A quantitative study of sovereign default with heterogenous borrowers[*]

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Abstract

We study a standard model of sovereign default, but we assume that governments with different discount factors alternate in power. Our model generates the default probability observed in the data, and consequently it generates higher spreads than the benchmark without heterogeneity. The alternation in power of different government types is crucial to generate a higher default probability. The default probability is higher in an economy where patient and impatient governments alternate in power than in an economy with only impatient (or patient) governments. The model also generates higher volatility in spreads than a model without heterogeneity. Moreover, difficulties in market access after a default episode appear endogenously. The paper also provides insights on how do changes in government stability impact on the default risk, and thus, on spreads. We shall also describe the strategic interaction of governments with different patience. JEL classification: F34, F41. *Keywords:* Sovereign Default, Political Risk, Strategic Behavior, Endogenous Borrowing Constraints, Markov Perfect Equilibrium.

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1   Introduction

Many nations have experienced episodes of sovereign default, some of the most recent being Russia in 1998, Ecuador in 1999, and Argentina in 2001. Quantitative models have been developed (based on the willingness to pay approach) to study these episodes (see, for example, Aguiar and Gopinath (2006), Arellano (2005), Bai and Zhang (2005), Cuadra and Sapriza (2006a, 2006b), Lizarazo (2005), and Yue (2005)). In these models, the government maximizes the utility of a representative agent by deciding how much to save and whether to default on its debt. In particular, the government issues bonds that are priced in a competitive market. In general, it is assumed that two costs follow a default episode. First, there is an output loss. Second, countries are exogenously excluded from capital markets. The exogenous exclusion assumption may appear to be at odds with the assumption of competitive lenders. It is unlikely that after a default episode, competitive creditors could coordinate to cut off credit to defaulting countries (see, for example, Cole, Dow, and English (1995), and Athreya and Janicki (2006)). Moreover, empirical evidence indicates that once other variables are control for, market access is not significantly influenced by previous defaults (see, for example, Gelos, Sahay, and Sandleris (2003)).

In this paper, we study a standard quantitative model of sovereign default but, in contrast with previous quantitative studies, we do not assume that a defaulting country is exogenously excluded from capital markets. A borrower's ability to obtain loans depends only on the probability of repay. However, we find that when we consider that governments with different willingness to pay alternate in power, difficulties to access capital markets are likely to appear endogenously after a default episode.

Emerging markets typically face large political uncertainty. In particular, fragile and often unstable political institutions have been the norm in Latin America. Most Latin American countries have weaker public institutions than those found in the investment-grade countries of Europe and Asia, a factor that contributes to their more volatile economic performance. Significant increases in short term interest rate volatility from political risk has been evidenced in Brazil, Bolivia, Ecuador and Venezuela in 2005. These features suggest that political uncertainty may have a significant role on economic fluctuations. Moreover, empirical studies point out the importance of political factors in determining spreads (see, for example, Citron and Nickelsburg (1987), Balkan (1992), Li (1992), Rivoli and Brewer (1997), Catao and Sutton (2002), and Jahjah and Yue (2004)). These studies find a significant relationship between the probability of default and political indicators, thus underscoring the relevance of political factors as a determinant of a country's credit worthiness. This suggests that there is heterogeneity in the governments' willingness to repay.

Our paper models such feature and shows how considering government heterogeneity can improve the quantitative performance of the standard model of sovereign default. We allow the composition of the government or the distribution of power within the government to change each period. We model this in a very stylized way by assuming that two types of governments with different patience alternate in power—the patient government discounts future utility flows less than the impatient government.

With our benchmark parametrization, even though impatient governments assign more weight to current consumption, they decide to borrow less than patient governments. This is the case because impatient governments are more likely to default and, therefore, face higher borrowing costs. Consequently, in their first period in of-

fice, impatient governments typically find themselves with a debt level higher than the maximum debt level they would choose to pay. Therefore, in our simulations, most default episodes—approximately 75%—arise when a patient government is replaced by an impatient one.

Consequently, even without assuming exogenous exclusion, the model generates difficulties in market access after a default episode—after default episodes, because impatient governments are more likely to be in power, for the same level of debt, average spreads are higher; moreover, the equilibrium debt level is lower. That is, after default, capital inflows are initially decreased, and later recover slowly. This is consistent with the observed governments' reluctance to issue new debt after defaulting.

Furthermore, consistently with historical evidence, in our model, market access improves after the defaulting government loses power. A clear example is discussed by Cole, Dow, and English (1995); they explain that *"the ability of Reconstruction governments in Florida and Mississippi to borrow after the Civil War suggests that the old creditors could not block new loans once the states' reputations had been restored by an observable change in regime."*

It will also be shown that considering heterogeneous borrowers allows the standard model to generate the default probability observed in the data—the default probability is much lower in the benchmark without heterogeneity. Consequently, our model generates higher spreads—closer to the data. The alternation in power of different government types is crucial to generate a higher default probability. The default probability is higher in an economy where patient and impatient governments alternate in power than in an economy with only impatient (or patient) governments.

