PhD Thesis

RISK TAKING IN FINANCIAL MARKETS: A BEHAVIORAL PERSPECTIVE

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September, 2007
Acknowledgements

This thesis studies behavioral characteristics of human beings and how they influence the risk taking decisions of individuals in financial markets. In a broad sense, any human action involves consequences, which are typically uncertain, and so we are constantly assuming risks. More than an alternative way to approach financial problems, we propose a novel view of how people deal with risks. This work wouldn’t be possible without the help of several people.

First of all, I have to thank my supervisors Juan Ignacio Peña and Benjamin Tabak. Ignacio believed in my ideas from the very first day and was able to guide me positively throughout the creation process. Benjamin was also essential, notably in the scope definition and in the experiment performed in Brazil. I am eternally in debt with them.

I am also grateful to my professors from the doctoral programme, and particularly to Luis Gomez-Mejia and Luis Ubeda Rives, who provided me the first insights, related to the behavioral topic and first motivated me to accept this challenge to join psychology and finance in the same framework. Professors Jaime Ortega, Mikel Tapia, Margarita Samartín and Josep Tribó were also of fundamental importance teaching me the financial and economic tools to address the finance subject.

My friends from the doctoral programme, Sabi, Jordi, Eduardo, Augusto, Kike, Peter, Kremena, Silviu, Santiago, Rômulo, among others, were also crucial in my motivation process.

I want to specially thank my wife Luciana for her support and understanding that a PhD is a joint project for a couple. To love is to share emotions and thanks for sharing these years with me.
The help of my family was also infinite. I always have to thank them for giving me the confidence to go for my objectives. My mother Maria Luiza gave me the wings to fly away and discover the world. Mom, this thesis is for you!

This thesis also benefited from financial support from the Business Administration Department of Universidad Carlos III de Madrid. With it I was able to take part in several international conferences in which I could discuss and better define my ideas. I also could carry through the experiment which is an important part of the thesis.

Finally I want to thank the Central Bank of Brazil for the opportunity to take part in its specialization programme. I am sure that my knowledge is going to be helpful in my daily tasks at the Central Bank. I want to give a special thank to Isabela Maia, Antônio Francisco, Paulo Cacella, José Renato Ornelas and my other workmates at the Executive Office for Monetary Policy Integrated Risk Management for several fruitful discussions which surely enhanced this thesis.
“Behavior Finance is a fascinating area, a course of self analysis. The more we learn about it, more we realize that each of us fail in traditional tests of rationality in an unsuspected way. Von Newmann-Morgenstern, despite of their brilliant analysis, omitted relevant pieces of the history.”

*Peter Bernstein (1997)*

*Against the Gods: The Remarkable Story of Risk.*
ABSTRACT

Since the innovative work of Kahneman and Tversky (1979), behavioral finance has become one of the most active areas in financial economics. As compared to traditional models in this area, behavioral models often have the degree of flexibility that permits reinterpretation to fit new facts. Unfortunately, this flexibility makes it hard either to disprove or to validate behavioral models. In the present thesis we try to overcome this problem, by proposing a general framework based on stylized facts of human behavior (invariants); and applying it to three financial contexts. Related to the individual risk taking decision, we focus: on the role of incentives; on how prior outcomes influence future decision; and on the portfolio choice problem. Different from traditional models where risk aversion is usually assumed, in our behavioral framework, the risk preference of the investor varies depending on how he frames his choices. Our main conclusion is that absolute evaluations based on final wealth are limited and the relativity of risk taking decisions, where the perception of gains and losses drives the process, is a requirement to understand individual’s decisions. As a reply to behavioral critics, we reach propositions that can be falsified and make several predictions.

Summing up, under the same theoretical framework we make the following contributions: (1) Shed more light on how incentives affect risk taking behavior, proposing a model where the reference plays a central role in the previous relationship; (2) Empirically test our incentives model considering a comprehensive world sample of equity funds (cross-section data); (3) Evaluate which effect is stronger in multi-period risk taking decisions for individual investors: house-money or disposition effect, proposing a novel experiment treatment; (4) Compare survey to experimental results finding that they are not necessarily aligned. (5) Generalize the myopic loss aversion
behavior performing an experiment in Brazil and Spain; (6) Propose a theoretical model of how behavioral individuals choose their portfolios in terms of risk taking behavior; (7) Include estimation error in the behavioral portfolio analysis; (8) Evaluate whether theoretical models of portfolio choice should adapt or moderate the individual behavior characteristics. In this case we used a sample of several country equity indices.

Our propositions suggest that managers in passive managed funds tend to be rewarded without incentive fee and be risk averse. On the other hand, in active managed funds, whether incentives will reduce or increase the riskiness of the fund will depend on how hard it is to outperform the benchmark. If the fund is (un)likely to outperform the benchmark, incentives (increase) reduce the manager’s risk appetite. Furthermore, the evaluative horizon influences the trader’s risk preferences, in the sense that if traders performed poorly (well) in a period, they tend to choose riskier (conservative) investments in the following period given the same evaluative horizon. Empirical evidence, based on a comprehensive world sample of mutual funds is also presented in this chapter and gives support to the previous propositions. Related to the house-money and disposition effect, we employ a survey approach and find evidence of house money effect. However, in an experiment performed in a dynamic financial setting, we show that the house money effect disappears, and that disposition is the dominant effect. We report supportive results related to existence of myopic loss aversion across countries and some evidence of country effect. Finally, in terms of asset allocation, our results support the use of our behavioral model (BRATE) as an alternative for defining optimal asset allocation and posit that a portfolio optimization model may be adapted to the individual biases implied by prospect theory without efficiency loss. We also explain why investors keep on holding, or even buy, loosing investments.
RESUMEN

Desde el trabajo innovador de Kahneman y de Tversky (1979), las finanzas del comportamiento se han convertido en una de las áreas más activas en la economía financiera. Comparados con los modelos tradicionales en esta área, los modelos del comportamiento tienen a menudo el grado de flexibilidad que permite su reinterpretación, ajustándoles a nuevos hechos empíricos. Desafortunadamente, esta flexibilidad hace difícil refutar o validar modelos del comportamiento. En la actual tesis intentamos superar este problema, proponiendo un marco general basado en hechos estilizados del comportamiento humano (invariables); y aplicándolo a tres contextos financieros. Relacionado con la decisión individual de toma de riesgo, nos enfocamos: en el papel de los incentivos financieros; en cómo los resultados anteriores influencian la decisión futura; y en el problema de la gerencia de cartera. Diferente de los modelos tradicionales donde la aversión al riesgo se asume generalmente, en nuestro marco del comportamiento, la preferencia del riesgo del inversionista varía dependiendo de cómo él enmarca sus opciones. Nuestra conclusión principal es que las evaluaciones absolutas basadas en la riqueza final son limitadas y la relatividad de las decisiones de riesgo, donde la percepción de las ganancias y de las pérdidas conduce el proceso, son un requisito para entender las decisiones del individuo. Como contestación a los críticos del comportamiento, alcanzamos proposiciones que pueden ser contrastadas y hacemos varias predicciones.

Resumiendo, bajo el mismo marco teórico hacemos las contribuciones siguientes: (1) Verificamos cómo los incentivos afectan el comportamiento de toma de riesgo, proponiendo un modelo donde la referencia desempeña un papel central en la relación anterior; (2) Empíricamente contrastamos nuestro modelo de incentivos en
vista de una muestra mundial representativa de los fondos de acciones; (3) Evaluamos qué efecto es más fuerte en un estudio dinámico del comportamiento individual de toma de riesgo: house-money o disposition effect, proponiendo un nuevo tratamiento experimental; (4) Comparamos los resultados de una encuesta a los experimentales y verificamos que no están necesariamente alineados. (5) Generalizamos el comportamiento miope de la aversión de la pérdida (myopic loss aversion) con un experimento realizado en Brasil y España; (6) Propone un modelo teórico de cómo los individuos sesgados eligen sus carteras en términos de toma de riesgo; (7) Incluimos el error de la estimación en el análisis de la cartera; (8) Evaluamos si los modelos teóricos de la selección de carteras deben adaptarse a las características sesgadas de los individuos o moderar las características individuales del comportamiento. En este caso utilizamos una muestra de varios índices de acciones de diversos países.

Nuestras proposiciones sugieren que los gerentes en fondos manejados pasivamente tiendan a ser recompensados sin el honorario variable y sean adversos al riesgo. Por otra parte, en fondos manejados activamente, si los incentivos reducirán o aumentarán la toma de riesgo del fondo dependerá de su posibilidad de superar la rentabilidad de su cartera de referencia. Si es probable que el fondo supere la cartera de referencia, los incentivos (aumentan) reducen el apetito del riesgo del gerente. Además, el horizonte evaluativo influencia las preferencias del riesgo del gerente, en el sentido que si han obtenido mal (buenos) resultados en un período, ellos tienden para elegir inversiones (conservadoras) más arriesgadas en el periodo siguiente dado el mismo horizonte evaluativo. La evidencia empírica presentada en la tesis considerando una base representativa de fondos de acciones soporta esas predicciones. Relacionado con el house money y disposition effect, empleamos una encuesta y verificamos la existencia de house money. Sin embargo, en un experimento demostramos que desaparece el
efecto house money, y que disposition es el efecto dominante. Nuestros resultados también soportan la existencia de la aversión miope de la pérdida a través de países y una cierta evidencia del efecto del país. Finalmente, en términos de gerencia de carteras, nuestros resultados apoyan el uso de nuestro modelo del comportamiento (BRATE) como alternativa para definir la asignación óptima de los activos y postulamos que un modelo optimización de cartera se puede adaptar a los sesgos individuales implicados por la teoría (prospect theory) sin pérdida de la eficacia.
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Chapter One

General Introduction

Since the seminal work of Kahneman and Tversky (1979) two different approaches are being used to understand and forecast human behavior in terms of the decision making process, applied to economy and other social sciences: the traditional rational approach and behavioral theories.

The traditional finance paradigm seeks to understand financial markets using models in which agents are “rational”. Barberis and Thaler (2003) suggest that rationality is a very useful and simple assumption, which means that when agents receive new information, they update their beliefs and preferences instantaneously in a coherent and normative way such that they are consistent, always choosing alternatives which maximize their expected utility. Economists have traditionally used the axioms of expected utility and Bayes’ rule to serve both the normative and descriptive purposes.

The paradigm of individual behavior in traditional finance theory is that of expected utility maximization, combined with risk aversion. Ever since it was founded by von Neumann and Morgenstern (1944) the expected utility assumption has been
under severe fire as a descriptive\(^1\) theory of investor’s behavior. Allais (1953), Ellsberg (1961) and Kahneman and Tversky (1979) are three prominent examples of this critique. Traditional finance has also been empirically rejected in explaining several financial phenomena as the growing behavioral finance literature shows.

Agency theory has its foundations in traditional economic theory considering stable risk preferences. In this perspective the existence of a separation between ownership and management in organization creates conflict as some decisions taken by the agent may be in his own interest and considered not to be maximizing the principal’s welfare, which is called “moral-hazard”, and it is a consequence of the information asymmetry between the agent and the principal. An agency relationship has arisen between two (or more) parties when one, designated as the agent, acts for the other, designated the principal, in a particular domain of decision problems.

Behavioral finance suggests an alternative approach to expected utility and agency theory using models in which agents are not fully rational – a theory that is consistent with the psychology of the investor. The field has two building blocks: limits to arbitrage, which argues that it can be difficult for rational traders to undo the dislocations caused by less rational traders; and psychology, which catalogues the kinds of deviations from full rationality we might expect to see impacting investors’ beliefs and preferences.

As pointed out by Shefrin (2005), financial economists are in the midst of a debate about a paradigm shift, from neoclassical-based paradigm to one that is behaviorally based. In the essence of the debate, there is the unsolved problem of how individuals make decisions and specifically, how comfortable they are in assuming risks. This is the main point we want to address in the present thesis: How and in what

\(^1\) Normative theories characterize rational choice, while descriptive theories characterize actual choices.
situations individuals are more willing to take risks? We also evaluate the practical consequences of a behavioral risk decision in terms of financial efficiency. Our work belongs to the field of behavior finance, which has the objective to enhance the explanatory power of traditional economic models leading to more realistic psychological foundations.

Behavioral biases can roughly be grouped in two categories: cognitive and emotional; though both types yield irrational decisions. Because cognitive biases (heuristics like anchoring, availability and representative biases) stem from faulty reasoning, better information and advice can often align them with the traditional rational theory. Conversely, emotional biases such as regret and loss aversion originate from impulsive feelings or intuition, rather than conscious reasoning and are hardly possible to be aligned to traditional rationality. Kahneman and Tversky\(^2\) (1979) were the first to propose the use of behavioral lens to evaluate individuals’ decision-making process. Their prospect theory is a psychologically based theory of choice under risk and uncertainty.

In general terms, prospect theory or its latter version cumulative prospect theory\(^3\) posits four novel concepts in the framework of individuals risk preference, which can be classified as emotional biases that cannot be eliminated: investors evaluate assets according to gains and losses and not according to final wealth (mental accounting), individuals are more averse to losses than they are attracted for gains (loss aversion); individuals are risk-seeking in the domain of losses and risk averse in the domain of

\(^2\) In 2002 their work has been rewarded with the Nobel prize in economics.

\(^3\) CPT (Cumulative Prospect Theory) was proposed by Tversky and Kahneman (1992) as an improvement on, and development from, their earlier Prospect Theory (Kahneman and Tversky, 1979) and is a combination of Rank Dependent Utility (first proposed by Quiggin, 1982) with reference point to differentiate between gains and losses.
gains (asymmetric risk preference); individuals evaluate extreme probabilities in a sense of overestimating low probabilities and underestimating high probabilities (probability weighting function). The theory holds that individuals do not utilise objective probabilities in their decisions, but rather transform the objective probabilities using non-linear decision weighting function.

The choice process under prospect theory starts with the editing phase, followed by the evaluation of the edited prospects and at the end the alternative with the highest value is chosen. During the editing phase agents code outcomes into gains and losses and implement mental calculations over the probabilities. In the valuing phase the agents attach a subjective value to the lottery and then chose the prospect which generates the highest value.

All previous concepts were already identified in several experiments and we consider them stylized facts or invariants of human behavior. Under this perspective, we develop a general theoretical framework to infer the risk taking behavior of human beings. The main goal of this thesis is to analyze risk taking by individual investors in financial markets under a behavioral perspective. We approach this topic through three special cases: the role of incentives in risk taking behavior, risk attitudes in a multi period analysis; and behavioral risk taking in portfolio choice. The main message we want to convey is that we live in a behavioral world and we should study financial markets as such. It doesn’t mean that people are completely irrational or that financial markets are not efficient. But instead, standard models based on the traditional paradigm are not capable to fully describe the human nature in terms of decision-making.

Standard models of moral hazard predict a negative relationship between risk and incentives, however empirical studies on mutual funds present mixed results. In
Chapter 2 we propose a behavioral principal-agent model to the professional manager’s context, focusing on the situation of active and passive investment strategy. In this general framework we evaluate how incentives affect the manager’s risk taking behavior, where the standard moral hazard model is just a special case.

Our propositions suggest that managers in passive managed funds tend to be rewarded without incentive fee and be risk averse. On the other hand, in active managed funds, whether incentives will reduce or increase the riskiness of the fund will depend on how hard it is to outperform the benchmark. If the fund is (un)likely to outperform the benchmark, incentives (increase) reduce the manager’s risk appetite. Furthermore, the evaluative horizon influences the trader’s risk preferences, in the sense that if traders performed poorly (well) in a period, they tend to choose riskier (conservative) investments in the following period given the same evaluative horizon. Empirical evidence, based on a comprehensive world sample of mutual funds is also presented in this chapter and gives support to the previous propositions.

Recent literature has advocated that risk-taking behavior is influenced by prior monetary gains and losses. On one hand, after perceiving monetary gains, people are willing to take more risk. This effect is known as the house-money effect. Another stream of the literature, based on prospect theory and loss aversion, suggests that people are risk averse/seeking in the gain/loss domain – disposition effect. Also, behavior economic research has tended to ignore the role of country differences in financial and economic decision-making.

The objective of Chapter 3 is twofold: first to clarify which effect is dominant in a dynamic setting: disposition or house-money; and second to verify the existence of myopic loss aversion across countries. We employ a survey approach and find evidence of house money effect. However, in an experiment performed in a dynamic financial
setting, we show that the house money effect disappears, and that disposition is the
dominant effect. This finding indicates that care should be taken in generalizing survey
results. We report supportive results related to existence of myopic loss aversion across
countries and some evidence of country effect.

Chapter 4 has two main objectives. The first is to incorporate mental accounting,
loss aversion, asymmetric risk-taking behavior and probability weighting in a multi-
period portfolio stochastic optimization for individual investors. The previous
behavioral biases have already been identified in the literature; however their overall
impact during the process of determining optimal asset allocation in a multi-period
analysis is still missing. And second, we also take into account the estimation risk in the
analysis. Considering 26 daily index stock data, from the period from 1995 to 2007, we
empirically evaluate our model (BRATE – Behavior Resample Adjusted Technique) to
the traditional Markowitz. Our results support the use of BRATE as an alternative for
defining optimal asset allocation and posit that a portfolio optimization model may be
adapted to the individual biases implied by prospect theory without efficiency loss. We
also explain why investors keep on holding, or even buy, losing investments.

Finally, in Chapter 5 we provide a brief review of the main findings and
contributions of the thesis and propose several lines for further research. Behavioral
finance is surely an open avenue and we speculate that future paradigms should be
centred on this stream of research.

Our main conclusion is that absolute evaluations based on final wealth are
limited and the relativity of risk taking decisions, where the perception of gains and
losses drives the process, is a requirement to understand individual’s risk decisions. As
a reply to behavioral critics, we reach propositions that can be falsified and make
several predictions. Summing up, under the same theoretical framework we make the following contributions:

- Shed more light on how incentives affect risk taking behavior, proposing a model where the reference plays a central role in the previous relationship;
- Empirically test our incentives model considering a comprehensive world sample of equity funds (cross-section data);
- Evaluate which effect is stronger in multi-period risk taking decisions for individual investors: house-money or disposition effect, proposing a novel experimental treatment;
- Compare survey to experimental results finding that they are not necessarily aligned.
- Generalize the myopic loss aversion behavior performing an experiment in Brazil and Spain, also verifying some country effect;
- Propose a theoretical model of how behavioral individuals choose their portfolios in terms of risk taking behavior;
- Include estimation error in the behavioral portfolio analysis;
- Evaluate whether theoretical models of portfolio choice should adapt or moderate the individual behavior characteristics. In this case we used a sample of several country equity indices.

As a last remark, this thesis was elaborated in a way that any of the following 3 chapters can be read independently. In this sense, Chapters 2, 3 and 4 present complete researches that, although sharing the behavioral assumptions explained previously, consider different contexts and lead to independent conclusions.
Chapter Two

Delegated Portfolio Management and Risk Taking Behavior

2.1. Introduction

This Chapter deals with a relevant financial phenomenon that occurs in several markets. There has been tremendous and persistent growth in the prominence of mutual funds and professional investors over the recent years, which is relevant for both academics and policy makers (Bank for International Settlements, 2003). Nowadays, most real world financial market participants are professional portfolio managers (traders), which means that they are not managing their own money, but rather are managing money for other people (e.g. pension funds, hedge funds, central banks, mutual funds, insurance companies). The value of the assets managed by mutual funds rose from $50 billion in 1977 to $4.5 trillion in 1997. Similarly, the assets managed by pension plans have grown from around $250 billion in 1977 to 4.2 trillion in 1997 (Cuoco and Kaniel, 2003). Considering only the United States market during the nineties, assets managed by the hedge fund industry experienced exponential growth; assets grew from about US$40 billion in the late eighties to over US$650 billion in
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2003. Assets managed by mutual funds exceed those of hedge funds, as total assets managed by mutual funds are in excess of US$6.5 trillion \(^4\) (2003). US equity mutual funds had total net assets of US$ 4.4 trillion at the end of 2004 (Sensoy, 2006).

The main reasons for the investor to delegate the right of investing their money to traders include: customer service (including record keeping and the ability to move money around among funds); low transaction costs; diversification; and professional management (traders task). Individual investors expect to receive better results, as they are provided a professional investment service. However, an important stylized fact of the delegated portfolio management industry is the poor performance of active funds compared to passive ones (Stracca, 2005). Fernández et al. (2007a,b) found that just 23 of 649 Spanish funds outperformed their benchmarks. Gil-Bazo and Ruiz-Verdú (2007) found that for active US funds, the ones that charge higher fees often obtained lower performance. Thus, active management appears to subtract, rather than add value \(^5\). A way to justify the previous empirical evidence is to assume that the delegated portfolio management context generates an agency feature that has relevant negative consequences. As investors usually lack specialized knowledge (information asymmetry), they may evaluate the trader just based on his performance, generating early liquidation of the trader’s strategy, and can lead to mispricing. This is called the “separation of capital and brains” (Shleifer and Vishny, 1997). Also, Rabin and Vayanos (2007), show that investors move assets too often in and out mutual funds, and exaggerate the value of financial information and expertise.

\(^4\) Data provided by HedgeCo.net

\(^5\) Fernandez et al. (2007) show that during the last 10 years (1997-2006), the average return of mutual funds in Spain (2.7%) was smaller than average inflation (2.9%).
Despite relevant research on incentives produced in both scientific areas, management and economics, the search for integrative models has been neglected. In general, management papers usually provide good intuition and interpretation but lack a more precise methodology and often reach ambiguous results. On the other hand, economic papers are usually tied to classical rationality assumptions and just capture one side of an issue. Moreover, standard models of moral hazard predict a negative relationship between risk and incentives, but empirical work has not confirmed this prediction (Araújo, Moreira and Tsuchida, 2004).

Building on agency and prospect theory, Wiseman and Gomez-Mejia (1998) first proposed a behavioral agency model (BAM) of executive risk taking suggesting that the executive risk propensity varies across and within different forms of monitoring, and that agents may exhibit risk seeking as well as risk averse behaviors. However, this study considered only a single period model applied to the case of company CEOs.

In this Chapter, considering BAM to the professional portfolio manager’s context, and using the theory of contracts and behavior-inspired utility functions, we propose an integrative model that aims to explain the risk taking behavior of the traders with respect to active or passive investment strategies. Our focus is on relative risk taking measured against a certain benchmark. We argue that BAM can better explain the situation of professional portfolio managers, elucidating the way incentives in active or passive investment strategies affect the attitudes of traders towards risk. A portfolio manager decides the scale of the response to an information signal (he also decides the required effort) and so influences both the level of the risk and the portfolio returns. As pointed out in Stracca (2005), in a standard agency problem, the agent controls either the return or the variance, but not both. The previous specific characteristic offers its own challenges as the fact that the agent controls the effort and can influence risk makes it more difficult for the principal to write optimal contracts.
propositions suggest that managers in passively managed funds tend to be rewarded without an incentive fee and are risk averse. On the other hand, in actively managed funds, whether incentives reduce or increase the riskiness of the fund will depend on how hard is to outperform the benchmark. If the fund is likely to outperform the benchmark, incentives reduce the manager’s risk appetite, while the opposite is true if the fund is unlikely to outperform the benchmark. Furthermore, the evaluative horizon influences the trader’s risk preferences, in the sense that if traders performed poorly in a period, they tend to choose riskier investments in the following period given the same evaluative horizon. Conversely, if traders performed well in a given time period, they tend to choose more conservative investments following that period. We test our propositions in a world sample of equity mutual funds, finding supportive results.

The remainder of this Chapter is structured as follows. In Section 2, we first offer a brief literature review. Section 3 describes the professional portfolio manager’s context and formally presents the model, positing the propositions. Section 4 provides some empirical evidence supporting the model and Section 5 concludes with a summary of the main findings.

### 2.2. Literature Review

The traditional finance paradigm seeks to understand financial markets using models in which agents are “rational”. Barberis and Thaler (2003) suggest that rationality is a very useful and simple assumption. This means that when agents receive new information, they instantaneously update their beliefs and preferences in a coherent and normative way such that they are consistent, always choosing alternatives which
maximize their expected utility. Unfortunately, this approach has been empirically challenged in explaining several financial phenomena, as demonstrated in the growing behavioral finance literature\(^7\). The increase in price of a stock which has been included in an Index (Harris and Gurel, 1986) and the case of the twin shares which were priced differently (Barberis and Thaler, 2003) are examples of the empirical market anomalies found in the literature.

Agency theory has its foundations in traditional economics assuming the previous “rationality” paradigm. The perspective of a separation between ownership and management creates conflict as some decisions taken by the agent may be in his own interest and may not maximize the principal’s welfare (Jensen and Meckling, 1976). This is known as “moral-hazard”, and it is a consequence of the information asymmetry between the agent and the principal. We say that an agency relationship has arisen between two (or more) parties when one, designated as the agent, acts for the other, designated as the principal, in a particular domain of decision problems (Eisenhardt, 1989).

Related to the main assumptions, agency theory considers that humans are rationally bound, self-interested and prone to opportunism. It explores the consequences of power delegation and the costs involved in this context characterized by an agent which has much more information than the principal about the firm (information asymmetry). The delegation of decision-making power from the principal to the agent is problematic in that: (i) the interests of the principal and agent will typically diverge; (ii) the principal cannot perfectly monitor the actions of the agent without incurring any costs; and (iii) the principal cannot perfectly monitor and acquire information available to or possessed by the agent without incurring any costs. If agents could be induced to

\(^7\) Allias paradox and Ellsberg paradox.
internalize the principal’s objectives with no associated costs, there would be no place for agency models (Hart and Homstrom, 1987).

Moreover, while focusing on divergent objectives that principals and agents may present, agency theory considers principals as risk neutrals in the individual actions of their firms, because they can diversify their shareholding across different companies. Formally, principals are assumed to be able to diversify the idiosyncratic risk but they still bear market risk. On the other hand, since agent employment and income are tied to one firm, they are considered risk averse in order to diminish the risk they face to their individual wealth. (Gomez-Mejia and Wiseman, 1997).

Hence, current agency literature considers that principals and agents have predefined and stable risk preferences and that risk seeking attitudes are irrational. Highlighting this fact, Grabke-Rundell and Gomez-Mejia (2002) posit that agency theorists give little consideration to the processes in which individual agents obtain their preferences and make strategic decisions for their firms. Some empirical studies have shown that people systematically violate previous risk assumptions when choosing risky investments, and depending on the situation, risk seeking attitudes may be present. This occurrence of risk seeking behavior was already identified by several studies related to choices between negative prospects, and the most prominent of these studies is that of Kahneman and Tversky (1979) which proposes the prospect theory.

In general, prospect theory\(^8\) posits four novel concepts in the framework of individuals risk preferences: investors evaluate financial alternatives according to gains and losses and not according to final wealth (mental accounting); individuals are more averse to losses than they are attracted to gains (loss aversion); individuals are risk seeking in the domain of losses, and risk averse in the gains domain (asymmetric risk

\(^8\) And in its latter version (Kahneman and Tversky, 1992) known as cumulative prospect theory.
preference); and individuals evaluate extreme events in a sense of overestimating low probabilities and underestimating high probabilities (probability weighting function). In this Chapter, we consider a behavior inspired utility function, in the framework of delegated portfolio managers, which takes into account the first three stated concepts.

