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# PORTFOLIO SELECTION WITH MINIMUM TRANSACTION LOTS: AN APPROACH WITH DUAL EXPECTED UTILITY

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**Abstract.** *In this paper we analyse the portfolio selection problem with minimum transaction lots in the context of non-expected utility theory. We assume that the decision maker ranks the alternatives by using a specific Dual Expected Utility. This function allows portfolio values less or equal a fixed benchmark to be weighted in a different way from values greater than the fixed benchmark. Under normally distributed returns and opportune choice of the benchmark, the suggested approach leads to an NP-complete problem and has the advantage of using mixed linear programming to obtain the optimal portfolio. We also show results obtained by implementing the model on the Italian stock market.*
*(keywords: dual expected utility, portfolio selection, NP-completeness, linear programming with mixed variables)*

## INTRODUCTION

In this paper we analyse the stock portfolio selection problem in the context of minimum transaction lots. We assume that the decision maker uses subjective selection criteria based on the dual expected utility. The dual expected utility proposed by Yaari [20] is a particular non-expected utility and such it was developed to overcome the violations of the independence axiom ([1], [4], [7], [17], [18]), which, together with the completeness, transitive and continuity axioms, characterizes Von Neuman Morgenstern's expected utility (EU).

From an axiomatic viewpoint, the dual expected utility is characterized by the completeness, transitivity and continuity axioms; however, it replaces the independence axiom with the comonotonic axiom. As well known in literature, such a theory does not allow diversification to select portfolio formed of a risk and a risk-free asset. However, in a 1995 article, Hadar and Kun Seo [6] illustrate that in the case of a portfolio formed of only risk assets, the problem does not exist. Its use, therefore, is suitable in the field of selecting portfolio assets.

The use of the particular form of dual expected utility, introduced in Cenci-Filippini [3], allows, under opportune assumptions, the problem of optimal portfolio selection to be lead back to a linear programming problem easily resolvable even with high dimensional problems.

By introducing the minimum transaction lot constraints in the optimisation problem of the dual expected utility quoted above, we obtain a mixed linear programming problem that we prove to be NP-complete. In applying the model, we can examine if and how the risk perception of the decision maker, the capital owned by the decision maker and the constraint concerning minimum lots influence the composition of the optimal portfolio.

This paper is structured as follows. In Section 1 we describe the dual expected utility. In Section 2 we formalize the model for the resolution of portfolio selection problem with minimum transaction lots. Section 3 illustrates the NP-complete problem. In Section 4, by applying the model to assets included in the S&P MIB index, we analyse how the decision maker's risk perception influences the solution and we compared the results obtained with those reached assuming relaxed constraints. Section 5 draws on some final conclusions.

## 1. DUAL EXPECTED UTILITY

Yaari's dual theory (DEU) is a particular rank dependent utility theory (RDEU).

RDEU is a generalisation of expected utility theory (EU) based on probability weighting. The RDEU may be regarded as an "Expected utility with respect to a transformed probability distribution"[19].

Let X be a random variable whose outcomes $x_i$ occur with probability $p_i$. Furthermore, we assume $x_1 \le x_2 \le ..... \le x_n$.

With RDEU the decision maker chooses a separable utility function whose functional form is

$$RDEU(X) = \sum_{i=1}^{n} u(x_i)\gamma(p_1, p_2, ... p_i)$$

where $u(x)$ is the usual utility function,

$$\gamma(p_1, p_2, ... p_i) = g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)$$

and g(.) is a non decreasing function such that g(0)=0 and g(1)=1.

Note that, when $g(x) = x$, $\gamma(p_1, p_2, ... p_i) = p_i$ and $DEU(X) = EU(X)$.

If $g(x)$ is concave, it overweights the worst outcomes and if $g(x)$ is convex it underweights these outcomes with respect to the best ones.

Moreover, Quiggin ([15]) shows that, if $g(x)$ is monotonic, the choices made according to the RDEU are coherent with stochastic dominance principle.

We propose a generalization of the RDEU in which the ordering of the values of the random variable portfolio returns divides in two separable classes, the returns greater than a fixed benchmark and the outcomes less or equal to a fixed benchmark. We then can express the function $g(x)$ in that, it overweights the worst outcomes and underweights the positive ones.

