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Proposals for a needed adjustment of the VaR-based market risk charge of Basle II

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Abstract

We analyze around 200 different financial time series, i.e. components of Dow Jones, Nasdaq, FTSE and Nikkei with seven different VaR approaches. We differentiate our analysis according to characteristics that can be observed. Our analysis shows that in high risk situations in which the time series show high volatility risk and high fat tail risk the current Basle II guidelines fail in the attempt to cushion against large losses by higher capital requirements. One of the factors causing this problem is that the built-in positive incentive of the penalty factor resulting from the Basle II backtesting is set too weak. Therefore, we propose adjustments regarding the Basle II penalty factor that take different risk situations into account and lead to higher capital buffers for forecast models with a systematic risk underestimation.

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Keywords: Risk evaluation, Value-at-risk, Basle II backtesting, GARCH

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1 Motivation

Value-at-risk (VaR) approaches are the most common concepts to evaluate the market risk of financial instruments. In many cases very simple approaches based on the quantile of historical observations or normal distribution assumptions are used. However, a lot of papers come to the result that more sophisticated methods are needed to cope with the risks in real world problems. Performance analysis is often limited to specific financial time series, a broader differentiation of risk situations based on different characteristics of financial time series has rarely been done.

In our view, it is exactly this gap that matters the most, especially when analyzing high risk financial time series. We expect that the poor performance of common VaR approaches will be most obvious in high risk situations. The VaR-based market-risk charge in the Basle II regulation seems not to be prepared for such high risk situations. Therefore, a deeper analysis including the reliability of VaR forecasts as well as the desired built-in positive incentive of the penalty factor resulting from Basle II backtesting procedures is necessary. As the underestimation risk of VaR forecasts can be considerably high and as the intended built-in incentive to use better models in the current Basle II regulation turns out to be too weak potential adjustments have to be proposed and evaluated. In the future, this will lead to a higher protection against possible risk underestimation and potential real losses.

The paper is structured as follows: Together with the current Basle II guidelines for market risk analysis, the different VaR approaches will be briefly discussed in section 2. In section 3, we present our empirical analysis using the Basle II capital requirement. In section 4, we propose adjustments to the current Basle II capital requirement (market risk charge) while section 5 enhances our empirical analysis by incorporating the proposed adjustments. Finally, section 6 summarizes our results.

2 VaR approaches and Basle II guidelines

Various approaches to calculate VaR forecasts are extensively discussed, see Alexander (2001). We will give a brief introduction on the main concept and the most relevant differences between the approaches we will use in our empirical analysis.

In univariate approaches, a series of 1-day relative losses y_t (negative log returns) on a given financial asset or portfolio in period t is typically modelled as follows:

$$y_t = \mu_t + \sqrt{h_t}\varepsilon_t, \qquad t = 1, ..., T, \tag{2.1}$$

where μ_t is the conditional mean of y_t and h_t is its conditional variance (conditional on I_{t-1}, the information about the loss process available up to time $t-1$). The sequence $\{\varepsilon_t\}$ is an independently and identically distributed (i.i.d.) process with mean zero, variance one, and distribution function F.

The seven approaches we will investigate in the empirical analysis show differences in two major assumptions, the volatility process and the distribution of ϵ.

The volatility $\sqrt{h_t}$ could be assumed as constant over a period of time. In contrary the volatility dynamics could be modeled by a stationary GARCH(1,1) model for h_t:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \delta h_{t-1}, \quad 0 < \alpha_1 + \delta < 1. \tag{2.2}$$

Given this model the volatility is changing from day to day. With the parameters $\alpha_0 = 0$, $\alpha_1 = 0.06$ and $\delta = 0.94$ we get the EWMA approach from Risk Metrics which determines the parameters without estimation.

For the distribution function F of the sequence of ϵ the assumption of a gaussian distribution is common. But real financial time series often show fat tails, i.e. a higher probability of extreme observations compared to the gaussian distribution. Therefore a lot of different distributions have been proposed. We focus on the generalized pareto distribution (GPD) and the Hill approach based on extreme value theory (EVT) and the bootstrapping from standardized losses in a certain historical time period, referred to as filtered historical simulation (FHS).

In the next paragraphs we outline the VaR-based market risk charge within the Basle II regulation and discuss the different modeling approaches in more detail.

Current Basle II guidelines for the market risk charge

The capital requirement CR is based on the $(1-p)$-VaR of 1-day relative losses y_t and on 10-day relative losses $y_{t,10}$, the $(1-p)$-quantile of the distribution of y_t and $y_{t,10}$. The relative h-day loss $y_{t,h}$ is the sum of the 1-day losses:

$$y_{t,h} = \sum_{j=0}^{h-1} y_{t-j}. \tag{2.3}$$

The relative h-day $VaR_{t,h}^{1-p}$ - for \$ 1 invested - is the $(1-p)$-quantile of the distribution of losses y_{t+h}, conditional on information I_{t-h^*} available up to time $t-h^*$:

$$P(y_{t,h} > VaR_{t,h}^{1-p} | I_{t-h^*}) = p, \tag{2.4}$$

where h^* indicates the forecast horizon and h the number of cumulative losses.

The Basle II regulation sets p equal to 0.01, $h = h^*$ and information is assumed to be available up to period T. For practical reasons we use for 1-day VaR forecasts $VaR_{0.99}(1|T)$ instead of $VaR_{T+1,1}^{0.99}$ given I_t and for 10-day VaR forecasts $VaR_{0.99}(10|T)$ instead of $VaR_{T+10,10}^{0.99}$ given I_t. Furthermore, we set $y_{t,10}$ equal to $\mathbf{y}_t$ and notice that $y_{t,1} = y_t$.

The 10-day forecasts $VaR_{0.99}(10|T)$ basically determine the CR which additionally depends on 1-day VaR forecasts via a penalty factor, compare Jorion (2007, I.3 VaR-based Regulatory Capital). This penalty factor which should ensure the built-in positive incentive to improve the predictive quality results from the backtesting procedure with 1-day forecasts $VaR_{0.99}(1|T-i)$, $i = 1, ..., 250$, see Basle Committee on Banking Supervision (2006, D. Market Risk - The Internal Models Approach, 4 Quantitative Standards) as well as Danielsson et al. (1998, 5 Incentives and the Basle multiplication factor).

For the penalty factor the number of exceptions is considered. In a correct model the probability that the loss y_{T-i+1} exceeds the corresponding forecast $VaR_{0.99}(1|T-i)$ is $p = 0.01$. A loss y_{T-i+1} greater than the $VaR_{0.99}(1|T-i)$ can be regarded as an exception. Given (2.1) we can express the $VaR_{1-p}(1|T)$ using the quantile q_{1-p} of the distribution of the i.i.d. error ε_t:

$$VaR_{1-p}(1|T) = \mu_{T+1} + \sqrt{h_{T+1}}q_{1-p}, \quad (2.5)$$

where q_{1-p} is defined in $P(\varepsilon \leq q_{1-p}) = 1 - p$. According to (2.1) and (2.5) we can determine the number of exceptions H_1 by examining how often standardized losses $\tilde{y}_{T-i+1}$ exceed q_{1-p}, i.e. whether

$$\tilde{y}_{T-i+1} = \frac{y_{T-i+1} - \mu_{T-i+1}}{\sqrt{h_{T-i+1}}} = \varepsilon_{T-i+1} > q_{1-p}, \qquad i = 1, ..., 250. \quad (2.6)$$

With i.i.d. errors ε, the number of exceptions H_1 for the 1-day VaR forecasts has a Binomial Distribution, $H_1 \sim \mathcal{B}(n, p)$ with $n = 250$ and $p = 0.01$.

