Banco de México
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Banco de México
Working Papers

N° 2011-04

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July 2011

Documento de Investigación
2011-04

Working Paper
2011-04

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Abstract

We present a model of imperfect price competition where not all firms can sell to all consumers. A network structure models the local interaction of firms and consumers. We find that aggregate surplus is maximized with a fully connected network, which corresponds to perfect competition, and decreases monotonically as the network becomes less connected until firms become local monopolists. When we study which networks are likely to form in equilibrium, we find that stable networks are not fully connected but are connected enough to rule out local monopolists. Our results extend to oligopolistic competition when consumers can either buy from a single firm or from all firms.
Keywords: Network markets, price competition, oligopoly competition, Bertrand competition.
JEL Classification: D43, D85, L11, L13.

Resumen

Presentamos un modelo de competencia imperfecta donde no todas las empresas pueden venderle a todos los consumidores. Usamos un modelo de red para modelar la estructura de interacciones locales entre empresas y consumidores. Encontramos que el bienestar total se maximiza cuando la red está completamente conectada, lo cual corresponde a competencia perfecta, y se reduce monótonamente a medida que se eliminan conexiones hasta que las empresas se vuelven monopolios locales. Cuando estudiamos cuáles redes se forman en equilibrio encontramos que las redes estables no están completamente conectadas pero están lo suficientemente conectadas para descartar que existan monopolios locales. Nuestros resultados se extienden a competencia oligopolística cuando los consumidores sólo pueden comprar a una empresa o a todas las empresas.
Palabras Clave: Mercados de redes, competencia en precios, competencia oligopolística, competencia de Bertrand.

*I am greatly in debt with Matthew Jackson for his guidance and support during this project. I also thank Doug Bernheim, Jon Levin, Ilya Segal, Andy Skrzypacz, Juuso Toikka, Ben Golub, Matt Elliot, Michihiro Kandori, Preston McAfee, Luis Rayo, attendants at the theory lunch and the members of the networks working group for their useful feedback.

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1 Introduction

Networks have become popular to model markets where buyers can only purchase from some (but not all) sellers. These models seek to understand how the structure of possible transactions influences prices and welfare, as well as studying how market incentives shape the structure of the network.

Current network models for markets are still striving to find a balance between modeling a realistic description of the market while at the same time keeping the models tractable for complicated network structures. In this paper we model the market through a standard decentralized mechanism: Bertrand price competition. We find that aggregate surplus is only maximized when the network is fully connected, which corresponds to perfect competition, and decreases monotonically as the network becomes less connected. The maximum inefficiency occurs when firms become monopolists over separate components of the network. This contrasts with previous models of networks which find that the allocation is always efficient conditional on the structure of the network.[1]

The two seminal papers in the literature are Kranton and Minehart (2001) and Corominas-Bosch (2004). Kranton and Minehart use a stylized ascending-bid auction where markets sequentially clear for sub-components of the network. Although similar in spirit to the idea of a Walrasian auctioneer, it's hard to see how this mechanism could be decentralized. Corominas-Bosch instead uses an alternating-offer bargaining protocol, but she requires that proposals be announced simultaneously by all members on each side of the market, which also implies some sort of centralization in the price setting process.

Both papers reach different conclusions about which networks are likely to form in equilibrium. Kranton and Minehart find that networks are guaranteed to be efficient, while Corominas-Bosch finds that networks tend to be over-connected. We find that stable networks are underconnected relative to the efficient network structure.[2]

Key for our results is the presence of *locked-in consumers*: consumers who can only purchase from a single firm. Without locked-in consumers, sellers always play the standard Bertrand strategies, the allocation is always efficient and the details of the network structure don't matter. This is a corollary of an important result by Byford (2007) in a setting closely related to ours. His result is strong because it can apply to any subset of sellers in any network. To be precise, take any subset S such that all buyers connected to a seller in S

[1]See Kranton and Minehart (2001), Corominas-Bosch (2004), Blume et al. (2007) and Elliot (2010).

[2]Except if the cost of connecting members is high. As we discuss below, if a regulator knows the firms' costs and connecting the network is costly, it would prefer to regulate prices and mandate that the network become less connected.

are also connected to at least one other seller in S. Byford then shows that the sellers in S always post a price equal to marginal cost.[3] For Byford's condition to hold, it's necessary that no seller in S has locked-in consumers. And if there are no locked-in consumers at all, S equals the entire set of sellers.

Another paper similar in spirit to ours is Nava (2010), who studies quantity (Cournot) competition on networks where the flow of goods is endogenous. In contrast to our model, he finds that adding links might decrease the aggregate surplus and that sellers profit from being able to discriminate in the price they charge to their buyers.

Our model is closely related to Varian's model of sales, but his main objective was to study pricing patterns when some consumers where informed and some were not, and hence, focused on a symmetric model.[4] We wish to relate prices and welfare to different network structures.

After solving the pricing game for a fixed network, we study which networks are likely to form. To do so we analyze a series of entry games where firms decide to form links with consumers. We find that stable networks are partially connected. Firms create do create links to compete over some consumers, but refrain from creating enough links to fully connect the network. Since price competition is very aggressive, firms decrease their aggressiveness by refraining from forming links to all consumers.

Most of our results are based on a model of duopoly competition because it's hard to get tractable results with more firms. We view this as a useful benchmark to clarify the results of other network models. As we show in Section 4, our results extend to oligopoly competition as long as consumers can only buy from either one firm or all firms.

The rest of the paper is structured as follows: Section 2 sets up the basic duopoly model; Section 2.1 solves an example of competition on a road network which contains most of the intuition for solving the model; Section 2.2 solves for the unique Nash equilibrium of the duopoly model; Section 2.3 does comparative static exercises for welfare; Section 2.4 analyzes what would change if we allow for price-discrimination; Section 3 analyzes which networks are likely to form; Section 4 extends the results to oligopolistic competition; and Section 5 discusses some implications of the model for standard measures of network centrality.

