A Model of Money and Credit, with

Application to the Credit Card Debt Puzzle[*]

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Abstract

Many individuals simultaneously have significant credit card debt and money in the bank. The *credit card debt puzzle* is, given high interest rates on credit cards and low rates on bank accounts, why not pay down the debt? While economists have recently gone to elaborate lengths to explain this observation, we argue that it is nothing more than the venerable *rate of return dominance puzzle* from monetary economics. We therefore analyze the issue by extending standard monetary theory to incorporate consumer debt. This seems interesting in its own right, since developing models where money and credit coexist is a long-standing challenge, and it helps put into context recent discussions of consumer debt.

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1 Introduction

A large number of households simultaneously have significant credit card debt and a significant amount of money in their checking and savings accounts. Although there are many ways to measure this, a simple summary statistic is that 27% of U.S. households in 2001 had credit card debt in excess of $500 and over $500 in the bank; and the median such household revolved around $3,800 of credit card debt and had $3,000 in the bank (see Telyukova 2005). The so-called *credit card debt puzzle* is this: given 14% interest rates on credit cards, and 1 or 2% on bank accounts, why not pay down the debt? "Such behavior is puzzling, apparently inconsistent with no-arbitrage and thus inconsistent with any conventional model." (Gross and Souleles 2001).

Economists have gone to elaborate lengths to explain this type of phenomena. For example, some people assume that consumers cannot control themselves (Laibson et al. 2000); others assume they cannot control their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003); and still others hypothesize that such households are typically on the verge of bankruptcy (Lehnert and Maki, 2001). While these ideas are interesting, and may contain elements of truth, we think it is useful to point out that the credit card debt puzzle is actually not a new observation. Rather, it is "simply" another ramification of the venerable *rate of return dominance puzzle* from monetary economics, and hence, insights may be gained by using models and ideas from monetary theory. In particular, the relevant notion is *liquidity*.[1]

Our hypothesis is the following. Households need money – generally, relatively liquid assets – for contingencies where it is difficult or costly to use credit. It is important to note that there are many big-ticket items for which this is the case, over and above the usual examples like taxis and cigarettes. For instance, usually rent or mortgage payments cannot be made by credit card. Also, many less perfectly anticipated events such as household repairs (plumbing, heating, air conditioning, etc.) often require cash, for whatever reason, and getting caught short can be

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[1]The idea that agents may hold assets with low rates of return because they are relatively liquid – i.e. because they have potential advantages as a medium of exchange – underlies much of modern monetary theory going back to Kiyotaki and Wright (1989). And, as we discuss below, it goes back much further in the less formal literature.

very costly.[2] Even if agents are revolving credit card debt, they need to have some cash easily accessible to meet these contingencies. The point may be obvious – but this does not mean that it would not be interesting to analyze it in detail.

The rate of return dominance question and the idea that some notion of liquidity ought to be part of the solution goes back a long time. Hicks (1935) is well known for challenging monetary economists to "look frictions in the face" when framing "the central issue in the pure theory of money" as the need for an explanation of the fact that people hold money when rates of interest are positive. One perhaps better known version of the challenge is to explain "the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities." But the same issue arises in reverse: "So long as interest rates are positive, the decision to hold money rather than lend it, or *use it to pay off old debts*, is apparently an unprofitable one" (Hick 1935, p.5, emphasis added).

We believe there is something to be gained by analyzing the credit card debt puzzle in the context of monetary economics, and bringing to bear some of the ideas from modern theory. However, to our knowledge – or maybe, in our opinion – there does not exist an appropriate off-the-shelf model of money and credit that can be used to address the issue. So we build one. While this is *not* meant to constitute the last word on rate of return dominance, because we do need some strong assumptions, we think we have a logically consistent economic environment that is useful for this purpose. While our framework builds on some recent results, we also extend existing monetary theory along several dimensions. This is interesting in its own right, since clearly getting coexistence of consumer credit and money in a logically consistent theory is not easy. And from a substantive point of view, it allows us to interpret the coexistence of credit card debt and money in the bank in a very different light vis a vis the literature.[3]

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[2] According to the U.S. Statistical Abstract, 77% of consumer transactions in 2001 used liquid assets (defined to include cash, bank deposits, and closely related instruments). According to the Consumer Expenditure Survey, the median household described above (with $3,800 of credit card debt and $3,000 in the bank) spent $1,993 per month on goods purchased with liquid assets (Telyukova 2005).

[3] This paper is about *theory*; whether the approach is able to account *quantitatively* for the salient aspects of the data is the subject of ongoing research. See Telyukova (2005).

2   The Basic Model

We build on Lagos and Wright (2005), hereafter LW. That model gives agents periodic access to centralized markets, in addition to the decentralized markets where due to frictions money is essential for trade. Having some centralized markets is interesting for its own sake, and also makes the analysis much more tractable than what one finds in much of the literature on the microfoundations of money.[4] However, existing versions of the model have nothing that resembles consumer debt, and as we explain below, it is not easy to get consumer debt into the framework without extending it in just the right way. Here we describe the basic physical environment, for now focusing on a special case; later, we generalize.

A $[0,1]$ continuum of agents live forever in discrete time. There is one nonstorable consumption good at each date that agents may (stochastically) be able to produce using labor. There is also money in this economy, a perfectly divisible and storable object that is intrinsically worthless, but potentially could have use as a medium of exchange. The money supply is fixed for now at M, but later we allow it to vary over time. Although we use the word money, we do not necessarily mean cash literally. It is not hard to recast the model with agents depositing cash into bank accounts and paying for goods using checks or debit cards, as in He, Huang and Wright (2005). We discuss this further below. It is relevant because what we have in mind is not money *per se*, but relatively liquid assets more generally.

In LW, each period is divided into two subperiods. In one, there is a centralized Walrasian market, and in the other, there is a decentralized market where agents meet according to a random bilateral-matching process. With the additional assumption that agents are anonymous in the decentralized market, a medium of exchange becomes essential.[5] After each meeting of this market agents go to the centralized market, where they engage in various activities that include

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[4] See Molico (1997), Green and Zhou (1998), Camera and Corbae (1999), Zhou (1999) or Zhu (2003) for models where all trade is decentralized, and the analysis is much more difficult. Earlier models, like Shi (1995) or Trejos and Wright (1995), were also tractable, but only because money was assumed to be indivisible and agents were allowed to hold at most 1 unit.

[5] See Kocherlakota (1998) and Wallace (2001) for formal discussions of what makes money essential.

working and rebalancing their money holdings. If utility is linear in hours worked, all agents take the same amount of money out of the centralized market, which is what keeps the analysis relatively simple; without these assumptions, there is generally an endogenous distribution of money across traders in the decentralized market that one has to track as a state variable.