Coval and Shumway (2005) found strong evidence that CBOT traders were highly loss-averse, assuming high afternoon risk to recover from morning losses. In an interesting experiment, Haigh and List (2005) used traders recruited from the CBOT and found evidence of myopic loss aversion, supporting behavioral concepts. They conclude that expected utility theory may not model professional trader behavior well, and this finding lends credence to behavioral economics and finance models as they relax inherent assumptions used in standard financial economics. Aveni (1989) in a study about organizational bankruptcy posit that creditors wish to avoid recognizing losses and thus tend to assume more risk then they would otherwise take.

Wiseman and Gomez-Mejia (1998) argue that prospect and agency theories can be understood as complementing each other for reaching better predictions of risk taking by managers. Fernandes et al. (2007), in an analysis of risk factors in forty-one international stock markets, show that tail risk is a relevant risk factor. We argue that tail risk can be associated with loss aversion and therefore the BAM offers more fruitful results in the professional managers’ context.

Now, we will comment on the main criticism received by this approach. Traditional rational theorists believe that: (i) people, through repetition, will learn their way out of biases; (ii) experts in a field, such as traders in an investment institution, will make fewer errors; and (iii) with more powerful incentives, the effects will disappear. While all these factors can attenuate biases to some extent, there is little evidence that
they can be completely eliminated. Thaler (2000) suggests that “homo economicus” will become a slower learner. In this Chapter, we address the argument of incentives (iii), showing that in some cases, compensation contracts may even induce risk seeking attitudes.

As noted by Hart and Holmstrom (1987), underlying each agent model is an incentive problem caused by some form of asymmetric information. The literature on incentives and compensation contracts is very extensive, both on theoretical and empirical studies. Among them there is a consensus about the usefulness of piece-rate contracts in order to increase productivity. In our study, we approach the professional portfolio manager's setting considering a widely used piece-rate contract.

Baker (2000) concludes that most real-world incentive contracts pay people on the basis of risky and distorted performance measures. This is powerful evidence that developing riskless and undistorted performance measures is a costly activity. We extend the previous argument showing that the use of risky performance measures might be in the interest of companies to induce risk seeking behavior of the agent.

Araujo, Moreira and Tsuchida (2004) discuss the negative relationship between risk and incentives, predicted by conventional theory but not verified by empirical

---

9 Behavioral literature suggests two types of biases: cognitive and emotional. Cognitive biases (representativeness, anchorism, etc) are related to misunderstanding and lack of information about the prospect, and can be mitigated through learning. On the other hand, emotional biases (loss aversion, asymmetric risk taking behavior, etc) are human intrinsic reactions and may not be moderated.

10 Lazear (2000a), analyzing a data set for the Safelite Glass Corporation found that productivity increased by 44% as the company adopted a piece-rate compensation scheme. Bandiera, Barankay and Ransul (2004) found that productivity is at least 50% higher under piece rates, considering the personnel data from a UK soft fruit farm for the 2002 season. Lazear (2000b) stresses that the main reason to use piece-rate contracts is to provide better incentives when the workforce is heterogeneous.
studies. They propose a model with adverse selection followed by a moral hazard, where the effort and degree of risk aversion is the private information of an agent who can control the mean and the variance of profits, and conclude that more risk adverse agents provide more effort in risk reduction.

Palomino and Prat (2002) develop a general model of delegated portfolio management, where the risk neutral agent can control the riskiness of the portfolio. They show that the optimal contract is simply a bonus contract. In an empirical study, Kouwenberg and Ziemba (2004) evaluate incentives and risk taking in hedge funds, finding that returns of hedge funds with incentive fees are not significantly more risky than the returns of funds without such a compensation contract.

Our approach is distinguished from the previous approaches as we consider changes in risk preference of the agents depending on how they frame their optimization problem rather than assuming risk aversion or risk neutrality from the beginning. Agents are still considered to be value maximizers, but we are using behavior-inspired utility functions, based on prospect theory. We also focus on relative risk measured against a certain benchmark (tracking error), instead of total risk, as this is the relevant variable of interest for individual investors to decide whether to put their money in passive or active funds.

The key element to apply prospect theory to our context is to identify what the trader perceives as a loss or a gain, in other words, to determine what their reference point should be. In the mutual funds industry, benchmarks are widely used and are published in their prospects. It is safe to assume the return of the benchmark as the trader’s reference point. If he can anticipate a negative frame problem, his loss aversion behavior will lead him to go on riskier actions in order to avoid his losses even if there are other less risky alternatives which could minimize the loss. This is based on a
behavioral effect called "escalation of commitment". The intuition is that, due to the convex shape of the value function in the range of losses, risk seeking behavior will prevail in the case of prior losses.

Daido and Itoh (2005) propose an agency model with reference-dependent preferences to explain the Pygmalion effect (if a supervisor thinks her subordinates will succeed, they are more likely to succeed) and the Galatea effect (if a person thinks he will succeed, he is more likely to succeed). They show that the agent with high expectations about his performance can be induced to choose a high effort with low-powered incentives. Empirical evidence of the escalation situation can be found in Odean (1998) and Weber and Camerer (1998). They found that investors sell stocks that trade above the purchase price (winners) relatively more often than stocks that trade below the purchase price (losers). Both papers interpreted this behavior as evidence of decreased risk aversion after a loss and increased risk aversion after a gain.

2.3. The Decision Making Model

We consider professional portfolio managers to be traders who are responsible for managing the financial resources of others who work for financial institutions such as: pension funds, mutual funds, insurance companies, banks, and central banks. Their jobs consist of investing financial resources, selecting assets (e.g. stocks, bonds), and often using an index as a reference. Despite high competition in financial markets, we argue that traders, as any human beings, are continuously dealing with their own emotional biases which make their attitudes toward risk different depending on how they frame the situation they face.
A characteristic that can affect trader behavior is if the funds they manage have a passive or active investment strategy. Under active management, securities in the portfolio and other potential securities are regularly evaluated in order to find specific investment opportunities. Managers make buy/sell decisions based on current and projected future performance. This strategy, while tending toward more volatile earnings and transaction costs, may provide above-average returns. In this case, traders must be much more specialized because results are directly related to how they choose among different assets and allocate the resources of the fund in order to obtain better profits.

On the other hand, in the passive strategy, part or the entire portfolio is settled to follow a predetermined index, such as the S&P500 or the FTSE100, with the idea of mimicking market performance (tracking the index). Traders are much more worried about constructing a portfolio similar to the index than in trying to find investment opportunities. In this situation, a trader’s activity can be specified in advance as it consists of allocating the resources closely to a predetermined public index, and then it is much more programmable and predictable, which raises the possibility for better control. This strategy requires less administrative costs, tends to avoid under-market returns and lessens transaction costs. However, because of their commitment to maintaining an exogenously determined portfolio, managers of these funds generally retain stocks, regardless of their individual performance.

The approach suggested by Eisenhardt (1985) yields task programmability, information systems, and uncertainty as determinants of control strategy (outcome or behavior based). Outcome-based contracts transfer risk from the principal to the agent and it is viewed as a way of mitigating the agency costs involved. But this rewarding package has a side effect, as appropriate behaviors can lead to good or bad outcomes. It
Chapter Two: Delegated Portfolio Management and Risk Taking Behavior

is a very complex problem to isolate the effect of the specific agent’s behavior on the outcome, especially in businesses with high risk. Contingent pay will be more effective in motivating agents when outcomes can be controlled or influenced by them. Bloom and Milkovich (1998) posit that higher levels of business risk not only make it more difficult for principals to determine what actions agents take, but also make it more difficult for principals to determine what actions agents should take.

In line with the agency literature (Holmstrom and Milgrom, 1987; 1991), we model the interaction between a risk neutral, profit maximizer principal and a value-maximizing agent in a competitive market. The principal delegates the management of his funds to the agent, whose efforts can affect the probability distribution of the portfolio excess return - differential return for a given portfolio, relative to a certain benchmark\(^{11}\), \(x = x_p - x_b \rightarrow N(\mu(t), \sigma^2(t))\). The agent's task is related to obtaining information about expected returns and defining portfolio strategies. The agent chooses an effort level \(t\) incurring in a personal cost \(C(t)\). We consider the general differential assumptions for \(C(t)\): \(C'(t) > 0\) and \(C''(t) > 0\). Also, let’s call \(C_0\) the agent’s minimum cost of effort required to follow a passive strategy and just replicate the benchmark\(^{12}\).

Consider:

\[
C(t) = C_0 + \frac{t^2}{2} \quad \text{(Eq. 01)}
\]

And, the portfolio excess return is given by:

\[
x = \mu(t) + \varepsilon(t) \quad \text{(Eq. 02)}
\]

\(^{11}\) \(x_p\) is the portfolio return and \(x_b\) is the return of the benchmark.

\(^{12}\) This cost is related to the index tracking activity and can be estimated considering the ETF’s (Exchange Traded Funds) total management fee.
where $\mu(t)$ is concave and increasing, referring to the part of the return due to his level effort $(t)$. Also take $\varepsilon(t) \sim N(0, \sigma^2(t))$. In order to simplify, we assume that the performance of the trader has a linear relationship with his efforts plus a random variable, so that: $\mu(t) = \mu t$, and then:

$$x = \mu t + \varepsilon(t) \quad \text{(Eq. 03)}$$

Moreover, the timing of the proposed principal-agent game is: (i) the principal proposes a contract to the agent; (ii) the agent may or may not accept the contract, and if he accepts, he receives an amount of funds to invest; (iii) the reference point of the agent is defined; (iv) the agent chooses the level of effort (related to his personal investment strategy) to spend; (v) the outcome of the investment is realized and the principal pays the agent using part of the benefits generated by the chosen strategy and keeps the remaining return.

In this case the certainty equivalent of the agent’s utility, as proposed in Holmstrom and Milgrom (1991), can be given by:

$$CE_a = E[w(x)] - C(t) + \frac{1}{2} \sigma^2 \alpha^2 \frac{v''(x)}{v'(x)} \quad \text{(Eq. 04)}$$

where $E[w(x)]$ is the expected wage of the trader, considered as a function of the information signal (excess return), $\alpha$ is the performance pay factor, and $v(x)$ is the trader’s value function, which depends on $x$, the agent’s perceived gain or loss related to his reference point (benchmark). In the previous model, $w(x) = \alpha x + \beta = \alpha \mu t + \alpha \varepsilon(t) + \beta$, and so $\text{Var}[w(x)] = \alpha^2 \sigma^2(t)^2$.

The value function was proposed in the prospect theory of Kahneman and Tversky (1979) and is an adaptation of the standard utility function in the case of the
behavior approach. The ratio $\frac{\mu'}{\mu}$ is the coefficient of absolute risk aversion. For a risk averse agent, this ratio is negative and the certainty equivalent is less than the expected value of the gamble as he prefers to reduce uncertainty. This is the origin of the negative relationship between risk and incentives in moral hazard models.

Let $t^*$ denote the agent’s optimal choice of effort, given $\alpha$. Note that $t^*$ is independent of $\beta$. The resulting indirect utility is given by: $V(\alpha, \beta) = \beta + v(\alpha)$, where $v(\alpha) = \alpha \mu(t^*) - C(t^*) + \frac{1}{2} \alpha^2 \frac{v''(x)}{v'(x)} \sigma^2(t^*)$ is the non-linear term. The marginal utility of incentives can then be derived:

$$\frac{\partial v}{\partial \alpha} = v_a = \mu(t^*) + \alpha \frac{v''(x)}{v'(x)} \sigma^2(t^*)$$  \hspace{1cm} (Eq. 05)

and if we were considering risk averse agents, it would represent the mean of the excess profits minus the marginal risk premium.

The effort of the agent leads to an expected benefits function $B(t)$ which accrues directly to the principal. Let’s consider $B(t) = x_b + x$. The principal’s expected profit (which equals certainty equivalent as he is risk neutral) is given by:

$$CE_p = B(t) - E[w(x)]$$  \hspace{1cm} (Eq. 06)

Hence the total certainty equivalent (our measure of total surplus) is:

$$TCE = CE_a + CE_p = x_b + x - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 \frac{v''(x)}{v'(x)} \sigma^2(t)$$  \hspace{1cm} (Eq. 07)

The optimal contract is the one that maximizes this total surplus subject to the agent’s participation constraint ($CE_a \geq 0$). Adapting the previous model to the professional manager’s case and considering mental accounting, loss aversion and asymmetric risk taking behavior, we assume the value function as follows:
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\[ v(x) = \begin{cases} 1 - e^{-rx}, & \text{if } x \geq 0 \\ \lambda e^{rx} - \lambda, & \text{if } x < 0 \end{cases} \quad \text{(Eq. 08)} \]

where \( r \) is the coefficient of absolute risk preference, \( \lambda \) is the loss aversion factor which makes the value function steeper in the negative side; and \( x \) is the perceived gain or loss, rather than final states of welfare, as proposed by Kahneman and Tversky (1979). It is useful to consider the previous form for the value function because of the existence of a CAPM equilibrium (Giorgi et al., 2004) and because we reach constant coefficients of risk preference. The following graph indicates \( v(x) \) when \( \alpha = 0.88, \lambda^- = 2.25 \) and \( \lambda^+ = 1 \) (using values suggested by Kahneman and Tversky).

![Graph showing the prospect theory value function](image)

**Figure 1 – Prospect theory value function for** \( \alpha = 0.88, \lambda^- = 2.25 \) and \( \lambda^+ = 1 \)

We assume a general symmetric compensation contract applied to the situation presented in this paper. Starks (1987) shows that the “symmetric” contract, while it does not necessarily eliminate agency costs, dominates the convex (bonus) contract in aligning the manager’s interests with those of the investor. Also, Grinblatt and Titman (1989) posit that penalties for poor performance should be at least as severe as the rewards for good performance.
This indicates that the agent is paid a base salary $\beta$ plus an incentive fee calculated as a proportion $\alpha$ of the total return of the fund (the performance indicator). The previous contract arrangement follows the optimal compensation scheme defined in Holmstrom and Milgrom (1987), and was also used in Carpenter (2000). Lazear (2000b) argues that continuous and variable pay is appropriate in case of worker heterogeneity as in the case of professional portfolio managers. Finally, let’s call $\psi$ the probability that the fund outperforms the benchmark and $(1 - \psi)$ the likelihood that it performs poorly. So, we can re-write the TCE, $CE_a$ and $CE_p$ as follows:

\[
TCE = \psi \left( x_b + x - C_0 - \frac{t^2}{2} - \frac{1}{2} \alpha^2 r \sigma^2 (x) \right) + (1 - \psi) \left( x_b + x - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 \lambda \sigma^2 (x) \right)
\]

(Eq. 10)

\[
CE_a = \psi \left( \alpha x + \beta - C_0 - \frac{t^2}{2} - \frac{1}{2} \alpha^2 r \sigma^2 (x) \right) + (1 - \psi) \left( \alpha x + \beta - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 \lambda \sigma^2 (x) \right)
\]

(Eq. 11)

\[
CE_p = x_b + (1 - \alpha)x - \beta
\]

(Eq. 12)

2.3.1. The Case of Passive Funds

Investors in passive funds have expectations of receiving average market returns ($E(x_p) = E(x_b)$ and $E(x) = 0$), and trader actions are limited and tied in relation to the process of buying and selling assets to adjust stock weights in the portfolio in order to follow the benchmark. The agent’s task is more programmable and his behavior is easy to monitor (“$t$” is observable by the principal). As the principal has no interest that the agent goes on riskier strategies than that of the benchmark, he should set $\alpha = 0$. Thus, the certainty equivalent of the agent would be given by:
which implies that $t^* = 0$. Optimally, the agent will make no effort to beat the benchmark. An important aspect considered in this paper is the competitive situation in the market of professional portfolio managers, which is of crucial importance in determining who extracts the surplus from the agency contract. We considered, as it is usual in the delegated portfolio managers' literature, a perfect competition among agents with the entire surplus accrued to the principal. This situation implies that: $CE_a = 0$ and $\beta = C_0$. The certainty equivalent of the principal would be given by: $CE_p = x_b - C_0$.

The principal pays the agent a base salary which is equal to the agent’s cost of effort to ensure the investor receives the return of the benchmark (say the agent's choice of effort represents the minimum level needed to replicate the benchmark portfolio). Moreover, if the agent chooses a level of effort different from $C_b$, the performance of the fund will not be tied to the performance of the benchmark and so $\sigma^2(x) > 0$ (increased the risk). If the agent just receives the base salary alone, he doesn’t have any incentive to choose a level of effort different from 0 and so performs in a risk averse way. Also, because of employment risk, managers tend to decrease risk in order to prevent potential job loss (Kempf et al, 2007).

In this incentive scheme, there’s no risk premium associated with the agent’s decisions. Recall that in this case $t$ is observable, and then if the trader chooses $t \neq 0$, the investor will notice and just fire him. Finally, in this case, there is no reason for using incentive fees, as the trader is not responsible for the earnings of the fund, which should be equal to the performance of the benchmark. Observe that the previous result is robust for different levels of risk preference as it is independent of the value function of the
agent, regardless of whether he is risk averse or risk seeking. Summing up, we can construct the following table:

<table>
<thead>
<tr>
<th>Agent's choice of ( t )</th>
<th>Compensation ( w = \beta )</th>
<th>Performance ( x = xb )</th>
<th>Risk ( \sigma^2 = 0 )</th>
<th>Result</th>
<th>( t \neq 0 ) ( w = \beta \neq xb ) ( \sigma^2 &gt; 0 )</th>
<th>agent is fired</th>
</tr>
</thead>
</table>

**Proposition One:** Traders in passively managed funds tend to be rewarded with a base salary \( (\alpha = 0) \).

**Proposition Two:** Traders in passively managed funds are more likely to perform as risk adverse agents \( (t = t_p^*) \).

### 2.3.2. The Case of Active Funds

In the case of an active fund, investors are usually expecting to receive above-average risk adjusted returns as they consider it linked to the expertise of the traders. The trader has to make investment decisions, and a great number of these decisions are based on his own point of view of the market, raising a relevant problem of information asymmetry (moral hazard). In this case, the first best results are no longer feasible and outcome-based rewards are often used as part of their contracts and the agent is stimulated to go on risky alternatives in order to reach above-average returns. Hence, the idea of the contract is to reduce objective incongruence between the principal (investor) and agent (trader), and to transfer risk to the agent.

We now examine two cases. In the single task case, the agent’s effort affects only the mean of the excess return. In the multitask case, the agent’s effort influences both the expected return and the risk of the portfolio.
a) Single Task

We first analyze the case in which the agent’s effort controls only the mean of the excess profits and so the risk is exogenous: $\sigma^2(x) = \sigma^2$. Consider a loss averse agent with a value function given by (Eq. 08). The main point in applying prospect utility is to define the reference point by which the manager measures his gains and losses. It seems reasonable in the funds industry to assume the returns of the public benchmark published by the fund as a reference point, since it is the one used by individual investors when deciding which fund to invest in. Thus, the total certainty equivalent would be given by:

$$TCE = \nu \left( x_b + \mu t - C_0 - \frac{t^2}{2} - \frac{1}{2} \alpha^2 r \sigma^2 \right) + (1 - \psi) \left( x_b + \mu t - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 r \lambda \sigma^2 \right).$$

(Eq. 14)

Taking into account the agent’s maximization problem, we reach the following results:

$$\max_{x} \left[ CE_{\mu t} \right] = \nu \left( \alpha \mu t + \beta - C_0 - \frac{t^2}{2} - \frac{1}{2} \alpha^2 r \sigma^2 \right) + (1 - \psi) \left( \alpha \mu t + \beta - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 r \lambda \sigma^2 \right)$$

(Eq. 15)

so, $t^* = \alpha \mu$ and $C(t^*) = C_0 + \frac{t^*}{2}$. As expected, efforts in outperforming the benchmark increases with incentives. The agent’s marginal utility of incentives is given by:

$$v_{x} = \alpha \mu^2 - \alpha r \sigma^2 \left( \psi - (1 - \psi) \lambda \right)$$

(Eq. 16)

So the effect of incentives on the agent’s utility will depend on whether the benchmark is likely to be outperformed. Suppose that the fund can easily outperform the benchmark. In this case, the probability that the return of the fund is greater than the
benchmark, \( \psi \), is close to one and \( \mu > 0 \). Then, \( \nu = \alpha \mu^2 - \alpha r \sigma^2 \), which is the usual solution found by moral hazard models. This implies that an increase in incentives has both positive and negative effects on the utility of the agent. The positive effect results from the share of the positive excess return, and the negative effect comes from the increased risk of the wage. Finally, when we maximize the total surplus:

\[
\max \, TCE = \psi \left( x_h + \mu t - C_0 - \frac{t^2}{2} - \frac{1}{2} \alpha^2 r \sigma^2 \right) + (1 - \psi) \left( x_b + \mu t - C_0 - \frac{t^2}{2} + \frac{1}{2} \alpha^2 r \lambda \sigma^2 \right)
\]

\[
\frac{\partial TCE}{\partial t} = \mu - \alpha \mu + \frac{(1 - \psi) \alpha \lambda r \sigma^2}{\mu} - \frac{\psi \alpha \sigma^2}{\mu} = 0
\]

then

\[
\alpha = \frac{\mu^2}{\mu^2 + \sigma^2 r [\psi - (1 - \psi) \lambda]} \quad \text{(Eq. 17)}
\]

So the relationship between risk and return is ambiguous, depending on how likely it is to outperform the benchmark. As previous experiments have shown that the value for \( \lambda \) is around 2 (Kahneman and Tversky, 1979) if \( \psi \) is higher than 67\%, then a negative relationship between risk and incentives is predicted by the model. However, as we decrease \( \psi \), a positive relation between risk and return appears.

If we consider a benchmark that is easy to be outperformed, then \( \psi \) approaches 1 and so

\[
\alpha = \frac{\mu^2}{\mu^2 + \sigma^2 r} \quad \text{(Eq. 18)}
\]

and therefore, increases in \( \sigma^2 \) and \( r \) imply decreases in \( \alpha \). The previous negative relationship between risk (\( \sigma^2 \)) and incentives (\( \alpha \)) is the usual standard result obtained by
moral hazard models. However our model generalizes this, and the previous result is simply a special case. If we consider a benchmark that is difficult to outperform, then \( \psi \) approaches 0 and so

\[
\alpha = \frac{\mu^2}{\mu^2 - \sigma^2 r \lambda}
\]  
(Eq. 19)

and, therefore, increases in \( \sigma^2 \) and \( r \) imply increases in \( \alpha \). Some empirical papers have found previous positive relationships between risk and return.

Recall that \( \sigma^2 \) in our model represents a variance in the differential portfolio which uses the benchmark as its reference (tracking error). The performance of the benchmark \( X_B \) and the performance of the chosen portfolio \( X_P \) are respectively given by:

\[
X_B = E(X_B) + \varepsilon_B, \quad \text{with} \quad \varepsilon_B \sim N(0, \sigma_B^2)
\]

\[
X_P = E(X_P) + \varepsilon_P, \quad \text{with} \quad \varepsilon_P \sim N(0, \sigma_P^2)
\]

Also we consider that the expected return of the benchmark is normalized to zero \( \{E(X_B) = 0\} \) and the expected return of the portfolio equals the agent’s choice of effort \( \{E(X_P) = \mu_t\} \). Therefore, the return of the differential portfolio is given by:

\[
\left( X_P - X_B \right) = \mu_t + (\varepsilon_P - \varepsilon_B)
\]

then

\[
\left( X_P - X_B \right) \sim N(\mu_t, \sigma_B^2 + \sigma_P^2 - 2 \rho \sigma_B \sigma_P)
\]

so

\[
\sigma_B^2 + \sigma_P^2 - 2 \rho \sigma_B \sigma_P = 2 \sigma_P^2 (1 - \rho)
\]  
(Eq. 20)
Finally, consider the simplified assumption that $\sigma_u^2 = \sigma_r^2$ (i.e. the total risk of the portfolio selected by the manager is the same as the total risk of the benchmark portfolio, so based on portfolio theory, both portfolios should be equivalent in terms of risk/return trade-off), where $\rho$ is the correlation coefficient between the chosen portfolio and the benchmark. Therefore, $\alpha$ can be rewritten as:

$$\alpha = \frac{\mu^2}{\mu^2 + 2\sigma_r^2(1-\rho)r[\psi-(1-\psi)\lambda]}$$  \hspace{1cm} \text{(Eq. 21)}

which suggests that increases in $\alpha$ imply increases in $\sigma_r^2(1-\rho^2)$, and also implies a decreasing correlation ($\rho$), for low values of $\psi$. Thus, using the benchmark as a filter reduces uncontrollable risk by $(1-\rho)$. If the agent just reproduces the benchmark (passive strategy), the correlation is equal to 1 (perfect correlation), all risk can be filtered out, and the first best can be achieved. Because of the agency problem, we see that the agent’s choice will depend on the degree of idiosyncratic risk associated with his contract, as measured by $\sigma_r^2(1-\rho)$. Unlike standard portfolio theory (Markowitz, 1952), idiosyncratic risk will play a role in incentive schemes.

**Proposition Three:** Traders in actively managed funds tend to be rewarded in incentive-base pay ($\alpha > 0$).

**Proposition Four:** The relationship between incentives and risk can either be positive or negative depending on the likelihood $\psi$ of outperforming the benchmark. High (low) values of $\psi$ imply a negative (positive) relationship.

**Table 2 - Agent’s Choice of Effort in an Active Fund**

<table>
<thead>
<tr>
<th>Agent’s choice of $t$</th>
<th>Compensation</th>
<th>Performance</th>
<th>Risk $\sigma^2$</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$w = \beta$</td>
<td>$x = 0$</td>
<td>$\sigma^2 = 0$</td>
<td>agent is fired</td>
</tr>
<tr>
<td>$t = t^*$</td>
<td>$w = \alpha x + \beta$</td>
<td>$x &gt; 0$</td>
<td>$\sigma^2 &gt; 0$</td>
<td>optimum</td>
</tr>
<tr>
<td>$t = t^*$</td>
<td>$w = \alpha x + \beta$</td>
<td>$x &lt; 0$</td>
<td>$\sigma^2 &gt; 0$</td>
<td>agent is fired</td>
</tr>
</tbody>
</table>
b)  Multi Task

We now introduce the possibility that the agent can also influence the risk of the portfolio’s excess return. Let $t_{\mu}$ and $t_{\sigma}$ be the effort in mean increase and in variance reduction. We assume the cost is quadratic and separable: $C(t) = C_0 + \frac{t_{\mu}^2}{2} + \frac{t_{\sigma}^2}{2}$. Also, let $\mu(t) = \mu_{t_{\mu}}$ and $\sigma^2(t) = (\sigma_0 - t_{\sigma})^2$, where the exogenous variance $\sigma_0^2$ is the variance of the excess return when no effort is provided to change it. Taking into account the agent’s maximization problem, we reach the following results:

$$\max_{t_{\mu}, t_{\sigma}} \{ CE_{t_{\mu}, t_{\sigma}} \} = \psi \left( \alpha \mu_{t_{\mu}} + \beta - C_0 - \frac{t_{\mu}^2}{2} - \frac{t_{\sigma}^2}{2} - \frac{1}{2} \alpha^2 r (\sigma_0 - t_{\sigma})^2 \right)$$

$$+ (1 - \psi) \left( \alpha \mu_{t_{\mu}} + \beta - C_0 - \frac{t_{\mu}^2}{2} - \frac{t_{\sigma}^2}{2} + \frac{1}{2} \alpha^2 r \lambda (\sigma_0 - t_{\sigma})^2 \right)$$

so, $t_{\mu}^* = \alpha \mu$ and $t_{\sigma}^* = \frac{\alpha^2 r (\psi - (1 - \psi) \lambda)}{1 + \alpha^2 r (\psi - (1 - \psi) \lambda)} \sigma_0$. As expected, efforts to outperform the benchmark increase with incentives. The endogenous variance is then given by:

$$\sigma^2(t) = \left( \frac{1}{1 + \alpha^2 r (\psi - (1 - \psi) \lambda)} \right)^2 \sigma_0^2 \quad \text{(Eq. 22)}$$

which implies that endogenous risk can be lower or greater than exogenous risk depending on whether the agent is framing a gain or loss situation. If the benchmark is easily outperformed, $\psi$ approaches 1 and $\sigma^2(t) = \left( \frac{1}{1 + \alpha^2 r} \right)^2 \sigma_0^2$, and so endogenous risk and incentives are negatively related. On the other hand, if the agent is framing a loss situation, the endogenous risk would be given by $\sigma^2(t) = \left( \frac{1}{1 - \alpha^2 r \lambda} \right)^2 \sigma_0^2$, and a positive relationship between risk and incentives is predicted.
Summing up, our model predicts that when the fund manager is facing a situation of high likelihood to outperform the benchmark, he will frame the portfolio construction problem in the gain domain, and will act in a risk averse way, and incentives will stimulate him to exert efforts to reduce risk and improve the expected excess return. Incentives are lower in riskier portfolios. On the other hand, when he is facing a situation of low likelihood to outperform the benchmark, the agent is likely to frame the investment problem in the loss domain, and incentives will make him look for riskier alternatives. Incentives are higher in riskier portfolios.

c) Multi-Period Analysis

In this section, we discuss the effect of previous outcomes in the future risk appetite of the agent. Wright, Kroll and Elenkov (2002) posit that institutional owners exerted a significant positive influence on risk taking in the presence of growth opportunities. Gruber (1996) showed that in the American economy, actively managed funds assumed greater risk, but reached lower average returns compared to passively managed funds.