The relative number of exceptions $\hat{p}_1 = H_1/250$, the failure rate $\hat{p}_1$ (compare Jorion (2007), 6.2.1. Model Verification Based on Failure Rates), should be an unbiased estimate of p in a correct model. When this failure rate $\hat{p}_1$ is significantly higher than the target rate $p = 0.01$ in an applied model, we can presume that the estimate $\hat{p}_1$ has a bias which results in a systematic underestimation of the $VaR_{0.99}(1|T)$. As the use of models with underestimation risk should be avoided, the Basle test procedure regulates how to decide whether a model is suitable for risk analysis or not.

Taking into account the distribution of $n\hat{p}_1 = H_1$, the Basle test accepts the model as correct with a failure rate $\hat{p}_1 < 0.02$, i.e. with $H_1 < 5$. The model is rejected when the failure rate $\hat{p}_1 \geq 0.04$, when $H_1 \geq 10$. For $0.02 \leq \hat{p}_1 < 0.04$ $(5 \leq H_1 < 10)$, the model is put under surveillance and the resulting capital charge is increased by a penalty factor.

The penalty factor k is based on the Gaussian assumption, that ε has a standard normal distribution, i.e. $\varepsilon = \tilde{y} \sim N(0, 1)$. The factor k takes into account the underestimation $\hat{p}_1 > 0.02$ in form of

$$k = \left\{ \begin{array}{ll} \dfrac{z_{0.99}}{z_{1-\hat{p}_1}}, & 0.02 \leq \hat{p}_1 < 0.04 \\ \dfrac{z_{0.99}}{z_{0.96}}, & \hat{p}_1 \geq 0.04 \end{array} \right\}, \quad (2.7)$$

where the 0.99-quantile $z_{0.99}$ is referred to the $(1 - \hat{p})$-quantile $z_{1-\hat{p}}$ of the $N(0, 1)$, compare Stahl (1997). The higher the underestimation the higher is the penalty factor, see Table 2.1. A purpose of our paper is to analyze whether the penalty factor k is sufficient enough to ensure the desired built-in incentive.

The penalty factor k in (2.7) can be criticized:
(1) Its construction is based on the normal distribution which insufficiently captures the fat-tail risk inherent in financial time series.
(2) The increase in k will only compensate the underestimation.
(3) Analyses with real data show that models with a high underestimation risk may have relative low CR compared to reliable models, as will be seen later.

In the capital requirement formula

$$CR(T+1) = \max\left((\mathcal{M} + k^*) \frac{1}{60} \sum_{j=1}^{60} VaR_{0.99}(10|T-j+1), VaR_{0.99}(10|T) \right),$$

the factor k is transformed in an add-on component k^* within the Basle II framework, which is added to the multiplication factor $\mathcal{M} = 3$.

Table 2.1: Backtesting and penalty

Zone	H_1	$\hat{p}_1$	k	$\mathcal{M} \times k \cong \mathcal{M} + k^*$
Green	< 5	< 0.02	1.000	3.00
Yellow	5	0.020	1.133	3.40
	6	0.024	1.176	3.50
	7	0.028	1.217	3.65
	8	0.032	1.256	3.75
	9	0.036	1.293	3.85
Red	≥ 10	≥ 0.04	1.329	4.00

We complete the evaluation of the forecast performance of selected models by the number of 10-day exceptions. Here, H_{10} is the number of losses $\mathbf{y}_{T-i+1}$ greater than the $VaR_{0.99}(10|T-i-9)$, $i = 1, ..., 250$. Although the CR essentially depends on 10-day forecasts of VaR the test procedure is not based on H_{10} as the nice property of the binomial distribution no longer holds for H_{10} since the overlapping effects in the cumulative losses $\mathbf{y}_{T-i+1}$ yield dependent variables. Nevertheless, also the failure rate $\hat{p}_{10} = H_{10}/250$ can be regarded as an estimate of the target value $p = 0.01$ and this rate can be considered as an additional characteristic value for risk evaluation.

The multiplication factor $\mathcal{M} = 3$ can be justified by the Chebyshev Inequality, see Stahl (1997). In Danielsson et al. (1998) the fixed component $\mathcal{M} = 3$ has been criticized as too high. They recommend an extension of the range of the variable add-on factor k in order to increase the built-in incentive for financial institutions to use reliable models. We propose in section 4 "Proposals for a new multiplication factor" an extension by a further additional factor which is based on the expected shortfall ES. It takes into account the expected loss y in the 1% worst cases, i.e., when the loss y exceeds the $VaR_{0.99}(1|T)$,

$$\begin{aligned} ES_{0.99}(1|T) &= E(y_{T+1}|y_{T+1} > VaR_{0.99}(1|T)) \\ &= \mu_{T+1} + \sqrt{h_{T+1}} ES_{0.99}, \end{aligned} \tag{2.8}$$

where $ES_{0.99} = E(\epsilon|\epsilon > q_{0.99})$.

One-step forecasting

Historical Simulation (HS) and Historical Volatility (HV)

The HS calculates the $VaR_{1-p}(1|T)$ and the $ES_{1-p}(1|T)$ using the empirical distribution of past losses. The HS forecast is

$$\widehat{VaR}^{HS}_{1-p}(1|T) = \hat{q}^{HS}_{1-p}(\{y_T\}), \tag{2.9}$$

where $\hat{q}_{1-p}(\{y_T\})$ denotes the $(1-p)$-th empirical quantile of the losses data $\{y_t\}_{t=1}^{T}$. The HS forecast of $ES_{1-p}(1|T)$ is given by

$$\widehat{ES}^{HS}_{1-p}(1|T) = \frac{1}{|\{y_t > \widehat{VaR}^{HS}_{1-p}(1|T)\}|} \sum_{y_t > \widehat{VaR}^{HS}_{1-p}(1|T)} y_t, \tag{2.10}$$

where $|\{y_t > \widehat{VaR}^{HS}_{1-p}(1|T)\}|$ denotes the number of losses $\{y_t\}_{t=1}^{T}$ that are above the forecasted $\widehat{VaR}^{HS}_{1-p}(1|T)$. We also assume that the forecast is based on T observations for the other forecast models.

Assuming constant mean, $\mu_t = \mu$, and volatility, $h_t = h = \sigma^2$, and errors ε_t with a standard normal distribution, the sequence of losses $\{y_t\}_{t=1}^{T}$ is an i.i.d. process with mean μ and variance σ^2, $y_t \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \sigma^2)$.

Thus, with $\hat{u} = \bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$ and $\widehat{\sigma^2} = s_y^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2$ the $VaR_{1-p}(1|T)$ is

$$\widehat{VaR}^{HV}_{1-p}(1|T) = \hat{\mu} + \hat{\sigma} z_{1-p}, \tag{2.11}$$

where z_{1-p} is the $(1-p)$-th quantile of the standard normal distribution, and

$$\widehat{ES}^{HV}_{1-p}(1|T) = \hat{\mu} + \hat{\sigma}\frac{\phi(z_{1-p})}{p}, \tag{2.12}$$

where ϕ is the density function of the standard normal distribution.

