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48, Bd JOURDAN – E.N.S. – 75014 PARIS
TEL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
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Bubbles and self-fulfilling crises

Edouard Challe

Xavier Ragot

JEL Codes : G12, G33

Keywords : Credit market imperfections, self-fulfilling expectations, financial crises.

Bubbles and Self-fulfilling Crises

Edouard Challe* and Xavier Ragot†

Abstract

Financial crises are often associated with an endogenous credit reversal followed by a fall in asset prices and serious disruptions in the financial sector. To account for this sequence of events, this paper constructs a model where the excessive risk-taking of portfolio investors leads to a bubble in asset prices (in the spirit of Allen and Gale, ‘Bubbles and Crises’, *Economic Journal*, 2000), and where the supply of credit to these investors is endogenous. We show that the interplay between the risk shifting problem and the endogeneity of credit may give rise multiple equilibria associated with different levels of lending, asset prices, and output. Stochastic equilibria lead, with positive probability, to an inefficient liquidity dry-up at the intermediate date, a market crash, and widespread failures of borrowers. The possibility of multiple equilibria and self-fulfilling crises is showed to be related to the severity of the risk shifting problem in the economy.

Keywords: Credit market imperfections; self-fulfilling expectations; financial crises.

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*CNRS, DRM-CEREG, Université de Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16; email: edouard.challe@dauphine.fr.

†CNRS, Paris-Jourdan Sciences Economiques, 48 bd Jourdan 75014 Paris; email: ragot@pse.ens.fr.

1 Introduction

The resurgence of financial crises in past the fifteen years, both in OECD and emerging countries, has sparked a renewal of interest in the potential sources of financial fragility and market imperfections in which they originate. Although each crisis had, of course, its own specificities (depending, in particular, on the variety of exchange rate regimes that were adopted), it is now widely agreed that they all were characterised by a typical underlying pattern involving destabilising developments in credit and asset markets. Amongst OECD countries in the 80s and early 90s, like Japan or Scandinavian countries, financial crises were an integral part of a broader ‘credit cycle’, whereby financial deregulation led to an increased amount of available credit, fuelled a period of overinvestment in real estate and the stock market, leading to high asset-price inflation. These events were then followed by a credit contraction (or ‘crunch’), the bursting of the asset bubble, causing the actual or near bankruptcy of the financial institutions which had initially levered their asset investment (see Borio *et al.* (1994), Allen and Gale (1999) for a more detailed account of these events). In many emerging countries, particularly in Asia and Latin America, capital account liberalisation allowed large inflows of capital, with a similar effect of raising asset prices to unsustainable levels; This phase of overlending usually ended in a brutal capital account reversal from large deficits to small ones (or sometimes small surpluses), accompanied by a market crash and a banking crisis, also often (but not necessarily) coupled with the collapse of the prevailing exchange rate regime (see Kaminsky and Rheinart (1998, 1999), and Calvo (1998) for the evidence about this pattern, sometimes referred to as ‘sudden stop’).

An important theoretical issue, yet largely unanswered, is whether the credit reversal that typically accompanies such crisis is the outcome of an autonomous, ‘extrinsic’, reversal of expectations on the part of economic agents, or simply the natural outcome of building up macroeconomic imbalances and/or policy mistakes, i.e., the intrinsic fundamentals of the economy. For a time, the consensus was to interpret financial crisis as the mere outcome of extraneous ‘sunspots’ hitting the beliefs of investors, regardless of the underlying fundamental soundness of the economy. For example, early models of banking crises would emphasise the inherent instability of the banking system, whose provision of liquidity insurance made them sensitive to self-fulfilling runs, as the ultimate source of vulnerability to crises (see Diamond and Dybvig (1983), and Chang and Velasco (2000) for an open economy version of a similar model). In a similar vein, ‘second-generation’ models of currency crisis would insist on the potential existence of multiple equilibria in models of exchange rate determination, where the defense of a pre-announced peg by the central bank is too costly to be

fully credible (e.g., Velasco (1996)).

Although expectational factors certainly play a rôle in triggering financial crises, theories based purely on self-fulfilling expectations clearly do not tell the full story. In virtually all the recent episodes that were just briefly referred to, specific macroeconomic or structural sources of fragility preceded the actual occurrence of the crisis. In OECD countries, for example, financial crises usually followed periods of excessively loose monetary policy and/or poor exchange rate management (see Borio *et al.* (1994)). In emerging countries, the culprit was often to be found in the weakness of the banking sector due to poor financial regulation, as well as other factors such as unsustainable fiscal or exchange rate policies (Summers (2000)). In the specific case of emerging countries crises, the empirical evidence clearly indicates that, while indicators of fundamental weaknesses clearly explain a large part of the probability that a crisis will occur, a sizeable non fundamental component remains (see Kaminsky (1999), and the discussion of this piece of evidence by Chari and Kehoe (2003)). We interpret such evidence as suggesting that *both* aspects (fundamental and extrinsic) are at work when a financial crisis triggers, and that both ingredients should be part of any theoretical model trying to explain the recent crises in developed and developing countries.

The present paper aims to offer a model of this kind. We draw on Allen and Gale's (2000) (AG, henceforth) theory of financial crises, which in our view best grasps a central feature of all recent crisis, i.e., a credit-fuelled asset bubble, followed by a market crash and the failures of the financial institutions that had borrowed to buy speculative assets. In AG, financial crises are the natural outcome of credit relations where portfolio investors borrow to buy risky assets, and are protected against a bad realisation of their payoff by the use of simple debt contract with limited liability. Investors' twisted incentives then lead them to overinvest in risky assets (i.e., a *risk shifting* problem arises), whose price consequently rises to high levels (leading to an *asset bubble*), with the possibility that they go bankrupt if asset payoffs turn out badly (a *financial crisis* occurs). While AG focus on the partial equilibrium case where the total amount of credit available to portfolio investors is exogenous, we allow the supply of credit to vary according to an optimal consumption-savings plan by lenders. We regard this alternative assumption as not only more realistic, but also particularly relevant to our understanding the recent crisis episodes, where the endogeneity of the credit supply was frequently blamed for being an important cause of financial instability.

Analysing the interdependence between the risk shifting problem and the endogeneity of credit turns out to yield a whole new set of predictions, which can be summarised as follows. First, we show that when the risk shifting problem is present, the equilibrium

return that lenders expect from their loans to investors may be non monotonic and *increase* with the aggregate quantity of loans – rather than *decrease* with total loans, as standard marginal productivity arguments would seem to suggest. The reason for this can intuitively be explained as follows; Under risk shifting, higher lending to investors tends to alleviate their excessive risk-taking by lowering the optimal share of their resources that they devote to risky asset investment. This in turn tends to decrease the average riskiness of investors’ portfolio, and thus to increase the *ex ante* return on the loans that are made to them. In certain circumstances, which we derive and explain in the paper, this ‘portfolio composition effect’ dominates the usual marginal productivity effect (at least for some range of aggregate savings), causing the *ex ante* loan return to increase with total loans. The resulting strategic complementarity between individual lending decisions then naturally leads to the existence of multiple equilibria associated with different levels of aggregate credit, asset prices, interest rates, and output. We relate the intensity of these strategic complementarities, and the implied possibility of multiple equilibria, to the severity of the risk shifting problem in the economy, i.e., its ‘financial fragility’.