Hence, in some sense, we have the investment strategy and the contract arrangements disciplining the risk taking behavior of the agent. However we are aware that the trader’s cognitive biases moderate this relationship. In this study, we do not deal with the way these biases moderate the relationship as a deeper psychological analysis of the trader in his context is required, and we also assume that cognitive biases can be moderated.

Going further in the analysis of the relationship with risk-return, we can apply Miller and Bromiley’s (1990) multiperiod approach to the professional investor environment, taking into account the evaluative period. We assume that a company has a target performance level which for instance corresponds to the performance of a
chosen index and the firm provides a report annually to the investors. Investors and traders are likely to consider this target as the reference point for gain/loss analysis.

Supposing that in the first semester, this company performed poorly and so the likelihood of outperforming the benchmark is lower, the loss aversion of the agent will make him choose risky projects in the second semester hoping to convert losses into gains until the end of the year. On the other hand if the company performed well in the first period, the agent will only accept an increase in risk if the investment opportunity offers high expected returns. In this case, the trader tends to reduce his relative risk exposure and follow the index in the second semester in order to guarantee the return obtained in the previous period. This is based on a behavioral effect called "escalation of commitment". In other words, if the fund performed well in the first period, the likelihood of outperforming the benchmark is higher (greater $\psi$) and the trader is more likely to perform in a risk averse way (gain domain). Weber and Zuchel (2003) found that subjects in the "portfolio treatment" take significantly greater risks following a loss than a gain.

Deephouse and Wiseman (2000) found supportive evidence to these risk-return relationships in a large sample of US manufacturing firms. Odean (1998) and Weber and Camerer (1998) provide empirical evidence of the escalation situation; these studies found that investors sell stocks that trade above the purchase price (winners) relatively more often than stocks that trade below purchase price (losers). Both works interpreted this behavior as evidence of decreased risk aversion after a loss and increased risk aversion after a gain. Chevalier and Ellison (1997) also found supportive empirical evidence that an agent with a low interim result is tempted to look for high-risk investments.
Chapter Two: Delegated Portfolio Management and Risk Taking Behavior

**Proposition Five:** If traders performed (well) poorly in a period, they tend to choose (less risky) riskier investments in the following period, considering both in the same evaluative horizon.

Basak et al. (2003) state that as the year-end approaches, when the fund's year-to-date return is sufficiently high, fund managers set strategies to closely mimic the benchmark; however they argue that this is because of the convexities in the manager's objectives. We extend this approach, stating that the previous proposition is a direct consequence of the individual behavior inspired utility function.

d) **Asymmetric Contract**

Despite the fact that most mutual funds adopt a symmetric compensation contract, there are a few which use asymmetric option-based contract as follows:

\[
  w(x) = \begin{cases} 
  (\alpha + \gamma)x + \beta & \text{if } x \geq 0 \\
  \alpha x + \beta & \text{if } x < 0 
  \end{cases} \tag{Eq. 23}
\]

where \( \gamma \) is usually called the performance fee. If we considered an incentive scheme, as defined in Eq. 23, the main conclusions of our model would remain with the expression (21) now given by:

\[
  \alpha = \frac{\mu^2 (1 - \psi \gamma) - \psi \gamma \sigma^2}{\mu^2 + \sigma^2 \left[ \psi - (1 - \psi)\lambda \right]} \tag{Eq. 24}
\]

As can be seen, the only effect of \( \gamma \) would be to increase the negative relation between incentives and risk, in the case of an easy to be outperformed benchmark. If the benchmark is difficult to outperform, the performance fee has no effect. Probably due to its diminished effect on risk, the performance fee is not common. Empirical papers (Kouwenberg and Ziemba, 2004; Golec and Starks, 2002) found mixed results related to the impact of performance fees on risk taking behavior.
Table 3 provides a summary of the main formulas for $\alpha$ in all the cases considered. From the model, we can state the following predictions to be tested in the empirical section of this chapter:

1. Passive funds have lower management fees than active funds\(^{13}\);  
2. Asymmetric contracts are less common than symmetric contracts;  
3. Active funds which are likely to outperform the benchmark show a negative relationship between relative risk (tracking error) and incentives.  
4. Active funds which are likely to underperform the benchmark will show a positive relationship between relative risk (tracking error) and incentives;  
5. Active funds under performing the benchmark in one given period tend to increase their relative risk (tracking error) in the subsequent period.

\(^{13}\) We can consider the management fee of a passive fund as a proxy for the $\beta$ in our compensation scheme, and in this case, as predicted by the model, the management fee for passive funds should be lower than for active funds.
### Table 3 - Summary of the Equations

<table>
<thead>
<tr>
<th>Passive Funds</th>
<th>( \alpha = 0 )</th>
<th>Active Funds</th>
<th>Out Performing Bench</th>
<th>Under Performing Bench</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric Contract</td>
<td>( \alpha = \frac{\mu^2}{\mu^2 + \sigma^2 r \left[ \gamma - (1 - \gamma) \lambda \right]} )</td>
<td></td>
<td>( \alpha = \frac{\mu^2}{\mu^2 + \sigma^2 r \lambda} )</td>
<td>( \alpha = \frac{\mu^2}{\mu^2 - \sigma^2 r \lambda} )</td>
</tr>
<tr>
<td>Asymmetric Contract</td>
<td>( \alpha = \frac{\mu^2 \left( 1 - \gamma \gamma \right) - \gamma \gamma \sigma^2}{\mu^2 + \sigma^2 r \left[ \gamma - (1 - \gamma) \lambda \right]} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.4. Empirical Analysis

#### 2.4.1. Data

For our empirical investigation, we used the Bloomberg cross-sectional equity mutual funds database for February 2007. There are 4584 funds using 26 different equity benchmarks (stock indices) from 15 countries\(^{14}\). The database includes emerging markets (Brazil, Mexico) as well as developed countries (United States, United Kingdom, Germany)\(^ {15} \). As the theoretical model proposed in this paper is always

\(^{14}\) All funds in the database are alive as of January, 2007. Unfortunately data on the dead funds for the sample used was not available.

\(^{15}\) We also used daily data from January 2002 to February 2007 for 739 funds from France, United States, Brazil and Japan in a total of 787,216 day-fund-return data from the Bloomberg time series database in order to validate the results based on the cross-sectional Bloomberg data.
dependent on the reference considered, all the analysis is performed separately for each benchmark. As in the funds market, the use of a public benchmark is widespread (Elton et al., 2003), we assume that fund managers tend to be evaluated and compensated using the benchmark as the reference point, to which gains and losses are defined.

Table 4 provides some descriptive statistics of the funds in the database. The funds were grouped by the benchmark they use to evaluate their performance, and so we can consider that they compete for the same class of investors. The number of funds for each benchmark varies from 32 (Austrian Stock Exchange) to 1332 (S&P500). From the list of funds, just 261 (5.67%) use performance fees indicating that this sort of asymmetric compensation contract is not common, except for Brazil (IBOVESPA), where 18.10% of the funds charge performance fees.

The mean management fee among the entire sample is 1.36% (median 1.25%). Mexican funds charge the highest management fees (mean 4.84%) and U.S. funds, which use the Russell 3000 index benchmark, have the lowest management fee (mean 0.68%). The average volatility of management fees is 0.97, highest in Brazil (1.77) and lowest in Taiwan (0.25). In terms of net asset value (given in the country’s currency), the mean is usually much higher than the median indicating the concentration of the market, with few large funds and many small ones. In terms of fund age, we have a sample of established funds with an average age of 10.55 years and a median ranging from 5.99 (IBOVESPA) to 16.79 (Germany REX Total Index).
Table 4 Descriptive Statistics of the Funds’ Data

Descriptive statistics of the funds in the database are displayed. The cross-sectional mean, median and standard deviation of the management fee (values in %), the net asset value (millions unit in the country’s currency), and the age (in years) are listed, respectively, for each country. The number of funds for each country, and those using a performance fee are also displayed.

We have a representative cross-country sample of established mutual funds, diversified in terms of size. The global nature of the data can surely provide good insight for testing the theoretical model proposed in Section 3 of this chapter.

2.4.2. Empirical Results

In order to test our propositions, we will first distinguish between active and passive funds. From our predictions, passive funds should have a very small variable pay factor (Propositions 1 and 3). Typically, funds may charge investors management fees as a proportion of the total assets value, and performance fees, paid if the return of the fund outperforms the one obtained by the benchmark. We already observed that performance fees (asymmetric contracts) are not common. Thus, funds charge the management fee and use it to compensate the traders and face other operational costs. As we previously discussed in the theoretical model, management fees and performance fees act in the same direction in terms of influencing trader behavior, and thus the latter
is not really a requirement. We expect that passive funds charge lower management fees as they use it to set the trader’s base salary ($\beta$), since incentives ($\alpha$) are not necessary.

Table 5 provides the average mean of the management fee for the funds for each benchmark, distinguishing between active and passive\textsuperscript{16} funds. The last column shows the t statistics and p-values for the differences among means.

<table>
<thead>
<tr>
<th>Index</th>
<th>Management Fee</th>
<th>t-Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
<td>Active</td>
<td>t-stat</td>
</tr>
<tr>
<td>'ASX'</td>
<td>0.88</td>
<td>1.18</td>
<td>2.00</td>
</tr>
<tr>
<td>'CAC'</td>
<td>1.39</td>
<td>1.68</td>
<td>3.27</td>
</tr>
<tr>
<td>'DAX'</td>
<td>0.86</td>
<td>1.25</td>
<td>3.29</td>
</tr>
<tr>
<td>'DJIST'</td>
<td>1.57</td>
<td>1.52</td>
<td>0.17</td>
</tr>
<tr>
<td>'IBEX'</td>
<td>1.25</td>
<td>1.46</td>
<td>0.39</td>
</tr>
<tr>
<td>'IBOV'</td>
<td>2.07</td>
<td>2.09</td>
<td>0.15</td>
</tr>
<tr>
<td>'INDU'</td>
<td>0.81</td>
<td>2.19</td>
<td>0.41</td>
</tr>
<tr>
<td>'KLCI'</td>
<td>1.33</td>
<td>1.46</td>
<td>1.55</td>
</tr>
<tr>
<td>'MEXBOL'</td>
<td>4.21</td>
<td>4.14</td>
<td>-0.20</td>
</tr>
<tr>
<td>'MID'</td>
<td>0.42</td>
<td>0.88</td>
<td>0.51</td>
</tr>
<tr>
<td>'NKY'</td>
<td>0.78</td>
<td>1.58</td>
<td>4.04</td>
</tr>
<tr>
<td>'RTY'</td>
<td>0.40</td>
<td>1.10</td>
<td>3.91</td>
</tr>
<tr>
<td>'SPX'</td>
<td>0.66</td>
<td>1.08</td>
<td>3.71</td>
</tr>
<tr>
<td>'TPX'</td>
<td>0.56</td>
<td>1.38</td>
<td>7.90</td>
</tr>
<tr>
<td>'UKX'</td>
<td>1.18</td>
<td>1.52</td>
<td>0.04</td>
</tr>
<tr>
<td>All</td>
<td>1.06</td>
<td>1.37</td>
<td>3.57</td>
</tr>
</tbody>
</table>

From the results, it can be seen that active funds charged higher management fees in 14 of the 16 cases and that this difference is significant at the 5% level in 7 of the 16 indices considered. If we consider the entire sample of mutual funds (All), evidence suggests that active funds charge higher fees. In this sense, propositions 1 and 3 are given empirical support, and the level of the management fee for passive funds can be used as a proxy for the base compensation considered in the model. For instance, considering the benchmark SPX, funds charge 0.66% of the total assets value to pay the

\textsuperscript{16} Passive funds are the mutual funds classified as Index Funds by Bloomberg.
trader’s base salary and other operational costs, and any increments on the management fee are used as incentives\(^{17}\).

Consider the implications of proposition 2, which implies that in general, active managed funds assume a higher risk than passive funds (passive fund managers are risk averse). Recall, that the risk we are considering in the model is relative to the benchmark (tracking error). We use the variable \((\beta - 1)^2\) as a proxy of the tracking error\(^{18}\). Table 6 provides the average mean of the previous risk variable for the funds for each benchmark, distinguishing between active and passive funds. We considered both a short term beta calculated over the previous 6 months (using daily data) and a long term beta calculated over the previous 2 years (using monthly data).

The results indicate that both the short-term and long-term tracking error for passive funds are lower than for active funds, as predicted by proposition 2\(^{19}\). The difference is statistically significant (5% level) for 22 out of 32 cases. If we consider the entire sample, the results are similar.

\(^{17}\) In the case of benchmarks DJST (US) and MEXBOL (Mexico), the management fee for passive funds is higher than for active funds; however the difference is not statistically significant.

\(^{18}\) This proxy is valid if we assume CAPM and the same market portfolio (benchmark) for all funds. \(TE = \sqrt{(\beta - 1)^2 \sigma_{Bench}^2}\). (Carroll et al., 1992)

\(^{19}\) The two indexes where the tracking error was greater for passive funds than for active funds is in the case of the Dow Jones Industrial Index (INDU) (the difference is not significant) and in the case of MEXBOL (significant difference).
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Table 6 Passive vs. Active Funds: Tracking Error

The tracking error proxy \((\beta - 1)^2\) for active and passive funds for the various benchmarks considered. Indices with less than 10 passive funds were excluded from the analysis. The proxy used was non-negative by definition, and in this case we run the test on the natural logarithm of the measure in order to improve the normality of the variable.

<table>
<thead>
<tr>
<th>Index</th>
<th>Passive ((\text{Beta6M} - 1)^2)</th>
<th>Active ((\text{Beta6M} - 1)^2)</th>
<th>t-Test</th>
<th>p-value</th>
<th>Passive ((\text{Beta2Y} - 1)^2)</th>
<th>Active ((\text{Beta2Y} - 1)^2)</th>
<th>t-Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>'ASX'</td>
<td>0.05</td>
<td>0.12</td>
<td>2.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.08</td>
<td>1.24</td>
<td>0.22</td>
</tr>
<tr>
<td>'CAC'</td>
<td>0.07</td>
<td>0.17</td>
<td>3.23</td>
<td>0.00</td>
<td>0.07</td>
<td>0.16</td>
<td>2.34</td>
<td>0.02</td>
</tr>
<tr>
<td>'DAX'</td>
<td>0.08</td>
<td>0.11</td>
<td>1.09</td>
<td>0.28</td>
<td>0.05</td>
<td>0.08</td>
<td>1.48</td>
<td>0.14</td>
</tr>
<tr>
<td>'DJST'</td>
<td>0.11</td>
<td>0.15</td>
<td>2.48</td>
<td>0.01</td>
<td>0.07</td>
<td>0.11</td>
<td>1.38</td>
<td>0.17</td>
</tr>
<tr>
<td>'IBEX'</td>
<td>0.00</td>
<td>0.12</td>
<td>6.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>5.42</td>
<td>0.00</td>
</tr>
<tr>
<td>'IBOV'</td>
<td>0.07</td>
<td>0.13</td>
<td>2.57</td>
<td>0.01</td>
<td>0.00</td>
<td>0.04</td>
<td>3.37</td>
<td>0.00</td>
</tr>
<tr>
<td>'INDU'</td>
<td>0.11</td>
<td>0.09</td>
<td>-0.57</td>
<td>0.57</td>
<td>0.22</td>
<td>0.38</td>
<td>-0.69</td>
<td>0.49</td>
</tr>
<tr>
<td>'KLCI'</td>
<td>0.02</td>
<td>0.07</td>
<td>2.18</td>
<td>0.03</td>
<td>0.01</td>
<td>0.07</td>
<td>3.17</td>
<td>0.00</td>
</tr>
<tr>
<td>'MEXBOL'</td>
<td>0.40</td>
<td>0.26</td>
<td>-2.52</td>
<td>0.01</td>
<td>0.02</td>
<td>0.16</td>
<td>3.93</td>
<td>0.00</td>
</tr>
<tr>
<td>'MID'</td>
<td>0.02</td>
<td>0.18</td>
<td>1.92</td>
<td>0.06</td>
<td>0.06</td>
<td>0.26</td>
<td>-0.79</td>
<td>0.44</td>
</tr>
<tr>
<td>'NKY'</td>
<td>0.06</td>
<td>0.21</td>
<td>14.28</td>
<td>0.00</td>
<td>0.05</td>
<td>0.12</td>
<td>15.88</td>
<td>0.00</td>
</tr>
<tr>
<td>'RTY'</td>
<td>0.05</td>
<td>0.12</td>
<td>10.63</td>
<td>0.00</td>
<td>0.13</td>
<td>0.13</td>
<td>-1.76</td>
<td>0.08</td>
</tr>
<tr>
<td>'SPX'</td>
<td>0.08</td>
<td>0.13</td>
<td>7.23</td>
<td>0.00</td>
<td>0.07</td>
<td>0.11</td>
<td>7.73</td>
<td>0.00</td>
</tr>
<tr>
<td>'TPX'</td>
<td>0.01</td>
<td>0.09</td>
<td>16.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>16.39</td>
<td>0.00</td>
</tr>
<tr>
<td>'UKX'</td>
<td>0.02</td>
<td>0.07</td>
<td>11.15</td>
<td>0.00</td>
<td>0.07</td>
<td>0.14</td>
<td>1.16</td>
<td>0.25</td>
</tr>
<tr>
<td>All</td>
<td>0.08</td>
<td>0.13</td>
<td>13.84</td>
<td>0.00</td>
<td>0.05</td>
<td>0.10</td>
<td>19.24</td>
<td>0.00</td>
</tr>
</tbody>
</table>

From our model (proposition 4), the relationship between incentives and risk depends upon the likelihood of outperforming the benchmark. In order to test this, we assume that, considering the sample of funds for each benchmark, the group of funds with better past-performance is more likely to frame a gain situation and so risk and incentives should have a negative relationship. On the other hand, the group of funds with the worse past-performance is more likely to frame a loss situation and act in a risk seeking way (risk and incentives should have a positive relationship). In this sense, we evaluated both the short term and long term relative risk strategies of the fund managers.

For the short term strategy we considered the returns in the first half of the year. The funds classified as winners are those with a previous return in the top 25%
percentile, and the losers are those with returns in the bottom 25%\(^{20}\). We then regressed\(^{21}\) the following 6-month tracking error, taking the management fee as the explanatory variable, as follows\(^{22}\):

\[
\log(\beta_i - 1)^2 = C_0 + C_1 \cdot dl_i \cdot mf_i + C_2 \cdot dw_i \cdot mf_i + \epsilon_i
\]

for

\[i = 1, \ldots, N\]

where \(i\) is the reference the fund, \(N\) is the number of funds for a specific benchmark, \(C_0, C_1\) and \(C_2\) are the regression coefficients, \(\beta\) is the 6-month beta for the second half of the year, \(mf\) is the fund’s management fee, \(dl\) is a dummy variable which equals 1 if the fund is a loser (considering the past 6 month return) and zero otherwise, and \(dw\) is a dummy variable which equals 1 if the fund is a winner (considering the past 6 month return) and zero otherwise. This is a cross-sectional regression for all funds in each benchmark index. The results are presented in Table 7.

We can observe that \(C_1\) is positive in 24 out of 26 cases and significant in 14 cases. \(C_2\) is negative in 12 out of 26 cases and significant in 11 out of 26. Therefore the data suggest that for loser funds, the relationship between incentives and risk is positive, and for winners this relationship is negative. The empirical results give some support to proposition 4, especially in the case of loser funds. However, both \(C_1\) and \(C_2\) were significant with the expected signs in only four cases.

---

\(^{20}\) This definition for the dummy variables will be the same for the remaining regressions. Observe that they are not complementary as there are funds which are classified neither as losers nor as winners.

\(^{21}\) We used White’s (1980) heteroskedasticity consistent covariance matrix.

\(^{22}\) Our measure of tracking error is non-negative by definition and then we run the regression on the natural logarithm of the measure in order to improve the normality of the residuals.
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Table 7 Active Funds: Short Term Tracking Error

Tracking error proxy $\left( \beta - 1 \right)^2$ for active funds for the various benchmarks considered. The explanatory variables are management fee and dummies for past performance which equals 1 if the fund is a loser (considering the past 6-months return) and zero otherwise, and equals 1 if the fund is a winner (considering the past 6-months return) and zero otherwise.

<table>
<thead>
<tr>
<th>Index</th>
<th>C0</th>
<th>p-value</th>
<th>C1</th>
<th>p-value</th>
<th>C2</th>
<th>p-value</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>'ASE'</td>
<td>-3.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.99</td>
<td>-0.26</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.23</td>
<td>0.08</td>
<td>0.09</td>
<td>0.44</td>
<td>0.01</td>
</tr>
<tr>
<td>'ATX'</td>
<td>-1.00</td>
<td>0.00</td>
<td>0.06</td>
<td>0.56</td>
<td>0.05</td>
<td>0.62</td>
<td>0.01</td>
</tr>
<tr>
<td>'CAC'</td>
<td>-4.29</td>
<td>0.00</td>
<td>1.22</td>
<td>0.00</td>
<td>-0.60</td>
<td>0.20</td>
<td>0.14</td>
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<tr>
<td>'COMP'</td>
<td>-2.78</td>
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<td>0.28</td>
<td>0.01</td>
<td>-0.03</td>
<td>0.90</td>
<td>0.05</td>
</tr>
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<td>0.00</td>
<td>0.75</td>
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<td>0.15</td>
<td>0.05</td>
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<td>'DJST'</td>
<td>-3.84</td>
<td>0.00</td>
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<td>0.06</td>
<td>0.15</td>
<td>0.05</td>
</tr>
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<td>'E100'</td>
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<td>0.00</td>
<td>0.84</td>
<td>0.01</td>
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<td>0.25</td>
</tr>
<tr>
<td>'IISX'</td>
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<td>0.00</td>
<td>1.87</td>
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<td>0.34</td>
</tr>
<tr>
<td>'INDU'</td>
<td>-2.39</td>
<td>0.00</td>
<td>0.03</td>
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For the long term strategy we considered the returns obtained in the first 2 years of the last 4 years, and classified them as winners and losers, depending on whether the fund was in the top 25% or in the bottom 25% of funds. We then regressed the tracking error for the following 2 years taking the management fee as the explanatory variable, as follows:

$$\log(\beta - 1)^2 = C_0 + C_1 \cdot d_{l_i} \cdot mf_i + C_2 \cdot d_{w_i} \cdot mf_i + \varepsilon_i$$

for $i = 1..N$
where \( i \) refers to the fund, \( N \) is the number of funds for a specific benchmark, \( C_0, C_1 \) and \( C_2 \) are the regression coefficients, \( \beta \) is the 2 years beta for the last 2 years, \( mf \) is the fund’s management fee, \( dl \) is a dummy variable which equals 1 if the fund is a loser (considering the past 2 years return) and zero otherwise, and \( dw \) is a dummy variable which equals 1 if the fund is a winner (considering the past 2 years return) and zero otherwise. This is a cross sectional regression for all funds in each benchmark index.

The results are presented in the following Table.

### Table 8 Active Funds: Long Term Tracking Error

Tracking error proxy \((\beta - 1)^2\) for active funds for the various benchmarks considered. The explanatory variables are the management fee and dummy variables for past performance, which equal 1 if the fund is a loser (considering the past 2 years return) and equal 1 if the fund is a winner (considering the past 2 years return).

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<th>p-value</th>
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</table>
We may observe that $C_1$ is positive in 24 out of 26 cases and significant in 16 cases, while $C_2$ is negative in 16 out of 26 cases and significant in only 2 out of 26. Therefore the data suggest that for loser funds, the relationship between incentives and risk is positive, but for winners the evidence is weaker. The empirical results give some support to proposition 4, especially in the case of loser funds. However, only in two cases are both $C_1$ and $C_2$ significant with the expected signs.

If we add to control variables for size and the age\textsuperscript{23} of the fund the regression as the following equation:

\[
\log(\beta_i - 1)^2 = C_0 + C_1 \cdot dl_i \cdot mf_i + C_2 \cdot dw_i \cdot mf_i + C_3 \cdot AGE_i + C_4 \cdot SIZE_i + \varepsilon_i
\]

for $i = 1..N$

we reach the results presented in Table 9:

$C_1$ is positive in 24 out of 26 cases and significant in 17 cases, while $C_2$ is negative in 17 out of 26 cases and significant in only 5 out of 26. Results are similar to Table 8. The empirical results give some support to proposition 4, especially in the case of loser funds. However, only in three cases are both $C_1$ and $C_2$ significant with the expected signs. Also, $C_3$ is negative (positive) in 16 (10) out of 26 cases and significant in 4 (3) cases, so no clear pattern emerges. $C_4$ is negative in 19 out of 26 cases and significant in 10 out of 26, which suggests that smaller funds have larger tracking errors.

\textsuperscript{23} We actually used the natural logarithm of age and size.
Table 9 Active Funds: Long Term Tracking Error

Tracking error proxy \((\beta - 1)^2\) for active funds for the various benchmarks considered. The explanatory variables are the management fee and the dummy variables for past performance which equal 1 if the fund is a loser (considering the past 2 years return) and zero otherwise, and equal 1 if the fund is a winner (considering the past 2 years return) and zero otherwise. Control variables for log(age) and log(size) are included.