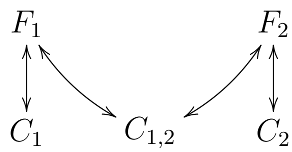

Figure 1: Consumer C_1 can only buy from Firm 1, C_2 can only buy from F_2; and a $C_{1,2}$ can buy from any firm.

2 The Duopoly Model

Our model has a finite number consumers and two firms, labeled F_1 and F_2. Firms and consumers are represented by nodes on a network. (See Figure 1.) Let C_1 denote the **number** of consumers who can buy only from Firm 1, C_2 the number who can buy only from F_2, and $C_{1,2}$ the number who can buy from both. Consumers must be linked to a firm to be able to buy its product or service.

For now, we assume the network structure is exogenously determined before any transactions are carried out and is common knowledge between the players. We will discuss how the network is formed in Section 3.

We refer to the C_1 and C_2 consumers as *locked-in consumers* and to the $C_{1,2}$ as *mobile consumers*. We label the firms such that **Firm 1 has weakly more locked-in consumers**: $C_1 \geqslant C_2$.

Consumers buy at most one item and have a value that is private information. Products from different firms are perfect substitutes. Values are drawn independently from a common distribution function. Let $Q(p)$ be the probability that a consumer has a value higher or equal to p. That is, Q is defined over $[0, \infty) \rightarrow [0, 1]$ and is decreasing. We assume Q is differentiable. For tractability, we assume that $Q(p) - pQ'(p)$ has the *single-crossing condition*, as defined below.

$$Q(p) - pQ'(p) \textit{ crosses zero once and from above.}$$

A sufficient condition to have the single-crossing condition is that Q has a decreasing elasticity (it is log-concave) and that the monopolist's problem has a well defined solution (i.e. $Q(p) - pQ'(p)$ crosses zero at least once). This assumption simplifies the proof and

[3] Byford shows this for pure strategy Nash equilibria, but his proof can be extended to mixed strategy equilibria.

[4] See Varian (1981).

the equilibrium pricing strategies, but is not essential. To get some intuition on why it is useful, we note that in equilibrium firms face a trade-off between decreasing the rents they extract from the locked-in consumers and increasing the probability they sell to the mobile consumers. The single-crossing condition ensures any decrease in price decreases the revenue from the locked-in consumers, so the trade-off is present over the entire relevant price range.

Firms engage in Bertrand competition. They simultaneously announce prices they will charge to any consumer who buys their product. Firms cannot price discriminate. (We relax this in Section 2.4.) Consumers first observe all prices and then, with probability $Q(p)$, chose to buy from the cheapest firm that's linked to them. They randomize equally across firms in case of a tie. Firms can quote any non-negative price. To find equilibria we need to allow firms to play a mixed strategy.

Definition 1. A **mixed strategy pricing scheme** for Firm j is a distribution function $\sigma_j(p) : [0, \infty) \rightarrow [0, 1]$.

Firms maximize their expected profits, which we label π_j. A subscript $-j$ denotes the firm who is not j. In the oligopoly section it denotes all firms that are not j. We assume firms have a constant marginal cost for selling their products which we normalize to zero. Firms solve the following problem.

$$\max_{\sigma_j} \pi_j(\sigma_j, \sigma_{-j}) = \max_{\sigma_j} E_{p_j}\Big[p_j\Big(C_j + C_{1,2}\big(1 - \sigma_{-j}(p_j)\big)\Big)Q(p_j)\Big]$$

To solve the game we look for Nash equilibria. Because the behavior of consumers is trivially captured by $Q(p)$, we focus on the strategies for the firms.

Definition 2. A **Nash Equilibrium** of the pricing game is a strategy profile, (σ_1, σ_2), such that no player j can deviate to an alternative mixed strategy σ'_j and get a higher payoff:

$$\pi_j(\sigma_j, \sigma_{-j}) \geqslant \pi_j(\sigma'_j, \sigma_{-j}); \forall \sigma'_j \neq \sigma_j, \forall j \in \{1, 2\}$$

We denote equilibrium strategies by a star superscript, (σ^*_1, σ^*_2), and denote by (π^*_1, π^*_2) the corresponding expected profits.

Next we develop an example to preview how to highlight the results of the paper. Those interested in going directly to the solution of the model can skip to Section (2.2).

2.1 An example

Two consumers, A and B, wish to cross a river to go to an Irish pub. To do so, they must cross through one of two available bridges. Each bridge is independently operated by a firm that charges a toll to any consumer who wishes to cross. Consumers are willing to pay up to 1 to cross the river. The only cost they incur is that of the toll. That is

$$Q(p) = \begin{cases} 1 & \textit{if } p \leqslant 1 \\ 0 & \textit{if not} \end{cases}$$

An exogenous road network limits the choice of bridges for each consumer. For example, in Figure (4), Consumer A can only cross through the bridge operated by F_1 while Consumer B can choose to cross through any bridge. Firms simultaneously post their tolls (prices). After observing them consumers choose which bridge to cross, if any.

In the Monopolistic Network, Figure (2), each firm knows it has a monopoly and quotes prices to extract all surplus:

$$p_1^* = p_2^* = 1$$

$$\pi_1^* = \pi_2^* = 1$$

The Competitive Network, Figure (3), is a standard Bertrand competition game where firms undercut each other until their profits vanish.

$$p_1^* = p_2^* = 0$$

$$\pi_1^* = \pi_2^* = 0$$

In the Mixed Case Network, Figure (4), F_1 has a locked-in consumer, C_A, from where it can extract all the valuation by charging $p_1 = 1$. Alternatively it can try to compete against F_2 for the mobile consumer, C_B, at the cost of reducing the price charged to its locked-in consumer. No pure strategy Nash equilibrium exists in this game: if any firm charges a positive price, at least one of the two firms would want to charge slightly lower price. If both firms charged zero, F_1 could profitably deviate to charging 1, extracting all the surplus of its locked-in consumer. This same logic rules out pure strategy equilibria in our duopoly model.

