpha = 0$ are frequently complemented by means-testing.

3. Analytical results

In this Section we consider simple cases rather than a general framework which can be studied analytically.

Logarithmic utility functions

Most studies (cf. Feldstein, 1987) apply logarithmic specification, although it eliminates certain subtler relations. In order to exclude the anomaly that the worker can increase his subjective utility by overreporting his true earning, a linear term had to be added to the logarithmic function $z(w, p, \cdot)$. Here are the parametric functions:

$$u(c) = \log c, \qquad z(w, p, v) = p(\log v - v/w), \qquad q(\bar{a}) = \kappa \log \bar{a},$$

where $p > 0$ is the parameter of the preference to report and $\kappa > 0$ is the parameter of the efficiency of public goods and services. Note that $u'(c) = 1/c$, $z'_v(w, p, v) = p(1/v - 1/w)$, $q'(\bar{a}) = \kappa/\bar{a}$. They are all positive and decreasing in the interval $w_m < v \leq w$.

We again distinguish the cases of slack and binding credit constraints but temporarily neglect the macrobalance (7).

Slack credit constraint

We start our analysis with the case of slack credit constraint. Introducing notion $t_1 = t - \mu\beta$, (3S) and (4S) imply

$$d(c) = \delta c \quad \text{and} \quad v(c) = \frac{pwc}{pc + t_1 w}, \qquad t > \mu\beta. \tag{10S}$$

Inserting (10S) into (5S),

$$c + \delta\mu c + \frac{t_1 pwc}{pc + t_1 w} = w + \mu\alpha + \varepsilon. \tag{12S}$$

(Note that if $p \to +\infty$, then $v \to w$.)

After rearrangement, we obtain a quadratic equation for c:

$$h_2 c^2 + h_1 c + h_0 = 0,$$

where

$$h_2 = p(1 + \mu\delta), \quad h_1 = t_1(1 + \mu\delta)w + t_1 pw - p(w + \mu\alpha + \varepsilon), \quad h_0 = -t_1 w(w + \mu\alpha + \varepsilon).$$

The equation has a unique positive root and under suitable specification, it is less than or equal to the earning: $v^S \leq w$.

Hence the other variables can also be expressed with the parameters of the model. Note that $s^S = w - tv^S - c^S - \varepsilon$ should be nonnegative. In case of negative value, we must put zero, as is done in the next subsection.

Binding credit constraint

We turn to the analysis of the binding credit constraint. In view of (2B), the equation of the optimal earnings report simplifies to

$$Z'_v(w,p,v,0) = -\frac{t}{w-tv+\varepsilon} + \frac{\delta\mu\beta}{\alpha+\beta v} + \frac{p}{v} - \frac{p}{w} = 0. \qquad (13B)$$

This equation is cubic in v, which can be solved in a closed form, but its treatment is prohibitively difficult. There are two special cases, when the equation reduces to a quadratic one: (i) the purely proportional system and (ii) the purely flat pension: either

$$-\frac{t}{w-tv+\varepsilon} + \frac{\delta\mu+p}{v} - \frac{p}{w} = 0 \quad \text{if} \quad \alpha = 0; \qquad (13B1)$$

or

$$-\frac{t}{w-tv+\varepsilon} + \frac{p}{v} - \frac{p}{w} = 0 \quad \text{if} \quad \beta = 0. \qquad (13B2)$$

Fixing the other parameter values, it is obvious that the report is higher in the proportional than in the flat system.

The quadratic equation implied by (13B1) is as follows:

$$e_2v^2 + e_1v + e_0 = 0,$$

where

$$e_2 = pt, \quad -e_1 = tw + t(p+\mu\delta)w + p(w+\varepsilon), \quad e_0 = (p+\mu\delta)(w+\varepsilon)w.$$

Similar formulas can be obtained for (13B2).

Returning to the general (13B), note that $Z'_v(w,p,0,0) = +\infty$, but as a matter of fact, $v \ge w_m$ implies $Z'_v(w,p,w_m,0) \ge 0$. Consequently, it is sufficient to assume that

$$Z'_v(w,p,w,0) = -\frac{t}{w-tw+\varepsilon} + \frac{\delta\mu\beta}{\alpha+\beta w} \le 0.$$

This inequality determines another domain of the admissible parameter values than its S counterpart.

