Health, Development, and the Demographic Transition

Matteo Cervellati
University of Bologna

Uwe Sunde*
IZA, Bonn
University of Bonn

January 20, 2006

Abstract

This paper provides a unified theory of the economic and demographic transition. The main mechanism is based on optimal decisions about fertility and time investments in heterogeneous types of human capital. These decisions depend on different dimensions of health, which themselves are endogenously determined in the process of development. By disentangling the distinct roles that different dimensions of health, such as adult longevity, child mortality, and overall healthiness, play for education and fertility decisions, the model is able to generate dynamics that can replicate the historical development pattern in the Western world. The model generates an endogenous economic transition from a situation of sluggish growth in incomes and productivity to a modern growth regime. Closely related, a demographic transition from high mortality and high fertility to low mortality and low fertility arises, with an intermediate phase of increasing fertility despite falling mortality rates. The model can generate a positive correlation between income and fertility during early stages of development, as well as a decline net fertility in the last phase of the demographic transition, and it provides a rationale for fertility declines that precede drops in child mortality.

JEL-classification: E10, J10, O10, O40, O41

Keywords: Long-term development, endogenous life expectancy, child mortality, health, heterogeneous human capital, technological change, industrial revolution

*The authors wish to thank Javier Birchenall, Raouf Boucekkine, Matthias Doepke, Omar Licandro as well as seminar participants at the Workshop on Demographics, Health and Economic Development at Stanford University, and the SAET 2005 conference in Vigo, Spain, for helpful discussions and comments. The usual disclaimer applies.
Cervellati, Contact: Universitá di Bologna, Strada Maggiore 45, Bologna, Email: cervellati@spbo.unibo.it. Sunde, Contact: Schaumburg-Lippe Str. 7-9, D-53113 Bonn, Tel. +49-228-3894-221, Fax. +49-228-3894-510, Email: sunde@iza.org (corresponding author).

1 Introduction

Transitions from stagnant economic environments to developed economies with permanent growth and improving living conditions are often characterized by substantial changes in many dimensions of human life. In the Western world, for example, aggregate and per capita income displayed a virtual explosion from the second half of the 18th century onwards, after almost stagnant development during the entire history. The changes that this transition brought about, however, were not confined to the economic domain only. Rather, this transition to modern societies also involved changes in other important dimensions of the human environment and indicators of general living conditions and health reflected in fertility behavior, population density, educational attainment, child mortality and adult longevity. In fact the economic transition from stagnation to growth was closely related to a demographic transition from a regime characterized by high child mortality, little longevity, large fertility, and a positive correlation between living conditions and fertility, to a Modern Growth regime with large improvements in terms of adult longevity, reduced child mortality and higher educational penetration, but also lower total and even net fertility.[1] The more recent episodes of escape from stagnation are characterized by similar changes with the improvements in economic and living conditions being associated with demographic transitions.

While human capital acquisition is usually seen as the main engine of economic development, empirical findings suggest that health and longevity are the crucial determinants for human capital acquisition. Several authors have emphasized the role of health and improved health conditions for labor market participation and productivity, as well as for school attendance and human capital acquisition, see e.g. Fogel (1994), Bleakley (2003) and Miguel and Kremer (2004). Life expectancy of adults seems to be the key determinant of human capital acquisition and income differences across countries, see Reis-Soares (2005) and Shastry and Weil (2003). This claim is substantiated by the empirical findings of Lorentzen, McMillan, and Wacziarg (2005), who provide evidence that life expectancy has a causal impact on economic development.

Closely related is the effect of health on fertility behavior. While bad health conditions reflected in high child mortality imply a high gross fertility in order to sustain a certain net fertility rate and thus a stable population, improving child survival rates decrease gross fertility, but have no clear a priori impact on net fertility. Most existing theories of fertility choice assume an income-driven trade-off between quantity and quality of children along the lines of Barro and Becker (1988 and 1989). In these models, declining child mortality predicts an increasing net fertility rate along the economic transition. Moreover, there is an a priori ambiguous effect of adult longevity on fertility.

However, existing theories of the economic transition generate dynamics that are inconsistent with empirical and historical evidence on the economic and demographic transition. Clark (2005) argues that theories considering income changes as the driving force behind changes in fertility behavior are problematic because they treat changes in human capital formation as consequence, and not as a cause, of the economic and demographic transition. Increased human capital accumulation *before* the economic transition is therefore difficult to rationalize with existing theories. Moav (2005) and Hazan and Zoabi (2005) point out that the role of adult longevity for fertility behavior is difficult to rationalize with a quantity-quality approach, because changes in parent life expectancy leave the main trade-off essentially unchanged. But this cannot explain the observed fall in net fertility. Similarly, Doepke (2005) argues that the decline in net fertility rates is difficult to rationalize by reductions in child mortality. Because of the income effect involved with optimal fertility choice, net fertility increases in that framework as mortality of

[1] Compare e.g. the data provided by Maddison (1991). We provide and discuss the empirical evidence for England and Wales in section 5.

children decreases.[2] Kalemli-Ozcan (2002, 2003) investigates the role of uncertainty about child survival in inducing forward looking parents to 'hoard' children due to a precautionary motive. In her model, a substitution effect implies that gross fertility declines if child survival chances increase. Still, this approach does not explain a fall in net fertility.[3]

A further criticism concerns the observed timing of events. Galor (2005) notes that in some countries like England, fertility declined even before child mortality fell. This observation casts doubt on causality running from from life expectancy to fertility.[4] Moreover, adult longevity began to increase well before the economic transition, preceding reductions in fertility by as much as a century in some countries (see Lorentzen *et al.*, 2005, and Galor, 2005). Also, in some cases, fertility actually increased before the emergence a regime with low mortality and declining fertility.[5]

Summing up, conventional theories based on the quantity-quality trade-off are difficult to reconcile with empirical observations. On top of that, the distinct roles played by different dimensions of health, reflected by child mortality, general health status, and adult longevity, for development have not been investigated in a unified growth model. To our knowledge, there exists no theoretical unified growth framework that investigates the the central role that health and life expectancy seems to have for the transition according to the available empirical evidence, and the interdependent feedback effects with fertility behavior and economic development.

The object of this paper is to provide a unified theory in which the economic and demographic transitions endogenously arise from the interactions between economic variables, in particular human capital formation and technological improvements, and health in various contexts, reflected by adult longevity, child mortality and overall health status. The model investigates the role of health for education and fertility as well as for the evolution of the economy. The model builds on the framework proposed by Cervellati and Sunde (2005), which studies how life expectancy and development affect individual education decisions. We generalize this framework in several directions and investigate the dynamic evolution of fertility and human capital formation by focussing attention to the role of the different dimensions of health.

The different dimensions of health considered in this paper affect different decisions. In particular, health conditions reflected by child mortality are the major determinant of fertility decisions. Health of adults, reflected by adult longevity, determines the horizon over which human capital investments can amortize. Finally, the overall health status of an individual, is reflected in his ability to use time for productive or recreative activities, and particularly determines the time the individual needs to acquire human capital or to perform a certain task, i.e., his or her productivity.

We consider an overlapping generations setting in which heterogenous adults, who have successfully survived childhood, maximize their utility from consumption and from surviving offspring by making decisions on education and fertility. In particular, these adults decide which type of human capital to acquire, how much time to spend on education and how many children they want to raise. Both education and fertility are crucially affected by adult longevity, which individuals take as given. The probability that their children survive, which is also a state variable, affects fertility decisions. Dynamically, the human capital acquired by a generation

[2] This is the case since a reduction in child mortality increases the returns to quality and, given the assumed complementarity, also the returns to quantity.

[3] Doepke (2005) shows that allowing for sequential fertility weakens the effect of declining fertility from precautionary demand for children as child mortality can be compensated by subsequent child births. He argues that the precautionary motive cannot account for a sizable decline in net fertility and concludes that child mortality cannot explain the fertility drop alone. Lorentzen, McMillan, and Wacziarg (2005) point out that sequential fertility does not help replacing offspring if children die as adults. If the demand for children is for old age support a decline in (adult) mortality still induces a "hoarding" motive.

[4] See alsoDoepke (2005) for a historical discussion.

[5] We discuss these inconsistencies and problems in more detail in section 3.

of adults affects the technology as well as living conditions in terms of adult longevity and child mortality in the future. Changes in life expectancy and technology, in turn, affect the profitability of investing in the different types of human capital for those children, who survive until adulthood.

As result, we present an analytical characterization of the transition from stagnation to growth, which is based on the interplay between human capital formation, technological progress, and health. A feedback between human capital, fertility and adult longevity is eventually triggered when there are sufficiently high returns to human capital for large fractions of the population to outweigh the cost in terms of lifetime spent on higher education. As a consequence of a changing environment and education behavior, fertility behavior also changes. We identify several opposing effects. The *income* effect, related to longevity, induces adults to have more children, while the *substitution* effect, related to child mortality, allows them to reduce the number of children. Finally, a *differential fertility* effect arises because high skilled individuals, for whom the opportunity cost of raising children is higher, give birth to fewer children. This effect gains momentum as larger fractions of the population decide to get higher education. The dynamic equilibrium path is consistent with the empirical and historical evidence on the economic and demographic transitions. By simulating the model for illustrative purposes we show that the behavior of key variables like adult longevity, infant mortality, gross and net fertility, and literacy and income per capita are in line with empirical observations and historical evidence.

This paper thereby contributes to the recent literature on unified theories studying the development process as being endogenously driven by the interaction between fertility behavior and human capital.[6] The seminal contribution by Galor and Weil (2000), which initiated the literature on unified theories, adopted a modelling approach based on a quantity-quality trade-off while Galor and Moav (2002) investigate the role of natural selection forces. According to these theories, the decline in fertility came with a more intensive caring for children, a shift from more quantity to more quality.[7] In a more recent paper, Galor and Moav (2005) investigate the role of changing environmental conditions, like population density, for longevity. They illustrate a mechanism of natural selection, which leads to more robust types of humans surviving when environmental conditions are characterized by higher extrinsic mortality. At the same time, fertility declines as more robust types spend more resources on somatic investment, like maintenance and repair, and less on bearing children, which is consistent with the patterns of the demographic transition. Our paper provides an alternative explanation for the transition that explicitly accounts for the distinct yet crucial roles of different dimensions of longevity for economic and demographic development. Other than the quantity-quality approach of earlier contributions where parents face a trade-off between the quantity and the quality of their offspring, we concentrate attention to the trade-off between resources devoted to child bearing and the parents' own education investment in terms of time devoted to different types of human capital.[8] Our contention is that the explicit consideration of the shift in the *quantity* and *quality* of human capital can help to shed some light on the puzzling evidence cited above. In that, the model is largely complementary to previously studied channels for the economic and

[6] Earlier contributions to the literature have concentrated on the existence of multiple steady state equilibria and have explained the transition from a stagnant regime to an environment of sustained growth by scale effects, see Goodfriend and McDermott (1995), exogenous technological change, see Hansen and Prescott (2002), or shocks, that move the dynamic system from one steady state to another, see Blackburn and Cipriani (2002) and Boucekkine, de la Croix, and Licandro (2003).

[7] Other contributions studying the role of fertility for long-term development and the demographic transition, include Kogel and Prskawetz (2001) (2001), Hazan and Berdugo (2002), Lucas (2002), Kalemli-Ozcan (2002), and Lagerlof (2003), among others. See also the extensive survey by Galor (2005) for a detailed discussion of the theoretical literature and empirical evidence regarding these issues.

[8] In this respect the trade-off considered here is fundamentally different from the parental choice between quantity and quality of children as studied by Barro and Becker (1988, 1989) , Galor and Weil (2000), and the features of which have been discussed by Doepke (2005).

demographic transition. However, the model does nor rely on subsistence consumption levels, on precautionary demand for children or a different rate of risk aversion with respect to consumption and children. Another novelty is the explicit consideration of distinct dimensions of longevity. A similar distinction was emphasized by Reis-Soares (2005). But while he studies the response of fertility to exogenous changes in longevity and mortality, our focus is rather on the study of the endogenous interactions between these different domains in equilibrium, and the endogenous economic and demographic transition. We limit attention to a deterministic environment to highlight the role of the different dimensions of health.[9]

The paper proceeds as follows. Section 2 introduces the basic framework, discusses the individual decision problem concerning education choice and fertility behavior, and derives the equilibrium allocation within a given generation. Section 3 then introduces the dynamic interactions between generations through life expectancy and technological progress. Section 3 derives the dynamic equilibrium, as well as a characterization of the long-term development patterns implied by the setup. In section 5, we provide an illustrative simulation of the model and briefly discuss the implications of the model in light of empirical and historical evidence. Section 6 concludes. For convenience, all proofs are relegated to the appendix.

2 A Model of Human Capital and Fertility Choice with Health

2.1 The Framework

Individual Endowments and Timing. The timing of the model is illustrated in Figure 1. Time is continuous, $\tau \in \mathbb{R}^+$. The economy is populated by an infinite sequence of overlapping generations of individuals, which are denoted with subscript t, where $t \in \mathbb{N}^+$. As we want to concentrate attention on the endogeneity of fertility choice and hence family size, for the moment being we neglect any choices regarding the timing of fertility.[10] A generation of individuals t, born at some moment in time τ_t enjoys a childhood of length $k_t = k$ after which individuals turn adult. Not all children of generation t survive childhood because of infant and child mortality. The fraction of children surviving to adulthood is denoted by π_t. At the beginning of their life as adults (i.e. once a time k has passed from their birth) agents make decisions concerning fertility and education. We assume that individuals reproduce as soon as they become adults. Consequently, every generation is born $k_t = k$ periods after the birth of the respective previous generation.[11] Reproduction is asexual and we abstract from issues of non-divisibility, i.e. the number of children as a continuous choice variable, $n \in \mathbb{R}_0^+$.[12] In order to highlight the mechanism we restrict attention to a deterministic framework without sequential child birth.[13] A generation of adults consists of a continuum of agents with population size N_t, which is determined by size and fertility of the previous generation, as well as the survival probability of children. Adults face a (deterministic) remaining life expectancy T_t specific to their generation

[9] We abstract from a precautionary motive for fertility. As discussed below, the results are complementary to the ones by Kalemli-Ozcan (2002, 2003).

[10] See Blackburn and Cipriani (2002) for a paper on long-term development that deals with changes in the timing of fertility.

[11] Instead of assuming a fixed frequency of births, the length of the time spell between the births of two successive generations, hence the timing of fertility, could be modelled as a function of the life expectancy of the previous generation. This would modify the results concerning population size, but would leave the main results concerning the economic and demographic transition unchanged.

[12] As investigated by Doepke (2005), accounting for the fact that in reality the number of children is discrete can affect the optimal choice if the parents have a precautionary demand for children. In the current framework the assumption of n as a continuous variable is only made for simplicity.

[13] Uncertainty giving rise to precautionary motives in fertility behavior is realistic, but strictly complementary to our analysis of fertility. Sequential fertility decisions would complicate the analysis without adding any additional insights.

t. The determinants of both child survival probability π_t and adult longevity T_t are investigated below.