As we said above, there is no role for credit in LW, and it is important to understand why it is not easy to create such a role. Credit is not possible in the decentralized market because of the assumption that agents are anonymous, which we cannot relax since this is what makes money essential. And credit is not necessary in the centralized market because of the assumption that all agents can work and have utility that is linear in hours, which we do not want to relax since this is what keeps the analysis tractable. How to proceed? Our idea is to introduce another subperiod (we generalize later to many subperiods) with a market where agents may want to consume but cannot produce, which makes credit useful, and where we do not assume anonymity, which makes credit feasible. We determine whether agents use cash or credit in this market endogenously, while maintaining an essential role for money plus analytic tractability due to the other two markets.[6]

All agents want to consume in subperiod (market) 1, and $u_1(x_1)$ is their common utility function. A random subset want to consume in $s = 2, 3$, and conditional on this, $u_s(x_s)$ is their utility function. Assume $u_s(x_s)$ is strictly increasing and concave. All agents are able to produce in $s = 1$, and the disutility of working h_1 hours is $c_1(h_1) = h_1$. A random subset are able to produce in $s = 2, 3$, and conditional on this, the disutility of working is an increasing and convex function $c_s(h_s)$. When they can produce, agents transform labor one-for-one into goods, $x_s = h_s$.[7] For simplicity, at any $s = 2, 3$ a random set of agents chosen in an i.i.d. manner want to consume but cannot produce and vice-versa; no one can do both, but this would be easy to

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[6]Berentsen, Camera and Waller (2005a) also add a third subperiod to LW, but it is another round of decentralized exchange and so there is no possibility of credit. Berentsen et al. (2005b) and Chiu and Meh (2006) introduce a third subperiod with a centralized market in order to discuss banking, as in He et al. (2005); the focus here is on an entirely different set of issues, however.

[7]One can assume there is a real wage w that is constant and can be normalized to 1 because there are competitive firms with linear technologies; it is easy to extend this and introduce firms with general technlologies to determine w endogenously.

relax. Let x_s^* denote the solution to $u_s'(x_s^*) = c_s'(x_s^*)$. Let β_s be the discount factor between s and the next subperiod, with $\beta_1 \beta_2 \beta_3 < 1$.

An individual's state is (m_{st}, b_{st}), denoting money and debt in subperiod s of period t, but we drop the t when there is no risk of confusion; e.g. we write $m_{st} = m_s$, $m_{s,t+1} = m_{s,+1}$, etc. Let $W_s(m_s, b_s)$ be the value function. At $s = 1, 2$, the market value of money is ϕ_s, so $p_s = 1/\phi_s$ is the nominal price level; there there is no ϕ_3 since there is no centralized market at $s = 3$, although prices will implicitly be defined by trades. Similarly, the real interest rate in the centralized markets at $s = 1, 2$ is r_s, but there is no r_3. Our convention for notation is as follows: if you bring debt b_s into subperiod $s = 1, 2$ you owe $(1 + r_s) b_s$. The plan now is to consider each subperiod (market) in turn. After this, we put the pieces together and describe equilibrium.

2.1 Market 1

At $s = 1$, there is a centralized market where agents solve[8]

$$
\begin{aligned}
W_1(m_1, b_1) &= \max_{x_1, h_1, m_2, b_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2)\} \\
\text{s.t. } x_1 &= h_1 + \phi_1(m_1 - m_2) - (1 + r_1)b_1 + b_2.
\end{aligned}
$$

Substituting h_1 from the budget constraint into the objective function, we have

$$
W_1(m_1, b_1) = \max_{x_1, m_2, b_2} \{u_1(x_1) - [x_1 + \phi_1(m_2 - m_1) + (1 + r_1)b_1 - b_2] + W_2(m_2, b_2)\}. \tag{1}
$$

The first-order conditions are

$$
1 = u_1'(x_1) \tag{2}
$$

$$
\phi_1 = \beta_1 W_{2m}(m_2, b_2) \tag{3}
$$

$$
-1 = \beta_1 W_{2b}(m_2, b_2). \tag{4}
$$

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[8]To rule out Ponzi schemes, one normally imposes a credit limit $b_j \leq \bar{B}$, either explicitly or implicitly. We do not need this here because we can explicitly impose that agents pay off past debts at $s = 1$ each period without loss in generality (due to quasi-linear utility). Also, we always assume an interior solution for h; see LW for conditions to guarantee this is valid in these types of models.

Notice (2) implies $x_1 = x_1^*$ for all agents, while (3)-(4) imply (m_2, b_2) is independent of x_1 and (m_1, b_1) – a feature of quasi-linearity and a generalization of results in LW. As long as W_2 is strictly concave, there is a unique solution for (m_1, b_1). It is simple to check that the same conditions that guarantee strict concavity in m used by LW also apply here, and so $m_2 = M$ for all agents. However, we will see that W_2 is actually linear in b_2, which means we cannot pin down b_2 for any individual. This is no surprise with a competitive market and quasi-linear utility: in equilibrium, agents are indifferent between the allocation they have and an alternative where they work a little more now, save the proceeds, and work a little less later.

Although this is true for any individual, it cannot be true in the aggregate, since the average labor input $\bar{h}_1$ must equal total output x_1^*. Given this, one can resolve the indeterminacy for individuals in two ways. First, one can focus on symmetric equilibria where all agents choose the same solution when they have the same set of solutions to a maximization problem, which is innocuous since other equilibria are payoff equivalent and observationally equivalent at the aggregate level; this pins down $b_2 = \bar{b}_2$ for all agents. Alternatively, we could impose an arbitrarily small but positive transaction cost on borrowing in subperiod 1, which would break agents' indifference and refine away all but the symmetric equilibria. In what follows we take the former route, and simply focus on symmetric equilibria.

Aggregating budget equations across agents, we have

$$\bar{x}_1 = \bar{h}_1 + \phi_1(\bar{m}_1 - \bar{m}_2) - (1 + r_1)\bar{b}_1 + \bar{b}_2. \tag{5}$$

In equilibrium, $\bar{h}_1 = \bar{x}_1 = x_1^*$, $\bar{m}_1 = \bar{m}_2 = M$, and $\bar{b}_1 = 0$ (average debt must be 0). Hence (5) implies $b_2 = \bar{b}_2 = 0$ for all agents.[9] We close the analysis of market 1 with the envelope conditions

$$W_{1m}(m_1, b_1) = \phi_1 \tag{6}$$

$$W_{1b}(m_1, b_1) = -(1 + r_1), \tag{7}$$

which imply that W_1 is linear in (m_1, b_1), another generalization of LW.

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[9]This is the aforementioned condition that rules out Ponzi schemes.

2.2 Market 2

At $s = 2$, a measure π of agents want to consume but cannot produce, while a measure π can produce but do not want to consume. In equilibrium, $x_2^C = h_2^P$, where x_2^C is the consumption of consumers and h_2^P the production of producers. The expected value of entering market 2 is therefore

$$W_2(m_2, b_2) = \pi W_2^C(m_2, b_2) + \pi W_2^P(m_2, b_2) + (1 - 2\pi)W_2^N(m_2, b_2), \tag{8}$$

where W_2^C, W_2^P and W_2^N are the value functions for a consumer, a producer and a nontrader. We study their problems one at a time, which is slightly tedious, but useful.