Exponentially weighted moving average: EWMA-N and EWMA-FHS

Following a widely used method proposed by Risk Metrics, which sets an industry-wide standard, the variances h_t are considered as changing over time and modelled using an exponentially weighted moving average (EWMA) approach. Formally, the forecast for time $T+1$ is a weighted average of the previous forecast and the latest loss in form of

$$h_{T+1} = (1-\lambda)(y_T - \mu)^2 + \lambda h_T \qquad 0 < \lambda < 1$$

with starting value $h_1 = \sigma^2$. In substituting h_T by observable losses, the λ parameter places geometrically declining weights on past observations. Therefore, λ is called the decay factor. In Risk Metrics, the decay factor λ has been chosen to be equal to 0.94. With innovation $\varepsilon_t \sim \mathcal{N}(0,1)$, the forecast of VaR is

$$\widehat{VaR}_{1-p}^{E-N}(1|T) = \hat{\mu} + \sqrt{\hat{h}_{T+1}} z_{1-p} \tag{2.13}$$

and of ES is

$$\widehat{ES}_{1-p}^{E-N}(1|T) = \hat{\mu} + \sqrt{\hat{h}_{T+1}} \frac{\phi(z_{1-p})}{p}, \tag{2.14}$$

where $\hat{h}_{T+1} = 0.06(y_T - \bar{y})^2 + 0.94\hat{h}_T$ with $\hat{h}_1 = s_y^2$.

Dropping the assumption of the normal distribution, the distribution-free FHS forecast of VaR is

$$\widehat{VaR}_{1-p}^{E-FHS}(1|T) = \hat{\mu} + \sqrt{\hat{h}_{T+1}} \hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\}). \tag{2.15}$$

Here $\hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\})$ is the $(1-p)$-th empirical quantile of EWMA standardized losses $\{\tilde{y}_t^E\}_{t=1}^T$, where

$$\tilde{y}_t^E = \frac{y_t - \hat{\mu}}{\sqrt{\hat{h}_t}}, \qquad t = 1, ..., T,$$

with $\hat{h}_t = 0.06(y_{t-1} - \bar{y})^2 + 0.94\hat{h}_{t-1}$ and $\hat{h}_1 = s_y^2$. The distribution-free forecast of ES is

$$\widehat{ES}_{1-p}^{E-FHS}(1|T) = \hat{\mu} + \sqrt{\hat{h}_{T+1}} \widehat{ES}_{1-p}^{HS}(\{\tilde{y}_T^E\}), \tag{2.16}$$

where

$$\widehat{ES}_{1-p}^{HS}(\{\tilde{y}_T^E\}) = \frac{1}{|\{\tilde{y}_t^E > \hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\})\}|} \sum_{\tilde{y}_t^E > \hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\})} \tilde{y}_t^E$$

with $|\{\tilde{y}_T^E > \hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\})\}|$ the number of exceedances of $\hat{q}_{1-p}^{HS}(\{\tilde{y}_T^E\})$.

GARCH based forecasts: GARCH-N and -FHS as well as GARCH-GPD and -Hill

According to (2.2), the volatility h_{T+1} is forecasted by

$$\tilde{h}_{T+1} = \tilde{\alpha}_0 + \tilde{\alpha}_1 (y_T - \bar{y})^2 + \tilde{\delta}\tilde{h}_T$$

with the QML estimates $\tilde{\alpha}_0, \tilde{\alpha}_1, \tilde{\delta}$ and the starting value $\tilde{h}_1 = s_y^2$.

Compared to the EWMA-N approach, the GARCH-N forecasts $\widehat{VaR}_{1-p}^{G-N}(1|T)$ and $\widehat{ES}_{1-p}^{G-N}(1|T)$ only differ by the estimation of the volatility h_t, $t = 2, \ldots, T+1$.

For the distribution-free approach, we build GARCH standardized losses

$$\tilde{y}_t^G = \frac{y_t - \bar{y}}{\sqrt{\tilde{h}_t}} \quad ,t = 1, \dots, T.$$

With these losses we compute the $(1-p)$-th quantile $\hat{q}_{1-p}^{HS}(\{\tilde{y}_T^G\})$ and the Expected Shortfall $\widehat{ES}_{1-p}^{HS}(\{\tilde{y}_T^G\})$ to calculate the forecasts

$$\widehat{VaR}_{1-p}^{G-FHS}(1|T) = \hat{\mu} + \sqrt{\tilde{h}_{T+1}}\hat{q}_{1-p}^{HS}(\{\tilde{y}_T^G\}) \tag{2.17}$$

and

$$\widehat{ES}_{1-p}^{G-FHS}(1|T) = \hat{\mu} + \sqrt{\tilde{h}_{T+1}}\widehat{ES}_{1-p}^{HS}(\{\tilde{y}_T^G\}). \tag{2.18}$$

In Extreme Value Theory (EVT), the focus of interest is not the entire distribution but the relevant tail of the distribution. The tail (or peak-over-threshold) approach considers the exceedances over a high threshold. Following McNeil and Frey (2000), we consider two different EVT estimators of q_{1-p} and of ES_{1-p}, respectively. The first one is based on the Generalized Pareto distribution (GPD), the so-called GPD-based estimation. The second one has been proposed by Hill and is related to the Fréchet extreme value distribution, the so-called Hill-based estimation. As we look for an estimation of q_{1-p} and of ES_{1-p}, the GPD-based and the Hill-based estimators are based on i.i.d. variables, i.e. on the standardized losses $\tilde{y}_t = (y_t - \mu)/\sqrt{h_t} = \varepsilon_t$ with distribution function F. In the GARCH-GPD approach, we fix a high threshold u and assume that the excess residuals $\omega_t = \varepsilon_t - u$ over the threshold u have a GPD with distribution function

$$F_{\xi,\beta}(\omega) = \begin{cases} 1 - (1 + \xi\omega/\beta)^{-1/\xi}, & \xi \neq 0, \\ 1 - exp(-\omega/\beta), & \xi = 0, \end{cases} \tag{2.19}$$

where $\beta > 0$, and the support is $\omega \neq 0$ and $0 \leq \omega \leq -\beta/\xi$ when $\xi < 0$.

The choice of the GPD is motivated by a limit result in EVT. According to it, the function $F_{\xi,\beta}(\omega)$ is approximately equal to the corresponding function $F_u(\omega)$ of the excesses ω_t, i.e. $F_{\xi,\beta}(\omega) \approx F_u(\omega)$, where $F_u(\omega)$ is given by

$$\begin{aligned} F_u(\omega) &= P(\varepsilon - u < \omega | \varepsilon > u) \\ &= \frac{F(u+\omega) - F(u)}{1 - F(u)}, \quad \omega = \varepsilon - u > 0. \end{aligned} \tag{2.20}$$

From the approximation $F_{\xi,\beta} \approx F_u$ and a transformation of (2.20), we get

$$1 - F(u - \omega) = [1 - F(u)][1 - F_u(\omega)] \approx [1 - F(u)][1 - F_{\xi,\beta}(\omega)]. \tag{2.21}$$

Using this result, we compute the GPD-based estimators $\hat{q}_{1-p}^G$ and $\widehat{ES}_{1-p}^G$.

