



DEPARTMENT OF ECONOMICS

ISSN 1441-5429

DISCUSSION PAPER 07/11

## Golden-rule social security and public health in a dynastic model with endogenous longevity and fertility

Siew Ling Yew<sup>a</sup> and Jie Zhang<sup>b</sup>

### Abstract:

In this paper we investigate long-run optimal social security and public health and their effects on fertility, longevity, capital intensity, output per worker and welfare in a dynastic model with altruistic bequests. Under empirically plausible conditions, social security and public health reduce fertility and raise longevity, capital intensity and output per worker. The effects of social security, except that on longevity, are stronger than those of public health. Numerically, they can improve welfare (better when they are used together than used separately). We also illustrate numerically that there exists a unique convergent solution in the dynamic system at the steady state.

**Key words:** Social security; Public health; Life expectancy; Fertility

**JEL Classification:** H55; J13; O41

---

<sup>a</sup> Acknowledgement: Siew Ling Yew gratefully acknowledges financial support from Monash University (Grant No. B03005-1758108).

<sup>b</sup> **Correspondence:** Jie Zhang, Department of Economics, National University of Singapore, Singapore 117570; Fax: +65 6775 2646; Tel: +65 6516 6024; E-mail: [ecszj@nus.edu.sg](mailto:ecszj@nus.edu.sg)

## **1. Introduction**

Social security and public health have been two key concerns of many societies because of their possible important impacts on economic growth, population growth, and welfare. In fact, most of the developed nations have instituted pay-as-you-go (PAYG) social security and public health programs for several decades (see, e.g., Aaron, 1985; Lee and Tuljapurkar, 1997). At the same time, these countries have observed dramatic increases in life expectancy and significant declines in fertility, leading to rapid population aging. In OECD countries, total fertility rates have declined dramatically from an average 2.7 in 1970 to 1.7 in 2008, while life expectancy at birth has increased by more than 10 years since 1960, reaching 79.1 in 2007 (OECD, 2010a, 2010b). Government expenditure on social security and public health, therefore, increased substantially in tandem with the population aging.

Indeed, the gross public social expenditure on average across the OECD countries has increased from 16% of GDP in 1980 to 19% in 2007, of which public pensions and public health expenditures were over 7% and 6.4 % of GDP respectively (OECD, 2010c). By contrast, private health spending accounted only for about 2.6% of GDP on average across the OECD countries in 2008 (OECD, 2010d). In the United States, for example, there were upward trends in the ratio of public to private health expenditure and in life expectancy in the time series data for the period 1870-2000 as noted in Tang and Zhang (2007). These patterns suggest that increases in social security and public health may be closely linked to declines in fertility and increases in life expectancy in developed countries.

The steady population aging has caused serious concerns about future economic growth, the pressure on funding social security and public health care, and the wellbeing of a greyer population. A particular challenge is: increasing spending on social security and public health may increase longevity and reduce fertility further, but the resultant increase in population aging may in turn call for more spending on social security and public health. The welfare consequence of the challenge may depend largely on how such policies affect capital accumulation and economic growth. Therefore, it is important and relevant to explore the implications of PAYG social security and public health together for fertility, life expectancy, capital accumulation, economic growth and welfare.

We will carry out this task in a dynastic model of neoclassical growth with altruistic bequests, endogenous fertility, endogenous longevity, and actuarially fair annuity markets. In our model, longevity depends positively on per worker public health spending. A rise in the tax rate for social security in our model has opposing effects on fertility and capital intensity, and hence on longevity. On the one hand, by increasing the bequest cost of having a child, the tax rise tends to reduce fertility and raise capital intensity and longevity. On the other hand, by reducing the after-tax wage rate, the opportunity cost of spending time rearing a child falls and hence, the tax rise tends to increase fertility and reduce capital intensity and longevity. Moreover, under a system that links social security benefits to earnings, the forgone social security benefits of spending time rearing a child rises with the tax rate, thereby adding to the cost of a child to channel a negative effect of social security on fertility and a positive effect on capital intensity and longevity.

A rise in the tax rate for public health care also exerts conflicting effects on fertility, capital intensity, and longevity. On the one hand, when the tax rate for public health increases, the time cost of spending time rearing a child falls and thus fertility may rise and capital intensity and longevity may fall. When higher public health spending drives up longevity on the other hand, agents may shift their focus from the number of children and middle-age consumption toward old-age consumption, thereby tending to reduce fertility and raise capital intensity (and hence, to raise longevity further).

Our main finding is that the net effect of a tax rise for social security or for public health on fertility will depend on the taste for the number, relative to the welfare, of children. A stronger taste for the welfare of children tends to strengthen the negative effect on fertility and the positive effects on longevity and capital intensity. When the taste for the welfare of children is not weaker than the taste for the number of children, social security and public health reduce fertility and thus raise capital intensity, output per worker, and longevity, as long as the productivity parameter is large enough. Under the same condition, increasing social security increases public health spending per worker and vice versa. It is also important to compare the magnitudes of these effects between social security and public health. In this comparison, social security has a stronger

positive effect on capital intensity and a stronger negative effect on fertility than public health does, other things being equal.

When the tax rate rises, the opposite movements of fertility on the one hand and capital intensity and longevity on the other hand inevitably affect welfare. A reduction in fertility reduces welfare as households obtain utility from the number of children. The welfare loss of falling fertility is increasing with the tax rate at the margin. However, an increase in capital intensity increases labor productivity and welfare. An increase in longevity also increases welfare. The welfare gains of rising capital intensity and rising longevity are decreasing with the tax rate at the margin. The net welfare effect will depend on the relative strength of the tastes for the welfare and number of children and on the tax rate. We illustrate numerically that when the taste for the welfare of children is not weaker than the taste for the number of children, social security and public health can be welfare enhancing before reaching an optimal scale by reducing fertility and raising capital intensity and longevity as long as the productivity parameter is large enough. When the tax rate is beyond the optimal scale, a further increase in the tax rate for social security and public health will reduce welfare.

By a numerical comparison for plausible parameterizations, public health obtains a higher welfare level than social security when using them separately. When public health is used alone, its welfare-improving role works through the channels of fertility, capital intensity and longevity, but when social security is used alone instead, its welfare-improving role works through only the first two channels when public health is absent. However, since public health has weaker effects on fertility and capital intensity than social security does, other things being equal, using them together achieves even higher welfare. The optimal tax rates for social security and public health in our numerical example with plausible parameterizations are close to their realistic values in developed countries.

Due to the complexity of the model, most of our analysis and results focus on the steady state. Therefore, our optimal social security and public health should be associated with the notion of the "modified golden rule" in the literature on neoclassical growth. Since the model does not possess an analytical solution for the dynamic path, we shall use the linearization approach to approximate the solution to this non-linear dynamic

general equilibrium model. Our numerical result shows that in the linearized model there exists a unique convergent path leading to the steady state.

Our model differs from a large body of related literature. For example, many study social security or the consequences of rising longevity by assuming exogenous longevity. Among them, Samuelson (1958), Diamond (1965), Barro (1974), Feldstein (1974), Hubbard and Judd (1987), Zhang (1995), Rosati (1996), and Corneo and Marquardt (2000) investigate the impact of social security on savings. Kaganovich and Zilcha (1999), Cooley and Soares (1999), Zhang and Zhang (2007), and Yew and Zhang (2009) study the welfare implications of social security. Zhang (1995), Sanchez-Losada (2000), and Kemnitz and Wigger (2000) show that with human and physical capital, social security can promote growth, which is consistent with the empirical evidence in Zhang and Zhang (2004) that social security has positive effects on human capital investment and on the growth rate of per capita income. Ehrlich and Lui (1991), Hu (1999), de la Croix and Licandro (1999), and Zhang and Zhang (2001) conclude that higher life expectancy increases the rate of return to human capital investment and leads to higher human capital investment and faster per capita growth. Barro (1997) and Barro and Sala-i-Martin (1995) find empirical evidence that life expectancy has a positive effect on economic growth when income is low, and that the growth effect fades away when income is high. Zhang, Zhang and Lee (2001) show that a rise in longevity tends to reduce fertility and increase human capital investment and growth in a dynastic family model with social security in line with the empirical evidence in Zhang and Zhang (2005).

Some empirical studies provide evidence against the hypothesis of exogenous longevity. For instance, Preston (1975) empirically shows that in aggregate data average income contributes positively to life expectancy. Focusing on developing countries, Anand and Ravallion (1993) find considerable cross-country evidence that the positive relationship between life expectancy and income per capita works mainly via the impact of income on public health spending. More recently, Lichtenberg (2004) provides empirical evidence that public health expenditure contributed to higher longevity in the U.S. during the period 1960-2001. According to the observed stylized facts, the inclusions of life expectancy as an endogenous variable and its positive association with

public health expenditure in a model of income growth are highly relevant in the analysis of social security.

Though there are studies that consider endogenous longevity, these studies usually do not consider social security at the same time. For instance, Ehrlich and Chuma (1990) concern the role of endowed wealth, health, and other initial conditions in determining the demand for health and longevity, among others. Blackburn and Cipriani (2002) combine endogenous fertility and longevity to explain multiple development regimes. Leung, Zhang and Zhang (2004) consider gender-specific factors in the determination of longevity. Chakraborty and Das (2005) show that in the absence of perfect annuities markets, the interplay between income and mortality can generate poverty traps by assuming a positive relationship between the probability of survival and private health investment. Tang and Zhang (2007) investigate health investment, human capital investment, and life cycle savings and show that subsidies on health and human capital investment can improve welfare. Most of them assume non-altruistic preferences and thus would have different policy implications from our altruistic model.

There are a few exceptions that model endogenous longevity in the studies on social security and health. Davies and Kuhn (1992) consider the intake of health related goods that endogenously affect longevity and show that a social security system would encourage suboptimal health investment, leading to excessive longevity, in the presence of a moral hazard problem. Philipson and Becker (1998) consider longevity under the influence of public programs, such as health care and social insurance and pointed out that all forms of old-age income annuity, such as private life insurance or social security programs, would have a similar effect on life prolongation. By analyzing the relationships between life-cycle saving and health investments in different stages of life, Zhang, Zhang and Leung (2006) show that public pensions and health subsidies tend to retard capital accumulation but may improve life expectancy and welfare. However, these studies have ignored the combination of such important factors as altruistic intergenerational transfers and endogenous fertility that may lead to very different results.

Overall, our joint consideration of social security and public health in a rich neoclassical growth model with endogenous longevity and endogenous fertility has not only empirical relevance but also advantages in the analysis of their effects. In particular,

it allows us to compare the relative strengths of social security and public health across cases when they are used separately or jointly in terms of their effects on capital intensity, fertility, longevity, and welfare. The joint consideration also allows us to achieve higher welfare than those when focusing on just one of them.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 provides analytical and numerical results. Section 4 concludes.

## 2. The model

Time is discrete in this model, extending from period 0 to infinity ( $t = 0, 1, \dots, \infty$ ). The model economy is inhabited by overlapping generations of a large number of identical agents who live for three periods. In the first period of life, agents do not make any decision. In their second period of life, they work and make decisions on life-cycle savings, the number of children, the amount of bequests to children and their own consumption; they retire when old. Survival is certain from childhood through middle-age, but each middle-aged agent faces a probability  $\bar{p} \in (0, 1)$  to survive to old age.

