Adoption Lags, Implementation Gaps, and Economic Growth

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Abstract

We introduce a model of endogenous growth in which the returns to innovation are determined by the technology adoption decisions of the users of the new innovative technologies. The technology adoption decisions in our model consist of two dimensions. The first is when to adopt a new technology. The second is at what initial productivity level to adopt it and which part of its productivity potential to learn by doing. Our model economy is one with realistic adoption curves for each technology, the shape of which are an important determinant of the return to innovations and thus of economic growth.

keywords: endogenous growth, learning by doing, technology adoption, vintage capital.

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1 Introduction

In recent years, the theoretical literature on economic growth and technological progress has basically struggled with two main questions.

The first is: "What mechanisms and incentives move the world technology frontier?". This question is at the heart of the literature on endogenous growth.

The second question is: "Why do not all countries use the most advanced technologies that they have access to?".

The two leading answers to this question that have emerged from the literature are (*i*) that institutional barriers in many countries prevent users from adopting the best practice technologies, as in Parente and Prescott (1994), and (*ii*) that the new and best practice technologies are developed to complement a skill set that is available in less-developed countries and are thus not appropriate for them to use, as in Basu and Weil (1998) and Acemoglu and Zilibotti (????).

Even though thinking about these two main questions separately is interesting in itself, the answers to them are inherently intertwined. That is, the speed with which technologies are adopted and implemented affects the profits made using them which in turn finance the research and development needed to invent them.

In this paper, we introduce a model that considers both endogenous invention and adoption of technologies. We use this model to show how incentives to adopt technologies affect the return to the introduction of such technologies and thus the amount of R&D done in an economy which determines the pace of economic growth.

What emerges from our joint consideration of incentives to invent and adopt is that what used to look like barriers to riches in the context of Parente and Prescott (1994) are also barriers to invention. This significantly changes the welfare impact of policies that move countries closer to the technological frontier and speed up their adoption of technologies. Such policies do not only have the potential to substantially increase the standard of living of the current generation they also affect the pace of technological change and therefore have an incrementally important effect going forward.

The structure of our paper is as follows. In the next section we introduce the four sectors of economic activity that make up our model economy. In Section 3, we solve for the equilibrium path of this economy and derive the dynamic system that describes this path. In Section 4, we calibrate our model and illustrate and quantify the importance of the rate of technology adoption for the incentives to innovate and the rate of growth.

2 The model

The model economy that we consider consists of four sectors. The first is the household sector. The second is the final goods sector. The third is the intermediate goods sector, while the final sector is the R&D sector that innovates.

In this section, we introduce each of these sectors separately. We consider the equilibrium outcome when they interact in the next section.

2.1 Household sector

The representative household in our model economy is endowed with one unit of time each period and supplies this unit of time of labor inelastically. It aims to maximize the present discounted value of the flow of its utility. Throughout, we will assume that is present discounted value is of the form

$$\int_t^{\infty} e^{-\rho s} \frac{\sigma}{\sigma - 1} c_s^{\frac{\sigma-1}{\sigma}} ds \tag{1}$$

where c_t is final goods consumption.

It does so subject to the following flow budget constraint

$$\dot{a}_t = r_t a_t + w_t + \pi_t - c_t \tag{2}$$

Here a_t denotes the assetholdings of the household in terms of the final good, r_t is the real interest rate, w_t is the real wage rate that is paid to the household in compensation for its supply of its one unit of labor, and π_t are the profits the are redistributed from firms to the households.

The resulting optimal consumption Euler equation is of the form

$$\frac{\dot{c}_t}{c_t} = \sigma \left(r_t - \beta \right) \tag{3}$$

As Caselli and Ventura (2000) have shown, because the growth rate of consumption for a household is the same independent of its level of wealth, the distribution of profits among households does not matter for aggregate household behavior. Therefore, we will ignore distributional effects in the rest of this paper and assume that all households receive the same dividend payments, π_t.