A suitable $g(x)$ which represents this is as follows

$$g(x)=\begin{cases} Bx & if \quad 0 \le x \le \Pr(X \le V) \\ Ax + C & if \quad \Pr(X \le V) < x \le 1 \end{cases}.$$

where V is the benchmark value.

Under concavity and monotonicity of the function, we have:

(1) $$1 \le B \le \frac{1}{\Pr(X \le V)}, \qquad A = \frac{1 - B\Pr(X \le V)}{1 - \Pr(X \le V)}, C = (B - A)\Pr(X \le V).$$

For each outcome of the random variable less or equal to the benchmark, this implies $\gamma(p_1, p_2, \ldots p_i) = Bp_i$, while for outcomes over the benchmark we have $\gamma(p_1, p_2, \ldots p_i) = Ap_i$

The function $g(x)$ is given by a piece-wise linear graph, the dashed line of Fig.1.

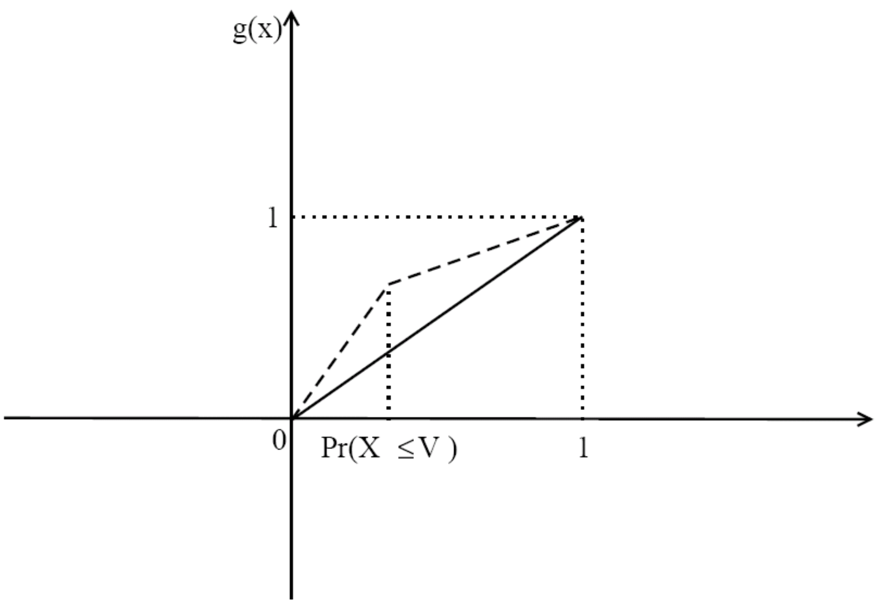


Fig.1: Function $g(x)$

Using this function $g(x)$, DEU will be distribution dependent. The different slopes in the two classes of outcomes depend on the psychological impact of overperformance and underperformance outcomes compared to the benchmark.

Subsequently, we consider the particular case of Yaari's dual theory (DEU) for RDEU with $u(x)=x$. The risk-aversion is represented by the curvature of g(p), rather than by the curvature of the utility function. In Yaari's theory, attitudes toward risk are characterised only by a distortion applied to probability distribution function [19]. A concave g(p) weights low-rank outcomes more, just as a concave utility function weights low-rank outcomes more heavily.

Let $\chi$ denote the set of alternatives which the decision maker has to choose from. According to the dual expected utility, the decision maker's preferences have to (must) verify the following axioms [19]:

A1) completeness

$\forall\, X \text{ and } Y \in \chi$ is $X \succ= Y$ or $Y \succ= X$ where $\succ=$ equals preferred or indifferent

A2) transitivity

if $X \succ= Y$ and $Y \succ= Z \Rightarrow X \succ= Z$

A3) continuity

if $X \succ= Y \succ= Z \Rightarrow \exists\, p \in (0,1) \mid Y = pX + (1-p)Z$

A4) comonotonicity

if $X, Y, Z$ are paired comonotonic risk variables and if $X \succ= Y \Rightarrow \forall\, p \in (0,1)\; pX + (1-p)Z \succ= pY + (1-p)Z$

Where X and Y are comonotonic if a risk Z variable and the two real value nondecreasing functions f and h exist such that $X = f(Z)$ and $Y = h(Z)$.