In general, equilibrium spreads depend on the equilibrium probability of default

which depends on the probability of facing a government likely to pay in the future. In our stylized model, the probability of facing a government likely to pay (patient) in the future depends on the type of the government in power which changes over time generating volatility in spreads. Thus, the model produces higher spread volatility—closer to the volatility in the data—than a model without heterogeneity.

The paper also provides insights on how do changes in government stability impact on the default risk, and thus, on spreads. We shall also describe the strategic interaction of governments with different patience.

1.1 Related literature

Cuadra and Sapriza (2006b) also present a quantitative model of sovereign default in which different governments alternate in power. However, these governments disagree on the optimal allocation of resources within each period (see also Amador (2005), and Azzimonti Renzo (2005)), but assign the same weight to the future and, therefore, they do not differ in their willingness to repay. That is, governments are homogeneous from a lender's point of view. Their analysis also imposes exogenous restrictions to credit market access after default.

Cole, Dow, and English (1995) study sovereign default with heterogeneous borrowers and asymmetric information about the borrower's type, but their study is not quantitative. They focus on equilibria in which the default history reveals the type. As in our framework, default episodes are associated with impatient borrowers, but in their paper, only impatient governments default. They assume that resources or obligations cannot be transfer from one period to another period and, therefore, there is no room for strategic interaction between government types. Moreover, governments do not de-

cide the amount they borrow, which is decided by the lenders—who offer either a fixed positive amount (when they believe the government may be patient) or zero (when they know the government is impatient). In their paper, the impatient government always defaults on obligations it contracted on before its type was known. In our framework, a more impatient government may default on obligations contracted by a more patient government in the past, because the patient government borrowed more than what the impatient government is willing to pay. Chatterjee, Corbae, and Rios-Rull's (2005) study bankruptcy with heterogenous borrowers. In their model, the borrower's type is its private information while in our environment types are public information. They also assume that a borrower's type changes over time while we assume borrowers of different types alternate in power—this seems more appropriate for studying sovereign default. They also assume that the impatient type always default, and they only allow for three levels of savings. Besides considering heterogeneous borrowers, and removing the assumption on exogenous exclusion, we follow exactly the standard model in recent quantitative studies of sovereign default (see, for example, Aguiar and Gopinath (2006), Arellano (2005), Cuadra and Sapriza (2006a, 2006b), Lizarazo (2005), and Yue (2005)).

Difficulties in market access after a default episode may also be an equilibrium outcome in studies focusing on infinitely repeated games (see Miller, Tomz, and Wright (2005) and the references therein). These studies assume that competitive creditors can coordinate, and do not present quantitative assessments.

2   The model

Our model builds on the standard framework with trend shocks presented in Aguiar and Gopinath (2006) (hereafter denoted by AG). There is a single tradable good. The

economy receives an stochastic endowment stream of this good, $y_t = g_t y_{t-1}$, where

$$ln\left(g_t\right) = \left(1 - \rho_g\right)\left(ln\left(\mu_g\right) - z\right) + \rho_g ln\left(g_{t-1}\right) + \varepsilon_t,$$

$|\rho_g| < 1$, $\varepsilon_t \sim N\left(0, \sigma_g^2\right)$, and $z = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.[1] To ensure a well defined problem it is assumed that $E\left\{\lim_{t\to\infty}\beta^t\left(y_t\right)^{(1-\sigma)}\right\} = 0$, where σ denotes the coefficient of relative risk aversion and β denotes the discount factor.

The objective of the government is to maximize the present value of future utility flows of a representative agent. The representative agent has CRRA preferences over consumption:

$$u\left(c\right) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma}.$$

We allow the composition of the government or the distribution of power within the government to change each period. We model this by assuming that two types of governments alternate in power. Patient governments discount future utility flows at a rate β_h, and impatient governments discounts future utility flows at a rate β_l, where $\beta_h > \beta_l$. There is a constant exogenous probability π of a change in government type. The government in power makes two decisions. First it decides whether to refuse to pay previously issued debt. Second, it decides how much to borrow or save for the following period.

As in previous quantitative studies, we assume there are two costs of defaulting. First, after default, the country may be excluded from capital markets. We assume that in each period (including the default period), the country regains access to capital

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[1]The endowment process is motivated by the work of Aguiar and Gopinath (2004). They find that shocks to trend growth (rather than transitory fluctuations around a stable trend) are the primary source of fluctuations in emerging markets. Aguiar and Gopinath (2006) show that the ability of their model of sovereign default to match the data is improved when trend shocks are included.

markets with probability $\phi \in [0,1]$.[2] In contrast with previous quantitative studies, we will consider the case of $\phi = 1$ (no exogenous exclusion). Second, we assume that if a country has defaulted on its debt, it faces an exogenous "output loss" of λ percent in the following period.[3]

The government can choose to save or borrow using one period bonds. These assets are priced in a competitive market. There is a large number of identical, infinitely lived foreign lenders. Each lender can borrow or lend at the risk free rate r and can lend in a perfectly competitive market to the small open economy. The individual lender is risk neutral. Creditors have perfect information regarding the economy's endowment.