2.2 Final goods sector

The final goods that are consumed by the households are produced using a set of intermediate goods combined using a CES technology. Let y_t denote final goods output at time t and y_{vt} be the inputs of intermediate good of vintage v at time t. The range of intermediate goods used in production increases over time. This increase is the major source of technological progress in this economy. In this sense, the model here is a model of increasing varieties and quality ladders in the tradition of Romer (1990)[1]. The production function for final goods is given by

$$y_t = \left(\int_{-\infty}^{\overline{v}_t} y_{vt}^{\theta} dv \right)^{1/\theta} \quad \text{where } 0 < \theta < 1 \tag{4}$$

Here $\overline{v}_t$ is the newest vintage of intermediates used in production. We will be more specific about what we mean by 'newest' in the next section when we introduce the intermediate goods producing sector. This newest vintage will be endogenously determined in our model in the sense that an increase in $\overline{v}_t$ reflects the adoption of new technologies at time t.

The market for final goods is assumed to be perfectly competitive, such that its factor allocation can be represented as resulting as that of a single firm choosing its factor demands to equate marginal products to their corresponding factor prices.

These factor prices are given by the prices of the intermediate goods, which are given by $p_{v,t}$. We normalize the price of the final good to unity by using it as the numeraire good throughout our analysis.

Given these prices, the intermediate goods demand for goods of vintage v at time t as a function of its own price is given by

$$y_{vt} = \left(\frac{1}{p_{vt}} \right)^{\frac{1}{1-\theta}} y_t \tag{5}$$

[1] In principle, one can add a scrappage margin such that intermediate goods that are not productivity enough are not used in production anymore. This would add a creative destruction element to the model. This margin would dramatically complicate the math and not add a lot to the main intuition, though.

which reflects the demand curve faced by the producers of the intermediate goods.

2.3 Intermediate goods producers

The intermediate goods producers are at the heart of our economy. They are the ones that make the technology adoption decisions that determine the return to innovations invented in the R&D sector that we consider in the next subsection.

Each type of intermediate good is provided by a single supplier. Because of the fact that these goods are only imperfect substitutes in final goods production, the suppliers of these goods compete monopolistically. The decisions of these intermediate goods suppliers can be divided into two parts.

The first set of decisions pertains to the price set as well as the factor demand choices. The second set of choices has to do with the choice of adoption of the intermediate goods producing technology, i.e. the entry in the market, as well as with the implementation of this technology and the path of its productivity level.

In this section we will solve for these two parts of the intermediate goods producer's problem sequentially. We will first solve for its optimal factor demands and price setting choice as a function of the paths of its own productivity level, factor prices, and final goods demand. Subsequently, we will then introduce and solve the optimal technology adoption and implementation choice that the intermediate goods producers make.

2.3.1 Factor demands and price setting

Intermediate goods are produced using capital and labor that are combined using a Cobb-Douglas technology of the form

$$y_{vt} = z_{vt} k_{vt}^{\alpha} l_{vt}^{1-\alpha} \quad \text{where } 0 < \alpha < 1 \tag{6}$$

Labor is assumed to be homogenous and each intermediate goods producer hires workers at the competitive real wage rate w_t.

What makes the producer of a particular intermediate goods vintage special in this economy is that it is the sole firm that has access to the production technology of the vintage. This production technology is embodied in the type of capital that the firm uses.

Each intermediate goods producer has the sole knowledge on how to convert a unit of the final good into a unit of the vintage specific capital good k_{vt}. This process is reversible. It can then use the available vintage specific capital stock, k_{vt}, to produce intermediate goods at the embodied productivity level z_{vt}.

Capital goods used in production are assumed to depreciate at a constant rate δ. This implies that the capital stock used in the production of intermediate good of vintage v follows the perpetual inventory rule of the form

$$\dot{k}_{vt} = i_{vt} - \delta k_{vt} \tag{7}$$

where i_{vt} is the level of gross investment by supplier v at time t and δ reflects the physical depreciation rate, which we will assume to be constant across vintages.

In each period, the flow profits of the intermediate goods producer of vintage v, denoted by π_{vt}, are given by the difference between its revenue and its factor costs. That is

$$\pi_{vt} = p_{vt}y_{vt} - w_t l_{vt} - i_{vt} \tag{8}$$

At time t, given its level of capital, k_{vt}, and the paths of the factor prices, w_t and r_t, its productivity level, z_{vt}, as well as final goods demand, y_t, the firm chooses its path of gross investment i_{vt}, its labor input, l_{vt}, its output level, y_{vt}, and its price p_{vt} to maximize the present discounted value of its flow profits. That is, the objective of the firm is to maximize

$$V_{vt} = \int_t^{\infty} e^{-\int_t^s r_j dj} \pi_{vs} ds \tag{9}$$

It does so subject to the definition of flow profits, (8), the vintage production function, (6), the capital accumulation equation, (7), and the demand function, (5).

