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FINE VALUE ALLOCATIONS IN LARGE EXCHANGE ECONOMIES WITH DIFFERENTIAL INFORMATION**

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Abstract

We show that the set of fine value allocations of a pure exchange economy with a continuum of traders and differential information coincides with the set of competitive allocations of an associated symmetric information economy in which each trader has the "joint infomation" of all the traders in the original economy.

Keywords: Atomless exchange economies, differential information, fine value.

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1 Introduction

The equivalence of value and competitive allocations was first established by Shapley (1964) in the context of a replicated finite economy with money. Using the extension of the definition of value allocation to exchange economies with non-transferable utilities due to Shapley (1969), a number of authors have studied this equivalence for exchange economies with full information; see, e.g., Shapley and Shubik (1969), Champsaur (1975), Mas-Colell (1977), Aumann and Dreze (1986), Wooders and Zame (1987) for replica of finite economies, and Aumann (1975), Hart (1977), Dubey and Neyman (1984 and 1997) for economies with a continuum of traders.

Radner (1968 and 1982) introduces a model of exchange economy with differential information in which every trader is characterized by a state dependent utility function, a random initial endowment, an information partition, and a prior belief. In this framework, traders arrange contingent contracts for trading commodities before they obtain any information about the realized state of nature. Radner (1968) extends the notion of Arrow-Debreu competitive equilibrium to this model. In the definition of competitive equilibrium (in the sense of Radner), the information of an agent places a restriction on his feasible trades (i.e., his budget set): better information allows for more contingent trades (i.e., enlarges the agent's budget set). Thus, in a Radner competitive equilibrium better informed agents are generally, ceteris paribus, better off (and they are never worse off) than those with worse information; i.e., a competitive equilibrium rewards the information advantage of a trader.

Allen (1991 and 1997) and Krasa and Yannelis (1994) extend Shapley's definition of value allocation to differential information economies with a finite number of traders by associating with the economy a cooperative game (a market game) with differential information. This approach is based on the presumption that agreements within coalitions are reached ex-ante. Krasa and Yannelis (1994) concentrate mainly in studying the private value; in this approach the traders of a coalition use only their private information (i.e., there is no information exchange). Einy and Shitovitz (1998) show that in a Radner type economy with a continuum of traders the set of private value allocations coincides with the set of Radner competitive equilibrium allocations.

Thus, as pointed out by Krasa and Yannelis (1994), the private value rewards the information advantage of a trader.

An interesting question is whether the information advantage of a trader is rewarded when we consider the possibility that the members of a coalition may share some of their information. Wilson (1978) introduces the notion of fine core for an economy with differential information. Einy, Moreno and Shitovitz (1998) show that the set of (weak) fine core allocations of a Radner type economy with a continuum of traders coincides with set of competitive allocations of an associated economy with symmetric information in which each trader has the "joint information" of all the traders in the original economy. Krasa and Yannelis (1994), using Wilson's ideas about information exchange, define the notion of fine value allocation for a Radner type economy by considering a market game associated with the economy in which each trader of a coalition is given the joint information of all the members of the coalition.

In this work we study the relation between fine value and competitive allocations in a Radner type economy with a continuum of traders and a finite number of traders' types. We show that, under appropriate assumptions, the set of fine value allocations of the economy coincides with the set of competitive allocations of an associated symmetric information economy in which the traders' information is the joint information of all the traders in the original economy. Thus, whereas when there is no information exchange, as established by Einy and Shitovitz (1998), (private) value allocations reward the information advantage of a trader, when the possibility of sharing information is introduced the information advantage is worthless; e.g., if two traders A and B have identical characteristics, except that A is better informed than B (i.e., A's information partition is finer than B's) then in a private value allocation trader A is as well off as trader B, and he may be better off than B; in a fine value allocation, however, because fine value allocation are competitive allocations of the associated symmetric information economy, both traders are equally well off.

A difficulty in studying this issue when agents share information is that, unlike in the full information case studied by Aumann (1975) or the private information case studied by Einy and Shitovitz (1998), the market game associated with the economy may not be continuous even when the utility functions of the traders satisfy Aumann's (1975) smoothness assumptions (see Example 3.4). Therefore the market game may not be in any of the classical spaces of non-atomic games studied in Aumann and Shapley (1974). In order to overcome this difficulty we use a recent result of Neyman (1998) who proves the existence of a value on a very general space of non-atomic games which includes the market games we encounter in our framework. It is possible to obtain similar results for economies with an infinite set of traders' types, under somewhat more restrictive assumptions on the traders' utility functions and endowments, using the value defined in Merterns (1980). The proofs of these results, however, would be considerably more cumbersome.

Aumann (1975) conjectured that in a full information economy with a continuum of traders the fact the every value allocation is competitive can be proved without the differentiability assumption on the traders' utilities (which is assumed both in Aumann (1975) and in Einy and Shitovitz (1998)). Here using Neyman's value we are able to show, without this differentiability assumption, that the set of fine value allocations is included in the set of competitive allocations of the associated symmetric information economy. In proving the converse, however, we do use the assumption that the utility functions of the traders are differentiable (but in a weaker sense than Aumann (1975)).

2 The Model

We consider a Radner-type exchange economy $\mathcal{E}$ with differential information (e.g., Radner (1968 and 1982)).

The space of traders is a measure space (T, Σ, μ), where T is a set (the set of *traders*), Σ is a σ-field of subsets of T (the set of *coalitions*), and μ is a non-atomic measure on Σ. The commodity space is $\Re^l_+$. The space of states of nature is a finite set Ω. The economy extends over two time periods, $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty over the state of nature; in this period traders arrange contracts that may be contingent on the realized state of nature at $\tau = 1$. At $\tau = 1$ traders do not necessarily know which state of nature $\omega \in \Omega$ actually occurred, although they know their own endowments, and may also have some additional information about the state of nature. We do not assume, however, that traders know their own utility function.