We focus on a particular model example where two possible lending equilibria exist at the intermediate date, and where the selection of the equilibrium with low lending follows a ‘sunspot’, i.e., an extraneous signal of any *ex ante* probability on which agents coordinate their expectations. We show that this events generates a *self-fulfilling crisis* at the intermediate date, which has the following characteristics; i) lending to portfolio investors drops down as lenders choose to consume, rather than save, a large share of their goods (credit contraction), ii) this causes a fall in investors’ financial resources and a drop in the demand for risky assets, whose price consequently falls to low levels (market crash), and iii) this fall in asset prices forces into bankruptcy investors who had previously borrowed to buy them, as the total value of their assets falls short of their liabilities (financial sector disruptions). Importantly, such crises follow a reversal of expectations on the part of economic agents, and are thus not restricted to situations where uncertainty about the amount of available credit is induced by policy (as in times of uncertain financial liberalisation, the example emphasised by AG). We end the paper by an analysis of the welfare properties of these stochastic equilibria, and show that the probability that a self-fulfilling crisis occurs at the intermediate date unambiguously decreases *ex ante* welfare.

The remainder of the paper is organised as follows. Section 2 introduces the model and derives its unique fundamental (i.e., first-best efficient) equilibrium. Section 3 shows how the interdependence between endogenous lending and the excessive risk-taking of portfolio investors may give rise, at the intermediate date, to multiple equilibria associated with

different levels of lending, interest rate, asset prices, and output. Section 4 derives the stochastic equilibria of this economy (i.e., equilibria featuring self-fulfilling financial crises), and analyses their welfare properties. Section 6 concludes the paper.

2 The model

2.1 Timing and assets

Since our model builds on AG's framework, we shall use similar notations whenever is possible in order to ease the comparison between our results and theirs.

There are three dates, 0, 1 and 2, and two real assets. One asset, safe and in variable supply, is two-period lived and yields $f(x)$ units of the (all-purpose) good at date $t + 1$ for $x \geq 0$ units invested at date t , $t = 0, 1$. It is assumed that $f(\cdot)$ is a twice continuously differentiable production function satisfying $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. Moreover, the following standard assumption is made to limit the curvature of $f(\cdot)$, for all $x > 0$:

$$\eta(x) \equiv -xf''(x)/f'(x) < 1 \quad (1)$$

The other asset is risky, in fixed supply (normalised to 1), and three-period lived – it is available for buying at date 0 and delivers a terminal payoff R at date 2, where R is a random variable at date 0 and 1 that takes on the value R^h with probability $\pi \in (0, 1]$, and 0 otherwise, at date 2. The market price of the risky asset at date t , in terms of the all-purpose good (which is taken as the numeraire), is denoted P_t , $t = 0, 1$.

Although more general distributions for the fundamental uncertainty affecting the asset payoff can be considered, we choose this simple one in order to focus on 'extrinsic' uncertainty generated by the presence of multiple equilibria. Note that multiple equilibria very similar to those analysed in this paper also exist if the risky asset is in variable supply, so that its quantity, rather than its price, adjusts over time to clear markets. The interpretation of the present specification is that the supply of the risky asset responds slowly to changes for its demand (as it is the case for stocks or real estate, for example), while that of the safe asset adjusts quickly, and we analyse the way markets clear in the short run.

2.2 Agents and market structure

The economy is populated by four types of risk-neutral agents in large numbers. There is a continuum of three-period lived *lenders* in mass 1, who enter the market at date 0 and

leave it at date 2. Their intertemporal utility is $u(c_1, c_2) = c_1 + \beta c_2$, where c_t , $t = 1, 2$, is date- t consumption and $\beta > 0$ is the discount factor (i.e., lenders do not enjoy date-0 consumption). Lenders receive an endowment $e_0 > 0$ at date 0 and e_1 at date 1, about which the following technical assumption is made:

$$e_1 > f'^{-1}(1/\beta) + \beta\pi R^h \quad (2)$$

As will become clear in the following, condition (2) is necessary and sufficient for all the equilibria that we analyse in the paper to correspond to interior solutions (i.e., where both c_1 and c_2 are positive). Given the assumed utility function, lenders save their entire endowment e_0 at date 0 (provided the *ex ante* return on savings at date 0 is non negative, as is always the case), while savings decisions at date 1 depend on the comparison between the *ex ante* return on savings then and the gross rate of time preference, $1/\beta$. As will become clear shortly, this possibility that lenders consume rather than lend their wealth at date 1 renders aggregate lending endogenous at that date, and is the novel and crucial feature of our model.

Lenders face overlapping generations of two-period lived *investors* and *entrepreneurs* in positive mass, entering the economy at dates 0 and 1 and maximising final consumption. In the remainder of the paper, we shall refer to ‘date- t investors (entrepreneurs)’ as the investors (entrepreneurs) who enter the economy at date t , $t = 0, 1$, and leave it at date $t+1$. Neither investors nor entrepreneurs receive any endowment. Finally, the stock of risky asset is initially held by a class of one-period lived *initial asset holders*, who sell them to date-0 investors and leave the market.

There is *market segmentation* (i.e., restrictions on agents’ asset holdings), in the two following senses. First, only entrepreneurs have access to the production technology $f(\cdot)$; Entrepreneurs’ utility maximisation under perfect competition then ensures that the gross interest rate on corporate bonds at date t ($= 0, 1$), called r_t , is equal to the marginal product of capital, $f'(X_{St})$, where X_{St} is the amount invested in production at date t . Second, lenders cannot directly buy risky assets or corporate bonds, and must thus lend to investors to finance future consumption. These restrictions imply that market equilibria at date 0 and 1 are *intermediated*, with lenders first entrusting investors with their savings, and investors then lending to entrepreneurs (i.e., buying X_{St} corporate bonds at the normalised price 1) and investing in risky assets (i.e., buying X_{Rt} assets at price P_t). We denote B_t the demand for loans by date- t investors, $t = 0, 1$ (which, in equilibrium, equals lenders’ savings at the same date). Finally, we follow AG in assuming that lenders and investors are restricted to use simple debt contracts, where the contracted rate on loans at date t ,

denoted r_t^l , cannot be conditioned on the loan size or, due to asymmetric information, on investors' portfolio.

2.3 Fundamental equilibrium

In the intermediated economy just described, investors are granted exclusive access to the markets for risky assets and corporate bonds. Before analysing the implied market outcome in more details, it is useful to derive first the equilibrium that would prevail if these restrictions were removed, i.e., if lenders could directly buy both real assets. The corresponding 'fundamental' equilibrium, along which prices and quantities are first-best efficient, will provide a natural benchmark against which the intermediated equilibrium(a) can be compared (see AG, p. 244). As is usual with finite horizon economies, we work backwards equilibrium prices and quantities, using date-1 outcomes to feed date-0 equilibrium conditions. We index fundamental values by using the superscript f .

Equilibrium at date 1. Since lenders' date-1 savings, B_1 , sum up to safe asset investment, X_{S1} , plus risky asset investment, $X_{R1}P_1$, lenders' expected date-2 consumption from saving B_1 and choosing a portfolio (X_{S1}, X_{R1}) at date 1 is

$$E_1 \left(r_1^f X_{S1} + R X_{R1} \right) = r_1^f B_1 + X_{R1} \left(\pi R^h - r_1^f P_1^f \right), \quad (3)$$

where E_1 denotes expectations formed on the basis of information available at date 1, and P_1^f and r_1^f denote the date-1 fundamental values of the risky asset and the interest rate, respectively. Given B_1 , the price of the asset in the fundamental equilibrium must be:

$$P_1^f = \pi R^h / r_1^f \quad (4)$$

If the fundamental value of the risky asset were lower than $\pi R^h / r_1^f$, then $\pi R^h - r_1^f P_1^f$ would be positive for all positive values of X_{R1} and lenders would want to buy an infinite quantity of risky assets; If it were higher than $\pi R^h / r_1^f$, then the net unit return on holding risky assets would be negative and the demand for them would be zero. Since the risky asset is in positive and finite supply, neither $P_1^f < \pi R^h / r_1^f$ nor $P_1^f > \pi R^h / r_1^f$ can be an equilibrium situation.