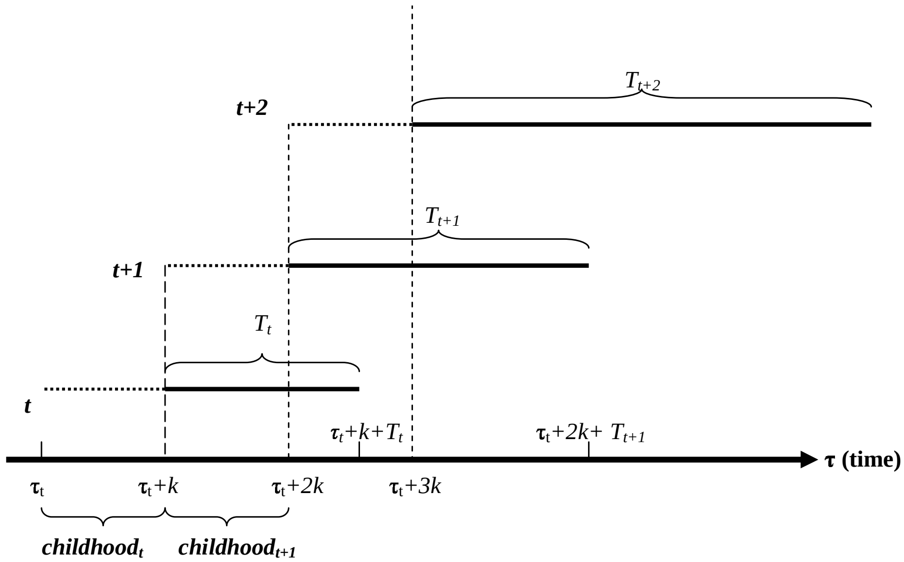


Figure 1: Timing of Events

At birth, every individual is endowed with ability $a \in [0,1]$, which is distributed uniformly over the unit interval. Since child mortality affects every child the same way, the ability distribution of adults is also uniform.[14]

Individual Preferences. Adults care about their own consumption as well as about the number of their surviving offspring. Lifetime utility is given by

$$u_t(c_t, n_t) = c_t^{(1-\gamma)} (\pi_t n_t)^{\gamma} \ , \tag{1}$$

where c_t is total lifetime consumption, n_t is the total number of offspring, $\pi_t \in (0,1)$ denotes the probability that a child survives childhood, and a parameter $\gamma \in (0,1)$. This formulation reflects the trade-off between the resources devoted to own consumption and to raising children. Individual utility is concave in consumption and the number of surviving offspring. We abstract from life cycle considerations, the choice of the optimal consumption and savings path over the life cycle, discounting etc.[15]

Production of Human Capital. In order to make an income to consume, individuals have to acquire human capital, which they can then supply on the labor market. Every generation has to build up the stock of human capital from zero, since the peculiar characteristic of human capital is that it is embodied in people. Human capital acquisition requires an education process which requires foregoing some fraction of an individual's available life time in education. In order to isolate the development effects related to the various dimensions of health and human capital

[14] The uniform distribution is for simplicity since the central results can be generated with any distribution of ability $f(a)$ among the surviving adults. The *ex ante* distribution of innate ability or intelligence does not change over the course of generations. This issue is discussed below.

[15] The formulation also implies that individuals can perfectly smooth consumption as well as the utility from children over their life. But at the same time, individuals cannot perfectly substitute utility from their own consumption by utility derived from their offspring.

formation, any links between generations through savings or bequests are excluded.[16]

Agents can decide to acquire one of two types of human capital, which differ in terms of the education process they require, and the returns they generate. The first type is interpreted as high-quality human capital characterized by a high content of abstract knowledge. We refer to this as skilled or theoretical human capital and denote it by h. The second type is labelled unskilled, or practical human capital, denoted by l, and contains less intellectual quality, but more manual and practical skills that are important in performing tasks related to existing technologies.[17] Education time, denoted by e and individual ability a are the only inputs of the process of production of human capital. Nonetheless, as clarified below, health facilitates the acquisition of human capital. In particular the health status may affect the amount of effective human capital which is acquired for any given time spent in education.

Both types of human capital are inherently different with respect to the time intensity of their acquisition, the effectiveness of ability and the role of health in the education process. To acquire h, it is necessary to first spend time on the building blocks of the elementary concepts without being productive in the narrow sense. Once the basic concepts are mastered, the remaining time spent on education is more productive. This is captured by a fix cost $\underline{e}$ measured in time units, which an agent needs to pay when acquiring h. The characteristics of this education process are formalized adopting the formulation,

$$h = \begin{cases} \alpha(s)\,(e - \underline{e})\,a & \text{if } e \geq \underline{e} \\ 0 & \text{if } e < \underline{e} \end{cases} . \tag{2}$$

In contrast, the time devoted to acquire l is immediately effective, albeit with a lower overall productivity, so there is no fix time cost involved when acquiring l.[18]

$$l = \beta(s)e\,. \tag{3}$$

A unit time of education produces $\alpha(s)a$ units of h and $\beta(s)$ units of l with $\alpha(s) \geq \beta(s)$. Health affects both the probability of offspring surviving childhood and the adult longevity, i.e. the quantity of time but it also affects the quality of lifetime. This third dimension is considered by assuming that a better health status increases the productivity education $\partial\alpha(s)/\partial s \geq 0, \partial\beta(s)/\partial s \geq 0$.

This formulation captures three crucial features of the human capital formation process: Ability is relatively more important (and effective) when acquiring advanced skills h; a larger adult longevity T induces individuals to acquire more of any type of human capital and makes skilled human capital relatively more attractive for individuals of any level of ability; and a better health status s increases the effectiveness of time spent in human capital acquisition, particularly for advanced skills h. We adopt a linear specification to disentangle the different effects of health, longevity and ability in the subsequent analysis.[19]

2.2 Optimal Education and Fertility Decisions

We now turn to the choice problem of adult members of a given generation t. As is discussed in more detail now, to maximize their utility, individuals optimally choose their education and

[16] We also abstract from real resources as input for the human capital formation process, as well as issues related to capital market development and public provision of education.

[17] Hassler and Rodriguez-Mora (2000) use a related perception of abstract versus applied knowledge.

[18] A sufficient condition for the results below is that for applied human capital l the fix cost in terms of time is smaller, so for simplicity it is normalized to zero. We abstract from other costs of education, like tuition fees etc. Moreover, the fix cost is assumed to be constant and the same for every generation. Costs that increase or decrease along the evolution of generations would leave the qualitative results of the paper unchanged.

[19] Any alternative model of human capital formation reproducing these features would be entirely equivalent for the purpose of this paper. Alternative settings like learning on-the-job could similarly be used to illustrate the importance of longevity and health for human capital formation.

their fertility.[20]

The individual choice problem. Investments in human capital as well as in children imply costs. In particular, while acquiring human capital, an individual cannot work and therefore earns no income. With respect to fertility, parents face (time equivalent) cost both to give birth and to raise children. We assume that the birth of each child entails a cost b so that the total cost of births is given by bn_t. While each birth is costly, not all children survive. Bringing up the surviving children requires additional care in the sense of time spent with them, inflicting opportunity costs in terms of foregone working time. We incorporate these additional opportunity costs by assuming that the raising of surviving children implies an additional time cost, so that the total cost of raising children is given by $r(\pi_t)\, n_t$, where $r(\cdot)$ is non-decreasing in the child survival probability π_t.

Denoting by w_t^j the wage rate paid at any moment in time to every unit of human capital of type $j = h, l$ acquired by generation t, the respective lifetime budget constraint of an individual of generation t is then given by

$$c_t + (bn_t + r(\pi_t)n_t)\, w_t^l l_t \leq (T_t - e_t) w_t^l l_t \tag{4}$$

in case of acquiring p human capital and

$$c_t + (bn_t + r(\pi_t)n_t)\, w_t^h h_t \leq (T_t - e_t) w_t^h h_t \tag{5}$$

in the case of h human capital.[21]

The problem of an agent with ability a born in generation t is therefore, while taking as given life expectancy T_t, child survival probability π_t health status s_t and the wage w_t^j, to choose the type of human capital $j \in \{h, l\}$, the optimal education time spent on its accumulation, e^j and the optimal fertility n^j.[22] Formally, lifetime utility is maximized according to

$$\max_{\{j, e_t^j, n_t^j\}} u_t^j(j, e_t^j, n_t^j; a, T_t, s_t, w_t^j) \tag{6}$$

$$\text{subject to } j \in \{h, l\} \text{ , and conditions (4) and (5) .} \tag{7}$$

The solution of the individual maximization problem (6) is given by the vector $\{j^*, e_t^{j*}, n_t^{j*}\}$ satisfying all constraints in (7). In order to characterize this vector, we first characterize the optimal education and fertility choices for both types of human capital $j = h, l$, and then compare the utility an agent derives from acquiring theoretical human capital h to that from acquiring practical human capital l. The optimal choice j^* is finally given by that type of human capital that offers the highest lifetime utility given the optimal choices of education time and fertility.

Optimal education time and fertility for l-type human capital. The optimization problem is strictly globally concave so that first order conditions uniquely identify the optimal choices made by any individual, conditional on the acquisition of any type of human capital. Substituting the budget constraint, the optimal choices of education time and number of children for an individual of ability a acquiring human capital type l are given by the solution to the following optimization problem

$$\{e_t^{p*}, n_t^{l*}\} = \arg\max \left((T_t - e_t^l)\beta(s) e^l w_t^l - (b + r(\pi_t)) n_t \beta e^l w_t^l \right)^{(1-\gamma)} (\pi_t n_t)^\gamma \ . \tag{8}$$

[20] We abstract from decisions about retirement and leisure.

[21] In general, one can think of the possibility that the costs for nourishing children and taking care of them differ between individuals that acquire theoretical human capital and individuals specializing in practical human capital, but for simplicity we abstract from this possibility.

[22] Allowing individuals to acquire both types of human capital would not change the formal arguments, but would imply a somewhat different interpretation of human capital.

The first order conditions to this problem lead to the following expressions for the optimal education time and fertility behavior,

$$e_t^l = \frac{T_t - (b + r(\pi_t))n_t^l}{2} \qquad \text{and} \qquad n_t^l = \gamma \frac{T_t - e_t^l}{b + r(\pi_t)} \,. \tag{9}$$

The optimal time investment in education decreases in the number of children, and in the time invested in raising them. Likewise, the number of offspring is lower the more time is invested in education and the higher the cost for giving birth and raising them. Solving for the optimal values of e^p and n^p we have,

$$e_t^{l*} = \frac{1-\gamma}{2-\gamma} T_t \qquad \text{and} \qquad n_t^{l*} = \frac{\gamma}{2-\gamma} \frac{T_t}{b + r(\pi_t)} \,. \tag{10}$$

Optimal education time and fertility for h-type human capital. Similarly an individual of ability a acquiring theoretical human capital chooses the education time and the number of children optimally so that

$$\{e_t^{h*}, n_t^{h*}\} = \arg\max \left((T_t - e_t^h)\alpha(s)a\left(e_t^h - \underline{e}\right) w_t^h - (b + r\pi_t)n_t^h \alpha(s)a\left(e_t^h - \underline{e}\right) w_t^h \right)^{(1-\gamma)} (\pi_t n_t)^\gamma \,. \tag{11}$$

The first order conditions to this problem read

$$e_t^h = \frac{T_t + \underline{e} - (b + r(\pi_t))n_t}{2} \qquad \text{and} \qquad n_t^h = \gamma \frac{T_t - e_t^h}{b + r(\pi_t)} \,. \tag{12}$$

The inspection of these first order condition makes clear that, again, ceteris paribus more children decrease the time invested in own education. A higher fix cost $\underline{e}$ involved with the acquisition of theoretical human capital requires a larger time investment in education, however. Solving the two equations for the expressions of the two optimal choices, one obtains

$$e_t^{h*} = \frac{(1-\gamma)T_t + \underline{e}}{2-\gamma} \qquad \text{and} \qquad n_t^{h*} = \frac{\gamma}{2-\gamma} \frac{T_t - \underline{e}}{b + r(\pi_t)} \,. \tag{13}$$

Optimal Choice of Human Capital Types. Summing up the previous results, larger adult longevity facilitates both the acquisition of human capital and child rearing, regardless of the type of human capital. This is the case since a longer working life relaxes the budget constraint and allows to increase the number of children. This income effect implies a positive relationship between longevity and fertility. The second dimension of health, child mortality, has an opposite effect on fertility, however. A larger survival probability π_t implies a larger total cost of child raising and leads to a reduction in optimal fertility. This substitution effect implies that an improvement of health conditions in the form of lower child mortality implies a reduction of fertility.

Still, from inspection of (8), (10), (11) and (13) it becomes apparent that, for any level of T_t and π_t the optimal education and fertility choices are different depending on the type of human capital acquired. In particular agents acquiring h type human capital invest more in education and have fewer children.[23] For any set of market wages and fix cost of education, $\left\{w_t^h, w_t^l, \underline{e} > 0\right\}$, an individual endowed with ability a and life expectancy T_t optimally chooses a

[23] This differential fertility behavior is well documented. See de la Croix and Doepke (2003) for a theoretical investigation of the role of differential fertility in explaining the timing of the demographic transition.

longer education period $e_t^{h*} > e_t^{l*}$ and a lower fertility $n_t^{h*} < n_t^{l*}$ if he decides to acquire human capital h as compared to human capital l.

The previous results illustrate the direct roles of the different dimensions of health in affecting directly both education and fertility. In order to investigate the total effect of T_t and π_t, however, we need to consider the optimal choice of the type of human capital acquired. This choice depends, as shown below, on the level of wages which are determined in general equilibrium on the labor markets and depend on the aggregate supply of both types of human capital.

In order to characterize the equilibrium vector $\{j^*, e_t^{j*}, n_t^{j*}\}$ we now turn attention to the individual problem of choosing the type of education, l or h. Given the level of wages, this choice depends on which type of education promises the higher lifetime utility, that is

$$u^{l*}\left(e_t^{l*}, n_t^{l*}, a, w_t^l\right) \gtrless u^{h*}\left(e_t^{h*}, n_t^{h*}, a, w_t^h\right) . \tag{14}$$

Using e_t^{l*} and e_t^{h*} from equations (10) and (13) to determine the optimal supplies of either type of human capital in (3) and (2), respectively, one obtains the respective levels of human capital in the two cases,

$$l_t^*\left(T_t, a\right) = \frac{(1-\gamma)}{(2-\gamma)}\beta(s)T_t \quad \text{and} \quad h_t^*\left(T_t, a\right) = \alpha(s)\frac{(1-\gamma)}{(2-\gamma)}\left(T_t - \underline{e}\right)a . \tag{15}$$

Comparing the respective indirect utilities from investment in h and l and rewriting condition (14) we have,

$$\begin{aligned} u_t^{l*} &\gtrless u_t^{h*} \\ \left(w_t^l \beta(s) T_t^2\right)^{1-\gamma}\left(\frac{T_t \pi_t}{b + r(\pi_t)}\right)^{\gamma} &\gtrless \left(w_t^h \alpha(s) a \left(T_t - \underline{e}\right)^2\right)^{1-\gamma}\left(\frac{\left(T_t - \underline{e}\right)\pi_t}{b + r(\pi_t)}\right)^{\gamma} . \end{aligned}$$

While the utility from low skilled human capital l on the left is independent of ability, the utility from high skilled human capital h on the right is monotonically increasing in a. Agents with higher ability therefore have a comparative advantage in the acquiring h, and the lifetime utility for those investing in h increases monotonically in the ability parameter. For any individual of ability a, there is a unique time investment and level of fertility which maximizes lifetime utility from any type of human capital. The intra-generational allocation of time and ability resources on the two types of human capital, and consequently also fertility behavior, is fully characterized by a unique threshold ability that splits the population. For every vector of wage rates there is only one level of ability $\widetilde{a}_t$ for which the indirect utilities are equal. Due to the monotonicity of u_t^{h*} in ability, all agents with $a < \widetilde{a}_t$ optimally choose to acquire p, while those with ability $a > \widetilde{a}$ optimally choose to obtain h. This ability threshold $\widetilde{a}_t$ is implicitly characterized by

$$\left(\frac{w_t^l}{w_t^h}\frac{\beta(s)}{\alpha(s)}\frac{1}{\widetilde{a}_t}\left(\frac{T_t}{T_t - \underline{e}}\right)^2\right)^{1-\gamma} = \left(\frac{T_t - \underline{e}}{T_t}\right)^{\gamma} . \tag{16}$$

In order to be able to determine the wages, and thus characterize the general equilibrium allocation within a given generation of individuals, we close the model by introducing the production sector .