The information of a trader $t \in T$ is described by a partition Π_t of Ω. We denote by $\mathcal{F}_t$ the field generated by Π_t. If ω_0 is the true state of nature, at $\tau = 1$ trader t observes the member of Π_t which contains ω_0. Every trader $t \in T$ has a probability distribution q_t on Ω which represents his *prior beliefs.* The preferences of a trader $t \in T$ are represented by a *state dependent utility function,* $u_t : \Omega \times \Re^l_+ \to \Re$. If x is a random bundle (i.e., a function from Ω to $\Re^l_+$) we denote by $h_t(x)$ the *expected utility from the random bundle* x of trader $t \in T$. That is,

$$h_t(x) = \sum_{\omega \in \Omega} q_t(\omega) u_t(\omega, x(\omega)).$$

Traders' initial endowments are described by a function $\mathbf{e} : T \times \Omega \to \Re^l_+$ such that for every $\omega \in \Omega$, $\mathbf{e}(\cdot, \omega)$ is μ-integrable on T, and for every $t \in T$, $\mathbf{e}(t, \cdot)$ is $\mathcal{F}_t$-measurable; $\mathbf{e}(t, \omega)$ represents the *initial endowment* of trader $t \in T$ in the state of nature $\omega \in \Omega$.

Two traders in T are of the same type if they have the same information partition, the same prior, the same initial endowment, and the same utility function. We assume that there is a finite number n of different types of traders. For $i = 1, \ldots, n$ we denote by T_i the set of all traders of type i. We assume that for all $1 \leq i \leq n$, $T_i \in \Sigma$ and $\mu(T_i) > 0$. The information field of traders of type i will be denoted by $\mathcal{F}_i$, their prior by q_i, their utility function by u_i, and their initial endowment by e_i. We assume that for all $1 \leq i \leq n$ and each non-empty event $A \in \bigvee_{i=1}^n \mathcal{F}_i$, we have $q_i(A) > 0$. If $x : \Omega \to \Re^l_+$ is a random bundle we denote by $h_i(x)$ the expected utility of a trader of type i from x.

Henceforth an economy $\mathcal{E}$ is a differential information economy with a continuum of traders and a finite set of traders' types as described above. We use the following notations. For two vectors $x = (x_1, \ldots, x_l)$ and $y = (y_1, \ldots, y_l)$ in $\Re^l$ we write $x \geq y$ when $x_k \geq y_k$ for all $1 \leq k \leq l$, $x > y$ when $x \geq y$ and $x \neq y$, and $x \gg y$ when

$x_k > y_k$ for all $1 \leq k \leq l$. A function $u : \Re^l_+ \to \Re$ is *strictly increasing* if for all $x, y \in \Re^l_+$, $x > y$ implies $u(x) > u(y)$.

Let $\mathcal{E}$ be an economy. An *assignment* is a function $\mathbf{x} : T \times \Omega \to \Re^l_+$ such that for every $\omega \in \Omega$ the function $\mathbf{x}(\cdot, \omega)$ is μ-integrable on T. A *private allocation* is an assignment $\mathbf{x}$ such that

(2.1) for all $t \in T$, $\mathbf{x}(t, \cdot)$ is $\mathcal{F}_t$-measurable, and

(2.2) $\int_T \mathbf{x}(t, \omega) d\mu \leq \int_T \mathbf{e}(t, \omega) d\mu$ for all $\omega \in \Omega$.

A *price system* is a non-zero function $p : \Omega \to \Re^l_+$. Let $t \in T$ and let M_t be the set of all $\mathcal{F}_t$-measurable functions from Ω to $\Re^l_+$. For a price system p, define the *budget set* of $t \in T$ by

$$B_t(p) = \left\{ x \mid x \in M_t \text{ and } \sum_{\omega \in \Omega} p(\omega) \cdot x(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot \mathbf{e}(t, \omega) \right\}.$$

A *competitive equilibrium* (in the sense of Radner) is a pair $(p, \mathbf{x})$ where p is a price system and $\mathbf{x}$ is private allocation such that

(2.3) for almost all $t \in T$, $\mathbf{x}(t, \cdot)$ maximizes h_t on $B_t(p)$, and

(2.4) $\sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{x}(t, \omega) d\mu = \sum_{\omega \in \Omega} p(\omega) \cdot \int_T \mathbf{e}(t, \omega) d\mu$.

A *competitive allocation* is a private allocation $\mathbf{x}$ for which there exists a price system p such that $(p, \mathbf{x})$ is a competitive equilibrium.

Radner (1982) noted that, unlike in full information economies, the inequality (2.2) cannot be replaced with an equality even if there is free disposal, as the amount to be disposed of might not be measurable with respect to the information partition of any single agent. See Einy and Shitovitz (1998) for an example of an economy with differential information which has no competitive equilibrium for which (2.2) is satisfied with equality. Condition (2.4) ensures that in an equilibrium the price of a commodity which is in excess supply is zero. It is implied by Walras' Law, which is satisfied in our framework. We included it as an equilibrium condition to facilitate comparison to Radner's (1982) definition.

Throughout the paper we refer to the following conditions:

$(A.1)$ For every $\omega \in \Omega$, $\int_T \mathbf{e}(t,\omega)d\mu \gg 0$.

$(A.2)$ For all $1 \leq i \leq n$ there exists $\omega \in \Omega$ such that $e_i(\omega) \neq 0$.

$(A.3)$ For all $1 \leq i \leq n$ and $\omega \in \Omega$, $u_i(\omega, 0) = 0$.

$(A.4)$ For all $1 \leq i \leq n$ and $\omega \in \Omega$, $u_i(\omega, \cdot)$ is continuous, strictly increasing and concave on $\Re^l_+$.

$(A.5)$ For all $1 \leq i \leq n$ and $\omega \in \Omega$, the partial derivative $\frac{\partial u_i(\omega,\cdot)}{\partial x_j}$ exists at each $x = (x_1, \ldots, x_l) \in \Re^l_+$ such that $x_j > 0$.

Condition $(A.1)$ assures that every commodity is actually present in the market in every state of nature. Condition $(A.2)$ implies that every trader is potentially an active trader at some state of nature. Condition $(A.3)$ is just a normalization assumption. Condition $(A.4)$ (together with conditions $(A.1) - (A.3)$) is used below to establish that every (fine) value allocation is competitive; whereas concavity of the traders' utility functions is not needed to establish the equivalence of the core and the set of competitive allocations, it is required to show the equivalence of value and competitive allocations–see Aumann (1975) and Dubey and Neyman (1997). Conditions $(A.3) - (A.5)$ are weaker than those of Aumann (1975) and Einy and Shitovitz (1998). Aumann (1975) assumes, in addition, that traders utility functions have partial derivatives which are bounded on every compact subset of $\Re^l_+$. The utility function $u(x, y) = \sqrt{x} + \sqrt{y}$, for example, satisfies conditions $(A.3) - (A.5)$, but do not satisfy the assumptions of Aumann (1975). However, Aumann (1975) does not assume a finite number of traders' types. Einy and Shitovitz (1998) assume that the traders satisfy the Aumann-Perles Condition (Aumann and Perles (1965)), which is not implied by conditions $(A.3) - (A.5)$.