This dynamic profit maximization problem yields that the firm will set its price equal to its marginal cost times a constant factor that represents its gross markup. Mathematically, this yields

$$p_{vt} = \frac{1}{\theta} mc_{vt} \tag{10}$$

where $1/\theta > 1$ is the gross markup factor and

$$mc_{vt} = \left[\left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \right] \frac{1}{z_{vt}} w_t^{1-\alpha} uc_t^{\alpha} \tag{11}$$

Here, $uc_t = r_t + \delta$ reflects the real user cost of capital.

The resulting factor demands of the firm satisfy

$$w_t = \theta (1 - \alpha) p_{vt} \frac{y_{vt}}{l_{vt}} \tag{12}$$

and

$$uc_t = \theta \alpha p_{vt} \frac{y_{vt}}{k_{vt}} \tag{13}$$

The resulting level of flow profits in each period is given by

$$\pi_{vt} = (1 - \theta) p_{vt} y_{vt} \tag{14}$$

Such that the resulting value of the firm equals

$$V_{vt} = (1 - \theta) \int_t^{\infty} e^{-\int_t^s r_j dj} p_{vs} y_{vs} ds \tag{15}$$

which will simplify after we aggregate all firms in the intermediate and final goods sector.

As we show in the mathematical appendix, the firms' decisions in the final and intermediate goods sector allow us to use an aggregate production function representation of the form

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha} \tag{16}$$

where the aggregate capital and labor inputs are given by

$$k_t = \int_{-\infty}^{\overline{v}_t} k_{vt} dv \text{ and } l_t = \int_{-\infty}^{\overline{v}_t} l_{vt} dv \tag{17}$$

Furthermore, the aggregate level of total factor productivity, is given by a CES aggregate of the underlying vintage specific productivity levels. In particular,

$$z_t = \left[\int_{-\infty}^{\overline{v}_t} z_{vt}^{\frac{\theta}{1-\theta}} dv \right]^{\frac{1-\theta}{\theta}} \tag{18}$$

Furthermore, the shares of output and inputs of the individual firms in the aggregate are given by

$$\frac{y_{vt}}{y_t} = \left(\frac{z_{vt}}{z_t} \right)^{\frac{1}{1-\theta}}, \frac{k_{vt}}{k_t} = \frac{l_{vt}}{l_t} = \left(\frac{z_{vt}}{z_t} \right)^{\frac{\theta}{1-\theta}}, \text{ and } \frac{p_{vt}}{p_t} = p_{vt} = \left(\frac{z_t}{z_{vt}} \right) \tag{19}$$

The aggregate factor demands turn out to satisfy the same optimality conditions as the factor demands of the individual firms in the sense that

$$w_t = \theta (1 - \alpha) \frac{y_t}{l_t} \tag{20}$$

and

$$uc_t = \theta \alpha \frac{y_t}{k_t} \tag{21}$$

This aggregate production function representation now allows us to rewrite the value of the firm as

$$V_{vt} = (1 - \theta) \int_t^\infty e^{-\int_t^s r_j dj} \left(\frac{z_{vs}}{z_s} \right)^{\frac{\theta}{1-\theta}} y_s ds \tag{22}$$

which is only a function of the path of the firm's productivity level z_{vs}, as well as that of aggregate productivity, the real interest rate, and output levels.

2.3.2 Technology adoption and implementation

The technology adoption and implementation decisions that an intermediate goods producer makes determine the time the intermediate goods vintage is brought online and at which productivity level it is initially implemented respectively. In this section we will introduce the costs and benefits related to these two decisions and solve for the implied optimal choices.

In order to do so, we first have to describe the path of productivity after the adoption and implementation of the technology. This is what we will do in the first part of this subsection. In the second part we then consider what determines the optimal choices for these decisions.