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<td>-0.97</td>
<td>0.12</td>
<td>-0.29</td>
<td>0.08</td>
<td>0.26</td>
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</table>

The previous result sheds light on the problem of relating incentives to risk taking behavior, indicating that the mixed results found in previous empirical papers are probably due to a framing problem. Sensoy (2006) found that tracking error is greater, among funds with stronger incentives, while the agency theory predicts a negative relationship. The relationship between incentives and risk seems to depend on the reference, and this result is robust over various financial markets (developed and emerging markets). The asymmetry in the risk taking behavior is likely to be an invariant of the decision making process. Another interesting result is that typically the
coefficient for $C_1$ is higher than that for $C_2$, indicating the loss of aversion in human behavior.\footnote{The mean value for abs($C_1$) in Table 7 is 0.69 while that of abs($C_2$) is 0.51 (the p value for the difference = 0.05).}

Empirical studies of incentives and risk taking in the literature typically test if funds with poor performance in the first half of the year increase risk in the second half of the year (Chevalier and Ellison, 1997). In the framework of prospect theory, this will happen because loss averse managers will always increase risk as their wealth drops below the threshold, and this effect will be more pronounced for funds with higher fees.

Related to proposition 5, we implemented the following cross-sectional regression:

$$\log(\beta_i - 1)^2 = C_0 + C_1 \cdot dl_i + C_2 \cdot dw_i + \varepsilon_i$$

where $C_0$, $C_1$ and $C_2$ are the regression coefficients, $\beta$ is the 6 month beta for the second half of the year, $dl$ is a dummy variable which equals 1 if the fund is a loser (considering the past 6 month return), and $dw$ is a dummy variable which equals 1 if the fund is a winner (considering the past 6 month return). The results are presented in Table 10.

Typically, we verify the relationship that a fund increases the risk if it underperforms in the first half of the year and decreases risk if it outperforms. This supports proposition 5, and is in line with other empirical papers (Elton et al., 2003).
Table 10 Active Funds: Short Term Tracking Error

Tracking error proxy \((\beta - 1)^2\) for active funds for the various benchmarks considered. The explanatory variables are dummies for past performance which equals 1 if the fund is a loser (considering the past 6 month return) and equals 1 if the fund is a winner (considering the past 6 month return).

<table>
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<tr>
<th>Index</th>
<th>C0</th>
<th>p-value</th>
<th>C1</th>
<th>p-value</th>
<th>C2</th>
<th>p-value</th>
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<td>0.85</td>
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</table>

Finally, in terms of the use of the performance fee, we already commented that it is not common in the sample and that less than 6% of funds use this fee. However, for Brazil (IBOVESPA) and the United States (S&P500), we could observe more funds using the performance fee. From the model, funds with a performance fee should have a higher tracking error. We tested this for the previous two indices and found supportive results.\(^{25}\)

\(^{25}\) Funds that use performance fees have a higher tracking error, indicating that the performance fee acts to magnify the management fee.
2.5. Conclusions

In this study we applied the Behavior Agent Model to the professional investor environment, using the theory of contracts, and focused on the situation of active or passive investment strategies. In a deductive way, we formulated five propositions linking investment strategy, compensation and risk taking in a professional investor’s context.

Our propositions suggest that managers in passively managed funds tend to be rewarded without incentive fee and are risk averse. On the other hand, in actively managed funds, whether incentives that reduce or increase the riskiness of the fund depends on how hard it is to outperform the benchmark. If the fund is likely to outperform the benchmark, incentives reduce the manager’s risk appetite; conversely, if the benchmark is unlikely to be outperformed, incentives increase the manager’s risk appetite. Furthermore, the evaluative horizon influences the trader’s risk preferences, in the sense that if traders performed poorly in a period, they tend to choose riskier investments in the following period given the same evaluative horizon. On the other hand, more conservative investments are chosen after a period of good performance by a trader. To the best of our knowledge, this is the first time these results have been illustrated in the literature using a behavioral framework.

We tested the model in an empirical analysis over a large world sample of mutual equity funds, including developed and emerging markets, and we reached supportive results to the propositions established in the theoretical model.

Further extensions of this work may include the type of financial institution the trader works for (banks, insurance companies, pension funds) to take into account
regulatory and institutional effects. Also, delegated portfolio management often involves more than one agency layer and future work could examine how this feature affects incentives? More generally, studies about general equilibrium implications and price impact should be interesting, especially for policy-makers, given the relevance of these funds in all developed financial markets. Also, the consideration of other contract schemes should be of interest (Sundaram and Yermack, 2006, suggest the use of debt contracts). Indeed, Manthei and Mohnen (2004) show that factors other than the performance-dependent part of the compensation influence an individual’s effort decision. Their experimental data show significantly higher effort levels for very low or very high fixed payments.

Despite the fact that we use prospect theory assumptions, the impact of cognitive biases in an agent’s risk preference still needs to be better understood in order to understand the way psychological states may affect risk preferences in this context. We applied BAM specifically to the trader’s situation. An extension of this study to other institutional contexts would be interesting in order to find some external validity to the propositions settled. Also, in the compensation analysis, only financial compensations were considered, and we think that including non-financial rewards like recognition and prestige would enrich the theory and enable better predictions. Finally, inclusion of career concerns in the model could also improve multi-period analysis. Kempf et al. (2007) suggest that when employment risk is high, managers that lag behind tend to decrease risk relative to leading managers in order to prevent potential job loss. All of these aspects are left for future research.
Chapter Three

House-Money and Disposition Effect Overseas: An Experimental Approach

3.1. Introduction

Behavioral concepts have been used to provide explanations for several financial market inefficiencies found in empirical studies. Behavioral economists argue that the prospect of losses seems more damaging than looking at the entire wealth, even if the average outcome is the same. This sort of myopia in the face of losses may explain much of the irrationality some agents display in financial markets.

In this sense, several empirical studies (Tversky and Kahneman, 1992) have shown that when dealing with gains, agents are risk averse, but when choices involve losses, agents are risk seeking. Moreover in a wide variety of domains, people are significantly more averse to losses than they are attracted to same-sized gains (Rabin,
Loss aversion is a relevant psychological concept that has been imported to financial and economic analysis, which represents the foundation of prospect theory.\(^{26}\)

In the finance literature, there are conflicting results about the effect of prior outcomes in future risky choices. Thaler and Johnson (1990) show that when faced with sequential gambles, people are more risk-taking if they earned money on prior gambles than if they lost. The intuition is that losses are less painful to people when they occur after a gain rather than when losses happen after a previous loss. This effect is known in the financial literature as the house-money effect. Gneezy et al. (2003) investigate the house-money effect in a market experiment, and find a significant positive effect of lagged profits on expenditures on assets.

On the other hand, another stream of the literature based on individuals’ loss aversion suggests that subjects take significantly greater risk following a loss than a gain (disposition effect).\(^{27}\) Weber and Zuchel (2003) provided empirical evidence of the previous risk behavior in his portfolio treatment. In chapter two, in a theoretical study about traders’ compensation, we proposed that if traders performed poorly in a period, they tend to choose riskier investments the following period. However, considering the house-money effect, traders who have earned profits in the first period that exceed some benchmark level would become less risk-averse in the following period, because they would feel that they were gambling with the house-money (Coval and Shumway, 2005).

Benartzi and Thaler (1995) proposed another behavioral bias—myopic loss aversion (MLA) as an explanation for the equity premium puzzle (Mehra and Prescott, 1985). MLA combines two behavioral concepts, loss aversion and mental accounting,

\(^{26}\) Coval and Shumway (2005) found strong evidence that traders from the Chicago Board of Trade (CBOT) appear highly loss-averse, assuming high afternoon risk to recover from morning losses.

\(^{27}\) The increase in risk taking following a loss is also referred as “escalation of commitment”.

51
where the latter is related to how individuals employ implicit methods to evaluate the consequence of their decisions. Myopic Loss Aversion is related to the behavior of individuals to evaluate prospects considering a shorter horizon than it should be given his original investment horizon. The evaluation period tends to fit to the feedback frequency of the financial strategy.

Gneezy and Potters (1997), Thaler et al. (1997), Gneezy et al. (2003) and Haigh and List (2005) have provided experimental evidence supporting MLA. In their experiments, they change the feedback frequency of investment decisions and find that agents tend to invest more when the performance of their decisions is assessed more infrequently.

Another relevant aspect addressed in this paper is that behavioral economic research has tended to ignore the role of country differences in financial and economic decision-making, usually making universalistic assumptions of human behavior. As pointed out by Levinson and Peng (2007), despite the progress of behavioral economics and its increasing importance in explaining international financial markets, behavior theory has failed to answer how do systematic cultural and social differences affect economic, financial, and legal decision-making. Under cultural psychology, people perceive life through different lenses and interact with diverse environment. It’s expected that this fact might affect individuals’ decision making process.

The objective of this Chapter is twofold: first to verify the existence of myopic loss aversion across countries; and second to clarify which effect is dominant in a

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28 The reader is also referred to Langer and Weber (2005), who extend the concept of myopic loss aversion to myopic prospect theory, predicting that for specific risk profiles, where the chance of winning is much high, myopia will not decrease, but increase the attractiveness of a sequence.
dynamic setting: loss aversion or house-money. We test the house-money effect following Thaler and Johnson (1990), and find strong evidence of it. We then modify the experiment conducted in Gneezy and Potters (1997) and Haigh and List (2005) to test which effect is dominant: the house-money effect or the disposition effect. This experiment was conducted on students in Brazil and Spain, and results suggest that loss aversion dominates the house-money effect. Therefore, in an experiment that replicates the dynamics of decision making, the house-money effect disappears. We also report supportive results related to existence of myopic loss aversion (MLA) across countries (Brazil and Spain).

The chosen countries, despite their economic differences, have similar cultural characteristics. Gouveia and Clemente (2000) have shown that Brazil and Spain are similar in terms of the level of individualism-collectivism, with Spanish people been slightly more individualists.

In a framework close to ours, Weber and Zuchel (2003) investigate the influence of prior outcomes on risk attitude. They found that the prospect presentation might influence the risk attitude of subjects after prior outcomes. They observed that in the “portfolio treatment”, subjects take significantly greater risk following a loss than a gain. On the other hand, in the “lottery treatment,” there was greater risk taking after a gain than after a loss. However, in their experiment, subjects participated in only one sequence of two periods, so they were not given the opportunity to learn by participating in several rounds. This could influence the perception of the subjects about the likelihood of the outcomes (law of small numbers). Also, their within-subject design aiming to get the impact of prior outcomes is unnatural and unrealistic. There’s clearly a difference in asking for the behavior of a subject in the face of gains or losses; and the
direct observation of his attitude facing risky dynamic decisions. Our experimental design overcomes these previous limitations.

The remainder of this Chapter proceeds as follows. In the next section, we provide brief theoretical background of loss aversion, MLA and house-money effects. In section 3 we describe our experimental design. In section 4 we present the results and section 5 concludes the chapter, highlighting the main findings.

3.2. Theoretical Background

Benartzi and Thaler (1995) were the first to propose the myopic loss aversion (MLA) concept to elucidate the equity premium puzzle raised by Mehra and Prescott (1985). This puzzle refers to the fact that despite stocks have provided a more favorable risk-return profile; investors were still willing to buy bonds. Benartzi and Thaler took advantage of two behavioral concepts to solve the puzzle: loss aversion and mental accounting. They included the impact of anticipated emotions on decision-making. Anticipated emotions are those that decision-makers expect to experience given a certain outcome. They demonstrated that the size of the equity premium is consistent with the investors evaluating their investments annually, and weighting losses about twice as heavily as gains.

The approach of Benartzi and Thaler (1995) was not direct (experimental) evidence of MLA. However, Gneezy and Potters (1997) conducted an experiment that produced evidence to support the behavioral hypothesis of myopic loss aversion. Haigh and List (2005), in another experiment with undergraduate students and traders from the CBOT, in a design similar to the one proposed by Gneezy and Potters (1997), found evidence of myopic loss aversion (MLA), and that traders exhibit behavior consistent
with MLA to a greater extent than students. In the experiment, they evaluated the participation of the agents in a certain lottery, changing the frequency of information provided to them. Bellamare et al. (2005), in another experiment similar to Gneezy and Potters (1997), distinguish the effects of information feedback and investment flexibility over the myopic loss aversion. They found that varying the amount of information, alone, suffices to induce the behavior that is in line with the myopic loss aversion.

However, Langer and Weber (2003) found a strong MLA effect depending on the length of commitment, a much less pronounced effect of feedback, and a strong interaction between both variables. The effect of the feedback frequency was reversed for a long binding period. Huang and Liu (2007) suggest that an investor with a higher risk aversion or a longer investment horizon chooses less frequent but more accurate periodic news updates. Charness and Gneezy (2003) found that myopic loss aversion treatment participants were willing to pay money to have more freedom to choose, even though (in line with the documented bias) they invested less when having more freedom to change their investment.

There is ample evidence that feelings do significantly influence decision-making, especially when the decision involves conditions of risk or uncertainty (see Lucey and Dowling (2005) for a recent review). The consideration of risk as feeling can be presented as in the figure 1. The idea is to incorporate the fact that the emotions people experience at the time of making a decision influence their eventual decision. Edmans et al. (2007) find significant market decline after soccer losses, which is assumed to affect negatively the investors’ mood. Supported by psychologist’s research, the risk as feeling model is based on three premises: cognitive evaluations induce emotional reactions; emotions inform cognitive evaluations; and feelings can affect behavior.
In our study, we extend the previous model, by considering a multi-period analysis, suggesting a feedback process in which the experienced outcomes (experienced utility), whether they were positive or negative, will influence the following decision-making process (decision utility) inducing a different risk taking behavior. In other words, realized outcomes have an impact on the anticipated outcomes, emotions and subject probabilities; which will deviate the risk taking decision of the subject from what is predicted by standard utility functions.

Figure 1
Risk as Feelings framework

The reasons why decision utility and experienced utility fail to coincide are related to the following empirical facts: the financial decisions are cognitively hard to make; people usually have poor forecasts of preference dynamics (failure to anticipate adaptation and visceral effects); and individuals have inaccurate memories of past hedonic experiences.
Assume the following example to highlight the influence of MLA in the individual decision making process. Suppose an agent is loss averse, and weights losses relative to gains at a rate of $\lambda > 1$. Consider also that the agent is evaluating the possibility of taking part in a lottery, in which he has probability $p$ of losing $L$, and probability $(1 - p)$ of winning $W$. So the expected utility of the lottery is $E(U) = (1 - p)(W) + \lambda p(-L)$, and the agent should accept the gamble if $E(U) > 0$ or $\lambda < \frac{(1 - p)(W)}{p(L)}$. Now, if the agent is evaluating the possibility of participating in an $n$-round lottery, each with the same payoff as the previous one, the expected utility is now given by:

$$E(U_n) = nW(1 - p)^n + [(n - 1)W - \lambda L](1 - p)^{n-1} p + \ldots + \lambda(p)^n(-nL). \quad (\text{Eq. 01})$$

For instance, for values $n = 1$, $L = 1$, $p = 2/3$ and $W = 2.5$, the agent should accept the gamble if $\lambda < 1.25$. When $n = 3$ the agent should accept the gamble if $\lambda < 1.56$. As the number of rounds in the lottery increases, the gamble becomes more attractive to increasingly loss-averse agents.

Translating the previous framework to the financial market, if agents evaluate their investments more often, there will be more occasions when the risky asset (stocks) underperforms the safe asset (bonds). As individuals are loss averse, those occasions would generate to them a greater dissatisfaction, so they will tend to avoid the risky asset. Conversely, if agents evaluate their investments over longer time periods, the risky asset will rarely underperform the safe asset, and the investors will face losses more seldom. In this case, the loss averse agent will be more comfortable taking more risk.

**Hypothesis 1 (MLA):** Within a dynamic environment, decision makers who receive feedback more (less) often will take less (more) risk.
The house-money effect, proposed in Thaler and Johnson (1990), suggests that the risk tolerance of an agent increases as his wealth increases as a consequence of previous gains. To explain this effect, they developed Quasi-Hedonic Editing (QHE) Theory, which considers that decision makers segregate recent gains from their initial position, but they do not segregate recent losses. Because they have segregated their prior winnings from their initial wealth, future losses will appear less damaging, as they are losing someone else's money (the house-money). Slattery and Ganster (2002) found that decision maker who had failed to reach their goals, set lower, less risky goals in subsequent decisions.

On the other hand, disposition effect would predict that traders with profitable mornings would reduce their exposure to afternoon risk trying to avoid losses and so guaranteeing the previous gains. One major explanation for this effect is based on the value function and loss aversion from prospect theory. When subjects use the wealth they bring to the experiment plus any initial endowment in the experiment as reference point, any loss in the experiment will be perceived as a loss, not as a reduced gain as in the hedonic-editing case. The intuition is that, due to the convex shape of the value function in the range of losses, risk-seeking behavior will prevail in the case of prior losses. As empirical evidence of the previous effect, Odean (1998) and Weber and Camerer (1998) have shown that investors are more willing to sell stocks that trade above the purchase price (winners) than stocks that trade below purchase price (losers) – disposition effect. Both works interpreted this behavior as evidence of decreased risk aversion after a loss, and increased risk aversion after a gain.

It’s important to highlight that myopic loss aversion describes the bias when evaluating a prospect before taking an investment decision, while house-money and loss
aversion (as considered here in a dynamic setting) describe behavioral biases after persons experienced gains or losses. We evaluate both situations in this work.

Ackert et al. (2006) pointed out the previous contradiction between loss aversion and house-money, however they argue that prospect theory was developed for one-shot games, and so it cannot be applied to a multi-period setting. They provide the results of a multi-period experiment that gives evidence of house-money instead of disposition effect. Nevertheless, the way they test house-money is by changing the initial amount given to the subjects, and so the change in the individual wealth is not a result of the individual’s choices. We argue here, based on the risk as feeling ideas, that an experienced outcome will impact future individual’s decisions, and so we could perfectly observe the disposition effect. In this study, we want to address the influence of previous losses (or gains) in future risk-taking decisions. The following two competing hypotheses can be formulated, based on disposition and on the house-money effect, respectively:

**Hypothesis 2a(DE):** Within a dynamic environment, decision makers who receive negative (positive) feedback in a period will take more (less) risk in the following period.

**Hypothesis 2b(HM):** Within a dynamic environment, decision makers who receive negative (positive) feedback in a period will take less (more) risk in the following period.

As pointed out by Brennan (2001), the house-money effect is just one of at least three behavioral stories about how an investor will respond to good or bad news. Biased self-attribution causes the investor to become more confident as his previous positive assessment of the investment is confirmed, and therefore, he will be willing to take more risk; an effect similar to the one proposed by the house-money effect. On the other
hand, the disposition effect suggests that investors become more risk averse as their stocks increase in value, in line with the loss aversion prediction.

Related to MLA, our experimental design is similar to the experiment proposed in Gneezy and Potters (1997) and Haigh and List (2005). On the other hand, in order to test which effect is dominant, loss aversion or house-money, we introduce a novel treatment allowing participants to accumulate previous gains. We explain our experiment in depth in the next section.

In terms of the cross-country analysis, based on psychological theories, it is quite plausible that fundamental differences in how people perceive the world might predict fundamental differences in how people make financial estimations, economic decisions, and exhibit cognitive biases. One prominent theory has helped social psychologists explain systematic cultural differences: the individualism/collectivism model (Triandis, 1995). In general terms, the individualism conveys two aspects: segregation of the individual from the social group which he participates; and self-confidence. The same happens to collectivism: integration with the social group and interdependence.

Previous psychological studies have related the level of individualism to age (Hui and Yee, 1994), gender (Cha, 1994), educational level (Han and Choe, 1994), economic level (Freeman, 1997) and economic independency (Kagitçibasi, 1994). Age is considered one of the main factors that explain the level of individualism/collectivism, in the sense that elder people present a tendency towards collectivism. In terms of gender, women are more social and centred in the family, when compared to men, and this might explain their higher level of collectivism. The educational level has also being shown to influence the level of collectivism. Previous researches have described that one with lower educational level is more likely to present
friendship actions and favorable attitudes towards a member of his social group. Conversely, individuals with a higher educational level usually adopt individualistic values. Finally, a collectivism (individualism) model is produced when an economic or emotional dependency (independency) is observed.

The conclusion of previous social psychological studies is that someone who is young, man, has a high educational and economic level, and is economic independent, tends to be more individualist. Individualism is related to behavioral biases like optimism and overconfidence, which leads to less aversion to risk.

The chosen countries, despite their economic differences, have similar cultural characteristics. Gouveia and Clemente (2000) have shown that Brazil and Spain are similar in terms of the level of individualism-collectivism, with Spanish people been slightly more individualists. The previous difference is probably due to the Spanish higher economic level and economic independency. So we would expect Spanish subjects to be more comfortable in assuming more risks. Also young and men subjects are also supposed to be less risk averse.

3.3. Experiment Design

We have two behavioral biases to investigate: the MLA and the house-money (against disposition) effect. To study the MLA effect we followed closely the experimental design proposed by Haigh and List (2005). We used a straightforward 2 X 2 experimental design (see Table 1). With this setting, it’s not only possible to verify the existence of MLA, but also to investigate if there’s any country effect. Both groups of students were selected from undergraduate courses of Management and Economics.
They were evaluated in two distinct treatments: Treatment F (denoting frequent feedback) and Treatment I (denoting infrequent feedback).

<table>
<thead>
<tr>
<th>Subject Type</th>
<th>Treatment F</th>
<th>Treatment I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students from Brazil</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>Students from Spain</td>
<td>32</td>
<td>32</td>
</tr>
</tbody>
</table>

Note: Number of students in each treatment (F: frequent feedback; I: infrequent feedback) grouped by country (Brazil and Spain).

In Treatment F, our control group, subjects took part in a twelve-round lottery. In each round, they were endowed with 100 units\(^{29}\) and had to decide how much to bet in a lottery, which pays two and a half times the amount invested with one-third of probability, and zero with two-thirds of probability. The subjects were previously informed about the probabilities, payoffs and mechanism of the lottery, and so they were aware that they could win from zero to 350\(^{30}\) in each round, depending on the amount invested. At the end of each round they were informed about the lottery results and had to decide the next round bet. The experimental instructions for students in each treatment are given in the Appendix A. After the twelve rounds, all individual results were summed and the final amount was paid.

In opposite of Treatment F, we used the Treatment I (infrequent feedback), which is similar to the frequent treatment, except that subjects place their bets in blocks of three. Agents decide just before round \(t\), how much to invest in rounds \(t, t + 1\) and \(t + \)

\(^{29}\) For subjects in Brazil, 1 unit represents 1 cent of Brazilian Real, and for students in Spain, 1 unit represents 1 cent of Euro.

\(^{30}\) If the subject invests 100 and wins, his total earnings would be \(100 \times 2.5 = 250 + 100\) (his initial endowment) = 350.
2. Following the suggestion made in the literature, we restricted the bets to be homogeneous across the three rounds. After all the subjects placed their blocks of bets, they were informed about the combined results.

Based on the MLA effect, it’s expected that students in the Treatment F will invest less in the lottery, compared to those in Treatment I, as they face losses more often and are loss averse. Subjects in Treatment I face losses less often, and so should be willing to take more risk.

In order to check the house-money effect against loss aversion, we again used a 2 x 2 experimental design summarized in Table 2. With this setting it’s not only possible to verify the existence of loss aversion or house-money, but also to investigate if there’s any country effect. As in the previous setting, both groups of students were selected from undergraduate courses of Management and Economics. They were evaluated in two distinct treatments: Treatment IR (Isolated Results) and Treatment C (Cumulative Results).

<table>
<thead>
<tr>
<th>Subject Type</th>
<th>Treatment IR</th>
<th>Treatment C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students from Brazil</td>
<td>24</td>
<td>33</td>
</tr>
<tr>
<td>Students from Spain</td>
<td>32</td>
<td>33</td>
</tr>
</tbody>
</table>

Note: Number of students in each treatment (IR: isolated results; C: cumulative results) grouped by country (Brazil and Spain).

In fact, Treatment IR is the same as Treatment F and so the group is the same (control group). In opposite of Treatment IR, we used Treatment C (cumulative results), which is similar to the IR treatment, except that the amount available to invest in each pair round (t = 2, 4, 6, 8, 10 and 12) was the result individually obtained in the previous round (t = 1, 3, 5, 7, 9 and 11) respectively, plus the 100 units corresponding to that...
round. In this setting, subjects tend to place their reference point in the beginning of each two rounds, in opposite to Treatment F where each rounds’ endowment represent their reference point. The results were given to the subjects and checked after each round.

The purpose of this novel treatment arrangement is to capture the real effect on the subject’s risk preference after perceiving a gain or a loss due to his previous risk choice. We argue that this framework can definitely elucidate the house-money, loss aversion contradiction. Observe that in Treatment F, the amount available to be invested by the subject is the same for all rounds, independent of his choices, while in Treatment C, this value is affected by the previous outcome. In other words, the subject is effectively experiencing the effects of his previous risky choice. In this framework, very close to a real investment decision situation, we expect to distinguish between house-money and loss aversion.

Evidence that past winners invest more in the following period is consistent with the house-money effect. Otherwise, we have an indication that the loss aversion effect is more relevant, since the past winners are willing to take less risk to avoid future losses. It could be argued that the decisions in our proposed economic experiment could be distorted, because the money the subjects risk comes from the experimenter, rather than their own pockets. However, Clark (2002) found no evidence of the previous distortion, suggesting that use of "free" initial money endowments does not have an impact on the final experimental results. On the other hand, there are good reasons for supplying subjects with starting funds in experiments. Specifically, it facilitates recruitment, and allows subjects to make decisions in the realm of losses without leaving the experiment in debt. Also, since we are investigating future risk decisions after experiencing a loss
(gain), our results tend to be more reinforced than if subjects were using their own money.

Some final remarks about the experiment: we recruited 85 subjects for the Brazilian group from the Universidade Católica de Brasília and 97 subjects for the Spanish group from the Universidad Carlos III de Madrid. The treatment was run from January to April 2006, in a classroom in Spain and another in Brazil. Subjects were seated apart from each other avoiding communication. No more than 15 students were evaluated at a time. Each subject could take part in only one treatment group. All treatments were run using pencil and paper. After each round, experimenters circulated to check whether the subjects were calculating the payoffs correctly. To determine if a subject wins or loses in a round depends on the winning color previously settled for each participant: green, yellow or red. If the individual winning color is equal to the one randomly defined by the experimenter for that round, the subject wins; otherwise she loses. In order to compare our results with the work of Thaler and Johnson (1990), we included two questions taken from their questionnaire in our experiment. We also included two control variables: sex and age.

3.4. Experiment Results

The descriptive statistics of the sample are presented in Table 3. In total, we had 182 subjects taken from Brazil (85 students) and Spain (97 students). In terms of sex distribution, the sample is equitative except for Treatment C (Brazil), which had 75.8% of men. The average age of the participants was similar across the treatments, with the
Brazilian subjects being slightly older. The average earnings in each treatment was 1084 units (Treatment F), 1076 (Treatment I) and 1611 (Treatment C)\textsuperscript{31}.

### Table 3 - Descriptive Statistics

Descriptive statistics given for each treatment (F: frequent feedback; I: infrequent feedback; C: cumulative results). The Table provides the number of subjects in each group (# Students), the percentage of men (% Men), the average age of the students (Avg. Age), and the average bet as a percentage over the total amount available for the subject (Avg. Bet).