For a nontrader,

$$
\begin{aligned}
W_2^N(m_2, b_2) &= \max_{m_3, b_3} \ \beta_2 W_3(m_3, b_3) \\
\text{s.t. } 0 &= \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.
\end{aligned}
$$

Note that although nontraders neither consume nor produce, they can adjust their portfolios.[10] The solution (m_3^N, b_3^N) satisfies

$$W_{3m}(m_3^N, b_3^N) = -\phi_2 W_{3b}(m_3^N, b_3^N), \tag{9}$$

plus the budget equation. The envelope conditions are

$$W_{2m}^N(m_2, b_2) = \beta_2 W_{3m}(m_3^N, b_3^N) \tag{10}$$

$$W_{2b}^N(m_2, b_2) = \beta_2(1 + r_2)W_{3b}(m_3^N, b_3^N). \tag{11}$$

For a consumer,

$$
\begin{aligned}
W_2^C(m_2, b_2) &= \max_{x_2, m_3, b_3} \ \{u_2(x_2) + \beta_2 W_3(m_3, b_3)\} \\
\text{s.t. } x_2 &= \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.
\end{aligned}
$$

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[10]It might therefore appear that calling them nontraders is inaccurate, but we will see that in equilibrium they choose not to rebalance their portfolios.

The solution (x_2^C, m_3^C, b_3^C) satisfies

$$\phi_2 u_2'(x_2^C) = \beta_2 W_{3m}(m_3^C, b_3^C) \tag{12}$$

$$-u_2'(x_2^C) = \beta_2 W_{3b}(m_3^C, b_3^C) \tag{13}$$

plus the budget equation. Using these, the envelope conditions are

$$W_{2m}^C(m_2, b_2) = \phi_2 u_2'(x_2^C) = \beta_2 W_{3m}(m_3^C, b_3^C) \tag{14}$$

$$W_{2b}^C(m_2, b_2) = -(1 + r_2)u_2'(x_2^C) = (1 + r_2)\beta_2 W_{3b}(m_3^C, b_3^C). \tag{15}$$

For a producer,

$$W_2^P(m_2, b_2) = \max_{h_2, m_3, b_3} \{-c_2(h_2) + \beta_2 W_3(m_3, b_3)\}$$

$$\text{s.t.} \, 0 = h_2 + \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.$$

The solution (h_2^P, m_3^P, b_3^P) satisfies

$$\phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{16}$$

$$-c_2'(h_2^P) = \beta_2 W_{3b}(m_3^P, b_3^P) \tag{17}$$

plus the budget equation. The envelope conditions are

$$W_{2m}^P(m_2, b_2) = \phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{18}$$

$$W_{2b}^P(m_2, b_2) = -(1 + r_2)c_2'(h_2^P) = (1 + r_2)\beta_2 W_{3b}(m_3^P, b_3^P). \tag{19}$$

We cannot conclude that (m_3, b_3) is independent of (m_2, b_2), the way we could conclude that (m_2, b_2) is independent of (m_1, b_1) in the previous subperiod. If x_2^C depends on (m_2, b_2) then so will (m_3^C, b_3^C), unless u_2 is linear; and if h_2^P depends on (m_2, b_2) then so will (m_3^P, b_3^P), unless c_2 is linear. In any case, we have

$$W_{2m}(m_2, b_2) = \beta_2[\pi W_{3m}(m_3^C, b_3^C) + \pi W_{3m}(m_3^P, b_3^P) + (1 - 2\pi) W_{3m}(m_3^N, b_3^N)] \tag{20}$$

$$W_{2b}(m_2, b_2) = \beta_2(1 + r_2)[\pi W_{3b}(m_3^C, b_3^C) + \pi W_{3b}(m_3^P, b_3^P) + (1 - 2\pi) W_{3b}(m_3^N, b_3^N)] \tag{21}$$

2.3 Market 3

In market 3 trade occurs via anonymous bilateral meetings and bargaining.[11] Because of anonymity, you cannot use credit: I will not take your IOU because I understand you could renege, without fear of punishment, given that I do not know who you are. However, one may ask why some institution that is not anonymous cannot issue interest-bearing claims to goods next period that might circulate in market 3. The simplest answer is to assume such claims can be counterfeited. Thus, the government here has a monopoly on the production of non-counterfeitable notes, and chooses to issue only non-interest-bearing money. These assumptions are clearly strong. As suggested above, we do not presume to provide a definitive solution to the rate of return dominance problem; we do think we have a logically consistent environment in which there is a role for money plus credit.[12]

There is one more issue to address. As we said above, there is a version of the model where agents deposit money in banks and pay with checks or debit cards in decentralized markets, along the lines of He et al. (2005). Checks work even though consumers are anonymous, because they are claims on the bank and not on the consumer personally (think of travellers' checks as the purest example). In such a model, interest paid on checking accounts is determined endogenously, but it will not equal the market interest rate on consumer credit, as it would in a frictionless market, as long as we adopt one of several assumptions. We can simply assume government prohibition of interest on checking, as was the case for much of U.S. history. Or we can assume the bank has some operating costs. Or we can assume that banks need to hold reserves, either to meet legal requirements or to facilitate settlement.

For example, suppose we have 100% reserve requirements (so-called *narrow banking*). Then banks earn no revenue from and hence pay no interest on deposits. Indeed, if there are operating costs, they pay negative interest – i.e. they charge a fee for checking privileges. In He et al.

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[11]Bargaining is not a crucial part of the specification – versions of related models exist with price taking and price posting (as in Rocheteau and Wright 2005), and with auctions (once one allows some multilateral meetings, as in Kircher and Galenianos 2006 or Julien, Kennes and King 2006).

[12]In Section 3 we discuss what happens when non-government securities *can* circulate as media of exchange.

(2005), agents may be willing to deposit money in banks even at negative interest rates for safety reasons (again think of travellers' checks). As long as banks keep some reserves for whatever reason and/or have some operating costs, the equilibrium interest rate on checking accounts is below the market interest rate. The point is that liquidity comes at a cost. The pure monetary model without banks captures this in an economical way, but it ought to be clear that the underlying ideas apply more generally.

Consider a meeting where one agent wants to consume and the other can produce. Call the former agent the *buyer* and the latter the *seller*. They bargain over the amount of consumption for the buyer x_3 and labor by the seller h_3, and also a dollar payment d from to the former to the latter. Since feasibility implies $x_3 = h_3$, we denote their common value by q. If (m_3, b_3) is the state of a buyer and $(\tilde{m}_3, \tilde{b}_3)$ the state of a seller, the outcome satisfies the generalized Nash bargaining solution,

$$(q, d) \in \arg\max \ S(m_3, b_3)^\theta \tilde{S}(\tilde{m}_3, \tilde{b}_3)^{1-\theta} \text{ s.t. } d \le m_3, \tag{22}$$

where θ is the bargaining power of the buyer, and the surpluses are given by

$$S(m_3, b_3) = u_3(q) + \beta_3 W_{1,+1}(m_3 - d, b_3) - \beta_3 W_{1,+1}(m_3, b_3)$$
$$\tilde{S}(\tilde{m}_3, \tilde{b}_3) = -c_3(q) + \beta_3 W_{1,+1}(\tilde{m}_3 + d, \tilde{b}_3) - \beta_3 W_{1,+1}(\tilde{m}_3, \tilde{b}_3).$$

Using (6) and (7), these simplify to

$$S(m_3, b_3) = u_3(q) - \beta_3 \phi_{1,+1} d$$
$$\tilde{S}(\tilde{m}_3, \tilde{b}_3) = -c_3(q) + \beta_3 \phi_{1,+1} d.$$

The constraint $d \le m_3$ in (22) simply says a buyer cannot transfer more money than he has. Given all this, we have the following generalization of LW (proof is in the Appendix).