The utility function of a middle-aged agent,  $V_t$ , is defined over own middle-age consumption,  $c_t \in R_+$ , own old-age consumption,  $d_{t+1} \in R_+$ , the number of children,  $n_t \in R_+$ , and the utility of each identical child,  $V_{t+1}$ :<sup>1</sup>

$$V_t = \ln c_t + \bar{p}_t \beta \ln d_{t+1} + \eta \ln n_t + \alpha V_{t+1}, \quad \alpha, \beta, \in (0, 1), \quad \eta > 0 \quad (1)$$

where  $\alpha$  is the discounting factor,  $\beta$  is the taste for utility derived from own old-age consumption, and  $\eta$  is the taste for utility derived from the number of children. We assume that the survival rate is increasing in public health spending per worker,  $\bar{M}_t \in R_+$ , at a decreasing rate:  $\bar{p}_t = a_0 - a_1 / e^{a_2 \bar{M}_t}$ , where  $a_0, a_1, a_2 > 0; 1 \geq a_0 > a_1$ .<sup>2</sup> The assumption of a logarithmic utility function helps to ensure tractability.

In period  $t$ , a middle-aged agent devotes  $vn_t$  units of time endowment to rearing children where  $0 < v < 1$  is fixed. The remaining  $(1 - vn_t)$  units of time are devoted to

---

<sup>1</sup> Our use of an altruistic model is consistent with some of the existing empirical evidence. See Tomes (1981), Laitner and Juster (1996), and Laitner and Ohlsson (2001), for instance.

<sup>2</sup> The assumption that the survival rate is increasing in public health expenditure is consistent with empirical evidence in Lichtenberg (2004) as mentioned earlier.

working that earns  $(1 - \tau_t^T - \tau_t^M)(1 - \nu n_t)w_t$  where  $w_t \in R_+$  is the wage rate per unit of labor,  $\tau^T < 1$  is the contribution rate for social security, and  $\tau^M < 1$  is the tax rate for public health. This agent receives a bequest  $b_t$  with earned interest  $r_t \in R_+$ ,  $b_t(1 + r_t)$ , from his or her old parent at the beginning of period  $t$ , and leaves a bequest,  $b_{t+1} \in R_+$ , to each child at the end of period  $t$  so that children receive bequests regardless of their parents' survival status at old age. He or she spends the earnings and the received bequest with earned interest on own middle-age consumption,  $c_t$ , retirement savings via actuarially fair annuity markets  $s_t \in R_+$ , and bequests to children  $b_{t+1}n_t$ . An old agent spends his or her savings plus interest income and social security benefits on own consumption,  $d_{t+1}$ . The budget constraints can be written as:

$$c_t = b_t(1 + r_t) + (1 - \tau_t^T - \tau_t^M)(1 - \nu n_t)w_t - s_t - b_{t+1}n_t, \quad (2)$$

$$d_{t+1} = (1 + r_{t+1})s_t / \bar{p}_t + T_{t+1}, \quad (3)$$

where  $T_{t+1} \in R_+$  is the amount of social security benefits per retiree.

As practiced in many countries such as France and Germany, the amount of social security benefits received by a retiree depends on his or her own earnings in working age according to a replacement rate  $\phi$ . The government budget constraints are given by

$$T_t = \phi(1 - \nu n_{t-1})w_{t-1} = \bar{n}_{t-1}\tau_t^T(1 - \nu \bar{n}_t)w_t / \bar{p}_{t-1},$$

$$\bar{M}_t = \tau_t^M(1 - \nu \bar{n}_t)w_t$$

where the bar above a variable indicates its average level in the economy. With this formula linking the amount of one's social security benefits to his or her own past earnings, a worker who has more children (hence more time for rearing children and less time for working) will not only earn less wage income today but also receive less social security benefits in old age. With identical agents in the same generation, in equilibrium we have  $\bar{n} = n; \bar{p} = p; \bar{M} = M$  by symmetry. In this model, we focus on public healthcare systems that are available in many industrial nations.

The production of the single final good is

$$Y_t = AK_t^\theta(1 - \nu n_t)^{1-\theta} \bar{K}_t^\delta, \quad A > 0, \quad \theta \in (0,1), \quad \delta \in [0,1-\theta) \quad (4)$$



where  $Y_t \in R_+$  and  $K_t \in R_+$  are output per worker and physical capital per worker, respectively;  $A$  is the total factor productivity parameter,  $\theta$  is the share parameter of capital, and  $\delta$  measures the strength of spillovers from average capital per worker  $\bar{K}_t$ . Since one period in this model corresponds to about 30 years, it is reasonable to assume that physical capital depreciates fully within one period. When  $\delta = 0$ , there is no externality from average physical capital in this model. However, when  $\delta > 0$ , the externality takes the form of positive spillovers from average physical capital to the production of the final good.<sup>3</sup> However, the exact degree of this externality is unclear. When  $\delta = 1 - \theta$ , the externality is strong enough to generate endogenous growth in an AK-style model. However, Jones (1995), using time series data in OECD countries, finds empirical evidence against this AK-type model. Moreover, an AK-style model has been criticized based on the empirical evidence on convergence (see, e.g., Barro and Sala-i-Martin, 1995, 2004). We therefore limit our attention to  $1 - \theta > \delta \geq 0$ .

Factors are paid by their marginal products; and the price of the sole final good is normalized to unity. The wage rate per unit of labor and the real interest factor are then given by

$$w_t = (1 - \theta)Y_t / (1 - v n_t), \quad (5)$$

$$1 + r_t = \theta Y_t / K_t, \quad (6)$$

The physical capital market clears when

$$K_{t+1} = [s_t + b_{t+1} n_t] / n_t. \quad (7)$$

### 3. The equilibrium and results

We now solve the dynastic family's problem and track down the equilibrium allocation. The problem of a dynastic family is to maximize utility in (1) subject to budget constraints (2) and (3), knowing the earnings dependent benefit formula, and taking the prices, the probability to survive to old age, the taxes and replacement rates as given. This problem can be rewritten as the following:

---

<sup>3</sup> The investment externality has been emphasized in the literature on economic growth (e.g. Arrow, 1962; Romer, 1986; Lucas, 1993). Based on cross-country data, DeLong and Summers (1991) argued that the spillovers from equipment investment are very substantial. See also Bernstein and Nadiri (1988, 1989), and Nakanishi (2002)) for examples of externalities found in studies of research and development (R&D) stock.

$$\max_{b_{t+1}, n_t, s_t} \sum_{t=0}^{\infty} \alpha^t \{ \ln[b_t(1+r_t) + (1-\tau_t^T - \tau_t^M)(1-vn_t)w_t - s_t - b_{t+1}n_t] + \bar{p}_t \beta \ln[(1+r_{t+1})s_t / \bar{p}_t + \phi_{t+1}(1-vn_t)w_t] + \eta \ln n_t \}$$

where we have used the budget constraints and the earnings dependent benefit formula for substitution. For  $t \geq 0$ , the first-order conditions are given as follows:

$$b_{t+1} : \frac{n_t}{c_t} = \frac{\alpha(1+r_{t+1})}{c_{t+1}}, \quad (8)$$

$$s_t : \frac{1}{c_t} = \frac{\beta(1+r_{t+1})}{d_{t+1}}, \quad (9)$$

$$n_t : \frac{vw_t(1-\tau_t^T - \tau_t^M) + b_{t+1}}{c_t} + \frac{\beta \bar{p}_t \phi_{t+1} vw_t}{d_{t+1}} = \frac{\eta}{n_t}. \quad (10)$$

In (8), the marginal loss in utility from giving a bequest to each child is equal to the marginal gain in children's utility. In (9), the marginal loss in utility from saving is equal to the marginal gain in utility in old age through receiving the return to saving. Such public policies do not create wedges in the intertemporal conditions with respect to bequests and lifecycle savings. In (10), the marginal loss in utility from having an additional child, through forgoing a fraction of wage income and earnings-dependent social security benefits and leaving a bequest to this child, is equal to the marginal gain in utility from enjoying the child. In (10), increasing the tax rates for public health and social security will reduce the forgone after-tax wage cost of a child but raise the forgone pension benefit cost of a child via  $\bar{p}_t$  and  $\phi_{t+1}$ , respectively. We thus expect that social security and public health will affect the allocations via fertility: without the choice of fertility, social security would be neutral overall and public health would be neutral towards capital intensity and output per worker although it would still affect life expectancy. These first-order conditions hold for all  $t \geq 0$ .

The equilibrium of the economy is described below.

**Definition.** *Given an initial state  $(s_{-1}, n_{-1}, b_0)$ , a competitive equilibrium in the economy with PAYG social security and public health is a sequence of allocations*

$\{b_{t+1}, c_t, d_{t+1}, K_{t+1}, n_t, s_t, \phi_t, \tau_t^T, \tau_t^M, T_{t+1}, M_t, p_t, Y_t\}_{t=0}^{\infty}$  and prices  $\{1+r_t, w_t\}_{t=0}^{\infty}$  such that (i) taking prices and government policies  $\{\tau_t^T, \tau_t^M, \phi_t, M_t, T_{t+1}\}_{t=0}^{\infty}$  as given, firms and households optimize and their solutions are feasible, (ii) the government budgets are balanced in every period, (iii) all markets clear with  $K_{t+1} = [s_t + b_{t+1}n_t] / n_t$  and per worker labor being equal to  $(1 - vn_t)$ , and (iv)  $\bar{n} = n; \bar{p} = p; \bar{M} = M$  by symmetry.

Specifically, these equilibrium conditions correspond to the first-order conditions of firms and households, the budget constraints of households and the government, the production technology, the capital market clearing condition, and the amount of labor supply per worker equal to  $(1 - vn_t)$ , for  $t \geq 0$ . In addition, as mentioned earlier, we have  $\bar{n} = n; \bar{p} = p; \bar{M} = M$  in equilibrium by symmetry. Because the model is too complex to be tractable for its full dynamic path, we will mainly focus on the analysis of the steady state equilibrium. In what follows, we first characterize the steady state equilibrium and investigate the implication of social security and public health for fertility, capital intensity, longevity and welfare at the steady state. We then use the linearization approach to approximate the solution to our model, as the model cannot be solved analytically.