Using Eq.(4) and the fact that in equilibrium $X_{R1} = 1$ and thus $r_1^f = f'(X_{S1}) = f'(B_1 - P_1^f)$, clearing of the market for corporate bonds implies:

$$f'^{-1}(r_1^f) + \pi R^h / r_1^f = B_1 \quad (5)$$

The above equation defines r_1^f uniquely for all positive values of B_1 , and can thus be inverted to yield the interest rate function $r_1^f(B_1)$. Given the properties of $f(\cdot)$ specified in Sec. 2.1, $r_1^f(B_1)$ is continuous, strictly decreasing, and such that $r_1^f(0) = \infty$ and $r_1^f(\infty) = 0$.

Substituting (4) into (3), we can see that lenders' (expected) date-2 consumption is $B_1 r_1^f(B_1)$. Given lenders' assumed linear utility (see Sec. 2.2) and our assumption of high initial endowment (inequality (2)), lenders increase savings up to the point where the rate of return on savings, $r_1^f(B_1)$, is equal to the rate of time preference, $1/\beta$ (see figure 1 below). Using Eqs. (4) and (5), this implies that asset prices and aggregate savings in the fundamental equilibrium are uniquely determined and given by:

$$P_1^f = \beta\pi R^h \quad (6)$$

$$B_1^f = f'^{-1}(1/\beta) + \beta\pi R^h \quad (7)$$

In short, lenders' risk neutrality implies that the fundamental value of the asset, P_1^f , is equal to the discounted expected dividend stream, $\beta\pi R^h$, while capital investment, X_{S1}^f , settles at the point where its rate of return equals lenders' rate of time preference, $f'^{-1}(1/\beta)$.

Equilibrium at date 0. The fundamental price vector at date 1, (P_1^f, r_1^f) , can now be used to derive that at date 0, (P_0^f, r_0^f) , by simply noting that the equilibrium price of risky assets at date 1, P_1^f , is also the payoff from holding them from date 0 to date 1. Lenders' total (deterministic) payoff at date 1 from choosing a portfolio (X_{S0}, X_{R0}) at date 0 is then $r_0^f X_{S0} + P_1^f X_{R0}$, which they maximise subject to the portfolio choice constraint $X_{S0} + P_0^f X_{R0} = e_0$. Thus, they maximise:

$$r_0^f X_{S0} + P_1^f X_{R0} = e_0 r_0^f + X_{R0} (P_1^f - r_0^f P_0^f)$$

Given $e_0 r_0^f$, the fundamental value of the risky asset at date 0 cannot be higher (lower) than P_1^f / r_0^f , since asset demand would then be equal to zero (infinity). It must thus be:

$$P_0^f = P_1^f / r_0^f = \beta\pi R^h / r_0^f \quad (8)$$

Finally, using Eq. (8), the properties of $f(\cdot)$, and the fact that $X_{R0} = 1$ and thus $r_0^f = f'(X_{S0}) = f'(e_0 - P_0^f)$ in equilibrium, r_0^f is uniquely determined by the following equation:

$$f'^{-1}(r_0^f) + \beta\pi R^h / r_0^f = e_0 \quad (9)$$

Equations (8) and (9) fully characterise equilibrium prices at date 0 and complete our derivation of the fundamental equilibrium of this economy. The remainder of the paper works out equilibrium prices and quantities for the intermediated case, i.e., where lenders no longer have direct access to the markets for risky assets and corporate bonds.

3 Endogenous lending and multiple equilibria

The remainder of the paper derives the intermediated equilibrium(a) of the economy, in a way similar to that used in the derivation of the fundamental one. The present Section solves for the equilibrium at the intermediate date (i.e., date 1), and shows how the interplay between endogenous lending and the risky shifting problem (due to market segmentation) may lead to multiple equilibria. Sec. 4 uses date-1 outcomes to derive the stochastic equilibria of the full model.

3.1 Market clearing at date 1

Contracted loan rate. Date-1 investors borrow B_1 (≥ 0) from lenders, which they use to buy X_{S1} units of corporate bonds at price 1 and X_{R1} units of the risky asset at price P_1 (so that $B_1 = X_{S1} + X_{R1}P_1$). The use of debt contracts with limited liability allows investors to default, and earn 0, when their total payoff at date 2, $r_1X_{S1} + RX_{R1}$, is less than the amount owed to lenders, $r_1^l B_1$. Thus, the terminal consumption of date-1 investors is:

$$\sup [r_1X_{S1} + RX_{R1} - r_1^l B_1, 0] = \sup [X_{R1} (R - r_1P_1) + B_1 (r_1 - r_1^l), 0]$$

Note from the latter equation that the contracted rate on loans between lenders and investors, r_1^l , must be equal to the interest rate on corporate bonds, r_1 . If $r_1 > r_1^l$, then investors would want to borrow an unlimited amount of funds from lenders (to invest them in the safe asset at rate r_1); they would then reach the (finite) limit of available funds, and from then compete for loans until $r_1 = r_1^l$. If $r_1 < r_1^l$ then investors' loan demand would be nil, implying that the return on safe assets would be $r_1 = f'(0) = \infty$, a contradiction. Thus, any equilibrium in the markets for loans and corporate bonds must satisfy $r_1^l = r_1 = f'(X_{S1})$.

Asset prices and interest rate. Since $B_1 (r_1 - r_1^l) = 0$, investors' terminal consumption is simply $\sup [X_{R1} (R - r_1P_1), 0]$. Because $X_{R1} (0 - r_1P_1) < 0$ for all $P_1 > 0$, investors default on loans when the asset payoff is 0, and this occurs with probability $1 - \pi$. Their expected date-2 consumption is thus $\pi X_{R1} (R^h - r_1P_1)$, provided they do not default when the asset payoff is R^h (i.e., $X_{R1} (R^h - r_1P_1)$ is non negative, as is always the case in equilibrium). Given their objective of maximising expected consumption, clearing of the market for the risky asset implies that its equilibrium price must be:

$$P_1 = R^h / r_1 \tag{10}$$

If the price of the asset were lower (higher) than R^h/r_1 , then $R^h - r_1P_1$ would be positive (negative) for all positive values of X_{R1} and date-1 investors would want to buy infinitely many (zero) risky assets. Note from Eq. (10) that the competition of risk-neutral investors for the risky asset implies that their expected gain is zero even when the asset payoff is R^h . Thus, date-1 investors' profits and consumption levels are zero under both possible realisations of R at date 2.

Using Eq. (10) and the fact that in equilibrium $X_{R1} = 1$ and $r_1 = f'(X_{S1})$, we have $r_1 = f'(B_1 - P_1)$. Clearing of the market for corporate bonds at date 1 then implies:

$$f'^{-1}(r_1) + R^h/r_1 = B_1 \tag{11}$$

Eq. (11) defines the equilibrium interest rate uniquely for all positive values of B_1 . From the assumed properties $f(\cdot)$, the interest rate function $r_1(B_1)$ is continuous and such that $r'_1(B_1) < 0$, $r_1(0) = \infty$ and $r_1(\infty) = 0$. Eqs. (10)–(11) then fully characterise the intermediated equilibrium price vector at date 1, (P_1, r_1) , *conditionally* on the amount of aggregate lending, B_1 (the latter is endogenised in Sec. 3.3 below).