3 The market game

In this section we define a class of non-atomic coalitional games (market games) associated with the economies described in Section 2, and we derive some properties of this class of games.

Let $\mathcal{E}$ be an economy. A *coalitional game,* or simply a *game,* on (T, Σ) is a function $v : \Sigma \to \Re$ with $v(\emptyset) = 0$. Let $S \in \Sigma$. Define

$$I(S) = \{j \mid 1 \leq j \leq n \text{ and } \mu(S \cap T_j) > 0\}.$$

A *fine S-allocation* is an assignment $\mathbf{x}$ such that

(3.1) for all $t \in S$, $\mathbf{x}(t, \cdot)$ is $\bigvee_{j \in I(S)} \mathcal{F}_j$-measurable, and

(3.2) $\int_S \mathbf{x}(t,\omega) d\mu \leq \int_S \mathbf{e}(t,\omega) d\mu$ for all $\omega \in \Omega$

Denote by $\mathcal{A}(S)$ the set of all fine S-allocations. The *market game associated with $\mathcal{E}$* is given by

(3.3) $$v(S) = \sup \left\{ \sum_{i=1}^{n} \int_{S \cap T_i} h_i(\mathbf{x}(t, \cdot)) d\mu \mid \mathbf{x} \in \mathcal{A}(S) \right\}.$$

For $y = (y_1, \ldots, y_n) \in \Re^n_+$ we denote by $I(y)$ its support, i.e.,

$$I(y) = \{j \mid 1 \leq j \leq n \text{ and } y_j > 0\}.$$

Also write

$$M(y) = \left\{ x : \Omega \to \Re^l_+ \mid x \text{ is } \bigvee_{j \in I(y)} \mathcal{F}_j\text{-measurable} \right\}.$$

The *market function associated with $\mathcal{E}$*, $f : \Re^n_+ \to \Re$ is given by

(3.4) $f(y) = \max \{ \sum_{i=1}^{n} y_i h_i(x_i) \mid x_i \in M(y), \text{ and } \forall \omega \in \Omega, \sum_{i=1}^{n} y_i x_i(\omega) \leq \sum_{i=1}^{n} y_i e_i(\omega) \}$.

For every $S \in \Sigma$ write $\xi(S) = (\mu(S \cap T_1), \ldots, \mu(S \cap T_n))$.

Lemma 3.1. *Let $\mathcal{E}$ be an economy satisfying condition (A.4). Then $v = f \circ \xi$.*

Proof: See the Appendix.

Lemma 3.2. *Let $\mathcal{E}$ be an economy satisfying conditions (A.3) and (A.4). Then f is concave, homogeneous of degree one, non-decreasing on $\Re^n_+$ and continuous at 0 and at $\xi(T)$.*

Proof: See the Appendix.

Lemma 3.3. *Let $\mathcal{E}$ be an economy satisfying conditions (A.3) to (A.5). Then f is continuously differentiable in the interior of $\Re^n_+$.*

Proof: See the Appendix.

We note that in the case of full information (e.g., Aumann (1975)), or when there is no information exchange (that is, when for each $S \in \Sigma$, $\mathcal{A}(S)$ is taken to be the set of assignments $\mathbf{x}$ which are feasible for S at every state of nature and is such that for all $t \in S$, $\mathbf{x}(t, \cdot)$ is $\mathcal{F}_t$-measurable), a case studied in Einy and Shitovitz (1998), the market function f is continuous on the range of the vector measure ξ. As the following example shows, in our model (where there is information exchange) the market function f may not be continuous.

Example 3.4. Consider an economy $\mathcal{E}$ in which the space of traders is $([0,3], \mathcal{B}, \lambda)$, where $\mathcal{B}$ is the σ-field of Borel subsets of $[0,3]$ and λ is the Lebesgue measure. The commodity space is $\Re_+$. The space of states of nature is $\Omega = \{\omega_1, \omega_2\}$. Let $T_1 = [0,1], T_2 = (1,2]$, and $T_3 = (2,3]$. The information partition of a trader. $t \in T_1 \cup T_2$ is $\Pi_1 = \Pi_2 = \{\Omega\}$, and that of a trader $t \in T_3$ is $\Pi_3 = \{\{\omega_1\}, \{\omega_2\}\}$. The priors of the traders are $q_1 = (\frac{1}{3}, \frac{2}{3})$ and $q_2 = (\frac{2}{3}, \frac{1}{3})$ for the traders in T_1 and T_2, respectively, and the prior of traders in T_3 is q_3 arbitrary. All traders have the same initial endowments, $e(\omega) = 2$ for all $\omega \in \Omega$, and the same utility function, $u(\omega, x) = \ln(1+x)$ for all $(\omega, x) \in \Omega \times \Re_+$. (Note that u satisfies the assumptions of Aumann (1975) and Einy and Shitovitz (1998).) For this economy, the market function f is not continuous at $\xi(T_1 \cup T_2) = (1,1,0)$. Indeed a direct computation yields

$$f(1,1,0) = 2\ln 3 \neq \frac{10}{3}\ln 2 = \lim_{\substack{y_3 \to 0 \\ y_3 > 0}} f(1,1,y_3).$$

Let v be a coalitional game on (T, Σ). The *core* of the game v, denoted by $Core(v)$, is the set all finitely additive measures λ on (T, Σ) such that $\lambda(T) = v(T)$, and $\lambda(S) \geq v(S)$ for all $S \in \Sigma$. The following proposition will be useful in the sequel.

Proposition 3.5. *Let $\mathcal{E}$ be an economy satisfying conditions (A.3) to (A.5). Then*

$$Core(v) = \{\nabla f(\xi(T)) \cdot \xi\}.$$

Proof: By Lemma 3.1, $v = f \circ \xi$. Since f is homogeneous of degree one on $\Re^n_+$, by Euler's Theorem we have $\nabla f(\xi(T)) \cdot \xi(T) = f(\xi(T))$. Therefore by Corollary

4.2 in Einy, Moreno and Shitovitz (1999), $Core(v) \neq \emptyset$, and moreover, $Core(v) = \{\nabla f(\xi(T)) \cdot \xi\}$. □

4 Fine Value Allocations

In this section we extend to our economy the definition of fine value allocations of Krasa and Yannelis (1994). We start with some standard definitions in the theory of non-atomic games.