### 3.1. Analysis of the steady state equilibrium

#### 3.1.1. The implications of social security and public health for fertility, life expectancy, capital per worker, and output per worker

Since labor income is a constant fraction,  $(1 - \theta)$ , of output per worker in this model, let us define  $\gamma_c = c_t / (1 - vn_t)w_t$ ,  $\gamma_d = d_{t+1} / (1 + r_{t+1})(1 - vn_t)w_t$ ,  $\gamma_b = b_{t+1}n_t / (1 - vn_t)w_t$ , and  $\gamma_s = s_t / (1 - vn_t)w_t$  for convenience. We use them to transform variables in the budget constraints and first-order conditions into their relative ratios to labor income in order to achieve the steady state solution:

$$\gamma_c = \frac{\gamma_b \theta}{(1 - \theta)(\gamma_s + \gamma_b)} + (1 - \tau^T - \tau^M - \gamma_s - \gamma_b), \quad (11)$$

$$\gamma_d = \frac{\gamma_s \theta + \tau^T (1 - \theta)(\gamma_s + \gamma_b)}{\bar{p} \theta}, \quad (12)$$

$$\gamma_s + \gamma_b = \frac{\alpha \theta}{1 - \theta}, \quad (13)$$

$$\beta \gamma_c = \gamma_d, \quad (14)$$

$$\frac{v(1 - \tau^T - \tau^M)}{(1 - vn)\gamma_c} + \frac{\gamma_b}{n\gamma_c} + \frac{\tau^T v \alpha}{(1 - vn)\gamma_c} = \frac{\eta}{n}. \quad (15)$$

Equation (15) can be derived by using  $1 + r_{t+1} = \theta Y_{t+1} / K_{t+1}$ , where  $Y_{t+1} = (1 - vn_{t+1})w_{t+1} / (1 - \theta)$ , and  $K_{t+1} = [s_t + b_{t+1}n_t] / n_t$ . The left-hand side of (15) contains three cost components of a child. The first cost component is the forgone wage income of spending time rearing a child, which falls with the tax rates for social security or public health other things being equal. Higher tax rates for social security and public health may also reduce the ratio of middle-age consumption to output, through which social security and public health spending may indirectly increase all the cost components of having a child according to (15). Formal results will be given later.

The second cost component is the bequest cost of a child, which should rise with the tax rates for social security but may rise or fall with the tax rates for public health. When the tax rate for social security increases, altruistic parents are tempted to reduce the tax burdens of social security on their children by leaving more bequests to them, and thus higher tax rates for social security increase the bequest cost of a child and tend to reduce fertility. When the tax rate for public health increases, the provision of public health per worker increases and hence, life expectancy increases, for any fertility level. With higher life expectancy, the annuity income from private savings and social security benefits will fall, other things being equal, so that agents have stronger incentives to save more for their old-age consumption and reduce bequests and middle-age consumption. However, altruistic parents, who expect their children to live longer as well, are tempted to leave more bequests for their children's old-age consumption, thereby tending to reduce their life-cycle savings and increase the amount of bequests. Hence, when a rise in the tax rate for public health drives up life expectancy, agents may shift focus from the

number of children and their middle-age consumption toward either their own old-age consumption or children's old-age consumption and thus, fertility may fall.

The third cost component is the forgone social security benefit of spending time rearing a child. It increases with the tax rate for social security through the linkage between the replacement rate and the tax rate for social security under a balanced social security budget.

Overall, when the tax rate for social security rises, the subsequent rise in the third cost component partially offsets the fall in the first cost component, and the total time cost of having a child is likely to fall. However, the possible rise in the bequest cost of a child due to higher tax rates for social security may reduce fertility. When the tax rate for public health rises, a fall in the time cost of having a child also tends to increase fertility but the subsequent rise in life expectancy tends to reduce fertility via a possible reduction in middle-age consumption.

The net effect of social security on fertility will depend on the taste for the number, relative to the welfare, of children. When the taste for the welfare of every child,  $\alpha$ , becomes stronger, the third cost component of a child in (15) becomes larger and hence it is more likely that social security reduces fertility. By contrast, when the taste for the number of children,  $\eta$ , becomes stronger, the marginal benefit of a child becomes larger and hence it is more likely for a rise in the tax rate for social security to raise fertility. This is similar to Zhang and Zhang (2007) where survival is certain.

What is new here is that the net effect of public health on fertility also depends on the taste for the number, relative to the welfare, of children. When the taste for the welfare of children becomes stronger relative to that for the number of children, it is more likely that public health may reduce fertility. In addition, the net effect of public health also depends on the productivity parameter  $A$ . When the productivity parameter is larger, average public health spending is greater for any positive  $\tau^M$ , and therefore it is more likely that public health may raise life expectancy and reduce fertility. We now establish the results in the steady state. From these equilibrium conditions, we obtain the following steady-state allocation rules:

$$\gamma_b = \frac{\alpha \left\{ \theta(\alpha + \beta p(\tau^T, \tau^M)) - \beta p(\tau^T, \tau^M) \left[ 1 - (1 - \theta)(\tau^T + \tau^M) - \alpha \theta \right] + (1 - \theta)\alpha \tau^T \right\}}{(1 - \theta)(\alpha + \beta p(\tau^T, \tau^M))}$$

(16)

$$\gamma_c = \frac{\alpha[\theta(1-\alpha) + (1-\tau^M)(1-\theta)]}{(1-\theta)(\alpha + \beta p(\tau^T, \tau^M))}, \quad (17)$$

$$\gamma_s = \frac{\alpha \left\{ \beta p(\tau^T, \tau^M) \left[ 1 - (1-\theta)(\tau^T + \tau^M) - \alpha\theta \right] - (1-\theta)\alpha\tau^T \right\}}{(1-\theta)(\alpha + \beta p(\tau^T, \tau^M))}, \quad (18)$$

$$n = \frac{n_n}{v \left\{ n_n + (1-\theta)(\alpha + \beta p(\tau^T, \tau^M)) [1 - \tau^T(1-\alpha) - \tau^M] \right\}}, \quad (19)$$

where the numerator of  $n$  is

$$\begin{aligned} n_n &= \alpha \left\{ \eta \left[ \theta(1-\alpha) + (1-\theta)(1-\tau^M) \right] - \theta(\alpha + \beta p(\tau^T, \tau^M)) + \right. \\ &\quad \left. \beta p(\tau^T, \tau^M) \left[ 1 - (1-\theta)(\tau^T + \tau^M) - \alpha\theta \right] - (1-\theta)\alpha\tau^T \right\}, \\ \gamma_d &= \beta \frac{\alpha[\theta(1-\alpha) + (1-\tau^M)(1-\theta)]}{(1-\theta)(\alpha + \beta p(\tau^T, \tau^M))}. \end{aligned} \quad (20)$$

Note that life expectancy  $p(\tau^T, \tau^M)$  is a constant function in the steady state equilibrium:  $p(\tau^T, \tau^M) = a_0 - a_1 / e^{a_2 M(\tau^T, \tau^M)}$  where  $M$  is a function of  $\tau^T$  and  $\tau^M$  via  $n$  in (19):

$$M(\tau^T, \tau^M) = \tau^M (1-\theta) \left( \frac{\alpha\theta}{n} \right)^{\frac{\theta+\delta}{1-(\theta+\delta)}} \left[ A(1-vn)^{1-\theta} \right]^{\frac{1}{1-(\theta+\delta)}}.$$

We can easily observe that if  $n_n > 0$  then fertility  $n$  is positive in (19). However, since the log utility function excludes corner solutions for fertility, the presence of non-convexity in the form of  $b_{t+1}n_t$  in the budget constraint (2) may lead to a situation in which there is no solution for fertility for some parameter values. As shown in Zhang et al. (2001) and Zhang (1995), the sufficient condition for the solution to be optimal is a sufficiently large taste parameter for the number of children ( $\eta$ ) such that an interior solution for fertility exists. In order to obtain positive fertility in (19), we assume  $\eta > \underline{\eta} \equiv [\alpha\theta - \beta(a_0 - a_1)(1-\theta - \alpha\theta)] / (1-\alpha\theta)$ . Further, we assume a strong

enough taste for the welfare of children ( $\alpha$ ) such that bequests are positive:  
 $\alpha > [\beta(a_0 - a_1)(1 - \theta)] / \{\theta[1 + \beta(a_0 - a_1)]\}$ .<sup>4</sup>

Now, we investigate the impact of rises in tax rates for social security and public health on the fractions of middle-age earnings spent on savings and bequests.

**Proposition 1.** *A rise in the tax rate for unfunded social security,  $\tau^T$ , or for public health,  $\tau^M$ , has no effect on the fraction of middle-age earnings spent on the sum of savings and bequests ( $\gamma_s + \gamma_b$ ).*

**Proof.** This is obvious in equation (13).  $\square$

A rise in the tax rate for social security has the following effects on bequests and savings: a higher tax rate for social security increases the burden of children in paying higher social security contributions and hence, altruistic parents leave more bequests to children as in Barro (1974) and Zhang (1995). At the same time, parents expect to receive higher social security benefits and therefore, they tend to save less such that the fraction of middle-age earnings spent on the sum of savings and bequests ( $\gamma_s + \gamma_b$ ) is unaffected by social security. On the other hand, when the tax rate for public health increases, life expectancy increases, thereby reducing annuity incomes from savings and social security. Thus, agents may either save more for their old-age consumption and leave less bequests to their children at the same time, or do the opposite. By doing so, the fraction of middle-age earnings spent on the sum of savings and bequests ( $\gamma_s + \gamma_b$ ) is unaffected by the tax rate for public health. This differs from the result in lifecycle models without altruistic bequests. For example, public health spending increases lifecycle savings and accelerates capital accumulation in Tang and Zhang (2007) without bequests.

We now investigate how fertility, capital per worker and output per worker respond to rises in tax rates for unfunded social security and public health:

---

<sup>4</sup> Kotlikoff and Summers (1981) find empirical evidence that bequests are an important element in accounting for capital accumulation.

**Proposition 2.** *If  $\alpha \geq \eta$ , then a rise in the tax rate for unfunded social security,  $\tau^T$ , reduces fertility, raises capital per worker, and raises output per worker. If  $\alpha \geq \eta$  and  $A$  is large enough, then a rise in the tax rate for public health,  $\tau^M$ , reduces fertility, raises capital per worker, and raises output per worker.*

**Proof.** The proof is relegated to Appendix A.

As discussed earlier, with a stronger taste for the welfare of children, it is more likely that the tax rise reduces fertility and leads to a rise in both capital and output per worker. When a rise in the tax rate for social security reduces fertility under  $\alpha \geq \eta$  without any effect on the ratio of savings and bequests to output in Proposition 1, it must increase capital and output per worker. This extends a similar result in Zhang and Zhang (2007) to be applicable to a model with endogenous longevity. The negative effect of social security is consistent with the empirical finding in the literature (e.g. see Zhang and Zhang, 2004, and some other papers cited therein). So we regard the condition in Proposition 2 as empirically plausible.

The new finding in Proposition 2 is: when the taste for the welfare of children is not weaker than the taste for the number of children,  $\alpha \geq \eta$ , and when the productivity parameter,  $A$ , is large enough, a rise in the tax rate for public health reduces fertility and thus raises capital and output per worker.<sup>5</sup> Intuitively, with a stronger taste for the welfare of children and with a larger productivity parameter, the negative effect of public health spending on fertility via a rise in life expectancy (to be established next) is more likely to outweigh the positive effect of public health on fertility via a fall in the time cost of spending time rearing a child.

Let us now investigate the effects of a rise in the tax rates for social security and public health on the provision of public health per worker, life expectancy, the ratio of middle-age consumption to income and the ratio of old-age consumption to income.

---

<sup>5</sup> This relationship between fertility and output per worker accords well with the empirical evidence that fertility is negatively related to output per worker, except, perhaps, at very low levels of income (see, e.g., Barro and Sala-i-Martin, 1995; World Bank, 1984).