Note from Eqs. (5) and (11) that, for a given quantity of savings, B_1 , the intermediated interest rate, r_1 , is higher than the fundamental one, r_1^f . The reason for this is the following; For that value of B_1 the expected asset payoff that accrues to investors in the intermediated equilibrium, R^h , is higher than the expected payoff to lenders in the fundamental equilibrium, πR^h . In consequence, risky assets are bid up in the intermediated equilibrium, safe asset investment, X_{S1} , is crowded out, which in turn raises the equilibrium interest rate, r_1 (with respect to the fundamental one, r_1^f). The intermediated equilibrium is thus characterised by *risk shifting*, in the sense that portfolio delegation to debt-financed investors leads to an excessive share of risky asset investment, and too little safe asset investment, with respect to the efficient portfolio (i.e., the fundamental equilibrium). The implications of this distortion for equilibrium asset prices and savings are analysed in Sec. 3.4 below.

3.2 Expected loan return

Given our assumed lenders' utility function, individual lending decisions at date 1 simply depend on the gross expected return on loans to portfolio investors, denoted ρ_1 , as compared to the gross rate of time preference, $1/\beta$. Note that ρ_1 generally differs from the contracted loan rate, r_1 , because of the possibility that date-1 investors default on loans at date 2.

When date-1 investors do not default on loans, which occurs with probability π , the contracted loan rate applies and they repay lenders $B_1r_1(B_1)$. When they do default,

lenders gather the residual value of investors' portfolio, $f'(X_{S1}) X_{S1} = r_1(B_1)(B_1 - P_1)$. The *ex ante* unit loan return is thus $\pi r_1(B_1) + (1 - \pi) r_1(B_1)(1 - P_1/B_1)$ or, using Eq. (10),

$$\rho_1(B_1) = r_1(B_1) - \frac{(1 - \pi) R^h}{B_1} (> 0) \quad (12)$$

Note from Eqs. (5), (11) and (12) that the probability that investors go bust at date 2, $1 - \pi$, indexes the distance between the fundamental and the intermediated returns on savings, r_1^f and ρ_1 . When $\pi = 1$ the risk shifting problem disappears since portfolio investors never default; The intermediated loan return function, $\rho_1(B_1)$, is then identical to the fundamental return function, $r_1^f(B_1)$, so that the intermediated equilibrium becomes uniquely determined by Eqs. (6)–(7). When $\pi < 1$, the distance between these two return functions, for a given level of aggregate savings, is easily shown to be:

$$r_1^f(B_1) - \rho_1(B_1) = \frac{r_1^f f^{-1}(r_1^f) - r_1 f^{-1}(r_1)}{B_1},$$

which is positive since $x f^{-1}(x)$ decreases with x (by assumption (1)) and $r_1 > r_1^f$ due to the crowding out of safe asset investment (see Sec. 3.1 above). Because the extend of this crowding out depends on π , the probability that the asset payoff turns out badly, $1 - \pi$, measures both the severity of the risk shifting problem in the economy at date 1 and the implied distortion in the intermediated return on loans.

To analyse the existence and properties of the intermediated equilibrium(a) when $\pi < 1$, one must characterise the behaviour of $\rho_1(B_1)$ as B_1 varies over $(0, \infty)$. First, note that $\rho_1(B_1)$ is continuous and such that $\rho_1(\infty) = 0$ and $\rho_1(0) = \infty$.¹ Although this implies that $\rho_1'(B_1)$ must be negative somewhere, the two terms in the right-hand side of equation (12) indicate that, over a given interval $[B_a, B_b] \subset (0, \infty)$, changes in $\rho_1(B_1)$ following variations in B_1 are of ambiguous sign.

The first term of the right-hand side of (12), $r_1(B_1)$, is the decreasing interest rate function characterised in Sec. 3.1 above; An increase in B_1 raises the amount invested in the safe asset, X_{S1} , which tends to lower the equilibrium interest rate, $r_1 = f'(X_{S1})$, and thus the average return on loans; This is the usual 'marginal productivity effect' of aggregate savings on the loan return. In contrast, the second term, $-(1 - \pi) R^h/B_1$, *increases* with B_1 ; This latter effect reflects the impact of the total amount of loan on the average riskiness

¹That $\rho_1(0) = \infty$ can be seen by solving (11) for R^h and substituting the resulting expression into (12) to obtain $\rho_1(B_1) = r_1(B_1)(\pi + (1 - \pi) X_{S1}/B_1)$. Since $r_1(0) = \infty$ and $X_{S1}/B_1 > 0$, we have $\rho_1(0) = \infty$.

of loans as the *composition* of the optimal portfolio varies with B_1 . To see this use Eq. (11) again to write the relation between safe asset investment, X_{S1} , and aggregate lending, B_1 , as follows:

$$B_1 = X_{S1} + R^h/f'(X_{S1}) \quad (13)$$

From equation (13) and assumption (1) about the concavity of $f(\cdot)$, it is easy to check that an increase in B_1 raises *both* the quantity of safe assets, X_{S1} , and the share of safe asset investment in investors' portfolio, X_{S1}/B_1 (i.e., it lowers $B_1/X_{S1} = 1 + R^h/X_{S1}f'(X_{S1})$). In other words, even though an increase in B_1 lowers r_1 and thus raises asset prices, R^h/r_1 , the relative size of risky asset investment, $P_1/B_1 = 1 - X_{S1}/B_1$, tends to *decrease* as B_1 increases. This 'portfolio composition effect' in turn limits the loss to lenders in case of investor's default and increases the *ex ante* return on loans.

Given these two effects at work, the crucial question is, Are there intervals of B_1 over which $\rho_1(B_1)$ may be increasing? Taking the derivative of (12) with respect to B_1 , this is the case if there are intervals of B_1 over which

$$-r'_1(B_1) B_1^2 < (1 - \pi) R^h \quad (14)$$

When $\pi < 1$ condition (14) may be satisfied if $-r'_1(B_1) (> 0)$ is small enough for some values of B_1 , that is, if the equilibrium interest rate is not very responsive to changes in the implied level of safe asset investment, X_{S1} . This in turn is true if $f(X_{S1})$ is 'flat enough' for the relevant range of X_{S1} , so that $r_1 = f'(X_{S1})$ responds little to changes in X_{S1} . Using Eq. (11), together with the fact that $\partial f'^{-1}(r_1)/\partial r_1 = 1/f''(X_{S1})$, the left-hand side of (14) can be written as follows:

$$-r'_1(B_1) B_1^2 = \frac{(R^h + X_{S1}f'(X_{S1}))^2}{R^h + f'(X_{S1})^2 / (-f''(X_{S1}))} (> 0)$$

For $X_{S1} \in [X_a, X_b]$, which occurs when $B_1 \in [X_a + R^h/f'(X_a), X_b + R^h/f'(X_b)]$, $-r'_1(B_1) B_1^2$ can be made smaller and smaller by decreasing the curvature of $f(\cdot)$ over $[X_a, X_b]$; In this case $f'(X_{S1})$ is bounded above and below, $-f''(X_{S1})$ can be made arbitrarily small, making $-r'_1(B_1) B_1^2$ as small as necessary for (14) to hold (provided $\pi \neq 1$). Importantly, the larger $1 - \pi$ (i.e., the more severe the risk shifting problem), the more likely inequality (14) is satisfied, for a given interest rate function, $r_1(B_1)$.