To determine if S or B is valid, one must check inequality $d^B > \delta c^B$ ($Z'_s < 0$).

At this point we specify our earlier results on the means-tested pension, where the utility function is Cobb–Douglas and the underlying benefit is proportional: $\alpha = 0$. We confine our attention to the simplest case, where there are only two types with earnings w_L and w_H, $w_L < 1 < w_H$. Let the value of the means-tested benefit lie between the two proportional benefits: $\mu^{-1}\tau v_L^{(P)} < \gamma < \mu^{-1}\tau v_H^{(P)}$. Then type L receives a means-tested benefit, while type H does not. In the slack case, the two lifetime budget constraints are respectively

$$c_L + \delta\mu c_L + \frac{tpw_Lc_L}{pc_L+tw_L} = w_L + \mu\gamma + \varepsilon \qquad (12SML)$$

and

$$c_H + \delta\mu c_H + \frac{tpw_H c_H}{pc_H + (t - \mu\beta)w_H} = w_H + \varepsilon. \qquad (12SMH)$$

In the binding case, however, (13B1) refers to H and (13B2) refers to L. Note that in the optimality condition (13B2), β becomes 0, but in the pension balance equation (7M)

$$\tau(f_L v_L + f_H v_H) = \mu(f_L\gamma + f_H\beta v_H),$$

it remains positive.

We have a typical problem of general equilibrium. There are three equations: (7M), (12SML), (12SMH), and three unknowns: v_L, v_H, β.

We have determined the conditional individual decisions and the relevant variables. The resulting formulas, however, are so complex that the analytical treatment of the balance equation and socially optimal rates and others seems hopeless.

It can easily be shown that under mild conditions, in the proportional pension- and tax system ($\varepsilon = \alpha = 0$) the report is positive and less than the earning: $w_m \le v_i < w_i$, $i = 1, 2, \ldots, I$. The other variables are also meaningful. We only conjecture but cannot prove that for socially optimal rates, the higher the preference to report, the tighter the credit constraint is.

White economy

Before giving up analytical hopes, it is worthwhile considering the *white* economy, where every worker reports his full wage: $p = +\infty$, i.e. $v = w$. (Note that we have dropped the indices of type.) Now (12S) simplifies to the linear equation $c + \delta\mu c = \mu\alpha(1 - w) + \varepsilon + w(1 - \theta)$, i.e. to

$$c^S = \frac{\mu\alpha(1 - w) + \varepsilon + w(1 - \theta)}{1 + \delta\mu} \quad \text{and} \quad d^S = \delta c^S. \qquad (14S)$$

Similarly, (5B) reduces to

$$c^B = (1 - \tau - \theta)w + \varepsilon, \qquad d^B = \alpha + (\mu^{-1}\tau - \alpha)w \quad \text{and} \quad a^B = \frac{\theta w - \varepsilon}{1 + \mu}. \qquad (14B)$$

In both cases, $\bar{v} = \bar{w} = 1$ makes the two balance equations rather trivial: $\mu(\alpha + \beta) = \tau$ and $\bar{a} = (\theta - \varepsilon)/(1 + \mu)$. It is quite easy to show that the credit constraint is only binding if and only if

$$\alpha(1 - w) > \delta[(1 - \tau - \theta)w + \varepsilon] - \mu^{-1}w.$$

We can immediately check that in case of positive flat benefit ($\alpha > 0$) only earners below average ($w < 1$) can have slack credit constraints, but only if α and τ are large enough.

Note that due to the assumed perfect substitution between private and public savings, the slack system's optimal variables are independent of the contribution rate τ. This suggests that in investigating socially optimal rates, it is sufficient to consider the binding system.