Let $\tilde{y}_{(t)} = \varepsilon_{(t)}$ denote the t-th order statistics of ε_t($i.e.$ $\varepsilon_{(t)} \geq \varepsilon_{(t-1)}$ for $t = 2, \dots, T$) and let T_u denote the number of standardized losses $\tilde{y}$ that exceed u. A natural estimate of $F(u)$ is given by

$$\hat{F}(u) = \frac{T - T_u}{T},$$

where T_u is the number of exceedances above the threshold u. We specify the threshold u indirectly by fixing a probability value p^* near p, say $p^* = 0.02$ or 0.05. With p^* we calculate the $(1 - p^*)$-th empirical quantile $\hat{u}_{1-p^*}(\{\tilde{y}_T^G\})$ of the standardized losses $\{\tilde{y}_T^G\}_{t=1}^T$. Here, $1 - F(\hat{u}_{1-p^*}) = p^*$. With the estimated threshold $\hat{u}$, we get estimations $\hat{q}_{1-p}^G$ for the GPD quantile and $\widehat{ES}_{1-p}^G$ for the GPD Expected Shortfall, for further details see McNeil and Frey (2000), see also McNeil, Frey and Embrechts (2005).

In the GARCH-Hill approach, we suppose that the tail of the distribution of ε is well approximated by the distribution function

$$F(\varepsilon) = 1 - L(\varepsilon)\varepsilon^{-1/\xi} \approx 1 - c\varepsilon^{-1/\xi}, \varepsilon > u, \xi > 0, \tag{2.22}$$

where $L(\varepsilon)$ is a slowly varying function. As in the GPD approach we also indirectly fix the threshold. With the estimated threshold, we get estimations $\hat{q}_{1-p}^H$ and $\widehat{ES}_{1-p}^H$ for the Hill method, for details compare Hill (1975) as well as Christoffersen and Gonçalves (2005).

Ten-step forecasting with bootstrap techniques

Historical Simulation (HS) and Historical Volatility (HV)

The sequence ϵ_t in $y_t = \mu_t + \sqrt{h_t}\epsilon_t$ is an independently and identically distributed (*i.i.d.*) process with mean zero and variance one. The HS procedure is based on the assumption that μ and h are constant, i.e.,

$$\mu_t = \mu, \qquad h_t = h = \sigma^2. \tag{2.23}$$

Thus, the sequence y_t is an *i.i.d.* process and the set $\{y_t\}_{t=1}^T$ can be regarded as a sample space for the bootstrap technique.
For each period $T + \tau, \tau = 1, ..., 10$, we generate a bootstrap pseudo series of losses $\{y_{T+\tau}^{HS}(j)\}_{j=1}^T$ by resampling from the finite sample space $\{y_t\}_{t=1}^T$, compare Davison and Hinkley (1997). Thus we build a random sample of $10T$ variables and with them the 10-day loss distribution $\{\mathbf{y}_{T+10}^{HS}(j)\}_{j=1}^T$ with *i.i.d.* variables $\mathbf{y}_{T+10}^{HS}(j) = \sum_{\tau=1}^{10} y_{T+\tau}^{HS}(j)$. The HS forecast of the (relative) value at risk is

$$VaR_{1-p}^{HS}(10|T) = q_{1-p}(\{\mathbf{y}_{T+10}^{HS}(T)\}), \tag{2.24}$$

where $q_{1-p}\{\mathbf{y}_{T+10}^{HS}(T)\}$ denotes the $(1-p)$-th empirical quantile of the bootstrap data $\{\mathbf{y}_{T+10}^{HS}(j)\}_{j=1}^T$.

Assuming errors ϵ_t with a standard normal distribution, the relative 10-day loss $\mathbf{y}$ has a normal distribution, $\mathbf{y} \sim N(10\mu, 10\sigma^2)$, and the HV-forecast of the 10-day VaR is

$$VaR_{1-p}^{HV}(10|T) = 10\mu + \sqrt{10}\sigma z_{1-p}. \tag{2.25}$$

Exponentially weighted moving average (EWMA)

According to the EWMA approach, the loss is

$$y_t^E = \mu + \sqrt{h_t^E}\epsilon_t^E \tag{2.26}$$

with

$$h_t^E = \lambda(y_{t-1} - \mu)^2 + (1 - \lambda)h_{t-1}^E. \tag{2.27}$$

Assuming that ϵ_t^E is an *i.i.d.* process, the standardized losses, $\tilde{y}_t^E = (y_t^E - \mu)/\sqrt{h_t^E} = \epsilon_t^E, t = 1, ..., T$, can serve as sample space. Similar to the HS approach, we generate a bootstrap pseudo series of *i.i.d.* losses $y_{T+\tau}^E(j) = \mu + \sqrt{h_T^E}\epsilon_{T+\tau}^E(j), j = 1, ..., T$, by resampling with replacement from the sample space $\{\epsilon_t^E\}_{t=1}^T$. With the *i.i.d.* 10-day losses $\mathbf{y}_{T+10}^E(j) = \sum_{\tau=1}^{10} y_{T+\tau}^E(j)$, the EWMA-FHS forecast of the VaR is

$$VaR_{1-p}^{E-FHS}(10|T) = q_{1-p}\{\mathbf{y}_{T+10}^E(T)\}, \tag{2.28}$$

where $q_{1-p}\{\mathbf{y}_{T+10}^E(T)\}$ denotes the $(1-p)$-th empirical quantile of the bootstrap data $\{\mathbf{y}_{T+10}^E(j)\}_{j=1}^{10}$. In the empirical analysis we set $\lambda = 0.06$, $u_0^2 = (y_0 - \mu)^2 = h_0 = s_y^2$ and we estimate μ by $\hat{\mu} = \overline{y}$ in the EWMA-FHS as well as in the EWMA-N approach. There, the assumption of standard normal distributed errors ϵ_t^E yields the EWMA-N forecast of VaR in form of

$$VaR_{1-p}^{E-N}(10|T) = 10\mu + 10\sqrt{h_T^E}z_{1-p}. \tag{2.29}$$

GARCH based forecasts

Following the GARCH approach, the loss is specified as

$$y_t^G = \mu + \sqrt{h_t^G}\epsilon_t^G, \tag{2.30}$$

where

$$h_t^G = \alpha_0 + \alpha_1(y_{t-1} - \mu)^2 + h_{t-1}^G \tag{2.31}$$

With *i.i.d.* errors ϵ_t^G the standardized losses $\tilde{y}_t^G = (y_t^G - \mu)/\sqrt{h_t^G} = \epsilon_t^G, t = 1, ..., T$, are the sample space for generating bootstrap GARCH losses

$$y_{T+\tau}^G(j) = \mu + \sqrt{h_{T+\tau}^G(j)}\epsilon_{T+\tau}^G(j), \tau = 1, ..., 10, \qquad j = 1, ..., T, \tag{2.32}$$

where

$$h_{T+\tau}^G(j) = \alpha_0 + \alpha_1(\alpha_1\epsilon_{T+\tau-1}^G(j) + \delta)h_{T+\tau-1}^G(j), \qquad \tau = 2, ..., 10 \tag{2.33}$$

and

(2.34) $$h^G_{T+1}(j) = \alpha_0 + \alpha_1 u_T^2 + \delta h_T^G.$$

With the *i.i.d.* 10-day losses $\mathbf{y}^G_{T+10}(j) = \sum_{\tau=1}^{10} y^G_{T+\tau}(j)$ the GARCH-FHS forecast of the 10-day VaR is