Since there may be several intervals of B_1 over which (14) is satisfied, $\rho'_1(B_1)$ potentially changes signs many times as B_1 increases. In the remainder of the paper, we shall focus on a particularly simple case of non-monotonicity by assuming that $\rho_1(B_1)$ has a single increasing

interval, as is depicted in figure 1 (all our results are easily generalised to the case of multiple increasing intervals). To give a simple example of a class of production technologies generating this property, Appendix A shows that so looks the loan return function if $f(x)$ is isoelastic, where $\eta(x)$ in inequality (1) is a constant that is close enough to zero (formally, $\rho_1(B_1)$ has exactly one increasing interval if $\eta < (1 - \sqrt{\pi})/2$, none otherwise).

3.3 Loan market equilibrium

The possibility that the expected loan return be an increasing function of the total quantity of loans is an example of ‘strategic complementarity’ (in the sense of Cooper and John (1988)) in lending decisions, since the choice by other lenders to increase savings may then lead any individual lender to vary savings in the same direction. Lenders utility function imply that they increase savings as long as $\rho_1(B) > 1/\beta$, but decrease savings whenever $\rho_1(B) < 1/\beta$; Any equilibrium must thus satisfy $\rho_1(B) = 1/\beta$. We focus on symmetric Nash equilibria, where consumption/savings plans are identical across lenders and no lender finds it worthwhile to individually alter his own plan. Then, our normalisation of a unit mass of lenders implies that individual and aggregate quantities coincide in equilibrium.

Figure 1: Loan market equilibrium at date 1

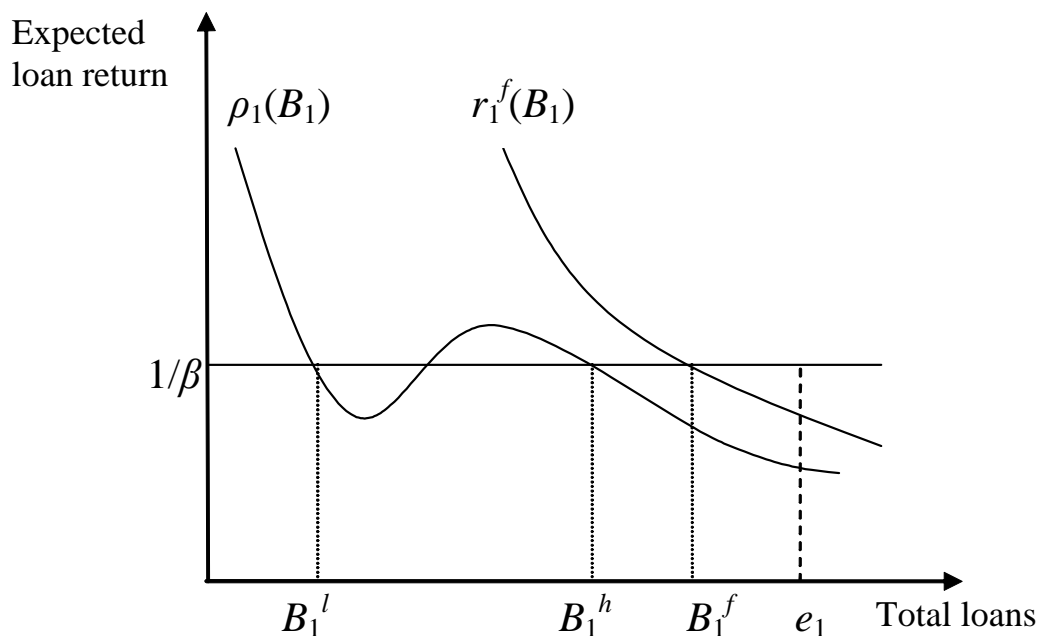


Figure 1 shows how multiple crossings between the $\rho_1(B)$ -curve and the $1/\beta$ -line, when they occur, give rise to multiple equilibria (this phenomenon is robust since there are infinitely many production functions, $f(\cdot)$ and associated gross rates of time preference, $1/\beta$, that generate such multiple crossings). B_1^l and B_1^h represent two stable levels of aggregate lending, i.e., where a symmetric marginal move away from equilibrium by all lenders alters the loan return in a way that favours the restoration of the equilibrium. The value of B_1 where the $\rho_1(B_1)$ -curve crosses the $1/\beta$ -line from below is not stable and will not be discussed any further (starting from there, an arbitrarily small increase (decrease) in B_1 tends to increase (decrease) $\rho_1(B_1)$, triggering a further move away from equilibrium). In both stable equilibria the *ex ante* return on loans is $1/\beta$, and lenders (expected) date-2 consumption, conditional on the selection of equilibrium j , $j = l, h$, is $\rho_1(B_1)B_1^j = B_1^j/\beta$. Note that assumption (2) ensures that both B_1^l and B_1^h are interior solutions that are independent from the amount of goods that lenders receive from the loans they have made at date 0. Any income coming from these loans is thus consumed at date 1 (the effects of date-0 loans on lenders' date-1 wealth and consumption are analysed in Sec. 4 below).

Recall from the previous Section that an increase in B_1 lowers marginal productivity but also reduces the average riskiness of investors' portfolio. The low-lending equilibrium is thus characterised by a high safe return but a high share of risky assets in investors' portfolio, while the high-lending equilibrium has a low safe return but a safer average portfolio. Finally, notice that even though both equilibria yield the same *ex ante* return on loans, $1/\beta$, they are always associated with different levels of interest rates, asset prices and (expected) date-2 output. Indeed, Eq. (11) and the fact that $B_1^h > B_1^l$ implies that $r_1(B^h) < r_1(B^l)$ and $X_{S1}^h > X_{S1}^l$, where X_{S1}^j , $j = l, h$, denotes the level of safe asset investment when B_1^j is selected. Then calling P_1^j the asset's price and $E_1(Y|j)$ expected date-2 output (in the sense of the total quantity of goods available for agents' consumption) when total lending is B_1^j , we have:

$$\begin{aligned} P_1^h &= R^h/r_1(B^h) > P_1^l = R^h/r_1(B^l) \\ E_1(Y|h) &= f(X_{S1}^h) + \pi R^h > E_1(Y|l) = f(X_{S1}^l) + \pi R^h \end{aligned}$$

To summarise, the selection of the equilibrium with low lending raises the interest rate and depresses asset prices, productive investment, and future output, with respect to the equilibrium with high lending. (More generally, there may be more than two stable equilibria if $\rho_1(B_1)$ has more than one increasing interval, but their properties are similar to the 2-equilibrium case, i.e., the higher B_1 , the lower $r_1(B_1)$, and the higher P_1 , X_{S1} and $E_1(Y|j)$).

3.4 Asset bubble and crowding out

We emphasised in Sec. 3.1 that the risk shifting problem that arises under market segmentation leads investors to overinvest in risky assets, with respect to the fundamental equilibrium. We now analyse the implications of this distortion for the price of the risky asset and the amount of aggregate savings in equilibrium. We prove in Appendix B the following inequalities:

$$P_1^j > P_1^f, \quad j = l, h \quad (15)$$

$$B_1^j < B_1^f, \quad j = l, h \quad (16)$$

Equation (15) indicates that assets are overpriced at date 1 in both intermediated equilibria, i.e., both of them are associated with a positive *bubble* in asset prices (the bubble being larger, the larger aggregate credit). This bubble is an immediate consequence of the fact that investors, who are protected against a bad realisation of the asset payoff by the use of simple debt contracts, find it worthwhile to bid up the asset, and thus to overinvest in it, with respect to the fundamental equilibrium.