Let b denote the current position in bonds. Each bond delivers one unit of the good next period for a price of q this period. A negative value of b denotes that the country was an issuer of bonds in the previous period.

The government can issue as many bonds as it wants, but the issue price of these bonds is not constant because it depends on how likely is that the country defaults on the following period. Competitive lenders will offer a price $q_{jd}(b', y, g)$ for each bond if the country decides to issue $-b'$ bonds. This price satisfies the lenders' zero profit condition. It depends on the government type, j, and on its default decision, d. The former is due to the fact that the type conveys information about the probability distribution of future types, and therefore it affects the probability distribution of next period default decisions. The latter is due to the fact that a current default decreases

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[2] Previous quantitative studies assume that the government cannot borrow in the period it defaults. We allow the government to access the capital market in the default period with probability ϕ so that when we want to eliminate the exogenous exclusion (and we set $\phi = 1$), the government is not exogenously excluded in the default period.

[3] Previous quantitative studies assume that after default, the country suffer the output loss for a stochastic number of periods (the periods in which the country is excluded from capital markets). For simplicity, we do not do the same. Otherwise, when we assume that there is no exogenous exclusion ($\phi = 1$) and, therefore, the country can default in consecutive periods, we would have to keep track of the number of output losses the country is suffering.

future output and affects future default decisions.

The exclusion state is determined at the beginning of each period. If the country was not excluded from financial markets at the end of the previous period, it is not excluded at the beginning of the current period. If the country was excluded at the end of the previous period, it is excluded at the beginning of the current period with probability $1 - \phi$. When participating in financial markets, the government compares two value functions: $V_{j1}(y, g, h)$ and $V_{j0}(b, y, g, h)$. The former denotes the value function under default of a government of type j, when the country has a credit history h. The variable h takes a value of 1 when the country defaulted in the previous period, and takes a value of 0 when the country did not default in the previous period. The second value function denotes the value function of government j when it did not default, and it has to pay $-b$ at the beginning of the period.

Let x denote the exclusion state. The variable x takes a value of 1 when the country is excluded, and takes a value of 0 otherwise. Let $V_j(b, y, g, h, x)$ denote j's value function at the beginning of a period if j is in power, and $W_j(b, y, g, h, x)$ denote j's value function at the beginning of a period if j is not in power—since a government's decisions are influenced by its type, V_j and W_j do not need to coincide. After default, the value function of the government is given by

$$V_{j1}(y, g, h) = \phi \tilde{V}_{j1}(y, g, h, 0) + (1 - \phi) \tilde{V}_{j1}(y, g, h, 1)$$

where

$$\tilde{V}_{j1}(y,g,h,1) = \beta_j \left[\begin{array}{l} u\left(y\left(1-h\lambda\right)\right) + ... \\ \pi \int \left[\begin{array}{l} \phi W_j(0,y',g',1,0) + ... \\ (1-\phi)\, W_j(0,y',g',1,1) \end{array} \right] F_g\left(dg' \mid g\right) ... \\ + (1-\pi) \int \left[\begin{array}{l} \phi V_j(0,y',g',1,0) + ... \\ (1-\phi)\, V_j(0,y',g',1,1) \end{array} \right] F_g\left(dg' \mid g\right) \end{array} \right]$$

is the value function for a defaulting government that is excluded, and

$$\tilde{V}_{j1}(y,g,h,0) = \max_{b'} \left\{ \beta_j \left[\begin{array}{l} u\left(y\left(1-h\lambda\right) - q_{j1}\left(b',y,g\right)b'\right) + ... \\ \pi \int W_j(b',y',g',1,0)F_g\left(dg' \mid g\right) ... \\ + (1-\pi)\int V_j(b',y',g',1,0)F_g\left(dg' \mid g\right) \end{array} \right] \right\}.$$

gives the dynamic programming problem for a defaulting government that is not excluded.

The value function of j when it has decided to pay back the debt is obtained from the following Bellman equation

$$V_{j0}(b,y,g) = \left\{ \max_{b'} \left\{ \beta_j \left[\begin{array}{l} u\left(y\left(1-h\lambda\right) + b - q_{j0}\left(b',y,g\right)b'\right) + ... \\ \pi \int W_j(b',y',g',1,0)F_g\left(dg' \mid g\right) ... \\ + (1-\pi)\int V_j(b',y',g',1,0)F_g\left(dg' \mid g\right) \end{array} \right] \right\} \right\}.$$

The function $V_j(b,y,g,h,x)$ is computed as follows:

$$V_j(b,y,g,h,0) = \max\{V_{j1}(b,y,g,h), V_{j0}(b,y,g,h)\},$$

and

$$V_j(b,y,g,h,1) = \beta_j \left[\begin{array}{l} u\left(y\left(1-h\lambda\right)\right) + ... \\ \pi \int \left[\begin{array}{l} \phi W_j(0,y',g',0,0) + ... \\ (1-\phi)\, W_j(0,y',g',0,1) \end{array} \right] F_g\left(dg' \mid g\right) ... \\ + (1-\pi) \int \left[\begin{array}{l} \phi V_j(0,y',g',0,0) + ... \\ (1-\phi)\, V_j(0,y',g',0,1) \end{array} \right] F_g\left(dg' \mid g\right) \end{array} \right].$$