Aggregate Production and Wages. The unique consumption good is produced with a production technology that uses the stocks of human capital of both types available in the economy at any moment in time, i.e. embodied in all generations alive at that date, as the only factors of production. The production technology is characterized by constant returns to scale, and different intensities with which they use different types of human capital.[24] We adopt a simple vintage model in which technologies are characterized by total factor productivity and the shares of high skilled and low skilled human capital in production. For simplicity, vintages are generation specific. This means that a given generation t can only operate the respective vintage t.[25] This allows us to concentrate attention to the production process during a given generation when determining the labor market equilibrium. Total production of generation t is given by a CES technology

$$Y_t = A_t \left(x_t L_t^{\eta} + (1 - x_t) H_t^{\eta} \right)^{\frac{1}{\eta}} \ , \tag{17}$$

with $\eta \in (0,1)$ and production shares $x_t = x(A_t) \in (0,1)\ \forall t$ where $x'(A_t) < 0$. L_t and H_t denote the aggregate levels of the two types of human capital supplied by generation t, where

$$L_t = \int_0^{\widetilde{a}_t} l_t(a) f(a) da \ , \qquad \text{and} \qquad H_t = \int_{\widetilde{a}_t}^1 h_t(a) f(a) da \ .$$

Wage rates are determined in the macroeconomic competitive labor market and equal the respective marginal productivity.[26] Wages are therefore given by

$$w_t^h = \frac{\partial Y_t}{\partial H_t} \ , \text{ and } w_t^l = \frac{\partial Y_t}{\partial L_t} \ . \tag{18}$$

The corresponding ratio of instantaneous wage rates is then given by: [27]

$$\frac{w_t^l}{w_t^h} = \frac{x_t}{1 - x_t} \frac{H_t^{1-\eta}}{L_t^{1-\eta}} \ . \tag{19}$$

2.3 Intra-generational Equilibrium

We now solve for the equilibrium allocation of the economy for a given generation of adult individuals. An intragenerational equilibrium is defined as follows:

Definition 1. [Intragenerational Equilibrium] *An intragenerational equilibrium for generation t of adult individuals is a vector:*

$$\left\{ \lambda_t := 1 - \widetilde{a}_t^*, e_t^{h*}, e_t^{l*}, n_t^{h*}, n_t^{l*}, \{h^*(\widetilde{a}_t^*)\}, \{l^*(\widetilde{a}_t)\}, H_t^*, L_t^*, w_t^{h*}, w_t^{l*} \right\}$$

such that, for any given $\{T_t, \pi_t, A_t, x_t\}$, the individual choices of human capital investments and fertility, (10) *and* (13)*, the implied optimal individual levels of human capital given by* (15)*, the corresponding population structure defined by the threshold $\widetilde{a}_t$ in condition* (16)*, and the resulting aggregate levels of human capital and wages in* (19) *are mutually consistent.*

[24] The focus of the paper is not on the macroeconomic role of demand for different consumption goods. The role of different income elasticities for different goods for structural change from agriculture to industry has been studied by Laitner (2000).

[25] Human capital is inherently heterogenous across generations, because individuals acquire it in an environment characterized by the availability of different vintages of technologies. Human capital acquired by agents of a generation allows them to use technologies up to the latest available vintage. This implies that a generation's stock of human capital of either type is not a perfect substitute of that acquired by older or younger generations, and is sold at its own price.

[26] Empirical evidence supports the view that different sectors competed for labor, and wage payments reflected productivity even at early stages of industrial development, see e.g. Magnac and Postel-Vinay (1997).

[27] Decreasing marginal productivity of human capital of any type and Inada conditions insure interior equilibria, but they are not necessary to obtain the key results.

The equilibrium threshold $\widetilde{a}_t$ determines the fractions of the population of adults of a given generation that acquire low and high skilled human capital. For notational convenience, we denote these fractions in the following as

$$\lambda_t = \lambda(\widetilde{a}_t^*) := \int_{\widetilde{a}_t^*}^{1} f(a)da = (1 - \widetilde{a}_t^*) \quad \text{and} \quad (1 - \lambda_t) = (1 - \lambda(\widetilde{a}_t^*)) := \int_0^{\widetilde{a}_t} f(a)da = \widetilde{a}_t^* \, .$$

The solution of the individual decision problem in Section 2.2 has shown that, for each ratio of wages w_t^p/w_t^h, there exists a unique ability threshold $\widetilde{a}_t$ that splits the population such that individuals with higher ability acquire human capital of type h, while individuals with lower ability acquire p. Moreover, from condition (19) in the last section we know that the wage ratio is determined by the relevant aggregate levels of human capital available in the economy for a given generation that arise from the equilibrium allocation of adults to the two types of human capital. Since for a given T_t the wage ratio is decreasing in the unique ability threshold, there exists a unique intragenerational equilibrium that can be characterized as follows. Substituting for the optimal human capital supplies and the resulting wages into condition (16), the intragenerational equilibrium can implicitly be characterized by

$$\left(\frac{T_t - e}{T_t}\right)^{1-\eta(1+\gamma)} = \left[\left(\frac{\beta(s)}{\alpha(s)}\right)^{\eta} \frac{x_t}{1 - x_t} 2^{-2(1-\eta)} \frac{(1 - \widetilde{a*}_t^2)^{\eta}}{\widetilde{a*}_t^{1+\eta}}\right]^{1-\gamma} . \tag{20}$$

After some manipulations, one can solve for T_t as an implicit function of $\widetilde{a}_t^*$, the ability cut-off separating the population into individuals acquiring practical and theoretical human capital, respectively, as

$$T_t = \frac{e}{1 - \Omega g(\widetilde{a}_t^*)} \, , \tag{21}$$

where $g(\widetilde{a}_t^*) = \left[\frac{(1-\widetilde{a*}_t^2)^{\eta}}{\widetilde{a*}_t^{1+\eta}}\right]^{\frac{1-\gamma}{1-\eta(1+\gamma)}}$ and $\Omega = \left[\left(\frac{\beta(s)}{\alpha(s)}\right)^{\eta} \frac{x_t}{1-x_t} 2^{-2(1-\eta)}\right]^{\frac{1-\gamma}{1-\eta(1+\gamma)}}$. The discussion so far implies that the problem of determining the equilibrium vector is well defined. An equilibrium defines an implicit function in $(\widetilde{a}_t^*, T_t)$ linking the equilibrium cut-off level of ability $\widetilde{a}_t^*$ to lifetime duration T_t, as in condition (21). Moreover, all variables characterizing the equilibrium human capital formation of each generation are uniquely identified, since the implicit function relating the cut off $\widetilde{a}_t^*$ to life expectancy T_t is monotonically decreasing in T_t. Moreover, the relationship exhibits (at most) one inflection point. We can therefore summarize our findings in the following

Proposition 1. [Intragenerational Equilibrium] *For any generation t with $T_t \in [e, \infty)$, the equilibrium fraction of individuals acquiring human capital, $\lambda_t(T)$, is implicitly defined by* (20) *and satisfies Definition 1. This fraction is unique; is an increasing, S-shaped function of expected lifetime duration T_t, with zero slope for $T \longrightarrow 0$ and $T \longrightarrow \infty$, and exactly one inflection point; is increasing in the level of technological development, A; and is bounded from above by $\underline{\lambda}_t(A) < 1$ with $\frac{\partial \lambda_t(A)}{\partial A} > 0$, $\lim_{A \to 0} \underline{\lambda}_t(A) = 0$, $\lim_{A \to \infty} \underline{\lambda}_t(A) = 1$.*

The S-shape relation between life expectancy T and the fraction of population acquiring p, $\widetilde{a}_t^*$, is a first central result, and generalizes the results presented in Cervellati and Sunde (2005). The higher the life expectancy, the more people will invest in the time-consuming human capital acquisition of h in equilibrium. However, this relation is stronger and more pronounced for intermediate values of T and $\widetilde{a}_t^*$. For low levels of life expectancy, the share of population investing in h is small due to the fix cost involved with acquiring h, which prevents a large part of the population to receive sufficient lifetime earnings to be worth the effort. The larger the fix cost, the more pronounced is the concavity of the equilibrium locus. In this situation, it takes sufficiently large increases in life expectancy to incentivate a significant fraction of individuals to switch from l-acquisition to h-acquisition. On the other hand, when the ability threshold is very low, and a substantial share of the population is engaged in h, very large increases in T are necessary to make even more individuals acquire h instead of l: Due to decreasing returns in both sectors, the relative wage w_h/w_l is low when only few individuals decide to invest in l. This dampens the attractiveness of investing in h for the individuals with low ability, even though life expectancy is very high, rendering the equilibrium locus convex.

The equilibrium split of the generation into h- and l-skilled individuals, determines the average fertility. In particular, the fertility behavior by h- and l-skilled individuals given by the optimal individual decisions (10) and (13) implies an overall average *gross* fertility rate of

$$n_t^*(T_t) = (1-\lambda_t)n_t^{l*} + \lambda_t n_t^{h*} = (1-\lambda_t)\frac{\gamma}{2-\gamma}\frac{T_t}{b+r(\pi_t)} + \lambda_t \frac{\gamma}{1-\gamma}\frac{(T_t-\underline{e})}{b+r(\pi_t)} \,. \tag{22}$$

Taking child mortality into account, the net fertility rate, and therefore the size of the next generation of adults is given by $n_t^*(T_t)\pi_t$. The size of the adult generation t is therefore given by $N_{t+1} = N_t(1+n_t\pi_t)$.[28]

As a result the health status affects gross and net fertility in the population both directly, by inducing a change in the optimal choice of education time and individual fertility, and indirectly by inducing agents to acquire different types of human capital. Concerning the role of longevity these results are summarized in,

Proposition 2. [Effects of Adult Longevity] *For any given π_t, a longer adult longevity T_t implies: (1) An increase in the optimal education time for all types of human capital:*

$$\frac{\partial e_t^{l*}}{\partial T_t} > 0 \qquad \text{and} \qquad \frac{\partial e_t^{p*}}{\partial T_t} > 0 \,. \tag{23}$$

(2) An increase in fertility for any type of acquired human capital:

$$\frac{\partial n_t^{l*}}{\partial T_t} > 0 \qquad \text{and} \qquad \frac{\partial n_t^{p*}}{\partial T_t} > 0 \,. \tag{24}$$

(3) An increase in the fraction of individuals acquiring h type human capital:

$$\frac{\partial \lambda(\widetilde{a}_t^*)}{\partial T_t} > 0 \,, \tag{25}$$

which implies an ambiguous effect on total gross fertility,

$$\frac{\partial n_t}{\partial T_t} = \frac{\partial n_t^l}{\partial T_t} + \frac{\partial \lambda_t}{\partial T_t}\left(n_t^h - n_t^l\right) \gtrless 0 \Leftrightarrow \frac{\partial \lambda_t}{\partial T_t} \lessgtr \frac{1}{\underline{e}} \,. \tag{26}$$

Adult longevity thus induces all individuals to spend more time on education. The partial equilibrium effect (income effect) of adult longevity on fertility induces all agents to increase the number of offspring, taking child mortality as given. The general equilibrium additionally implies a shift toward acquisition of h type human capital, which tends to reduce average fertility through the differential fertility effect. The first term in condition (26) reflects the income effect, which is always positive and does not depend on T_t, while the second effect reflects the differential fertility effect, which is always negative. The differential fertility effect depends on T_t and is

[28]To allow for more realism, below we consider the possibility that not all surviving adults are able to reproduce themselves, where this reproduction probability depends positively on the general health status s of the society. Then $N_{t+1} = N_t(1+n_t\pi_t\zeta(s))$ with $\zeta(s) \in (0,1]$ and $\zeta'(s) > 0$. In what follows, we simply assume that $\zeta(s) = 1\ \forall s$.

particularly strong for intermediate values of T because of the s-shape relationship between λ and T. If the latter effect is sufficiently small, in particular for small or large values of T (for which $\partial\lambda_t/\partial L_t$is small), the income effect implies increasing gross fertility as longevity increases. However, if the change in population structure is sufficiently pronounced, i.e. the relationship between λ and T is strong, average gross fertility actually declines if adult longevity increases. Note that a similar result holds for net fertility rates, taking child survival π_t as given.

Analogous results concerning the role of child mortality are summarized in

Proposition 3. [Effects of Child Survival Probability] *For any T_t a larger survival probability of children π_t implies: (1) A reduction in fertility for any type of acquired human capital:*

$$\frac{\partial n_t^{l*}}{\partial \pi_t} < 0 \qquad \textit{and} \qquad \frac{\partial n_t^{p*}}{\partial \pi_t} < 0\,. \tag{27}$$

(2) A reduction in total gross fertility,

$$\frac{\partial n_t^{*}}{\partial \pi_t} < 0\,. \tag{28}$$

(3) An ambiguous effect on net fertility,

$$\frac{\partial\left[n_t^{*}\pi_t\right]}{\partial \pi_t} = \frac{\partial n_t^{*}(T_t)}{\partial \pi_t}\pi_t + n_t^{*}(T_t) \gtrless 0\,. \tag{29}$$

Gross fertility of individuals, regardless of their education, is negatively related to child survival rates. This is the substitution effect, since children become more costly the more of them survive until adulthood. Net fertility, on the other hand, may increase or decrease with child survival probability. This effect depends on whether the negative gross fertility effect of an increase in child survival, the first term of condition (29), outweighs or not the positive effect of an increase in child survival on the number of surviving children, the second term of condition (29). Technically, this condition is equivalent to $\frac{\partial(n_t^*\pi_t)}{\partial\pi_t} \lesseqgtr 0 \Leftrightarrow 1 \lesseqgtr \frac{\pi_t r'(\pi_t)}{b+r(\pi_t)}$. That is, if the costs of raising children, $r(\pi_t)$ increase strongly in child survival, then net fertility actually falls in π when adult longevity T is high, i.e. the substitution effect dominates.

This discussion concludes the analysis of the economy for a given generation t. We now turn to the dynamics of the economy over the course of generations in order to investigate the mutual interactions between the process of economic development, the acquisition of human capital and the various dimensions of health.[29]

3 Dynamic Interactions

Each generation t of individuals takes adult longevity T_t, the survival probability of children π_t and the level of technological advancement, as expressed by A_t and $x(A_t)$ as given. Before being able to analyze the dynamics of the economy, we introduce the dynamics of these state variables across generations.