## 2. FORMALIZING THE PROBLEM WITH TRANSACTION LOTS

We are going to solve a portfolio selection problem on a mono-period horizon, having a capital C to invest in N assets whose unit price is $P_i$ where i=1,..,N.

In the presence of constraints on minimum transaction lots, the price of a minimum lot to purchase will be indicated with $L_i$, for each asset. Obviously, $L_i = P_i N_i$, where $N_i$ indicates the number of assets which constitute the relative minimum lot. In the event that minimum lot constraints do not exist $L_i = P_i$.

We indicate with

- $m_i \in N$ the number of minimum lots purchased for each asset
- $\widetilde{R}_i$ the random variable which represents the return rate of the i^th asset whose determinations will be obtained on the basis of price time series and will be indicated by $R_{it}$,t=1,…M.

The immediate growth from the capital investment $C = \sum_{i=1}^{N} m_i L_i$ in the period examined will be identified by

$$\Delta C = \sum_{i=1}^{N} m_i L_i \widetilde{R}_i$$

.

We shall assume that the decision maker determines the values of $m_i$ so that the dual expected utility is at its maximum, as introduced in the previous paragraph, associated to final wealth.

In other words, the decision maker will solve the following integer optimisation problem

(P1)

$$\max \frac{1}{M}\sum_{t=1}^{M} A(\sum_{i=1}^{N} m_i L_i R_{it} - V)^{+} + \frac{1}{M}\sum_{t=1}^{M} B(\sum_{i=1}^{N} m_i L_i R_{it} - V)^{-} + V$$

$s.t.$

$$H \leq \sum_{i=1}^{N} m_i L_i \leq K$$

$$m_i \in N$$

Considering

$$\left(\sum_{i=1}^{N} m_i L_i R_{it} - V\right) = \left(\sum_{i=1}^{N} m_i L_i R_{it} - V\right)^{+} + \left(\sum_{i=1}^{N} m_i L_i R_{it} - V\right)^{-}$$

$$\left(\sum_{i=1}^{N} m_i L_i R_{it} - V\right)^{-} = -\left(V - \sum_{i=1}^{N} m_i L_i R_{it}\right)^{+}$$

and introducing the variables

$$z_t = (V - \sum_{i=1}^{N} m_1 L_i R_{it})^{+}$$

the problem (P1) can be written in this form

(P2)

$$\max \frac{A}{M}\sum_{t=1}^{M}\left(\sum_{i=1}^{N} m_i L_i R_{it} - V\right) - \frac{B-A}{M}\sum_{t=1}^{M} z_t + V$$

$s.t.$

$$z_t \geq V - \sum_{i=1}^{N} m_i L_i R_{it}$$

$$z_t \geq 0$$

$$H \leq \sum_{i=1}^{N} m_i L_i \leq K$$

$$m_i \in N$$

Assuming that $V = E(\Delta C)$ and that the returns of single assets are normally distributed, we have:

- $\Delta C$ is normally distributed as a sum of normally distributed random variables;
- $pr(\Delta C \leq V) = \frac{1}{2}$ because the normal distribution is symmetrical regarding its mean;
- A=2-B for the relation (1)

Therefore, (P2) is a linear programming problem with mixed variables. In this case, dual expected utility is a linear combination between the portfolio expected increase $E(\Delta C)$ and its standard deviation $\sigma$

$$DEU = E(\Delta C) - \frac{\sigma}{\sqrt{2\pi}}(2B-2)$$

### 3. NP-COMPLETENESS

We want to prove that the problem (P2) is an NP-complete problem when asset returns are normally distributed.

In proving this, the constraint concerning variables $m_i$ is solely considered. Actually, the real variables $z_t$ do not create any problems. As suggested in Garey-Jonson [5], in order to prove that a problem is NP-complete, we exploit the analogy between the problem considered and the problems in which such a property is verified.