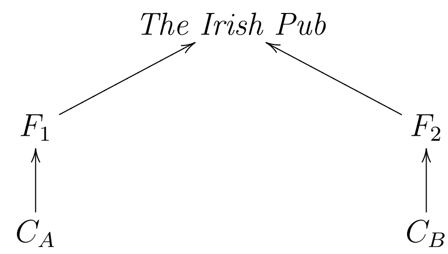

Figure 2: **(The Monopolistic Network)** Each Firm is a monopoly in their local market

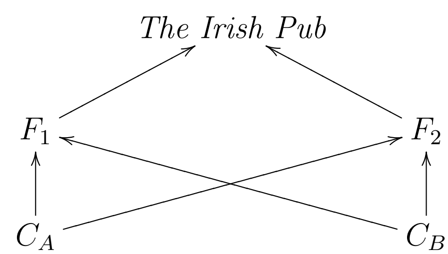

Figure 3: **(The Competitive Network)** A network with perfect competition

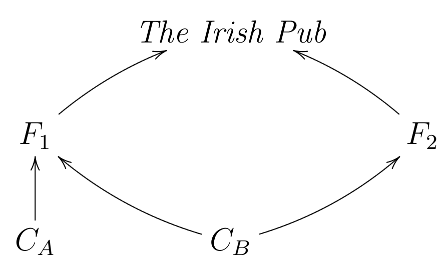

Figure 4: **(The Mixed Case Network)** An intermediate case

In this network, there is unique a mixed strategy equilibrium, as described below.

$\mathbf{F_1}$: Post $p = 1$ with probability $\frac{1}{2}$. With probability $\frac{1}{2}$ chose $p < 1$ and then price according to:

$$\sigma_1^*(p|p < 1) = 2 - \frac{1}{p}; \forall p \in [\frac{1}{2}, 1)$$

$\mathbf{F_2}$: Price according to:

$$\sigma_2^*(p) = 2 - \frac{1}{p}; \forall p \in [\frac{1}{2}, 1]$$

Equilibrium payoffs are

$$\pi_1^* = 1; \pi_2^* = \frac{1}{2}$$

From the example we can see that firms are not always better off by having access to more consumers. In the Mixed Case Network, Firm 2 receives a payoff strictly larger than in the Competitive Network. Adding a link from F_2 to C_A increases the competition and reduces profits.

Equilibrium strategies for our duopoly model are qualitatively similar to those in the example. Firm 1 quotes the price that maximizes its monopoly payoffs with a discrete probability and both firms mix continuously over prices below. We now proceed to solve the model.

2.2 Solving the Duopoly Model

Let's return to analyzing an arbitrary network with two firms and any number of consumers. To solve the model it is useful to derive two useful properties from the *single-crossing condition*. First, there exists a unique price, p^M, that maximizes revenue from locked-in consumers. Second, decreasing the price in the range below p^M monotonically decreases the revenue from locked-in consumers.

Proposition 3. For any firm with $C_j > 0$ there exists a unique price, p^M, that solves the monopolist's problem:

$$\max_{p \in [0,\infty)} pQ(p)C_j$$

This price is independent of C_j. Furthermore, for any prices p, p' such that $p < p' \leqslant p^M$, p' yields a higher revenue than p.

Proof. By the *single-crossing condition* we know the derivative of the objective function crosses zero only once and from above. Call p^M the value where it crosses zero. This is the unique maximum and does not depend on C_j, which is only a scale factor. Furthermore, for values below p^M, the derivative is positive, so any decrease in the price decreases revenues. Therefore revenue increases monotonically until it reaches its maximum. □

A direct implication of Proposition 3 is that any price above p^M is strictly dominated: decreasing the price to p^M would strictly increase the revenue from the locked-in consumers and would weakly increase the probability of winning the mobile consumers.

Proposition 4 shows that there are no pure strategy Nash equilibria except for two extreme cases. One is the standard Bertrand competition which corresponds to $C_1 = C_2 = 0$; the other is when each firm is a separate monopolist, $C_{1,2} = 0$.

Proposition 4. There is no pure strategy equilibria for the model with $C_1 > 0$ and $C_{1,2} > 0$.

Proof. A pure strategy equilibrium with a positive price can be ruled out by the standard Bertrand argument as follows. First, take a strategy profile (p_j, p_{-j}) such that $p_{-j} < p_j \leqslant p^M$. Player $-j$ would wish to deviate to pricing arbitrarily closer to p_j. Second, assume $p_j = p_{-j} > 0$, then any player could charge an arbitrarily smaller price obtaining a discrete gain from avoiding the probability of a tie while charging essentially the same price. Finally, assume $(p_j, p_{-j}) = (0, 0)$, then F_1 could deviate to extracting a positive profit by quoting p^M and selling to its locked-in consumers. □

We therefore look for *mixed strategy nash equilibria.* Proposition 5 solves for the unique equilibrium.

Proposition 5. There is a unique equilibrium of the game. Its associated payoffs are

$$\pi_1^* = p^M Q(p^M) C_1 \quad \pi_2^* = \pi_1^* \tfrac{C_2 + C_{1,2}}{C_1 + C_{1,2}}$$

For $p < p^M$, firms mix according to

$$\sigma_j(p) = 1 + \frac{C_{-j}}{C_{1,2}} - \frac{\pi_{-j}}{pQ(p)C_{1,2}}$$

Firm 1 charges p^M with probability $\left(\frac{C_1 - C_2}{C_1 + C_{1,2}}\right)$. Firm 2 charges p^M with probability 0.