Let BV be the space of all coalitional games on (T, Σ) that can be represented as the difference of two monotonic games. (A game on (T, Σ) is monotonic if for every two coalitions $S_1, S_2 \in \Sigma$, $S_1 \supseteq S_2$ implies $v(S_1) \geq v(S_2)$.) A non-decreasing sequence of sets in Σ of the form $\Lambda : S_0 \subseteq S_1 \subseteq \ldots \subseteq S_m$ is called a *chain.* For $v \in BV$, the *variation of* v over a chain Λ is defined by $\|v\|_\Lambda = \sum_{i=1}^{m} |v(S_i) - v(S_{i-1})|$, and the *variation norm of* v is defined by $\|v\|_{BV} = \sup\{\|v\|_\Lambda \mid \Lambda \text{ is a chain}\}$. It is well know that $(BV, \| \quad \|_{BV})$ is a Banach space (see, e.g., Proposition 4.3 in Aumann and Shapley (1974)).

Let Q be a subset of BV. A mapping from Q into BV is called *positive* if it maps each monotonic game in Q to a monotonic game in BV. An automorphism on (T, Σ) is a one to one mapping θ from (T, Σ) into itself such that for every $S \subseteq T$, $S \in \Sigma$ iff $\theta(S) \in \Sigma$. Each automorphism θ on (T, Σ) induces a linear mapping θ_* from BV onto itself defined by $(\theta_* v)(S) = v(\theta(S))$. A subset Q of BV is called symmetric if $\theta_*(Q) \subseteq Q$ for every automorphism θ on (T, Σ). A mapping ψ from a symmetric subset Q of BV into BV is called *symmetric* if for every automorphism θ on (T, Σ) we have $\theta_* \circ \psi = \psi \circ \theta_*$. A mapping ψ from a subset Q of BV into BV is called *efficient* if for every $v \in Q$ we have $(\psi v)(T) = v(T)$. Let Q be a symmetric linear subspace of BV. A value on Q (in the sense of Aumann and Shapley (1974)) is a linear, positive, efficient and symmetric mapping from Q into the space FA of all bounded finitely additive measures on (T, Σ).

Let η be a finite dimensional vector of non-atomic measures on (T, Σ). Denote by $Q(\eta)$ the linear subspace of BV of all games of the form $g \circ \eta$, where g is a real-valued function defined on the range of the vector of measures η which is continuous at $0 = \eta(\emptyset)$ and $\eta(T)$. Throughout the rest of the paper, Q will denote the union of all the spaces $Q(\eta)$ where η ranges over all finite dimensional vectors of non-atomic measures on (T, Σ). Neyman (1998) showed that there is a value on Q, which we refer to as Neyman value. A Neyman value on Q may not be unique, but if v is a game in Q of the form $v = g \circ \eta$, where η is a finite dimensional vector of non-atomic measures on (T, Σ) and g is a real-valued function which is concave and homogeneous of degree one on the range of η, then for every two Neyman values φ_1 and φ_2 on Q we have $\varphi_1 v = \varphi_2 v$. Moreover, if φ is a Neyman value on Q, then $\varphi v \in Core(v)$ (see Proposition 4 in Neyman (1998)).

Let $\mathcal{E}$ be an economy, and let $\lambda = (\lambda_1, \ldots, \lambda_n) \in \Re^n_{++}$. Write $\mathcal{E}_\lambda$ for the economy identical to $\mathcal{E}$ except that the utility function of every trader of type i, $1 \leq i \leq n$, is replaced by $\lambda_i u_i$. It is clear that if the utility functions of the traders in $\mathcal{E}$ satisfy any one of the conditions $(A.3)$ to $(A.5)$, then this condition is also satisfied by the utility functions of the traders in $\mathcal{E}_\lambda$. Let v_λ be the market game associated with $\mathcal{E}_\lambda$, and let f_λ be the corresponding associated market function. Then by Lemma 3.1, $v_\lambda = f_\lambda \circ \xi$, where $\xi(S) = (\mu(S \cap T_1), \ldots, \mu(S \cap T_n))$ for all $S \in \Sigma$. Moreover, if an economy $\mathcal{E}$ satisfies conditions $(A.3)$ and $(A.4)$, then by lemmas 3.1 and 3.2, $v_\lambda \in Q$. Further, since f_λ is homogeneous of degree one and concave on $\Re^n_+$, if φ_1 and φ_2 are two Neyman values on Q then $\varphi_1 v_\lambda = \varphi_2 v_\lambda$, and also if φ is a Neyman value on Q then $\varphi v_\lambda \in Core(v_\lambda)$.

We are now ready to introduce the notion of fine value allocation. Let $\mathcal{E}$ be an economy. A *fine allocation* $\mathbf{x}$ is a T-allocation; that is, $\mathbf{x}$ is a fine allocation if

(4.1) for all $t \in T$, $\mathbf{x}(t, \cdot)$ is $\bigvee_{i=1}^n \mathcal{F}_i$-measurable, and

(4.2) $\int_T \mathbf{x}(t, \omega) d\mu \leq \int_T \mathbf{e}(t, \omega)$ for all $\omega \in \Omega$.

A fine allocation $\mathbf{x}$ is called a *fine value allocation* if there exists $\lambda = (\lambda_1 \ldots, \lambda_n) \in \Re^n_{++}$ such that for all $S \in \Sigma$

$$(\varphi v_\lambda)(S) = \sum_{i=1}^n \int_{S \cap T_i} \lambda_i h_i(\mathbf{x}(t, \cdot)) d\mu,$$

where φ is a Neyman value on Q. As we noted above, the allocation $\mathbf{x}$ does not depend on the choice of φ.

A fine value allocation can be interpreted as a maximizer of a "social welfare function" which is a weighted average of the traders expected utilities. Thus, for every $1 \leq i \leq n$, the number λ_i can be interpreted as the weight in the society of a trader of type i. This definition of fine value allocation may suggest that we are implicitly assuming "equal treatment," as traders of the same type have the same weights. But as suggested by Champsaur (1975) and Aumann and Dreze (1986), each λ_i can be reinterpreted as the average weight of the traders of type i. The results below establish that equal treatment is a property of value allocations.