For the time being, we consider proportional transfer systems. Under this assumption, we are able to obtain analytical results for socially optimal contribution and tax rates. To avoid lengthy notations, we only consider the case when every worker has a unit wage: $w = 1$ and a binding credit constraint. Now the social welfare function is reduced to

$$g(\tau, \theta) = \log(1 - \tau - \theta) + \mu \log(\mu^{-1}\tau) + \kappa^* \log \theta, \quad \text{where} \quad \kappa^* = (1 + \mu)\kappa.$$

Taking the partial derivatives with respect to the two rates and setting them to zero, yields the optimality conditions:

$$-\tau + \mu(1 - \tau - \theta) = 0 \quad \text{and} \quad -\theta + \kappa^*(1 - \tau - \theta) = 0.$$

Solving for the optimal rates:

$$\tau^o = \frac{1}{1 + \mu^{-1}(1 + \kappa^*)} \quad \text{and} \quad \theta^o = \frac{\mu^{-1}\kappa^*}{1 + \mu^{-1}(1 + \kappa^*)}.$$

What happens if we relax the proportionality assumption and give way to $\alpha > 0$ or $\varepsilon > 0$? We obtain a surprising result, at least for the two-type model, where w_L and w_H are the low and high earnings, respectively. Then the absolute optimum can be achieved: $c_H^o = d_H^o = c_L^o = d_L^o$.

For the time being, let us neglect public services: ($\kappa = 0$) and take into account the relation $\beta = \mu^{-1}\tau$ in the formulas $c_H = c_L$, $c_L = d_L$ and $c_H = d_H$. The resulting equations are

$$\tau + \theta = 1 \quad \text{and} \quad w_H = (w_H + \mu^{-1})\tau + (w_H - 1)\theta,$$

that yield the optimal contribution and tax rates:

$$\tau^o = \frac{1}{1 + \mu^{-1}} \quad \text{and} \quad \theta^o = \frac{\mu^{-1}}{1 + \mu^{-1}}.$$

Taking also into account the public services ($\kappa > 0$), these rates should be reduced to make room for the *net tax rate* $\omega = \theta - \varepsilon$. Then $c^o = d^o = (1 - \omega)/(1 + \mu)$ and $a^o = \omega/(1 + \mu)$. Now the social welfare function coincides with the utility function. Leaving aside the logged variables's denominator $1 + \mu$ and their common multiplier $1 + \mu$, we end up with the formula $g(\omega) = \log(1 - \omega) + \kappa \log \omega$.

Taking the derivative of $g(\omega)$ and expressing the optimal net rate from $g'(\omega) = 0$, yields the general optimum:

$$\omega^o = \frac{\kappa}{1 + \kappa}, \qquad \tau^o = \frac{1 - \omega^o}{1 + \mu^{-1}} = \frac{1}{(1 + \kappa)(1 + \mu^{-1})} \quad \text{and} \quad \theta^o = \mu^{-1}\tau^o.$$

Black economy

Another analytically tractable case is the *black economy*, where workers only report the minimum earnings. For the government it is superfluous to struggle with the pension system and the tax return: $\tau = 0$, $\varepsilon = 0$. Only the slack system has any relevance now:

$$c^S = \frac{w - \theta w_m}{1 + \mu\delta}, \qquad d^S = \delta c^S \quad \text{and} \quad a^S = \frac{\theta w_m}{1 + \mu}. \tag{15S}$$

In this case, the socially optimal tax rate can be derived analytically again. Due to the logarithmic utility functions, the constant denominators as well as the common multiplier $(1 + \mu)$ can be dropped. Therefore V can be replaced by

$$g(\theta) = \sum_{i=1}^{I} f_i \log(w_i - w_m\theta) + \kappa \log(w_m\theta).$$

Taking the derivative and setting it to zero, yields the equation for the unique equilibrium tax rate:

$$-\sum_{i=1}^{I} f_i \frac{w_m}{w_i - w_m\theta} + \frac{\kappa}{\theta} = 0.$$

Taking the common denominators, an I-degree polynomial obtains. As an illustration, take $I = 1$, when (dropping the index 1) $w = 1$, $f = 1$:

$$-\frac{w_m}{1 - w_m\theta} + \frac{\kappa}{\theta} = 0, \quad \text{i.e.} \quad \theta^o = \frac{\kappa}{(1 + \kappa)w_m}.$$

Of course, reality is neither white nor black but grey. It would be pleasant to have simple solutions for the grey economy as well, but this is a futile hope. It is time to turn on the computer.