Proof. See Appendix (A). □

2.3 Welfare

We can use the payoffs in Proposition 5 to do comparative statics on welfare by changing the number of consumers. Firm 1's payoff does not depend on the size of the mobile market, $C_{1,2}$, nor on the size of Firm 2's locked-in market, C_2. F_1 gets a profit which is exactly the same as if it extracted the monopoly surplus from its locked-in consumers. F_1 does sell to the mobile consumers with a positive probability, but since in equilibrium F_1 must be indifferent between quoting the monopoly price and quoting prices below, any increase in expected sales must be exactly offset by the decrease in price.

Firm 2 has a profit that is strictly increasing the number of its locked-in consumers and strictly increasing in the number of mobile consumers, $C_{1,2}$. Its profits are also strictly increasing in the number of locked-in consumers Firm 1 has, because a larger C_1 makes Firm 1 less aggressive, which is beneficial for Firm 2. Firm 2's profits are smaller than those of Firm 1, but larger than the monopoly rents it could get from its locked-in consumers.

$$\pi_1^* \geqslant \pi_2^* \geqslant C_2 p^M Q(p^M)$$

With strict inequality whenever $C_1 > C_2$ and $C_{1,2} > 0$, respectively.

Every consumer is weakly better off by any increase in the number of mobile consumers, $C_{1,2}$, because the distribution of prices shifts downwards. The old distribution first-order stochastically dominates the new distribution.

Aggregate surplus is maximized by equating demand to marginal cost. Because of we normalized marginal cost to zero this is equivalent to producing to fulfill all demand. Higher prices create deadweight loss. Therefore increasing $C_{1,2}$ also increases aggregate surplus.

2.4 Price Discrimination

Firms prefer not to be able to price-discriminate between mobile and locked-in consumers. If both firms could discriminate in prices, profits from the mobile consumers would completely dissipate. F_1 is indifferent between being able to discriminate or not, but F_2 strictly prefers not to be able. If there were an arbitrarily low cost to be able to discriminate, F_1 would also prefer it were not possible.

To be clear, each firm would wish to be able to discriminate if the other firm could not. If a firm is the only one with the ability to discriminate, it can compete for the mobile consumers without sacrificing any rents from its locked-in consumers. The discriminating firm would get the same profit from the mobile consumers as Firm 2 in our duopoly model with $C_2 = 0$. The firm's total profit would be that plus the monopoly rents from its locked-in consumers. As long as the other firm had some locked-in consumers, the discriminating firm would get a positive gain from this.

The table below summarizes this. All payoffs have to multiplied by $p^M Q(p^M)$. The table is not symmetric for the case where both firms cannot price-discriminate because Firm 2 is able to compete more aggressively.

Equilibrium Payoffs	F_2 can discriminate	F_2 cannot discriminate
F_1 can discriminate	C_1,C_2	$C_1 + C_2\frac{C_{1,2}}{C_2+C_{1,2}}$, C_2
F_1 cannot discriminate	C_1, $C_2 + C_1\frac{C_{1,2}}{C_1+C_{1,2}}$	C_1,$C_1\frac{C_2+C_{1,2}}{C_1+C_{1,2}}$

The table is not necessarily a game that firms play. Whether price discrimination is possible or not, and how costly it would be to implement, depends on the application at hand. For example, in the bridge example it might not be cost-effective to identify the geographical origin of each driver when a large number of them is crossing at rush hour.

3 Which networks are likely to form?

Our analysis up to now has taken the network as given. Given that the network structure can have a big influence on welfare it is important to know which networks are more likely to arise. To do so we will proceed in two different ways.

We will first ignore the precise protocol of how links are formed and look for a solution from cooperative game theory. We will look for networks that are *pairwise-stable* with respect to the payoffs induced by the pricing game. We find that stable networks are always between the two extreme cases of networks with local monopolists and networks with perfect competition.

We will next analyze two "entry" games where we precisely specify how and when firms can form links. These games will consist of two stages, an entry stage and a pricing stage. The network will be determined at the entry stage and will be fixed during the pricing stage. The pricing stage will correspond exactly to our pricing model. It will turn out that the equilibria of the entry game are a subset of the pairwise-stable networks.

Entry games are more than a device to understand which networks are likely to form. They are interesting in their own right and have a long tradition in game-theoretic models of industrial organization.[5] Their aim is to understand under what circumstances and by what actions, if any, an incumbent firm can **credibly** deter competitors from entering its market.

The entry games considered here are different from the traditional ones because entry can be partial: we allow firms to establish links with some consumers without having to grant access to all consumers in the incumbent's niche.

3.1 Pairwise-stable networks

Fix the total number of consumers,[6] for a given network we ask if a firm has an incentive to add or delete links to the existing consumers. By adding and deleting links, firms change the network. The payoff for a firm from a network will be its expected payoff of the pricing game that would be carried out under such a network. When we say firms have incentives to add or delete links, we are doing a comparative statics across the payoffs from different networks in our pricing model. We assume there is no cost to forming links. Unlinked firms and consumers get a payoff of zero. We want to characterize all the networks that are *pairwise-stable.*

Definition 6 (Pairwise-stable Networks)**.** A network is pairwise-stable if :

1. No disconnected firm-consumer pair can become linked and increase the payoff of both members.

2. No firm or consumer can delete one of their links and increase their individual payoff.

The Competitive Network of the tollway example we analyzed in Section 2.1 was not pairwise-stable. Any firm could increase its profit by deleting a link. Both the Monopolistic Network and the Mixed Network were pairwise stable, even though Firm 2 receives a strictly smaller payoff in the later. This because Firm 1 has no strict incentive to add or remove the link that connects it to Consumer B.