5 The Equivalence Result

Let us be given an economy $\mathcal{E}$, and denote by $\mathcal{E}^*$ an economy identical to $\mathcal{E}$, except for the information fields of the traders which for each $t \in T$ is taken to be $\mathcal{F}_t^* = \bigvee_{i=1}^n \mathcal{F}_i$. Note the information in $\mathcal{E}^*$ is symmetric.

Proposition 5.1. *Assume that an economy $\mathcal{E}$ satisfies $(A.1)$ to $(A.4)$. Then every fine value allocation of $\mathcal{E}$ is a competitive allocation of $\mathcal{E}^*$.*

Note that unlike in the analogous results of Aumann (1975) (see Proposition 5.1 in Aumann (1975)) and of Einy and Shitovitz (1998), in our Proposition 5.1 we do not assume that the utility functions of the traders are differentiable. Aumann (1975) raised the conjecture (in the full information case) that value allocations are competitive even without differentiability of the traders utility functions (see footnote 17 in Aumann (1975)). Proposition 5.1 establishes this conjecture for economies with a finite number of traders' types.

Proposition 5.2. *Assume that an economy $\mathcal{E}$ satisfies $(A.1)$ to $(A.5)$. Then every competitive allocation of $\mathcal{E}^*$ is a fine value allocation of $\mathcal{E}$.*

Theorem A below is a direct corollary of propositions 5.1 and 5.2.

Theorem A. *Assume that an economy $\mathcal{E}$ satisfies $(A.1)$ to $(A.5)$. Then the set of fine value allocations of $\mathcal{E}$ coincides with the set of competitive allocations of $\mathcal{E}^*$.*

For the proof of Proposition 5.1 we need the notion of weak fine core allocation of an economy introduced in Allen (1991) and Koutsougeras and Yannelis (1993). Let $\mathcal{E}$ be an economy. An assignment $\mathbf{x}$ is called a *weak fine core allocation of* $\mathcal{E}$ if

(5.1) $\mathbf{x}$ is a fine allocation of $\mathcal{E}$, and

(5.2) there does not exists a coalition $S \in \Sigma$ with $\mu(S) > 0$ and a fine S-allocation $\mathbf{y}$ such that $h_t(\mathbf{y}(t,\cdot)) > h_t(\mathbf{x}(t,\cdot))$ for almost all $t \in S$.

Einy, Moreno and Shitovitz (1998) showed that if an economy $\mathcal{E}$ satisfies assumptions $(A.1)$ to $(A.4)$, then the set of weak fine core allocations of $\mathcal{E}$ coincides with the set of competitive allocations of $\mathcal{E}^*$ (see Theorem C in Einy, Moreno and Shitovitz (1998)).

Proof of Proposition 5.1: Let $\mathbf{x}$ be a fine value allocation of $\mathcal{E}$, and assume, contrary to our claim, that $\mathbf{x}$ is not a competitive allocation of $\mathcal{E}^*$. Then by the above mentioned result of Einy, Moreno and Shitovitz (1998), $\mathbf{x}$ is not a weak fine core allocation of $\mathcal{E}$. Therefore there exists a coalition $S \in \Sigma$ with $\mu(S) > 0$ and a fine S-allocation $\mathbf{y}$ of $\mathcal{E}$ such that $h_t(\mathbf{y}(t,\cdot)) > h_t(\mathbf{x}(t,\cdot))$ for almost all $t \in S$. Since $\mathbf{x}$ is a fine value allocation of $\mathcal{E}$, there exists $\lambda = (\lambda_1, \ldots \lambda_n) \in \Re^n_{++}$ such that

$$(\varphi v_\lambda)(S) = \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{x}(t,\cdot))d\mu,$$

where φ is a Neyman value on Q. Hence

$$(\varphi v_\lambda)(S) < \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{y}(t,\cdot))d\mu.$$

But as noted in Section 4, $\varphi v_\lambda \in Core(v_\lambda)$. Therefore

$$v_\lambda(S) \leq (\varphi v_\lambda)(S) < \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{y}(t,\cdot))d\mu.$$

Since $\mathbf{y}$ is a fine S-allocation, this contradicts the definition of v_λ. □

Proof of Proposition 5.2: Let $\mathbf{x}$ be competitive allocation of $\mathcal{E}^*$. We show that $\mathbf{x}$ is a fine value allocation of $\mathcal{E}$. Denote by M the set of all functions $x : \Omega \rightarrow \Re^l_+$ that are $\bigvee_{j=1}^n \mathcal{F}_j$-measurable, and let p be a price system such that $(p, \mathbf{x})$ is a competitive equilibrium of $\mathcal{E}^*$. The budget set of a trader $t \in T$ in $\mathcal{E}^*$ for the price system p is

$$B_t^*(p) = \left\{ x \in M \mid \sum_{\omega\in\Omega} p(\omega) \cdot x(\omega) \leq \sum_{\omega\in\Omega} p(\omega) \cdot \mathbf{e}(t, \omega) \right\}.$$

For every $1 \leq i \leq n$ let

$$S_i = \{t \in T_i \mid \mathbf{x}(t, \cdot) \text{ maximizes } h_i \text{ on } B_t^*(p)\}.$$

Then $\mu(S_i) = \mu(T_i)$. Let $1 \leq i \leq n$, and consider the problem

$$(P_i) : \quad \begin{array}{l} \max_{x\in M} h_i(x) \\ s.t. \\ \sum_{\omega\in\Omega} p(\omega) \cdot x(\omega) \leq \sum_{\omega\in\Omega} p(\omega) \cdot e_i(\omega). \end{array}$$

We show that the problem (P_i), $1 \leq i \leq n$, satisfies the Slater Condition (see, e.g., page 276 in Duffie (1996)). As $0 \in M$, it suffices to show that $\sum_{\omega\in\Omega} p(\omega) \cdot e_i(\omega) > 0$. By $(A.2)$, there exists $\omega_0 \in \Omega$ such that $e_i(\omega_0) \neq 0$. Let A be the atom of the field $\bigvee_{j=1}^n \mathcal{F}_j$ containing ω_0. Since e_i is $\mathcal{F}_i$-measurable, we have $e_i \in M$. Therefore e_i is constant on A. For $a \in \Re^l_+$ denote by a^j the jth coordinate of a. Let $1 \leq j \leq l$ be such $e_i^j(\omega_0) > 0$. We claim that $\sum_{\omega\in A} p^j(\omega) > 0$. Suppose not; let $t \in S_i$ and let δ_j be the jth unit vector in $\Re^l_+$. Define $x : \Omega \rightarrow \Re^l_+$ by

$$x(\omega) = \begin{cases} \mathbf{x}(t, \omega) + \delta_j & \omega \in A \\ \mathbf{x}(t, \omega) & \text{otherwise.} \end{cases}$$