Lemma 1. $\forall (m_3, b_3)$ *and* $(\tilde{m}_3, \tilde{b}_3)$, *the solution to the bargaining problem is*

$$q = \begin{cases} g^{-1}(\beta_3 m_3 \phi_{1,+1}) & \text{if } m_3 < m_3^* \\ q^* & \text{if } m_3 \ge m_3^* \end{cases} \quad \text{and } d = \begin{cases} m_3 & \text{if } m_3 < m_3^* \\ m_3^* & \text{if } m_3 \ge m_3^* \end{cases} \tag{23}$$

where q^* solves $u_3'(q^*) = c_3'(q^*)$, the function $g(\cdot)$ is given by

$$g(q) = \frac{\theta u_3'(q)c_3(q) + (1-\theta)u_3(q)c_3'(q)}{\theta u_3'(q) + (1-\theta)c_3'(q)}, \tag{24}$$

and $m_3^* = g(q^*)/\beta_3\phi_{1,+1}$.

Clearly, the bargaining solution (q, d) depends on the buyer's money holdings m_3, but on no other element of (m_3, b_3) or $(\tilde{m}_3, \tilde{b}_3)$; hence we write $q = q(m_3)$ and $d = d(m_3)$ from now on. Of course, q and d at t also depend on ϕ_1 at $t+1$, but this is left implicit in the notation. We show in the Appendix that, exactly as in LW, $m_3 < m_3^*$ in any equilibrium. Hence, from Lemma 1, buyers always spend all their money in market 3, and receive $q = g^{-1}(\beta_3 m_3\phi_{1,+1})$. Notice $\partial q/\partial m_3 = \beta_3\phi_{1,+1}/g'(q) > 0$. It will be useful to define

$$e(q) = \frac{u_3'(q)}{g'(q)}, \tag{25}$$

and to assume $e'(q) < 0$. Sufficient conditions on preferences that guarantee $e'(q) < 0$ can be found in LW; a simple condition that works for any preferences is $\theta \approx 1$, since $\theta = 1$ implies $g(q) = c(q)$.

Let σ denote the probability of a meeting between a buyer and a seller in market 3. Then

$$W_3(m_3, b_3) = \sigma\{u_3[q(m_3)] + \beta_3 W_1[m_3 - d(m_3), b_3]\}$$
$$+ \sigma\mathbb{E}\{-c_3[q(\tilde{m}_3)] + \beta_3 W_1[m_3 + d(\tilde{m}_3), b_3]\} + (1-2\sigma)\beta_3 W_1[m_3, b_3], \tag{26}$$

where $\mathbb{E}$ is the expectation of $\tilde{m}_3$ (the money holdings of a random agent one meets, which we will later show to be degenerate at $\tilde{m}_3 = M$). Differentiating (26) and using (6) and (7),

$$W_{3m}(m_3, b_3) = \beta_3\phi_{1,+1}\{\sigma e[q(m_3)] + 1 - \sigma\} \tag{27}$$

$$W_{3b}(m_3, b_3) = -\beta_3(1 + r_{1,+1}). \tag{28}$$

As described by (27), the marginal value of money in market 3 is a weighted average of the values of spending it and of carrying it forward to next period, while (28) gives the marginal value of debt as the value of simply rolling it over.

2.4 Equilibrium

Our definition of equilibrium is relatively standard, except that there is no market-clearing condition for market 3: since trade is bilateral in this market, it clears automatically. To reduce notation, we describe every agent's problem at $s = 1, 2$ in terms of choosing $(x_s, h_s, m_{s+1}, b_{s+1})$, which are implicitly functions of the state, where it is understood that for producers $x_2^P = 0$, for consumers $h_2^C = 0$, and for nontraders $x_2^N = h_2^N = 0$.[13]

Definition 1. *An* equilibrium *is a set of (possibly time-dependent) value functions* $\{W_s\}$, *$s = 1, 2, 3$, decision rules* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, *$s = 1, 2$, bargaining outcomes* $\{q, d\}$, *and prices* $\{r_s, \phi_s\}$, *$s = 1, 2$, such that:*

1. *Optimization: In every period, for every agent,* $\{W_s\}$, *$s = 1, 2, 3$, solve the Bellman equations (1), (8) and (26);* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, *$s = 1, 2$, solve the relevant maximization problems; and* $\{q, d\}$ *solve the bargaining problem.*

2. *Market clearing: In every period,*

$$\bar{x}_s = \bar{h}_s, \ \bar{m}_{s+1} = M, \ \bar{b}_{s+1} = 0, \ s = 1, 2$$

   where for any variable y, *$\bar{y} = \int y^i di$ denotes the aggregate.*

Definition 2. *A steady state equilibrium is an equilibrium where the endogenous variables are constant across time periods (although not generally across subperiods within a period).*

We are mainly interested in equilibria where money is valued, which means that it is valued in all subperiods in every period.

Definition 3. *A monetary equilibrium is an equilibrium where, in every period,* $\phi_s > 0$, *$s = 1, 2$, and* $q > 0$.

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[13] We do not include the distribution of the state variable in the definition of equilibrium, but it is implicit: given an initial distribution $F_1(m, b)$ at the start of subperiod 1, the decision rules generate $F_2(m, b)$; then the decision rules at $s = 2$ generate $F_3(m, b)$; and the bargaining outcome at $s = 3$ generates $F_{1,+1}(m, b)$. Also, as we said above, we only consider equilibria where we have an interior solution for h.

We now characterize steady-state equilibria (the steady state requirement is relaxed below). First, recall that in equilibrium we impose that in market 1, if two agents have multiple solutions for b_2 they choose the same one. As we discussed above, due to quasi-linear utility, any other equilibria are payoff equivalent for individuals, and observationally equivalent at the aggregate level. Further, in any of the other equilibria, prices and consumption are exactly the same as stated in the following theorem.

Theorem 1. *In any steady state monetary equilibrium:*

1. At $s = 1$, all agents choose $x_1 = x_1^*$, $m_2 = M$, $b_2 = 0$, and

$$h_1 = h_1(m_1, b_1) = x_1^* - \phi_1(m_1 - M) + (1 + r_1)b_1,$$

which implies $\bar{h}_1 = x_1^*$.

2. At $s = 2$,

consumers choose $x_2 = x_2^*$, $m_3 = M$ and $b_3 = x_2^*$;

producers choose $h_2 = x_2^*$, $m_3 = M$ and $b_3 = -x_2^*$;

nontraders choose $m_3 = M$ and $b_3 = 0$.

3. At $s = 3$, in every trade $d = M$ and q solves

$$1 + \frac{\rho}{\sigma} = e(q), \tag{29}$$

where $e(q)$ is given by (25) and ρ is defined by $\dfrac{1}{1 + \rho} = \beta_1\beta_2\beta_3$.