The reason why savings are lower in both intermediated equilibria than in the fundamental one (Eq. (16)) naturally follows; Excess risky asset investment by portfolio investors implies that, for any given level of savings B_1 , the intermediated return, $\rho_1(B_1)$, is *lower* than the fundamental one, $r_1^f(B_1)$ (see our analysis in Sec. 3.1). Lenders must thus reduce credit in the intermediated equilibrium (with respect to the fundamental one) up to the point where the intermediated *ex ante* return, $\rho_1(B_1)$, is back to the fundamental one, i.e., the gross rate of time preference $1/\beta$ (see figure 1 again). Notice, as a consequence of this analysis, that a ‘double crowding out’ is in fact at work on X_{S1} in the intermediated equilibrium. First, for a given level of aggregate savings B_1 , bubbly asset prices crowd out safe asset investment, X_{S1} , and raise the equilibrium interest rate, $r_1 = f'(X_{S1})$ (see Sec. 3.1 again). Second, lenders’ optimal reaction to the resulting price distortion is to reduce savings, B_1 , which lowers X_{S1} (and raises r_1) even further.

The crowding out of productive investment by bubbly asset prices is the basic source of output loss in the intermediated economy, with respect to one where fundamental outcomes would prevail. The implications of this loss as to the welfare ranking of the (many) intermediated equilibria are analysed in the context of the full stochastic model below.

4 Self-fulfilling financial crises

The previous Section has shown that the excessive risk taking of portfolio investors may lead, under endogenous credit, to the existence of multiple equilibria at date 1 associated with different levels of aggregate lending, interest rates, and asset prices. We now analyse the full time span of the model to demonstrate the possibility of a *self-fulfilling financial crisis* associated with the risk that the low-lending equilibrium is selected.

We construct equilibria with self-fulfilling crises by randomising over the two possible lending equilibria that may prevail at date 1. To do this, assume that at date 1 high lending is selected with probability $p \in (0, 1)$, so that a ‘sunspot’ causes lending and asset prices to drop down to low levels with probability $1 - p$.² With this specification for extraneous uncertainty about which level of aggregate lending will prevail at date 1, the model potentially has a *continuum* of stochastic equilibria indexed by the *ex ante* probability of a market crash, $1 - p$. Since the asset price at date 1 is the asset payoff for date-0 investors, this extraneous uncertainty about asset prices creates a risk shifting problem at date 0 similar to that created at date 1 by intrinsic uncertainty about the terminal payoff of the asset. This causes the asset to be bid up at date 0, with the possibility that a self-fulfilling crisis (i.e., a drop in asset prices forcing date-0 investors into bankruptcy) occurs at the intermediate date if the low-lending equilibrium is selected.

4.1 Market clearing at date 0

Contracted loan rate. Call (P_0, r_0) the equilibrium price vector, r_0^l the contracted borrowing rate, and (X_{S0}, X_{R0}) the portfolio of date-0 investors, all at date 0. Limited liability of investors and the portfolio constraint $B_0 = X_{S0} + P_0 X_{R0}$ imply that their terminal consumption is:

$$\sup [r_0 X_{S0} + P X_{R0} - r_0^l B_0, 0] = \sup [X_{R0} (P - r_0 P_0) + B_0 (r_0 - r_0^l), 0],$$

where, given our specification for extraneous uncertainty at date-1, P_1 is a random variable at date 0 taking on the value P_1^h with probability p (i.e., B_1^h is selected), and P_1^l otherwise (B_1^l is selected).

²We choose to focus on equilibria where financial crises may actually occur at date 1 (i.e., where date-0 investors may go bankrupt), and thus leave out of the analysis equilibria with deterministic date-1 outcomes. The $p = 1$ case (high lending is selected for sure) has similar date-0 prices and quantities than the $0 < p < 1$ case, while $p = 0$ (low lending for sure) entails different date-0 equilibrium values than the $p \in (0, 1]$ case.

The contracted rate on loans at date 0, r_0^l must necessarily be equal to the rate on corporate bonds at the same date, r_0 . If the former were lower (higher) than r_0 , then date-0 investors would want to borrow infinitely many (zero) units of goods and use them to buy corporate bonds, while the loan supply at date 0 is exactly e_0 (provided the expected loan return at date 0 is non negative, as is always the case since, even in case of investors' default, lenders get some positive repayment, i.e., the liquidation value of date-0 investors' portfolio). Thus, any equilibrium must satisfy $r_0 = r_0^l$ and $B_0 = e_0$.

Asset prices and interest rate. In the equilibria that we are considering, date-0 investors default on loans when the asset price at date 1 is P_1^l , but do not default when it is P_1^h . Since $B_0(r_0 - r_0^l) = 0$, their terminal consumption is then $X_{R0}(P_1^h - r_0P_0) \geq 0$ with probability p , and 0 otherwise. Date-0 investors choose X_{R0} that maximises expected terminal consumption, $pX_{R0}(P_1^h - r_0P_0)$, while any potential solution to their decision problem must be such that they do not default on loans if the asset price at date 1 is P_1^h , but do default if it is P_1^l , i.e.,

$$P_1^h - r_0P_0 \geq 0, P_1^l - r_0P_0 < 0 \quad (17)$$

The demand for risky assets by date-0 investors is infinite (zero) if $P_1^h - r_0P_0 > 0 (< 0)$. Market clearing thus requires that the equilibrium price of the risky asset be such that $P_1^h - r_0P_0 = 0$, i.e.,

$$P_0 = P_1^h/r_0, \quad (18)$$

which satisfies both inequalities in (17). Here again the interpretation of this equilibrium price is straightforward. The perfect competition for the risky asset by investors implies that the asset's price must be such that they make zero expected profit. Because they make zero profit when the realisation of the asset payoff is P_1^l (i.e., when they default), they must also earn zero even when it is P_1^h ; This is exactly what the equilibrium price P_1^h/r_0 ensures.

Aggregate lending from date 0 to date 1 is e_0 . In equilibrium we have $X_{R0} = 1$ and $r_0 = f'(X_{S0}) = f'(e_0 - P_0)$. Thus, r_0 is uniquely determined by the following equation:

$$f'^{-1}(r_0) + P_1^h/r_0 = e_0, \quad (19)$$

where $P_1^h = R^h/r_1(B_1^h)$ is independent of e_0 , due to the interiority of B_1^h allowed by assumption (2). Note from (18)-(19) that the equilibrium price vector at date 0, (P_0, r_0) , is uniquely determined and does not depend on the probability of a crisis, $1 - p$; Because date-0 investors are protected against a bad shock to the value of their portfolio by the use of debt contracts, they simply disregard the lower end of the payoff distribution altogether (i.e., the payoff P_1^l with probability $1 - p$).

Asset bubble and crowding out. Finally, we complete this Section by showing that the risk-shifting problem due to date-1 extraneous uncertainty and limited liability of date-0 investors implies that assets are also overvalued at date 0, and that they crowd out real investment then, X_{S0} . From Eqs. (8), (9), (18) and (19), the mispricing of risky assets at date 0 is given by:

$$P_0 - P_0^f = f'^{-1}(r_0^f) - f'^{-1}(r_0)$$

From Eqs. (9) and (19), together with the fact that $P_1^h > P_1^f$ (see Sec. 3.4), it is easily seen that $r_0 > r_0^f$. Since $f'^{-1}(\cdot)$ is decreasing, $P_0 - P_0^f > 0$ and there is a positive asset price bubble at date 0. Note that e_0 being exogenously given, the amount of crowding out caused by this bubble is simply $X_{S0}^f - X_{S0} = P_0 - P_0^f$. The implied lower level of capital investment at date 0 in turn lowers date-1 output, $f(X_{S0})$, in the same way as date-2 (expected) output, $f(X_{S1}) + \pi R^h$, was lowered by bubbly asset prices at date 1.