(2.35) $$VaR^{G-FHS}_{1-p}(10|T) = q_{1-p}\{\mathbf{y}^G_{T+10}(T)\},$$

where $q_{1-p}\{\mathbf{y}^G_{T+10}(T)\}$ denotes the $(1-p)$-th empirical quantil of the bootstrap data $\{\mathbf{y}^G_{T+10}(j)\}_{j=1}^T$. Similar to the 1-day forecast of VaR, the data is used to build the GARCH-GPD forecast $VaR^{G-GPD}_{1-p}(10|T)$ and the GARCH-Hill forecast $VaR^{G-Hill}_{1-p}(10|T)$, too. In the empirical analysis we estimate μ by $\hat{\mu} = \overline{y}$, h_o^G and u_0^2 by $h_0^G = u_0^2 = s_y^2$. The coefficients α_0, α_1 and δ are replaced by ML-estimates.

3 Empirical analysis with Basle II capital requirements

As data basis we collected around 200 financial time series. These are stocks that are listed within the four major indices Dow Jones, Nasdaq, FTSE and Nikkei. We adapt the procedures of Basle II. Therefore we analyze the time series within an rolling sample (the number of exceptions H and the failure rate $\hat{p} = H/250$ calculated quarterly for the last twelve months) and conduct around 2000 1- and 10-day VaR forecasts for each time series with the discussed seven approaches.

For each time series we compute:
- the average $VaR_{0.99}(1|T)$ and $VaR_{0.99}(10|T)$ forecasts,
- the average failure rates $\hat{p}_1$ and $\hat{p}_{10}$,
- the number of exceptions in the yellow zone (YZ) and red zone (RZ),
- the average penalty term $MB = \mathcal{M} \times k$, i.e. the Basle II multiplication factor,
- the average capital requirement CR.

As a first introductory example we would like to show the results for the EWMA-N and the GARCH-FHS model applied to the Dow Jones index in the following table:

Table 3.1: Results Dow Jones

Model	$VaR(1\|T)$	$VaR(10\|T)$	$\hat{p}_1$	$\hat{p}_{10}$	YZ	RZ	MB	CR
EWMA-N	0.022	0.066	0.019	0.027	7	1	3.142	0.205
GARCH-FHS	0.023	0.075	0.013	0.014	2	0	3.024	0.227

In both models, the failure rates $\hat{p}_1$ and $\hat{p}_{10}$ are higher than the target rate $p = 0.01$. With $\hat{p}_1 = 0.019$ and $\hat{p}_{10} = 0.027$ the overestimation in the EWMA-N model is considerably higher compared to the distribution-free GARCH-FHS model. There, the failure rates $\hat{p}_1 = 0.013$ and $\hat{p}_{10} = 0.014$ are clearly closer to the nominal rate $p = 0.01$. Thus, the GARCH-FHS model leads to a better result which is confirmed by its relative low number of exceptions in the YZ. The high degree of overestimation in the EWMA-N model, i.e. the underestimation of VaR, should be punished by the penalty factor k which attains according to $MB = 3k$ and $k = 1.047$ indeed a higher value than the penalty factor $k = 1.008$ of the GARCH-FHS model. However, the increase in the penalty factor is insufficient. Despite the high failure rates $\hat{p}_1 = 0.019$ and $\hat{p}_{10} = 0.027$ the capital requirement $CR = 0.205$ of the EWMA-N model is clearly lower than the corresponding value $CR = 0.227$ of the GARCH-FHS model. That means the bad performance of the approach does not lead to a strong enough punishment in terms of the capital requirement CR. Looking only on the low CR a financial institution will use the unreliable EWMA-N model.

We did the same analysis for all 200 financial time series (Dow Jones, Nasdaq, FTSE, Nikkei). To summarize the results we cluster the time series based on their characteristics in terms of volatility excess and quantile surplus which allows us to select time series with high risk.

Following the EWMA-approach of Risk Metrics, we estimate the volatility h_t of each considered real financial time series $\{y_t\}_{t=1}^{T}$ by $\hat{h}_t = 0.06(y_{t-1} - \bar{y})^2 + 0.94\hat{h}_{t-1}$ with $\hat{h}_1 = s_y^2$. Thereby, we compute standardized losses $\tilde{y}_t^E = (y_t - \bar{y})/\sqrt{\hat{h}_t}$ and the quantile estimation $\hat{q}_{1-p}^{HS}(\{\tilde{y}_t^E\})$. As a high volatility can lead to a high risk, we measure the degree of **v**olatility by the relative mean **e**xcess (VE) over $\overline{\sqrt{\hat{h}}}$ as follows:

$$VE = \frac{1}{|\{\sqrt{\hat{h}_t} > \overline{\sqrt{\hat{h}}}\}|} \sum_{\sqrt{\hat{h}_t} > \overline{\sqrt{\hat{h}}}} \frac{\sqrt{\hat{h}_t} - \overline{\sqrt{\hat{h}}}}{\overline{\sqrt{\hat{h}}}} \times 100,$$

with $|\{\sqrt{\hat{h}_t} > \overline{\sqrt{\hat{h}}}\}|$ the number of exceedances of $\overline{\sqrt{\hat{h}}}$.

The size of heavy tails is measured by the relative **s**urplus of the **q**uantile estimation $\hat{q}_{1-p}^{HS}(\{\tilde{y}_t^E\})$ over the corresponding normal distribution quantile z_{1-p} (QS) as follows

$$QS = \frac{\hat{q}_{1-p}^{HS}(\{\tilde{y}_t^E\}) - z_{1-p}}{z_{1-p}} \times 100.$$

As Basle II fixes p at 0.01, the corresponding normal quantile is $z_{0.99} = 2.326$.

Applying the Risk Metrics EWMA-approach to all 200 financial time series of Dow Jones, Nasdaq, FTSE and Nikkei, we can describe each of them by their characteristics (QS, VE). The scatter diagram in Figure 3.1 informs about the volatility risk VE and the tail risk QS of all time series.

The scatter diagram in figure 3.2 represents selected time series in more detail with values higher than the 70% quantile of QS and VE. This selection contains 24 time series that are highly risky, 10 FTSE, 8 Nasdaq, 4 Nikkei and 2 Dow Jones series. These time series should receive a high attention from the perspective of a risk manager.

For all considered time series the average volatility risk $\overline{VE}$ is 36.0% and the average tail risk $\overline{QS}$ is 12.5%. In the selected high risk cluster these averages increase to $\overline{VE} =$ 49.5% and to $\overline{QS} = 19.5\%$. Here, the underestimation risk augments. Measured with HV the average failure rate $\hat{p}_1$ raises from $\hat{p}_1 = 0.019$ for all time series to $\hat{p}_1 = 0.021$ in the cluster with high risky time series. Whereas the historical volatility remains at the same level $\bar{s} = 0.023$ in both cases. With regard to the underestimation the historical volatility is an inappropriate indicator of risk.