**Proposition 3.** *If  $\alpha \geq \eta$  and  $\tau^M > 0$ , then a rise in the tax rate for unfunded social security,  $\tau^T$ , raises public health spending per worker, raises life expectancy, reduces the ratio of middle-age consumption to income, and reduces the ratio of old-age consumption to income. If  $\alpha \geq \eta$  and  $A$  is large enough, then a rise in the tax rate for public health,  $\tau^M$ , raises public health spending per worker, raises life expectancy, reduces the ratio of middle-age consumption to income, and reduces the ratio of old-age consumption to income.*

**Proof.** The proof is relegated to Appendix A.

In the conventional dynastic model without health spending, social security is neutral with regard to consumption pattern over life stages via saving, which is well known as the Ricardian equivalence hypothesis (Barro, 1974; Zhang, 1995). When public health is present in our model, however, social security increases public health spending per worker by increasing output per worker (and hence life expectancy as well) for any given positive tax rate for public health spending, if the taste for the welfare of children is not weaker than the taste for the number of children. The positive effect of social security on public health spending per worker, and hence on life expectancy, works through the negative effect of social security on fertility and the positive effect on capital intensity in Proposition 2 for any positive tax rate for public health  $\tau^M > 0$ . With higher public health spending per worker, life expectancy increases. The increases in public health spending and life expectancy driven by social security lead to lower ratios of middle-age and old-age consumption to income according to equations (17) and (20). Intuitively, higher life expectancy induces a shift from middle-age consumption to either savings or bequests and reduces annuity incomes for old-age consumption to income.

There exist both direct and indirect effects of a rise in the tax rate for public health on public health spending per worker. The direct effect is simple: when the tax rate for public health increases, so does public health spending per worker increases, given any fertility level. The indirect effect of a rise in the tax rate for public health on public health spending per worker works through its effect on fertility. As shown in Proposition 2, if

the taste for the welfare of children,  $\alpha$ , is not weaker than the taste for the number of children,  $\eta$ , and if the productivity parameter,  $A$ , is large enough, then a rise in the tax rate for public health reduces fertility and hence, increases output per worker and public health spending per worker. Since both the direct and indirect effects of a rise in the tax rate for public health increase public health spending per worker under  $\alpha \geq \eta$  and for a large enough  $A$ , life expectancy increases. As a consequence, a rise in the tax rate for public health leads to lower ratios of middle-age consumption and old-age consumption to income according to equations (17) and (20) for  $\alpha \geq \eta$  and large enough  $A$ .

We now compare the magnitude of the effects of a tax rise for social security on fertility, capital per worker and output per worker with those of a tax rise for public health.

**Proposition 4.** *Let us start from zero taxes. If  $\alpha \geq \eta$ , then the decrease in fertility due to an increase in the tax rate for unfunded social security,  $\tau^T$ , is larger than that due to an equal increase in the tax rate for public health,  $\tau^M$ , at the margin. At the same time, the increase in capital per worker and output per worker due to an increase in the tax rate for unfunded social security,  $\tau^T$ , is higher than the counterpart due to an equal increase in the tax rate for public health,  $\tau^M$ , at the margin.*

**Proof.** The proof is relegated to Appendix A.

According to Proposition 4, if the taste for the welfare of children is not weaker than the taste for the number of children,  $\alpha \geq \eta$ , and if the productivity parameter,  $A$ , is large enough, then social security has stronger negative effects on fertility and stronger positive effects on both capital and output per worker than public health does, starting with zero taxes. This implies that a rise in the tax rate for social security may be more effective in reducing fertility and increasing both capital and output per worker than that for public health even starting at higher tax rates. The intuition is that a tax rise for social security engenders an additional cost component of a child in terms of forgone social security benefits of spending time rearing a child in equation (15), compared to a tax rise

for public health. Therefore, social security exerts larger effects on fertility, capital per worker and output per worker than public health. The task next is to investigate how social security and public health affect welfare numerically with endogenous life expectancy and fertility.

### 3.1.2. Welfare implications of social security and public health through simulations

Due to the complexity of tracking down the full dynamic path for a complete welfare analysis in this complicated model, we only focus on the steady state for the welfare analysis. Such a steady-state welfare analysis yields results corresponding to what is coined as the "modified golden rule of capital accumulation" in the conventional neoclassical growth model. At the steady state, the welfare level  $V$  in (1) is given as follows:

$$V_{ss} = \left\{ (1 + p\beta) \ln \gamma_c + (1 + p\beta) \ln Y + (\eta + p\beta) \ln n + \ln(1 - \theta) + p\beta \ln[\beta(1 - \theta) / \alpha] \right\} / (1 - \alpha) \quad (21)$$

where  $\gamma_c, Y, p, n$  are at their respective steady state levels and are functions of  $\tau^T$  and  $\tau^M$  with

$$\begin{aligned} \gamma_c &= \frac{\alpha[\theta(1 - \alpha) + (1 - \tau^M)(1 - \theta)]}{(1 - \theta)(\alpha + \beta p)}, \\ p &= a_0 - a_1 / e^{a_2 M}, \\ M &= \tau^M (1 - \theta) \left( \frac{\alpha \theta}{n} \right)^{\frac{\theta + \delta}{1 - (\theta + \delta)}} \left[ A(1 - vn)^{1 - \theta} \right]^{\frac{1}{1 - (\theta + \delta)}}, \\ Y &= A^{\frac{1}{1 - (\theta + \delta)}} (\alpha \theta)^{\frac{\theta + \delta}{1 - (\theta + \delta)}} n^{\frac{-(\theta + \delta)}{1 - (\theta + \delta)}} (1 - vn)^{\frac{1 - \theta}{1 - (\theta + \delta)}}, \end{aligned} \quad (22)$$

$$n = \frac{\tilde{n}_n}{\tilde{n}_d}, \quad (23)^6$$

---

<sup>6</sup> By substituting  $p(\tau^T, \tau^M) = a_0 - a_1 / e^{a_2 M(\tau^T, \tau^M)}$  into the equation for fertility in (19), we obtain  $n = \tilde{n}_n / \tilde{n}_d$ .

where

$$\begin{aligned}\tilde{n}_n &= e^{a_2 M} \alpha \left\{ \eta \left[ \theta(1-\alpha) + (1-\tau^M)(1-\theta) \right] - \alpha \left[ \theta + (1-\theta)\tau^T \right] \right\} + \\ &\quad \alpha \beta \left[ (1-\theta)(1-\tau^T - \tau^M) - \alpha \theta \right] \left[ e^{a_2 M} a_0 - a_1 \right], \\ \tilde{n}_d &= v \left\{ e^{a_2 M} \alpha \left[ (1-\theta)(1-\tau^T - \tau^M) - \alpha \theta + \eta \left( \theta(1-\alpha) + (1-\tau^M)(1-\theta) \right) \right] + \right. \\ &\quad \left. \beta \left[ (1-\theta)(1-\tau^T - \tau^M) + \alpha \left( -\alpha \theta + (1-\tau^M)(1-\theta) \right) \right] \left[ e^{a_2 M} a_0 - a_1 \right] \right\}.\end{aligned}$$

We now investigate the optimal tax rates of social security and public health at the steady state. We first differentiate the welfare function in (21) with respect to the tax rate for social security or public health and obtain the following first-order conditions:

$$\begin{aligned}\frac{\partial V_{ss}}{\partial \tau^T} = 0 &\Leftrightarrow (1+p\beta) \left[ \frac{1}{\gamma_c} \frac{\partial \gamma_c}{\partial n} + \frac{1}{Y} \frac{\partial Y}{\partial n} \right] + \\ &\quad \frac{\partial p}{\partial n} \beta \left[ \ln \left( \frac{\beta(1-\theta)}{\alpha} \right) + \ln \gamma_c + \ln n + \ln Y \right] + \frac{1}{n} (\eta + p\beta) = 0, \\ \frac{\partial V_{ss}}{\partial \tau^M} = 0 &\Leftrightarrow \left[ \frac{1+p\beta}{\theta(1-\alpha) + (1-\tau^M)(1-\theta)} \right] + \\ &\quad \frac{\partial p}{\partial M} \beta Y \left[ \left( \frac{1+p\beta}{\alpha + p\beta} \right) - \ln \left( \frac{\beta(1-\theta)}{\alpha} \right) - \ln \gamma_c - \ln n - \ln Y \right] = 0.\end{aligned}$$

The above first-order conditions implicitly determine the optimal tax rates of social security and public health (see Figure 1 for a numerical illustration). We next explore the implications for welfare in equation (21) using a numerical approach for plausible parameterizations.

The values of parameters are either in line with those in the literature if any (e.g.,  $\alpha = 0.65$ ,  $\theta = 0.25$ ), or they are chosen to yield plausible values for fertility and the survival probability to old-age (e.g.  $v = 0.1$ ,  $\beta = 0.5$ ,  $\eta = 0.5$ ,  $a_0 = 0.95$ ,  $a_1 = 0.45$ ,  $a_2 = 0.9$ , and  $A = 25$ ). Also, we set a low value for  $\delta$  at 0.01, measuring the degree of the externality, which can generate realistic values for the tax rates. We later will

examine whether the existence of positive investment externalities is essential for social security or public health to improve welfare by setting  $\delta$  at zero.

The numerical results show that the optimal tax rates for social security and public health together are  $(\tau^T, \tau^M) = (0.21, 0.09)$  as reported in Case 1 in Table 1 under the conditions  $\alpha > \eta$  and  $A$  is large enough such that higher tax rates for social security and public health reduce fertility. Given the parameterization, we can compare the impacts of a tax rise for social security or public health on fertility, the ratio of middle-age consumption to income, the ratio of old-age consumption to income, the provision of public health per worker, life expectancy, capital per worker, output per worker and the welfare level in cases with or without social security or public health in Table 1. Case 2 of Table 1 reports the numerical results when both social security and public health are absent. Case 3 reports the effect of social security when public health is absent. Case 4 reports the effect of public health when social security is absent. Case 5 reports the effects of social security and public health when their tax rates are equal. Finally, Case 6 provides the effect of social security when the tax rate for public health is held constant.

[Table 1 goes here.]

Using Case 2 as a benchmark for comparisons, Table 1 illustrates Proposition 2 in that a rise in the tax rate for social security or public health reduces fertility and raises both capital and output per worker when the parameterizations satisfy  $\alpha > \eta$  and a large enough productivity parameter,  $A$ .<sup>7</sup> In Case 3, when public health is absent, a rise in the tax rate for social security has no effect on the provision of public health per worker, life expectancy, the ratio of middle-age consumption to income and the ratio of old-age consumption to income. This special case is in line with the existing literature on social security with exogenous longevity (e.g. Zhang, 1995, Yew and Zhang, 2009). However, as shown in Case 6, a rise in the tax rate for social security raises the provision of public health per worker and life expectancy but reduces the ratio of middle-age consumption to income and the ratio of old-age consumption to income when the tax rate for public health is positive and held constant. On the other hand, in Case 4 a rise in the tax rate for

---

<sup>7</sup> The numerical results are also true qualitatively for the case  $\alpha = \eta$  and therefore, we only focus on the case  $\alpha > \eta$  for simplicity.

public health raises the provision of public health per worker and life expectancy but reduces the ratio of middle-age consumption to income and the ratio of old-age consumption to income. These results are consistent with Proposition 3.