Then $x \in M$. Moreover, $x \in B_t^*(p)$. Since $q_i(A) > 0$ and $u_i(\omega, \cdot)$ is strictly increasing for all $\omega \in \Omega$, we have $h_i(x) > h_i(\mathbf{x}(t, \cdot))$, which is impossible because $t \in S_i$. Now

$$\sum_{\omega\in\Omega} p^j(\omega) e_i^j(\omega) \geq \sum_{\omega\in A} p^j(\omega) e_i^j(\omega) = e_i^j(\omega_0) \sum_{\omega\in A} p^j(\omega) > 0.$$

Therefore $\sum_{\omega\in\Omega} p(\omega) \cdot e_i(\omega) > 0$.

For every $1 \leq i \leq n$ let x_i be a solution to (P_i) (such solution exists because $\mathbf{x}(t, \cdot)$ is a solution to (P_i) for $t \in S_i$). Then by the Saddle Point Theorem, for every

$1 \leq i \leq n$ there is $\alpha_i \geq 0$ such that (α_i, x_i) is a saddle point on $\Re^l_+ \times M$ of the Lagrangian L_i of (P_i). Moreover,

(5.3) $\qquad \alpha_i(\sum_{\omega\in\Omega} p(\omega) \cdot x_i(\omega) - \sum_{\omega\in\Omega} p(\omega) \cdot e_i(\omega)) = 0.$

Note that since $u_i(\omega, \cdot)$ is strictly increasing for all $\omega \in \Omega$ and $1 \leq i \leq n$, we must have $\alpha_i > 0$. Therefore by (5.3), we have

(5.4) $\qquad \sum_{\omega\in\Omega} p(\omega) \cdot x_i(\omega) = \sum_{\omega\in\Omega} p(\omega) \cdot e_i(\omega).$

For all $1 \leq i \leq n$ let $\lambda_i = \frac{1}{\alpha_i}$, and let $x \in M$. Then as $L_i(x_i, \alpha_i) \geq L_i(x, \alpha_i)$ for all $1 \leq i \leq n$, (5.4) yields

(5.5) $\qquad \lambda_i h_i(x_i) \geq \lambda_i h_i(x) - \sum_{\omega\in\Omega} p(\omega) \cdot (x(\omega) - e_i(\omega)),$

for all $1 \leq i \leq n$. For every $S \in \Sigma$ let

$$\sigma(S) = \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{x}(t, \cdot)) d\mu.$$

We show that $\sigma = \varphi v_\lambda$, where $\lambda = (\lambda_1, \ldots, \lambda_n)$ and φ is a Neyman value on Q. Since $\varphi v_\lambda \in Core(v)$ and by Proposition 3.5 $|Core(v_\lambda)| = 1$, it suffices to show that $\sigma \in Core(v_\lambda)$. Let $S \in \Sigma$. We show that $\sigma(S) \geq v_\lambda(S)$. In order to prove we show that if $\mathbf{y}$ is a fine S-allocation of $\mathcal{E}$, then

$$\sigma(S) \geq \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{y}(t, \cdot)) d\mu.$$

Indeed, let $\mathbf{y}$ be a fine S-allocation of $\mathcal{E}$. Then $\mathbf{y}(t, \cdot) \in M$ for all $t \in S$. Hence (5.5) yields

(5.6) $\qquad \lambda_i h_i(\mathbf{x}(t, \cdot)) \geq \lambda_i h_i(x_i) \geq \lambda_i h_i(\mathbf{y}(t, \cdot)) - \sum_{\omega\in\Omega} p(\omega)(\mathbf{y}(t, \omega) - e_i(\omega)),$

for all $1 \leq i \leq n$ and $t \in S \cap S_i$. Since $\mathbf{y}$ is a fine S-allocation we have

$$\sum_{\omega\in\Omega} p(\omega) \cdot \int_S (\mathbf{y}(t, \omega) - e_i(\omega)) d\mu \leq 0.$$

Thus by (5.6),

$$\sigma(S) = \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{x}(t, \cdot)) d\mu \geq \sum_{i=1}^{n} \int_{S\cap T_i} \lambda_i h_i(\mathbf{y}(t, \cdot)) d\mu.$$

Hence $\sigma(S) \geq v_\lambda(S)$. From the definition of v_λ it is clear that $\sigma(T) \leq v_\lambda(T)$. This completes the proof that $\mathbf{x}$ is a fine value allocation of $\mathcal{E}$. □

6 Concluding Remarks

In this section we discuss an approach that would allow one to obtain an equivalence result analogous to Theorem A for economies with an infinite set of traders' types, under somewhat more restrictive assumptions on traders' utility functions and initial endowments.

Let $\mathcal{E}$ be an economy with a finite or an infinite set of traders' types. The market game associated with $\mathcal{E}$ is defined analogously to the case of finite types as

$$v(S) = \sup\left\{ \int_S h_t(\mathbf{x}(t,\cdot))d\mu \mid \mathbf{x} \in \mathcal{A}(S) \right\}.$$

Let Λ denote the set of all functions $\lambda : T \to \Re_+$ that are bounded and positive almost everywhere in T. For $\lambda \in \Lambda$ write $\mathcal{E}_\lambda$ for the economy obtained from $\mathcal{E}$ by replacing the utility function of every trader $t \in T$ with $\lambda(t)u_t$; also write v_λ for the market game associated with $\mathcal{E}_\lambda$.

It can be shown that if traders' utility functions satisfy the assumptions in Aumann (1975) (see also Dubey and Neyman (1997)), and if their initial endowments are strictly positive at every state of nature, then every market game v_λ belongs to a space of coalitional games (called DIFF) studied by Mertens (1980), who shows that on this space of games there is a value. Moreover, the value of a market game v_λ is the unique point in its core (see Proposition 4 in Mertens (1980)).