Only the firms' incentives determine if a network is pairwise-stable because adding a link between any firm and any consumer increases the expected payoff for **all** consumers.

Firms would never want to delete links to their locked-in consumers and would always want to add a link to consumers that are not linked to the other firm.

[5] Tirole (1988) is the required reference.

[6] This is one of the only parts of the paper where the fact that the number of consumers are integers matters. Multiplying the number of all consumers by a constant does not change the strategic incentives. Thus, allowing the number of consumers to be a continuous variable can be thought as an approximation to large numbers of consumers with some adequate rescaling.

Firm 1 is always indifferent between adding links or not to consumers that are linked to Firm 2. This would only increase $C_{1,2}$ and decrease C_2 by one, but would not affect its equilibrium payoffs.

Firm 2 would have an incentive to add a link to a consumer of type C_1 if and only if:

$$C_2 + C_{1,2} + 1 < C_1$$

That is, only if F_2's potential consumers are not larger than Firm 1's locked-in market after the links are added.

Except for integer constraints, Firm 2 would want to increase its links at low levels of $C_{1,2}$ (i.e. for those with $C_2 < C_1$) but would want to decrease them at high levels ($C_2 > C_1$). In this sense, pairwise-stable networks are bounded away from the networks with local monopolists and from those with perfect competition.

3.2 Entry Game 1

Suppose Firm 1 is the incumbent and initially has links with all potential consumers. Firm 2 enters by forming links with the consumers. It can decide to form any number of links. After Firm 2 makes its decision, the network is fixed and the pricing game is played out.

How many links would Firm 2 wish to form? What would be the final network formed? Firm 2 solves:

$$\max_{0 \leqslant E \leqslant C_1} (C_1 - E) p^M Q(p^M) \frac{E}{(C_1 - E) + E}$$

Where E is the level of entry measured by the number of links formed to enter.

Using the results in Section 3.1 we know that Firm 2 would continue to add links while the following condition holds:

$$E + 1 < C_1 - E \Rightarrow E^* = \frac{C_1 - 1}{2}$$

Therefore Firm 2 would stop forming links much before it takes away all of Firm 1's locked-in market, to keep Firm 1 from being too aggressive in the subsequent pricing game. Roughly speaking it would only enter half of the potential market.

A similar effect has been previously shown in the entry games literature. It was labeled by Fudenberg and Tirole (1984) as the "fat cat effect". The "fat cat effect" refers to a situation of strategic over-investment in capital by the incumbent firm to act less aggressive conditional on an entry it knows it cannot deter. The effect is also present in the multi-market oligopoly model of Bulow et al. (1985).

Our focus on the entry stage is different than the previous models where the incumbent chose to be a "fat cat" by over-investing in capital. In our model it's the entrant who choses to keep the incumbent as a "fat cat" by only entering partially.

Would Firm 1, given a chance, wish to delete links to its initial consumers to accommodate Firm 2 in better terms? Could it perhaps even deter entry by sustaining a "lean and hungry" stance described by Fudenberg and Tirole (1984)? The answer is no. Suppose Firm 1 preemptively broke C_2 links, Firm 2 would form links to any unlinked consumer and then continue to form links while the following condition holds.

$$C_2 + E + 1 < C_1 - C_2 - E \Rightarrow E^* = \frac{C_1 - 1}{2} - C_2$$

Firm 1 ends up with the same amount of locked-in consumers regardless of how many links it initially deletes.[7] Since Firm 1's payoff only depends on this amount, it would never wish to break links to accommodate Firm 2.

3.3 Entry Game 2

Suppose now that initially Firm 1 and Firm 2 have two separate monopolies. That is, at the initial stage there are only locked-in consumers. We keep the convention that Firm 1 starts out as the dominant firm: $C_1 > C_2$. The game starts with an entry stage where firms are allowed to simultaneously form links with consumers in the other firms locked-in market. Once all links are formed, the network is fixed and the pricing game is carried out.

This could be the case of two separate regional monopolies held in place by a restriction to sell across borders. The entry stage occurs after a free trade agreement lifts restrictions across borders, but firms still have to decide in which locations across the border they want to open a point of sale. It could also represent two similar products that in the pre-entry stage could not reach the same consumers because of regulation or technological restrictions. This happened in the phone and cable service markets where technological advances allowed cable providers to supply phone services and vice-versa, although companies still had the ability to decide in which regions they would operate.

In the entry stage, firms simultaneously announce the number of links they will form with the other firm's locked-in consumers. This announcement is their level of entry. We seek to find how many links each firm would form in a Nash equilibrium.

[7] Unless Firm 1 broke links with more than half its locked-in consumers, in which case it would have an even smaller payoff!

Proposition 7. The set of Nash Equilibria of the Entry Game 2 is:

$$E_1^* \in Integer\big([0, C_2]\big); \quad E_2^* = Integer\big((\tfrac{C_1-C_2}{2} - 1, \tfrac{C_1-C_2}{2}]\big)$$

Where the *Integer* function maps intervals to the set of integers inside that interval.

Proof. Their profits from a strategy profile (E_1, E_2) are $\pi_1(E_1, E_2)$ as described below.

$$\pi_1(E_1, E_2) = \begin{cases} (C_1 - E_2)p^M Q(p^M); & \text{if } C_2 - E_1 \leqslant C_1 - E_2 \\ (C_2 - E_1)\frac{C_1+E_1}{C_2+E_2} p^M Q(p^M); & \text{if } C_2 - E_1 > C_1 - E_2 \end{cases}$$

$$\pi_2(E_1, E_2) = \begin{cases} (C_1 - E_2)\frac{C_2+E_2}{C_1+E_1} p^M Q(p^M); & \text{if } C_2 - E_1 \leqslant C_1 - E_2 \\ (C_2 - E_1)p^M Q(p^M) & \text{if } C_2 - E_1 > C_1 - E_2 \end{cases}$$

First we rule out any strategy profile that makes Firm 2 the dominant firm after entry. Assume we have an equilibrium such that:

$$C_2 - E_1 > C_1 - E_2$$

This can only be an equilibrium if Firm 1 sets $E_1 = 0$. For this strategy Firm 2's best response is $Integer[\frac{C_1-C_2}{2}]$, which is not enough to make Firm 2 the dominant firm after entry. This is a contradiction. We conclude there are no equilibria where this happens.