4. Prices are given by:

$$
\begin{aligned}
r_1 &= \frac{u_2'(x_2^*) - \beta_2\beta_3}{\beta_2\beta_3}, \; r_2 = \frac{\rho - r_1}{1 + r_1}, \\
\phi_1 &= \frac{g(q)}{\beta_3 M}, \; and \; \phi_2 = \frac{\phi_1\left[\sigma e(q) + 1 - \sigma\right]}{1 + r_1}.
\end{aligned}
$$

Proof: To begin, insert the envelope condition for W_{3b} from (28) into (13) and (17) to get

$$u_2'(x_2^C) = \beta_2\beta_3(1 + r_{1,+1}) \tag{30}$$

$$c_2'(h_2^P) = \beta_2\beta_3(1 + r_{1,+1}). \tag{31}$$

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This implies $u_2'(x_2^C) = c_2'(h_2^P)$, and hence $x_2^C = h_2^P = x_2^*$. Similarly, insert the envelope condition for W_{3m} from (27) in to the first order conditions (12) and (16) to get

$$\phi_2 u_2'(x_2^C) = \beta_2 \beta_3 \phi_{1,+1} \left\{ \sigma e \left[q(m_3^C) \right] + 1 - \sigma \right\} \tag{32}$$

$$\phi_2 c_2'(h_2^P) = \beta_2 \beta_3 \phi_{1,+1} \left\{ \sigma e \left[q(m_3^P) \right] + 1 - \sigma \right\}. \tag{33}$$

Given $e'(q) < 0$ and $q'(m) > 0$ for all $m < m_3^*$, plus $x_2^C = h_2^P = x_2^*$, we conclude $m_3^C = m_3^P$.

Similarly, inserting (28) and (27) into the first order condition for a nontrader, we get

$$\phi_{1,+1} \left\{ \sigma e \left[q(m_3^N) \right] + 1 - \sigma \right\} = \phi_2 (1 + r_{1,+1}) \tag{34}$$

Exactly the same condition results from combining (30) and (32) for a consumer, or (31) and (33) for a producer. Hence, we conclude $m_3^N = m_3^C = m_3^P = M$. From the budget equations, this means debt is given by

$$b_3^C = x_2^* + (1 + r_2) b_2$$

$$b_3^P = -x_2^* + (1 + r_2) b_2$$

$$b_3^N = (1 + r_2) b_2.$$

This completes the description of market 2. Moving back to market 1, clearly (2) implies $x_1 = x_1^*$. Inserting the envelope conditions for W_2 and W_3 into (3) and (4), we have

$$\phi_1 = \beta_1 \beta_2 \beta_3 \phi_{1,+1} \{ \sigma e[q(M)] + 1 - \sigma \} \tag{35}$$

$$1 = \beta_1 \beta_2 \beta_3 (1 + r_2)(1 + r_{1,+1}), \tag{36}$$

where we use in the first case the result that W_{3m} depends on m_3 but not b_3, and $m_3 = M$. Notice (36) is an arbitrage condition between r_2 and $r_{1,+1}$: if it does not hold there is no solution to the agents' problem at $s = 1$; and if it does hold then any choice of b_2 is consistent with optimization. Hence we can set $b_2 = 0$. On the other hand, (35) implies

$$(1 + \rho) \frac{\phi_1}{\phi_{1,+1}} = \sigma e[q(M)] + 1 - \sigma. \tag{37}$$

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In steady state this implies (29).

The only things left to determine are the prices. We get r_1 from (30) with $x_2 = x_2^*$, and then set r_2 in terms of r_1 to satisfy the arbitrage condition (36). Given q, Lemma 1 tells us $\phi_1 = g(q)/\beta_3 M$, and (34) gives

$$\phi_2 = \frac{\phi_1[\sigma e(q) + 1 - \sigma]}{(1 + r_1)}.$$

This completes the proof. ∎

The central result of Theorem 1 is that at $s = 2$, consumers buy on credit $(b_3 = x_2^*)$ even though they are holding $m_3 = M$ and even though buying on credit entails a cost in terms of interest. The reason, of course, is that they know they may need the money at $s = 3$.

We now discuss rates of return. Condition (34) equates the value of a dollar's worth of cash and a dollar's worth of credit coming out of market 2. The left side is a weighted average of the marginal gain if the dollar is spent in market 3, $u'(q)q'(m_3) = \beta_3\phi_{1,+1}e(q)$, and the return if it is not spent but carried forward to the next period, $\beta_3\phi_{1,+1}$. The right side of (34) is the real return (the interest saving) from using the same dollar to pay down debt, $\beta_3\phi_2(1+r_{1,+1})$. Notice the return to money includes a *liquidity premium*: we show below that $e(q) > 1$ in equilibrium, and hence the value to spending a dollar is higher than the value to carrying it to the next period. If one ignores the liquidity premium, and simply considers the return on carrying money across periods, then it looks like – indeed, *it is* – rate of return dominance.

Theorem 2. *(Rate of Return Dominance) In any steady state monetary equilibrium,*

$$\frac{\phi_{1,+1}}{\phi_2} < 1 + r_{1,+1}.$$

Proof: By (34),

$$\frac{\phi_{1,+1}}{\phi_2} = \frac{1 + r_{1,+1}}{1 - \sigma + \sigma e(q)}.$$

The result follows if $1 - \sigma + \sigma e(q) > 1$, or $e(q) > 1$. However, by (29), in steady state $e(q) = 1 + \rho/\sigma$. ∎

Two brief comments are in order. First, we can price nominal bonds via the Fisher equation, which is simply a no-arbitrage condition, to get the nominal rate $1+i_{1,+1} = (1 + r_{1,+1})\, \phi_2/\phi_{1,+1}$. Then Theorem 2 can be equivalently stated as $i_{1,+1} > 0$. Second, the above results are framed in terms of rates of return between $s = 2$ at t and $s = 1$ at $t+1$, because this seems most natural given it is at $s = 2$ that the key decision is made (whether to pay with cash or credit). But we could use returns over the entire period. From $s = 1$ at t to $s = 1$ at $t + 1$, the gross return on money in steady state is 1, while the return on credit (paying down debt) is $(1 + r_2)(1 + r_{1,+1})$. We readily get $(1 + r_2)(1 + r_{1,+1}) > 1$ from (36), given $\beta_1\beta_2\beta_3 < 1$, so obviously rate of return dominance holds across the entire period.

3   Additional Discussion

As we discuss here, the analysis is easily extended in several ways.[14] First, in any equilibrium, and not just in steady state, essentially everything in Theorems 1 and 2 holds, except that (37) does not reduce to (29). However, we can insert $g(q) = \beta_3 m_3 \phi_{1,+1}$ from Lemma 1 to get

$$(1 + \rho)\frac{g(q)}{g(q_{+1})} = \sigma e(q_{+1}) + 1 - \sigma. \tag{38}$$

A monetary equilibrium is a bounded, positive, solution $\{q_t\}$ to (38), along with values for the other objects satisfying the same conditions as above. There exist many equilibrium paths for $\{q_t\}$ (see Lagos and Wright 2003 for details), but in all equilibria, x_s, b_s, and r_s are exactly as given in Theorem 1. And although ϕ_1 and ϕ_2 vary over time with q, Theorem 2 still holds exactly as stated.

Second, suppose the money supply changes at constant rate γ: $M_{+1} = (1 + \gamma)M$. Then it is natural to look for equilibria where all real variables, including q and ϕM, are constant. Hence, $\phi_1/\phi_{1,+1} = 1 + \gamma$, and (37) becomes

$$(1 + \rho)(1 + \gamma) = \sigma e[q(M)] + 1 - \sigma.$$

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[14] We already mentioned that bargaining can be replaced with price taking, posting, or auctions, and the main results go through.