Learning by doing Suppose that the firm uses its technology at productivity level z_{vt} at time t. We will assume that it gets more productive the longer it uses the technology. That is, we will assume that intermediate goods firms learn-by-doing. The specific functional form that we use for the learning-by-doing mechanism is similar to the one used by Parente (1994) and Basu and Weil (1998).

We will assume that, ultimately, the productivity of each technology vintage converges to a maximum feasible level, $\overline{z}_v$. Denote the normalized relative distance of current productivity from the feasibility frontier as

$$x_{vt} = (z_{vt}/\overline{z}_v)^{\frac{\theta}{1-\theta}} \tag{23}$$

then we assume that z_{vs} converges to $\overline{z}_v$ according to

$$\dot{x}_{vt} = \lambda (1 - x_{vt}) \tag{24}$$

This learning pattern implies that

$$z_{vs}^{\frac{\theta}{1-\theta}} = \left(1 - e^{-\lambda(s-t)}\right) \overline{z}_v^{\frac{\theta}{1-\theta}} + e^{-\lambda(s-t)} z_{vt}^{\frac{\theta}{1-\theta}} \text{ for } s > t \tag{25}$$

and thus, the whole future path of vintage specific productivity z_{vs} is determined by its current level z_{vt}. Hence, the choice of productivity at the time the technology is adopted determines the future path of productivity for the rest of the lifetime of the intermediate goods vintage.

Given this path of productivity, the value of the firm can be written solely as a function of its current productivity level and the paths of aggregate productivity, the real interest rate, and output. That is, the value function simplifies to

$$V_{vt}(x_{vt}) = (1-\theta)\overline{z}_v^{\frac{\theta}{1-\theta}} \int_t^\infty e^{-\int_t^s r_j dj} \left(1 - e^{-\lambda(s-t)}\right) \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds + \tag{26}$$

$$(1-\theta)\overline{z}_v^{\frac{\theta}{1-\theta}} x_{vt} \int_t^\infty e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \tag{27}$$

where the relative distance to the frontier, x_{vt}, is the only decision variable that the firm has control over. Note that, given the other variables, the value function is linear in x_{vt}.

Adoption and implementation Technologies are licensed to the intermediate goods producers. They pay a licensing fee that allows them to use the technology vintage infinitely long into the future. This licensing fee depends on the level of advancement, i.e. $\overline{z}_v$, of the technology that they adopt.

The licensing fee is not the only thing that the firms have to pay to start using the technology, though. Before starting to use a technology, a firm first has to implement it. This implementation requires the use of the final good as an input. We will assume that there are increasing marginal costs to implementing a more advanced technology and that technologies that are just invented require an infinite implementation cost.

Formally, a firm that adopts a new technology at time t has to make two decisions. The first is which technology to adopt, i.e. the choice of vintage v. The second is at which productivity level to implement it, i.e. the choice of the relative productivity level x_{vt}.

We will assume that each of these two decisions are associated with their own specific costs. The adoption cost part of this decision is given by

$$c_t^{adopt}(v) = c_{at} y_t \left(\frac{1}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left(\frac{\overline{z}_v^2}{\overline{z}_t^* - \overline{z}_v}\right) \tag{28}$$

Here $\overline{z}_t^*$ represents the maximum feasible productivity level of the best technology invented at time t, while c_{at} is the time varying adoption cost parameter. This functional form is just a conjecture. We will show that it is consistent with equilibrium in R&D sector that we consider in the next section.

The second cost, the implementation cost, is given by

$$c_t^{implement}(z_{vt}) = c_i y_t \left(\frac{1}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left(\frac{z_{vt}^2}{\overline{z}_v - z_{vt}}\right) \tag{29}$$

where c_i is the implementation cost parameter.

The problem of the entrant can be solved sequentially. In the first step we solve for the optimal level of technology implementation conditional on the technology vintage, v, being implemented. In the second step, we solve for the free entry condition, which implies that all technologies are implemented up till the one for which the new entrants make zero profits in the sense that the implementation and adoption costs exhaust the value of the firm, derived in (26).