4.2 The wealth effect of crises

Having shown the existence of a continuum of stochastic equilibria indexed by the probability of a self-fulfilling crisis at the intermediate date, we are now in a position to study the welfare properties of these equilibria in more details. The present Section analyses the way crises affect lenders' wealth and intertemporal consumption flow, while the next One computes the effect of crises on the consumption of other agents.

To see why lenders' wealth at date 1 is contingent on whether a crisis occurs at date 1 or not, let us compute the way it is affected by the possible default of date-0 investors. When these investors do not default, they owe lenders the capitalised value of outstanding debt at date 1, $r_0 e_0$. As lenders receive an endowment e_1 at date 1, their date-1 wealth if no crisis occurs is simply $W^h = e_1 + r_0 e_0$. When investors do default, on the contrary, lenders wealth at date 1 is their date-1 endowment, e_1 , *plus* the residual value of date-0 investors' portfolio, i.e., $W^l = e_1 + r_0 X_{0S} + X_{0R} P_1^l$. Using the fact that in equilibrium we have $X_{R0} = 1$, $X_{S0} = e_0 - X_{R0} P_0$, and $P_0 = P_1^h / r_0$, we find that lenders' date-1 wealth, W^j , conditional on the fact that a crisis occurs ($j = l$) or not ($j = h$), is given by:

$$W^j = e_1 + r_0 X_{S0} + P_1^j, \quad j = l, h. \quad (20)$$

Obviously, the total quantity of goods available at date 1 is the same across equilibria, because initial capital investment, X_{S0} , is uniquely determined (i.e., does not depend on p). This quantity amounts to lenders' date-1 endowment, e_1 , *plus* entrepreneurs' production, $f(X_{S0})$, the latter being shared between date-0 entrepreneurs, who get $f(X_{S0}) - r_0 X_{S0}$

in competitive equilibrium, and lenders, who get $r_0 X_{S0}$ (recall that date-0 investors always consume zero, as was shown in Sec. 4.1 above).³

From condition (2) and inequality (16), we have $B_1^j < B_1^f < W_1^j$, $j = l, h$, implying that both possible levels of wealth give rise to interior solutions for consumption/savings plans at date 1 where $\rho_1(B_1^j) = 1/\beta$. If a crisis occurs at date 1, then lenders' wealth and savings are W^l and B_1^l , respectively, while their date-1 and (expected) date-2 consumption levels are $W^l - B_1^l$ and B_1^l/β , respectively; It follows that their discounted utility flow from date 1 on is simply W^l . Similarly, if a crisis does *not* occur at date 1, then lenders date-1 and date-2 consumption levels are $W^h - B_1^h$ and B_1^h/β , respectively, yielding a discounted utility from date 1 on of W^h . Weighing these possible outcomes with the probabilities that they actually occur, we find that lenders *ex ante* utility (i.e., from the point of view of date 0) depends on the probability of a crisis, $1 - p$, as follows:

$$EUL = pW^h + (1 - p)W^l = e_1 + r_0 X_{S0} + pP_1^h + (1 - p)P_1^l$$

EUL is decreasing in $1 - p$, since $P_1^h > P_1^l$ and $e_1 + r_0 X_{S0}$, P_1^l and P_1^h do not depend on p . Note that it is the selection of the low lending equilibrium itself that triggers the crisis which lowers lenders' wealth and discounted utility. Thus, the utility loss incurred by lenders when a crisis occurs is akin to a pure coordination failure in consumption/savings decisions – rather than an exogenously assumed destruction of value associated with the early liquidation of the long asset, as is typically assumed in liquidity-based theories of financial crises (e.g., Diamond and Dybvig (1983), Allen and Gale (2000)).

4.3 Aggregate welfare

We may now complete the welfare analysis of the model by studying the effect of the *ex ante* crisis probability on other agents' consumption. *Investors*; Secs. 3.1 and 4.1 have established that both date-0 and date-1 investors' consume zero in equilibrium, whatever the realisation of extrinsic (date-1) and fundamental (date-2) uncertainty. Investors' *ex ante* welfare is thus zero in all equilibria. *Entrepreneurs*; The terminal consumption of date-1 entrepreneurs is $f(X_{S1}) - X_{S1}f'(X_{S1})$, which is increasing in X_{S1} . Since $X_{S1}^h > X_{S1}^l$ (see Sec. 3.3),

³There are two equivalent ways of characterising lenders' budget set at date 1. Looking at their *wealth*, W^j is assigned to date-1 consumption and date-1 lending, so that, using Eq. (20), $W^j = e_1 + r_0 X_{0S} + P_1^j = c_1^j + B_1^j$, $j = l, h$. Looking at the total quantity of *goods* that accrues to lenders at date 1, these are ultimately shared between date-1 consumption, c_1^j , and date-1 capital investment, X_{S1}^j , so that $e_1 + r_0 X_{0S} = c_1^j + X_{S1}^j$, $j = l, h$. Since $B_1^j = X_{S1}^j + P_1^j$, these two formulations are, obviously, mutually consistent.

their *ex ante* welfare, from the point of view of date 0, is $p(f(X_{S1}^h) - X_{S1}^h f'(X_{S1}^h)) + (1-p)(f(X_{S1}^l) - X_{S1}^l f'(X_{S1}^l))$, which decreases with $1-p$. Date-0 entrepreneurs consume $f(X_{S0}) - f'(X_{S0})X_{S0}$, where $X_{S0} = f'^{-1}(r_0)$ does not depend on p . Finally, *Initial asset holders'* consumption is just the selling price of the asset at date 0, P_0 , which is independent of p . To summarise, neither investors nor initial asset holders or date-0 entrepreneurs are affected by the probability that a crisis occurs at date 1. Lenders are, because the crisis cuts their asset wealth and discounted consumption flow. Date-1 entrepreneurs are, because low lending reduces their profit and terminal consumption. Thus, the higher the *ex ante* probability that a self-fulfilling crisis occurs at date 1, the lower aggregate welfare.

5 Concluding remarks

This paper has offered a simple theory of self-fulfilling financial crises based on the excess risk-taking of debt-financed portfolio investors. In our model, the interplay between the amount of funds available to investors, the composition of their portfolio, and the return that they are able to offer in competitive equilibrium, creates a strategic complementarity between lenders' savings decisions, which may in turn give rise to multiple equilibria associated with different levels of lending, interest rates, asset prices and future output. Expectations-driven financial crises may then occur with positive probability as soon as the intermediate date has (at least) two possible equilibrium levels of lending, and lenders' coordination on a particular equilibrium follows an extraneous 'sunspot'. We showed that such crisis where associated with a self-fulfilling credit contraction followed by a market crash, widespread failures of investors, and a contraction in productive investment.

Besides demonstrating that credit intermediation based on debt contracts is a source of purely endogenous financial instability, the model developed above also gives new insights into the potential welfare costs of financial crises. In our model, the dramatic reduction in savings associated with the selection of the crisis equilibrium at the intermediate date has two implications. First, it causes a reduction in lenders' wealth as the total value of their capitalised investment drops down, which lowers their discounted consumption flow from the time of the crisis onwards. Second, the credit contraction associated with the crisis causes a fall in productive investment and output, with the consequence of lowering entrepreneurs' profits and consumption levels. Thus, both savers and final producers are hurt by the financial crisis, while intermediate investors, whose risk is 'hedged' by the use of debt contracts, are ultimately left unharmed.