<table>
<thead>
<tr>
<th>Country</th>
<th>Treatment</th>
<th># Students</th>
<th>% Men</th>
<th>Avg. Age</th>
<th>Avg. Bet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>F</td>
<td>24</td>
<td>50.0%</td>
<td>24.08</td>
<td>40.10</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>28</td>
<td>57.1%</td>
<td>25.64</td>
<td>55.38</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>33</td>
<td>75.8%</td>
<td>21.36</td>
<td>44.45</td>
</tr>
<tr>
<td>Spain</td>
<td>F</td>
<td>32</td>
<td>56.3%</td>
<td>21.59</td>
<td>54.42</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>32</td>
<td>37.5%</td>
<td>22.09</td>
<td>66.02</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>33</td>
<td>54.5%</td>
<td>21.82</td>
<td>47.24</td>
</tr>
</tbody>
</table>

The main variable of interest is the amount invested by the students in the lottery, since we want to infer their risk aversion (the results for the Treatment C were normalized for a range between 0 and 100, to be comparable with the other treatments). From Table 3, we observe that the average bet varied from 40.10 to 66.02 among the treatment groups. Figure 2 shows the average bets for all groups. To maintain consistency with previous literature, we first make a non-parametric analysis of the results.

In line with our predictions, we observe a country effect, with Spanish students being less risk averse than Brazilian subjects in all treatment groups. Since MLA predicts that subjects in the F treatment should invest less than those in the I treatment, we directly observe their means in each country. Related to Brazil, the group F had an

\textsuperscript{31} Throughout our analysis we considered relative bets in order to evaluate the treatment C under the same norm of the remaining treatments. We argue that house-money and disposition effects might remain in relative terms.
average investment of 40.10, while the I treatment had 55.38. The results for the Spanish subjects follow the same tendency, with the F group having 54.42, and the I treatment 66.02. In both samples, the predictions of MLA were verified and our results were close to the ones found by Haigh and List (2005): 50.89 (group F) and 62.5 (group I).

**Figure 2 Average Bets**

Average bets given by the percentage invested over the total amount available, grouped by country (Brazil and Spain) and treatment (F – frequent feedback; I – infrequent feedback; C – cumulative results).

To determine the significance of the differences, we used the non-parametric Mann-Whitney test. Table 4 presents the results segregated by groups of three rounds. The Table can be read as follows: row 1, column 1, at the intersection of “Rounds 1-3” and “Treatment F”, denotes that the average student in Treatment F bet 45.9 (with a standard deviation of 30.9) units in rounds 1-3. As a comparison, column 2 in the same

---

32 We cannot use the parametric t-test. Given the fact that subjects are confronted with an upper and lower bound for investing, the distribution must be non-normal. Moreover, results from the Kolmogorov-Smirnov test confirm the non-normality of data.
row indicates that the average student in Treatment I bets 55.7 (standard deviation of 28.1) in the same rounds.

Considering the whole sample, the average investment of group F was 48.3 and that of group I was 61.1, with a Mann-Whitney statistic of -7.49 (p-value of 0.000), indicating that these averages are statistically different. When we consider the time evolution of the bets, we observe the same pattern, with group F investing consistently less than group I. Also, subjects in Treatment I are investing more in latter rounds than in the first ones, indicating some learning effect. When we compare Treatments F and C, the previous difference disappears. The average student in Treatment C invested 46.8 (standard deviation of 30.9) over all rounds, and the Mann-Whitney statistic when compared to the F group equals to 0.71 (p-value of 0.480), hence indicating no statistical difference. Another interesting observation from Table 4 is that subjects tend to bet more in latter rounds and this pattern remains among all treatments. Our intuition is that there is some learning effect and as the rounds are played, subjects realize the positive expected value of the lottery and are motivated to invest more.

**Table 4 - Treatments F, I and C**

Note: Columns 1-3 summarize student’s betting behavior over rounds. Standard deviations are provided in parentheses and p-values in brackets. Columns 4-5 summarize Mann-Whitney tests of the differences in behavior across treatment type.

<table>
<thead>
<tr>
<th>Rounds 1-12</th>
<th>Treatment F</th>
<th>Treatment I</th>
<th>Treatment C</th>
<th>Mann-Whitney z</th>
</tr>
</thead>
<tbody>
<tr>
<td>F vs. I</td>
<td>-7.49 [0.000]</td>
<td>-7.49 [0.000]</td>
<td>0.71 [0.480]</td>
<td></td>
</tr>
<tr>
<td>F vs. C</td>
<td>0.64 [0.520]</td>
<td>0.64 [0.520]</td>
<td>0.71 [0.480]</td>
<td></td>
</tr>
</tbody>
</table>

In order to investigate age and gender effects, we grouped the results accordingly, reaching the numbers presented in Tables 5 and 6. The results are not clear,
with some gender effect appearing in the groups I and C, and age effect being significant in group F. Despite the previous results on gender and age were not very robust, they go in line with the cultural behavioral prediction that a man and young subject is more individualist and so less risk averse. Johnson et al. (2006), experiment results show that age increases loss aversion.

**Table 5 - Treatments grouped by gender**

Columns 1-2 summarize student’s betting behavior over treatments, grouped by gender. Standard deviations are provided in parentheses and p-values in brackets. Column 3 summarizes Mann-Whitney tests of the differences in behavior across age groups.

<table>
<thead>
<tr>
<th>Rounds 1-12</th>
<th>Women</th>
<th>Men</th>
<th>Mann-Whitney z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment F</td>
<td>46.6 (32.0)</td>
<td>49.8 (36.7)</td>
<td>-0.57 [0.570]</td>
</tr>
<tr>
<td>Treatment I</td>
<td>59.4 (25.9)</td>
<td>62.9 (32.9)</td>
<td>-2.52 [0.012]</td>
</tr>
<tr>
<td>Treatment C</td>
<td>41.7 (26.2)</td>
<td>48.1 (32.9)</td>
<td>-2.33 [0.020]</td>
</tr>
</tbody>
</table>

**Table 6 - Treatments F and I grouped by age**

Columns 1-2 summarize student’s betting behavior over treatments, grouped by gender. Standard deviations are provided in parentheses and p-values in brackets. Column 3 summarizes Mann-Whitney tests of the differences in behavior across age groups.

<table>
<thead>
<tr>
<th>Rounds 1-12</th>
<th>Age &lt;= 23</th>
<th>Age &gt; 23</th>
<th>Mann-Whitney z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment F</td>
<td>50.6 (35.0)</td>
<td>40.7 (32.1)</td>
<td>-2.98 [0.003]</td>
</tr>
<tr>
<td>Treatment I</td>
<td>62.8 (28.9)</td>
<td>59.0 (32.9)</td>
<td>-1.24 [0.216]</td>
</tr>
<tr>
<td>Treatment C</td>
<td>46.0 (31.1)</td>
<td>46.3 (30.9)</td>
<td>0.10 [0.923]</td>
</tr>
</tbody>
</table>

Table 7 presents the investment results of Treatments F, C and I, grouped by country. We can observe a significant country effect in Treatments F and I with subjects in Spain being consistently less risk averse, while group C didn’t present a statistically
significant difference\textsuperscript{33}. This result goes in line with our predictions based on the fact that Spanish students are wealthier and slightly more individualists. Observe that all subjects were undergraduate students and so sharing approximately the same educational level, which could be influencing our results. That’s why the differences in the risk appetite of Spanish and Brazilian students are probably due to economic factors and not to the educational level. Also, Spanish students were slightly younger than the Brazilian subjects which can help explaining the results.

Table 7 - Treatments F and I grouped by country

Columns 1-2 summarize student’s betting behavior over treatments, grouped by country. Standard deviations are provided in parentheses and p-values in brackets. Column 3 summarizes Mann-Whitney tests of the differences in behavior across country groups.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Rounds 1-12 & Brazil & Spain & Mann-Whitney z \\
\hline
Treatment F & 48.7 (36.1) & 54.4 (35.0) & 2.56 [0.011] \\
Treatment I & 58.1 (28.5) & 66.0 (29.9) & 3.97 [0.000] \\
Treatment C & 45.5 (31.0) & 46.5 (29.3) & 0.49 [0.625] \\
\hline
\end{tabular}
\end{table}

To infer the house-money effect against loss aversion, Table 8 presents the average investment decision in each group, considering the decision taken after a loss and after a gain. Contradicting the predictions of the house-money effect, subjects invested more after a loss and less after a gain, supporting loss aversion, in Treatment C. However house-money prevails in Treatment I. So it seems that if we don’t make the

\textsuperscript{33} We believe that these results could in part be due to a wealth effect. One unit in the experiment represented 1 Euro cent for students in Spain and 1 Brazilian Real cent for students in Brazil. Since 1 EUR = 2.2 BRL, European students received 2.2 as much as students in Brazil. However in terms of purchase power parity, this difference is much less. (1 BigMac = 4.50 BRL in Brazil = 3.50 EUR in Spain)
subject feel the monetary consequences of his previous decision affecting his available income for his future choices (as we did in Treatment C), the results are not clear.

An interesting result is that when we asked the subjects about their risk preferences, in line with Thaler and Johnson (1990), we found that 79% of the students would increase their bets after a gain and just 37% would invest after a loss; thereby supporting the house-money effect. We conclude from these findings that the prospect of a loss (gain) is different from the experience of a loss (gain), and in a dynamic environment, the outcome effect dominates the individual’s previous cognitive evaluation. Previous papers (Haigh and List, 2005; Bellamare et al, 2005; Gneezy and Potters, 1997) didn’t find conclusive results about this dynamic risk aversion behavior.

Table 8 - Treatments F and I grouped previous gain or loss

Columns 1-2 summarize student’s betting behavior over treatments, grouped by previous result. Standard deviations are provided in parentheses and p-values in brackets. Column 3 summarizes Mann-Whitney tests of the differences in behavior across previous result groups.

<table>
<thead>
<tr>
<th>Rounds 1-12</th>
<th>After Loss</th>
<th>After Gain</th>
<th>Mann-Whitney z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment F</td>
<td>50.1 (32.3)</td>
<td>49.5 (35.4)</td>
<td>0.61 [0.520]</td>
</tr>
<tr>
<td>Treatment I</td>
<td>61.2 (29.7)</td>
<td>59.1 (32.7)</td>
<td>1.27 [0.181]</td>
</tr>
<tr>
<td>Treatment C</td>
<td>53.1 (31.5)</td>
<td>43.5 (31.5)</td>
<td>5.09 [0.000]</td>
</tr>
</tbody>
</table>

Although the previous analysis already gives some support to hypothesis 2a, and rejects hypothesis 2b, we can implement a complementary statistical inference. As in Haigh and List (2005), to provide a test for robustness, we estimate a regression model, in which we regressed\textsuperscript{34} the individual bet on a dummy variable for the country, a dummy variable for the treatment and on the interaction between the two. The results are presented in Table 9, in which we can observe a significant country effect.

\textsuperscript{34} We estimated a Tobit regression (censored regression model) as the dependent variable (individual bet) is non-negative.
Considering students from groups F and I, the average student from Spain invests 10.40 more than a Brazilian student. The treatment effect is also significant among groups F and I, indicating that students from Treatment F, on average, invest 16.87 less than the students from group I, again supporting the MLA predictions. When we consider the students from Treatments F and C, the country effect is still significant but no treatment effect remains.

**Table 9 - Regression Results**

Tobit regression, p-values are provided in brackets. The dependent variable is the individual bet. “Brazil” is the omitted subject category, and therefore represents the baseline group. Country S (Treatment F) is the country (treatment) indicator variable that equals 1 if the subject was a Spanish student (in Treatment F), 0 otherwise. Country S*Treatment F is the country indicator variable interacted with the frequent feedback treatment variable. P-values of each regression coefficient are in brackets. The F-statistic, $R^2$ and the number of observations, N, are also provided.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Specification</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F and I</td>
<td>F and C</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>55.13 [0.000]</td>
<td>42.80 [0.000]</td>
<td></td>
</tr>
<tr>
<td>Country S</td>
<td>10.40 [0.000]</td>
<td>4.94 [0.050]</td>
<td></td>
</tr>
<tr>
<td>Treatment F</td>
<td>-16.87 [0.000]</td>
<td>-3.68 [0.182]</td>
<td></td>
</tr>
<tr>
<td>Country S*Treatment F</td>
<td>4.36 [0.227]</td>
<td>9.83 [0.009]</td>
<td></td>
</tr>
<tr>
<td>F stat</td>
<td>36.66</td>
<td>10.35</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>7.34%</td>
<td>2.08%</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1392</td>
<td>1464</td>
<td></td>
</tr>
</tbody>
</table>

Finally, to investigate the house-money effect, we regressed students’ bets on a dummy variable indicating if the student had a prior gain or loss, a dummy for the treatment and their interaction. Table 10 provides the results. When we consider students from groups F and I, no significant win/lose effect is found, which is in agreement with previous results found in the literature that use an experimental design close to ours. The treatment effect is significant among groups F and I, indicating that
students from Treatment F, on average, invest 9.88 less than the students from group I, again supporting the MLA predictions.

If we investigate groups F and C, the fact of having a prior gain induces a significant reduction in the amount invested by 9.68, gives support to our hypothesis 2a, and rejects the house-money effect. It also indicates that our experimental design (Treatment C) could elucidate the contradiction between house-money and loss aversion in dynamic settings.

Table 10 - Regression Results

Tobit regression, p-values are provided in brackets. The dependent variable is the individual bet. “Previous Loss” is the omitted subject category, and therefore represents the baseline group. Prior Gain (Treatment F) is the win (treatment) indicator variable that equals 1 if the subject won in the previous round (in Treatment F), 0 otherwise. Prior Gain*Treatment F is the win indicator variable interacted with the frequent feedback treatment variable. P-values of each regression coefficient are in brackets. The F-statistic, R^2 and the number of observations N are also provided.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Specification</th>
<th>F and I</th>
<th>F and C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td>60.852</td>
<td>52.084</td>
</tr>
<tr>
<td>Prior Gain</td>
<td></td>
<td>0.91</td>
<td>-9.68</td>
</tr>
<tr>
<td>Treatment F</td>
<td></td>
<td>-9.88</td>
<td>-1.17</td>
</tr>
<tr>
<td>Prior Gain*Treatment F</td>
<td></td>
<td>-1.33</td>
<td>8.19</td>
</tr>
<tr>
<td>F stat</td>
<td></td>
<td>15.25</td>
<td>28.47</td>
</tr>
<tr>
<td>R2</td>
<td></td>
<td>2.68%</td>
<td>5.58%</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td>1276</td>
<td>1342</td>
</tr>
</tbody>
</table>

Observe that the coefficient for Treatment F in the specification F and C was negative (-1.17) but not significant and the coefficient for the compounding effect of Prior Gain*Treatment F was even positive and significant which would lead to an opposite result. With Treatment C we could distinguish the effect of previous risk
decisions and outcomes over future decisions and in these cases loss aversion dominates house-money.

3.5. Concluding Remarks

In the behavior finance literature, experiments were used to identify or give support to the existence of behavioral biases, which could clarify market anomalies. Nevertheless, those experiments were mainly made in unique locations: the USA or Europe. Moreover, two of these behavioral effects - house-money and loss aversion - lead to controversial results when we analyze the risk taking behavior of agents in a dynamic setting.

Our results support the existence of MLA over the countries but a country effect is also found, indicating that care should be taken when generalizing behavioral findings over the international financial market. A slightly salient gender and age effect was found in the sample. Young and male subjects presented a lower level of risk aversion.

Finally, and most importantly, disposition effect is found to dominate the risk-taking behavior of subjects in dynamic settings, overcoming the house-money effect. Subjects that experienced a gain (loss) tend to assume less (more) risk in the following period. As possible extensions we suggest to go further in the investigation of which country characteristics are generating the effect found in this study, and also the use of other countries’ students replicating the experimental design C, since we have found that there seems to be a country effect as well, which could be due to cultural differences.
Chapter Four

Behavior Finance and Estimation Risk in Stochastic Portfolio Optimization

4.1. Introduction

In a standard asset allocation process, once the risk tolerance, constraints, and financial goals are set, the output is given by a mean-variance optimization (Markowitz, 1952; Feldman and Reisman, 2002). Unfortunately this procedure is likely to fail for individuals, who are susceptible to behavioral biases. For instance, in response to short-term market movements and to the detriment of the long-term investment plan, the individual investor may require his asset allocation to be changed. Fernandes et al. (2007) suggest that early liquidation of a long term investment may be the cause of momentum.

Moreover, the paradigm of individual behavior in finance theory is based on expected utility maximization and risk aversion, which has been under attack in recent years due to its descriptive inaccuracy. Experimental psychologists have demonstrated
that people systematically deviate from the choice predictions it implies as they are typically biased.

Behavioral biases can roughly be grouped in two categories: cognitive and emotional, though both types yield irrational decisions. Because cognitive biases (heuristics like anchoring, availability, and representative biases) stem from faulty reasoning, better information and advice can often correct them. Conversely, emotional biases, such as regret and loss aversion, originate from impulsive feelings or intuition, rather than conscious reasoning, and are hardly possible to correct. Lo et al. (2005) investigated several possible links between psychological factors and trading performance, finding that subjects whose emotional reaction to monetary gains and losses was more intense on both the positive and negative side exhibited significantly worse trading performance.

Shefrin (2005) posits that the portfolios selected by investors whose choices conform to prospect theory will differ in key aspects from the portfolios selected by investors whose choices conform to expected utility theory. In this sense, an optimal solution to the asset allocation problem should guide investors to make decisions that serve their best interest. This could be the recommendation of an asset allocation that suits the investor’s natural psychological preferences (emotional biases), even though it may not maximize expected return for a given level of risk. More simply, a client’s best practical allocation may be a slightly under-performing long-term investment program to which the investor can comfortably adhere. From a mean-variance optimization perspective, behavioral investors select portfolios that are stochastically dominated. This does not mean that the individual investors are irrational in any sense: it is not irrational for people to anticipate emotional reactions and take them into account when making decisions that try to synchronize their choices to their preferences. However,
portfolio managers lack the guidelines necessary for incorporating these biases during
the process of determining asset allocation. We address this issue by evaluating whether
managers should moderate the way clients naturally behave to counteract the effects of
behavioral biases so that they can fit a pre-determined asset allocation or they should
create an asset allocation that adapt to clients’ biases, so that clients can comfortable
adhere to the fund.

In terms of emotional biases, several empirical studies (Tversky and Kahneman, 1992) have shown that, when dealing with gains, agents are risk averse, but when
choices involve losses, agents are risk seeking (asymmetric risk taking behavior).
Moreover, in a wide variety of domains, people are significantly more averse to losses
than they are attracted to same-sized gains (Rabin, 1998). Loss aversion (Schmidt and
Zank, 2005) is a relevant psychological concept that has been imported to financial and
economic analysis, and it represents the foundation of prospect theory.

In general terms, prospect theory and its latter version cumulative prospect
theory 35 (Kahneman and Tversky, 1979, 1992) posits four novel concepts in the
framework of individuals’ risk preferences. First, investors evaluate assets according to
gains and losses and not according to final wealth (mental accounting). Second,
individuals are more averse to losses than they are attracted to gains (loss aversion).
Third, individuals are risk-seeking in the domain of losses and risk averse in the domain
of gains (asymmetric risk preference). Finally, individuals evaluate extreme
probabilities in a way that overestimates low probabilities and underestimates high
probabilities (probability weighting function). This study, as far as we know, is the first
to consider all those aspects in the framework of portfolio choice.

35 Tversky and Kahneman’s Cumulative Prospect Theory (CPT) (1992) combines the concepts of loss
aversion and a non linear rank dependent weighting of probability assessments.
There are conflicting results in the finance literature on how prior outcomes affect the risk taking behavior of investors in subsequent periods. Loss aversion would predict that traders with profitable mornings would reduce their exposure to afternoon risk, trying to avoid losses and thus guaranteeing the previous gains (Weber and Zuchel, 2003). Odean (1998) and Weber and Camerer (1998) have shown that investors are more willing to sell stocks that trade above the purchase price (winners) than stocks that trade below purchase price (losers) – a phenomenon termed the disposition effect (Schefrin and Statman, 1985). Both works interpreted this behavior as evidence of decreased risk aversion after a loss, and increased risk aversion after a gain. The standard explanation for the previous behavior is based on prospect theory, and particularly on the fact that individuals are risk-seeking in the domain of losses and risk averse in the domain of gains (asymmetric risk preference).

However, another stream of the literature found the opposite behavior. Thaler and Johnson (1990) name the house-money effect, the behavior of increasing risk appetite after a gain. Barberis et al. (2001) present a model where investors are less loss averse after a gain while they become more loss averse after prior losses.

Despite the vast literature confirming the behavioral biases associated with prospect theory, the consideration of all those biases in an asset allocation framework is still missing. Barberis and Huang (2001) and Barberis, Huang, and Santos (2001) use loss aversion and mental accounting (Thaler, 1999) to explain aspects of stock price behavior, but do not employ the full prospect theory framework and don’t examine optimal asset allocation. Benartzi and Thaler (1995) consider prospect theory to solve the equity premium puzzle when investors are loss averse and evaluate their portfolios myopically with a horizon of approximately one year. They also suggest an optimal allocation in equities from 30% to 55%. Magi (2005) uses behavioral preferences to
numerically solve a simple model of international portfolio choice, providing a possible explanation for the equity home bias puzzle.

Davies and Satchell (2004) provide a solution for the optimal equity allocation, and explore more thoroughly the cumulative prospect theory parameter space that is consistent with observed equity allocations given a financial market’s returns distributions over a one-month horizon. Shefrin (2005) considers heterogeneous investors to see the impact of behavioral concepts in the framework of asset pricing.

The first main goal of this study is to incorporate mental accounting, loss aversion, asymmetric risk taking, disposition effect, and probability weighting in portfolio optimization in a multi-period setting for individual investors. We provide a solution for the asset allocation problem, taking into account all behavioral biases associated with prospect theory and using a utility function (suggested in Giorgi et. al., 2004) consistent with both the experimental results of Tversky and Kahneman, and also with the existence of equilibrium. We also shed more light on the issue of how prior outcomes affect subsequent risk taking behavior, investigating the investor’s risk taking behavior following a rise, or a fall, in the price of the risky asset.

In line with prospect theory, investors derive utility from fluctuations in the value of their final wealth. In our framework, there is a financial market on which two assets are traded. A riskless asset, also called a bond, and a risky asset, also called a stock (under the assumption of normally distributed returns for the risky asset). As we are modeling the decision making process of an individual investor, short-selling is not allowed. In each period (we consider two periods), the investor chooses the weight of his endowment to be invested in the risky asset, in order to maximize his utility (prospect theory based). We assume that the investor acts myopically, and that the reference point relative to which he measures his gains and losses for the first period is
his initial endowment. Although all agents solve the same maximization problem in the first period, the second period decision depends on the reference point relative to which the agent measures the second period outcomes (gains or losses). We consider two possible reference points: the initial wealth or the current wealth, and analyze both cases. St-Amour (2006) results reveal that references are strongly relevant and state-dependent.

Another well-known issue in asset allocation problems, using Markowitz optimization, is that the output is strongly driven by the risk/return estimation, which usually generates very unstable portfolios. The most famous problem with this technique is the substitution problem, where two assets with the same risk but slightly different expected returns. The optimizer would give all the weight to the asset with the higher expected return, leading to a very unstable asset allocation.

Recent literature has tried to overcome the previous problem of leading to unfeasible portfolios. The main focus of those models is to find out how to create realistic portfolios considering that the values used for risk and return are not deterministic but instead just estimates (they are stochastic). It should be noted that the misspecification of expected returns is much more critical than that of variances (Zimmer and Niederhauser, 2003).

Jorion (1986) offers a simple empirical Bayes estimator that should outperform the sample mean in the context of a portfolio. His main idea is to select an estimator with average minimizing properties relative to the loss function (the loss due to estimation risk). Instead of the sample mean, an estimator obtained by “shrinking” the means toward a common value is proposed (the average return for the minimum variance portfolio), which should lead to decreased estimation error. Similar to Jorion, Kempf (2002) assumes that the prior mean is identical across all risky assets. However,
Kempf’s model considers estimation risk as a second source of risk, determined by the heterogeneity of the market and given by the standard deviation of the expected returns across risky assets.

Black and Litterman (1992) postulate that the consideration of the global CAPM equilibrium can significantly improve the usefulness of asset allocation models, as it can provide a neutral starting point for estimating the set of expected excess returns required to drive the portfolio optimization process. Horst et al. (2002) propose a new adjustment in mean-variance portfolio weights to incorporate the estimation risk. The adjustment amounts to using a pseudo risk-aversion, rather than the actual risk aversion, which depends on the sample size, the number of assets in the portfolio, and the curvature of the mean-variance frontier. The pseudo risk aversion is always higher than the actual one and this difference increases with the uncertainty in the expected return estimations. Maenhout (2004) also considers an adjustment in the coefficient of risk aversion to insure the investor against some endogenous worst case.

Finally, Michaud (1998) suggests portfolio sampling as a way to allow an analyst to visualize the estimation error in traditional portfolio optimization methods, and Sherer (2002) posits that sampling from a multivariate normal distribution (a parametric method termed Monte Carlo simulation) is a way to capture the estimation error. Markowitz and Usmen, 2003, compared the traditional approach to resampling and their results support the latter. Fernandes et al. (2007) evaluates several asset allocation models and suggests that resampling methods typically offer the best results.

This study presents a novel approach (BRATE – Behavioral Resampling Adjusted Technique) to incorporate behavioral biases and estimation risk into mean-variance portfolio selection. In a paper close to ours, Vlcek (2006) proposes a model to evaluate portfolio choice with loss aversion, asymmetric risk-taking behavior, and
segregation of risk less opportunities. His findings suggest that the changes in portfolio weights crucially depend on the reference point and the ratio between the reference point and the current wealth, and thus indirectly on the performance of the risky asset. Our work differs from his study as we explicitly consider all novel aspects of prospect theory: mental accounting, loss aversion, asymmetric risk-taking behavior, and probability weighting function. We also evaluate the inefficiency cost of the behavioral biases and consider a more general form for the risky asset return process, including estimation risk in the analysis.

Considering daily equity data from the period from 1995 to 2007, we empirically evaluate our model in comparison to the traditional Markowitz model. Our results support the use of BRATE as an alternative for defining optimal asset allocation and posit that a portfolio optimization model may be adapted to the individual biases implied in prospect theory.

The remainder of this Chapter contains the following sections. Section 2 discusses the behavioral biases considered and describes our model proposing the behavioral resampling adjusted technique (BRATE). Section 3 presents the empirical study, describing the data and implementation, and providing the results. Section 4 concludes the chapter by reviewing the main achievements.

4.2. The Behavioral Model

We present a two period’s model for portfolio choice in a stylized financial market with only two assets, where the investor’s preferences are described by cumulative prospect theory as suggested by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). In our framework, there is a financial market in which two assets
are traded. A riskless asset, also called the bond, and a risky asset, the stock. Let us consider the return of the stock in each period given by the following process: 

\[ R = \mu + \sigma n, \quad \text{with} \quad n \sim N(0,1). \]

The risk free bond yields a sure return of \( R_f \). We assume that the time value of the money is positive, i.e. that interest rates are non-negative.