Since Firm 1 must remain the dominant firm, Firm 2's unique best response is

$$E_2^* = Integer\Big(\Big(\frac{C_1 - C_2}{2} - 1, \frac{C_1 - C_2}{2}\Big]\Big)$$

For this profile, Firm 1's best response set is

$$E_1^* \in Integer\big([0, C_2]\big)$$

$\square$

Firm 2 enters partially in any equilibrium. The level of entry is always low enough such that Firm 1 still remains the dominant firm after entry. Firm 1 has multiple best responses but has no strict incentive to enter Firm 2's locked-in market. Doing so does not alter its payoff nor alter Firm 2's strategic incentives. Adding an arbitrarily small cost to forming links would reduce the set of equilibria to a unique strategy profile where only Firm 2 enters as described above.

Consumer surplus and aggregate surplus strictly increase with the partial entry. By definition, in an entry equilibrium firms have no incentive to change their links. Since only firms' incentives matter to determine which networks are pairwise stable, equilibrium networks must necessarily be pairwise stable.

3.4 Regulating the network

If adding links is costly, as is the case for infrastructure networks, it is wasteful to add links that do not increase the feasible outcomes. If a regulator had perfect information about $Q(p)$ and about costs, she should mandate that the network be composed of disconnected monopolies and then set the price equal to the marginal cost.

On the other hand, if it is easier for the regulator to control the connectivity of the network instead of the price, she should force a connectivity between the firms in excess of what they would chose to do on their own. Even though firms with low numbers of locked-in consumers have an incentive to build the infrastructure to partially compete with the other firms, in equilibrium they do no generate sufficient links. The regulator should therefore force the firms to be more interconnected.

4 Extending the results for oligopolistic competition

For applications it would be useful to extend the model to allow for an arbitrary number of firms. Unfortunately the model quickly becomes intractable, because the potential types of consumers include all possible subsets of firms, which grow exponentially.

Here we solve a special case of oligopolistic competition where consumers come in two types: locked-in consumers who can buy only from one firm and perfectly mobile consumers who can buy from all firms. We find that the equilibria are very similar to the duopoly pricing equilibrium.

There are several applications that fit this environment. For example, a situation where some people are already locked into a specific technology while other people are waiting for the prices to be determined before they decide which technology to adopt. Small fixed-costs for adopting technology could be easily included in the model. Another pertinent environment is one where some consumers buy online and can see the all the price information, while others just go to their local store and cannot react to prices from other firms. A third application is treating the network as a model of brand loyalty. Firms face a trade-off between tendering to completely loyal consumers or competing for extreme price seeking consumers.

The model we present here is very similar to Varian's model of sales, except that we allow firms to have different amounts of locked-in consumers. This will force the equilibria to be asymmetric, while Varian focuses on symmetric outcomes.

In this model the competition will only happen between the two most aggressive firms, those with the lowest opportunity cost for lowering their prices. All other firms will give up on capturing the mobile market and stay out by setting the monopoly price.

4.1 An arbitrary number of firms competing in a single market

The model we are considering has a finite number J of firms each with some of locked-in consumers. We label firms such that:

$$C_1 \geqslant C_2 \geqslant \ldots \geqslant C_J > 0$$

In addition to their locked-in consumers, all firms can compete to sell in a large global market that has a number C_G of consumers. Consumer values and the pricing game are as before.

There is always an equilibrium where Firms J and $J-1$ play as described in the duopoly model while all other firms price p^M. We refer to this pricing behavior as "staying out" because the equilibrium probability that they will capture the global market is zero. Proposition 8 proves this is an equilibrium.

The intuition behind the proof is that firms face a trade-off between extracting all the surplus from their locked-in consumers or lowering their price to try to capture the global consumers. There are two effects that keep firms with more locked-in consumers out. They face a higher opportunity cost from lowering prices; and if they deviate from the prescribed equilibrium they have to quote a lower price than all firms who go in to win the global market. This two forces move in the right direction to sustain the equilibrium.

These strategies turn out to be the unique equilibrium when we assume that only two firms have the smallest number of consumers. That is, when $C_{J-2} > C_{J-1}$. This is proven in Proposition 9. Uniqueness follows because there is a price interval where firms J and $J-1$ must *necessarily* mix as in the duopoly model. Because of this higher firms can never be made indifferent between quoting the monopoly price and a lower one.

Proposition 8. The following strategies always constitute an equilibrium of the oligopoly model: Firms $J-1$ and J play as in the duopoly model, all other firms just chose $p_j = p^M$ with probability one.

Proof. See Appendix (B). $\square$

Proposition 9. If $C_{J-2} > C_{J-1} > C_J$, the strategies in Proposition 8 constitute the unique equilibrium of the game.

Proof. See Appendix (C). $\square$

The assumption that $C_{J-2} > C_{J-1}$ is absolutely necessary for uniqueness. If it fails we can always construct symmetric equilibria where all firms that have the two lowest values for C_j enter with positive probability. All these equilibria are payoff equivalent. Assuming $C_{J-1} > C_J$ only simplifies the proof.

5 Standard network centrality measures are inappropriate

Standard measures of network centrality rank positions with more links as being better, but this is inappropriate in our model. The problem is that standard centrality measures only use the information on the topology of the network, but do no provide an explicit model for how agents would exploit their positions to their advantage.