The optimal implementation decision of a firm that adopts technology vintage v at time t is that level of normalized relative productivity, x_{vt}, that maximizes

$$V_{vt}\left(\frac{z_{vt}}{\overline{z}_v}\right) - c_t^{implement}(z_{vt}) \tag{30}$$

As we show in Appendix A, the solution to this choice is the root of a quadratic equation and yields

$$x_{vt} = \frac{z_{vt}}{\overline{z}_v} = 1 - \sqrt{\frac{c_i}{c_i + b_{xvt}}} \tag{31}$$

where

$$b_{xvt} = (1-\theta)\left(\frac{\overline{z}_t^*}{z_t}\right)^{\frac{\theta}{1-\theta}} \int_t^\infty e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds \tag{32}$$

2.4 R&D sector

3 Equilibrium

3.1 Definition

3.2 Steady state

3.3 Dynamics

4 Adoption and growth

References

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A Mathematical details

Derivation of equations (10) through (15):
The Lagrangian associated with the firm's profit maximization can be written as

$$\int_t^\infty e^{\int_t^s r_j dj} H_{vs} ds$$

Where the current value Hamiltonian, H_{vt}, is given by

$$\begin{aligned} H_{vt} = \pi_{vt} + \mu_{\pi t} \quad & (\pi_{vt} - p_{vt}y_{vt} + w_t l_{vt} + i_{vt}) + \mu_{yt+s}\left[y_{vt+s} - z_{vt+s}k_{vt+s}^{\alpha} l_{vt+s}^{1-\alpha}\right] \\ & + \mu_{kt+s}\left[i_{vt+s} - \delta k_{ivt+s}\right] + \mu_{dt+s}\left(y_{vt} - p_{vt}^{-\frac{1}{1-\theta}} y_t\right) \end{aligned} \quad (33)$$

The first order necessary conditions implied by this objective function are

$$\text{w.r.t. } \pi_{vt} \quad : \quad 1 = -\mu_{\pi t} \quad (34)$$

$$\text{w.r.t } y_{vt} \quad : \quad \mu_{yt} = p_{vt}\mu_{\pi t} - \mu_{dt} \quad (35)$$

$$\text{w.r.t. } l_{vt} \quad : \quad w_t \mu_{\pi t} = (1-\alpha)\,\mu_{yt}\frac{y_{vt}}{l_{vt}} \quad (36)$$

$$\text{w.r.t. } i_{vt} \quad : \quad \mu_{\pi t} = -\mu_{kt} \quad (37)$$

$$\text{w.r.t. } k_{vt} \quad : \quad -\alpha\mu_{yt}\frac{y_{vt}}{k_{vt}} - \mu_{kt}\delta = r_t\mu_{kt} - \dot{\mu}_{kt} \quad (38)$$

$$\text{w.r.t. } p_{vt} \quad : \quad \mu_{\pi t} y_{vt} = \mu_{dt}\frac{1}{1-\theta}\frac{y_{vt}}{p_{vt}} \quad (39)$$

Condition (39) implies that

$$\mu_{dt} = (1-\theta)\,p_{vt} \quad (40)$$

Combining this with (34) and (35) yields that

$$\mu_{yt} = -\theta p_{vt} \quad (41)$$

When we substitute this into the optimal labor demand conditions, (36), we find

$$w_t l_{vt} = \theta\,(1-\alpha)\,p_{vt}y_{vt} \quad (42)$$

The optimal investment condition which equates the marginal cost of a unit of profits with the marginal value of an additional unit of capital, i.e. (37), yields that

$$\mu_{kt} = 1 \quad (43)$$

Substituting this into the capital Euler equation,(38), we obtain that

$$\alpha\theta\frac{p_{vt}y_{vt}}{k_{vt}} = r_t + \delta = uc_t \quad (44)$$

We can use these results to solve for the optimal capital labor ratio, which satisfies

$$\left(\frac{k_{vt}}{l_{vt}}\right) = \left(\frac{w_t}{\theta\,(1-\alpha)\,p_{vt}z_{vt}}\right)^{\frac{1}{\alpha}} = \left(\frac{\theta\alpha p_{vt}z_{vt}}{uc_{vt}}\right)^{\frac{1}{1-\alpha}} \quad (45)$$

The price level, p_{vt}, that equates the capital labor ratio for the optimal capital and labor demand decisions, and thus solves the above equation, is

$$p_{vt} = \frac{1}{\theta} mc_{vt}$$

where mc_{vt} is the marginal production cost of vintage v producer and equals the unit production cost implied by the Cobb-Douglas production technology, i.e.