Adult Longevity and Child Mortality. The theoretical analysis so far implies quite distinct roles of different dimension of life expectancy. In particular, while adult longevity is a

[29]For illustration purposes, we postpone the discussion of the dynamic evolution of the third dimension of health s affecting the effectiveness of education for the acquisition of human capital, that is $\alpha(s)$ and $\beta(s)$. While, as seen below, adding this dimension of health provides additional insights, the main results to be discussed next remain unchanged.

key determinant for human capital acquisition, the probability that children survive until adulthood primarily affects fertility behavior. The empirical evidence discussed in the introduction also suggests somewhat different determinants for these two distinct concepts of life expectancy. Child survival seems primarily determined by the possibility to avoid diseases, that is the availability of adequate and sufficient nourishment and an environment that prevents or facilitates infectious diseases. Adult longevity, in turn, appears to rather depend on the ability to cure diseases, that is the level of medical knowledge, the availability of surgery and other medical treatments that allow to repair physical damage and extend the aging process. Reis-Soares (2005) reports evidence that suggests that adult longevity is barely affected by improvements in income or nutrition, but is rather related to 'structural' factors that depend on the knowledge available in a society. Ample empirical evidence suggests that better knowledge about diseases and better technological conditions as well as public policies help to avoid or cure them, thereby reducing mortality (see Mokyr, 1993, Schultz, 1993 and 1999, Easterlin, 1999). Empirical findings also suggest that income, wealth and particularly the level of education affect mortality and health, see Mirovsky and Ross (1998) and Smith (1999), and that a better educated society also invents and adopts more and better drugs (Lichtenberg, 1998, 2002, 2003). Taken together, the evidence implies that the level of knowledge and the amount of human capital is relatively more important for adult longevity than the level of development per se (reflected by, for example, the level of per capita income). In light of this evidence, we make the extreme assumption that adult life expectancy T_t increases in the amount of human capital embodied in a generation, reflecting its level of knowledge.[30] We formalize this positive externality by linking a generation t's life expectancy to the fraction of population the previous generation $t-1$ that acquired human capital of type h:

$$T_t = \Upsilon(\widetilde{a}^*_{t-1}) = \underline{T} + \rho \lambda^*_{t-1} \ , \tag{30}$$

where $\rho > 0$ is a parameter describing the strength of the positive externality. This formulation implies that the positive link and the dynamic process does not rely on scale effects. Because of the definition of $\widetilde{a}^*$ as a fraction, the lifetime duration is bounded from above and thus cannot be increased beyond a certain level. We take this biological limit to extending life expectancy as a commonly agreed empirical regularity (see also Vaupel, 1998). The minimum lifetime duration without any human capital of type h is given by $\underline{T}$. The functional form of this relation entails no consequences for the main results, and a (potentially more intuitive) concave relationship would not change the main argument.

The probability to survive childhood, in turn, is assumed to be a function of the level of development at the time of birth of children only. This is in line with empirical findings that suggest that higher incomes, public health expenditures, but also access to electricity or vaccines, increases the probability of children to survive to adulthood, see e.g. Wang (2003) for a recent survey. Capturing these findings, π is modelled as a function of per capita income, implicitly assuming that income is crucial in generating an environment that reduces child mortality. This link is formalized as

$$\pi_t = \Pi\left(\frac{Y_{t-1}}{N_{t-1}}\right) \ , \tag{31}$$

[30]Of course, in reality individuals can influence their life expectancy by a healthy life style, smoking habits, drug and alcohol consumption, sports and fitness behavior health care expenditures etc. Our model reflects the view that individuals lacked a detailed knowledge about which factors and activities are healthy or detrimental for average life duration during early phases of development. Moreover, beneficial factors, such as leisure, were simply not available. An explicit consideration of a positive correlation between life expectancy and the level of education would reinforce the results.

where $\Pi'(\cdot) > 0$.[31] While a larger total income Y_{t-1} in the population improves the probability of children reaching adulthood, this formulation also implies a Malthusian element since a larger population size N_{t-1} decreases living conditions and therefore child survival rates.[32] Equivalently for our results, life expectancy and child survival probability could be related to average or total human capital or total income of the previous generation(s).[33] If one accepts a positive effect of the level of human capital on aggregate income, our assumptions are also consistent with evidence indicating that the aggregate income share spent on health care increases with aggregate income levels.[34] Note that improvements in adult longevity and child survival chances involve no scale effects.[35]

Technological Progress. The second dynamic element is the endogenous evolution of the production technology. Technological improvements occur with the birth of a new generation of individuals. Two features of the specification of technological progress are crucial for our results. First, human capital is the engine of growth through an externality working towards higher productivity along the lines of Lucas (1988) and Romer (1990). New technological vintages are characterized by larger TFP A. Secondly, human capital induces a non-neutral technological process, as studied e.g. by Nelson and Phelps (1966), Acemoglu (1998), and Galor and Moav (2000), among others. In particular, technological progress is biased towards high-skill intensive production and depends on the stock of human capital already available in the economy. Empirical evidence, provided e.g. by Doms, Dunne, and Troske (1997) supports this feature. These assumptions imply that the more individuals of a generation acquire skilled human capital the more attractive is the accumulation of skilled human capital for future generations. Using a simple vintage representation, advances in technology embodied in the latest vintage evolve according to:

$$\frac{A_t - A_{t-1}}{A_{t-1}} = F\left(\widetilde{a}_t^*, A_{t-1}\right) = \delta H_{t-1}(\lambda_{t-1}) A_{t-1} \,, \tag{32}$$

where $\delta > 0$.[36] In order to incorporate the feature of a skill biased technical change we assume that the relative productivity of low-skilled human capital in production, x decreases with the

[31]In the illustrative simulations of the model presented below, we assume that

$$\pi_t = 1 - \frac{1 - \underline{\pi}}{1 + (my_{t-1})^{\mu}}$$

with $y_{t-1} = Y_{t-1}/N_{t-1}$, $m > 0$ and $\underline{\pi} \in (0, 1)$ being the baseline survival in a non-developed society, in order to ensure that π_t is bounded between zero and one.

[32]Considerable evidence documents the negative effect of population density and urbanization on child mortality, especially during the first stages of the demographic transition, see e.g. Galor (2005). The relevance of population density and potential alternative, more explicit effects of density for health and mortality are discussed in the extensions below.

[33]See Tamura (2002) and Boucekkine, de la Croix, and Licandro (2002), or Blackburn and Cipriani (2002) for papers taking this approach.

[34]See Getzen (2000) and Gerdtham and Jönsson (2000) and the references there for respective evidence.

[35]The differential role of human capital and income per capita for child mortality and adult longevity has potentially important dynamic implications as discussed below. While the extreme assumption is made for simplicity, all results are unchanged as long as knowledge is relatively more important for adults longevity.

[36]While highlighting the role of human capital for technological progress, the specific functional form of this relationship has little impact. Any specification implying a positive correlation between technological progress $(A_t - A_{t-1})/A_{t-1}$ and H_{t-1} would yield qualitatively identical results. In the simulations below, we adopt Jones' (2001) specification, which is a generalization of the original contribution of Romer (1990) allowing for decreasing returns,

$$A_t = \left(\delta H_{t-1}^{\psi} A_{t-1}^{\phi} + 1\right) A_{t-1} \,,$$

where $\delta > 0$, $\psi > 0$, and $\phi > 0$. As will become clearer below, assuming exogenous technical change would be equivalent for the main results of the model. The only consequence of assuming exogenous advances would be a missing reinforcing feedback effect as the economy develops.

level of technological advancement,

$$x_t = X(A_t) \text{ with } \frac{\partial X(A_t)}{\partial A_t} < 0 \tag{33}$$

Note that there are no scale effects involved in the specification of technological progress. The crucial relation is between the level of development and the fraction of the previous generation of adults investing in skilled human capital.[37]

The process of development of the economy, and in particular the economic and demographic transition, can be characterized as an interplay of individually rational behavior and macroeconomic externalities, as we now show.

4 The Process of Economic Development

4.1 Steady State Equilibria

The global dynamics of the economy are fully described by the trajectories of the fractions of the population acquiring either type of human capital, characterized by $\lambda := (1 - \widetilde{a}_t^*)$, adult longevity, T_t, the probability of children surviving to adulthood, π_t, the level of technological development, A_t, and the respective production shares of human capital, x_t. For notational simplicity, in the following we denote $\widetilde{a}^*$ simply as a. The dynamic path is fully described by the infinite sequence $\{a_t, T_t, \pi_t, A_t, x_t\}_{t\in[0,\infty)}$, resulting from the evolution of the five dimensional, nonlinear first-order dynamic system derived from equations (21), (30), (31), (32) and (33):

$$\begin{cases} a_t &= \Lambda(T_t, A_t) \\ T_t &= \Upsilon(a_{t-1}) \\ \pi_t &= \Pi(a_{t-1}, T_{t-1}, A_{t-1}) \\ A_t &= F(a_{t-1}, T_{t-1}, A_{t-1}) \\ x_t &= X(A_t) \ . \end{cases} \tag{34}$$

For illustrative purposes, we analyze the behavior of the economy by looking at the dynamic adjustment of human capital and lifetime duration conditional on the value of relative productivity. Also note that the intra-generational equilibrium does not depend on the mortality of children, π, as is illustrated by condition (21). We can therefore illustrate the dynamics of the economy by first considering the reduced conditional system:

$$\begin{cases} \lambda_t := (1 - a_t) &= \Lambda(T_t, A) \\ T_t &= \Upsilon(a_{t-1}) \end{cases} \tag{35}$$

which delivers the dynamics of human capital formation and life expectancy for any given level of technology $A > 0$. From the previous discussion we know that the first equation of the conditional system is defined for $T \in [e, \infty)$.

Denote the S-shaped locus $T_t = \Lambda(\lambda_t, A)$ in the space $\{T, a\}$, which results from the intra-generational equilibrium reflected by (21), by $\Lambda(A,)$, and the locus $T_t = \Upsilon(a_{t-1})$ representing the intergenerational externality on lifetime duration by Υ. Any steady state of the conditional system is characterized by the intersection of the two loci $\Lambda(A)$ and Υ:

Definition 2. [Steady State Equilibrium of the Conditional Dynamic System] *A steady state equilibrium of the dynamic system* (35) *is a vector* $E^z(A) \equiv \{\lambda_A^z, T_A^z\}$ *with* $\lambda_A^z \in [0, 1]$ *and* $T_A^z \in [e, \infty)$*, such that, for any* $A \in (0, \infty)$*:* $\lambda_A^z = \Lambda^{-1}(T_A^z, A)$ *and* $T_A^z = \Upsilon(\lambda_A^z)$*. The associated equilibrium aggregate levels of unskilled and skilled labor are denoted by* P_A^z *and* H_A^z*.*

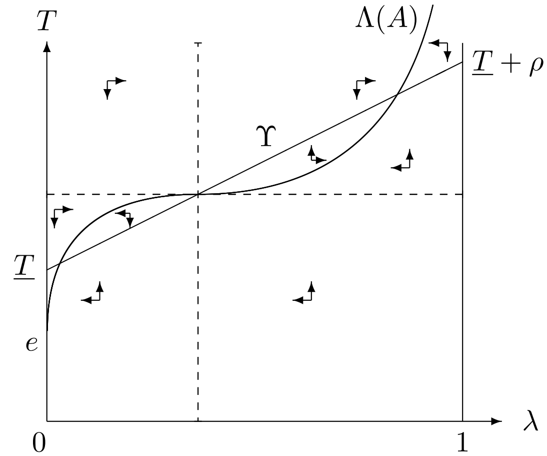


Figure 2: Phase Diagram of the Conditional Dynamic System

As a consequence of the s-shaped pattern of Λ, the system (35) displays at least one, and at most three steady state equilibria $E^z(A)$ denoted by $z = S, u, G$ with different properties. Figure 2 illustrates the system (35) in the case when three equilibria exist. Equilibria of type $E^u(A)$ are characterized by an intersection of the Υ-locus with the Λ-locus from below for intermediate levels of λ and T. These equilibria are unstable, so in the following we only consider equilibria of type S and G. The S-type equilibrium describes the situation of an economy in a nearly stagnant environment. Adult life expectancy T_A^S is low in such an equilibrium, and only a small share λ_A^S of the population acquires human capital of type h. An equilibrium of type G, on the other hand, is characterized by a large adult longevity T_A^G and a large fraction λ_A^G of the current adult population acquiring growth-enhancing human capital. Note that in any steady state equilibrium, the supply of both types of human capital is strictly positive. The set of equilibria $E^z(A) \equiv \{\lambda_A^z, T_A^z\}$ can be characterized as follows:

Proposition 4. [Properties of Steady State Equilibria] *For any $A \in (0, \infty)$, $a \in (\underline{a}(A), 1)$, and $T \in (\underline{e}, \infty)$, the conditional dynamic system* (35) *has steady state equilibria with the following properties: (1) There exists least one steady state equilibrium; if it is unique, it is globally stable; (2) there are at most three steady states denoted by $E^S(A)$, $E^u(A)$, and $E^G(A)$ with the following properties: (i) $\lambda_A^G \geq \lambda_A^u \geq \lambda_A^S$ and $T_A^G \geq T_A^u \geq T_A^S$; (ii) E_A^G and E_A^S are locally stable; (iii) E_A^u is unstable; (iv) in any steady state equilibrium $H_A^z > 0$ and $L_A^z > 0$ for $z = S, u, G$.*

We now turn to the analysis of the dynamic equilibrium of the economy.

4.2 The Economic Transition.

The analysis of the full dynamic system must account for the evolution of all variables of interest. Human capital h helps in adopting new ideas and technologies, and thus creates higher productivity gains than practical human capital p. This means that in the long run relative productivity A_t will tend to increase.

Lemma 1. *The technology index A_t increases monotonically over generations with $\lim_{t\longrightarrow\infty} A_t = +\infty$.*

The strict monotonicity of A_t over generations depends on the assumption that A is monotonically increasing as generations pass. However, this assumption is not necessary for the main argument. What is crucial is that productivity will eventually be increasing once a sufficiently

[37] In the simulations below we adopt the simple formulation $x_t = A_0/A_t$.

large fraction of the population acquires h. As A_t increases, the fraction of the population investing in h also increases. The levels of life expectancy necessary to make an agent of ability a indifferent between acquiring either types of human capital tend to decrease and the locus $HH(A)$ shifts down for any a (excluding the extremes):

Lemma 2. *The life expectancy required for any given level of ability to be indifferent between acquiring h or p decreases, as relative productivity A increases:* $\left.\frac{\partial T(a,A)}{\partial A}\right|_{\Lambda(A)} < 0, \ \forall\, a \in (0,1)$.

Thus, the more productive theoretical human capital h becomes relatively to applied human capital p, the less restrictive is the fix cost requirement of acquiring it, as the break-even of the investment in education is attained at a lower age.

The two previous results imply that technological development is monotonous, and that this monotonous development leads to continuous change in the allocation of ability and time towards human capital acquisition. The following result combines these findings with the result on the structure of steady state equilibria in the previous section.