Using the Fisher equation, the left side is $1 + i$, and so we have

$$1 + \frac{i}{\sigma} = e(q). \tag{39}$$

Thus, q is decreasing in i, but this does not affect the real allocation in markets 1 and 2. As is standard, the Friedman rule $i = 0$ is the lower bound on inflation, and also the optimal policy. At $i = 0$, the returns on money and credit are the same, and we lose rate of return dominance, but for any $i > 0$ everything is qualitatively the same.

The next point concerns our restriction that claims traded in the centralized market cannot circulate in the decentralized market, other than money, due to the assumption that individuals are anonymous and there is no institution can issue non-counterfeitable securities, other than the monetary authority. This is obviously an extreme assumption, meant to capture in a logically consistent way the idea that money is a relatively liquid asset. Consider the other extreme, where there is some perfectly safe, non-counterfeitable, security other than cash that can be used as a medium of exchange. To be concrete, consider a standard one-period, pure-discount security that sells for price ψ in the centralized market today and pays 1 unit of the consumption good in the next centralized market. Assume for now that there is no money and no other consumer credit, only this security, and let a be the amount of it brought into a period.[15]

To illustrate the idea, it suffices to consider a model with two rather than three subperiods – i.e. the basic LW set up – and it is convenient to add a little heterogeneity. Thus, there are now two types: type 1 are the agents in the benchmark LW model, while type 0 never go to the decentralized market (say, they do not consume or produce that good). Given this, it is convenient to change notation slightly. Let $U(x) - h$ be the common preferences in the centralized market, and let $u(q) - c(q)$ be type 1 preferences in the decentralized market. Let W and V be the value functions for type 1 in the centralized and decentralized markets, and J the

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[15]This security cannot be indexed by events that happen to you in the decentralized market, since they are not observable to others, and for simplcity there is no uncertainty in the centralized market, so it is not contingent on anything. We will show there is no role for any other securities. In particular, a takes the place of consumer debt b in the benchmark model, with $a = -b$, the difference being that now a can be traded in the decentralized market. As always, we need to rule out Ponzi schemes.

value function for type 0. Also, to reduce notation, assume agents discount across centralized markets at t and $t+1$, but not between centralized and decentralized markets.

For type 0,

$$J(a) = \max_{x,h,a_{+1}} \{U(x) - h + \beta J_{+1}(a_{+1})\}$$

$$\text{s.t. } x = h + a - \psi a_{+1},$$

since they do not participate in the decentralized market. For type 1,

$$W(a) = \max_{x,h,a_{+1}} \{U(x) - h + V(a_{+1})\}$$

$$\text{s.t. } x = h + a - \psi a_{+1},$$

where

$$V(a) = \sigma \{u[q(a)] + \beta W[a - d(a)]\} + \sigma \mathbb{E} \{-c[q(\tilde{a})] + \beta W[a + d(\tilde{a})]\} + (1 - 2\sigma)\beta W(a).$$

For type 0, the first order conditions are $U'(x) = 1$, which implies $x = x^*$, and $\psi = \beta W'_{+1}(a_{+1})$, which combined with the envelope condition implies the no-arbitrage condition $\psi = \beta$. For type 1, the first order conditions are $U'(x) = 1$ and $\psi = V'(a_{+1})$. Using the envelope condition, plus $V'(a) = \sigma u'(q)q'(a) + (1-\sigma)\beta$ and the bargaining solution $\beta a = g(q)$, we have

$$\psi = \beta \left[\sigma e(q) + 1 - \sigma \right].$$

Since we already established $\psi = \beta$, we conclude that $e(q) = 1$, and denote the solution by q_1. As is standard, if $\theta = 1$ then $q_1 = q^*$ and we get the first best, while if $\theta < 1$ then $q_1 < q^*$ and we do not, but we can do no better by introducing money. Indeed, no one would hold money, since it has a lower return than a, unless we run the Friedman rule, and does no better in terms of liquidity. It is clear that we need to make some assumption to give money an advantage in terms of liquidity if we are to get rate of return dominance.[16]

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[16] See Lagos and Rocheteau (2004), Lagos (2006), and Geromichalos, Licari and Suárez-Lledó (2006) for an extended analysis of related models, with multiple assets, all of which can be used as media of exchange.

It remains to discuss market clearing. Assume there is a measure 1 of type 0 agents and N_1 of type 1 agents, and that we start at the initial date where everyone has $a = 0$. From $\beta a = g(q_1)$, the demand for a by each type 1 agent in the first period is $g(q_1)/\beta$; hence the total demand by type 0 must be $-N_1 g(q_1)/\beta$, which is consistent with maximization, given $\psi = \beta$. From the budget constraint, in the first period a type 0 agent sets

$$h = x + \psi a_1 = x^* - N_1 g(q_1).$$

At every future t, he must make good on his negative a_t position, but again sets $a_{t+1} = -N_1 g(q_1)/\beta$, and ends up working $h = x^*$. He therefore has a one-time "seigniorage" gain (in terms of h) from selling short the asset that others use as a medium of exchange. In any case, it is clear that there is no role for money in this model as specified; and it should also be clear that government has an incentive to rule out the use of a as a medium of exchange so they can get the "seigniorage" for themselves.

Finally, to close this discussion section, we note the following. Although the baseline three-subperiod model clearly delivers on the main goal – agents carrying debt and money simultaneously – as long as we make some assumptions that give money a liquidity advantage, it does not have another feature that one might find desirable. Namely, our agents do not roll over debt for more than one period; they pay it off each period in market 1. Of course, they have to pay it off sometime (assuming no Ponzi schemes), and given utility is linear in h at $s = 1$, this is a good time to do so. But we show in the next section that in a generalized version of the model agents do generally roll over debt.

4 General Model

We pursue two generalizations. First, we allow any number n of subperiods per period (n could even change over time). Second, there are now centralized and decentralized markets open simultaneously in every subperiod. Agents visit one market or the other each subperiod according to the following process: an agent in the centralized market at s moves to the decentralized mar-
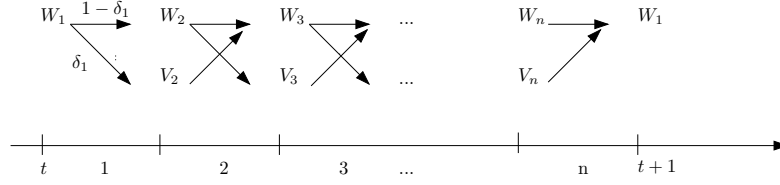
Figure 1: Market Structure

ket at $s+1$ with probability δ_s; and an agent in the decentralized market at s moves to the centralized market at $s+1$ with probability 1.[17] Now agents may want to hold money for every s, even if this is costly, since they might find themselves in need of it at $s+1$.

For convenience, we set $\delta_n = 0$, so that everyone is in the centralized market at $s = 1$, and assume all agents can produce with utility linear in hours, as in the benchmark model. For each $s \in \{2, ..., n\}$, the centralized markets are in the spirit of market 2 in the benchmark model, in the sense that credit is available, although we generalize the assumption that some agents cannot produce by now saying that productivity ω_s differs randomly in an i.i.d. manner across agents and s (for convenience, ω_1 is known and constant across agents). Agents in the centralized markets at $s > 1$ have general utility functions $U_s(x_s, h_s)$. The decentralized markets are in the spirit of market 3 in the benchmark, in the sense that some agents are buyers, some are sellers, and they meet and bargain bilaterally. Sellers can produce output one-for-one with labor in this market, that is, productivity is not random. Let $W_s(\omega_s, m_s, b_s)$ be the centralized and $V_s(m_s, b_s)$ the decentralized market value function at s. See Figure 1.