Comparisons for capital per worker across Case 2, Case 3, Case 4 and Case 5 reflect Propositions 2 and 4. For instance, when  $(\tau^T, \tau^M) = (0,0)$ , capital per worker is 1.188; but when public health is present at  $(\tau^T, \tau^M) = (0,0.1)$ , capital per worker increases to 1.257. On the other hand, when only social security is present at  $(\tau^T, \tau^M) = (0.1,0)$ , capital per worker increases to a higher level at 1.431. Hence, when both social programs are present at  $(\tau^T, \tau^M) = (0.1,0.1)$ , capital per worker is even higher at 1.572. These results show that the increase in capital per worker (and thus, output per worker) due to increases in both tax rates,  $\tau^T$  and  $\tau^M$ , are higher than the increase in capital per worker (and hence, output per worker) due to an increase in only one of these two tax rates. This is implied by Proposition 2.

By comparing fertility across Case 2, Case 3 and Case 4 in Table 1, it is also obvious that the decrease in fertility from 2.796 to 2.509 when the tax rate for social security increases from  $(\tau^T, \tau^M) = (0,0)$  to  $(\tau^T, \tau^M) = (0.1,0)$  is much larger than the decrease in fertility from 2.796 to 2.707 when the tax rate for public health increases from  $(\tau^T, \tau^M) = (0,0)$  to  $(\tau^T, \tau^M) = (0,0.1)$ . As a consequence, the increase in capital per worker or output per worker due to an increase in the tax rate for social security is higher than that due to the same amount of increase in the tax rate for public health. These results therefore reflect Proposition 4.

The simulation results also indicate that social security or public health can increase welfare by reducing fertility and raising capital per worker and longevity. When both social security and public health are absent as in Case 2, the benchmark welfare level is 10.998 in Table 1. By scaling up both social security and public health, welfare increases until it reaches the maximum at 11.611 at the optimal tax rates  $(\tau^T, \tau^M) = (0.21,0.09)$ . The maximum of welfare in this case is higher than the maximum welfare in all other cases in Table 1. This implies that it is more efficient when both social security and public health are implemented together than they are implemented separately. The optimal per worker public expenditure on health,  $M$ , at 1.668 is about

6% of the corresponding output per worker at 25.856. The optimal public expenditure on health at 6% of output and the optimal tax rate for social security at 21% are close to the observed rates in industrial nations. According to the World Health Statistics (2009), public expenditure on health may attain as high as 8% of income, and according to Social Security Administration and International Social Security Association (2006, 2008), payroll tax rates for social security may range from 10% to 20% or higher.

In Table 2, we examine whether the simulation results concerning the optimal tax rates for social security and public health are sensitive to variations in the parameters  $(a_0, a_1, a_2, \alpha, \eta, \theta, \beta, A, v)$  and to the existence of investment externalities by varying  $\delta$  from positive values to zero.<sup>8</sup> In doing so, we consider variations in one parameter at a time, starting from the parameterization in Table 1. First, a higher value of the taste for the welfare of children ( $\alpha$ ) yields a lower optimal tax rate of social security and the magnitude of the change in the optimal tax rate is large. This is because the more parents value their children's welfare than the number of children, the smaller the efficiency loss of the investment externalities for a given degree of investment externality ( $\delta$ ) and therefore, the lower the optimal social security.<sup>9</sup>

Second, a larger share parameter of capital ( $\theta$ ) leads to a higher optimal tax rate of social security and the magnitudes of the changes in the optimal tax rate are large as well. The reason for this result is that this share parameter measures the role of physical capital investment in the accumulation of physical capital. That is, with a larger share parameter  $\theta$ , physical capital investment becomes more important in the production of output and therefore the efficiency loss of the physical capital externality is larger for a given degree of investment externality ( $\delta$ ).

Third, a larger degree of investment externality also requires a higher optimal tax rate for social security due to a larger efficiency loss of the externality. Fourth, a higher value of the taste for the number of children ( $\eta$ ) yields a higher optimal tax rate for

---

<sup>8</sup> The taste for the number of children,  $\eta$ , and the taste for the welfare of children,  $\alpha$ , may change overtime due to cultural changes, government policies associated with children, increases in women's education attainment and labor participation rates.

<sup>9</sup> An investment externality causes an under-investment in capital and hence, over-reproduction of the population compare to their socially optimal levels, as a lower investment in capital reduces the marginal product of labor and therefore, the opportunity cost of spending time rearing a child. The reason is similar to that in Zhang and Zhang (2007) where survival is certain.

social security. This is because the more parents value the number of children relative to the welfare of children, the larger the efficiency loss of the investment externalities for a given degree of investment externality ( $\delta$ ) and therefore, the higher the optimal tax rate for social security. By contrast, variations in the other parameters produce relatively little changes in the optimal tax rate for social security in Table 2. This is because these parameters are less relevant for fertility and physical capital investment, which channel the efficiency loss of the physical capital externality, than  $(\alpha, \theta, \delta, \eta)$ .

Notice that the optimal tax rate of public health is insensitive to variations in the parameters  $(a_0, a_1, a_2, \alpha, \eta, \theta, \beta, \delta, A, v)$ . This is because the optimal tax rate for public health depends on the effect of the provision of public health per worker on average life expectancy which is taken as given by individuals in their optimization problem. So public health represents another form of externality in determining longevity.

[Table 2 goes here.]

Given the longevity externality, the investment externality is no longer essential for optimal policies. For example, when the investment externality is absent ( $\delta=0$ ), the optimal tax rates for social security and public health are still positive. This feature differs from Zhang and Zhang (2007) and Yew and Zhang (2009) where the optimal social security tax rate should be zero without the externality in production or education.

### ***3.2. Analysis of the equilibrium solution using linearization***

The model is complex and is non-linear, which, in turn, makes it difficult, if not impossible, to solve analytically. To deal with this problem, we will use the linearization approach to find a linear approximation to the dynamic system. Let us define  $\hat{x}_t = \log(x_t / x)$ , where  $x$  is the steady state value. We first linearize the budget constraints (2) and (3):



$$\begin{aligned}\hat{c}_t \approx & \left\{ b\theta A \left( \frac{n}{s+bn} \right)^{1-\theta-\delta} (1-vn)^{1-\theta} \left[ \hat{I}_{1,t-1} - (1-\theta) \left( \frac{vn\hat{n}_t}{1-vn} \right) \right] + \right. \\ & (1-\tau^T - \tau^M)(1-\theta) A \left( \frac{s+bn}{n} \right)^{\theta+\delta} (1-vn)^{1-\theta} \left[ \hat{I}_{2,t-1} - (1-\theta) \left( \frac{vn\hat{n}_t}{1-vn} \right) \right] - \\ & \left. bn(\hat{b}_{t+1} + \hat{n}_t) - s\hat{s}_t \right\} / (c)\end{aligned}$$

where

$$\begin{aligned}\hat{I}_{1,t-1} & \equiv \hat{b}_t - \left( \frac{1-\theta-\delta}{s+bn} \right) (s\hat{s}_{t-1} + bn(\hat{b}_t + \hat{n}_{t-1})) + (1-\theta-\delta)\hat{n}_{t-1} \\ \hat{I}_{2,t-1} & \equiv \left( \frac{\theta+\delta}{s+bn} \right) (s\hat{s}_{t-1} + bn(\hat{b}_t + \hat{n}_{t-1})) - (\theta+\delta)\hat{n}_{t-1}\end{aligned}$$

$$\begin{aligned}\hat{d}_{t+1} \approx & \left\{ \left( \frac{s\theta}{s+bn} \right) \left[ \hat{I}_{1,t} - \hat{b}_{t+1} - (1-\theta) \left( \frac{vn\hat{n}_{t+1}}{1-vn} \right) + \hat{s}_t - \hat{p}_t \right] + \right. \\ & \left. \tau^T (1-\theta) \left[ \hat{I}_{2,t} - (1-\theta) \left( \frac{vn\hat{n}_{t+1}}{1-vn} \right) + \hat{n}_t - \hat{p}_t \right] \right\} \left( \frac{A(1-vn)^{1-\theta} n^{1-\theta-\delta} (s+bn)^{\theta+\delta}}{dp} \right)\end{aligned}$$

with

$$\hat{p}_t \approx \left\{ \hat{I}_{2,t-1} - (1-\theta) \left( \frac{vn\hat{n}_t}{1-vn} \right) \right\} \left( \frac{a_1 a_2 M}{pe^{a_2 M}} \right).$$

Next, we linearize the first-order conditions (8), (9), and (10):

$$\begin{aligned}\hat{b}_{t+1} \approx & \left\{ \hat{c}_t - (\theta+\delta)\hat{n}_t - (1-\theta) \left( \frac{vn\hat{n}_{t+1}}{1-vn} \right) - \right. \\ & \left. \hat{c}_{t+1} - \left( \frac{1-\theta-\delta}{s+bn} \right) (s\hat{s}_t + bn\hat{n}_t) \right\} \left( \frac{s+bn}{(1-\theta-\delta)bn} \right)\end{aligned}$$

$$\begin{aligned}\hat{s}_t \approx & \left\{ \hat{c}_t + (1-\theta-\delta)\hat{n}_t - \hat{d}_{t+1} - (1-\theta) \left( \frac{vn\hat{n}_{t+1}}{1-vn} \right) - \right. \\ & \left. \left( \frac{1-\theta-\delta}{s+bn} \right) bn(\hat{b}_{t+1} + \hat{n}_t) \right\} \left( \frac{s+bn}{(1-\theta-\delta)s} \right)\end{aligned}$$

$$\hat{n}_t \approx \left\{ v(1-\tau^T - \tau^M)(1-\theta)A \left( \frac{s+bn}{n} \right)^{\theta+\delta} (1-vn)^{-\theta} \left[ \hat{c}_t - \hat{I}_{2,t-1} - \frac{\theta vn \hat{n}_t}{1-vn} \right] - \right. \\ \left. b(\hat{b}_{t+1} - \hat{c}_t) + \frac{v\tau^T(1-\theta)(s+bn)}{\theta(1-vn)} \left[ \hat{c}_t - \frac{s\hat{s}_t + bn(\hat{b}_{t+1} + \hat{n}_t)}{s+bn} - \frac{vn\hat{n}_t}{1-vn} \right] \right\} \left( \frac{n}{\eta c} \right)$$

Since the budget constraints and the first-order conditions are linear now, we conjecture a linear decision rule for each of these economic variables  $\hat{b}_{t+1}$ ,  $\hat{s}_t$ ,  $\hat{n}_t$  as a function of variables  $\hat{b}_t$ ,  $\hat{s}_{t-1}$ ,  $\hat{n}_{t-1}$ . The difference equation system can be written as follows:

$$X_t = ZX_{t-1}, \quad t \geq 0, \quad \text{given } X_{-1}$$

where

$$X_t = \begin{pmatrix} \hat{b}_{t+1} \\ \hat{s}_t \\ \hat{n}_t \end{pmatrix}, \quad Z = \begin{pmatrix} z_{bb} & z_{bs} & z_{bn} \\ z_{sb} & z_{ss} & z_{sn} \\ z_{nb} & z_{ns} & z_{nn} \end{pmatrix}, \quad X_{t-1} = \begin{pmatrix} \hat{b}_t \\ \hat{s}_{t-1} \\ \hat{n}_{t-1} \end{pmatrix}.$$