Proposition 5. [Dynamic Emergence of Steady State Equilibria] *Consider an economy with e sufficiently large, so that $\forall A \leq A^0$, the system (35) is characterized by a unique steady state equilibrium of type S. There exist $A^0 < A^1 < A^2 < \infty$ such that the system (35) is characterized by: (i) a unique type-S equilibrium $\forall A_t \leq A^1$; (ii) two steady states $E^u(A^1)$ and $E^S(A^1)$ at $A_t = A^1$; (iii) three steady states: $E^G(A_t)$, $E^u(A_t)$ and $E^S(A_t)$ $\forall A_t \in (A^1, A^2)$; (iv) two steady states $E^G(A^2)$ and $E^u(A^2)$ at $A_t = A^2$; (v) a unique G-type equilibrium $\forall A_t > A^2$.*

With these results, we can characterize the development path of the economy by describing the unique dynamic equilibrium. Consider a non-developed economy in which adult life expectancy is low.[38] Since A is low, investing in h is relatively costly for a large part of the population as the importance of the fix cost for education, $\underline{e}$, is large. This means that the concave part of the $\Lambda(A)$-locus is large and the conditional system is characterized by a unique dynamic equilibrium of type $\left\{\lambda_A^S, T_A^S\right\}$, exhibiting low life expectancy and a small fraction of individuals deciding to acquire theoretical human capital. This situation is depicted in Figure 3(a). During this early stage of development, the feedback effects on adult longevity and productivity are very minor.

As generations pass, productivity growth makes investing in h more profitable for everybody, and adult life expectancy increases slowly. Graphically, the locus $\Lambda(A)$ shifts downwards clockwise as time passes, and the importance of the concave part decreases. After a sufficiently long period of this early stage of sluggish development, $\Lambda(A)$ exhibits a tangency point, and eventually three intersections with Υ. From this generation onwards, in addition to E^S, also steady states of type E^u and E^G emerge. Since the S-type equilibria are locally stable, however, the economy remains trapped in the area of attraction of the S-type equilibria, as depicted Figure 3(b).

As generations pass, the dynamic equilibrium induced by initially low adult life expectancy moves along Υ. The consecutive downward shifts of $\Lambda(A)$, however, eventually lead to a situation in which the initial dynamic equilibrium lies in the tangency of the two curves, as shown in Figure 3(c). In the neighborhood of this tangency, the intragenerational equilibrium locus $\Lambda(A)$ lies below the linear Υ-locus and the equilibrium is not anymore stable. Already the following generation faces an adult longevity that is high enough to induce a substantially larger fraction

[38] As will become clear below, starting from this point is without loss of generality. However, even though the model is also capable of demonstrating the situation of developed economies, the main contribution lies in the illustration of the transition from low to high levels of development.

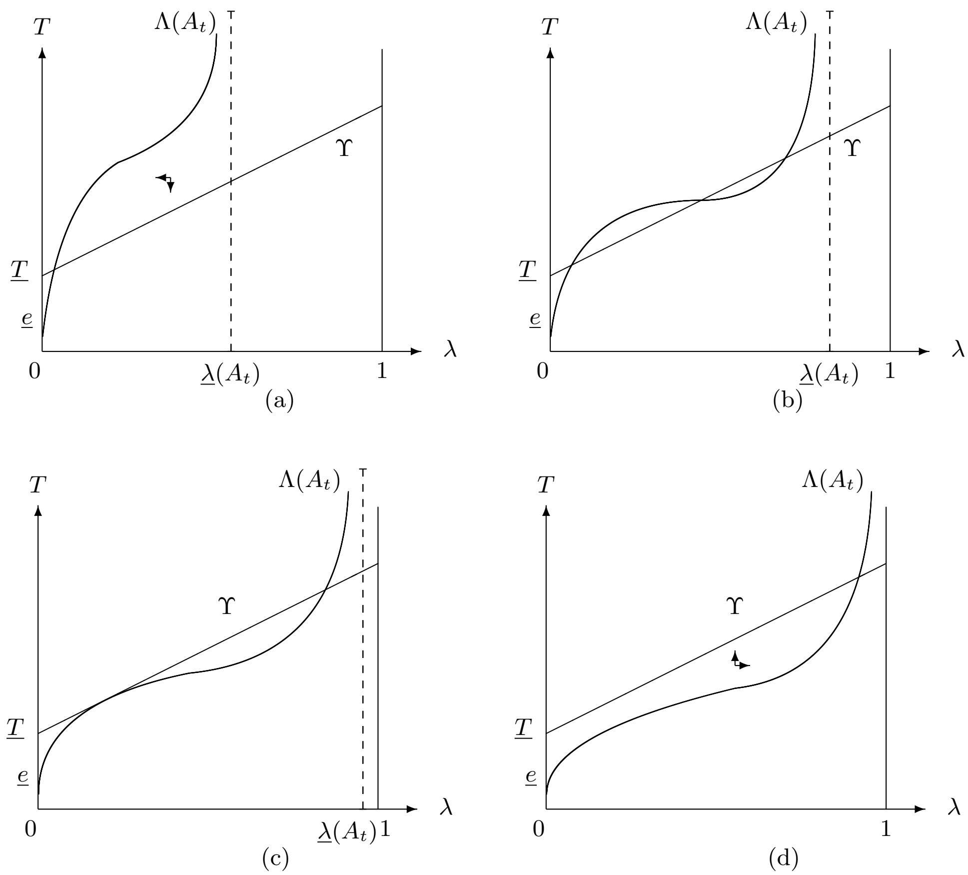


Figure 3: The Process of Development

to acquire human capital h than in the previous generation. At this point a unique E^G steady state emerges, as is shown in Figure 3(d). This triggers a period of rapid development, during which the fraction of population acquiring skilled human capital increases fast within few consecutive generations, and adult life expectancy increases rapidly as consequence of the intergenerational externality. This economic transition towards a path of permanent growth lasts for a few consecutive generations. After this transition, life expectancy converges asymptotically to its (biologically determined) upper bound $\rho + \underline{T}$. Even though the fraction of the population acquiring human capital h keeps growing, there is always some fraction of the population acquiring applied knowledge l, but due to standard endogenous growth mechanisms, economic development remains fast even though changes in adult longevity and human capital structure in the economy abate.

The following proposition summarizes these global dynamics. The evolution of the system is given by the sequence of ability thresholds, life expectancies and productivity levels $\{\lambda_t, T_t, A_t\}_{t\in[0,\infty)}$, starting in a situation of an undeveloped economy:

Proposition 6. [The Economic Transition] *There exists a unique generation t^2 such that for all $t \leq t^2$ the economy is characterized by a sequence of $S-$type equilibria, while for all $t > t^2$ the economy is characterized by a sequence of $G-$type equilibria. This generation t^2 is the first generation for which the level of technology exceeds A^2 as given by Proposition 5.*

It is important to note that the actual trajectory of the system depends on the initial conditions and cannot be precisely identified in general. Proposition 6 in fact states that the system moves generation by generation in the area of attraction of the locally stable steady state

E^S until this steady state disappears, and the system converges to a series of globally stable steady states E^G. In historical terms, the model therefore exemplifies the different stages of development. For example, Europe could be thought of as being trapped in a sequence of E^S equilibria during ancient times and the middle ages. At some point during the late 18th century development took off, as the development trap endogenously vanished.

4.3 The Demographic Transition

After characterizing the economic development of the economy, we now turn to studying the demographic development path in terms of fertility behavior and population dynamics, and the differential roles of child and adult longevity in this process.

In the current framework, population dynamics are characterized by several opposing mechanisms. As child mortality decreases, i.e. the survival probabilities of children increase, adults of either type of education optimally decide to have fewer offspring. This *substitution* effect arises since adults care about the total number of surviving children. In the current framework, there is no uncertainty that would require the hoarding of children and, as discussed above, if only this effect is at work no net fertility drop can be observed in the population following a reduction in child mortality. However, improvements in health conditions also have other effects. Consider again the individually optimal fertility decisions that are implied by the choice to acquire human capital of type l or h, conditions (10) and (13), respectively. All individuals increase their fertility as their adult life expectancy increases, regardless of which type of human capital they decide to acquire. This increase arises due to a standard *income* effect. The joint consideration of these two effects implies that, conditional on education choices, net fertility increases after improvements in living conditions. A third effect arises due to the shift toward the acquisition of h human capital. Conditional on facing the same adult longevity T_t and child survival probability π_t, adults who decide to become high-skilled optimally choose to have fewer children than adults who acquire human capital of type l. This *differential fertility* effect linked to the structural change, gains importance along the development path, since, as has been shown before, eventually the population structure shifts from acquisition of l-type human capital to the acquisition of $h-$type human capital as the economic transition takes off. As a result nearly the entire population eventually acquires skilled human capital which is associated to longer education and lower fertility. The joint consideration of three effects implies a non monotonic evolution of fertility and education and can rationalize the drop in net fertility rates after an initial increase in both gross and net fertility.

To characterize the overall fertility pattern, consider the two extremes of the development process described before. At the beginning of the development process, steady state equilibria are characterized by low adult longevity, high child mortality, and very few adults acquiring skilled human capital h. The alternative scenario is given by the conditions in developed economies with adult longevity approaching the biological upper limit, marginal child mortality and almost all adults acquiring h-type human capital. Denote total fertility in the population in the former scenario of a steady state E^S where virtually the entire population acquires l-type human capital, $\lambda_0 \approx 0$, as n_0. Total fertility under these conditions approximately corresponds to the fertility given by condition (10). In contrast, under the second scenario $\lambda_\infty \approx 1$, such that total fertility is approximately given by condition (13). The resulting total fertility is denoted by n_∞, and the gross fertility rates in the population in the two limits are given by,

$$n_0 = \frac{\underline{T}}{b + r(\underline{\pi})} \qquad \text{and} \qquad n_\infty = \frac{\overline{T} - \underline{e}}{b + r(1)} \, . \tag{36}$$

In terms of net fertility rates the condition for a drop in net fertility from the beginning until the end of the development process is then

$$n_0\pi_0 = \frac{\underline{T}\pi_0}{b + r(\underline{\pi})} > \frac{\overline{T} - \underline{e}}{b + r(1)} = n_\infty \, . \tag{37}$$

Since this paper is concerned with showing how the process of development can generate a fall in net fertility, in the following we restrict attention to specifications that in principle allow for such episodes, assuming that condition (37) is satisfied. As discussed above, a decline in net fertility can be observed in the model even in the absence of precautionary demand for children if the reduction in fertility associated with the switch from l to h type human capital acquisition is large enough. We can make this notion more precise. Note first that as a consequence of its definition and the fact that child survival increases monotonically along the development path, average net fertility $n_t\pi_t$ can only decline if average gross fertility n_t declines. For gross fertility to decline, the scope for increases in adult longevity must be limited, the fix time cost for education attainment must be sufficiently high, and the potential to increase in child survival probability, together with the associated increase in the time cost implied with bringing up surviving children, must be sufficiently high. These conditions are closely in line with historical data. For example, child mortality in England and Wales fell substantially from around 20 percent in the period 1550-1600 to less than 0.5 percent at the end of the 20th century. Adult longevity measured by life expectancy at the age of 30 experienced an increase from around 60 years to around 75 years.[39] Considering that the acquisition of higher education is currently associated to a time investment, $\underline{e}$, from 10 to 15 years this can help rationalize the reduction in fertility.[40] Hence, the model shows that the dramatic and fast changes in human capital acquisition that accompanied the demographic transition can rationalize and explain the substantial decline in gross fertility and the fact that net fertility eventually decreases. These observations are recorded in the following,

Proposition 7. [Gross and Net Fertility in the Process of Development] *(i) Gross fertility eventually declines if the positive income effect from improvements in T is outweighed by the negative substitution effect from increases in π, and the negative differential fertility effect from increases in λ, i.e. if $n_0 > n_\infty$ in condition* (36)*. (ii) Net fertility eventually declines if the negative differential fertility and substitution effects, and the consequential decrease in gross fertility is sufficiently pronounced compared to the increase in child survival probability, i.e. if $n_0\pi_0 > n_\infty\pi_\infty$ in condition* (37)*.*

The current framework therefore offers an explanation that can generate the observed drop in gross and net fertility, which is based on differential fertility choices depending on the education choice. Only in combination with the economic transition can a demographic transition arise, however, indicating the close interrelationship between economic and demographic development. In fact, as will become clearer below, the present framework constitutes a unified model which allows to investigate the endogenous transition from an equilibrium with low adult longevity, high child mortality and high fertility towards an equilibrium with high adult life expectancy, low child mortality and low fertility.[41]

After these quasi-static considerations using limit results, let us now turn to the full dynamic characterization of the demographic transition. The typical pattern of the demographic

[39] Data are from Wrigley and Schofield (1981) and UK national statistics.

[40] In the following section we discuss the relevant data with particular attention to the historical experience of England.

[41] In fact the present framework provides a unified theory which disentangles the role of changes in adult longevity and child mortality. This paper complements the quantity-quality framework used in earlier unified theories, including the seminal work by Galor and Weil (2000).

transition considered in the literature is the following. Development begins with an extended phase characterized by low living standards and low adult longevity, high child mortality and large fertility. The economic and demographic environment barely changes with technological progress being 'glacial' in the words of Galor and Weil (2000). Any increases in longevity (or income for that matter) directly funnel into higher levels of fertility. Eventually, when the economic development picks up momentum, more people become skilled, productivity growth accelerates and living standards improve. In this second phase of development, dubbed the 'Post-Malthusian Regime' by Galor and Weil (2000), adult longevity already increases, child mortality is still high, and fertility increases as incomes grow. Eventually, as child mortality falls and long adult longevity induces large fractions of the population to become skilled, also fertility starts declining, and the economy enters a modern growth path on which income grows while fertility declines to low levels.

From the previous discussion it appears reasonable to assume that condition (37) is satisfied, which we do in the following.[42] Then, recalling the results about the economic transition in Proposition 6 one can characterize the pattern of demographic development as follows:

Proposition 8. [The Demographic Transition] *The path of demographic development exhibits three phases: (i) an extended phase with low child survival and adult longevity, π and T, and high average fertility n (Malthusian equilibrium); this phase lasts until generation t^2; (ii) an intermediate phase with increasing child survival rates and adult longevity, together with an even higher gross fertility rate (post-Malthusian equilibrium); (iii) a phase of increasing child survival rates and adult longevity, together with a low gross and net fertility rates (modern growth equilibrium).*

Consider the dynamics of the economy as depicted in Figure 3. During the early phase of development, when the economy is *dynamically* trapped in the area of attraction of an E^S-type steady state, society is characterized by low levels of adult longevity, and very few adults in the population decide to become skilled. At the same time, living standards are low, and child mortality is high. The economic and demographic environment that is barely changing with technological progress. Any gains in longevity (or income for that matter) directly increase fertility. Larger populations, however, reduce per capita income, which, according to (31) increases child mortality. Thus, the early phase of development exhibits Malthusian features.[43] This situation reflects the first phase of demographic development as stated in the proposition.

As has been shown before, eventually an economic transition is triggered once adult longevity and the technological level provides incentives for a sufficiently large fraction of the population to acquire skilled human capital h. On the one hand, this increases adult longevity of subsequent generations due to the intergenerational externality (30). As a consequence of the income effect described before, this leads to increased fertility. Simultaneously, the aforementioned differential fertility and substitution effects tend to dampen fertility, as the education structure changes towards higher levels of λ, and as income gains associated with larger productivity reduce child mortality due to the external effects implied by (32) and (31), respectively. Which of the effects prevails depends on their relative strength as well as the timing of the delays involved in the intergenerational transmission mechanisms. Dynamically, the fertility reducing effects are likely to be minor at the onset of the economic transition and to be dominated by the income effect. In this case, the model predicts an increase in gross and net fertility during the early stages of the economic transition. This is reflected by phase (ii) of demographic development

[42]It should be noted, however, that qualitatively similar results arise even if (37) does not hold, as will become clear below.