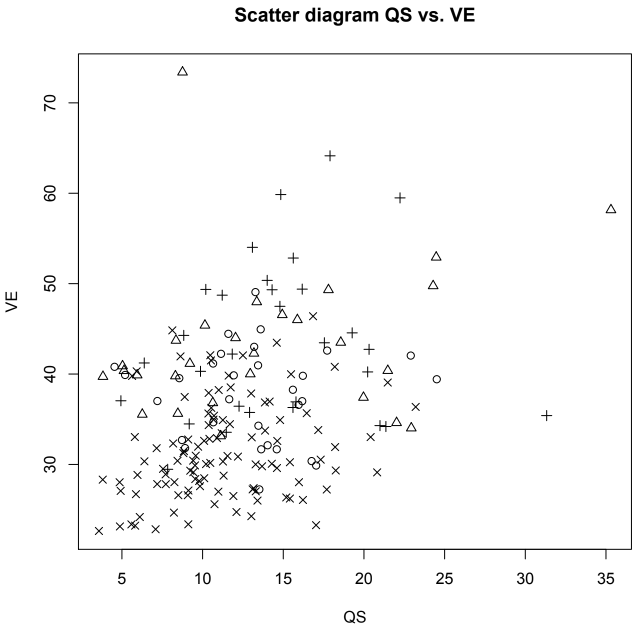


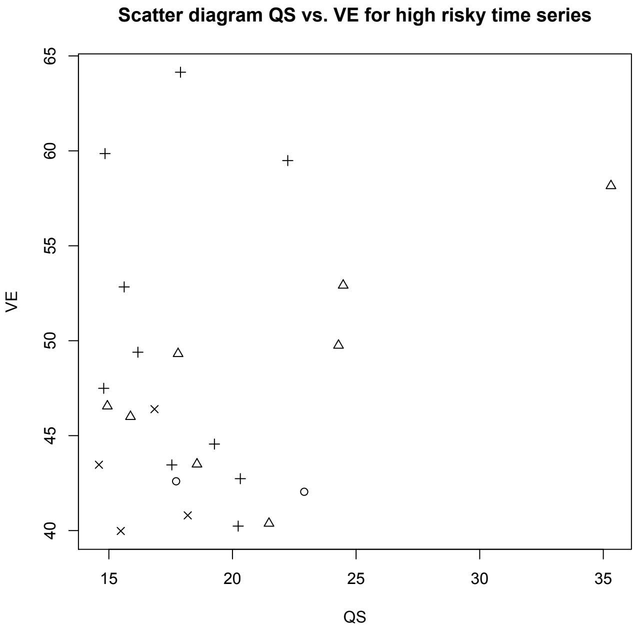
Figure 3.1

In the high risk cases, it is of special interest whether the Basle II CR construction has the desired built-in positive incentive, or not, i.e. whether the stimulus of the penalty is strong enough to prevent financial institutions from applying unreliable forecast models or not.

In Table 3.2 we summarize the results of seven models applied to the 24 high risky time series. As we have computed aggregate characteristics over time for the Dow Jones index, compare Table 3.1, we calculate now average values over all of the 24 time series based on aggregate data of each of the 24 time series. Here, we take into account that the averages of $VaR(1|T)$, $VaR(10|T)$ and the CR of each of the 24 time series vary with their volatility. Therefore we divide these values by their corresponding standard deviation SD.

Table 3.2: Results with all high risky time series

Model	$VaR1/SD$	$VaR10/SD$	$\hat{p}_1$	$\hat{p}_{10}$	YZ	RZ	MB	CR/SD
HV	1.633	4.769	0.021	0.024	5.083	0.792	3.178	15.290
HS	1.698	4.937	0.020	0.025	5.708	0.250	3.174	15.717
E-N	1.584	4.618	0.023	0.032	9.750	0.042	3.245	14.844
E-FHS	1.818	5.283	0.017	0.027	4.208	0.000	3.092	16.035
G-FHS	1.834	5.581	0.014	0.018	2.167	0.042	3.048	17.006
G-GPD	1.856	5.580	0.014	0.017	2.292	0.000	3.046	17.002
G-Hill	1.853	5.562	0.014	0.017	2.292	0.000	3.048	16.966



o	Dow Jones	+	FTSE
△	Nasdaq	x	Nikkei

Figure 3.2

The more extensive analysis of the high risky assets confirms the results of the Dow Jones index in the introductory example and shows a clear difference between the GARCH and Non-GARCH models.

All models have failure rates $\hat{p}_1$ and $\hat{p}_{10}$ that exceed the target rate $p = 0.01$ and underestimate the risk. The underestimation risk with the Non-GARCH models is considerably higher compared to the GARCH models. Here although, the increase of the penalty is not sufficient enough to counterbalance or punish the underestimation of the VaR in the CR formula. Here, the desired built-in positive incentive does not exist.

The results with real data in table 3.2 confirm the underestimation risk in the simulation study in Pauly and Fricke (2009) with nearly the same forecast models. Especially non-GARCH models yield unreliable forecasts for 1-day as well as for 10-day VaR.

4 Proposals for a new multiplication factor

As mentioned in section 2 ”Current Basle guidelines for market risk charge” the Basle II factor k can at best compensate the underestimation. The empirical analysis shows that the compensation will not be sufficient to cover the risks of equity portfolios. The aspired built-in positive incentive of the penalty factor to maintain the predictive quality can not be reached. Therefore, an adjustment of the Basle II guidelines is needed.

Two adjustments are proposed. The first one is based on the normal distribution

Table 4.1: MMN and its components for case a)

H_1	$\hat{p}_1$	k	f	$k \times f$	$M \times k$	$MMN = M \times k \times f$
4	0.016	1.00	1.00	1.00	3.0	3.0
5	0.020	1.13	1.12	1.27	3.4	3.8
6	0.024	1.18	1.11	1.31	3.5	3.9
7	0.028	1.22	1.10	1.34	3.7	4.0
8	0.032	1.26	1.10	1.38	3.8	4.1
9	0.036	1.29	1.09	1.41	3.9	4.2
10	0.040	1.33	1.09	1.45	4.0	4.3
11	0.044	1.36	1.09	1.48	4.1	4.4
12	0.048	1.40	1.08	1.51	4.2	4.5

and supplies an additional term to the Basle II factor k. This new factor takes the expected loss y in the 1% worst cases into account (expected shortfall ES), i.e., when the loss y exceeds the value at risk $VaR_{0.99}$, see (2.8). The additional factor enforces the compensation of the underestimation and increases the penalty strength.

In the second adjustment we suggest a distribution free approach which should be able to capture the ex-post underestimation due to fat-tail risk inherent in financial time series and should thereby ensure the built-in positive incentive in the market risk charge.