4. Numerical illustrations

We continue our investigation by numerical illustrations. We assume that the time spent in retirement is 1/2 of that spent in work: $\mu = 0.5$. The subjective discount factor is $\delta = 0.5$. First we neglect redistribution and consider the white economy. After some experimentation, we have chosen $\kappa = 1/3$ for the utility coefficient of the public services, yielding $\kappa^* = (3/2) \times (1/3) = 1/2$. This choice renders the results for the white economy reasonable: the socially optimal tax and contribution rates: $\theta^o = 0.25$ and $\tau^o = 0.25$ and the corresponding consumption pair is $c = 1/2$ and $d = 1/2$ with $a = 1/6$.

Switching on redistribution, the optimal contribution and tax rates become $\tau^o = 1/4$, $\theta^o = 3/4$ respectively, and the parameters of redistribution $\alpha^o = 1/2 = \varepsilon^o$. All these point to the obviously unrealistic assumption of the white economy, namely the lack of labor supply elasticity.

Turning to the black economy, for $\kappa = 1/3$ and $w_m = 0.4$, $\theta^{\rm o} = (1/3)/((1+1/3) \times 0.4) = 1/(4 \times 0.4) = 0.625$ is an extremely high value. Probably in the black economy, κ should be radically reduced, decreasing the optimal tax rate as well.

Next we move to the grey economy with various shades, represented by p, but for the time being, retain the previous rates. To begin with, we are studying the impact of preference to report parameter p on individual behavior in a proportional tax- and pension system with $\varepsilon = \alpha = 0$.

Table 1. *Preferences to report with proportional taxes and benefits:* $w = 1$

Preference to report p	Reported earning $v^{(P)}$	Benefit $b^{(P)}$	Saving $s^{(P)}$	Worker consumption $c^{(P)}$	Pensioner consumption $d^{(P)}$	Tax $a^{(P)}$
0.25	0.417	0.209	0.075	0.717	0.358	0.070
0.50	0.578	0.289	0.027	0.684	0.342	0.096
0.75	*0.667*	*0.333*	*0*	*0.667*	*0.333*	*0.111*
1.00	0.705	0.353	0	0.647	0.353	0.118
1.50	0.759	0.379	0	0.621	0.379	0.126
2.00	0.795	0.398	0	0.602	0.398	0.133
3.00	0.841	0.421	0	0.579	0.421	0.140
4.00	0.870	0.435	0	0.565	0.435	0.145
5.00	0.890	0.445	0	0.555	0.445	0.148
$+\infty$	1.000	0.500	0	0.500	0.500	0.167

Table 1 illustrates the fact that as the value of the preference to report rises, the reported earning rises, the worker consumption decreases. The behavior of pensioner consumption is more complex: it decreases in a slack system (for weaker preferences to report) but increases in a binding system (for stronger preferences to report). To display the white economy, we present c and d for $p = +\infty$, in the last row of Table 1. The separating preference to report is just $p^* = 0.75$, where s becomes 0, italicized. Presumably, in the former case, the gain in pensions does not make up for the loss in savings.

In Table 1, we assumed identical earnings. In the further calculations, we shall arbitrarily apply a very simple earning distribution: (w_L, w_H) with $f_H = 1/3$ and $f_L = 2/3$. To enhance its impact, we have chosen a rather extreme distribution with $w_L = 0.5$ and $w_H = 2$, yielding a unitary average wage.

Considering heterogeneous earnings, we start our investigation with proportional systems: $\alpha = 0$ and $\varepsilon = 0$. To illustrate the indeterminacy of the social optimum, we fix $\theta = 0.28$ and let τ change.

For the sake of brevity, we single out two parameter values of the preference to report: $p = 0.5$ for dark grey, $p = 2$ for light grey economy. Table 2 displays the result for the dark grey economy. As the contribution rate rises above a critical value 0.3, traditional savings disappear and aggregate life-cycle saving starts increasing. Therefore reported earnings start diminishing, and social welfare also drops. The highest socially optimal contribution rate and the corresponding variables are italicized.