At $s = 1$, everyone is in the centralized market, and solves a problem that is very similar to the centralized market problem in various related models:

$$W_1(\omega_1, m_1, b_1) = \max_{x_1, h_1, m_2, b_2} \{U_1(x_1) - h_1 + \beta_1(1-\delta_1)\mathbb{E}W_2(\omega_2, m_2, b_2) + \beta_1\delta_1 V_2(m_2, b_2)\}$$

$$\text{s.t. } x_1 = \omega_1 h_1 + \phi_1(m_1 - m_2) + b_2 - (1+r_1)b_1$$

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[17]Thus, agents are never in the decentralized market for two or more periods in a row, which is convenient because it guarantees they are always willing to spend all their money when they get there. This trick is borrowed from Williamson (2005). By the way, the baseline LW model is the special case where $n = 2$ and $\delta_1 = 1$.

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The first-order conditions are

$$1/\omega_1 = U_{1x}(x_1) \tag{40}$$

$$\phi_1/\omega_1 = \beta_1(1-\delta_1)\mathbb{E}W_{2m}(\omega_2,m_2,b_2) + \beta_1\delta_1 V_{2m}(m_2,b_2) \tag{41}$$

$$-1/\omega_1 = \beta_1(1-\delta_1)\mathbb{E}W_{2b}(\omega_2,m_2,b_2) + \beta_1\delta_1 V_{2b}(m_2,b_2). \tag{42}$$

The envelope conditions are

$$W_{1m}(\omega_1,m_1,b_1) = \phi_1/\omega_1 \tag{43}$$

$$W_{1b}(\omega_1,m_1,b_1) = -(1+r_1) \tag{44}$$

Again, as in the benchmark model, W_1 is linear, everyone chooses the same x_1, and (m_2,b_2) is independent of (m_1,b_1).

At $s > 1$, agents in the centralized market solve:[18]

$$W_s(\omega_s,m_s,b_s) = \max_{x_s,h_s,m_{s+1},b_{s+1}} \{U_s(x_s,h_s) + \beta_s(1-\delta_s)\mathbb{E}W_{s+1}(\omega_{s+1},m_{s+1},b_{s+1})$$
$$+ \beta_s\delta_s V_{s+1}(m_{s+1},b_{s+1})\}$$
$$\text{s.t. } x_s = \omega_s h_s + \phi_s(m_s - m_{s+1}) + b_{s+1} - (1+r_s)b_s$$

First-order conditions are

$$0 = \omega_s U_{sx}(x_s,h_s) + U_{sh}(x_s,h_s) \tag{45}$$

$$\phi_s U_{sx}(x_s,h_s) = \beta_s(1-\delta_s)\mathbb{E}W_{s+1,m}(\omega_{s+1},m_{s+1},b_{s+1}) + \beta_s\delta_s V_{s+1,m}(m_{s+1},b_{s+1}) \tag{46}$$

$$-U_{sx}(x_s,h_s) = \beta_s(1-\delta_s)\mathbb{E}W_{s+1,b}(\omega_{s+1},m_{s+1},b_{s+1}) + \beta_s\delta_s V_{s+1,b}(m_{s+1},b_{s+1}) \tag{47}$$

Note (40) is a special case of (45) where $U_{1h}(x_1,h_1) = -1$, and (41)-(42) are special cases of (46)-(47) where $U_{1x}(x_1,h_1) = 1/\omega_1$. The envelope conditions are

$$W_{sm}(\omega_s,m_s,b_s) = \phi_s U_{sx}(x_s,h_s) \tag{48}$$

$$W_{sb}(\omega_s,m_s,b_s) = -(1+r_s)U_{sx}(x_s,h_s). \tag{49}$$

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[18]We adopt the obvious notational convention of identifying subperiod $n+1$ at t with subperiod 1 at $t+1$.

In the decentralized market bargaining problem, for simplicity, we set $\theta = 1$ in this section. Hence, in equilibrium, $d_s = m_s$ and q_s solves

$$c(q_s) = \beta_s \mathbb{E} W_{s+1}[\omega_{s+1}, \tilde{m}_s + m_s, (1 + r_s)b_s] - \beta_s \mathbb{E} W_{s+1}[\omega_{s+1}, \tilde{m}_s, (1 + r_s)b_s], \qquad (50)$$

which can be used to compute $q'(m_s)$.[19] Also,

$$V_s(m_s, b_s) = \sigma \mathbb{E} \{u(q_s) + \beta_s W_{s+1}[\omega_{s+1}, 0, (1 + r_s)b_s]\} + (1 - \sigma)\beta_s \mathbb{E} W_{s+1}[\omega_{s+1}, m_s, (1 + r_s)b_s]$$

and, hence,

$$V_{sm}(m_s, b_s) = \sigma u'(q_s)q'(m_s) + (1 - \sigma)\beta_s \mathbb{E} W_{s+1,m}[\omega_{s+1}, m_s, b_s(1 + r_s)] \qquad (51)$$

$$V_{sb}(m_s, b_s) = \beta_s(1 + r_s)\mathbb{E} W_{s+1,b}[\omega_{s+1}, m_s, b_s(1 + r_s)]. \qquad (52)$$

By repeated substitution, the first-order conditions in the centralized market at s for m_{s+1} and b_{s+1} can be written

$$\phi_s U_{sx}(x_s, h_s) = \beta_s \beta_{s+1} \cdots \beta_n \phi_{1,+1}\{\delta_s[\sigma e(q_{s+1}) + 1 - \sigma] + (1 - \delta_s)\delta_{s+1}[\sigma e(q_{s+2}) + 1 - \sigma]$$

$$+ ... + (1 - \delta_s)(1 - \delta_{s+1}) \cdots (1 - \delta_{n-1})\} \qquad (53)$$

$$U_{sx}(x_s, h_s) = \beta_s \beta_{s+1} \cdots \beta_n (1 + r_{s+1})(1 + r_{s+2}) \cdots (1 + r_{1,+1}), \qquad (54)$$

where $e(q_s)$ is defined in (25). By (54) $U_{sx}(x_s, h_s)$ is constant across agents – i.e. independent of their (ω_s, m_s, b_s) – and hence by (48)-(49) W_{sm} and W_{sb} are too. That is, W_s is linear (m_s, b_s), for all s. Moreover, (50) now implies

$$q'_s(m_s) = \frac{\beta_s \beta_{s+1} \cdots \beta_n \phi_{s+1}(1 + r_{s+2}) \cdots (1 + r_{1,+1})}{c'(q_s)}. \qquad (55)$$

Also, (46) now has the following property: the left side is constant and the right side depends on m_{s+1}, since this is the only thing that influences q_{s+1} which is the only thing that influences $V_{s+1,m}$. In other words, (46) pins down m_{s+1}, independent of (ω_s, m_s, b_s). All agents carry the same amount of money, as in the benchmark model.

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[19]The expectation in this expression is with respect to both ω_{s+1} and $\tilde{m}_s$, the money of the seller one meets. We show below, however, that $\tilde{m}_s = M$, exactly as in the benchmark model, and W_s is linear in m_s, so the bargaining solution q does not actually depend on $\tilde{m}_s$.