We can therefore use Mertens's (1980) value on the space DIFF to define the notion of value allocation analogously to the case of finite types. Let $\mathcal{E}$ be an economy. A fine allocation $\mathbf{x}$ is a fine value allocation if there exists $\lambda \in \Lambda$ such that for all $S \in \Sigma$

$$(\varphi v_\lambda)(S) = \int_S \lambda(t)h_t(\mathbf{x}(t,\cdot))d\mu,$$

where φ is a Mertens's value on DIFF. By arguments essentially identical to those used in the proof of Theorem A it can be shown that the set of fine value allocations of the economy $\mathcal{E}$ coincides with the set of competitive allocations of the associated economy $\mathcal{E}^*$.

Providing formal proofs of these results would require to introduce the theoretical framework of Mertens (1980), and it will make the proofs considerably more cumbersome. Note also that, since the assumptions on the traders' utility functions and endowments required to work on this framework are more restrictive than those we have used for finite type economies, the result suggested above is not a straight generalization of that obtained in Section 5 for finite type economies.

7 Appendix

Proof of Lemma 3.1: Let $S \in \Sigma$. We first show that $v(S) \geq f(\xi(S))$. Note that $I(\xi(S)) = I(S)$. Let $x_1, \ldots, x_n$ be $\bigvee_{j\in I(S)} \mathcal{F}_j$-measurable functions from Ω to $\Re^l_+$ such that

$$f(\xi(S)) = \sum_{i=1}^{n} \mu(S \cap T_i) h_i(x_i).$$

For every $(t, \omega) \in T \times \Omega$ let

$$\mathbf{x}(t,\omega) = \begin{cases} x_i(\omega) & t \in T_i \\ 0 & \text{otherwise.} \end{cases}$$

Then for every $t \in T$, $\mathbf{x}(t,\cdot)$ is $\bigvee_{j\in I(S)} \mathcal{F}_j$-measurable, and for every $\omega \in \Omega$ we have

$$\int_S \mathbf{x}(t,\omega) d\mu = \sum_{i=1}^{n} \mu(S \cap T_i) x_i(\omega) \leq \sum_{i=1}^{n} \mu(S \cap T_i) e_i(\omega).$$

Therefore $\mathbf{x}$ is an S-allocation and thus

$$v(S) \geq \sum_{i=1}^{n} \int_{S\cap T_i} h_i(\mathbf{x}(t,\cdot)) d\mu = \sum_{i=1}^{n} \mu(S \cap T_i) h_i(x_i) = f(\xi(S)).$$

It remains to show that $v(S) \leq f(\xi(S))$. Let $\mathbf{x} \in \mathcal{A}(S)$. For every $1 \leq i \leq n$ and $\omega \in \Omega$ define

$$x_i(\omega) = \begin{cases} \frac{1}{\mu(S\cap T_i)} \int_{S\cap T_i} \mathbf{x}(t,\omega) d\mu & i \in I(S) \\ 0 & \text{otherwise.} \end{cases}$$

Since $\mathbf{x} \in \mathcal{A}(S)$, for all $t \in T$, $\mathbf{x}(t,\cdot)$ is constant on the atoms of the field $\bigvee_{j\in I(S)} \mathcal{F}_j$, and so is x_i for all is $1 \leq i \leq n$. Therefore x_i is $\bigvee_{j\in I(S)} \mathcal{F}_j$-measurable for all $1 \leq i \leq n$. Now for all $\omega \in \Omega$ we have

$$\begin{aligned} \sum_{i=1}^{n} \mu(S \cap T_i) x_i(\omega) &= \sum_{i=1}^{n} \int_{S\cap T_i} \mathbf{x}(t,\omega) d\mu = \int_S \mathbf{x}(t,\omega) d\mu \\ &\leq \int_S \mathbf{e}(t,\omega) d\mu = \sum_{i=1}^{n} \mu(S \cap T_i) e_i(\omega). \end{aligned}$$

Therefore

$$f(\xi(S)) \geq \sum_{i=1}^{n} \mu(S \cap T_i) h_i(x_i).$$

Since for all $1 \leq i \leq n$ and $\omega \in \Omega$ the function $u_i(\omega, \cdot)$ is concave on $\Re^l_+$, the function h_i is concave on $(\Re^l_+)^\Omega$. Therefore Jensen's inequality yields

$$h_i(x_i) \geq \frac{1}{\mu(S \cap T_i)} \int_{S \cap T_i} h_i(\mathbf{x}(t, \cdot)) d\mu,$$

for every $i \in I(S)$. Hence

$$f(\xi(S)) \geq \sum_{i=1}^{n} \mu(S \cap T_i) h_i(x_i) = \sum_{i \in I(S)} \mu(S \cap T_i) h_i(x_i) \geq \sum_{i=1}^{n} \int_{S \cap T_i} h_i(\mathbf{x}(t, \cdot)) d\mu.$$

Since $\mathbf{x}$ is an arbitrary member of $\mathcal{A}(S)$, we must have $f(\xi(S)) \geq v(S)$. □

Proof of Lemma 3.2: We first show that f is concave on $\Re^l_+$. Let $y^1, y^2 \in \Re^l_+$, and $0 < \alpha < 1$. Denote $y = \alpha y^1 + (1-\alpha) y^2$. Then $I(y) = I(y^1) \cup I(y^2)$. Let $x^1_1 \ldots, x^1_n$ be members of $(\Re^l_+)^\Omega$ such that $f(y^1) = \sum_{i=1}^n y^1_i h_i(x^1_i)$, and $f(y^2) = \sum_{i=1}^n y^2_i h_i(x^2_i)$. For every $\omega \in \Omega$ and $1 \leq i \leq n$ let

$$x_i(\omega) = \begin{cases} \frac{\alpha y^1_i x^1_i(\omega) + (1-\alpha) y^2_i x^2_i(\omega)}{y_i} & i \in I(y) \\ 0 & \text{otherwise.} \end{cases}$$

Now for all $1 \leq i \leq n$ the function x^1_i is $\bigvee_{j \in I(y^1)} \mathcal{F}_j$-measurable, and x^2_i is $\bigvee_{j \in I(y^2)} \mathcal{F}_j$-measurable. As $I(y) = I(y^1) \cup I(y^2)$, for all $1 \leq i \leq n$ the functions x^1_i and x^2_i are $\bigvee_{j \in I(y)} \mathcal{F}_j$-measurable, and thus x_i is $\bigvee_{j \in I(y)} \mathcal{F}_j$-measurable. Also for all $\omega \in \Omega$ we have

$$\begin{aligned} \sum_{i=1}^{n} y_i x_i(\omega) &= \alpha \sum_{i=1}^{n} y^1_i x^1_i(\omega) + (1-\alpha) \sum_{i=1}^{n} y^2_i x^2_i(\omega) \\ &\leq \alpha \sum_{i=1}^{n} y^1_i e^1_i(\omega) + (1-\alpha) \sum_{i=1}^{n} y^2_i e^2_i(\omega) = \sum_{i=1}^{n} y_i e_i(\omega). \end{aligned}$$