For this purpose, two cases are featured [12]:

1) H=K, from which C=H=K, in that case the NP-completeness descends from the analogy problem considered with a partition problem, which is defined as follows

Instance: A is a set $A=\{a_1,a_2,...,a_n\}$ with $a_i \geq 0$ given a dimension

$$2S=\sum_{i=1}^{n} s(a_n)$$

Question: Does a subset $A' \subset A$ exist such that $\sum_{a_i \in A'} s(a_i) = \sum_{a_i \in A-A'} s(a_i)$

Given the partition problem, we associate to it the particular case of portfolio selection problem where each asset is either not present or appears equal to the minimum purchase lot. We assume dimension S is equal to the capital to invest C ; we indicate:

- $n=N$ the number of assets (present) on the market;
- $s(a_i)$ the unit cost of the minimum lot associated with the $i^{th}$ asset.

If the partition problem has a solution, the admissible solution for the portfolio selection problem can be obtained by inserting $x_i$ minimum lots for each asset where $x_i = 1$ if $a_i \in A'$, while $x_i = 0$ if $a_i \in A - A'$.

2) $H \leq C \leq K$, in that case the NP-completeness descends from the analogy problem considered with a knapsack problem, which is defined as follows

Instance: Given a set U formed of n elements, for each $u_i \in U$ a dimension $s(u_i)$ is assigned and two positive integers H and K are set.

Question: Is there an assignement of a non-negative integer $c(u_i)$ to each $u_i \in U$ such that $\sum_{u_i \in U} c(u_i)s(u_i) \geq H$ e $\sum_{u_i \in U} c(u_i)s(u_i) \leq K$.

Given the knapsack problem, we associate to it the particular case of portfolio selection problem where each asset is either not present or appears equal to the minimum purchase lot. We indicate:

- $n=N$ the number of assets on the market;

- $s(u_i)$ the unit cost of the minimum lot associated with the i$^{th}$ asset.

It is easy to verify that the portfolio selection problem has an admissible solution if and only if the knapsack problem allows solutions.

## 4. APPLICATION

Here, we apply the model we propose in order to select the equity portfolio from the Italian stock market, included in the S&P/MIB index[1].

The monthly returns were computed on the monthly mid stock prices from June 2002 to May 2005 (font: Bloomberg).

Since the Italian Stock Exchange has eliminated the traditional minimum lots concerning stocks as of January 14, 2002, such instruments are negotiable for a quantitative equal to the unit or its multiple. Therefore, the portfolio selection problem has been solved assuming the price of each minimum lot equal to the average price of each share as at 01-06-2005 (font: Bloomberg).

The following results have been determined assuming the returns are normally distributed and the benchmark is the average value of the portfolio so DEU is distribution independent.

In order to analyse both the impact of C variability and the impact of the decision maker's risk perception on the solution, the mixed linear problem (P2) has been solved 28 times, taking into account 4 different variability intervals for C and, for each interval, 7 value pairs (A,B).

In order to determine the C variability intervals, 4 different K values have been set (50000,100000,150000, 250000) and the inferior limit H has been set as

$$H = K - \gamma K$$

where $\gamma$ =2%.

[1] In appendix A we show the stocks considered.

Excel Solver has been used to solve the problem.

Tables 5.1-5.4 show the results obtained in term of capital invested, maximum dual expected utility value and number of positive variables which specify the optimal solution as the decision maker's perception increases.

| A | B | C | DEU* | n. of positive variables |
|---|---|---|---|---|
| 1 | 1 | 50000 | 1644,985 | 3 |
| 0,8 | 1,2 | 49999 | 1012,832 | 3 |
| 0,6 | 1,4 | 49999,96 | 556,1404 | 2 |
| 0,5 | 1,5 | 49999,9 | 340,6617 | 2 |
| 0,4 | 1,6 | 49993,11 | 154,9211 | 5 |
| 0,2 | 1,8 | 49001,04 | -140,879 | 5 |
| 0 | 2 | 49000,82 | -396,591 | 7 |