Appendix

A. Shape of the date-1 loan return curve when $f(\cdot)$ is isoelastic

With $f(X_{S1}) = X_{S1}^{1-\eta}/(1-\eta)$, $\eta \in (0, 1)$ being a constant, we have $B_1(r_1) = r_1^{-1/\eta} + R^h r_1^{-1}$ (see Eq. (11)), which in turn implies:

$$r_1'(B_1) = \frac{1}{B_1'(r_1)} = \frac{1}{(-1/\eta)r_1^{-1-1/\eta} - R^h r_1^{-2}},$$

where $r_1 = r_1(B_1)$ is the interest rate function characterised in Sec. 2.3. From Eq. (12), $\rho_1(B_1)$ is increasing (decreasing) when $r_1'(B_1) + (1-\pi)R^h/B_1^2 > 0 (< 0)$, that is, when

$$\frac{1}{(-1/\eta)r_1^{-1-1/\eta} - R^h r_1^{-2}} + \frac{(1-\pi)/R^h}{(r_1^{-1/\eta} + R^h r_1^{-1})^2} > 0 (< 0)$$

Defining $Y \equiv r_1^{1-1/\eta}$ and rearranging the above inequality, we find that $\rho(B)$ is increasing (decreasing) when

$$\Psi(Y) = Y^2 + R^h \left(2 - \frac{1-\pi}{\eta}\right) Y + \pi (R^h)^2 < 0 (> 0)$$

$\Psi(Y)$ changes sign over $(0, \infty)$ if $\Psi(Y) = 0$ has two real roots, including one positive root at least. A necessary condition for this to be the case is that the discriminant of $\Psi(Y) = 0$ be positive, i.e., the following inequality must hold:

$$1 + 4\eta(\eta - 1) > \pi \tag{A1}$$

When (A1) holds, the roots Y_a, Y_b of $\Psi(Y) = 0$ are:

$$Y_{a,b} = \frac{R^h}{2} \left(\left(\frac{1-\pi}{\eta} - 2 \right) \mp \sqrt{\left(\frac{1-\pi}{\eta} - 2 \right)^2 - 4\pi} \right)$$

Both roots are positive (negative) if $1 - 2\eta > (<) \pi$. Combined with (A1), this means that $\Psi(Y)$ changes signs over $(0, \infty)$ if (and only if):

$$\eta < (1 - \sqrt{\pi})/2 \tag{A2}$$

$\Psi(Y)$ is negative for $Y \in (Y_a, Y_b)$, and positive for $Y \in (0, Y_a) \cup (Y_b, \infty)$. Since $Y = r_1^{1-1/\eta}$, this means that $\Psi(Y)$ is negative for intermediate values of r_1 and positive otherwise. Using Eq. (11), this in turn implies that, provided (A2) is fulfilled, $\rho_1(B_1)$ is strictly increasing for intermediate values of B_1 , and strictly decreasing otherwise. Note that when (A2) does not hold then $\Psi(Y)$ is strictly positive, and $\rho_1(B_1)$ strictly decreasing, over $(0, \infty)$.

B. Proof of inequalities (15) and (16)

Let us start with inequality (15). From Eqs. (10) and (6), the price difference $P_1^j - P_1^f = R^h(1/r_1(B_1^j) - \pi\beta)$, $j = l, h$, is positive if and only if

$$r_1(B_1^j) < \frac{1}{\pi} \times \frac{1}{\beta} = \frac{1}{\pi} \times \left(r_1(B_1^j) - \frac{(1-\pi)R^h}{B_1^j} \right),$$

where Eq. (12) and the fact that $\rho_1(B_1^j) = 1/\beta$, $j = l, h$, have been used to replace $1/\beta$ by a function of B_1^j . Rearranging the latter inequality, we find that a necessary and sufficient condition for $P_1^j > P_1^f$ is $B_1^j r_1(B_1^j) > R^h$. Using Eq. (11), the latter inequality is in turn satisfied if and only if

$$(f'^{-1}(r_1) + R^h/r_1) r_1 > R^h,$$

which is always true since $r_1 f'^{-1}(r_1) = r_1 X_{S1} > 0$ provided $B_1 > 0$ (see Eq. (13)).

Let us now turn to inequality (16). Since $\rho_1(B_1^j) = 1/\beta$ in both equilibria, we can rearrange Eq. (12) to obtain:

$$B_1^j = \beta B_1^j r_1(B_1^j) - \beta(1-\pi)R^h \tag{B1}$$

Comparing Eqs. (7) and (B1), we have that $B_1^j < B_1^f$, $j = l, h$, if and only if

$$B_1^j r_1(B_1^j) - R^h < (1/\beta) f'^{-1}(1/\beta),$$

or, using Eq. (11) again, if and only if

$$r_1 f'^{-1}(r_1) < (1/\beta) f'^{-1}(1/\beta)$$

$r_1 f'^{-1}(r_1)$ decreases with r_1 since $f'^{-1}(r_1) + r_1 f'^{-1\prime}(r_1) = X_{S1} + f'(X_{S1})/f''(X_{S1})$ is negative under assumption (1). Thus, the latter inequality is satisfied if and only if $r_1 > 1/\beta$. Solving (12) for $r_1(B_1^j)$ and imposing $\rho_1(B_1^j) = 1/\beta$, the necessary and sufficient condition for $B_1^j < B_1^f$ becomes

$$r_1(B_1^j) = 1/\beta + (1-\pi)R^h/B_1^j > 1/\beta,$$

which is always true since $B_1^j > 0$, $j = l, h$.

References

- Allen, F. and Gale, D. (2000), 'Bubbles and crises', *Economic Journal*, vol. 110, pp. 236-55.
- Allen, F. and Gale, D. (1999), 'Bubbles, crises, and policy', *Oxford Review of Economic Policy*, vol. 15, pp. 9-18.
- Allen, F. and Gale, D. (1998), 'Optimal financial crisis', *Journal of Finance*, vol. 53, pp. 1245-84.
- Borio, E.V., Kennedy, N. and Prowse, S. D. (1994), 'Exploring aggregate asset price fluctuations across countries: measurement, determinants and monetary policy implications', BIS Working Paper no 40, April
- Chang, R. and Velasco, A. (2000), 'Banks, debt maturity, and financial crises', *Journal of International Economics*, vol. 51, pp. 169-194.
- Chari, V.V. and Kehoe, P.J. (2003), 'Hot money', *Journal of Political Economy*, vol. 111, pp. 1262-92.
- Cooper, R. and John, A. (1988), 'Coordinating coordination failures in keynesian models', *Quarterly Journal of Economics*, vol. 103, pp. 441-464.
- Diamond, D.W. and Dybvig, P.H. (1983), 'Bank runs, deposit insurance, and liquidity', *Journal of Political Economy*, vol. 91, pp. 401-19.
- Kaminsky, G.L. (1999), 'Currency and banking crises: The early warnings of distress', IMF Working Paper 99/178, December.
- Kaminsky G.L. and Reinhart, C.M. (1999), 'The twin crises: The causes of banking and balance-of-payments problems', *American Economic Review*, vol. 89, pp. 473-500.
- Kaminsky G.L. and Reinhart, C.M. (1998), 'Financial crises in Asia and Latin America: Then and now', *American Economic Review*, vol. 88, pp. 444-448.
- Summers, L.H. (2000), 'International financial crisis: Causes, prevention, and cures', *American Economic Review*, 2000, vol. 90, pp.1-16.
- Velasco, A. (1996), 'Fixed exchange rates: credibility, flexibility and multiplicity', *European Economic Review*, 1996, vol. 40, 1023-1035.