Therefore

$$f(y) \geq \sum_{i=1}^{n} y_i h_i(x_i).$$

Since for all $1 \leq i \leq n$ the function h_i is concave on $(\Re^l_+)^\Omega$, we have

$$f(y) \geq \sum_{i=1}^{n} y_i h_i(x_i) \geq \sum_{i \in I(y)} y_i \left(\frac{\alpha y^1_i}{y_i} h_i(x^1_i) + \frac{(1-\alpha) y^2_i}{y_i} h_i(x^2_i)\right)$$

$$\begin{aligned} &= \alpha \sum_{i=1}^{n} y_i^1 h_i(x_i^1) + (1-\alpha) \sum_{i=1}^{n} y_i^2 h_i(x_i^2) \\ &= \alpha f(y^1) + (1-\alpha) f(y^2). \end{aligned}$$

This shows that f is concave on $\Re^n_+$.

It is clear that f is homogeneous of degree one. We show that f is non-decreasing on $\Re^n_+$. Let $y, z \in \Re^n_+$ such that $y \geq z$. Then $y - z \in \Re^n_+$. Since f is concave and homogeneous of degree one, it is superadditive. Therefore

$$f(y) = f((y-z)+z) \geq f(y-z) + f(z).$$

Since f is non-negative, we have $f(y) \geq f(z)$. Thus f is non-decreasing on $\Re^n_+$.

The continuity of f at $\xi(T)$ follows from the fact that f is concave on $\Re^n_+$ and $\xi(T)$ is in the interior of $\Re^n_+$. We show that f is continuous at 0. Let $\{y^k\}_{k=1}^{\infty}$ be a sequence in $\Re^n_+$ such that $\lim_{k\to\infty} y^k = 0$. Then there exists k_0 such that for all $k \geq k_0$ we have

$$\max\left\{y_i^k \mid 1 \leq i \leq n\right\} < 1.$$

For every k let $x_1^k, \ldots, x_n^k$ be members of $(\Re^l_+)^{\Omega}$ such that

$$f(y^k) = \sum_{i=1}^{n} y_i^k h_i(x_i^k).$$

Since for all $1 \leq i \leq n$ and $\omega \in \Omega$ the function $u_i(\omega, \cdot)$ is concave, for all $k \geq k_0$ we have

$$\begin{aligned} 0 &\leq f(y^k) = \sum_{i=1}^{n} \sum_{\omega\in\Omega} y_i^k q_i(\omega) u_i(\omega, x_i^k(\omega)) \\ &\leq \sum_{i=1}^{n} \sum_{\omega\in\Omega} q_i(\omega) u_i(\omega, y_i^k x_i^k(\omega)) \\ &\leq \sum_{i=1}^{n} \sum_{\omega\in\Omega} q_i(\omega) u_i(\omega, \sum_{j=1}^{n} y_j^k e_j^k(\omega)). \end{aligned}$$

As $\lim_{k\to\infty} \sum_{j=1}^{n} y_j^k e_j^k(\omega) = 0$ for all $\omega \in \Omega$, $(A.3)$ and $(A.4)$ imply that

$$\lim_{k\to\infty} f(y^k) = 0 = f(0).$$

Thus, f is continuous at 0. □

Proof of Lemma 3.3: The proof is based on the idea of the proof of Proposition 34.13 in Aumann and Shapley (1974).

Let $a \in \Re^n_{++}$, and let $1 \leq i \leq n$. We first show that the partial derivative $\frac{\partial f}{\partial y_i}$ exists at a. Without loss of generality, let $i = 1$. Let $\bar{x}_1, \ldots, \bar{x}_n$ be members of $(\Re^l_+)^\Omega$ such that

$$f(a) = \sum_{i=1}^{n} a_i h_i(\bar{x}_i).$$

Define the function $g : \Re_{++} \to \Re$ for $y_1 \in \Re_{++}$ by

$$g(y_1) = \sum_{\omega \in \Omega} y_1 q_1(\omega) u_1(\omega, \frac{a_1 \bar{x}_1(\omega)}{y_1}) + \sum_{i=2}^{n} \sum_{\omega \in \Omega} a_i q_i(\omega) u_i(\omega, \bar{x}_i(\omega)).$$

For every $y_1 \in \Re_{++}$ let

$$f_1(y_1) = f(y_1, a_2 \ldots, a_n).$$

Then $f_1(a_1) = f(a)$. Now for every $y_1 \in \Re_{++}$, $\frac{a_1}{y_1}\bar{x}_1$ is $\bigvee_{j \in I(a)} \mathcal{F}_j$-measurable, and for all $\omega \in \Omega$

$$y_1 \frac{a_1 \bar{x}_1(\omega)}{y_1} + \sum_{i=2}^{n} a_i \bar{x}_i(\omega) \leq \sum_{i=1}^{n} a_i e_i(\omega).$$

Therefore

$$g(y_1) \leq f_1(y_1).$$

Now since for every $\omega \in \Omega$, and every $1 \leq i \leq n$, $\frac{\partial u_i(\omega, \cdot)}{\partial x_j}$ exists at $(x_1 \ldots, x_l) \in \Re^l_+$ whenever $x_j > 0$ $(1 \leq j \leq l)$, the function g is differentiable at y_1. As f is concave on $\Re^n_+$, f_1 is concave on $\Re_{++}$. Therefore there exists an affine function l such that for all $y_1 \in \Re_{++}$, $f_1(y_1) \leq l(y_1)$, and $f_1(a_1) = l(a_1)$. Since $g \leq f_1 \leq l$ on $\Re_{++}$ and $g(a_1) = f_1(a_1) = l(a_1)$, g and l have the same derivative at a_1. Therefore f_1 is differentiable at a_1. Now by using the concavity of f on $\Re^n_+$, it can be proved (as in the proof of Proposition 39.1 in Aumann and Shapley (1974)) that the partial derivative $\frac{\partial f}{\partial y_1}$ is continuous in the interior of $\Re^n_+$. □

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