According to (2.6) we evaluate the forecasting quality by comparing standardized losses $\tilde{y}_{T-i+1}$ with their corresponding quantile $q_{0.99}$. Given the gaussian assumption, $\tilde{y}_{T-i+1} \sim N(0,1)$, the quantile $q_{0.99}$ equals $z_{0.99} = 2.3263$ and the number of exceptions H_1 results by counting how often $\tilde{y}_{T-i+1}$ exceeds the value $z_{0.99}$ for $i = 1, 2, ..., 250$. Or equivalently, we count how often the loss y_{T-i+1} exceeds the $VaR_{0.99}(1|T-i) = \mu_{T-i+1} + \sqrt{h_{T-i+1}} q_{0.99}$. As we use the value $z_{0.99}$ in the period $T-i$ for the forecast in period $T-i+1$ we can consider $z_{0.99}$ as an ex-ante value for $q_{0.99}$ in $T-i+1$, $q^a_{0.99} = z_{0.99}$. In the backtesting procedure we use information up to period $T-i+1$ to calculate $z_{1-\hat{p}}$ and $k = z_{0.99}/z_{1-\hat{p}}$. Therefore we can interpret $kz_{0.99}$ as an ex-post value for $q_{0.99}$, $q^p_{0.99} = kz_{0.99}$. Thus, the factor k in Table 2.1 can be written as $k = q^p_{0.99}/q^a_{0.99} \geq 1$ and can be interpreted as a compensation term for the ex-ante risk underestimation of $q^a_{0.99}$, $q^a_{0.99} < q^p_{0.99}$. To this underestimation factor k we add a penalty term. It is build with the expected shortfall $ES(q^p_{0.99}) = E[\tilde{y}|\tilde{y} \geq q^p_{0.99}] = \phi(q^p_{0.99})/(1-\Phi(q^p_{0.99}))$, which is related to $q^p_{0.99}$. The combination of the compensation terms $k = q^p_{0.99}/q^a_{0.99}$ for $\hat{p}_1 \geq 0.02$ with the penalty factor $f = ES(q^p_{0.99})/q^p_{0.99}$ leads to the modified multiplication factor MMN,

$$MMN = M \times k \times f \tag{4.1}$$

with f for

a) $\hat{p}_1 \geq 0.020$
b) $\hat{p}_1 \geq 0.024$
c) $\hat{p}_1 \geq 0.028$.

Table 4.1 informs about the multiplication factor MMN. Compared to the relatively low CR for the EWMA-N model in Table 3.2 the charge due to the penalty component f is rather small. The factor f decreases from 1.12 to 1.08 and its average value $\overline{f}$ is 1.1. Presumably, the new multiplication factor MMN based on the normal distribution is still too weak to ensure the aimed built-in positive incentive.

In the distribution-free approach we increase the penalty strength of the factor f by relating the expected shortfall ES to the ex-ante quantile $q^a_{0.99}$, $q^a_{0.99} \leq q^p_{0.99}$. Otherwise the construction of the distribution-free multiplication factor MMF is similar to the structure of MMN.

Here, we will outline how MMF is build for the GARCH-FHS model. Contrary to the gaussian approach where the backtesting is performed by comparing $\tilde{y}_{T-i+1}$ with the fixed value $z_{0.99} = 2.3263$, the comparison in the distribution-free approach is based on estimations of the unknown quantile q_{1-p}. The estimates are empirical quantiles with changing data sets, compare (2.6) and (2.17) and notice that q_{1-p} in (2.6) is estimated by $\hat{q}^{HS}_{1-p}(\{\tilde{y}^G_T\})$ in (2.17) with the data set $\{\tilde{y}^G_t\}^T_{t=1}$. Let us build the changing 0.99-th empirical quantiles $q^{HS}_{0.99}(\{\tilde{y}^G_{T-i}\})$ with standardized losses $\{\tilde{y}^G_t\}^{T-i}_{t=-i+1}$, $i = 1, 2, ..., 250$. Notice that in the distribution-free approach the number of exceptions H_1 results from the comparison of standardized losses $\tilde{y}^G_{T-i+1}$ with these changing empirical quantiles $q^{HS}_{0.99}(\{\tilde{y}^G_{T-i}\})$. With information up to period $T-1$ we can build the ex-ante quantile $q^a_{0.99}(T) = \frac{1}{250}\sum_{i=1}^{250} q^{HS}_{0.99}(\{\tilde{y}_{T-i}\})$ for the period T which corresponds to the fixed ex-ante quantile $z_{0.99}$ in the normal distribution based approach. For the sake of simplicity we suppress the index G in the ex-ante quantile $q^a_{0.99}(T)$. The corresponding ex-post quantile $q^p_{0.99}(T)$ is $q^{HS}_{0.99}(\{\tilde{y}_T\})$. Given the ex-post expected shortfall

(4.2) $$ES^p(q^p_{0.99}(T)) = \frac{1}{|\tilde{y}_{T-i+1} > q^p_{0.99}(T)|} \sum_{\tilde{y}_{T-i+1} > q^p_{0.99}} \tilde{y}_{T-i+1},$$

we can build the compensation factor

(4.3) $$k = q^p_{0.99}(T)/q^a_{0.99}(T) \qquad \text{for } \hat{p}_1 \geq 0.02$$

and with the penalty factor

(4.4) $$f = ES^p(q^p_{0.99}(T))/q^a_{0.99}(T)$$

the modified distribution-free multiplication factor MMF,

(4.5) $$MMF = M \times k \times f$$

with f for

a) $\hat{p}_1 \geq 0.020$
b) $\hat{p}_1 \geq 0.024$
c) $\hat{p}_1 \geq 0.028$.

5 Empirical analysis with modified capital requirements

With the described modifications of the multiplication factor we recalculate the capital requirement for the Dow Jones index. We used the modification where f is used in all cases with 5 or more exceedances, i.e. with f for $\hat{p}_1 \geq 0.02$. We added the values MMN and CRN of the gaussian approach as well as the values MMF and CRF of the distribution-free approach to the table from section 3, see Table 5.1.

Table 5.1: Results Dow Jones extended

Model	$\hat{p}_1$	$\hat{p}_{10}$	YZ	RZ	MB	CR	MMN	CRN	MMF	CRF
E-N	0.019	0.027	7	1	3.142	0.205	3.245	0.211	3.618	0.232
G-FHS	0.013	0.014	2	0	3.024	0.227	3.048	0.228	3.080	0.231

It is obvious that the distribution-free multiplication factor MMF leads to a substantial improvement. The capital requirement of the GARCH-FHS approach is nearly unchanged but the CR of the EWMA-N approach has significantly increased. The new factor punishes the bad performance and leads to a higher cushion against huge losses. As expected, the effect of the gaussian multiplication factor MMN is rather weak and insufficient.

In the next table we summarize the results for all high risky time series with f for $\hat{p}_1 \geq 0.020$. The penalty component f turns out to be insufficient for $\hat{p}_1 \geq 0.024$ and for $\hat{p}_1 \geq 0.028$, compare tables in the Appendix.