Tab5.1Results for $49000 \leq C \leq 50000$

| A | B | C | DEU* | n. of positive variables |
|---|---|---|---|---|
| 1 | 1 | 99999,87 | 3290,169 | 2 |
| 0,8 | 1,2 | 99999,87 | 2025,67 | 3 |
| 0,6 | 1,4 | 99999,92 | 1112,281 | 2 |
| 0,5 | 1,5 | 99999,81 | 681,3234 | 2 |
| 0,4 | 1,6 | 99983,85 | 309,8117 | 5 |
| 0,2 | 1,8 | 98000,98 | -281,751 | 5 |
| 0 | 2 | 98001,55 | -793,147 | 7 |

Tab5.2 Results for $98000 \leq C \leq 100000$

| A | B | C | DEU* | n. of positive variables |
|---|---|---|---|---|
| 1 | 1 | 149999,9 | 4935,294 | 2 |
| 0,8 | 1,2 | 149999,9 | 3038,507 | 3 |
| 0,6 | 1,4 | 149999,9 | 1668,421 | 2 |
| 0,5 | 1,5 | 149999,7 | 1021,985 | 2 |
| 0,4 | 1,6 | 149998,4 | 464,8454 | 4 |
| 0,2 | 1,8 | 147001,3 | -422,626 | 5 |
| 0 | 2 | 147002,6 | -1189,75 | 7 |

Tab5.3 Results for $147000 \leq C \leq 150000$

| A | B | C | DEU* | n. of positive variables |
|---|---|---|---|---|
| 1 | 1 | 249998,9 | 8225,46 | 2 |
| 0,8 | 1,2 | 249997,5 | 5063,671 | 4 |
| 0,6 | 1,4 | 249999,8 | 2780,702 | 2 |
| 0,5 | 1,5 | 249999,5 | 1703,308 | 2 |
| 0,4 | 1,6 | 249998,6 | 774,745 | 4 |
| 0,2 | 1,8 | 245001,9 | -704,375 | 4 |
| 0 | 2 | 245001,9 | -1982,98 | 7 |

Tab5.4 Results for $245000 \leq C \leq 250000$

From our findings, we can deduce that the decision maker chooses to invest sums which are very close to the limit superior K of the available capital only until his/her own risk perception sees that the admissible solutions exist with the positive values of the dual expected utility. However, when the dual expected utility never assumes positive values, the decision maker (aims at investing) invests the minimal capital. In fact in this latter case, being the decision maker very risk-averse, he prefers not to invest his own capital in risk assets. When the number of positive variables is constant for the same values (A,B), regardless of the C variability interval, the selected assets are always the same. When this does not happen, the portfolio, which for the same (A,B) values has a higher number of positive variables, contains apart from assets common to the other portfolios, only one unit of another asset whose presence is tied to the totality constraint.

From a financial point of view, we also notice that, as a confirmation of the diversification role in portfolio risk reduction, the maximum number of positive variables is obtained for A and B values which represent the maximum decision maker's risk aversion.

In order to evaluate the impact of minimum lots on the solution, we have solved the 28 problems examined above with relaxed constraints. Let z* be the optimal objective function value obtained with totality constraint and $z^R$ be the optimal solution value to the relaxed problem; the impact of the constraint concerning minimum lots has been measured as

$$\frac{z^* - z^R}{z^R}$$

and its values, in the C variability intervals for the various (A,B) values, are shown in Tab. 5.5

| | | 49000<=C<=50000 | 98000<=C<=100000 | 147000<=C<=150000 | 245000<=C<=250000 |
|---|---|---|---|---|---|
| A | B | | | | |
| 1 | 1 | -7,E-05 | -9,E-06 | -9,E-07 | -5,E-06 |
| 0,8 | 1,2 | -5,E-06 | -2,E-06 | -2,E-06 | -1,E-04 |
| 0,6 | 1,4 | -2,E-06 | -2,E-06 | -2,E-06 | -2,E-06 |
| 0,5 | 1,5 | -9,E-06 | -9,E-06 | -9,E-06 | -9,E-06 |
| 0,4 | 1,6 | -2,E-04 | -3,E-04 | -2,E-05 | -1,E-05 |
| 0,2 | 1,8 | 8,E-05 | 6,E-05 | 6,E-05 | 5,E-05 |
| 0 | 2 | 7,E-05 | 2,E-05 | 4,E-05 | 8,E-05 |

Tab.5.5 Values of $\frac{z^{*}-z^{R}}{z^{R}}$

As you can see, the results obtained can be considered satisfactory in all cases examined.