Let

$$d_j\left(b,y,g,h\right) = \begin{cases} 1 & \text{if } V_{j1}\left(y,g,h\right) > V_{j0}\left(b,y,g,h\right) \\ 0 & \text{if } V_{j1}\left(y,g,h\right) \le V_{j0}\left(b,y,g,h\right) \end{cases}.$$

denote the default decision of government j. The function $W_j\left(b,y,g,h,x\right)$ is computed

as follows:

$$W_j\left(b,y,g,h,1\right) = \beta_j \left[\begin{array}{c} u\left(y\left(1-h\lambda\right)\right)+\ldots \\ \pi \int \left[\begin{array}{c} \phi V_j(0,y',g',0,0)+\ldots \\ \left(1-\phi\right)V_j(0,y',g',0,1) \end{array} \right] F_g\left(dg' \mid g\right)\ldots \\ +\left(1-\pi\right)\int \left[\begin{array}{c} \phi W_j(0,y',g',0,0)+\ldots \\ \left(1-\phi\right)W_j(0,y',g',0,1) \end{array} \right] F_g\left(dg' \mid g\right) \end{array} \right].$$

If $d_{-j}\left(b,y,g,h\right) = 1$,

$$W_j\left(b,y,g,h,0\right) = \begin{array}{c} \left(1-\phi\right)\left[\beta_j \left[\begin{array}{c} u\left(y\left(1-h\lambda\right)\right)+\ldots \\ \pi \int \left[\begin{array}{c} \phi V_j(0,y',g',1,0)+\ldots \\ \left(1-\phi\right)V_j(0,y',g',1,1) \end{array} \right] F_g\left(dg' \mid g\right)\ldots \\ +\left(1-\pi\right)\int \left[\begin{array}{c} \phi W_j(0,y',g',1,0)+\ldots \\ \left(1-\phi\right)W_j(0,y',g',1,1) \end{array} \right] F_g\left(dg' \mid g\right) \end{array} \right] \right] + \\ \phi \left[\begin{array}{c} u\left(y\left(1-h\lambda\right)-q_{-j1}\left(b'_{-j1}\left(y,g,h\right),y\right)b'_{-j1}\left(0,y,g,h\right)\right)+\ldots \\ \beta_j \left[\begin{array}{c} \pi \int V_j(b'_{-j1}\left(y,g,h\right),y',1,0)F_g\left(dg' \mid g\right)\ldots \\ +\left(1-\pi\right)\int W_j(b'_{-j1}\left(y,g,h\right),y',g',1,0)F_g\left(dg' \mid g\right) \end{array} \right] \end{array} \right] \end{array},$$

where $b'_{j1}\left(y,g,h\right)$ denotes the optimal saving behavior of government j after default

(when j is not excluded), and $b'_{j0}\left(b,y,g,h\right)$ denotes the optimal savings of government

j when it has decided to pay back its debt. If $d_{-j}\left(b,y,g,h\right) = 0$,

$$W_j\left(b,y,g,h,0\right) = \beta_j \left[\begin{array}{c} u\left(y\left(1-h\lambda\right)+b-q_{-j0}\left(b'_{-j0}\left(b,y,g,h\right),y\right)b'_{-j0}\left(b,y,g,h\right)\right)+\ldots \\ \pi \int V_j(b'_{-j0}\left(b,y,g,h\right),y',0,0)F_g\left(dg' \mid g\right)\ldots \\ +\left(1-\pi\right)\int W_j(b'_{-j0}\left(b,y,g,h\right),y',g',0,0)F_g\left(dg' \mid g\right) \end{array} \right].$$

The price of a bond issued by government j if a default decision d was made in the current period satisfies the following zero profit condition:

$$q_{jd}\left(b',y,g\right) = \frac{1}{1+r}\left[1 - \pi E\left[d_{-j} \mid b',y,g,h'\right] - (1-\pi)E\left[d_j \mid b',y,g,h'\right]\right],$$

where

$$E\left[d_j \mid b',y,g,h'\right] = \int d_j\left(b',y',g',h'\right)F_g\left(dg' \mid g\right)$$

denotes the probability that government j decides to default if the current government purchases b' bonds, the current endowment is y, the growth rates is g, and the current default decision is d.

3 A benchmark with homogeneous borrowers and exogenous exclusion

In this section we present results with exogenous exclusion and homogeneous borrowers. That is, we assume that $\phi < 1$, and that $\beta_h = \beta_l = \beta$. We shall show that our benchmark produces results that are very similar to the results in previous studies— even though in our benchmark there is only one period of output loss, and a defaulting country may not be excluded in the default period.

When possible, we use the calibration in AG. Parameter values are presented in Table 1.