[43]Note that as a result of the intergenerational feedback effects, and the fact that technology growth is incremental, the economy never actually experiences stagnation in the strict sense since there are always improvements although they are negligible. We therefore refrain from calling the early phase a Malthusian regime of stagnancy.

in Proposition 8, and consistent with the second phase of development, dubbed the 'Post-Malthusian Regime' by Galor and Weil (2000). Note, however, that this transitional phase can be very short when convergence to the E^G-steady state is rapid. If the change in human capital structure is sufficiently swift over the course of a few generations, implying a strong differential fertility effect, or likewise if the substitution effect is strong since child survival rates improve strongly with little delay to the economic transition, the Post-Malthusian phase (ii) might be short as society quickly enter phase (iii) when the two fertility-reducing effects of increasing child survival rates and larger importance of differential fertility from investments in $h-$type human capital gain momentum. Thus, under condition (37) gross and net fertility eventually start declining despite adult longevity and per capita income still growing. The economy therefore enters a 'Modern Growth Regime', once more following the classification of Galor and Weil (2000).[44]

The joint consideration of different dimensions of health and the different timing of improvements in these dimensions may help to reconcile the empirical and historical observations with a unified theory based on endogenous life expectancy. Several authors pointed out problems of the existing theoretical literature aiming to explain the demographic transition and the economic take off with changes in life expectancy as the key state variable. As discussed before, the observation of a reduction in net fertility has proven difficult to rationalize in previous models. The joint consideration of income, substitution and differential fertility effects associated with human capital formation provides a framework that allows to generate a decline in fertility in a model with life expectancy as key state variable.

This completes the discussion of the theoretical results. Before presenting simulations of the model that illustrate the working of the model, we briefly discuss the effects when explicitly taking general health status into account.

4.4 The Role of Overall Health for Development

We can now come back to the role of health in terms of the quality of the lifetime available to individuals. As was argued in the beginning, one could think of health not only in terms of child survival probabilities or adult longevity, but also in terms of overall health status and the quality of life. A good health status allows to acquire human capital more efficiently, i.e. in less time, and also implies higher productivity than a bad health status with recurrent illness, as indicated by s in (2) and (3), with $\alpha'(s) \geq 0, \beta'(s) \geq 0$. Hence, changes in the overall health status unfold a differential effect on the productivity to acquire unskilled and skilled human capital.[45] The effect of s is different from both longevity and child mortality. Concerning the process of development larger longevity T_t induces an increases on both fertility and education (the income effect) but also induces a shift toward the acquisition of human capital. Child mortality, in turn, affects fertility via a substitution effect. The key ramification of taking health status s into account becomes obvious when considering the implicit condition for an intra-generational equilibrium (20). The larger the ration $\alpha(s)/\beta(s)$ the larger the fraction of the population which optimally decides to acquire human capital h.[46] This implies that the health status s has a direct impact

[44]Consider the dynamic pattern when condition (37) fails to hold. Even under this scenario, the observed fertility rates experience a marked slow-down due to the transition to a low-fertility high-education equilibrium allocation of large parts of the population, reflected by a E^G steady state.

[45]Available evidence documents how better health increases school attendance, see e.g. Alderman *et al.* (2001), Bleakley (2003), and Miguel and Kremer (2004).

[46]Concerning the intra-generational equilibrium health and technology are iso-morphic. The role played by health a similar role as productivity A, reflected by the production share of skilled human capital, $1 - x(A)$. Improvements in the overall health status have a similar effect on the intra-generational equilibrium locus as do productivity improvements.

on both fertility and human capital via the differential fertility effect. In particular we have,

Proposition 9. [Effects of Changes in Overall Health] *For any $\{T_t, \pi_t, A_t\}$ improvements in health s implies:*

1. *A change in the fraction of h-skilled individuals as,*

$$\frac{\partial \lambda_t^*}{\partial s} \gtrless 0 \Leftrightarrow \frac{\partial \left[\alpha\left(s\right)/\beta\left(s\right)\right]}{\partial s} \gtrless 0 \tag{38}$$

2. *A change in fertility,*

$$\frac{\partial n_t^*}{\partial s} \gtrless 0 \Leftrightarrow \frac{\partial \left[\alpha\left(s\right)/\beta\left(s\right)\right]}{\partial s} \lessgtr 0 \tag{39}$$

The effect of the health status depends on its impact on the relative productivity of the time invested in h education relative to the l. If the productivity of the education process is increasing more for h skilled human capital then health improvements facilitate the acquisition of h and accelerates the economic transition.[47] Dynamically, the effect of health implies a faster or slower transition process and crucially affects the strength of the differential fertility effect.

The overall health condition depends on several factors. Just as adult longevity, it is conceivable that health primarily depends on the possibility to treat and cure diseases, i.e. the state of medical knowledge that is reflected by the share of population of high skilled individuals in the past generation. On the other hand, health status may depend negatively on population density, which facilitates the spreading of infectious disease, so

$$s_t = \mathcal{S}(\lambda_{t-1}, N_t) \quad \text{with} \quad \frac{\partial \mathcal{S}}{\partial \lambda_{t-1}} > 0,\ \frac{\partial \mathcal{S}}{\partial N_t} < 0\,. \tag{40}$$

In this setting, health improvements work towards accelerating the escape from the development trap if
$\partial\left[\alpha\left(s\right)/\beta\left(s\right)\right]/\partial s > 0$ implied by an equilibrium in the area of attraction of an E^S steady state. If population density negatively affects health, however, this slows down the process of health improvement, adding another Malthusian feature to the model.

The main goal of this paper is to provide a first attempt to disentangle the different role of these different dimensions of health for the economic and the demographic transition. Depending on the assumptions one makes about the dynamics of overall health, the model generates development patterns which, despite being qualitatively similar, may differ with respect to the duration of the different phases. If health is modelled as incremental process, reflecting the idea that medical knowledge does build up as generations pass and record their experiences, it can substitute technological improvements as the engine behind the secular transition in the economy. In fact, as simulations of the model for this case show, an economic and demographic transition can arise entirely without technological improvements, just as a result of gradually improving health conditions.

5 Discussion

In this section we provide a brief discussion of the results in light of historical evidence, and then present an illustrative simulation of our model.

[47] Notice that the effect of health on $\alpha\left(s\right)/\beta\left(s\right)$ may depend on s itself being negative for initial improvements (i.e. more favorable to low-skilled human capital l) and positive once s is large enough.

5.1 The Economic and Demographic Transition: Stylized Facts

At the time when the economic transition took off in the late 18th century, mortality fell significantly and average life expectancy at birth as well as at later ages, which had virtually been unchanged for millennia, increased sharply within just a few generations.[48] The improved environment and living conditions associated with better health conditions, lower mortality and overall life expectancy, as well as changing economic demands triggered a more widespread acquisition of human capital. Interestingly, improvements in life expectancy at birth lags adult longevity in terms of life expectancy at age 30. This is an indication that infant mortality began to fall much later than mortality at later ages. Basic education and human capital indicators like average literacy rates, and years spent in formal schooling and year of apprenticeships improved substantially after the take off.[49] Also, along the process of development, fertility increased from high levels, before reproduction rates fell substantially. Nevertheless, population size increased, mostly due to increased life expectancy. The development in these dimensions is illustrated in Figure 4. [50]

The empirical and historical evidence suggests that the acquisition of human capital by part of parents and the increase in the lifetime devoted to education represents an first order determinant of fertility behavior.[51] Simultaneously, or slightly delayed compared to the improvements in health and life expectancy, fertility behavior changed substantially. Fertility rates dropped from high levels in less developed countries to substantially in the context of economic development and improvements in living conditions. This demographic transition typically lags health improvements somewhat, as during the second half of the 19th century, see Galor and Weil (2000), and as in developing countries today. Due to health improvements and despite fertility reductions, the size of population started to increase substantially in European countries. The increase in population size even after the decrease of fertility suggests that the reduction in reproduction is more than compensated by an increase in lifetime duration. Apart from changing overtime, both education and fertility choices differ substantially across the population, however. A large evidence documents that education choices depend on individual living conditions, in particular economic environment, health and life expectancy. Individual fertility decisions in turn depend on individual income and education.[52]

The majority of developed countries also exhibits a timing of the events in the different dimensions of the economic and demographic transition that is in line with the predictions of Proposition 8. In particular, improvements in adult longevity seem to have affected fertility as well as economic growth in a causal sense as has recently been documented by Lorentzen, McMillan, and Wacziarg (2005). Similarly, Reis-Soares (2005) reports that gains in adult longevity preceded the decline in net fertility. Several authors have pointed out, however, that in some historical experiences the timing of the transition is not compatible with the available theories linking mortality and fertility. Doepke (2005) and Galor (2005) cite evidence that the fertility decline started well before the onset of the substantial drop in child mortality in England. At the same time, the changes in fertility display a substantial time lag with respect to the increases in adult longevity at the onset of the Industrial Revolution.[53] While these empirical

[48] This is illustrated by the data provided by Floud and McCloskey (1994).

[49] Extensive evidence on this is reported by Cipolla (1969) and Floud and McCloskey (1994).

[50] Data are from the quoted sources. Life expectancy and fertility data are taken from Wrigley and Schofield (1981), Keyfitz and Flieger (1968) and the websites of the Office of National Statistics (`http://www.statistics.gov.uk`) and the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat (World Population Prospects: The 2004 Revision and World Urbanization Prospects: The 2003 Revision, `http://esa.un.org/unpp`). Missing values are obtained by linear intrapolation.

[51] This is true, *a fortiory*, once considering the increased education of work participation by women.

[52] de la Croix and Doepke (2003) provide theoretical and empirical arguments for which differential fertility affects the occurrence and the timing of the transition.

[53] See also the evidence provided by Boucekkine, de la Croix, and Licandro (2003).

Table 1: Parameter Values Used for Simulation

α	0.95	β	0.05	ρ	25.0	$\underline{\pi}$	0.75
γ	0.5	η	0.4	$\underline{e}$	15.0	$\widetilde{a}(0)$	0.995
ψ	0.42	ϕ	1.25	$\underline{T}$	50.0	$A(0)$	0.75
μ	2	ν	6	$\underline{r}$	1.5	b	1

finding are seemingly puzzling in the current theoretical literature they can be rationalized by considering the different dimensions of adults and child mortality and the possibility that improvements in these different domains where not always contemporaneous. Historically, one can argue that health, adult longevity and child mortality were affected differently by the process of development. Consider again Proposition 8. The duration and extent of the post-malthusian phase crucially depends on the relative strength and timing of the different effects at work. How could one generate a demographic transition with adult longevity improving well before child mortality declines?[54] A delay in the decline of child mortality implies that after the start of the transition the income effect is relatively stronger than the substitution effect. Under these conditions it is likely that gross fertility initially increases, and only declines with a lag once the differential fertility effect associated to the sharp changes in acquisition of human capital becomes sufficiently strong. This can be the case even if the substitution effect is not too strong since child mortality is still large. In the next section we discuss this issue in more detail and show that a decline in gross fertility can be observed even before the large improvement in child mortality (and accordingly of life expectancy at birth).

5.2 An Illustrative Simulation of the Development Path

We simulate the model for 600 generations, with new generations born at a frequency of one year.[55] The parametric specifications used for the intergenerational externalities are given in the text for T in equation (30), for π in footnote 31, for A in footnote 36, and for x in footnote 37, respectively. Parameter values and initial conditions used in the simulation are contained in Table 1. The time costs of surviving children is parameterized as function $r(\pi_t) = \underline{r} \cdot \pi_t^{\nu}$

Marginal productivity of time spent in education, given a specific level of ability, is assumed to be the same in the production of both types of human capital. A maximal life expectancy of 75 years cannot be exceeded, while the minimum adult longevity is assumed to be 60 years. Initial child survival probability is 0.8. All these numbers are roughly in line with historical data from England, where life expectancy at age 30 was around 60 in the 16th century, and mortality of infants between age 0 and 9 was around 220/1000. The fix cost of acquiring theoretical human capital h, $\underline{e}$, is 15 years. Initially, the share of production accruing to L, $x = 0.95$. Clearly, the model is capable of producing a deliberately long stagnancy period before the transition. The main patterns of development generated by the model are depicted in Figure 5. The patterns resemble closely those in the historical data presented before and illustrate the discussion of the main results in section 4. Figure 6 makes clear that the model is capable of generating complex fertility dynamics. In particular, during the demographic transition, fertility

[54] In fact there are several reasons why this could have been the case in the first experiences of demographic transition including the increase in population density and urbanization and the fact that living conditions (and per capita income) improved several decades after the increase in acquisition human capital and medical knowledge.

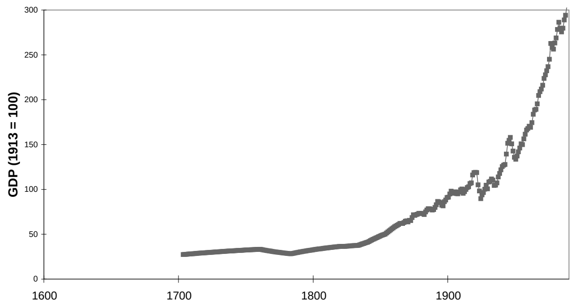
[55] Interpreting every year as the arrival of a new generation, this reflects a horizon from year 1500 to 2100, which includes the period of economic and demographic transition.

intermediately increases due to income effects. Eventually, fertility declines in the later stages of the demographic transition. However, net fertility peaks well after gross fertility has started to decline. This can be explained by the differential timing of the improvements in the different dimensions of health, i.e. adult longevity and child mortality.

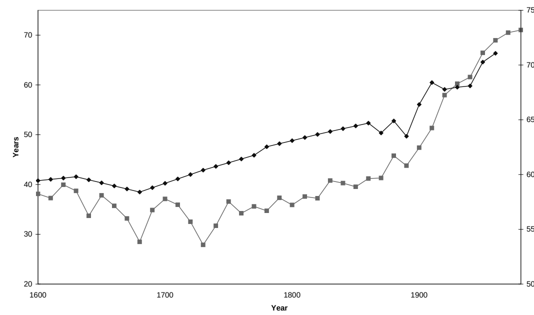
6 Conclusion

This paper provides a unified growth theory that disentangles the role of different dimensions of health for economic development. In particular, the framework distinguishes between the expected length of adult life, which constitutes the primary determinant for education decisions; the probability that children survive childhood, which crucially affects fertility decisions; and overall health status, which determines the overall capability to acquire human capital, and therefore productivity. Taking the different dimensions into account, agents choose optimally which type of human capital to acquire and the lifetime devoted to its acquisition. At the same time, they decide optimally about their fertility and the implicit time cost involved with bringing up children. Optimal choices of time investments in education and fertility behavior depend on the level of development and which education path is chosen. Agents choosing different education profiles exhibit different fertility behavior and spend a different amount of time in child rearing.

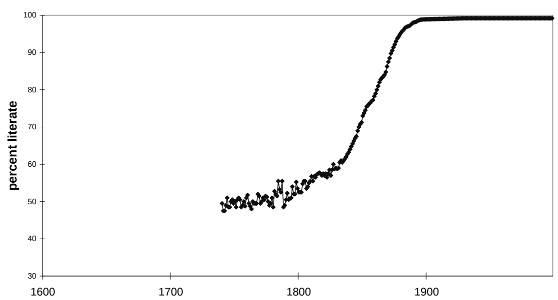
The dynamic equilibrium of the model exhibits patterns that are in line with historical evidence and stylized facts. In particular, the model can rationalize long periods of stagnancy followed by the simultaneous acceleration of income and productivity growth together with improved health conditions, and spreading education levels. The framework presented here generates a demographic transition, which is linked closely to the economic transition from a nearly stagnant to a fast growing environment. Unlike previous models, the separate consideration of different dimensions of health allows to generate a demographic development pattern that accounts for declines in net fertility rates. Moreover, it provides a rationale for the seemingly inconsistent timing of fertility, child mortality and adult longevity that has been observed in the data and proved difficult to rationalize in a model in which life expectancy is the crucial determinant of the economic transition. While the model is solved analytically, simulations of the model illustrate the different mechanisms at work.