Table 2. *Impact of contribution rate, dark grey economy, $p = 0.5$*

Contribution rate τ^o	Reported earning $\bar{v}$	Aggregate saving S	Tax $\bar{a}$	Social welfare V
0.28	0.547	0.016	0.102	–3.108
0.30	*0.547*	*0.005*	*0.102*	*–3.108*
0.32	0.540	0	0.101	–3.110
0.34	0.528	0	0.098	–3.113

Table 3. *Impact of contribution rate, light grey economy, $p = 2$*

Contribution rate τ^o	Reported earning $\bar{v}$	Aggregate saving S	Tax $\bar{a}$	Social welfare V
0.16	0.815	0.024	0.152	–4.859
0.18	0.815	0.008	0.152	–4.859
0.20	0.808	0	0.151	–4.852
0.22	0.795	0	0.148	–4.843
0.24	0.782	0	0.146	–4.838
0.26	*0.770*	*0.*	*0.144*	*–4.837*
0.28	0.757	0	0.141	–4.838
0.30	0.745	0	0.139	–4.842
0.32	0.733	0	0.137	–4.849

The situation is different in the light grey economy, shown in Table 3. For a given contribution rate, workers report higher earnings, therefore they need less traditional savings than in the dark grey economy. The critical contribution rate drops to 0.18, and passing it, the optimal contribution rate in the light grey economy is about 0.26, italicized, where a marginal rise in the contribution rate balances the marginal utility of higher old-age consumption and that of lower reported earnings and lower public services.

Having studied the proportional transfer systems, we turn to their generalizations, inserting flat benefits with varying size. We confine now our attention to the lowest optimal contribution and tax rates.

Table 4. *An optimal system with a given flat benefit, dark grey economy, $p = 0.5$*

Flat benefit α	Contri-bution rate τ^o	Tax rate θ^o	Marginal replace-ment rate β^o	Reported earning $\bar{v}^{(P)}$	Tax $\bar{a}$	Social welfare V
0.00	*0.22*	*0.28*	*0.440*	*0.547*	*0.102*	*–3.108*
0.04	0.35	0.29	0.620	0.500	0.097	–3.113
0.08	0.37	0.29	0.568	0.466	0.090	–3.124
0.12	0.36	0.29	0.447	0.439	0.085	–3.140
0.16	0.36	0.29	0.331	0.411	0.079	–3.161
0.20	0.35	0.29	0.181	0.385	0.074	–3.185

Table 4 suggests the following regularity: the higher the flat component, the lower is the average reported earnings and consequently, the lower is the social welfare, assuming that the government chose optimal tax rates. There is not much sense in differences between the values of social welfare functions, rather we must measure the decline by *consumption equivalencies*. For example, let $e > 1$ be such a scalar that by multiplying the earnings of both types' earnings with it, the social welfare with a flat component $\alpha = 0.2$ becomes identical with that of the original proportional system. We obtain $e = 1.03$, i.e. the economy should have earnings uniformly higher by 3% to achieve the original welfare of the proportional system in the foregoing flat system.

Note that contrary to our model, in reality, by increasing the value of the flat component, the contribution rate declines rather than rises0.

The situation is more complex in the light grey economy. The social welfare is maximal at a flat component $\alpha = 0.32$ (italicized row). Now it is the earnings in the proportional pension system which should be raised uniformly by 2.1%.

Table 5. *An optimal system with a given flat benefit, light grey economy, $p = 2$*

Flat benefit α	Contri-bution rate τ^o	Tax rate θ^o	Marginal replace-ment rate β^o	Reported earning $\bar{v}^{(F)}$	Tax $\bar{a}^{(F)}$	Social welfare $V^{(F)}$
0.00	0.26	0.26	0.520	0.782	0.136	–4.835
0.20	0.22	0.27	0.172	0.746	0.134	–4.779
0.24	0.21	0.27	0.094	0.735	0.132	–4.772
0.28	0.20	0.27	0.013	0.724	0.130	–4.767
0.32	*0.22*	*0.28*	*0*	*0.707*	*0.132*	*–4.755*
0.36	0.25	0.26	0	0.703	0.122	–4.760
0.40	0.28	0.24	0	0.699	0.112	–4.773

Table 6 provides the details of the four optimal solutions, namely those of the proportional and the flat systems in the dark and grey economies, respectively. The last (fifth) block contains the data of the means-tested system, to be outlined below. For comparability, the lifetime utility is broken down to four terms: utility of 1) worker consumption, 2) pensioner consumption, 3) public services and 4) reporting earnings.