The preferences of the investor are based on changes in wealth and are described by prospect theory. We assume that he owns an initial endowment, \( W_0 \) (normalized to 1 monetary unit), and that he earns no other income. The agent invests a proportion \( \theta \) of his wealth in the stock and \( (1 - \theta) \) in the bond. Since we want to model the individual investor behavior, we assume that short selling is not allowed \( (0 \leq \theta \leq 1) \). We also assume that the investor acts myopically, and the reference point relative to which he measures his gains and losses in the first period is his initial wealth. Then, the perceived gain or loss in the end of the first period is given by:

\[
x = \Delta W = \left[ (1 - \theta)W_0(1 + R_f) + \theta W_0(1 + R) \right] - W_0
\]

\[
\therefore x = (1 - \theta)R_f + \theta R
\]

\[
\therefore x = (1 - \theta)R_f + \theta(\mu + \sigma n) \quad \text{(Eq. 01)}
\]

As pointed out in Vlcek (2006) the choice process under prospect theory starts with the editing phase, followed by the evaluation of edited prospects, and finally the alternative with the highest value is chosen. During the editing phase, agents discriminate gains and losses. They also perform additional mental adjustments in the original probability function \( p = f(x) \), defining the probability weighting function \( \pi(p) \). Based on experimental evidence, individuals adjust the likelihood of outcomes such that small probabilities are overweighted and large probabilities are underweighted. We will consider the probability weighting function, as in Giorgi et al. (2004) given by:
\[
\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^\frac{2}{\gamma}}, \quad (\text{Eq. 02})
\]

where \(\gamma\) is the adjustment factor. The following graph compares the values of \(p\) and \(\pi(p)\), considering \(\gamma = 0.80\).\(^{36}\)

Figure 1 – Cumulative probability weighting function for \(\gamma = 0.80\).

In the valuing phase, the agents attach a subjective value to the gamble. Let us assume the value function proposed by Giorgi et al. (2004), as follows:

\[
v(x) = \begin{cases} 
\lambda^+ - \lambda^+ e^{-\alpha x}, & \text{if } x \geq 0 \\
\lambda^- e^{\lambda x} - \lambda^- & \text{if } x < 0
\end{cases} \quad (\text{Eq. 03})
\]

where \(\alpha\) is the coefficient of absolute risk preference, \(\lambda^- > \lambda^+ > 0\) makes the value function steeper in the negative side (loss aversion), and \(x\) is the change in wealth or welfare, rather than final states (mental accounting), as proposed by Kahneman and Tversky (1979). Also, the value function is concave above the reference point and convex below it (asymmetric risk preference). It is useful to consider the previous

\(^{36}\)Experiments suggest a value of \(\gamma\) between 0.80 and 0.90.
form for the value function because of the existence of a CAPM equilibrium\textsuperscript{37} and the ability to reach constant coefficients of risk preference (\( \alpha \)). The previous formulation is also supported by the laboratory results from Bosh-Domènech and Silvestre (2003). The following graph indicates \( v(x) \) when \( \alpha = 0.88, \lambda^- = 2.25 \) and \( \lambda^+ = 1 \) (Kahneman and Tversky suggested values).

![Figure 2 – Prospect theory value function for \( \alpha = 0.88, \lambda^- = 2.25 \) and \( \lambda^+ = 1 \)](image)

In our two-period model for portfolio choice, the investor chooses a weight in the risky asset to maximize his expected utility (V). His preferences are based on changes in his wealth (\( x \)) and are described by prospect theory. The total expected value he addresses to a given choice of \( \theta \) is given by:

\[
V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx \tag{Eq. 04}
\]

\textsuperscript{37} Under Cumulated Prospect Theory (CPT) with Tversky and Kahneman (1992) specifications, equilibria do not exist as at least one investor can infinitely increase his utility by infinitely leveraging the market portfolio (the utility index is almost linear for large stakes), while the Security Market Line Theorem holds (Giorgi et al., 2004).
where \( v(x) \) is the prospect value of the outcome \( x \), and \( \pi(f(x)) \) is the weighted cumulative probability associated with that outcome. Prospect theory is a descriptive theory, postulating that, in comparing alternatives, the investor will choose the alternative that makes \( V \) as high as possible. Let us then evaluate the investor’s problem in each period.

4.2.1 First Period

In the first period, the agent’s problem consists of defining the allocation of his initial wealth between the two assets traded in the financial market. He maximizes his utility in \( t = 0 \) by allocating a fraction, \( \theta_0 \), of his initial wealth \( W_0 \), in the risky asset and \( (1 - \theta_0) \) in the riskfree asset. We consider that the investor is a myopic optimizer in the sense that he takes into account only the first period result. For multi-period horizons, the choices at earlier dates impact the reference points at later dates. This feature allows for complex modeling. However, as pointed out in Shefrin (2005), prospect theory is a theory about investors who oversimplify, and so, assuming that individuals are sophisticated enough to perceive the link between their current choices and future reference points is something unreasonable. We also constrain short selling, as it is common for individual investors’ models. Thus, his problem can be given by

\[
\max_{\theta_0 \in [0, 1]} V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx \quad \text{(Eq. 05)}
\]

Let us make the following derivation: \( x = (1 - \theta_0)R_f + \theta_0(\mu + \sigma n) \). Rearranging the terms in \( x \), we get \( x = (1 - \theta_0)R_f + \theta_0\mu + \theta_0\sigma n \). We call \( (1 - \theta_0)R_f + \theta_0\mu = B \) and \( \theta_0\sigma = C \). Then, \( x = B + Cn \), and so \( x > 0 \) implies \( n > -\frac{B}{C} \). Then,

---

38 We will consider the investor’s initial wealth equals to 1.
\[
V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx
\]

\[
\therefore V = \int_{0}^{\infty} \left[-\lambda^+ e^{-\alpha x} + \lambda^+\right] d\pi(f(x)) + \int_{-\infty}^{0} \left[\lambda^+ e^{\alpha x} - \lambda^-\right] d\pi(f(x))
\]

\[
\therefore V = \int_{\frac{B}{C}}^{\infty} \left[-\lambda^+ e^{-\alpha \beta + \lambda^+} + \lambda^+\right] d\pi(f(n)) + \int_{-\infty}^{\frac{B}{C}} \left[\lambda^+ e^{\alpha \beta + \lambda^-} - \lambda^-\right] d\pi(f(n))
\]

\[
\therefore V = \lambda^+ \left[1 - \hat{\pi}(\frac{B}{C})\right] - \lambda^+ \pi(-\frac{B}{C}) + \lambda^+ e^{\hat{\pi}} \int_{\frac{B}{C}}^{\infty} e^{\alpha \beta} d\pi(f(n)) - \lambda^+ e^{-\alpha \beta} \int_{-\infty}^{\frac{B}{C}} e^{-\alpha \beta} d\pi(f(n))
\]

\[
\therefore V = \lambda^+ - \left(\lambda^+ + \lambda^+\right) \pi\left(-\frac{B}{C}\right) + \lambda^+ e^{\hat{\pi}} \int_{\frac{B}{C}}^{\infty} e^{\alpha \beta} d\pi(f(n)) - \lambda^+ e^{-\alpha \beta} \int_{-\infty}^{\frac{B}{C}} e^{-\alpha \beta} d\pi(f(n))
\]

\[
\therefore V = \lambda^+ - \left(\lambda^+ + \lambda^+\right) \pi\left(-\frac{B}{C}\right) + e^{\alpha \beta} \left[\lambda^+ e^{-\alpha \beta} \pi(-\frac{B}{C} - \alpha C) - \lambda^+ e^{-\alpha \beta} \pi(\frac{B}{C} - \alpha C)\right] \quad (Eq. 06)
\]

Where, for the last step, we used\(^{39}\):

\[
\int_{-\infty}^{\infty} e^{-\alpha \beta} d\phi(x) = e^{\frac{1}{2} \alpha^2 \sigma^2} \hat{\phi}(\alpha \sigma)
\]

Observe that, if we were considering a standard utility function (risk aversion over all possible outcomes), the value would be given by:

\[
V^S = \lambda^+ - \lambda^+ e^{-\alpha \beta + \frac{1}{2} \alpha^2 \sigma^2} \quad (Eq. 07)
\]

Moreover, the partial derivatives of V (Eq. 06) are:

\[
\frac{\partial V}{\partial \mu} = \{\alpha e^{\frac{1}{2} \alpha^2 \sigma^2} \left[\lambda^+ e^{\alpha \beta} \left[\left(1 - \theta_0\right) R_f + \theta_0 \mu\right] - \alpha \theta_0 \sigma\right] + \lambda^+ e^{-\alpha \beta} \left[\left(1 - \theta_0\right) R_f + \theta_0 \mu\right] - \alpha \theta_0 \sigma\} \cdot \theta_0 \quad (Eq. 08)
\]

\[
\frac{\partial V}{\partial \sigma} = \alpha^2 \theta_0 e^{\frac{1}{2} \alpha^2 \sigma^2} \left[\lambda^+ e^{\alpha \beta} \frac{\left[\left(1 - \theta_0\right) R_f + \theta_0 \mu\right]}{\theta_0 \sigma} - \alpha \theta_0 \sigma\right] - \lambda^+ e^{-\alpha \beta} \frac{\left[\left(1 - \theta_0\right) R_f + \theta_0 \mu\right]}{\theta_0 \sigma} \cdot \theta_0 \quad (Eq. 09)
\]

\(^{39}\) It’s valid for \(\gamma = 1\).
As a consequence, the following properties hold,

i)  \( \frac{\partial V}{\partial \mu} > 0 \);

ii)  \( \frac{\partial V}{\partial \sigma} = 0 \) for \( \sigma = 0 \) or \( \sigma = \infty \);

iii)  \( \frac{\partial V}{\partial \sigma} < 0 \) for \( \sigma > 0 \).

Equations 06 and 07 clearly yield different weights for the risky asset, considering the remaining parameters fixed. Thus, it is possible to evaluate the cost of inefficiency associated with the behavioral biases as compared to the standard utility solution.

\[ \text{Cost} = \left[ (1 - \theta^*_0 \mu) R_f + \theta^*_0 \sigma^2 \right] - \left[ (1 - \theta^*_{PT} \mu) R_f + \theta^*_{PT} \sigma^2 \right] \quad (\text{Eq. 10}) \]

where \( \theta^*_0 \) is the risky asset weight given by the standard utility maximization problem, and \( \theta^*_{PT} \) is the stock weight as defined in our model.

**Proposition 1.** The optimal asset allocation in \( t = 0 \), for the risky asset \( \theta^*_0 \), is such that it maximizes the value function given by:

\[ V = \lambda^* - \left( \lambda^* + \dot{\lambda} \right) \pi \left( -\frac{B}{C} \right) + e^{\frac{1}{2} \alpha^2 \sigma^2} \left[ \lambda^* e^{\alpha \beta} \pi \left( -\frac{B}{C} - \alpha C \right) - \dot{\lambda}^* e^{-\alpha \beta} \pi \left( \frac{B}{C} - \alpha C \right) \right] \]

where: \( B = \left[ (1 - \theta^*_0) R_f + \theta^*_0 \mu \right] \) and \( C = \theta^*_0 \sigma \).

If we were considering a standard utility function, the optimal allocation in \( t = 0 \), for the risky asset would then be given by:

\[ \theta^*_0 = \frac{\mu - R_f}{\alpha \sigma^2} \]

---

40 See Appendix B for the proofs.
Let us first consider standard values for the model’s parameters\(^{41}\). The risk free rate equals the historical annual return of the US three-month Treasury Bill (\(R_f = 2.73\%\)). The equity expected return and volatility equals the historical average of the MSCI global equity index and its standard deviation (\(\mu = 7.61\%\) and \(\sigma = 12.98\%\)). The adjustment factor in the probability weighting function equals \(\gamma = 0.90\). The coefficient of risk aversion equals \(\alpha = 3\). Also, as suggested by Kahneman and Tversky, \(\lambda^- = 2.25\) and \(\lambda^+ = 1\). The individual’s values (prospect theory and standard) as a function of the percentage of his wealth invested in the risky asset are given in Figure 3. The individual investor is expected to choose the allocation in the risky asset which maximizes his expected value.

\[\begin{align*}
V(\theta) & \quad \text{for prospect theory} \\
V_s(\theta) & \quad \text{for standard utility}
\end{align*}\]

\(\text{Figure 3 – Prospect value and standard utility value as function of } \theta\)

\(^{41}\) The riskfree rate, the expected return of the risky asset and the volatility of the risky asset were calculated, using daily data, over the period from 1995 and 2007. The results were annualized.
As can be observed from the graph, using a standard utility function, the allocation in the risky asset approaches 100% (theta for which the value function reaches its maximum), while using prospect theory utility, the investor should allocate 81% of his wealth in the stock. The shapes of the graphs are different, notably for large allocations in the stock. The value function using standard utility is equal to or greater than the one for prospect utility.

The reason for this difference comes from the fact that in prospect theory, negative outcomes are penalized more (as are risky portfolios) because individuals are loss averse ($\lambda^- > \lambda^+$). In the loss aversion literature evidence suggests that individuals are around twice more sensitive to losses than they are attracted to same size gains. For small allocations in stocks, the prospect of losses becomes less likely and the value functions tend to coincide.

Related to the effect of probability weighting, if we set $\gamma = 1$, thus canceling out its effect, we reach the following Figure representing the value function:

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Davies and Satchell (2004) found that the average proportion in domestic and foreign equities of large pension funds in 1993 was 83% in the UK, which is in line with the prospect theory results.
Note that the amount optimally invested by the behavioral investor in the risky asset decreases to 48%, and so probability weighting tends to increase the risk appetite. Kahneman and Tversky (1979) suggest that the overweighting of low probabilities has an ambiguous effect on risk taking, as it can induce risk aversion in the domain of losses, and risk seeking in the domain of gains. In our case, the overestimation of the extreme positive outcomes probabilities, shown in Figure 3, is inducing investors to take more risk.

However, despite the effects of loss aversion and probability weighting, even if we consider $\lambda^- = \lambda^+ = 1$ and $\gamma = 1$, keeping constant the remaining parameters, the value functions wouldn’t coincide, as can be seen in Figure 5:
Both models would predict that the investor should allocate 100% of his endowments in the stock. However, the value functions are different because, in prospect theory, individuals are risk seeking in the loss domain (asymmetric risk preference). Thus, they would be more comfortable in allocating a greater part of their wealth in the risky asset. The prospect value function is greater than the standard utility function.

Observe that the effect of the asymmetric risk preference goes in the opposite direction of loss aversion and probability weighting. When we diminish the coefficient of risk preference \( \alpha = 0.25 \) in both utility functions, we reduce the effect of asymmetry, and so the value functions are much closer, as can be seen in the following figure.
The effects of the behavioral biases can thus be summarized as follows: loss aversion reduces risk taking, and asymmetric risk taking behavior induces risky attitudes. Probability weighting has an ambiguous effect on risk. Our intuition is that, in the long run, as the value function parameters are changing, these biases tend to cancel out, eliminating the efficiency loss originated by each bias. That is why we argue that human biases do not need to be moderated to reach an efficient investment strategy. The experimental results of Blavatskyy and Pogrebna (2006) reveal that the effect of loss aversion is largely neutralized by the overweighting of small probabilities and underweighting of moderate and high probabilities.

In order to verify property (i), Let us evaluate $V$ while changing $\mu$ and keeping constant the other parameters (considering $\theta = 50\%$). Figure 7 presents the graph which indicates that over all positive values of $\mu$, the slope of $V$ is positive. The value
function is increasing in $\mu$. Thus, when the risky asset has a higher expected return, *ceteris paribus* implies a higher value for the investor.

![Graph showing prospect value as function of $\mu$](image)

**Figure 7 – Prospect value as function of $\mu$**

Considering properties (ii) and (iii), Let us evaluate $V$ while changing $\sigma$ and keeping constant the other parameters (considering $\theta = 50\%$). Figure 8 presents the graph indicating that over all positive values of $\sigma$, the slope of $V$ is negative, while for $\sigma = 0$, the slope is null. When $\sigma$ tends to infinity, the slope tends to null. The value function is decreasing in $\sigma$.

The intuition is that, if the volatility of the risky asset is higher, for the same allocation, this implies a higher probability of losses reducing the value of the prospect. In line with traditional rational investor, behavioral individuals also prefer higher return and lower risk; mainly because they are risk averse in the gain domain and also loss averse.
Now let us evaluate the values of $\theta_0$ when we change the riskfree rate and the expected return of the risky asset. Since many parameters are involved, it is not possible to find closed form solutions for $\theta_0$. Therefore, we present numerical results for the optimal allocation of wealth in $t = 0$. Figure 9 presents the results for $0% < \mu < 15%$ and $0 < R_f < 6%$. The remaining parameters are fixed ($\sigma = 12.98\%$, $\alpha = 3$, $\lambda^- = 2.25$, and $\lambda^+ = 1$).
As expected, when the risky asset offers more attractive returns, the agent gradually invests more in the stock. When the stock is very attractive, the investor chooses to allocate his entire wealth in the risky asset. Thus, we observe that $\theta_0$ is increasing in $\mu$ and decreasing in $R_f$. Also, when $R_f$ is higher, the changes in $\theta_0$ due to a variation in $\mu$ are smoother, because in these cases losses are less likely and we approach the standard utility solution. When $R_f$ is lower, the changes in $\theta_0$ due to a variation in $\mu$ are more abrupt, giving rise to extreme portfolio allocations. If we consider that $\mu$ is not known with certainty, the resulting portfolio would be very unstable. Gomes (2003), in a model with loss-averse investors, has found that individuals will not hold stocks unless the equity premium is quite high.

We can evaluate the expected cost of inefficiency related to the behavioral biases associated to the prospect theory function, for the same parameters considered in
the previous analysis, using equation 10. The result is presented in Figure 10, and its form is due to the fact that, in standard utility function, the investor is willing to take more risk than with the loss averse prospect utility. The cost is due to the fact that the expected return of the stock is greater than the bond, and the standard utility investor is allocating a greater part of his wealth in the risky asset than the prospect utility individual. Thus, the cost is increasing in $\mu$. However, it is worth noting that the previous cost is based on expected returns, which are stochastic in practice. The real cost can just be observed at the end of the first period with the realization of the stock’s return.

An important insight can be made from Figure 10 in terms of the best practice for asset allocation. As long as the risk free rate is lower and the expected return of the stock is higher, the optimal allocation should moderate the investor’s biases in order to reach a better performance. On the other hand, if the risk premium is lower, the moderation is less relevant, and the optimal allocation may adapt to the individual’s biases.
Figure 10 – Expected cost in the first period as function of $\mu$ and rf.

We can also analyze the change in the allocation of the stock when we vary the loss aversion in the risk taking behavior. The result is shown in Figure 11, for $2 < \lambda^- < 4$. Observe that, as long as the investor is much more averse to losses than he is attracted to gains, the allocation in the risky asset is lower. When $\lambda^- = 2.25$, the allocation in the risky asset corresponds to 81%, as previously mentioned.
Dimmock (2005) has already shown that a higher level of loss-aversion leads to lower equity exposure, and heterogeneity in the coefficient of loss aversion has the ability to explain puzzling features of household financial behavior.

### 4.2.2 Second Period

In order to evaluate the second period allocation choice of the investor, let us keep some parameters fixed: \((\sigma = 12.98, \; \alpha = 3, \; \lambda^- = 2.25 \; \text{and} \; \lambda^+ = 1)\). After the investor has made his first period decision in \(t = 0\), the state of nature realizes in \(t = 1\), when he is faced with his second period problem. Again, he must allocate his wealth in the two possible assets in the financial market, bond and stock, to maximize his utility. Let us consider the same normal distribution for the return of the risky asset. The investor’s wealth position at \(t = 2\) equals his position in \(t = 1\) plus the return of his portfolio in the second period.
While all agents solve the same maximization problem in the first period, in the second period, it will depend on the reference point to which he measures his gains and losses (in the framework of prospect theory). In our model, there are two candidates for the investor’s reference point at $t = 1$: his initial wealth at $t = 0 (W_0) = 1$ or his wealth at the end of the first period, $t = 1 (W_1)$. If he measures his gains and losses relative to his wealth at $t = 1$ (his current wealth), he treats each gain and loss separately. On the other hand, if he considers his initial wealth as the reference point, he adds up the outcomes (gains and losses), that is, he nets his positions. The previous distinction is relevant in prospect theory. The value function is concave in the domain of gains and convex in the loss domain (asymmetric risk behavior).

First, let us consider as the investor’s reference point his current wealth at $t = 1$. In this case, the maximization problem he will solve in the second period is the same as the one for the first period. Thus, we can state the following proposition.

**Proposition 2.** The optimal asset allocation in $t = 1$, for the risky asset $\theta^*_1$, if the agent measures his gains and losses relative to his current wealth, is such that maximizes the same value function of the first period. $\theta^*_1 = \theta^*_0$

We can observe that an individual who measures his gains and losses relative to his current wealth is actually solving the same maximization problem in each period. That is why the allocation in the risky asset might be the same. This is not surprising; as he is not using past information to update his beliefs about the assets, his preferences are similarly unaffected.

Next, let us analyze the investor’s maximization problem if he evaluates his gains and losses relative to his initial wealth. If he has an initial wealth position of $W_0$.
= 100 and his wealth rises in the first period to \( W_1 = 110 \) and falls in the next period to \( W_2 = 105 \), he values his position at \( t = 2 \) as a gain of 5, and not as a gain of 10 followed by a loss of 5.

In the second period, the agent’s problem consists of defining the allocation of his wealth \( (W_1) \) between the two assets traded in the financial market. He maximizes his utility in \( t = 1 \) by allocating a fraction, \( \theta_i \), of his wealth \( W_1 \) in the risky asset and \( 1 - \theta_i \) in the risk-less asset. As we did in the first period analysis, we also constrain short selling.

\[
\max_{0 \leq \theta \leq 1} V = \int_{-\infty}^{\infty} \nu(x) \frac{d}{dx} \pi(f(x))dx
\]

Let us make the following derivation:

\[
x = W_1 \left[ (1 - \theta_i)R_f + \theta_i(\mu + \sigma i) \right] + W_0 \left[ (1 - \theta_i)R_f + \theta_i R_1 \right]
\]

and \( W_i = W_0 \left[ 1 + (1 - \theta_i)R_f + \theta_i R_1 \right] \), where \( R_i \) is the return of the stock in the first period. So \( x = W_0 \left[ 1 + (1 - \theta_i)R_f + \theta_i R_1 \right] \cdot \left[ (1 - \theta_i)R_f + \theta_i(\mu + \sigma i) \right] + (1 - \theta_i)R_f + \theta_i R_1 \].

Rearranging the terms in \( x \) and considering \( W_0 = 1 \), we get

\[
x = \left[ \theta_i \sigma n \left( 1 + \left( 1 - \theta_i \right) R_f + \theta_i R_1 \right) \right] + \left( 1 + \left( 1 - \theta_i \right) R_f + \theta_i R_1 \right) \cdot \left[ (1 - \theta_i)R_f + \theta_i \mu \right] + \left( (1 - \theta_i)R_f + \theta_i R_1 \right)
\]

Let us call

\[
B = \left[ (1 + \left( 1 - \theta_i \right) R_f + \theta_i R_1 \right) \cdot \left[ (1 - \theta_i)R_f + \theta_i \mu \right] + \left( (1 - \theta_i)R_f + \theta_i R_1 \right)
\]

and

\[
C = \theta_i \sigma \left( 1 + \left( 1 - \theta_i \right) R_f + \theta_i R_1 \right)
\]

Then, \( x = B + Cn \), so \( x > 0 \) implies \( n > -\frac{B}{C} \). Then,

\[
V = \lambda^+ - \left( \lambda^+ + \lambda^- \right) \pi \left( -\frac{B}{C} \right) e^{-\frac{1}{2} \sigma^2 n^2 C^2} \left[ \lambda^+ e^{\alpha B} \pi \left( -\frac{B}{C} - \alpha C \right) - \lambda^- e^{-\alpha B} \pi \left( \frac{B}{C} - \alpha C \right) \right] \quad \text{(Eq. 11)}
\]
**Proposition 3.** The optimal asset allocation in $t = 1$, for the risky asset $\theta_1^*$, if the agent measures his gains and losses relative to his initial wealth, is such that it maximizes the value function given by:

$$V = \lambda^* - \left(\lambda^* + \lambda^*\right)\pi\left(-\frac{B}{C}\right) + e^{-\frac{1}{2}C^2}\left[\lambda^* e^{-ab \pi\left(-\frac{B}{C} - \alpha C\right)} - \lambda^* e^{-ab \pi\left(\frac{B}{C} - \alpha C\right)}\right]$$

where:

$$B = W_0\left[1 + (1 - \theta_0)R_f + \theta_0 R_1\right] \left[1 - \theta_1^*\right]R_f + \theta_1^* \mu, \quad C = W_0\left[1 + (1 - \theta_0)R_f + \theta_0 R_1\right] \theta_1^* \sigma,$$

$\theta_0$ is the amount allocated in the risky asset in the first period, and $R_1$ is the observed return of the risky asset in the previous period.

Observe that the value function to be maximized is close to the one of the first period, but with changes in the parameters B and C, which account for the previous period outcome (gain or loss). As we are interested in the investor’s risk taking behavior after realizing a gain or a loss, let us evaluate the values of $\theta_1^*$ when we change the total return obtained in the first period. Recall that the total return from $t = 0$ to $t = 1$ ($R_{tot_1}$), depends both on his allocation choice in $t = 0$ and on the realized return of the risky asset $R_1$.

$$R_{tot_1} = (1 - \theta_0^*)R_f + \theta_0^* R_1$$

Let us then, evaluate $\theta_1^*$ considering the realized return of the stock in the first period varying over the following range: $\mu - 2\sigma < R_1 < \mu + 2\sigma$. We present numerical results for the optimal allocation of wealth, $\theta_1^*$, at $t = 1$. The remaining parameters are fixed ($\mu = 7.61\%$, $\sigma = 12.98$, $\alpha = 3$, $W_0 = 1$, $\lambda^* = 2.25$ and $\lambda^* = 1$). Figure 12 shows the results. Recall that the optimal allocation in the risky asset for the first period,
considering the previous parameters, is 81%. Thus, we need to verify whether the allocation in the stock in the second period is greater or lower than 81%, indicating greater or lower risk appetite, respectively. First, observe that, for a total return in the first period equal to zero (no gains/losses), the situation replicates the same framework the investor faced in the first period. Then we reach the same optimal allocation in the risky asset (for $R_{t_1} = 0$ implies $\theta_r^* = 81\%$).

![Optimal equity allocation in the second period as function of the total return obtained in the first period.](image)

Figure 12 – Optimal equity allocation in the second period as function of the total return obtained in the first period.

Consider the surroundings of the net value ($R_{t_1} = 0$). If the investor experiences a gain in the first period, the model predicts that he should optimally invest less in the risky asset in the second period. This behavior prevails up to the point where the loss aversion effect is less pronounced. On the other hand, if a loss is observed in the first period, he should take more risk in the following period, allocating a greater part of
his wealth in the stock. This prediction is in line with several experiments, which have shown that disposition effect dominates house-money in dynamic settings (Weber and Zuchel 2003). When the investor experiences a gain in the first period, he tends to reduce his risk appetite in order to guarantee the previous outcome. On the other hand, if he experiences a loss in the first period, he will increase his bets on stocks, trying to avoid the previous loss. In the model, the pattern holds for the whole gain domain; however, in the loss domain, high losses in the first period induce less risk appetite in the second period. The intuition is that if the investor is facing a huge loss, the loss aversion effect will dominate the risk seeking behavior, inducing a reduction in the optimal allocation in the stocks.

When we evaluate the expected cost (Eq. 10) of the behavioral inefficiency in the second period as a function of the return of the risky asset in the first period (Figure 13), it is possible to observe that, depending on the previous outcome, the cost can be increasing or decreasing. If the value for $R_1$ is such that it implies a small loss in the first period, the cost is even negative, which means that the expected return in the second period under prospect theory is greater than the one associated with standard utility. This is related to a greater risk appetite of the prospect theory individual after a loss, implying a greater allocation in the stock, which has a greater expected return. If $R_1$ indicates a gain in the first period, then the cost is positive once the allocation in the stock for the standard utility investor is greater than for the prospect utility individual.
We can conclude that for losses in the first period, the optimal allocation should adapt to the individual’s biases to reach better performance as the cost comes out to be negative in this domain. For gains in the previous outcome, the allocation should moderate the biases (observe a positive value for the expected cost). For extreme losses in the first period, the allocation should also moderate the investor’s biases.