Table 1. Parameter values		
Risk aversion	σ	2
Interest rate	r	1%
Probability of redemption	ϕ	10%
Mean growth rate	μ_g	1.006
Autocorrelation coefficient	ρ_g	0.17
Standard deviation	σ_g	3%
Discount factor	β	0.8
Loss of output	λ	10%

The coefficient of relative risk aversion of 2 is standard. Each period refers to a quarter with a quarterly risk free interest rate of 1%. The probability of redemption implies an average stay in autarky of 2.5 years as in AG—in their paper, the probability of redemption is 10% but a defaulting country is excluded with probability one in the first period. The process of output is calibrated to match the process for Argentina (see AG). As in previous studies, high impatience is necessary to generate default in equilibrium. As explained above, in our framework, the costs of default are slightly different from the costs in previous studies—the country may not be excluded in the default period, and the loss of output occurs in only one period. In order to make our results comparable with the results in previous studies, we pick a value of λ that gives us the default probability in AG.

To solve the model numerically, we first recast the Bellman equations in detrended form. As in AG, to detrend, we normalize all variables by $\mu_g y_{t-1}$. Table 2 reports key business cycle moments in the data (Argentina; 1983.1-2000.2), from our benchmark simulations, and from Hatchondo, Martinez, and Sapriza (2006) (hereafter denoted by

HMS).[4] Standard deviations are denoted by σ (), and are reported in percentage terms; correlations are denoted by ρ (). We log the income (Y) and consumption series and compute the trade balance (TB) and (annualized) interest rate spread (R_s). All series are HP filtered with a smoothing parameter of 1600. The log of income and consumption are denoted by y and c respectively. Table 2 shows that our benchmark produces results that are consistent with the results in previous studies—even though in our benchmark there is only one period of output loss, and a defaulting country may not be excluded in the default period.

Table 2. Benchmark 1 (with exogenous exclusion)			
	Data	HMS	Benchmark 1
$\sigma(y)$	4.08	4.43	4.43
$\sigma(c)$	4.85	4.64	4.66
$\sigma(TB/Y)$	1.36	0.65	0.72
$\sigma(R_s)$	3.17	0.01	0.02
$\rho(c,y)$	0.96	0.99	0.99
$\rho(TB/Y,y)$	-0.89	-0.26	-0.25
$\rho(R_s,y)$	-0.59	-0.07	-0.11
$\rho(R_s,TB/Y)$	0.68	0.91	0.92
Rate of default (per 10,000 quarters)	75	22	22
Mean debt output ratio (%)		19	21
Maximum R_s (basis point)		97	96

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[4]HMS recalculate the business cycle moments in AG. The data for Argentina is from AG. We simulate the model for 10,000 periods and extract the last 500 observations to rule out any effect of initial conditions. We run 500 such simulations. We compute the business cycle moments from our simulations without considering the exclusion periods and without considering the first two periods after exclusion—in these period, the government does not borrow much and, consequently, it pays very low spreads (see HMS).

4 A benchmark with homogeneous borrowers and without exogenous exclusion (TO BE COMPLETED)

In this section we present results with homogeneous borrowers and without exogenous exclusion. That is, we assume that $\phi = 1$, and that $\beta_h = \beta_l = \beta$. Previous studies assume that after defaulting countries are exogenously excluded from capital markets ($\phi < 1$). The exogenous exclusion assumption may appear to be at odds with the assumption of competitive lenders. It is unlikely that after a default episode, competitive creditors could coordinate to cut off credit to defaulting countries (see, for example, Cole, Dow, and English (1995), and Athreya and Janicki (2006)). Moreover, empirical evidence indicates that once other variables are control for, market access is not significantly influenced by previous defaults (see, for example, Gelos, Sahay, and Sandleris (2003)).

Figure 1 shows that the average debt level is very similar before and after a default episode—the first period after default the government does not borrow much because its debt level is low. That is, the model does not generate difficulties in market access after a default episode. This is inconsistent with the observed governments' reluctance to issue new debt after defaulting.

Table 3 shows the business cycle statistics calculated without exogenous exclusion— to facilitate comparisons, we also report the statistics in the data and from the model with exogenous exclusion.[5]

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[5] The business cycle statistics without exogenous exclusion are calculated considering all periods. We plan to recalculate these statistics without considering outliers (see HMS).
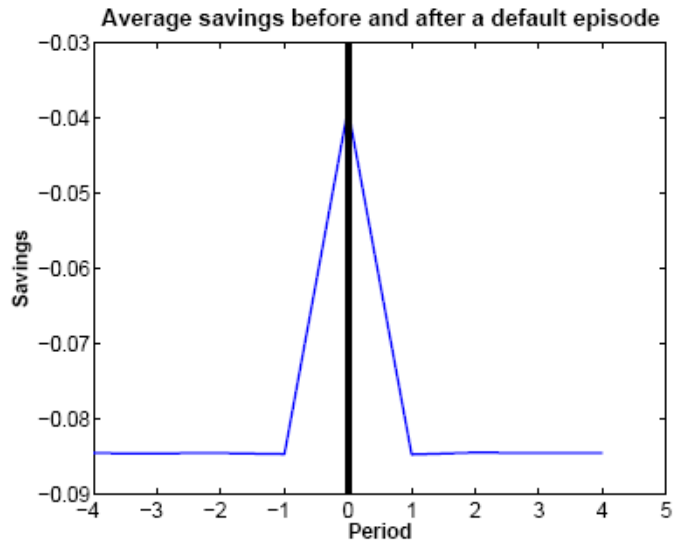
Figure 1:

Table 3. Benchmark 2 (with exogenous exclusion)			
	Data	Benchmark 1	Benchark2
$\sigma(y)$	4.08	4.43	4.45
$\sigma(c)$	4.85	4.66	4.51
$\sigma(TB/Y)$	1.36	0.72	0.35
$\sigma(R_s)$	3.17	0.02	0.03
$\rho(c,y)$	0.96	0.99	1
$\rho(TB/Y,y)$	-0.89	-0.25	-0.15
$\rho(R_s,y)$	-0.59	-0.11	0.20
$\rho(R_s,TB/Y)$	0.68	0.92	0
Rate of default (per 10,000 quarters)	75	22	18
Mean debt output ratio (%)		21	8
Maximum R_s (basis point)		96	115

Table 3 shows that the mean debt level generated by the model is lower without

exclusion. Moreover, when exogenous exclusion is not assumed, spreads are procyclical, and there is no correlation between spreads and trade balances.

5 Heterogenous borrowers (TO BE COMPLETED)

In this section, we allow the composition of the government or the distribution of power within the government to change each period. In particular, we allow the government's willingness to pay to change over time We model this by assuming that two types of governments alternate in power. The patient government discounts future utility flows less than the impatient government, that is, we assume that $\beta_h > \beta_l$. It is also assumed that $\phi = 1$, that is, the economy is never exogenously excluded from capital markets, and the government's ability to obtain loans depends only on the probability of repay.

We show how allowing for government heterogeneity can improve the quantitative performance of the standard model of sovereign default. We find that even without assuming exogenous exclusion, the model generates difficulties in market access after default episodes—for the same level of debt, average spreads are higher. Moreover, after default, capital inflows are initially decreased, and later recover slowly. This is consistent with the observed governments' reluctance to issue new debt after defaulting. Furthermore, consistently with historical evidence, in our model, market access improves after the defaulting government loses power. We also show that considering heterogeneous borrowers allows the standard model to generate the default probability observed in the data, and consequently it allows it to generate higher spreads than a model without heterogeneity. The model also generates higher volatility in spreads than a model without heterogeneity. We also provide insights on how do changes in government stability impact on the default risk, and thus, on spreads. We shall also

describe the strategic interaction of governments with different patience. As previous studies, we also capture some of the main empirical regularities regarding emerging markets.

First, we assume that the $\beta_h = 0.9$ and $\beta_l = 0.6$. That is, we assume that patient governments are more patient than governments in the benchmarks presented in sections 3 and 4, and impatient governments are more impatient than governments in those benchmarks. We also present results with alternative parametrizations and we explain the role of both the patience level and the patience variability.

If the model generates default episodes when a patient government is replaced with an impatient government, the probability of a change in the government type, π, is closely related with the default probability. We calibrate π to match the default probability in the data. Following AG, Table 2 reports a default probability in the data of 0.75% that is consistent with the average number of crisis for emerging markets with at least one default or restructuring episode between 1824 and 1999 presented by Reinhart, Rogoff, and Savastano (2003). First we assume that $\pi = 0.015$. With this value for π, if the model generates default episodes when a patient government is replaced with an impatient government, the probability of this type of default is 0.75% (this parametrization is preliminary, we find that both patient and impatient governments may default without a government change; the total default probability generated with this parametrization is 0.97%).

Figures 2 shows the levels of assets and growth for which each government type would choose to default. The grey area is the region for which both types would default, and the black area is the region for which only an impatient government would default. As one would expect, this figure shows that impatient governments are more
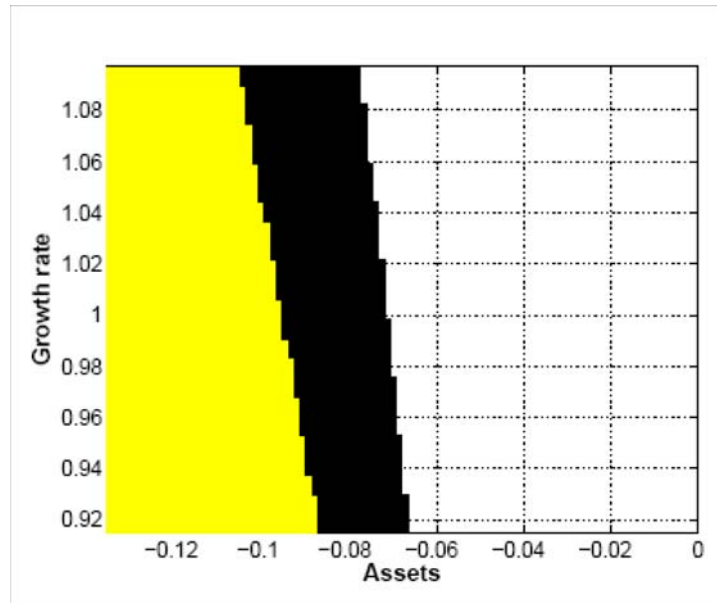
Figure 2:

likely to default.