For example, the *in-betweenness measure* counts all the shortest paths that cross a given node to measure the centrality of each position.[8] The implicit assumption is that the shortest paths are the cheapest paths and that nodes on the path can extract surplus from agents who wish to cross their position.

As our model shows, this is clearly inappropriate, because in equilibrium agents try to extract the maximum surplus from their position, which endogenously determines which paths are cheaper. Arbitrage considerations pushes the cost of the shortest paths toward something that should be similar to the cost of alternatives. Therefore, an explicit model of competition is required to determine the ability of each agent to extract the surplus from her position.

In our model, the number of links is a deceiving measure of profits. For example, compare Firm 2 in Figures 3 and 4. Firm 2 has a higher in-betweenness measure in the competitive

[8]See Jackson (2008) for the definition of this and other centrality measures.

network, because she has access to more consumers, but the increase in its connectivity is more than offset by the increase in competition, which drives profits to zero.

The intuition behind the in-betweenness measure is not completely wrong, being on more paths sometimes is beneficial. The problem is that Bertrand competition is very aggressive, so in our model there is a very real trade-off between connectivity and profits. Therefore, we need more models with economic foundations to evaluate when centrality measures based solely on the network topology are appropriate.

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A Solving for the unique equilibrium of the duopoly model. Proposition 5.

Proof. The *mixed strategy indifference condition* states that in equilibrium we must have

$$pQ(p)\Big(C_j + C_{1,2}\big(1 - \sigma^*_{-j}(p)\big)\Big) = \pi^*_j; \forall p \in support(\sigma^*_j) \tag{1}$$

$$pQ(p)\Big(C_j + C_{1,2}\big(1 - \sigma^*_{-j}(p)\big)\Big) \leqslant \pi^*_j; \forall p \notin support(\sigma^*_j) \tag{2}$$

The general strategy of the proof is to first find the equilibrium support, then find the equilibrium payoffs and then use the *mixed strategy indifference condition* to pin down the strategies.

Let p^{min} be the minimum price Firm 1 is willing to charge if it is guaranteed to sell to all consumers. This price solves the following equation.

$$p^{min}Q(p^{min})\big(C_1 + C_{1,2}\big) = p^M Q(p^M)C_1$$

The single-crossing condition guarantees that p^{min} is unique. The following properties are useful for the rest of the proof.

- **No mixing beyond p^M:** *Firms cannot charge a price above p^M with positive probability.* This is true because above this price the marginal revenue from their locked-in consumers is negative and firms can weakly increase their probability of selling to the mobile consumers by lowering their price.

- **The common support property:** *The support of each firm's strategies must be the same.* On the contrary, suppose (p', p'') is in the support of firm j but not of firm $-j$. Then Firm j is not maximizing because σ_{-j} is constant over this interval but $pQ(p)$ is strictly increasing (by the *single-crossing condition*). Therefore Firm j could profitably shift probability mass to p''.

- **The no gaps below the top property:** *In equilibrium there cannot be an interval $(p', p'']$ with $p'' \leqslant p^M$ such that p' is in the support of the equilibrium strategies but $(p', p'']$ is not.* Suppose there is such an interval, then $\sigma_j(p') = \sigma_j(p'')$ so $\pi_{-j}(p', \sigma^*_j) < \pi_{-j}(p'', \sigma^*_j)$ by the *single-crossing condition.* This contradicts the fact that p' is in the support of the equilibrium strategies.

- **The no ties property:** *Ties cannot occur with positive probability.* Ties at a price

p only happen if both firms have an atom at p. This cannot happen in equilibrium because any firm could profitably deviate to charging arbitrarily below of p, obtaining a discrete increase in the probability of winning the mobile consumers.

We now continue with the rest of the proof.

Step 1: Firm 2 makes profits above the monopoly rents from her locked-in consumers.

Since Firm 1 would never charge below p^{min} with a positive probability, Firm 2 can charge a price arbitrarily close (from below) to p^{min} and sell to all mobile consumers. From here we conclude that in equilibrium, Firm 2 has a strictly higher profit than the profit from only selling to its locked-in consumers.

$$\begin{aligned}
\pi_2^* &\geqslant \lim_{p\uparrow p^{min}} \pi_2(p,\sigma_1^*) = \big(C_2 + C_{1,2}\big)\frac{p^M Q(p^M) C_1}{C_1 + C_{1,2}} \\
&= \Big(\frac{C_1}{C_1 + C_{1,2}}\Big)\Big(\frac{C_2}{C_2 + C_{1,2}}\Big)^{-1} p^M Q(p^M) C_2 \\
&> p^M Q(p^M) C_2
\end{aligned}$$

Step 2: In equilibrium, Firm 1 has an atom at p^M.

From the *no ties property* and from *no mixing beyond* p^M, we know that $\lim_{p\uparrow p^M} \sigma_j(p) = 1$ holds for at least one firm . Therefore, from the *no gaps property* and the *common support property*, we know both firms actively mix in an interval below p^M. In particular, for Firm 2 we have.

$$\pi_2^* = \lim_{p\uparrow p^M} \pi_2(p,\sigma_1^*) = p^M Q(p^M)\Big(C_2 + C_{1,2} \lim_{p\uparrow p^M} \big(1 - \sigma_1^*(p)\big)\Big)$$

Since $\pi_2^* > p^M Q(p^M) C_2$ it must be that $\lim_{p\uparrow p^M} \sigma_1(p) < 1$. We conclude that F_1 must have an atom at p^M.

Step 3: The support of the equilibrium strategies is $[p^{min}, p^M]$.