If we accumulate the cost results in periods 1 and 2, we get the graph represented in Figure 14. It indicates that, for a negative stock result in the first period, or even a slightly positive one, the prospect theory individual outperforms the standard utility investor. And so, the allocation strategy should be adapted to the individual biases. The previous results should be taken with care as they refer to expected values. In section 3, we provide a more robust comparison, taking into account the performance of those individuals in an out-of-sample analysis.
4.2.3 Multi-Period Analysis

If we extend the two-period analysis to a multi-period one, by analogy, if the investor considers his current wealth as the reference to which he measures his gains/losses, he will solve the same maximization problem for each period and the optimal asset allocation is given as in proposition 1. In this situation, the agent acts myopically, just considering the following period possible gain/loss. In general, this result implies a smaller stock allocation if compared to a standard utility investor, generating an expected cost associated to the prospect theory biases.

On the other hand, if the individual’s reference point is his initial wealth (or his wealth in some moment in time \( t = t_0 \)), the allocation is defined as in proposition 3, but now considering the previous outcome as the total return obtained by him from \( t = 0 \) (or
from \( t = t_1 \) to the current time. As discussed in the two-period model, the allocation in the risky asset will depend on the previous gains/losses, and can be greater or smaller than the one chosen by the standard utility investor. Observe that the standard utility investor always chooses the same allocation in the risky asset, no matter what the reference point, as neither his decisions nor his beliefs are affected by previous outcomes.

4.2.4 Resampling

In sections 4.2.1, 4.2.2 and 4.2.3 we already evaluated the optimal asset allocation under prospect theory preferences and considering mental accounting, loss aversion, asymmetric risk taking behavior, and probability weighting. However, there is still an important issue in portfolio optimization missing: estimation error. Up to now, when solving the investor’s problem, we considered the expected return known with certainty, which is not the case in reality (especially in emerging markets where the uncertainty is higher). The assumed return for the risky asset is just an estimate, and so the real value can be different. This problem is relevant in any model of portfolio optimization and is crucial under prospect theory, where for lower values of the risk-free rate, a slightly increase in the risk premium of stocks can lead to extreme allocations. If the real return of the risky asset is lower, the likelihood of facing a loss is greater and should significantly reduce the value of that prospect.

In an attempt to overcome this estimation problem, Michaud (1998) proposed the resampling technique. Portfolio sampling allows an analyst to visualize the estimation error in traditional portfolio optimization methods. Suppose that we estimated both the variance and the excess return by using \( N \) observations. It is important to note that the point estimates are random variables and so another sample of the same size from the same distribution would result in different estimates.
Sherer (2002) suggests that sampling from a multivariate normal distribution (a parametric method termed Monte Carlo simulation) is a way to capture the estimation error. In this sense, return and variance would just be the expected values for a multivariate normal distribution. If we just consider two assets, the probability density function for a multivariate normal distribution would be given by

\[
f(x, y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left( -\frac{1}{2(1 - \rho^2)} \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} - \frac{2\rho xy}{\sigma_x \sigma_y} \right) \right),
\]

By repeating the sampling procedure \( n \) times, we get \( n \) new sets of optimization inputs, and then a different efficient allocation. The resampled weight for a portfolio would then be given by

\[
\theta_{\text{Resamp}} = \frac{1}{n} \sum_{i=1}^{n} \theta_i
\]

The resampled portfolios should reflect a greater diversification (more assets enter in the solution) than the classical mean-variance efficient portfolio, and should also exhibit less sudden shifts (smooth transitions) in allocations as return requirements change. Both characteristics are desirable for investors.

Recent literature has shown unambiguous results in favor of resampled portfolios in out of sample analysis (Pawley, 2005; Markowitz and Usmen, 2003; Wolf, 2006; Jiao, 2003). However, Harvey et al. (2006), evaluating Bayes vs. resampling methods, posit that the choice of risk aversion drives the results. Kohli (2005) concludes that, despite the fact that there are no conclusive advantages or disadvantages of using resampling as a technique to obtain better returns, resampled portfolios do seem to offer higher stability and lower transaction costs, two crucial features for long term investors’ choices.

We then propose the BRATE (Behavior Resampling Technique) as a novel methodology to define asset allocation, which incorporates behavioral ideas and
Chapter Four: Behavior Finance and Estimation Risk in Portfolio Optimization

resampling techniques into portfolio optimization, thus adapting to the individual’s preferences. In this case, the optimal asset allocation should be given by the previous propositions (1 and 2 or 3, depending on the reference point), but the procedure should be performed several times for different expected stock returns (given by a multivariate normal distribution). The population allocation is then given by the expected risky asset allocation. The procedure can be summarized as follows\(^{43}\):

Step 1: Estimate variance-covariance and return from the historical inputs.

Step 2: Resample from inputs (created in Step 1) by taking \(n\) draws from the input distribution. The number of draws reflects the degree of uncertainty in the inputs. Calculate new variance-covariance and return from sampled series. Estimation error will result in estimations that are different from those obtained in Step 1.

Step 3: Calculate the optimal allocation for inputs defined in Step 2, using the appropriate propositions (1 and 2 or 3, depending on the reference point considered).

Step 4: After repeating Steps 2 and 3 many times, calculate average portfolio weights. This is the BRATE portfolio allocation.

In the next section, we provide an empirical analysis comparing the BRATE allocation performance to a standard utility allocation.

4.3. Empirical Study

\(^{43}\) This methodology is an adaptation of the one proposed in Michaud (1998).
4. 3. 1. Data and Implementation

Our tests are based considering daily data from 26 countries’ MSCI stock indices and risk free rates, plus the MSCI World Index, for the period from April 4th, 1995 to January 5th, 2007. Developed countries and emerging markets (Brazil, Chile, South Africa, South Korea, Taiwan, Thailand, Turkey) were included in the analysis in order to find generalizable results. The total return time series are calculated on each country’s currency and also in US-Dollars. Thus, we are considering both hedged and unhedged investors. Table 1 presents some descriptive statistics of each market considered, for the whole sample period.

Table 1 - Descriptive Statistics

This Table provides descriptive statistics for the sample of world markets. For each market we present, the average risk free rate, the mean, standard deviation, skewness, and kurtosis of stock returns, as well as the Sharpe index (annualized values). The values are presented in the countries’ currency and also in USD. The risk free rate used to calculate the Sharpe Index in USD is the 3 month UST Bill rate for all markets.
From the table, we verify a risk premium associated with the stock market, both considering the values in each country’s currency and in USD, with the mean return of stocks being higher than the one of the corresponding riskfree rate.\textsuperscript{44}

Let us first consider the values in each country’s currency. The average annualized return of the risk free rate varied from 0.151\% (Japan) to 39.514\% (Turkey), while for the stock index, it ranges from 0.076\% (Thailand) to 47.804\% (Turkey). The annualized volatility (standard deviation) of the stock market varied from 12.976\% (World Index) to 45.171\% (Turkey). As expected, emerging markets tend to be more volatile than developed markets. While in Brazil, South Korea, Thailand, and Turkey the volatility was above 30 \%, in countries like United Kingdom and United States, its value was close to 16\%. In terms of skewness and kurtosis, usual results appear, indicating that daily stock index returns are negative skewed and have excess kurtosis (greater than 3). Finally, Table 1 presents the annualized Sharpe Index, which was greater in developed markets (around 0.35) than emerging markets (0.19). Our results are in line with previous literature (Mehra, 2003) which gives 0.34 as an estimation of the long-term Sharpe Ratio for the U.S. economy.

When we consider the values in USD, say in the perspective of a US based international investor who doesn’t currency hedge his investments, we find similar results. The average daily return in USD is close to the one in the country’s currency, which is evidence of the mean reverting aspect of the foreign exchange market. However, the standard deviation in USD is slightly greater than the one in the country’s currency, as the former includes both stock market risk and currency risk (the volatility of the foreign exchange rate). In terms of skewness and kurtosis, the previous results remain. However, now the Sharpe Indexes do not present relevant differences among

\textsuperscript{44} The only exception is Thailand where the sharpe index is negative (-0.109).
emerging and developed markets (for instance it is 0.430 for Brazil and 0.422 for the United States). Thus it seems that emerging stock markets are less interesting for domestic investors than for foreign unhedged investors.

Next we analyze the performance of the following optimization strategies: an investor with a standard utility preference - STU; an investor with prospect utility preference, with reference point given by his current wealth – PTU; an investor with prospect utility preference, with reference point given by his wealth in the previous period – CPT; an investor with a standard utility preference (resampled) – RSTU; an investor with prospect utility preference, with reference point given by his current wealth (resampled) – BRATEa; and an investor with prospect utility preference, with reference point given by his wealth in the previous period (resampled) – BRATEb. The utility function parameters are fixed ($\alpha = 3$, $\lambda^- = 2.25$ and $\lambda^+ = 1$). We vary the estimation period ($p$) in an out of sample analysis. The parameters are estimated using daily return observations of the past $p$ days. We define the efficient portfolio and hold it for the next ($e$) months, then re-estimate the parameters and adjust the portfolio weights. To judge the financial performance of the strategies, we compute their average return and empirical Sharpe-Ratios.

4.3.2. Results

The Sharpe Ratios of the different strategies are presented in Table 2 for the World Index and for the total period from 1995 to 2007, considering $p = 6$ months, 1, 2, and 4 years, and $e$ varying from 2 months to 1 year. We are evaluating the different strategies for a US based international stock investor. The risk free rate considered was the 3 month T Bill.
Table 2 – Sharpe Ratios

This Table presents the Sharpe-Ratio of the efficient portfolio generated by each estimation model. The Sharpe-Ratio is calculated by dividing the excess return observed by the standard deviation.

<table>
<thead>
<tr>
<th></th>
<th>STU</th>
<th>PTU</th>
<th>CPT</th>
<th>RSTU</th>
<th>BRATEa</th>
<th>BRATEb</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m-2m</td>
<td>0.189</td>
<td>0.134</td>
<td>0.136</td>
<td>0.207</td>
<td>0.154</td>
<td>0.156</td>
</tr>
<tr>
<td>6m-6m</td>
<td>0.101</td>
<td>0.080</td>
<td>0.083</td>
<td>0.125</td>
<td>0.102</td>
<td>0.114</td>
</tr>
<tr>
<td>1y-6m</td>
<td>0.439</td>
<td>0.392</td>
<td>0.392</td>
<td>0.438</td>
<td>0.400</td>
<td>0.401</td>
</tr>
<tr>
<td>2y-6m</td>
<td>0.462</td>
<td>0.426</td>
<td>0.421</td>
<td>0.464</td>
<td>0.434</td>
<td>0.423</td>
</tr>
<tr>
<td>4y-6m</td>
<td>-0.135</td>
<td>-0.023</td>
<td>-0.023</td>
<td>-0.122</td>
<td>-0.018</td>
<td>-0.019</td>
</tr>
<tr>
<td>1y-1y</td>
<td>0.413</td>
<td>0.347</td>
<td>0.389</td>
<td>0.420</td>
<td>0.354</td>
<td>0.393</td>
</tr>
<tr>
<td>2y-1y</td>
<td>0.456</td>
<td>0.428</td>
<td>0.431</td>
<td>0.461</td>
<td>0.444</td>
<td>0.465</td>
</tr>
<tr>
<td>4y-1y</td>
<td>-0.206</td>
<td>-0.126</td>
<td>-0.126</td>
<td>-0.193</td>
<td>-0.114</td>
<td>-0.113</td>
</tr>
<tr>
<td>mean</td>
<td>0.215</td>
<td>0.207</td>
<td>0.213</td>
<td>0.225</td>
<td>0.219</td>
<td>0.227</td>
</tr>
</tbody>
</table>

In general, we can state that the resampled models offered better results for a short selling constrained investor. It is an expected result as resampled models take into account the estimation risk, generating a more diversified portfolio which tends to outperform in out of sample studies. The highest Sharpe ratio was reached by the BRATEb model for an estimation period of 2 years and evaluation period of 1 year (0.465). On average resampled models increase the Sharpe ratio in around 0.10, when compared to the deterministic ones. Also, while the (R)STU investor seems to outperform (R)PTU, it doesn’t happen with (R)CPT.

If we consider just the total return obtained by each strategy, we find the results presented in Table 3. In this case, it’s possible to infer an inefficiency cost related to the behavioral investors, who tend to underperform the results of the standard utility investor in around 10 bps\(^{45}\). However if take into account the increment in risk (a risk adjusted measure like Shape Ratio), the inefficiency disappears.

\(^{45}\) 1 bps = 0.01%.
Table 3 – Average Total Return

This Table presents the Average Total Return of the efficient portfolio generated by each estimation model.

<table>
<thead>
<tr>
<th></th>
<th>STU</th>
<th>PTU</th>
<th>CPT</th>
<th>RSTU</th>
<th>BRATEa</th>
<th>BRATEb</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m-2m</td>
<td>4.302</td>
<td>3.781</td>
<td>3.822</td>
<td>4.447</td>
<td>3.935</td>
<td>3.976</td>
</tr>
<tr>
<td>6m-6m</td>
<td>3.654</td>
<td>3.449</td>
<td>3.476</td>
<td>3.883</td>
<td>3.643</td>
<td>3.749</td>
</tr>
<tr>
<td>1y-6m</td>
<td>6.670</td>
<td>6.211</td>
<td>6.211</td>
<td>6.632</td>
<td>6.238</td>
<td>6.242</td>
</tr>
<tr>
<td>2y-6m</td>
<td>7.377</td>
<td>6.987</td>
<td>6.875</td>
<td>7.345</td>
<td>6.910</td>
<td>6.775</td>
</tr>
<tr>
<td>4y-6m</td>
<td>1.065</td>
<td>2.083</td>
<td>2.083</td>
<td>1.064</td>
<td>1.992</td>
<td>1.993</td>
</tr>
<tr>
<td>1y-1y</td>
<td>6.711</td>
<td>5.981</td>
<td>6.295</td>
<td>6.630</td>
<td>5.979</td>
<td>6.341</td>
</tr>
<tr>
<td>2y-1y</td>
<td>7.247</td>
<td>6.935</td>
<td>6.966</td>
<td>7.289</td>
<td>7.068</td>
<td>7.194</td>
</tr>
<tr>
<td>4y-1y</td>
<td>0.419</td>
<td>1.226</td>
<td>1.226</td>
<td>0.530</td>
<td>1.287</td>
<td>1.297</td>
</tr>
<tr>
<td><strong>mean</strong></td>
<td><strong>4.681</strong></td>
<td><strong>4.582</strong></td>
<td><strong>4.619</strong></td>
<td><strong>4.727</strong></td>
<td><strong>4.631</strong></td>
<td><strong>4.696</strong></td>
</tr>
</tbody>
</table>

Based on the previous results, we can state that resampled models tend to outperform traditional models. Also, there is no clear advantage of standard utility investors over behavioral prospect theory investors at least to the CPT investor. Levy and Levy (2004) reached a similar result, positing that the practical differences between prospect theory and traditional mean-variance theory are minor. In this sense, behavioral biases should not be moderated, nor should standard models be adapted to include behavioral biases.

When we take into account each market separately, we find the results presented in Table 4 (in each country’s currency). Considering each country individually, there’s no clear dominance of a single strategy. Resampled models tend to outperform traditional models in emerging markets (observe the results for Brazil, Chile, South Africa, South Korea, Taiwan, Thailand and Turkey), where the uncertainty over the risk/return estimation is higher.
Chapter Four: Behavior Finance and Estimation Risk in Portfolio Optimization

Table 4 – Sharpe Ratios

This Table presents the Sharpe-Ratio of the efficient portfolio generated considering an estimation period of 1 year and evaluation period of 6 months (in each country’s currency). The Sharpe-Ratio is calculated by dividing the excess return observed by the standard deviation.

<table>
<thead>
<tr>
<th>Country</th>
<th>STU</th>
<th>PTU</th>
<th>CPT</th>
<th>RSTU</th>
<th>BRATEa</th>
<th>BRATEb</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Australia'</td>
<td>0.309</td>
<td>0.352</td>
<td>0.353</td>
<td>0.309</td>
<td>0.346</td>
<td>0.345</td>
</tr>
<tr>
<td>'Austria'</td>
<td>0.629</td>
<td>0.578</td>
<td>0.584</td>
<td>0.618</td>
<td>0.593</td>
<td>0.597</td>
</tr>
<tr>
<td>'Belgium'</td>
<td>0.977</td>
<td>0.982</td>
<td>0.973</td>
<td>0.982</td>
<td>0.986</td>
<td>0.977</td>
</tr>
<tr>
<td>'Brazil'</td>
<td>0.323</td>
<td>0.326</td>
<td>0.304</td>
<td>0.335</td>
<td>0.333</td>
<td>0.317</td>
</tr>
<tr>
<td>'Canada'</td>
<td>0.490</td>
<td>0.490</td>
<td>0.490</td>
<td>0.490</td>
<td>0.488</td>
<td>0.489</td>
</tr>
<tr>
<td>'Chile'</td>
<td>0.729</td>
<td>0.726</td>
<td>0.721</td>
<td>0.735</td>
<td>0.740</td>
<td>0.736</td>
</tr>
<tr>
<td>'Denmark'</td>
<td>0.914</td>
<td>0.914</td>
<td>0.914</td>
<td>0.908</td>
<td>0.910</td>
<td>0.909</td>
</tr>
<tr>
<td>'Finland'</td>
<td>0.696</td>
<td>0.685</td>
<td>0.638</td>
<td>0.691</td>
<td>0.658</td>
<td>0.665</td>
</tr>
<tr>
<td>'France'</td>
<td>0.778</td>
<td>0.790</td>
<td>0.755</td>
<td>0.780</td>
<td>0.785</td>
<td>0.764</td>
</tr>
<tr>
<td>'Germany'</td>
<td>0.619</td>
<td>0.614</td>
<td>0.619</td>
<td>0.619</td>
<td>0.616</td>
<td>0.616</td>
</tr>
<tr>
<td>'Ireland'</td>
<td>0.615</td>
<td>0.607</td>
<td>0.636</td>
<td>0.626</td>
<td>0.607</td>
<td>0.634</td>
</tr>
<tr>
<td>'Italy'</td>
<td>0.737</td>
<td>0.769</td>
<td>0.740</td>
<td>0.733</td>
<td>0.753</td>
<td>0.726</td>
</tr>
<tr>
<td>'Japan'</td>
<td>0.041</td>
<td>0.080</td>
<td>0.042</td>
<td>0.057</td>
<td>0.051</td>
<td>0.040</td>
</tr>
<tr>
<td>'Netherlands'</td>
<td>0.657</td>
<td>0.655</td>
<td>0.657</td>
<td>0.657</td>
<td>0.654</td>
<td>0.655</td>
</tr>
<tr>
<td>'Norway'</td>
<td>0.389</td>
<td>0.368</td>
<td>0.368</td>
<td>0.402</td>
<td>0.398</td>
<td>0.398</td>
</tr>
<tr>
<td>'Portugal'</td>
<td>0.751</td>
<td>0.685</td>
<td>0.728</td>
<td>0.764</td>
<td>0.716</td>
<td>0.738</td>
</tr>
<tr>
<td>'SouthAfrica'</td>
<td>0.161</td>
<td>0.206</td>
<td>0.218</td>
<td>0.167</td>
<td>0.208</td>
<td>0.224</td>
</tr>
<tr>
<td>'SouthKorea'</td>
<td>0.101</td>
<td>0.019</td>
<td>0.035</td>
<td>0.111</td>
<td>0.066</td>
<td>0.058</td>
</tr>
<tr>
<td>'Spain'</td>
<td>0.932</td>
<td>0.949</td>
<td>0.954</td>
<td>0.930</td>
<td>0.936</td>
<td>0.936</td>
</tr>
<tr>
<td>'Sweden'</td>
<td>0.634</td>
<td>0.631</td>
<td>0.631</td>
<td>0.643</td>
<td>0.634</td>
<td>0.633</td>
</tr>
<tr>
<td>'Switzerland'</td>
<td>0.773</td>
<td>0.720</td>
<td>0.739</td>
<td>0.773</td>
<td>0.739</td>
<td>0.748</td>
</tr>
<tr>
<td>'Taiwan'</td>
<td>-0.001</td>
<td>-0.003</td>
<td>0.000</td>
<td>-0.004</td>
<td>-0.005</td>
<td>-0.004</td>
</tr>
<tr>
<td>'Thailand'</td>
<td>0.041</td>
<td>-0.012</td>
<td>-0.048</td>
<td>0.055</td>
<td>0.033</td>
<td>0.018</td>
</tr>
<tr>
<td>'Turkey'</td>
<td>0.183</td>
<td>0.189</td>
<td>0.094</td>
<td>0.185</td>
<td>0.190</td>
<td>0.103</td>
</tr>
<tr>
<td>'UnitedKingdom'</td>
<td>0.411</td>
<td>0.428</td>
<td>0.423</td>
<td>0.411</td>
<td>0.429</td>
<td>0.426</td>
</tr>
<tr>
<td>'UnitedStates'</td>
<td>0.618</td>
<td>0.624</td>
<td>0.626</td>
<td>0.615</td>
<td>0.623</td>
<td>0.616</td>
</tr>
<tr>
<td>'World Index'</td>
<td>0.439</td>
<td>0.392</td>
<td>0.392</td>
<td>0.438</td>
<td>0.400</td>
<td>0.401</td>
</tr>
</tbody>
</table>

In terms of the comparison between the standard and the prospect utility investor, generally the former doesn’t outperform the latter, indicating no clear dominance of the traditional rational model. In this sense, there is no need for moderating the behavioral biases as described by prospect theory, as no extra financial efficiency is gained.

Generally speaking, an interesting finding is the fact that all previous allocation models outperform the 100% risky strategy. The Sharpe ratio of the 100% stock strategy was 0.383 while all resampled models reached, on average, a result above 0.509\(^46\).

\(^{46}\) A t-test over the Sharpe Ratio differences offered a significant result with a p-value of 0.0001.
Finally, if we take into account the values in USD and so considering that the investor is facing foreign exchange risk, we reach the results presented in Table 5.

**Table 5 – Sharpe Ratios**

This Table presents the Sharpe-Ratio of the efficient portfolio generated considering an estimation period of 1 year and evaluation period of 6 months (values in USD). The Sharpe-Ratio is calculated by dividing the excess return observed by the standard deviation.

<table>
<thead>
<tr>
<th></th>
<th>STU</th>
<th>PTU</th>
<th>CPT</th>
<th>RSTU</th>
<th>BRATEa</th>
<th>BRATEb</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Australia'</td>
<td>0.589</td>
<td>0.567</td>
<td>0.564</td>
<td>0.591</td>
<td>0.595</td>
<td>0.591</td>
</tr>
<tr>
<td>'Austria'</td>
<td>0.896</td>
<td>1.015</td>
<td>1.015</td>
<td>0.882</td>
<td>0.971</td>
<td>0.977</td>
</tr>
<tr>
<td>'Belgium'</td>
<td>0.904</td>
<td>0.904</td>
<td>0.904</td>
<td>0.886</td>
<td>0.886</td>
<td>0.899</td>
</tr>
<tr>
<td>'Brazil'</td>
<td>0.656</td>
<td>0.653</td>
<td>0.653</td>
<td>0.669</td>
<td>0.675</td>
<td>0.668</td>
</tr>
<tr>
<td>'Canada'</td>
<td>0.421</td>
<td>0.421</td>
<td>0.421</td>
<td>0.423</td>
<td>0.420</td>
<td>0.420</td>
</tr>
<tr>
<td>'Chile'</td>
<td>0.752</td>
<td>0.757</td>
<td>0.757</td>
<td>0.774</td>
<td>0.776</td>
<td>0.775</td>
</tr>
<tr>
<td>'Denmark'</td>
<td>0.781</td>
<td>0.754</td>
<td>0.820</td>
<td>0.776</td>
<td>0.773</td>
<td>0.803</td>
</tr>
<tr>
<td>'Finland'</td>
<td>0.612</td>
<td>0.596</td>
<td>0.597</td>
<td>0.613</td>
<td>0.597</td>
<td>0.593</td>
</tr>
<tr>
<td>'France'</td>
<td>0.654</td>
<td>0.643</td>
<td>0.625</td>
<td>0.649</td>
<td>0.625</td>
<td>0.629</td>
</tr>
<tr>
<td>'Germany'</td>
<td>0.533</td>
<td>0.509</td>
<td>0.521</td>
<td>0.537</td>
<td>0.502</td>
<td>0.516</td>
</tr>
<tr>
<td>'Ireland'</td>
<td>0.654</td>
<td>0.593</td>
<td>0.600</td>
<td>0.641</td>
<td>0.620</td>
<td>0.618</td>
</tr>
<tr>
<td>'Italy'</td>
<td>0.674</td>
<td>0.614</td>
<td>0.640</td>
<td>0.681</td>
<td>0.638</td>
<td>0.664</td>
</tr>
<tr>
<td>'Japan'</td>
<td>0.195</td>
<td>0.181</td>
<td>0.194</td>
<td>0.198</td>
<td>0.175</td>
<td>0.182</td>
</tr>
<tr>
<td>'Netherlands'</td>
<td>0.645</td>
<td>0.655</td>
<td>0.656</td>
<td>0.645</td>
<td>0.653</td>
<td>0.654</td>
</tr>
<tr>
<td>'Norway'</td>
<td>0.351</td>
<td>0.387</td>
<td>0.387</td>
<td>0.361</td>
<td>0.381</td>
<td>0.380</td>
</tr>
<tr>
<td>'Portugal'</td>
<td>0.666</td>
<td>0.641</td>
<td>0.641</td>
<td>0.669</td>
<td>0.647</td>
<td>0.660</td>
</tr>
<tr>
<td>'SouthAfrica'</td>
<td>0.465</td>
<td>0.441</td>
<td>0.456</td>
<td>0.479</td>
<td>0.459</td>
<td>0.460</td>
</tr>
<tr>
<td>'SouthKorea'</td>
<td>0.226</td>
<td>0.222</td>
<td>0.189</td>
<td>0.233</td>
<td>0.230</td>
<td>0.191</td>
</tr>
<tr>
<td>'Spain'</td>
<td>0.858</td>
<td>0.899</td>
<td>0.899</td>
<td>0.862</td>
<td>0.892</td>
<td>0.894</td>
</tr>
<tr>
<td>'Sweden'</td>
<td>0.562</td>
<td>0.558</td>
<td>0.566</td>
<td>0.563</td>
<td>0.554</td>
<td>0.565</td>
</tr>
<tr>
<td>'Switzerland'</td>
<td>0.599</td>
<td>0.531</td>
<td>0.552</td>
<td>0.596</td>
<td>0.607</td>
<td>0.612</td>
</tr>
<tr>
<td>'Taiwan'</td>
<td>-0.090</td>
<td>-0.100</td>
<td>-0.086</td>
<td>-0.083</td>
<td>-0.095</td>
<td>-0.076</td>
</tr>
<tr>
<td>'Thailand'</td>
<td>0.104</td>
<td>0.040</td>
<td>0.030</td>
<td>0.114</td>
<td>0.108</td>
<td>0.096</td>
</tr>
<tr>
<td>'Turkey'</td>
<td>0.288</td>
<td>0.250</td>
<td>0.238</td>
<td>0.296</td>
<td>0.267</td>
<td>0.246</td>
</tr>
<tr>
<td>UnitedKingdom'</td>
<td>0.625</td>
<td>0.574</td>
<td>0.605</td>
<td>0.632</td>
<td>0.597</td>
<td>0.626</td>
</tr>
<tr>
<td>'UnitedStates'</td>
<td>0.618</td>
<td>0.624</td>
<td>0.626</td>
<td>0.612</td>
<td>0.615</td>
<td>0.612</td>
</tr>
<tr>
<td>World Index'</td>
<td>0.439</td>
<td>0.392</td>
<td>0.392</td>
<td>0.438</td>
<td>0.400</td>
<td>0.401</td>
</tr>
</tbody>
</table>

Again, the results indicate a dominance of resampled models in emerging markets, while for developed countries, no clear dominance can be seen. The traditional rational model does not outperform the behavioral ones. Finally, all six dynamic models add value for the investor when compared to a 100% stock invested individual. Observe that the Sharpe Ratio found for the different markets (both in the country’s currency and in USD) are notably higher than the ones presented in Table 1.