Our task now is to solve for the undetermined coefficients in the matrix  $Z$ . As the linearized model is still very complicated, we solve for the undetermined coefficients numerically using the parameterization and the optimal tax rates for social security and public health in Table 1 and obtain the following solution:

$$Z = \begin{pmatrix} 0.528 & 0.113 & 0.148 \\ 0.256 & 0.211 & -0.111 \\ -0.076 & -0.081 & -0.019 \end{pmatrix}$$

In order to investigate the dynamic properties of the model (i.e., whether the economic variables  $\hat{b}_{t+1}$ ,  $\hat{s}_t$ ,  $\hat{n}_t$  converge and whether the equilibrium is unique and stable), we now solve for eigenvalues of the matrix  $Z$ . By solving the following characteristic equation:

$$|Z - \lambda I| = 0 \Leftrightarrow -0.006 - 0.071\lambda + 0.720\lambda^2 - \lambda^3 = 0,$$

where  $I$  is the identity matrix, we obtain eigenvalues  $\lambda_1 = 0.194, \lambda_2 = 0.580, \lambda_3 = -0.054$ . Since all eigenvalues are smaller than one in absolute value, the dynamic system is stationary and the equilibrium exists. Moreover, as the number of economic restrictions on the initial conditions in our model is three, i.e.,  $\hat{s}_{-1}, \hat{n}_{-1}, \hat{b}_0$ , and is equal to the number of eigenvalues less than one in absolute value, there exists a unique convergent path leading to the steady state in the dynamic system (see Krusell, 2004, p. 43).

#### 4. Conclusion

In this paper we have examined the implications of PAYG social security and public health for fertility, life expectancy, capital per worker, output per worker and welfare in a dynastic model with altruistic bequests, endogenous longevity, and endogenous fertility. We have shown analytically that if the taste for the welfare of children is not weaker than that for the number of children, scaling up social security reduces fertility, but raises capital per worker, output per worker, public health spending per worker and life expectancy. We have also shown analytically that if the taste for the welfare of children is not weaker than that for the number of children and the productivity parameter is large enough, scaling up public health reduces fertility, but raises capital per worker, output per worker, public health spending per worker and life expectancy. A comparison of tax policies between social security and public health shows that social security may be more effective than public health in reducing fertility and raising both capital and output per worker. This is because a tax rise for social security engenders an additional cost component of a child in terms of forgone social security benefits of spending time rearing a child compared to a tax rise for public health. However, without the implementation of the public health program at the same time (as in the existing literature), social security has no effect on public health spending per worker and life expectancy.

Our simulation results highlight that scaling up social security or public health improves welfare by reducing fertility and raising longevity and capital intensity in the presence of an investment externality or a longevity externality. Though social security and public health can be used separately to increase welfare, our simulation results show that the maximum welfare is reached when both social security and public health are implemented optimally together. It is also worth mentioning that investment externality is

no longer necessary in our model to justify positive optimal tax rates for social security and public health when these social programs improve life expectancy, and hence, welfare, via the longevity externality. Quantitatively, our model can generate the optimal tax rate of social security at 21% and optimal per worker public expenditure on health at 6% of output per worker at the same time. These optimal rates obtained jointly in this model are close to the observed rates for social security and for public expenditure on health as a percentage of output in industrial nations.

The combination of such important factors as altruistic intergenerational transfers, and endogenous life expectancy and fertility has not been used together in exploring the implications of PAYG social security and public health for fertility, life expectancy, capital per worker, output per worker and welfare, to the best of our knowledge. With these factors, our model has engendered some new insights. Our results may have useful policy implications. For instance, adopting both PAYG social security and public health may be appropriate for developing economies with high fertility, low life expectancy, and low levels of capital per worker and output per worker. Furthermore, our results may help explain the recent behavior of fertility and life expectancy, and the recent pattern of spending on social security and public health in developed countries. Our results also help to explain the popularity of PAYG social security and public health in developed economies but warn against possible excessive use of them.

## Appendix A

**Proof of Proposition 2.** First, we substitute  $p(\tau^T, \tau^M) = a_0 - a_1 / e^{a_2 M(\tau^T, \tau^M)}$  into the equation for fertility in (19) to obtain  $n = \tilde{n}_n / \tilde{n}_d$  as given in equation (23). We then differentiate  $n = \tilde{n}_n / \tilde{n}_d$  in (23) with respect to  $\tau^T$  and obtain

$$\frac{\partial n}{\partial \tau^T} = \frac{v\alpha(1-\theta) \left\{ -\Lambda_1 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right] + \Gamma_2 a_2 \left( \partial M / \partial \tau^T \right) (\alpha - \eta) \right\}}{(\tilde{n}_d)^2} \quad (24)$$

where

$$\Lambda_1 = \left[ 1 - \tau^M (1 - \theta) - \alpha \theta \right] \left[ e^{a_2 M} \alpha + \beta (e^{a_2 M} a_0 - a_1) \right] > 0,$$

$$\Gamma_1 = \alpha \left[ e^{a_2 M} \eta + \beta (e^{a_2 M} a_0 - a_1) \right] > 0,$$

$$\Gamma_2 = a_1 \beta e^{a_2 M} \left[ 1 - \tau^T (1 - \alpha) - \tau^M \right] \left[ 1 - \tau^M (1 - \theta) - \alpha \theta \right] > 0 \text{ if } \alpha \geq \eta, \text{ and}$$

$$\frac{\partial M}{\partial \tau^T} = -\frac{\tau^M \Pi_1 \Pi_2}{a_2} \frac{\partial n}{\partial \tau^T}.$$

Note that  $\Gamma_2 > 0$  if  $\left[ 1 - \tau^T (1 - \alpha) - \tau^M \right] > 0$  which is true if  $\alpha \geq \eta$ . Using the transformed budget constraint in equations (11) and (13), we obtain  $(\alpha \gamma_c - \gamma_b) / \alpha = 1 - \tau^T - \tau^M - [\alpha \theta / (1 - \theta)]$ . In addition, with positive fertility, the fertility equation in (15) implies  $\eta \gamma_c > \gamma_b$ . Thus, if  $\alpha \geq \eta$ ,  $(\alpha \gamma_c - \gamma_b) / \alpha = 1 - \tau^T - \tau^M - [\alpha \theta / (1 - \theta)] > 0$  and  $[\alpha \theta - (1 - \theta)(1 - \tau^T - \tau^M)] < 0$ . The condition  $[\alpha \theta - (1 - \theta)(1 - \tau^T - \tau^M)] < 0$  implies  $(1 - \tau^T - \tau^M) > 0$  which leads to  $\left[ 1 - \tau^T (1 - \alpha) - \tau^M \right] > 0$  and thus,  $\Gamma_2 > 0$ .

By substituting  $\partial M / \partial \tau^T$  into equation (24) and after rearranging equation (24), we obtain

$$\frac{\partial n}{\partial \tau^T} = \frac{-v\alpha(1-\theta)\Lambda_1 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right]}{(\tilde{n}_d)^2 + v\alpha(1-\theta)\tau^M \Pi_1 \Pi_2 \Gamma_2 (\alpha - \eta)} \quad (25)$$

where

$$\Pi_1 = a_2(1-\theta)(\alpha\theta)^{\frac{(\theta+\delta)}{1-(\theta+\delta)}} A^{\frac{1}{1-(\theta+\delta)}} > 0, \quad (26)$$

$$\begin{aligned} \Pi_2 = & \left[ \left( \frac{\theta+\delta}{1-(\theta+\delta)} \right) n^{\frac{-1}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}} \right. \\ & \left. + \left( \frac{1-\theta}{1-(\theta+\delta)} \right) n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{\delta}{1-(\theta+\delta)}} v \right] > 0. \end{aligned}$$

Therefore, if  $\alpha \geq \eta$ , then  $\partial n / \partial \tau^T < 0$  in equation (25). By equations (5), (7), and (13),  $K = [A(1-vn)^{1-\theta} \alpha\theta / n]^{1/[1-(\theta+\delta)]}$  in the steady state, and hence if  $\alpha \geq \eta$ , then  $\partial K / \partial \tau^T = (\partial K / \partial n)(\partial n / \partial \tau^T) > 0$ . By equation (4),  $Y = AK^{\theta+\delta}(1-vn)^{1-\theta}$  in the steady state, and with  $\partial K / \partial \tau^T > 0$  and  $\partial n / \partial \tau^T < 0$  if  $\alpha \geq \eta$ , we obtain  $\partial Y / \partial \tau^T = (\partial Y / \partial K)(\partial K / \partial \tau^T) + (\partial Y / \partial n)(\partial n / \partial \tau^T) > 0$ .

Similarly, by differentiating  $n = \tilde{n}_n / \tilde{n}_d$  in (23) with respect to  $\tau^M$ , we obtain

$$\frac{\partial n}{\partial \tau^M} = \frac{v\alpha(1-\theta) \left\{ -\Lambda_2 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right] + \Gamma_2 a_2 \left( \partial M / \partial \tau^M \right) (\alpha - \eta) \right\}}{(\tilde{n}_d)^2} \quad (27)$$

where

$$\Lambda_2 = \left[ \theta + (1-\theta)\tau^T \right] \left[ e^{a_2 M} \alpha + \beta \left( e^{a_2 M} a_0 - a_1 \right) \right] > 0$$

$$\frac{\partial M}{\partial \tau^M} = \frac{\Pi_1}{a_2} \left[ n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}} - \tau^M \frac{\partial n}{\partial \tau^M} \Pi_2 \right].$$

By substituting  $\partial M / \partial \tau^M$  into equation (27) and rearranging, we obtain

$$\frac{\partial n}{\partial \tau^M} = \frac{v\alpha(1-\theta) \left\{ -\Lambda_2 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right] + \Gamma_2 \Pi_1 n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}} (\alpha - \eta) \right\}}{(\tilde{n}_d)^2 + v\alpha(1-\theta)\tau^M \Pi_1 \Pi_2 \Gamma_2 (\alpha - \eta)} \quad (28)$$

It is obvious that  $(\partial n / \partial \tau^M) < 0$  if  $\alpha = \eta$ . Note that if  $\alpha > \eta$ , then the sign of  $\partial n / \partial \tau^M$  depends on the sign of the following component:

$$\left\{ -\Lambda_2 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right] + \Gamma_2 \Pi_1 n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}} (\alpha - \eta) \right\} \quad (29)$$

Recall that

$$M = \tau^M (1-\theta) \left( \frac{\alpha\theta}{n} \right)^{\frac{\theta+\delta}{1-(\theta+\delta)}} \left[ A(1-vn)^{1-\theta} \right]^{\frac{1}{1-(\theta+\delta)}} \quad (30)$$

By using the equation for  $M$  in (30) and  $\Pi_1$  in (26), expression (29) can be rewritten as:

$$\left\{ \left( \frac{\alpha - \eta}{\tau^M} \right) \left[ \Gamma_2 a_2 M - \tau^M \Lambda_2 e^{a_2 M} \right] - \Lambda_2 \Gamma_1 \right\}.$$