We can summarize what we now know about equilibrium as follows.[20] First denote the right side of (54) by k_s. Then given k_s, (54) for $s = 1, ...n$, (45) for $s = 2, ...n$, and (40) constitute $2n - 1$ equations in $2n - 1$ unknowns, pinning down $(\bar{x}_1, ...\bar{x}_n, \bar{h}_2, ...\bar{h}_n)$ (as functions of interest rates). Notice h_1 does not appear in these conditions. We also established that $m_{s+1} = M$ for all agents for all s. The centralized market budget equation therefore tells us

$$b_{s+1} = (1 + r_s)b_s + \bar{x}_s - \omega_s \bar{h}_s - \phi_s m_s + \phi_s M.$$

This says that agents' debt at $s + 1$ will be equal to their debt at s, all of which is rolled over (principle plus interest), plus consumption expenditure minus labor income, with an adjustment to maintain cash balances at the desired level.[21]

The key point is that some agents will quite generally carry debt, which is costly in terms of interest, while maintaining a stock of money. When they get to the start of the next period, they pay off their debts by adjusting $h_{1,+1}$. In particular, an agent that draws a low ω_s will not only have a low wage, he will additionally have a low h_s, since

$$\frac{\partial h_s}{\partial \omega_s} = \frac{-k_s U_{sxx}}{U_{sxx}U_{shh} - U_{sxh}^2} > 0$$

by virtue of (45). His consumption may be higher or lower, depending on the cross-derivative of U, since

$$\frac{\partial x_s}{\partial \omega_s} = \frac{k_s U_{sxh}}{U_{sxx}U_{shh} - U_{sxh}^2}.$$

For example, if U is separable in x_s and h_s, he will not lower x_s. He will often purchase part of x_s on credit while maintaining $m_{s+1} = M$. When ω_s is small and $m_s = M$, he will purchase most of x_s on credit while not adjusting his money holdings. In the worst-case scenario, when ω_s is small and $m_s = 0$, he purchases current goods on credit and also takes out a cash advance.

We collect some key results from this analysis as follows.

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[20] We do not provide a formal definition of equilibrium here since it is an obvious generalization of the definitions in the benchmark model.

[21] An individual's m_s may be above (below) his desired level for $s + 1$ if he just returned from the decentralized market where he acted as a seller (buyer). In general, if δ_s varies with s then desired real balances will, too; since we must have $m_s = M$ in equilibrium, this show up in the price ϕ_s.

Theorem 3. *In the model with n subperiods, in any monetary equilibrium,*

1. *For all s, every agent leaves the centralized market with the same m_{s+1}.*

2. *For all s, $U_{sx}(x_s, h_s) = k_s$ and $U_{sh}(x_s, h_s) = -k_s \omega_s$ where k_s is constant across agents in the centralized market.*

3. *If two agents have different (m_s, b_s) and the same ω_s, their h_s, x_s and m_{s+1} are the same, so they have different b_{s+1}; if two agents have the same (m_s, b_s) and different ω_s, their h_s will differ and they typically have different b_{s+1}.*

4. *Agents may roll over or run up debts between $s = 2$ and $s = n$ while maintaining, and sometimes even increasing, their holdings of m_s.*

Theorem 4. *(Rate of Return Dominance) In any monetary equilibrium, for all $s \neq n$,*

$$\frac{\phi_{1,+1}}{\phi_s} < (1 + r_{s+1})(1 + r_{s+2})...(1 + r_n)(1 + r_{1,+1}) \tag{56}$$

To say a little more about Theorem 4, a dollar held at s may be spent in the decentralized market in any of the subperiods that follow; or it may not, in which case it is brought into $t + 1$ where it yields ex post return $\phi_{1,+1}/\phi_s$. Alternatively, a dollar at s of consumer credit yields compound interest between then and $t + 1$ given by the right side of (56). The inequality indicates, as always, that the true value of money is greater than the return from simply carrying it into $t + 1$ with probability 1, since it has liquidity value. In particular, from the first-order conditions for m_{s+1} and b_{s+1} we get

$$\frac{\phi_{1,+1}}{\phi_s}\{\delta_s[\sigma e(q_{s+1}) + 1 - \sigma] + (1 - \delta_s)\delta_{s+1}[\sigma e(q_{s+2}) + 1 - \sigma]$$

$$+... + (1 - \delta_s)(1 - \delta_{s+1})...(1 - \delta_{n-1})\}$$

$$= (1 + r_{s+1})(1 + r_{s+2})...(1 + r_n)(1 + r_{1,+1}) \tag{57}$$

Theorem 4 then follows from the result that $e(q_s) > 1$ for all s.

5   Conclusion

In this paper we have tried to re-visit a classic issue: the coexistence of assets with different returns. An example of this issue is the so-called credit card debt puzzle, but more generally, it is known as rate of return dominance. We build on the recent monetary theory literature by allowing the option to sometimes trade using credit. Our model is tractable. It yields strong and interesting outcomes, including the prediction that agents may purchase on credit, even when this has a cost in terms of interest and they have liquid assets at hand. While we think that there is more theoretical work to be done on rate of return dominance, and certainly more quantitative work to be done, we hope this constitutes progress.

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Appendix

In this Appendix we do several things. First we derive the bargaining solution given in Lemma 1. The necessary and sufficient conditions for (22) are

$$\theta \left[\beta_3 \phi_{1,+1} d - c_3(q)\right] u_3'(q) = (1-\theta)\left[u_3(q) - \beta_3 \phi_{1,+1} d\right] c_3'(q) \tag{58}$$

$$\theta \left[\beta_3 \phi_{1,+1} d - c_3(q)\right] \beta_3 \phi_{1,+1} = (1-\theta)\left[u_3(q) - \beta_3 \phi_{1,+1} d\right] \beta_3 \phi_{1,+1} \tag{59}$$

$$-\lambda \left[u_3(q) - \beta_3 \phi_{1,+1} d\right]^{1-\theta} \left[\beta_3 \phi_{1,+1} d - c_3(q)\right]^{\theta}$$

where λ is the Lagrange multiplier on $d \leq m_3$. There are two possible cases: If the constraint does not bind, then $\lambda = 0$, $q = q^*$ and $d = m^*$. If the constraint binds then q is given by (58) with $d = m_3$, as claimed.

We now argue that $m_3 < m_3^*$. First, as is standard, in any equilibrium $\phi_{1,+1} \leq (1+\rho)\phi_1$; this just says the nominal interest rate i is nonnegative. In fact, again as is standard, although we allow $i \to 0$, we only consider equilibria where $i > 0$, so that $\phi_{1,+1} < (1+\rho)\phi_1$. Now suppose $m_3 > m_3^*$ at some date for some agent. Since the bargaining solution tells us he never spends more than m_3^*, he could reduce m_3 by reducing h_1 at t, then increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that this increases utility, so $m_3 > m_3^*$ cannot occur in any equilibrium.

Hence $m_3 \leq m_3^*$. To show the strict inequality, suppose $m_3 = m_3^*$ for same agent. Again he can reduce h_1 at t and carry less money. If he is a buyer in subperiod 3, he gets a smaller q, but the continuation value is the same since by the bargaining solution he still spends all his money. If he does not buy then he can increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that the net gain from carrying less money is positive, exactly as in LW.