$$mc_{vt} = \frac{1}{z_{vt}} \left[\frac{w_t}{1-\alpha}\right]^{1-\alpha} \left[\frac{uc_{vt}}{\alpha}\right]^{\alpha} \tag{46}$$

and the resulting profit level equals

$$\pi_{vt} = \frac{1-\theta}{\theta} mc_{vt} = (1-\theta)\, p_{vt} y_{vt} \tag{47}$$

Hence, the resulting value of the firm is given by

$$V_{vt} = \int_t^{\infty} e^{-\int_t^s r_j dj} \pi_{vs} ds = (1-\theta) \int_t^{\infty} e^{-\int_t^s r_j dj} p_{vs} y_{vs} ds \tag{48}$$

which corresponds to (15).

Derivation of equations (16) through (21):

Derivation of equation (31):
First of all, in order to see why the maximization problem (30) actually boils down to choosing x_{vt}, it is worthwhile to rewrite the implementation cost function, i.e. (29), as

$$\begin{aligned} c_t^{implement}(z_{vt}) &= c_i y_t \left(\frac{\overline{z}_v}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left(\frac{(z_{vt}/\overline{z}_v)^2}{1-(z_{vt}/\overline{z}_v)}\right) \\ &= c_i y_t \left(\frac{\overline{z}_v}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left(\frac{x_{vt}^2}{1-x_{vt}}\right) = \widetilde{c}_{vt}^{implement}(x_{vt}) \end{aligned} \tag{49}$$

Secondly, it is worthwhile to rewrite the expression for the value function, i.e. (26) as

$$\begin{aligned} V_{vt}(x_{vt}) &= (1-\theta)\,\overline{z}_v^{\frac{\theta}{1-\theta}} \int_t^{\infty} e^{-\int_t^s r_j dj} \left(1-e^{-\lambda(s-t)}\right) \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds + \\ & \quad (1-\theta)\,\overline{z}_v^{\frac{\theta}{1-\theta}} x_{vt} \int_t^{\infty} e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \\ &= (1-\theta) \left(\frac{\overline{z}_v}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left(\frac{\overline{z}_t^*}{z_t}\right)^{\frac{\theta}{1-\theta}} \times \\ & \quad \left[\int_t^{\infty} e^{-\int_t^s r_j dj} \left(1-e^{-\lambda(s-t)}\right) \left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds + \right. \\ & \quad \left. x_{vt} \int_t^{\infty} e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \right] \end{aligned} \tag{50}$$

then we can write the objective that maximized in the firms implementation decision as

$$V_{vt}\left(x_{vt}\right) - \widetilde{c}_{vt}^{implement}\left(x_{vt}\right) = y_t \left(\frac{\overline{z}_v}{\overline{z}_t^*}\right)^{\frac{\theta}{1-\theta}} \left[b_{0vt} + b_{xvt}x_{vt} - c_i\left(\frac{x_{vt}^2}{1-x_{vt}}\right)\right] \tag{51}$$

where

$$\begin{aligned} b_{0vt} &= (1-\theta)\left(\frac{\overline{z}_t^*}{z_t}\right)^{\frac{\theta}{1-\theta}} \int_t^\infty e^{-\int_t^s r_j dj}\left(1-e^{-\lambda(s-t)}\right)\left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds & (52) \\ b_{xvt} &= (1-\theta)\left(\frac{\overline{z}_t^*}{z_t}\right)^{\frac{\theta}{1-\theta}} \int_t^\infty e^{-\int_t^s r_j dj} e^{-\lambda(s-t)}\left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds & (53) \end{aligned}$$

The resulting necessary, and in this case sufficient, first order condition for the optimal choice of x_{vt} is that $(x_{vt}/(1-x_{vt}))$ is the unique positive real root of

$$0 = b_{xvt} - 2c_i\left(\frac{x_{vt}}{1-x_{vt}}\right) - c_i\left(\frac{x_{vt}}{1-x_{vt}}\right)^2 \tag{54}$$

which yields that

$$x_{vt} = 1 - \sqrt{\frac{c_i}{c_i + b_{xvt}}} \tag{55}$$

which is equation (31) in the main text.