It can be shown that:

(i)  $\left[ \Gamma_2 a_2 M - \tau^M \Lambda_2 e^{a_2 M} \right]$  is a decreasing function of  $M$  when  $M$  is large enough:

When  $M$  is large enough due to a large enough  $A$ ,<sup>10</sup>

$$\begin{aligned} & \frac{\partial \left[ \Gamma_2 a_2 M - \tau^M \Lambda_2 e^{a_2 M} \right]}{\partial M} \\ &= e^{a_2 M} a_2 \left\{ a_1 \beta \left[ 1 - \tau^T (1 - \alpha) - \tau^M \right] \left[ 1 - \tau^M (1 - \theta) - \alpha \theta \right] (1 + a_2 M) - \right. \\ & \quad \left. \tau^M \left[ \theta + (1 - \theta) \tau^T \right] 2 e^{a_2 M} (\alpha + \beta a_0) + \tau^M \left[ \theta + (1 - \theta) \tau^T \right] \beta a_1 \right\} \\ &< e^{a_2 M} a_2 \left\{ a_1 \beta (1 + a_2 M) - 2(1 + a_2 M) + \beta a_1 \right\} \\ &< e^{a_2 M} a_2 \left\{ a_1 \beta (2 + a_2 M) - (2 + a_2 M) \right\} \\ &< 0 \end{aligned}$$

(ii)  $\left[ \Lambda_2 \Gamma_1 \right]$  is an increasing function of  $M$ :

$$\begin{aligned} & \frac{\partial \left[ \Lambda_2 \Gamma_1 \right]}{\partial M} \\ &= \left\{ \left[ \theta + (1 - \theta) \tau^T \right] \alpha e^{a_2 M} a_2 \left[ (\alpha + \beta a_0) \left( e^{a_2 M} (\eta + \beta a_0) - \beta a_1 \right) + \right. \right. \\ & \quad \left. \left. (\eta + \beta a_0) \left( e^{a_2 M} (\alpha + \beta a_0) - \beta a_1 \right) \right] \right\} \\ &> 0, \text{ for } A \geq 0. \end{aligned}$$

<sup>10</sup> By holding all other factors constant in (30), public health,  $M$ , increases as the productivity parameter,  $A$ , increases, since  $M$  is positively related to the productivity parameter,  $A$ .

Hence, when  $M$  is large enough due to a large enough  $A$ , ceteris paribus,  $\left\{ \left[ (\alpha - \eta) / \tau^M \right] \left[ \Gamma_2 a_2 M - \tau^M \Lambda_2 e^{a_2 M} \right] - \Lambda_2 \Gamma_1 \right\} < 0$  by (i) and (ii) and as a consequence,  $(\partial n / \partial \tau^M) < 0$  in equation (28).

Therefore, if  $\alpha \geq \eta$  and  $A$  is large enough, then  $\partial n / \partial \tau^M < 0$  in equation (28), i.e., a rise in the tax rate for public health reduces fertility. As stated earlier,  $K = [A(1 - \nu n)^{1-\theta} \alpha \theta / n]^{1/[1-(\theta+\delta)]}$  and  $Y = AK^{\theta+\delta} (1 - \nu n)^{1-\theta}$  in the steady state. If  $\alpha \geq \eta$  and  $A$  is large enough, we therefore obtain

$$\begin{aligned} \partial K / \partial \tau^M &= (\partial K / \partial n)(\partial n / \partial \tau^M) > 0 \\ \partial Y / \partial \tau^M &= (\partial Y / \partial K)(\partial K / \partial \tau^M) + (\partial Y / \partial n)(\partial n / \partial \tau^M) > 0. \square \end{aligned}$$

**Proof of Proposition 3.** Recall that  $p(\tau^T, \tau^M) = a_0 - a_1 / e^{a_2 M(\tau^T, \tau^M)}$ . By differentiating  $p(\tau^T, \tau^M)$  with respect to  $\tau^T$ , we then obtain

$$\frac{\partial p(\tau^T, \tau^M)}{\partial \tau^T} = a_1 a_2 e^{-a_2 M(\tau^T, \tau^M)} \frac{\partial M(\tau^T, \tau^M)}{\partial \tau^T}$$

where

$$\frac{\partial M(\tau^T, \tau^M)}{\partial \tau^T} = \frac{-\tau^M \Pi_1 \Pi_2}{a_2} \frac{\partial n}{\partial \tau^T}.$$

By Proposition 2, if  $\alpha \geq \eta$ , then  $\partial n / \partial \tau^T < 0$ , and hence,  $\partial M / \partial \tau^T > 0$  and  $\partial p / \partial \tau^T > 0$  for  $\tau^M > 0$ . Consequently, by equations (17) and (20),  $\partial \gamma_c / \partial \tau^T < 0$  and  $\partial \gamma_d / \partial \tau^T < 0$  when  $\alpha \geq \eta$  and  $\tau^M > 0$ .

Similarly, by differentiating  $p(\tau^T, \tau^M) = a_0 - a_1 / e^{a_2 M(\tau^T, \tau^M)}$  with respect to  $\tau^M$ , we obtain

$$\frac{\partial p(\tau^T, \tau^M)}{\partial \tau^M} = a_1 a_2 e^{-a_2 M(\tau^T, \tau^M)} \frac{\partial M(\tau^T, \tau^M)}{\partial \tau^M}$$

where

$$\frac{\partial M(\tau^T, \tau^M)}{\partial \tau^M} = \frac{\Pi_1}{a_2} \left[ n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1 - \nu n)^{\frac{1-\theta}{1-(\theta+\delta)}} - \tau^M \frac{\partial n}{\partial \tau^M} \Pi_2 \right].$$



By Proposition 2, if  $\alpha \geq \eta$  and  $A$  is large enough, then  $\partial n / \partial \tau^M < 0$  and hence,

$\partial M / \partial \tau^M > 0$  and  $\partial p / \partial \tau^M > 0$ . When  $\partial p / \partial \tau^M > 0$ , then  $\partial \gamma_c / \partial \tau^M < 0$  and

$\partial \gamma_d / \partial \tau^M < 0$  by equations (17) and (20).  $\square$

**Proof of Proposition 4.** Proposition 2 implies that if  $\alpha \geq \eta$  and  $A$  is large enough, then

$\partial n / \partial \tau^T < 0$  and  $\partial n / \partial \tau^M < 0$ . If  $\alpha \geq \eta$ , the sign for  $(\partial n / \partial \tau^T - \partial n / \partial \tau^M)$  is given as

follows:

$$\text{sign}\left(\frac{\partial n}{\partial \tau^T} - \frac{\partial n}{\partial \tau^M}\right) = \left\{ \Gamma_4 \left[ \Gamma_1 + e^{a_2 M} (\alpha - \eta) \right] - \Gamma_2 \Pi_1 n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}} (\alpha - \eta) \right\} < 0$$

where

$$\Gamma_4 = \left[ e^{a_2 M} \alpha + \beta (e^{a_2 M} a_0 - a_1) \right] \left[ \alpha \theta - (1-\theta)(1-\tau^T - \tau^M) \right] < 0.$$

Note that from the proof of proposition 2,  $\left[ \alpha \theta - (1-\theta)(1-\tau^T - \tau^M) \right] < 0$  if  $\alpha \geq \eta$ .

With  $\text{sign}(\partial n / \partial \tau^T - \partial n / \partial \tau^M) < 0$ , we have  $\partial n / \partial \tau^T < \partial n / \partial \tau^M < 0$ .

Since  $K = [A(1-vn)^{1-\theta} \alpha \theta / n]^{1/(1-(\theta+\delta))}$  in the steady state, by combining

$\partial n / \partial \tau^T < \partial n / \partial \tau^M < 0$  with  $\partial K / \partial n < 0$ , we thus obtain

$$\frac{\partial K}{\partial \tau^T} = \frac{\partial K}{\partial n} \frac{\partial n}{\partial \tau^T} > \frac{\partial K}{\partial \tau^M} = \frac{\partial K}{\partial n} \frac{\partial n}{\partial \tau^M},$$

starting at  $\tau^M = \tau^T = 0$ .

By substituting  $K = [A(1-vn)^{1-\theta} \alpha \theta / n]^{1/(1-(\theta+\delta))}$  into  $Y = AK^{\theta+\delta} (1-vn)^{1-\theta}$ , we obtain

$$Y = A^{\frac{1}{1-(\theta+\delta)}} (\alpha \theta)^{\frac{\theta+\delta}{1-(\theta+\delta)}} n^{\frac{-(\theta+\delta)}{1-(\theta+\delta)}} (1-vn)^{\frac{1-\theta}{1-(\theta+\delta)}}$$

and obviously,  $\partial Y / \partial n < 0$ . Therefore, we have

$$\frac{\partial Y}{\partial \tau^T} = \frac{\partial Y}{\partial n} \frac{\partial n}{\partial \tau^T} > \frac{\partial Y}{\partial \tau^M} = \frac{\partial Y}{\partial n} \frac{\partial n}{\partial \tau^M},$$

starting at  $\tau^M = \tau^T = 0$ .  $\square$

## Reference

- Aaron, H., 1985. Social security: problems of maturity. Paper presented at International Institute of Public Finance Congress, Madrid.
- Anand, S., Ravallion, M., 1993. Human development in poor countries: on the role of private incomes and public services. *Journal of Economic Perspectives* 7, 133-150.
- Arrow, K.J., 1962. The economic implications of learning by doing. *Review of Economic Studies* 29, 155–173.
- Barro, R.J., 1974. Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1117.
- Barro R.J., 1997. *Determinants of Economic Growth: a Cross-country Empirical Study*, Cambridge MA: MIT Press.
- Barro, R. J., Sala-i-Martin, X., 1995. *Economic Growth*, New York: McGraw-Hill.
- Barro, R. J., Sala-i-Martin, X., 2004. *Economic Growth*, 2nd Edition, Cambridge MA: MIT Press.
- Bernstein, J., Nadiri, I., 1988. Interindustry R&D spillovers, rate of return, and production in high-tech industries. *American Economic Review* 78, 429-434.
- Bernstein, J., Nadiri, I., 1989. Research and development and intraindustry spillovers: an empirical application of dynamic duality. *Review of Economic Studies* 56, 249-267.
- Blackburn, K., Cipriani, G.P., 2002. A model of longevity, fertility and growth. *Journal of Economic Dynamics and Control* 26, 187-204.
- Chakraborty, S., Das, M., 2005. Mortality, human capital and persistent inequality. *Journal of Economic Growth* 10, 159-192.
- Cooley, T. F., Soares, J., 1999. A positive theory of social security based on reputation. *Journal of Political Economy* 107, 135-160.
- Corneo, G., Marquardt, M., 2000. Public pension, unemployment insurance, and growth. *Journal of Public Economics* 75, 293-311.
- Davies, J., Kuhn, P., 1992. Social security, longevity, and moral hazard. *Journal of Public Economics* 49, 91–106.
- de la Croix, D., Licandro, O., 1999. Life expectancy and endogenous growth. *Economics Letters* 65, 255-263.