Figures 3 shows the bond price that a patient government faces for the average growth rate. The price function in Figure 3 is close to a step function. With low debt levels, no government would default, and spreads are low (bond prices are high). With intermediate debt levels, only impatient governments would default, and spreads are intermediate. With high debt levels, both government types would default, and spreads are very high.

Figure 4 shows the objective function for this government for $b = -0.083$—given that this function is not concave, we use global search to find the optimal borrowing level; the government would choose a low level of debt for low initial debts and low output growth rates, but this does not occur in our simulations. It shows that patient governments would choose an "intermediate" level of debt and, therefore, would pay an
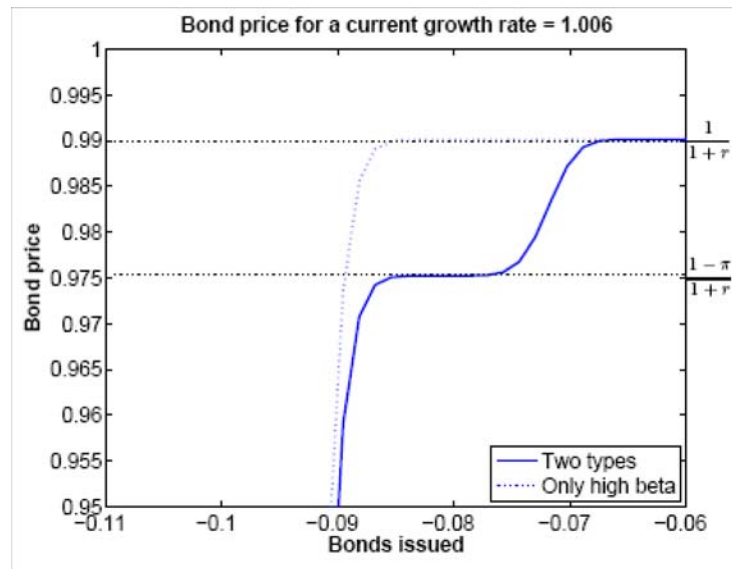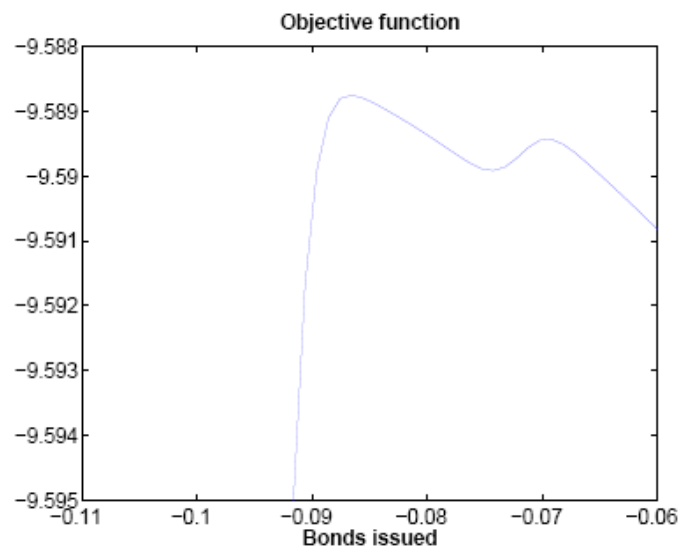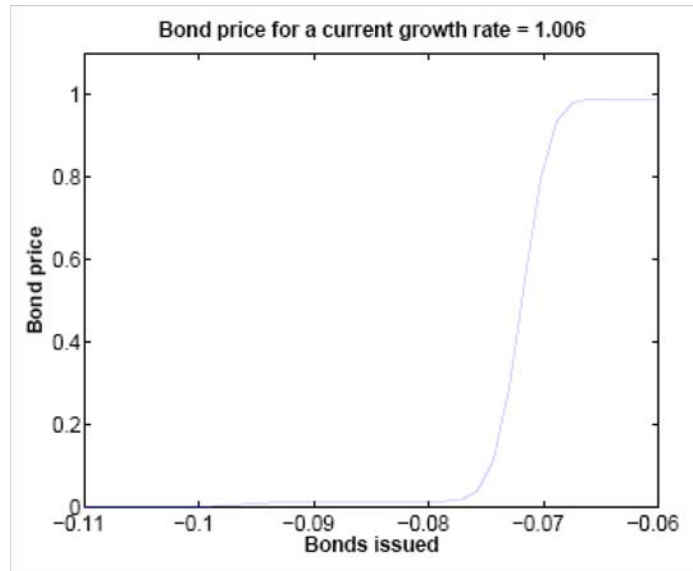
Figure 3:

Figure 4:

Figure 5:

intermediate spread.

In a model without heterogeneous governments, there are no "intermediate" spreads (see Figure 3). That is, bond prices fall sharply between the high price and the low price. Consequently, the government always chooses to pay low spreads.