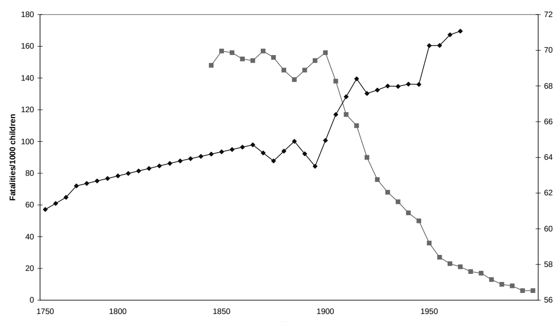
(a) GDP per capita (U.K.)



(b) Life Expectancy at Birth (left axis, lower graph) and at Age 30 (right axis, upper graph) (England and Wales)

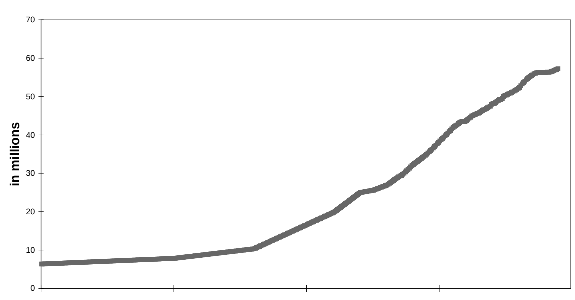


(c) Literacy Levels (England and Wales)



(d) Infant Mortality (left axis, short graph) and Life Expectancy at Age 30 (right axis, long graph) (England and Wales)

(e) Gross and Net Reproduction Rates (England and Wales)



(f) Population Size (U.K.)

Figure 4: The Stylized Facts of Long-Run Development

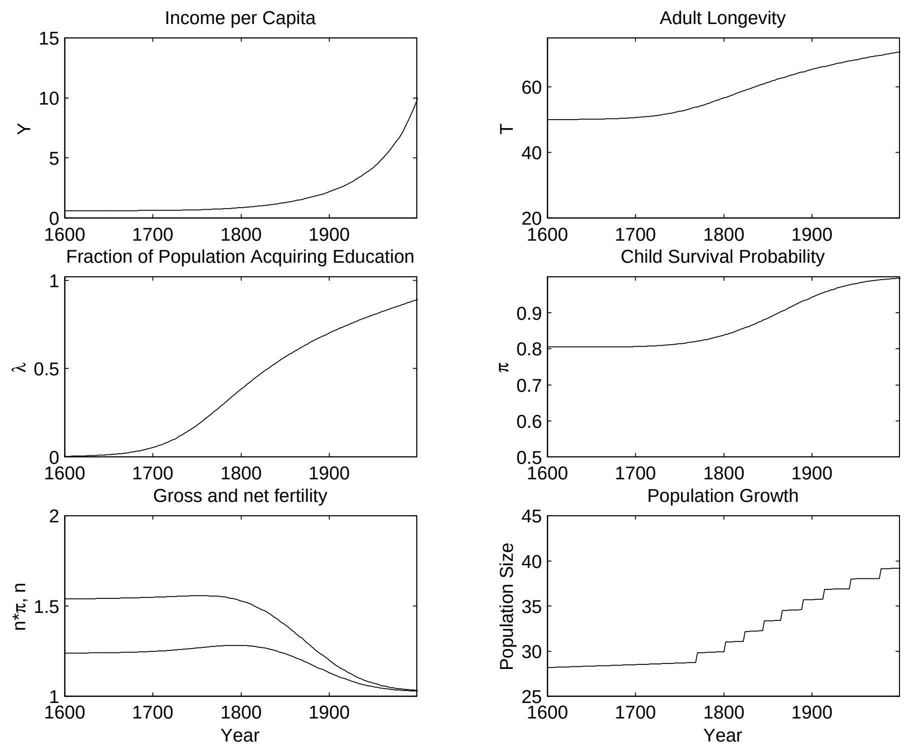


Figure 5: A Simulation of the Development Process

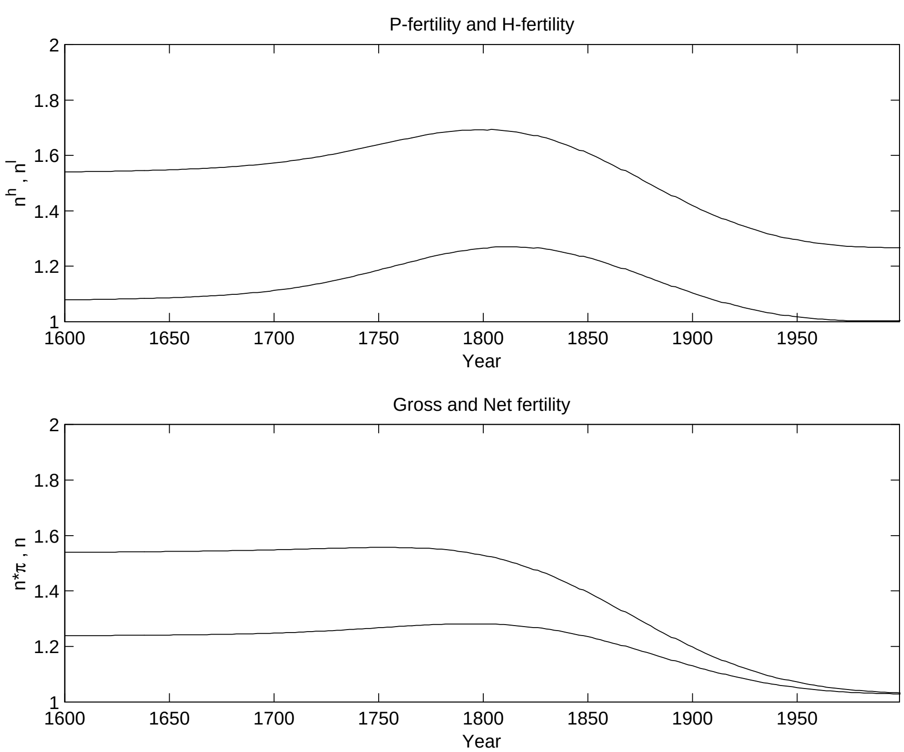


Figure 6: The Timing of the Demographic Transition

References

Acemoglu, D. (1998): "Why Do Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, 113(4), 1055–1089.

Alderman, H., J. Behrman, V. Lavy, and R. Menon (2001): "Child Health and School Enrollment - A Longitudinal Analysis," *Journal of Human Resources*, 36, 185–205.

Barro, R. J., and G. S. Becker (1989): "Fertility Choice in a Model of Economic Growth," *Econometrica*, 57(2).

Becker, G. S., and R. J. Barro (1988): "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 113(1).

Blackburn, K., and G. P. Cipriani (2002): "A Model of Longevity, Fertility and Growth," *Journal of Economic Dynamics and Control*, 26, 187–204.

Bleakley, H. (2003): "Desease and Development: Evidence the American South," *Journal of the European Economic Association*, 1(2).

Boucekkine, R., D. de la Croix, and O. Licandro (2002): "Vintage Human Capital, Demographic Trends, and Endogenous Growth," *Journal of Economic Theory*, 104(2), 340–375.

——— (2003): "Early Mortality Declines at the Dawn of Modern Growth," *Scandinavian Journal of Economics*, 105, 401–418.

Cervellati, M., and U. Sunde (2005): "Human Capital, Life Expectancy, and the Process of Development," *American Economic Review*, 95(5), 1653–1672.

Cipolla, C. M. (1969): *Literacy and Development in the West*. Penguin Books, Middlesex, England.

Clark, G. (2005): "Human Capital, Fertility and the Industrial Revolution," *Journal of the European Economic Association, forthcoming.*

de la Croix, D., and M. Doepke (2003): "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review*, 93(4), 1091–1113.

Doepke, M. (2005): "Child Mortality and Fertility Decline: Does the Barro-Becker Model Fit the Facts?," *Journal of Population Economics, forthcoming.*

Doms, M., T. Dunne, and K. Troske (1997): "Workers, Wages, and Technology," *Quarterly Journal of Economics*, 112(1), 253–290.

Easterlin, R. A. (1999): "How Beneficient is the Market? A Look at the Modern History of Mortality," *European Review of Economic History*, 3, 257–294.

Floud, R., and D. McCloskey (1994): *The Economic History of Britain Since 1700*. Cambridge University Press, Cambridge.

Fogel, R. W. (1994): "Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy," *American Economic Review*, 84(3), 369–395.

GALOR, O. (2005): "From Stagnation to Growth: Unified Growth Theory," in *Handbook of Economic Growth, forthcoming*, ed. by P. Aghion, and S. Durlauf. Elsevier Science B.V., Amsterdam.

GALOR, O., AND O. MOAV (2000): "Ability-Biased Technological Transition and Economic Growth," *Quarterly Journal of Economics*, 115(2), 469–497.

——— (2002): "Natural Selection and the Origin of Economic Growth," *Quarterly Journal of Economics*, 117(4), 1133–92.

GALOR, O., AND O. MOAV (2005): "Natural Selection and the Evolution of Life Expectancy," *Minerva Center for Economic Growth Paper*, 02-05.

GALOR, O., AND D. N. WEIL (2000): "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review*, 90(4), 807–828.

GERDTHAM, U., AND B. JÖNSSON (2000): "International Comparisons of Health Expenditure: Theory, Data and Econometric Analysis," in *Handbook of Health Economics*, ed. by A. Culyer, and J. Newhouse, chap. 1, pp. 11–53. Elsevier, Amsterdam.

GETZEN, T. E. (2000): "Health Care Is an Individual Necessity and a National Luxury: Applying Multi-Level Decision Models to the Analysis of Health Care Expenditures," *Journal of Health Economics*, 19, 259–270.

GOODFRIEND, M., AND J. MCDERMOTT (1995): "Early Development," *American Economic Review*, 85(1), 116–133.

HANSEN, G. D., AND E. C. PRESCOTT (2002): "Malthus to Solow," *American Economic Review*, 92(4), 1205–1217.

HASSLER, J., AND J. RODRIGUEZ-MORA (2000): "Intelligence, Social Mobility, and Growth," *American Economic Review*, 90(4), 888–908.

HAZAN, M., AND B. BERDUGO (2002): "Child Labor, Fertility, and Economic Growth," *Economic Journal*, 112, 810–828.

HAZAN, M., AND H. ZOABI (2005): "Does Longevity Cause Growth?," *CEPR Discussion Paper*, 4931.

JONES, C. I. (2001): "Was an Industrial Revolution Inevitable? Economic Growth Over the Very Long Run," *Advances in Macroeconomics*, 1(2).

KALEMLI-OZCAN, S. (2002): "Does Mortality Decline Promote Economic Growth?," *Journal of Economic Growth*, 7(4), 411–439.

——— (2003): "A Stochastic Model of Mortality, Fertility, and Human Capital Investment," *Journal of Development Economics*, 70(1), 103–118.

KEYFITZ, N., AND W. FLIEGER (1968): *World Population: An Analysis of Vital Data*. University of Chicago Press, Chicago.

KOGEL, T., AND A. PRSKAWETZ (2001): "Agricultural Productivity Growth and the Escape from Malthusian Trap," *Journal of Economic Growth*, 6, 337–357.

LAGERLOF, N.-P. (2003): "From Malthus to Modern Growth: The Three Regimes Revisited," *International Economic Review*, 44(2), 755–777.

Laitner, J. (2000): "Structural Change and Economic Growth," *Review of Economic Studies*, 67, 545–562.

Lichtenberg, F. R. (1998): "Pharmaceutical Innovation, Mortality Reduction, and Economic Growth," *NBER Working Paper*, 6569.

——— (2002): "Sources of U.S. Longevity Increase, 1960-1997," *NBER Working Paper*, 8755.

——— (2003): "The Impact of New Drugs Launches on longevity: Evidence from Longitudinal, Disease-Level Data from 52 Countries, 1982-2001," *NBER Working Paper*, 9754.

Lorentzen, P., J. McMillan, and R. Wacziarg (2005): "Death and Development," *mimeo, Stanford University.*

Lucas, R. E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.

——— (2002): *Industrial Revolution, Past and Future*. Harvard University Press, Cambridge, Massachusetts.

Maddison, A. (1991): *Dynamic Forces in Capitalist Development: A Long-Run Comparative View*. Oxford University Press, Oxford.

Magnac, T., and G. Postel-Vinay (1997): "Wage Competition Between Agriculture and Industry in Mid-Nineteenth Century France," *Explorations in Economic History*, 34(1), 1–26.

Miguel, E., and M. Kremer (2004): "Worms: Identifying the Impacts on Education and Health in the Presence of Treatment Externalities," *Econometrica*, 72(1), 159–217.

Mirovsky, J., and C. Ross (1998): "Education, Personal Control, Lifestyle and Health - A Human Capital Hypothesis," *Research on Ageing*, 20, 415–449.

Moav, O. (2005): "Cheap Children and the Persistence of Poverty," *Economic Journal, forthcoming*, 114.

Mokyr, J. (1993): "Technological Progress and the Decline of European Mortality," *American Economic Review*, 83(2), 324–330.

Nelson, R., and E. Phelps (1966): "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 56(2), 69–75.

Reis-Soares, R. (2005): "Mortality Reductions, Educational Attainment, and Fertility Choice," *American Economic Review, forthcoming.*

Romer, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98(5), S71–S102.

Schultz, P. T. (1993): "Mortality Decline in the Low-Income World: Causes and Consequences," *American Economic Review*, 83(2), 337–342.

Shastry, G. K., and D. Weil (2003): "How Much of the Cross-Country Variation in Income is Explained by Health?," *Journal of the European Economic Association*, 1(2-3), 387–396.

Smith, J. P. (1999): "Healthy Bodies and Thick Wallets: The Dual Relation Between Health and Economic Status," *Journal of Economic Perspectives*, 13(2), 145–166.

Tamura, R. (2002): "Human Capital and Economic Developoment," *Federal Reserve Bank of Atlanta Working Paper*, 2002-5.

Vaupel, J. W. (1998): "Demographic Analysis of Aging and Longevity," *American Economic Review*, 88(2), 242–247.

Wang, L. (2003): "Determinants of Child Mortality in LDCs - Empirical Findings from Demographic and Health Surveys," *Health Policy*, 65(3), 277–299.

Wrigley, E. A., and R. S. Schofield (1981): *The Population History of England 1541-1871*. Edward Arnold Publishers, London.

A Appendix: Proofs of Propositions

Proof of Proposition 1

The proof follows the same lines as the proof of Proposition 1 in Cervellati and Sunde (2005). Consider Equation (21). For notational simplicity, denote $\widetilde{a}^*$ simply as a, $\underline{e}$ as e. By standard calculus,

$$T'(a) = \frac{e\frac{g'(a)}{\Omega}}{\left[1 - \frac{g(a)}{\Omega}\right]^2} < 0\,, \tag{41}$$

since $g'(a) < 0$, $\forall a \in [0,1]$. The variable $\underline{a}$ denotes a lower bound on the ability threshold in the sense that even in the absence of biological constraints, i.e. when $T_t = \underline{T} + \rho$, the equilibrium ability cut-off cannot be lower. Therefore, we conclude that for a given set of parameters $\gamma \in (0,1)$, $A = \frac{A_H}{A_P}$, for every $a \in [\underline{a}, 1]$ there is one and only one $T > 0$ such that (21) is satisfied.