- De Long J.B., Summers L.H., 1991. Equipment investment and economic growth. *The Quarterly Journal of Economics* 106, 445-502.
- Diamond, P., 1965. National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.
- Ehrlich, I., and Chuma, H., 1990. A model of the demand for longevity and the value of life extension. *Journal of Political Economy* 98, 761-782.
- Ehrlich, I., Lui, F.T., 1991. Intergenerational trade, longevity, and economic growth. *Journal of Political Economy* 99, 1029-1060.
- Feldstein, M., 1974. Social security, induced retirement and aggregate capital accumulation. *Journal of Political Economy* 82, 905-926.
- Hu, S., 1999. Economic growth in the perpetual-youth model: implications of the annuity market and demographics. *Journal of Macroeconomics* 21, 107-124.
- Hubbard, R.G., Judd, K.L., 1987. Social security in individual welfare: precautionary saving, borrowing constraints, and the payroll tax. *American Economic Review* 77, 630-646.
- Jones, C.I., 1995. Time series tests of endogenous growth models. *Quarterly Journal of Economics* 110, 495–525.
- Kaganovich, M. and Zilcha, I., 1999. Education, social security, and growth. *Journal of Public Economics* 71, 289-309.
- Kemnitz, A., Wigger, B. U., 2000. Growth and social security. *European Journal of Political Economy* 16, 673-683.
- Kotlikoff, L.J., Summers, L.H., 1981. The role of intergenerational transfers in aggregate capital accumulation. *Journal of Political Economy* 89, 706-732.
- Krusell, P., 2004. *Lecture Notes for Macroeconomics I*.
- Laitner, J., Juster, F.T., 1996. New evidence on altruism: a study of TIAA-CREF retirees. *American Economic Review* 86, 893-908.
- Laitner, J., Ohlsson, H., 2001. Bequest motives: a comparison of Sweden and the United States. *Journal of Public Economics* 79, 205–236.
- Lee, Ronald, Tuljapurkar, Shripad, 1997. Death and taxes: longer life, consumption, and social security. *Demography* 34, 67-81.

- Leung, M. C., Zhang, J., Zhang, J., 2004. An economic analysis of life expectancy by gender with application to the United States. *Journal of Health Economics* 23, 737-759.
- Lichtenberg, F. R., 2004. Sources of U.S. longevity increase, 1960-2001. *The Quarterly Review of Economics and Finance* 44, 369-389.
- Lucas Jr., R.E., 1993. Making a miracle. *Econometrica* 61, 251–272.
- Nakanishi, Y., 2002. Empirical evidence of inter-industry R&D spillover in Japan. *Journal of Economic Research* 7, 91–104.
- OECD, 2010a. OECD Factbook. Economic, Environmental and Social Statistics. [www.oecd.org](http://www.oecd.org)
- OECD, 2010b. OECD Observer No 281 October. [www.oecd.org](http://www.oecd.org)
- OECD, 2010c. OECD Social Expenditure Database. [www.oecd.org](http://www.oecd.org)
- OECD, 2010d. OECD Health Data. [www.oecd.org](http://www.oecd.org)
- Philipson, T.J., Becker, G.S., 1998. Old age longevity and mortality contingent claims. *Journal of Political Economy* 106, 551-573.
- Preston, H. Samuel, 1975. The changing relation between mortality and level of economic development. *Population Studies* 29, 231-248.
- Romer, P.M., 1986. Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002–1037.
- Rosati, F.C., 1996. Social security in a non-altruistic model with uncertainty and endogenous fertility. *Journal of Public Economics* 60, 283-294
- Samuelson, P., 1958. An exact consumption–loan model of interest with or without the social contrivance of money. *Journal of Political Economy* 66, 467–482.
- Sánchez-Losada, F., 2000. Growth effects of an unfunded social security system when there is altruism and human capital. *Economics Letters* 69, 95-99.
- Social Security Administration and International Social Security Association, 2006. *Social Security Programs throughout the World: Europe, 2006*. [www.ssa.gov](http://www.ssa.gov)
- Social Security Administration and International Social Security Association, 2008. *Social Security Programs throughout the World: The Americas, 2007*. [www.ssa.gov](http://www.ssa.gov)

- Tang, K.K., Zhang, J., 2007. Health, education, and life cycle savings in the development process. *Economic Inquiry* 45, 615-630.
- Tomes, Nigel., 1981. The family, inheritance, and the intergenerational transmission of inequality. *Journal of Political Economy* 89, 928-958.
- World Bank, 1984. *Population Change and Economic Development*, Oxford: Oxford University Press.
- World Health Statistics, 2009. World Health Organization.
- Yew, S.L., Zhang, J., 2009. Optimal social security in a dynastic model with human capital externalities, fertility and endogenous growth. *Journal of Public Economics* 93, 605-619.
- Zhang, J., 1995. Social security and endogenous growth. *Journal of Public Economics* 58, 185-213.
- Zhang, J., Zhang, J., 2001. Longevity and economic growth in a dynastic family model with an annuity market. *Economics Letters* 72, 269-277.
- Zhang, J., Zhang, J., 2004. How does social security affect economic growth: evidence from cross-country data. *Journal of population Economics* 17, 473-500.
- Zhang, J., Zhang, J., 2005. The effect of life expectancy on fertility, saving, schooling and economic growth: theory and evidence. *Scandinavian Journal of Economics* 107, 45-66.
- Zhang, J., Zhang, J., 2007. Optimal social security in a dynastic model with investment externalities and endogenous fertility. *Journal of Economic Dynamics & Control* 31, 3345-3567.
- Zhang, J., Zhang, J., Lee, R., 2001. Mortality decline and long-run economic growth. *Journal of Public Economics* 80, 485-507.
- Zhang, J., Zhang, J., Leung, M.C.M., 2006. Health investment, saving, and public policy. *Canadian Journal of Economics* 39, 68-93.

**Table 1 Simulation results with the condition  $\alpha > \eta$** 

Parameterization:

$$a_0 = 0.95, a_1 = 0.45, a_2 = 0.9, \alpha = 0.65, \eta = 0.5, \theta = 0.25, \beta = 0.5, \delta = 0.01, A = 25, v = 0.1$$

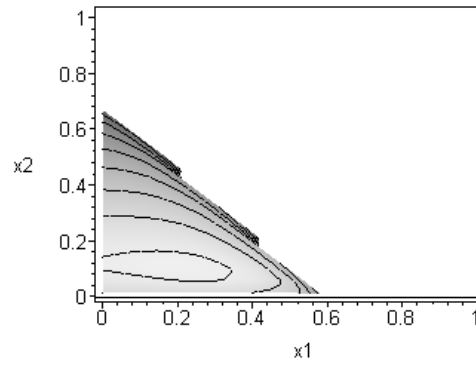
	$n$	$\gamma_c$	$\gamma_d$	$M$	$P$	$K$	$Y$	$V$
1. Optimal rates								
$\tau^T = 0.21,$ $\tau^M = 0.09$	<b>1.964</b>	<b>0.623</b>	<b>0.312</b>	<b>1.668</b>	<b>0.850</b>	<b>2.139</b>	<b>25.856</b>	<b>11.611</b>
2. $\tau^T = \tau^M = 0$	2.796	0.806	0.403	0.000	0.500	1.188	20.443	10.998
3. $\tau^T > 0, \tau^M = 0$								
$\tau^T = 0.1$	2.509	0.806	0.403	0.000	0.500	1.431	22.097	11.043
$\tau^T = 0.3$	1.775	0.806	0.403	0.000	0.500	2.511	27.430	11.074
$\tau^T = 0.4$	1.297	0.806	0.403	0.000	0.500	4.063	32.432	11.000
4. $\tau^M > 0, \tau^T = 0$								
$\tau^M = 0.05$	2.754	0.684	0.342	0.775	0.726	1.220	20.675	11.425
$\tau^M = 0.1$	2.707	0.617	0.309	1.570	0.841	1.257	20.939	11.522
$\tau^M = 0.15$	2.653	0.572	0.286	2.390	0.898	1.301	21.245	11.455
5. $(\tau^T = \tau^M) > 0$								
$\tau^T = \tau^M = 0.05$	2.606	0.682	0.341	0.807	0.732	1.342	21.519	11.460
$\tau^T = \tau^M = 0.1$	2.372	0.613	0.307	1.721	0.854	1.572	22.952	11.580
$\tau^T = \tau^M = 0.2$	1.703	0.532	0.266	4.212	0.940	2.679	28.080	11.271
6. $\tau^T > 0,$ $\tau^M = 0.05$								
$\tau^T = 0.1$	2.445	0.680	0.340	0.843	0.739	1.495	22.491	11.492
$\tau^T = 0.2$	2.080	0.674	0.337	0.936	0.756	1.951	24.972	11.540
$\tau^T = 0.3$	1.643	0.667	0.334	1.074	0.779	2.834	28.648	11.539

**Table 2 Simulated optimal tax rates: sensitivity analysis**

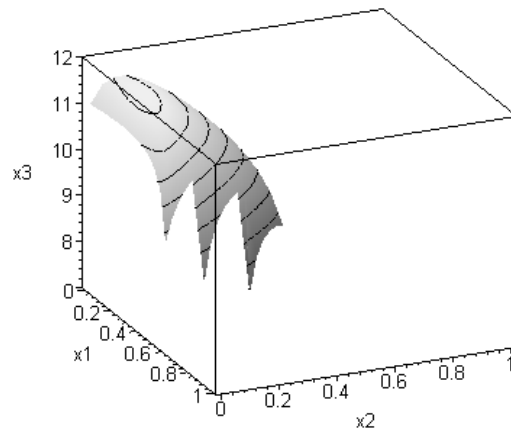
Parameter	$\tau^T$	$\tau^M$
Varying $\alpha$		
$\alpha = 0.6$	0.26	0.09
$\alpha = 0.7$	0.17	0.09
Varying $\delta$		
$\delta = 0$	0.18	0.09
$\delta = 0.02$	0.24	0.08
Varying $\eta$		
$\eta = 0.45$	0.19	0.08
$\eta = 0.55$	0.23	0.09
Varying $\theta$		
$\theta = 0.2$	0.16	0.09
$\theta = 0.3$	0.26	0.08
Varying $\beta$		
$\beta = 0.45$	0.22	0.08
$\beta = 0.55$	0.20	0.09
Varying $a_0$		
$a_0 = 0.9$	0.21	0.09
$a_0 = 1$	0.21	0.09
Varying $a_1$		
$a_1 = 0.4$	0.21	0.08
$a_1 = 0.5$	0.21	0.09
Varying $a_2$		
$a_2 = 0.85$	0.21	0.09
$a_2 = 0.95$	0.20	0.09
Varying $A$		
$A = 20$	0.21	0.09
$A = 30$	0.20	0.09
Varying $\nu$		
$\nu = 0.05$	0.22	0.11
$\nu = 0.10$	0.21	0.09

**Figure 1 Welfare with social security and public health**

a. Contour of welfare



b. Three-dimensional diagram of welfare



Note:  $\tau^T$  is  $x_1$ - axis,  $\tau^M$  is  $x_2$ - axis, and welfare is  $x_3$ - axis.

The welfare level refers to equation (21) with parameterization in Table 1.