Table 5.2: Results with all high risky time series extended, with $\hat{p}_1 \geq 0.020$

	HV	HS	E-N	E-FHS	G-FHS	G-GPD	G-Hill
$VaR(1\|T)/SD$	1.633	1.698	1.584	1.818	1.834	1.856	1.853
$VaR(10\|T)/SD$	4.769	4.937	4.618	5.283	5.581	5.580	5.562
$\hat{p}_1$	0.021	0.020	0.023	0.017	0.014	0.014	0.014
$\hat{p}_{10}$	0.024	0.025	0.032	0.027	0.018	0.017	0.017
YZ	5.083	5.708	9.750	4.208	2.167	2.292	2.292
RZ	0.792	0.250	0.042	0.000	0.042	0.000	0.000
MB	3.18	3.17	3.25	3.09	3.05	3.05	3.05
CR/SD	15.29	15.72	14.84	16.04	17.01	17.00	16.97
MMN	3.30	3.30	3.44	3.17	3.09	3.08	3.09
CRN/SD	15.92	16.38	15.78	16.46	17.25	17.25	17.22
MMF	3.66	3.63	3.99	3.35	3.19	3.18	3.18
CRF/SD	18.37	19.05	18.77	17.57	18.17	18.09	18.05

Here also the more extensive analysis of all high risky assets confirms the results of the Dow Jones example in 5.1. The gaussian multiplication factor MMN is insufficient and the distribution-free multiplication factor MMF works quite well. In the HV, HS and EWMA-N model the distribution-free approach with MMF clearly outweighs the underestimation of the 1-day forecast VaR. The bad forecasting performance is punished. Compared to the more reliable GARCH models the factor MMF leads to a higher market risk charge CRF. The stimulus of the new multiplication factor MMF is strong enough to prevent financial institutions from applying these unreliable models. Thus, for the HV, HS and EWMA-N model the new risk charge CRF has the desired built-in positive incentive.

In the EWMA-FHS model the risk charge CRF nearly reaches the level of the charge CRF of the GARCH models, but it remains under the corresponding GARCH level. This is due to the fact that compared to the GARCH models the underestimation risk of the 1-day forecast VaR is only slightly higher. However, the underestimation of the 10-day forecast VaR is clearly bigger. As this underestimation risk is not grasped by the market risk charge a great discrepancy between $\hat{p}_1$ and $\hat{p}_{10}$ may indicate an insufficient punishment of risk underestimation.

In Table 5.3 we summarize the results for the FTSE-cluster with the most high risky time series. Here, the built-in positive incentive works for all four Non-GARCH models.

Table 5.3: Results with high risky FTSE time series with $\hat{p}_1 \geq 0.020$

	HV	HS	E-N	E-FHS	G-FHS	G-GPD	G-Hill
$VaR(1\|T)/SD$	1.426	1.525	1.530	1.776	1.692	1.729	1.722
$VaR(10\|T)/SD$	4.209	4.345	4.490	5.062	5.103	5.144	5.123
$\hat{p}_1$	0.027	0.024	0.026	0.020	0.018	0.017	0.017
$\hat{p}_{10}$	0.031	0.032	0.038	0.033	0.021	0.021	0.020
YZ	3.9	5.2	7.3	2.6	1.2	1.1	1.1
RZ	0.9	0.2	0.1	0.0	0.1	0.0	0.0
MB	3.24	3.24	3.30	3.10	3.05	3.04	3.04
CR/SD	13.55	13.87	14.20	14.84	15.34	15.38	15.34
MMN	3.39	3.41	3.55	3.19	3.08	3.06	3.07
CRN/SD	14.28	14.68	15.32	15.28	15.53	15.53	15.50
MMF	3.79	3.73	4.13	3.41	3.18	3.13	3.14
CRF/SD	16.97	17.23	18.57	16.56	16.23	16.06	16.07

6 Conclusion

As the recent financial crisis has made clear, better risk management models are needed. We have detected a critical loophole in the Basel II regulation that we think can be closed by using the suggested adjustments. As a result, a financial institution has an incentive to choose a reliable model for their market risk evaluation. We believe that this step leads to a more reliable risk management and a more adequate capital buffer.

We have shown that the most common VaR approaches like historical volatility (HV), historical simulation (HS), EWMA or other methods with the assumption of a gaussian distribution are not reliable because they lead to a sytematic underestimation of risk, especially in high risk situations.

GARCH models with bootstrapped quantile distribution (FHS) or with extreme value distributions like GPD or Hill lead to a substantially better performance regarding the reliability of the forecasts. At the same time, they also tend to slightly underestimate risks. The capital requirements, however, are substantially higher compared to non-GARCH models.

Current Basle II guidelines are not sufficient to deal with high risk situations; the current Basle II penalty factor for the capital requirement is not powerful enough. The built-in multiplication factor does not compensate for the underestimation of risks. The intended built-in positive incentive to develop and use better models in the Basle II capital requirement is not strong enough. We have observed a great discrepancy between the 1-day failure rate $\hat{p}_1$ and the 10-day failure rate $\hat{p}_{10}$. Since this may indicate an insufficient punishment of risk underestimation we propose to include both values in a broader risk evaluation.

Given the missing built-in incentive, an adjustment of the Basle II capital requirement is needed. Our proposed adjustments of the penalty factor close that gap. As a result, this leads to higher capital requirements and consequently to a adequate cushion against large losses. Given our modified distribution-free multiplication factor the bad performance of non-GARCH models is compensated and the resulting capital requirements are higher than the corresponding values from the better performing GARCH models. As the distribution-free GARCH approach is reliable for VaR forecasts so is the distribution-free multiplication factor MMF efficient as a built-in incentive in the Basle II regulation of market risk.

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8 Appendix

Table 8.1: Results with all high risky time series extended, with $\hat{p}_1 \geq 0.024$

	HV	HS	E-N	E-FHS	G-FHS	G-GPD	G-Hill
$VaR(1\|T)/SD$	1.633	1.698	1.584	1.818	1.834	1.856	1.853
$VaR(10\|T)/SD$	4.769	4.937	4.618	5.283	5.581	5.580	5.562
$\hat{p}_1$	0.021	0.020	0.023	0.017	0.014	0.014	0.014
$\hat{p}_{10}$	0.024	0.025	0.032	0.027	0.018	0.017	0.017
YZ	5.083	5.708	9.750	4.208	2.167	2.292	2.292
RZ	0.792	0.250	0.042	0.000	0.042	0.000	0.000
MB	3.18	3.17	3.25	3.09	3.05	3.05	3.05
CR/SD	15.29	15.72	14.84	16.04	17.01	17.00	16.97
MMN	3.26	3.25	3.35	3.13	3.07	3.06	3.07
CRN/SD	15.75	16.18	15.37	16.21	17.13	17.13	17.10
MMF	3.53	3.51	3.70	3.20	3.12	3.12	3.12
CRF/SD	17.70	18.29	17.31	16.69	17.59	17.61	17.59

Table 8.2: Results with all high risky time series extended, with $\hat{p}_1 \geq 0.028$

	HV	HS	E-N	E-FHS	G-FHS	G-GPD	G-Hill
$VaR(1\|T)/SD$	1.633	1.698	1.584	1.818	1.834	1.856	1.853
$VaR(10\|T)/SD$	4.769	4.937	4.618	5.283	5.581	5.580	5.562
$\hat{p}_1$	0.021	0.020	0.023	0.017	0.014	0.014	0.014
$\hat{p}_{10}$	0.024	0.025	0.032	0.027	0.018	0.017	0.017
YZ	5.083	5.708	9.750	4.208	2.167	2.292	2.292
RZ	0.792	0.250	0.042	0.000	0.042	0.000	0.000
MB	3.18	3.17	3.25	3.09	3.05	3.05	3.05
CR/SD	15.29	15.72	14.84	16.04	17.01	17.00	16.97
MMN	3.24	3.23	3.29	3.10	3.06	3.06	3.06
CRN/SD	15.64	16.03	15.08	16.10	17.08	17.07	17.04
MMF	3.48	3.40	3.52	3.13	3.11	3.10	3.10
CRF/SD	17.43	17.62	16.48	16.34	17.49	17.44	17.41