We next show that the lower bound on the ability threshold, $\underline{a}$ is decreasing in Ω_t. The claim follows from the definition of $\underline{\widetilde{a}}^*(\Omega)$, and the fact that $g(\widetilde{a}^*)$ is strictly decreasing in $\widetilde{a}^*$.

The intuition of the following proof proceeds as follows: We solve equilibrium condition (20) for T as a function of $\widetilde{a}^*$ and investigate the behavior of this function. Due to the fact that $T(\widetilde{a}^*)$ is strictly monotonically decreasing within the admissible support the function is invertible within this range of support. We then show that there exists one and only one $\widetilde{a}^*$ for which the second derivative of this function equals zero. Since the condition for the second derivative to equal zero cannot readily be solved for $\widetilde{a}^*$, we decompose it into two components and show that one is strictly monotonically increasing within the support while the other is strictly monotonically decreasing, such that there must exist one and only one $\widetilde{a}^*$ for which the condition is satisfied by the intermediate value theorem. But if $T(a)$ has a single inflection point and is invertible, also $a(T)$ has a single inflection point and is therefore S-shaped.

Consider again Equation (21). Using standard calculus, one can now show that:

$$T'(a) = \frac{e\frac{g'(a)}{\Omega}}{\left[1 - \frac{g(a)}{\Omega}\right]^2}\,, \tag{42}$$

and

$$T''(a) = \frac{e\frac{g''(a)}{\Omega}\left[1 - \frac{g(a)}{\Omega}\right] + 2e\frac{[g'(a)]^2}{\Omega^2}}{\left[1 - \frac{g(a)}{\Omega}\right]^3}\,. \tag{43}$$

Due to the fact that $T'(a) < 0\ \forall a \in [\underline{a}, 1]$, we note that the function $T(a)$ is invertible in the range $a \in [\underline{a}, 1]$ of the support. Note also that $T(a) \geq \underline{T}\ \forall a \in [0,1]$, so the inverse function $a(T)$ is strictly monotonically decreasing for all positive T. The proof follows substituting a^2 with b and to re-write $g(a) \equiv h(b)$, $g'(a) \equiv h'(b)$, and $g''(a) \equiv h''(b)$, where $\underline{b} = (\underline{a})^2$. Thus define $T(a) = \mathcal{T}(b)$, so the derivatives $T'(a) = \mathcal{T}'(b)$ and $T''(a) = \mathcal{T}''(b)$ can be re-written in terms of b. Taking

$$\mathcal{T}'(b) = -\frac{[\mathcal{T}(b)]^2}{e}\frac{h'(b)}{\Omega}$$

Existence of an inflection point can already be inferred from a closer examination. Since

$$h'(b) < 0 \quad \forall b \in [\underline{b}, 1]$$

we know that also $\mathcal{T}'(b) < 0 \quad \forall b \in [\underline{b}, 1]$. Moreover, one immediately sees that $\lim_{b\to 1} h'(b) = -\infty \Leftrightarrow \lim_{a\to 1} T'(a) = -\infty$, such that T has infinitely negative slope at both boundaries of the admissible support, suggesting that there must exist at least one inflection point. From these arguments it is also clear that the slope of the inverse function, $a'(T)$, converges to zero at both boundaries of the support. Analysis of the second derivative $\mathcal{T}''(b)$ allows to show existence and uniqueness of an inflection point.

Proof of Proposition 2:

The claims *(1)* and *(2)* follow from taking partial derivatives of the respective expressions (10) and (13), leading to

$$\frac{\partial e_t^{l*}}{\partial T_t} = \frac{\partial e_t^{h*}}{\partial T_t} = \frac{1-\gamma}{2-\gamma} \quad \text{and} \quad \frac{\partial n_t^{l*}}{\partial T_t} = \frac{\partial n_t^{h*}}{\partial T_t} = \frac{\gamma}{2-\gamma}\frac{1}{b+r(\pi_t)} .$$

Claim *(3)* follows from Proposition 1 and the derivative of (22) since

$$\frac{\partial n_t^*}{\partial T_t} = \frac{\gamma}{2-\gamma}\frac{1}{b+r(\pi_t)}\left[1-\underline{e}\frac{\partial \lambda_t}{\partial T_t}\right] ,$$

which implies $\frac{\partial n_t^*}{\partial T_t} \gtrless 0 \Leftrightarrow \frac{\partial \lambda_t}{\partial T_t} \lessgtr \frac{1}{\underline{e}}$.

Proof of Proposition 3:

The claims *(1)* and *(2)* follow from taking partial derivatives of the respective expressions (10) and (13), leading to

$$\frac{\partial n_t^{l*}}{\partial \pi_t} = -\frac{\gamma}{2-\gamma}\frac{T_t}{b+r(\pi_t)}\frac{r'(\pi_t)}{b+r(\pi_t)} < 0 \quad \text{and} \quad \frac{\partial n_t^{h*}}{\partial \pi_t} = -\frac{\gamma}{2-\gamma}\frac{(T_t-\underline{e})}{b+r(\pi_t)}\frac{r'(\pi_t)}{b+r(\pi_t)} < 0$$

and the fact that $\frac{\partial n_t^*}{\partial \pi_t} = \lambda_t \frac{\partial n_t^{h*}}{\partial \pi_t} + (1-\lambda_t)\frac{\partial n_t^{l*}}{\partial \pi_t}$. Claim *(3)* follows from

$$\frac{\partial (n_t^* \pi_t)}{\partial \pi_t} = \frac{\gamma}{2-\gamma}\frac{1}{b+r(\pi_t)}\left[\lambda_t(T_t-\underline{e})+(1-\lambda_t)T_t\right]\left[1-\frac{\pi_t r'(\pi_t)}{b+r(\pi_t)}\right] = n_t^*\left[1-\frac{\pi_t r'(\pi_t)}{b+r(\pi_t)}\right] ,$$

which implies $\frac{\partial (n_t^* \pi_t)}{\partial \pi_t} \lesseqgtr 0 \Leftrightarrow 1 \lesseqgtr \frac{\pi_t r'(\pi_t)}{b+r(\pi_t)}$.

Proof of Proposition 4:

Note: As long as there is no danger of confusion, we suppress the subscripts '$_t$' for generation t for notational convenience (e. g. $T_t(a_t) = T(a)$, etc.).

***(i)*:** Existence of a dynamic equilibrium for the conditional system. Recall that the locus TT is linear with slope $-\rho$ and values $T(a=0) = \underline{T}+\rho$ and $T(a=1) = \underline{T}$. From the proof of Proposition **1** we know that, for any $A > 0$, the locus $HH\,(A)$ is such that $\lim_{a \downarrow \underline{a}(A)} T_t(a, A) = \infty$, and that its value is monotonically decreasing $\forall a > \underline{a}\,(A)$. Hence, if the value of this non-linear relation at $a = 1$ is smaller than that of the linear relation of the intergenerational externality, there must exist at least one intersection by the intermediate value theorem. However, note that $T(1) = \underline{e}\ \forall t$, and that by assumption $\underline{e} < \underline{T}$. That means the fix cost for theoretical education is always lower than any minimum life expectancy, otherwise theoretical education would never be an alternative, not even for the most able individual in the world. Hence a dynamic equilibrium exists for every generation t.

***(ii)*:** From the proof of *(i)* and noting that any steady state is characterized by an interior solution with $a < 1$, since $T(a=1) = \underline{T} > \underline{e}$, which in turn implies that $H_t > 0$ and $P_t > 0$ for any $t > 0$.

***(iii)*:** The claims follow from Proposition **1** : We know that $HH\,(A)$ has always a unique turning point and takes values above and below TT at the extremes $\underline{a}\,(A)$ and 1. Hence the two curves can intersect *at most* three times, while they intersect at least once by *(i)*. Claim

(a) follows from the negative slopes of both loci that allow to rank steady states. Claims *(b)* and *(c)* are true since, in the extreme equilibria E^G and E^S, $HH(A)$ intersects TT from above, which means that the system is locally stable, while the opposite happens in the intermediate equilibrium E^u, since $HH(A)$ must cut TT from below. Thus E^u is locally unstable. Claim *(d)* follows from the fact that if only one steady state exists it must be stable since $HH(A)$ starts above TT and ends below so it must cut from above. The concavity/convexity of $HH(A)$ in the stable equilibria is used to identify them since in case of multiplicity one must be in the concave and the other in the convex part.

Proof of Lemma 1.

By assumption, $\delta > 0$, $\phi > 0$, and $\chi > 0$ in equation (32), such that $\dot{A}_{H.t} > 0$, and $A_{H.t} > A_{H.t-1}$ $\forall t$. A_{t-1} and A_t are linked in an autoregressive way, and equation (32) as suggested in footnote 36 is of the form $A_t = (c_{t-1}+1)A_{t-1} = d_{t-1}A_{t-1}$, where $d_{t-1} = \delta H_t^{\phi} A_{.t-1}^{\chi} + 1 > 1$ for any t, since from the previous proposition $H_t > 0$ for any t and $\delta > 0$. This means that the process is positive monotonous and non stationary. Starting with any $A_0 > 0$ we can rewrite $A_t = \left(\prod_{i=1}^{t} d_{i-1}\right) A_0$, where $\left(\prod_{i=1}^{t} d_{i-1}\right) > 1$ and $\lim_{t\longrightarrow\infty}\left(\prod_{i=1}^{t} d_{i-1}\right) = \infty$.

As in the proof of Proposition **1** , solve equation (21) for $T(a)$ to get:

$$T_t(a_t) = \frac{e}{1-\frac{g(a_t)}{\Omega_t}} . \tag{44}$$

The claim follows by partial derivation of equation (44), $\frac{\partial}{\partial\Omega_t}T_t(a_t) = -\frac{g(a_t)e}{(\Omega_t-g(a_t))^2} < 0$ $\forall a_t \in [\underline{a}_t, 1]$.

Proof of Proposition 5:

Consider Equation (44), and denote the function characterizing the slope of the $HH(A)$ locus for any a by $HH'(A) \equiv \frac{\partial T(a,A)}{\partial a}$. Similarly, let $HH''(A) \equiv \frac{\partial^2 T(a,A)}{\partial a^2}$ denote the second derivative of the $HH(A)$ locus for any a. From Proposition 1, we know that $HH'(A)$ is U-shaped. It takes infinite value at the extremes of the support $\{\underline{a}(A), 1\}$, and exhibits a unique global minimum corresponding to the inflection point of $HH(A_t)$. In the following, we denote a_A^I as the level of a corresponding to the global minimum of the function $HH'(A)$ (or, equivalently corresponding to the unique inflection point of the function $HH(A)$), characterized by $HH''(A, a_A^I) = 0$. A useful intermediate result describes the effect of A on the slope of the HH-locus:

Lemma 3. *For any $a \in [\underline{a}(A), 1]$, $HH'(A)$ decreases as A increases.*

The result follows from

$$\frac{\partial}{\partial\Omega_t}\left|\frac{\partial T_t(a_t, A_t)}{\partial a_t}\right| = \frac{-\Omega_t + g(a_t) - 1}{\left(1-\frac{g(a_t)}{\Omega_t}\right)^3 \Omega_t^3} g'(a_t)e\frac{1}{\Omega_t} < 0 \ \forall a_t \in [\underline{a}_t(A_t), 1],$$

and the definitions of A and Ω. Note also that $\lim_{A\longrightarrow\infty}\left(\frac{\partial T(a,A)}{\partial a}\right) = 0 \ \forall a \neq \{0,1\}$, $\lim_{A\longrightarrow 0}\left(\frac{\partial T(a,A)}{\partial a}\right) = +\infty$, $\forall a \in [\underline{a}(A), 1]$ and $\lim_{A\longrightarrow 0}\underline{a}(A_0) = 1$, therefore $HH'(A)$ eventually take zero value in the interior of the bounded support as $A \longrightarrow \infty$, and $HH(A)$ is basically a vertical line at $a = 1$ (with infinite slope) as $A \longrightarrow 0$.

Since $HH'(A)$ shifts downwards monotonically as A increases, there exists a unique value A^0, and by Lemma 1 also a unique t^0, such that $HH'(A^0, a_{A^0}^I) = \rho$. For this level of $A = A^0$, HH' and TT' are tangent. Hence, $HH'(A, a_A^I) \gtrless \rho \iff A \lessgtr A^0$. Since $HH'(A)$ is globally convex, and by definition of a_A^I as extremum (or from graphical inspection of Figure 7), for any

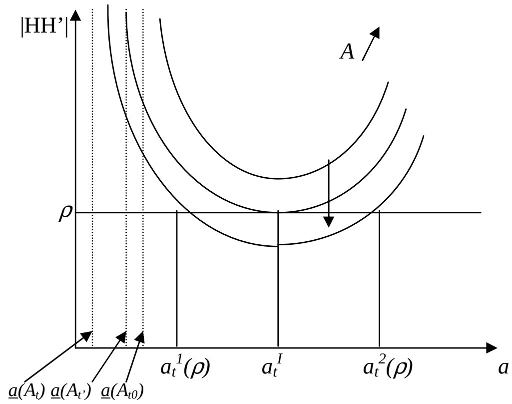


Figure A1: Emergence of Multiple Equilibria

Figure 7: Slope of the HH(A) Locus

$t \geq t^0$ there exist exactly two levels of a, $a^1_{A_t} \leq a^I_{A_t} \leq a^2_{A_t}$, where $a^1_{A_t}$ lies in the convex and $a^2_{A_t}$ in the concave part of $HH(A)$, such that $HH'\left(A_t, a^1_{A_t}\right) = \rho = HH'\left(A_t, a^2_{A_t}\right)$.

Existence of at least one equilibrium of the conditional dynamic system (35) has been shown in the previous proposition. For any $t < t^0$ the equilibrium is unique since $HH'(A_t) > \rho$ $\forall a \in [\underline{a}(A), 1]$ and the loci $HH(A_t)$ and TT necessarily intersect only once. For $t \geq t^0$, multiple equilibria may arise if $HH(A)$ is flatter than TT in some range of the support. Thus, at t^0 two scenarios are possible depending on the nature of the unique equilibrium. If $HH(A^0, a^I_{A^0}) < TT(a^I_{A^0})$, the unique equilibrium is of type H since, by definition of a^I_A as inflection point, the two loci TT and HH intersect in the convex part of $HH(A)$. This is the case *if and only if* the concave part of $HH(A)$ is sufficiently small, which is true if the fix cost of acquiring human capital H, $\underline{e}$ is sufficiently small. In this case, nothing prevents agents from acquiring high quality human capital from early stages on, and the economy develops smoothly as A increases overtime. In the other scenario, $HH(A_0, a^I_{A0}) \geq TT(a^I_{A^0})$, so the unique equilibrium at t^0 is of type S. In this case, acquiring H is individually costly in an underdeveloped economy, or, equivalently, $\underline{e}$ is sufficiently large to generate a development trap. The economy is characterized by a lengthy sequence of S-equilibria which is eventually followed by a development process. The proof follows from the incremental increase in TFP by Lemma 1 implying that the equilibria of the conditional system can be characterinzed in terms of A instead of t.