We know p^M is the upper bound of the support from the *no gaps below the top* property and the *no mixing beyond* p^M property. We also know Firm 1 would never quote a price below p^{min}. Since Firm 1 has an atom at p^M and there are no ties, her equilibrium payoff is $C_1 Q(p^M) p^M$. If the equilibrium strategies are not mixing in an interval above p^{min}, Firm 1 could charge a price arbitrarily close (from above) to p^{min} and get a higher payoff. We conclude that p^{min} is the lower bound of the support for the equilibrium strategies.

Step 4: Solving for equilibrium.

Since $[p^{min}, p^M]$ is the support of the equilibrium strategies, we know that the *mixed strategy indifference condition* must hold with equality over this range, so we only need to figure out the equilibrium payoff to get the equilibrium strategies. Since F_1 has an atom at p^M but F_2 does not, we know $\pi_1^* = C_1 p^M Q(p^M)$. As argued above, we know $\pi_2^* \geqslant p^{min} Q(p^{min})(C_2 + C_{1,2})$. Adding the *mixed strategy indifference condition* we get the following.

$$p^{min} Q(p^{min})\Big(C_2 + C_{1,2}\big(1 - \sigma_1^*(p^{min})\big)\Big) = \pi_2^* \geqslant p^{min} Q(p^{min})(C_2 + C_{1,2})$$

Which implies $\sigma_1^*(p^{min}) = 0$ and $\pi_2^* = p^{min} Q(p^{min})(C_2 + C_{1,2})$. Plugging this into the *mixed strategy indifference condition*, we get (σ_1^*, σ_2^*).

$$\sigma_1^*(p) = \begin{cases} 1 + \frac{C_2}{C_{1,2}} - \left(\frac{p^M Q(p^M) C_1}{pQ(p) C_{1,2}}\right)\left(\frac{C_2 + C_{1,2}}{C_1 + C_{1,2}}\right) & \text{if } p < p^M \\ 1 & \text{if } p = p^M \end{cases}$$

$$\sigma_2^*(p) = 1 + \frac{C_1}{C_{1,2}} - \frac{p^M Q(p^M) C_1}{pQ(p) C_{1,2}}$$

□

B The equilibrium of the duopoly model is also an equilibrium of the oligopoly model. Proposition 8.

Proof. Take any $p \in [0, p^M]$. For each firm we will show that the expected profit from quoting p is smaller or equal than the equilibrium payoff. From the duopoly model, Section 2.2, we know $\pi_j(p) \leqslant \pi_j^*$ for firms J and $J-1$ and for any price p. We want to show the same for any other firm j.

$$\begin{aligned} \pi_J(p; \sigma_{-J}) &\leqslant \pi_J^* \\ pQ(p^M)\left[C_J + C_G(1 - \sigma_{J-1}(p))\right] &\leqslant p^M Q(p^M) C_J \\ pC_G(1 - \sigma_{J-1}(p)) &\leqslant (p^M - p) C_J \\ \Rightarrow pC_G(1 - \sigma_{J-1}(p)) &\leqslant (p^M - p) C_j \\ \Rightarrow pC_G(1 - \sigma_{J-1}(p))(1 - \sigma_J(p)) &\leqslant (p^M - p) C_j \\ pQ(p^M)\left[C_j + C_G(1 - \sigma_{J-1}(p))(1 - \sigma_J(p))\right] &\leqslant p^M Q(p^M) C_j \\ \pi_j(p; \sigma_{-j}) &\leqslant \pi_j^* \end{aligned}$$

If $C_j > C_{J-1}$, the last inequality is strict.

□

C The equilibrium of the duopoly model is the unique equilibrium of the oligopoly model. Proposition 9.

Proof. To prove uniqueness we must use some of the properties of equilibrium strategies we found for the duopoly model. Below we restate the properties so they apply to the oligopoly model. The proofs are the same as before.

- **Common support:** For any price range where a firm is mixing, at least one other firm must also be mixing.

- **No gaps below the top:** For any $p' < p'' \leqslant p^M$ with p' in the support of at least one firm's strategy, it must be that at least two firms are actively mixing over $(p', p'']$.

- **No ties with positive probability:** Firms cannot have atoms below p^M and at least one firm must have $\lim_{p \uparrow p^M} \sigma_j(p) = 1$. This guarantees that if several firms have an atom at p^M they will not tie for the mobile consumers with positive probability.

Continuing with the proof, first we show that for a low enough interval, only Firms J and $Firm J-1$ can be actively mixing and do so according to the duopoly strategies. Define p_j^{min} as the unique price that makes Firm j indifferent between capturing the mobile market and extracting the monopoly rents from its locked-in consumers. We have

$$p_J^{min} < p_{J-1}^{min} < p_{J-2}^{min} \leqslant \ldots \leqslant p_1^{min}$$

Since there can't be ties at the top, there is at least one firm who doesn't have an atom at p^M. All other firms must have an expected utility of $p^M Q(p^M) C_j$, by the *no gaps below the top* property. Since only Firm J would be willing to price in $[p_J^{min}, p_{J-1}^{min})$ it can always guarantee herself a payoff higher than its monopoly rents. Therefore it cannot have an atom at the monopoly price. Firms J and $J-1$ are the only one who can be mixing in $[p_{J-1}^{min}, p_{J-2}^{min})$ and they must do so according to the duopoly strategies.

Next we show that all other firms strictly prefer to stay out. Let $p < p^M$ be a price such that below p only Firms J and $J-1$ are actively mixing. We know they must be mixing with the duopoly strategies. Firms J and $J-1$ are indifferent between charging p and p^M. Since $C_{J-2} > C_{J-1}$, from the proof in Proposition 8 we verify that all other firms **strictly** prefer to charge p^M. Therefore the other firms cannot be made indifferent between p^M and any price below. We conclude they all stay out with probability one and the strategies in Proposition 8 are the unique equilibrium. □