Summing up, resampled models, which take into account estimation risk, tend to outperform deterministic models, notably for emerging markets where the uncertainty
of the expected return estimation is higher. Moreover, prospect theory utility investors don’t reach worse returns if compared to the traditional rational ones, which indicate no need for addressing bias moderation in the portfolio allocation.

4.4 Conclusions

This study had two objectives: first to incorporate mental accounting, loss aversion, asymmetric risk-taking behavior, and probability weighting in portfolio optimization for individual investors; and second to take into account the estimation risk in the analysis.

Considering daily index stock data from 26 countries over the period from 1995 to 2007, we empirically evaluated our model (BRATE – Behavior Resample Technique) against the traditional Markowitz. Several estimation and evaluation periods were used and we also considered a foreign exchange hedged and an unhedged strategy.

Our results support the use of BRATE as an alternative for defining optimal asset allocation and posit that a portfolio optimization model may be adapted to the individual biases implied in prospect theory. Behavioral biases don’t seem to reduce efficiency when we consider a dynamic setting. This result is robust for different developed and emerging markets. Also, the previous optimization models add value for the individual investor when compared to a naive 100% risky strategy.

As further extensions of the present research, we suggest the inclusion of several risky assets in the analysis. In this case, the issue of multiple mental accounting is a crucial issue to address the problem. An investor who evaluates every security in their own mental account will not necessarily view additional securities as redundant, which dramatically increases the complexity of the problem.
We also leave unasked the question of how individuals arrive at the underlying return distribution. That is the model above is a proposed mechanism for how individuals might transform a given probability distribution (assumed to be an accurate representation of the underlying distribution) into decision weights. Once we introduce uncertainty, it can induce individual biases, subjectivity and error. There is evidence that people display considerable overconfidence when asked to provide a subjective assessment of a probability distribution\textsuperscript{47}. Moreover, it is questionable whether the weightings provided by CPT truly reflect the process by which individuals evaluate continuous probability distributions.

The agent who measures his gains and losses always relative to his actual wealth solves the same maximization problem each period, therefore selecting a fix-mix strategy. An open question remains, if a fix-mix strategy can be the cause of the disposition effect.

\textsuperscript{47} Their subjective distribution is too tightly centered on their estimated mean.
Chapter Five

General Conclusions, Contributions and Lines for Further Research

In this thesis, we assumed as invariants of human behavior: loss aversion; asymmetric risk taking behavior; probability weighting and mental accounting. Under the previous theoretical framework, we evaluated three financial contexts: the role of incentives in risk taking decisions (Chapter 2); the effect of prior outcomes in future risk decisions (Chapter 3); and finally the portfolio choice problem (Chapter 4).

Our main conclusion is that absolute evaluations based on final wealth are limited and the relativity of risk taking decisions, where the perception of gains or losses drives the process, is a requirement to understand individual’s decisions. As a reply to behavioral critics, we reach propositions that can be falsified and make several predictions. Our contributions are both on the theoretical and empirical streams of finance literature.

In Chapter 2 we formulated five propositions linking investment strategy, compensation and risk taking in professional investor’s context. We suggested that managers in passive managed funds tend to be rewarded without incentive fee and be
risk averse. On the other hand, in active managed funds, whether incentives will reduce or increase the riskiness of the fund will depend on how hard is to outperform the benchmark. If the fund is (un)likely to outperform the benchmark, incentives (increase) reduce the manager’s risk appetite. Furthermore, the evaluative horizon influences the trader’s risk preferences, in the sense that if traders performed poorly (well) in a period, they tend to choose riskier (conservative) investments in the following period given the same evaluative horizon. We tested the model in an empirical analysis over a sample of 4684 mutual equity funds (cross-section data), from developed and emerging markets, and we reached supportive results to the propositions established in the theoretical model.

In Chapter 3, we performed a cross-country (Brazil and Spain) experiment on how prior outcomes affects risk taking decisions trying to elucidate the controversial result found in house-money and loss aversion papers. Our results support the existence of MLA over the countries but a country effect is also found, indicating that care should be taken when generalizing behavioral findings over the international financial market. No salient sex or age effect was found in the sample; however, some cultural differences among countries seem to have influence on the risk taking decision. Disposition effect is found to dominate the risk-taking behavior of subjects in dynamic settings, overcoming the house-money effect. Subjects that experienced a gain (loss) tend to assume less (more) risk in the following period.

Chapter 4 had two objectives: first to incorporate mental accounting, loss aversion, asymmetric risk-taking behavior and probability weighting in portfolio optimization for individual investors; and second to take into account the estimation risk in the analysis. Considering daily index stock data, from 26 countries for the period from 1995 to 2007, we empirically evaluated our model (BRATE – Behavior Resample
Technique) to the traditional Markowitz. Our results support the use of BRATE as an alternative for defining optimal asset allocation and posit that a portfolio optimization model should be adapted to the individual biases implied in prospect theory. Behavioral biases don’t seem to reduce efficiency when we consider a dynamic setting. This result is robust for different developed and emerging markets.

Several extensions to the present research can be mentioned. Overall we considered several invariants of human behavior based on prospect theory. However, the impact of cognitive biases in agent’s risk preference still needs to be better understood, in order to understand the way psychological states may affect risk preferences in this context.

In terms of the delegated portfolio management model and the influence of incentives in the risk taking behavior of incentives, further research may include the type of financial institution the trader works for (banks, insurance companies, pension funds) to take into account regulatory and institutional effects. Also, delegated portfolio management often involves more than one layer of agency: how does this feature affect incentives? More generally, studies about general equilibrium implications and price impact of the agency aspects of professional portfolio management should be interesting especially for policy-makers, given the relevance of these funds in all developed financial market.

Also, in the compensation analysis, only financial compensations were considered and we think that including non-financial rewards like recognition and prestige would enrich the theory and enable better predictions. Finally, inclusion of career concerns in the model could also improve multi-period analysis. Kempf et al. (2007) suggest that when employment risk is high, managers that lag behind tend to decrease risk relative to leading managers in order to prevent potential job loss.
Related to the experiment performed in Chapter 3 and the influence of prior outcomes in risk taking behavior, as possible extensions we suggest to go further in the investigation of what country's cultural characteristics are generating the effect found in the research, and also the use of other countries’ students replicating the experimental design C, since we have found that there seems to be a country effect as well, which could be due to cultural differences.

Finally, related to Chapter 4 and the portfolio choice problem, as further extensions of the presented research, we suggest the inclusion of several risky assets in the analysis. In this case, the issue of multiple mental accounting is a crucial issue to address the problem. An investor who evaluates every security in its own mental account will not necessarily view additional securities as redundant. Also a deeper discussion of how individuals access returns’ underlying distribution is another crucial point which can be explored in the future.

Summing up, as can be seen, behavioral finance is an open avenue where several studies are still missing. We forecast that in the following years, behavioral models will be the rule and not the exception, and doing so, economists will be closer to the complexity of social reality, and their theories will explain much better what is going on in financial markets. As mentioned by Thaler (2000), it is the evolution from *homo economicus* to *homo sapiens*. 
Appendix A: Experiment Instructions (Chapter 3)

(Translated from Portuguese and Spanish)

Introduction [Read aloud only]

Welcome to our experimental study of decision-making. The experiment will last about 30 minutes. The instructions for the experiment are simple, and if you follow them carefully, you can earn a considerable amount of money. All the money you earn is yours to keep, and will be paid to you, privately and in cash, immediately after the experiment.

Before we start the experiment you will be asked to pick one envelope from this pile. In the envelope you will find your Registration Form. This form will be used to register your decisions and earnings. On the top of your Registration Form you will find your registration number. This number indicates behind which table you are to take a seat. When everyone is seated, we will go through the instructions of the experiment. After that, you will have the opportunity to study the instructions on your own, and to ask questions. If you have a question, please raise your hand, and I will come to your table. Please do not talk or communicate with the other participants during the experiment.

Are there any questions about what has been said until now? If not, then will the person on my left please be the first to pick an envelope, open it, and take the corresponding seat.

[Treatment F: Read aloud and distributed]

The experiment consists of 12 successive rounds. In each round you will start with an amount of 100 cents. You must decide which part of this amount (between 0 cents and 100 cents) you wish to bet in the following lottery.

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You have a chance of 2/3 (67%) to lose the amount you bet and a chance of 1/3 (33%) to win two and a half times the amount bet.

You are requested to record your choice on the Registration Form. Suppose that you decide to bet an amount of X cents (0 ≤ X ≤ 100) in the lottery. Then you must fill in the amount X in the column headed Amount in lottery, in the row with the number of the present round.

Whether you win or lose in the lottery depends on your personal win color. This color is indicated on the top of your Registration Form. Your win color can be Red, Yellow or Green, and is the same for all twelve rounds. In any round, you win in the lottery if your win color matches the round color that will be randomly selected and announced, and you lose if your win color does not match the round color.

The round color is determined as follows. After you have recorded your bet in the lottery for the round, the computer will randomly select one color that will be shown in the screen. If the round color matches your win color, you win in the lottery; otherwise you lose. Since there are three colors, one of which matches your win color, the chance of winning in the lottery is 1/3 (33%) and the chance of losing is 2/3 (67%).

Hence, your earnings in the lottery are determined as follows. If you have decided to put an amount X cents in the lottery, then your earnings in the lottery for the round are equal to - X if the round color does not match your win color (you lose the amount bet), and equal to (2.5X) if the round color matches your win color.

The round color will be shown in the screen and announced by the assistant. You need to record this color in the column Round color, under win or lose, depending on whether the round color does or does not match your win color. Also, you need to record your earnings in the lottery in the column Earnings in lottery. Your total earnings for the round are equal to 100 cents (your starting amount) plus your earnings in the
lottery. These earnings are recorded in the column *Total Earnings*, in the row of the corresponding round. Each time we will come by to check your Registration Form.

After that, you are requested to record your choice for the next round. Again you start with an amount of 100 cents, a part of which you can bet in the lottery. The same procedure as described above determines your earnings for this round. It is noted that your private win color remains the same, but that for each round the computer selects a new color randomly. All subsequent rounds will also proceed in the same manner. After the last round has been completed, your earnings in all rounds will be totaled. This amount determines your total earnings in the experiment, which you will receive in cash after the numbers are checked.

**[Treatment I: Read aloud and distributed]**

The experiment consists of 12 successive rounds. In each round you will start with an amount of 100 cents. You must decide which part of this amount (between 0 cents and 100 cents) you wish to bet in the following lottery.  

You have a chance of $\frac{2}{3} (67\%)$ to lose the amount you bet and a chance of $\frac{1}{3} (33\%)$ to win two and a half times the amount bet.

You are requested to record your choice on the Registration Form. Suppose that you decide to bet an amount of $X$ cents ($0 \leq X \leq 100$) in the lottery. Then you must fill in the amount $X$ in the column headed *Amount in lottery*. Please note that you fix your bet for the next three rounds. Thus, if you decide to bet an amount $X$ in the lottery for round 1, then you also bet an amount $X$ in the lottery for rounds 2 and 3. Therefore, three consecutive rounds are joined together on the Registration Form.

Whether you win or lose in the lottery depends on your personal *win color*. This color is indicated on the top of your Registration Form. Your win color can be Red, Yellow or Green, and is the same for all twelve rounds. In any round, you win in the
Appendix A: Experiment Instructions (Chapter 3)

lottery if your win color matches the *round color* that will be randomly selected and announced, and you lose if your win color does not match the round color.

The round color is determined as follows. After you have recorded your bet in the lottery for the next three rounds, the computer will randomly select one color that will be shown in the screen for each of the next three rounds. If the round color matches your win color, you win in the lottery; otherwise you lose. Since there are three colors, one of which matches your win color, the chance of winning in the lottery is $\frac{1}{3}$ (33%) and the chance of losing is $\frac{2}{3}$ (67%).

Hence, your earnings in the lottery for the three rounds are determined as follows. If you have decided to put an amount $X$ cents in the lottery, then your earnings in the lottery for the round are equal to $-X$ if the round color does not match your win color (you lose the amount bet), and equal to $(2.5X)$ if the round color matches your win color.

The three round colors will be shown in the screen and announced by the assistant. You need to record this color in the column *Round color*, under *win* or *lose*, depending on whether the round color does or does not match your win color. Also, you need to record your earnings in the lottery in the column *Earnings in lottery*. Your total earnings for the three rounds are equal to 300 cents (your starting amount) plus your earnings in the lottery. These earnings are recorded in the column *Total Earnings*, in the row of the corresponding round. Each time we will come by to check your Registration Form.

After that, you are requested to record your choice for the next three rounds (4-6). For each of the three rounds you again start with an amount of 100 cents, a part of which you can bet in the lottery. The same procedure as described above determines your earnings for these three rounds. It is noted that your private win color remains the
same, but that for each round the computer selects a new color randomly. All subsequent rounds will also proceed in the same manner, also grouped by three (i.e., 7-9 and 10-12). After the last round has been completed, your earnings in all rounds will be totaled. This amount determines your total earnings in the experiment, which you will receive in cash after the numbers being checked.

[**Treatment C: Read aloud and distributed**]

The experiment consists of 12 successive rounds. In each odd round (1, 3, 5, 7, 9 and 11) you will start with an amount of 100 cents. In each even round (2, 4, 6, 8, 10 and 12) you will start with an amount of 100 cents plus your **Total Earnings** in the previous round. You must decide which part of this amount (between 0 cents and 100 cents for odd rounds; and between 0 cents and 100 cents plus your **Total Earnings** in the previous round for even rounds) you wish to bet in the following lottery.

*You have a chance of 2/3 (67%) to lose the amount you bet and a chance of 1/3 (33%) to win two and a half times the amount bet.*

You are requested to record your choice on the Registration Form. Suppose that you decide to bet an amount of X cents in the lottery. Then you must fill in the amount X in the column headed **Amount in lottery**, in the row with the number of the present round.

Whether you win or lose in the lottery depends on your personal **win color**. This color is indicated on the top of your Registration Form. Your win color can be Red, Yellow or Green, and is the same for all twelve rounds. In any round, you win in the lottery if your win color matches the **round color** that randomly selected and announced, and you lose if your win color does not match the round color.

The round color is determined as follows. After you have recorded your bet in the lottery for the round, the computer will randomly select one color that will be shown
in the screen. If the round color matches your win color, you win in the lottery; otherwise you lose. Since there are three colors, one of which matches your win color, the chance of winning in the lottery is $1/3$ (33%) and the chance of losing is $2/3$ (67%).

Hence, your earnings in the lottery are determined as follows. If you have decided to put an amount $X$ cents in the lottery, then your earnings in the lottery for the round are equal to $-X$ if the round color does not match your win color (you lose the amount bet), and equal to $(2.5X)$ if the round color matches your win color.

The round color will be shown in the screen and announced by the assistant. You need to record this color in the column \textit{Round color}, under \textit{win} or \textit{lose}, depending on whether the round color does or does not match your win color. Also, you need to record your earnings in the lottery in the column \textit{Earnings in lottery}. Your total earnings for the round are equal to 100 cents in odd rounds; or 100 cents plus your \textit{Total Earnings} in the previous round for even rounds (your starting amount); plus your earnings in the lottery. These earnings are recorded in the column \textit{Total Earnings}, in the row of the corresponding round. Each time we will come by to check your Registration Form.

After that, you are requested to record your choice for the next round. Again, you start with an amount of 100 cents in odd rounds or 100 cents plus your \textit{Total Earnings} in the previous round for even rounds, a part of which you can bet in the lottery. The same procedure as described above determines your earnings for this round. It is noted that your private win color remains the same, but that for each round the computer selects a new color randomly. All subsequent rounds will also proceed in the same manner. After the last round has been completed, your earnings in all rounds will be totaled. This amount determines your total earnings in the experiment, which you will receive in cash after the numbers being checked.
**Appendix A: Experiment Instructions (Chapter 3)**

**Win Color: RED**

**REGISTRATION FORM**

*(Treatment F)*

Name: ________________________________________________________________

Date: ___/___/______ Gender: Male / Female        Age: ____

University: __________________________  Country: ______________

Course: _____________________________

Answer the following questions:

1) You have just won $30. Choose between:

   (a) A 50% chance to gain $9 and a 50% chance to lose $9

   (b) No further gain or loss

2) You have just lost $30. Choose between:

   (a) A 50% chance to gain $9 and a 50% chance to lose $9

   (b) No further gain or loss

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<th>Round</th>
<th>Amount in Lottery</th>
<th>Round Color</th>
<th>Win</th>
<th>Lose</th>
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### Appendix A: Experiment Instructions (Chapter 3)

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Recall:

**Amount in Lottery (X):** must be between 0 (zero) and 100 (one hundred) cents.

**Round Color:** write the color randomly selected for that round.

**Win / Lose:** check the corresponding box depending on if you won or lost that round.

**Earnings in Lottery:** equals (-X) if you lost and equals (2.5 X) if you won.

**Total Earnings:** equals 100 plus Earnings in Lottery.
Appendix A: Experiment Instructions (Chapter 3)

Win Color: RED

REGISTRATION FORM

(Treatment I)

Name: ________________________________________________________________

Date: ___/___/______ Gender: Male / Female        Age: ____

University: __________________________  Country: ______________

Course: _____________________________

Answer the following questions:

1) You have just won $30. Choose between:

   (a) A 50% chance to gain $9 and a 50% chance to lose $9

   (b) No further gain or loss

2) You have just lost $30. Choose between:

   (a) A 50% chance to gain $9 and a 50% chance to lose $9

   (b) No further gain or loss

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**Recall:**

**Amount in Lottery (X):** must be between 0 (zero) and 100 (one hundred) cents and it must be the same in the following 3 rounds.

**Round Color:** write the color randomly selected for that round.

**Win / Lose:** check the corresponding box depending on if you won or lost that round.

**Earnings in Lottery:** equals (-X) if you lost and equals (2.5 X) if you won.

**Total Earnings:** equals 100 plus Earnings in Lottery.
Win Color: RED

REGISTRATION FORM

(Treatment C)

Name: ________________________________________________________________
Date: ___/___/______ Gender: Male / Female        Age: ____
University: __________________________  Country: ______________
Course: _____________________________

Answer the following questions:

1) You have just won $30. Choose between:
   (a) A 50% chance to gain $9 and a 50% chance to lose $9
   (b) No further gain or loss

2) You have just lost $30. Choose between:
   (a) A 50% chance to gain $9 and a 50% chance to lose $9
   (b) No further gain or loss

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Recall:

**Amount in Lottery (X):** must decide between 0 cents and 100 cents for odd rounds (1, 3, 5, 7, 9); and between 0 cents and 100 cents plus your *Total Earnings* in the previous round for even rounds (2, 4, 6, 8, 10)

**Round Color:** write the color randomly selected for that round.

**Win / Lose:** check the corresponding box depending on if you won or lost that round.

**Earnings in Lottery:** equals (-X) if you lost and equals (2.5 X) if you won.

**Total Earnings:** equals 100 plus Earnings in Lottery for odd rounds (1, 3, 5, 7, 9); and equals Total Earnings in the *previous* round plus Earnings in Lottery for even rounds (2, 4, 6, 8, 10).
Appendix B: Proofs of the Value Function Properties (Chapter 4)

We want to prove that the following property hold:

\( i) \frac{\partial V}{\partial \mu} > 0; \)

The partial derivative of \( V \) (Eq. 06) is given by:

\[
\frac{\partial V}{\partial \mu} = \{\alpha e^{\frac{1}{2} \left(\theta_0, \sigma\right)^2} \left[ \lambda^+ e^{-\left(\theta_0, \mu\right)^2} \pi \left( \left(1 - \theta_0\right) R_f + \theta_0 \mu \right) \theta_0 \sigma - \alpha \theta_0 \sigma \right] + 
\lambda^+ e^{-\left(\theta_0, \mu\right)^2} \pi \left( \left(1 - \theta_0\right) R_f + \theta_0 \mu \right) \theta_0 \sigma - \alpha \theta_0 \sigma \right] \} \cdot \theta_0 \quad \text{(Eq. 08)}
\]

as

\[
\left[ \lambda^+ e^{-\left(\theta_0, \mu\right)^2} \pi \left( \left(1 - \theta_0\right) R_f + \theta_0 \mu \right) \theta_0 \sigma - \alpha \theta_0 \sigma \right] > 0 \quad \forall \ \mu
\]

so,

\[
\frac{\partial V}{\partial \mu} > 0
\]

Now, let’s prove properties (ii) and (iii)

\( ii) \frac{\partial V}{\partial \sigma} = 0 \) for \( \sigma = 0 \) or \( \sigma = \infty; \)

\( iii) \frac{\partial V}{\partial \sigma} < 0 \) for \( \sigma > 0. \)

The partial derivative of \( V \) (Eq. 06) is given by:

\[
\frac{\partial V}{\partial \sigma} = \{\alpha \theta_0 \sigma e^{\frac{1}{2} \left(\theta_0, \sigma\right)^2} \left[ \lambda^+ e^{-\left(\theta_0, \mu\right)^2} \pi \left( \left(1 - \theta_0\right) R_f + \theta_0 \mu \right) \theta_0 \sigma - \alpha \theta_0 \sigma \right] - 
\lambda^+ e^{-\left(\theta_0, \mu\right)^2} \pi \left( \left(1 - \theta_0\right) R_f + \theta_0 \mu \right) \theta_0 \sigma - \alpha \theta_0 \sigma \right] \} \cdot \theta_0
\]

It follows:
\(\partial_{\sigma}V(\mu, 0) = 0\) using that \(\pi(-\infty) = 0, \pi(\infty) = 1\) and \(\pi'(\infty) = 0\).

Let us consider \(f(\mu, \sigma) = \theta_0^{-1}\sigma^{-1}e^{\frac{1}{2}\sigma^2[1(\sigma+\theta_0)]}\partial_{\sigma}V(\mu, \sigma)\) for \(\sigma > 0\). We show that \(f(\mu, \sigma) < 0\).

Suppose that for some \(\mu^*\) and \(\sigma(\mu^*) > 0\), \(f(\mu, \sigma(\mu^*)) > 0\). Since \(f(\mu, \cdot)\) is continuous, \(\lim_{\sigma \to 0} f(\mu, \sigma) = -\lambda^+ e^{-2\alpha[(1-\theta_0)\theta_0 + \theta_0\mu]} < 0\) and \(\lim_{\sigma \to \infty} f(\mu, \sigma) = 0\) for all \(\mu > 0\), we can assume without loss of generality that \(\sigma(\mu^*) > 0\) is a local maxima of \(f(\mu^*, \cdot)\). We compute the partial derivative of \(f\) with respect to \(\sigma\). We have

\[
\partial_{\sigma}f(\mu, \sigma) = (\pi^{\left[\left[(1-\theta_0)\theta_0 + \theta_0\mu\right] + \alpha\theta_0\sigma\right]} \lambda \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \theta_0\sigma^{-2} + \alpha + \lambda^+ \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \theta_0\sigma^{-2} + \alpha - \alpha^2 \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \left(\theta_0\sigma^{(-2)} - \alpha^2 - (\theta_0\sigma^{-2})\right)\theta_0
\]

Let \(\eta = (\theta_0\sigma)^{-2}\) then

\[
\partial_{\sigma}f(\mu, \sigma) = 0 \Leftrightarrow \eta \left[-\frac{\lambda^+ - \lambda^+}{\alpha} \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \eta + \left(\lambda^+ + \lambda^\prime\right) \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \frac{\lambda^+ - \lambda^+}{\alpha}\right] = 0
\]

\[
\Leftrightarrow \eta \in \{0, \eta^*(\mu)\}
\]

where \(\eta^*(\mu) = \frac{\alpha \left[(1-\theta_0)\theta_0 + \theta_0\mu\right] \lambda^+ + \lambda^\prime\left[(1-\theta_0)\theta_0 + \theta_0\mu\right]}{\lambda^+ - \lambda^\prime\left[(1-\theta_0)\theta_0 + \theta_0\mu\right]}\). Moreover, for \(\eta > \eta^*(\mu), \partial_{\sigma}f(\mu, \sigma) < 0\) and for \(0 < \eta < \eta^*(\mu), \partial_{\sigma}f(\mu, \sigma) > 0\). It follows that \(\sigma^*(\mu) = \frac{\eta^*(\mu)^{1/2}}{\theta_0} > 0\) is the unique maximum/minumum of \(f(\mu, \cdot)\) and since for \(\sigma > \sigma^*(\mu), \partial_{\sigma}f(\mu, \sigma) > 0\) and for \(0 < \sigma < \sigma^*(\mu), \partial_{\sigma}f(\mu, \sigma) < 0, \sigma^*(\mu)\) is a minimum. This contradicts the existence of \(\mu^*\) and \(\sigma(\mu^*)\) local maxima of \(f(\mu^*, \cdot)\) such that \(f(\mu^*, \sigma(\mu^*)) > 0\). Hence, \(f(\mu, \sigma) < 0\) and therefore \(\partial_{\sigma}V(\mu, \sigma) < 0\).

Also,

\[
\lim_{\sigma \to \infty} \partial_{\sigma}f(\mu, \sigma) = 0\] for \(\mu > 0\) since
\[
\lim_{\sigma \to \infty} \left( (\theta_0 \sigma) e^{\frac{1}{2}(\theta_0 \sigma)^2 \sigma^2} e^{-\frac{1}{\sigma^2}[1-(1-\theta_0)R_f + \theta_0 \mu]} \right) \pi \left( -\frac{(1 - \theta_0)R_f + \theta_0 \mu}{\sigma} - \alpha(\theta_0 \sigma) \right) = \\
\lim_{\sigma \to \infty} \left( (\theta_0 \sigma) e^{\frac{1}{2}(\theta_0 \sigma)^2 \sigma^2} e^{-\frac{1}{\sigma^2}[1-(1-\theta_0)R_f + \theta_0 \mu]} \right) \pi \left( \frac{(1 - \theta_0)R_f + \theta_0 \mu}{\sigma} - \alpha(\theta_0 \sigma) \right) = \frac{1}{\alpha \sqrt{2\pi}}
\]

And

\[
\lim_{\sigma \to \infty} \pi \left( \frac{(1 - \theta_0)R_f + \theta_0 \mu}{(\theta_0 \sigma)} \right) = \frac{1}{\sqrt{2\pi}}
\]
References


References


References