Thus, we show that introducing government heterogeneity helps the standard framework to generate higher spreads—closer to the data. The maximum annualized spread generated in the benchmark is 112 basis points (see Table 3), and with heterogenous governments, the maximum spread is 670 basis points.

Figures 5 shows the bond price that a impatient government faces for the average growth rate. As the price function in Figure 3, the price function in Figure 4 is also close to a step function. The intermediate step in Figure 5 is very close to zero because the impatient government is very likely to stay in power and, therefore, the default probability is very high for the intermediate debt levels.
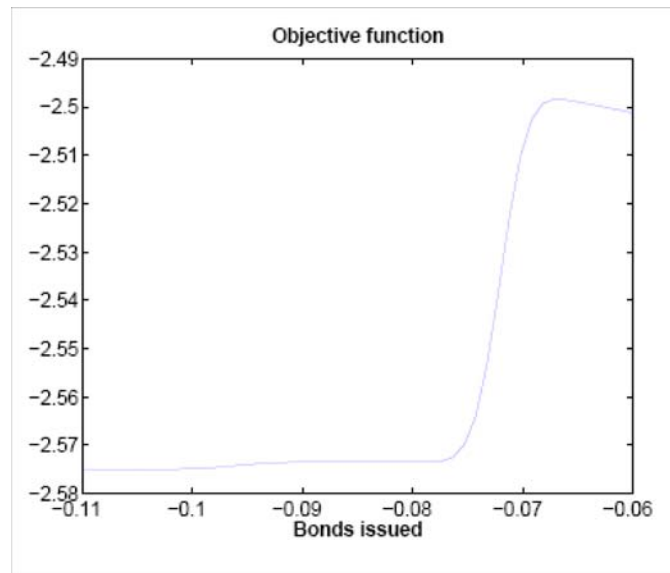
Figure 6:

Figure 6 shows the objective function for this government for a $b = -0.083$. It shows that impatient governments would choose a "low" level of debt and, therefore, would pay a low spread.

Consequently, with our benchmark parametrization, even though impatient governments assign more weight to current consumption, they decide to borrow less than patient governments. This is the case because impatient governments are more likely to default and, therefore, face higher borrowing costs.

In our simulations, in their first period in office, impatient governments typically find themselves with a debt level higher than the maximum debt level they would choose to pay. Approximately 75% of the default episodes arise when a patient government is replaced by an impatient one. The default probability is 0.99% considerably higher than the default probability with homogenous governments (0.18%; see Table 3). The alternation in power of different government types is crucial to generate a higher default
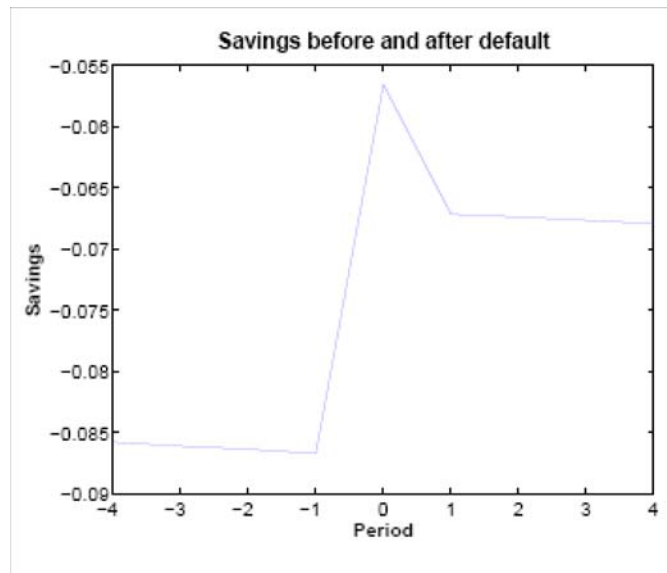
Figure 7:

probability. The default probability in an economy with only impatient governments $(\beta_l = \beta_h = 0.6)$ is 0.38%.

Even without assuming exogenous exclusion, the model generates difficulties in market access after a default episode. Figure 7 shows that the average debt level before and after a default episode. After default episodes, impatient governments are more likely to be in power and, therefore, the equilibrium debt level is lower—the first period after default the government borrow less because its debt level is low. That is, after default, capital inflows are initially decreased, and later recover slowly. This is consistent with the observed governments' reluctance to issue new debt after defaulting. Furthermore, consistently with historical evidence, in our model, market access improves after the defaulting government loses power. Given that in the model there is a cost of defaulting that does not depend on the level of debt (the output loss), there are low enough debt levels such that no government would default. Consequently, every government can

borrow these levels at the risk free rate, and there is never zero borrowing.

In general, equilibrium spreads depend on the equilibrium probability of default which depends on the probability of facing a government likely to pay in the future. In our stylized model, the probability of facing a government likely to pay (patient) in the future depends on the type of the government in power which changes over time generating volatility in spreads. Thus, the model produces higher spread volatility—closer to the volatility in the data—than a model without heterogeneity.

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