In order to analyse the impact of the minimum lots on the average portfolio returns, we examined the difference between portfolio expected returns $E\left(R_P^*\right)$, obtained solving the mixed linear problem, and portfolio expected returns $E\left(R_P^R\right)$, obtained solving the relaxed problem. The results are shown in Tab. 5.6

The minimum lot constraints**,** even if leads to dual expected utility always inferior compared to the ones obtained with the relaxed problem, does not always involve an optimal portfolio expected return lower than the one obtained with a relaxed constraint, as we can deduce from Tab. 5.6

| | | 49000<=C<=50000 | 98000<=C<=100000 | 147000<=C<=150000 | 245000<=C<=250000 |
|---|---|---|---|---|---|
| A | B | | | | |
| 1 | 1 | -2,28611E-06 | -2,55384E-07 | -1,11101E-08 | -1,90306E-07 |
| 0,8 | 1,2 | 6,14072E-06 | 1,42544E-06 | 7,70867E-06 | 4,28998E-06 |
| 0,6 | 1,4 | 3,53737E-08 | 3,53737E-08 | 3,19577E-08 | 3,53737E-08 |
| 0,5 | 1,5 | -5,54651E-08 | -5,54651E-08 | -5,32121E-08 | -5,54651E-08 |
| 0,4 | 1,6 | -1,29842E-05 | -6,07055E-06 | -2,03564E-06 | -3,18589E-07 |
| 0,2 | 1,8 | 1,33379E-05 | 3,35145E-06 | 1,20884E-06 | -0,00077571 |
| 0 | 2 | -1,16876E-05 | -8,01407E-08 | -6,93054E-06 | -1,36226E-06 |

Tab.5.6 Values of ( $E\left(R_P^*\right)$- $E\left(R_P^R\right)$)

Therefore, the optimal portfolio obtained with minimum lot constraints which have an expected return higher than the one obtained by relaxed constraint, must be riskier.

Tab.5.6, nevertheless, highlights that such differences are hardly significant.

## 5. CONCLUSION

This paper analyses the optimal portfolio selection problem with constraints on minimum purchase lots means of a particular dual expected utility. It has been demonstrated that, if the returns are normally distributed, the problem can lead back to a mixed linear programming problem having NP complexity. The suggested model allows to overcome some computational difficulties deriving from the solutions of a quadratic programming problem with integer variables, just as applying Markowitz's model.

Since the decision maker's risk aversion is implicitly contained in parameters that characterizes the particular dual expected utility examined, the portfolio selection suggested here is a subjective procedure that, on the basis of the decision maker's risk perception, allows to select the optimal portfolio composition.

The practical application has highlighted that the introduction of minimum lot constraints, when these correspond to only one asset, does not have a strong impact neither on the dual expected utility nor on the portfolio expected return. Nevertheless, if the quantities to purchase are considerable, it is possible to have significant differences among the solutions of the relaxed problem and those of the constrained problem with minimum lots.

# APPENDIX A

Stock list

ALLEANZA ASSICURAZIONI
AUTOGRILL SPA
AUTOSTRADE SPA
BANCA ANTONVENETA SPA
BANCA FIDEURAM SPA
BANCA INTESA SPA
BANCA POPOLARE DI MILANO
BANCO POPOLARE DI VERONA E N
BANCA NAZIONALE LAVORO-ORD
BULGARI SPA
CAPITALIA
ENEL SPA
FASTWEB
FIAT SPA
FINMECCANICA SPA
ASSICURAZIONI GENERALI
GRUPPO EDITORIALE L'ESPRESSO
ITALCEMENTI SPA
LUXOTTICA GROUP SPA
MEDIASET SPA
MEDIOBANCA SPA
MEDIOLANUM SPA
ARNOLDO MONDADORI EDITORE
BANCA MONTE DEI PASCHI SIENA
PIRELLI & C.
RAS SPA
RCS MEDIAGROUP SPA
SAIPEM
SANPAOLO IMI SPA
SNAM RETE GAS
STMICROELECTRONICS
TELECOM ITALIA SPA
TIM SPA
UNICREDITO ITALIANO SPA

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