Table 6. *Comparison of optimal systems with heterogeneous earnings*

Earning	Reported earning	Worker	Pensioner consumption	Saving	Lifetime 1	2	Utility 3	4
w	v	c	d	s	$U1$	$U2$	$U3$	$U4$
dark grey proportional pension: $p = 0.5$, $\alpha = 0$								
0.5	0.274	0.339	0.169	0.024	–1.083	–0.888	–1.140	–0.922
2.0	1.095	1.355	0.677	0.098	0.304	–0.195	–1.140	–0.228
dark grey pension with a flat component: $p = 0.5$, $\alpha = 0.2$								
0.5	0.198	0.373	0.236	0.000	–0.986	–0.722	–1.299	–1.008
2.0	0.760	1.346	0.673	0.168	0.297	–0.198	–1.299	–0.327
light grey proportional pension: $p = 2.0$, $\alpha = 0$								
0.5	0.391	0.297	0.203	0.000	–1.215	–0.796	–0.999	–3.442
2.0	1.564	1.187	0.813	0.000	0.171	–0.103	–0.999	–0.669
light grey pension with a flat component: $p = 2.0$, $\alpha = 0.32$								
0.5	0.360	0.320	0.320	0.000	–1.139	–0.570	–1.013	–3.484
2.0	1.400	1.168	0.584	0.132	0.155	–0.269	–1.013	–0.727
light grey means-tested pension: $p = 2.0$, $\gamma = 0.32$								
0.5	0.360	0.320	0.320	0.000	–1.139	–0.570	–0.991	–3.484
2.0	1.497	1.141	0.571	0.110	0.132	–0.281	–0.991	–0.690

Let us start with the dark grey economy. It is obvious that in an optimal proportional system almost every variable is proportional to earnings. The low-paid's pension as well as the corresponding pensioner consumption is inadmissibly low: $d_L^{(P)} = 0.169$. By introducing a flat benefit, this problem is ameliorated, but the report of earnings goes down so much that the level of public services drops and the utility of underreporting also shrinks. Nevertheless, the comparison needs detailed calculations.

Turning to the light grey economy, the average tax is much higher with flat benefits than without them. Though the pensioner's consumption is higher than it was in the dark grey economy, it is still insufficient: $d_L^{(P)} = 0.203$. For a flat component $\alpha = 0.32$, that value jumps to $d_L^{(F)} = 0.32$, quite satisfactory. At the same time, the average tax remains acceptable, while the quantity of public services and the utility level of underreporting are tolerable.

At programming the means-tested system, we have encountered difficulties, therefore we only compare our means-tested system with the optimal flat system, reserving the latter's parameter values. Since we have not yet reported aggregate data, we give them here: $V^{(M)} = -4.731$, is only better than its flat counterpart within the margin of calculation error. The average reported earnings is also greater than in the optimal flat system: $\bar{v} = 0.739 > 0.707$. Using the equivalence scale, one must raise the earnings by 0.6% in the optimal flat system. We have not yet succeeded to find the optimal means-tested system!

5. Conclusions

We have constructed a model, in which—reflecting mainly the characteristics of transitional and developing countries—the tax- and pension system influences the reported earnings rather than the labor supply. We have made just the first theoretical and numerical computations. The results are promising but a lot of further analytical investigations and numerical trials are needed to corroborate the temporary conclusions: the introduction of a significant flat component crowds out the public services, especially in economies with low reporting preference like Hungary is. But even in lighter grey economies the proportional system may remain superior to the progressive one if an appropriate means-tested (or a pension credit) system complements it. The impact of progressive earning tax requires further analysis (only appearing in the white economy in our paper) like that of the interaction of agents with different preferences to report0 (Simonovits, 2008).

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