The Valuation Channel of External Adjustment*

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Abstract

Onoing international financial integration has greatly increased foreign asset holdings across countries, enhancing the scope for a "valuation channel" of external adjustment (i.e., the changes in a country's net foreign asset position due to exchange rate and asset price changes). We examine this channel of adjustment in a dynamic stochastic general equilibrium model with international equity trading in incomplete asset markets. We show that the risk-sharing properties of international equity trading are tied to the distribution of income between labor income and profits when equities are defined as claims to firm profits in a production economy. We also find that, for given level of international financial integration (as measured by the size of gross foreign asset positions), the quantitative importance of the valuation channel of external adjustment depends on features of the international transmission mechanism such as the size of financial frictions, substitutability across goods, and the persistence of shocks. Finally, we find that, moving from less to more international financial integration, the overall amount of risk sharing that takes place through asset markets increases, and valuation changes are larger, but their relative importance in net foreign asset dynamics is smaller.

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1 Introduction

The cumulative total of the U.S. current account deficits since 1996 would have been sufficient to increase net foreign liabilities to about 30 percent of GDP. Yet, U.S. net foreign liabilities increased to only about 20 percent of GDP over the same period. The difference is due to valuation effects, namely, the effects of asset price and exchange rate movements on the stock of gross assets and liabilities of the United States and the rest of the world. The experience of the United States shows that external adjustment—i.e., changes in a country's net foreign asset position—can take place not only through changes in quantity and price of goods and services, but also through changes in asset prices, as argued by Gourinchas and Rey (2005), Obstfeld (2005) and Lane and Milesi-Ferretti (2005).

This paper explores the valuation channel of external adjustment in a two-country dynamic stochastic general equilibrium model (DSGE) with international equity trading. Specifically, we study the determinants of the valuation channel, its relative importance in external adjustment, and we illustrate its working. In the process, we explore the risk sharing implications of international trade in equity.

We introduce two-way international equity trading in an otherwise standard two-country, DSGE model with monopolistic competition and incomplete asset markets. To focus on household's consumption and equity holding behavior, we consider a very simple production structure. Output is produced using only labor subject to country-wide productivity shocks, and labor supply is inelastic. However, product differentiation across countries ensures that the consumption value of a country's output depends on its relative price, which is endogenous to the conditions of the economy. Monopolistic competition, based on product differentiation within countries, generates non-zero profits and firm values, essential for the asset dynamics we focus on.

Markets are incomplete across countries. Specifically, we assume that households can hold non-contingent bonds and shares in firms, but only the latter are traded internationally. Thus, different from most of the literature, we focus on equity rather than bond trading as the key mechanism for international consumption smoothing and risk sharing. Equity trades are subject to convex financial intermediation costs. As in models with bond trading, these costs are a technical device to ensures uniqueness of the deterministic steady state and stationary responses to temporary shocks.[1] The structure of costs enables us to determine endogenously the international distribution of wealth and the composition of country equity holdings in and off the steady state. This is a convenient feature of our model that we exploit in calibration exercises. However, several qualitative results are unaffected if the costs are removed. Importantly, even in the absence of financial

[1] As Schmitt-Grohé and Uribe (2003) point out, absent any stationarity-inducing device, once the model is log-linearized, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded.

frictions, international trade in shares does not generate perfect consumption risk sharing because international sharing is limited to the profit fraction of income, while labor cannot move across countries in response to shocks. So the extent of risk sharing in our model is crucially affected by the distribution of income between profits and labor income, in turn determined by substitutability across individual product varieties in consumer preferences.

The equity choices of the representative household in our model depend on time-varying expected return differentials adjusted for financial intermediation costs. In the log-linear solution of the model around a deterministic steady state, choices are not affected by standard risk diversification motives captured by conditional second moments of asset returns and consumption. Thus, our log-linearized setup does not help to explain the contribution of conditional risk diversification to portfolio changes over time. Nevertheless, this does not imply that there is no role for risk premia in the model. As illustrated by Lettau (2003), it remains possible in a log-linearized framework to define average premia based on unconditional second moments. Although we do not pursue the exercise here,it would be possible to address the risk diversification properties of our model – or extensions to a wider menu of assets – from an ex ante perspective and compare them to the data along the lines of Lettau's exercise.

Our main results are as follows. In our theoretical analysis, the magnitude and significance of the valuation channel of adjustment depends on initial asset positions and features of the economy such as the size of financial frictions, substitutability across goods, and the persistence of shocks. Starting from a fully symmetric steady state, relative productivity shocks induce larger asset equity price differentials, and hence valuation changes, the more persistent the shocks, the more substitutable home and foreign goods, the more impatient households, and the larger financial frictions (holding the steady state gross foreign asset positions unchanged). The degree of substitutability between home and foreign goods, however, has no effect on the relative share of valuation change and the current account in net foreign asset changes. We also find that, moving from less to more international financial integration, i.e., moving from a steady state with home bias in equity to one with larger, fully symmetric gross foreign asset positions, the overall amount of risk sharing that takes place through asset markets increases, and valuation changes are larger but their relative importance in net foreign asset dynamics is smaller. Larger trade imbalances and a less destabilizing income balance are a more important source of net foreign asset dynamics with more integrated, but still incomplete, international asset markets.

Our work is closely related to that of Kim (2002), Tille (2005), Blanchard, Giavazzi, and Sa (2005), and Devereux and Saito (2005). All these contributions focus on the role of the exchange rate in the valuation channel. We focus on the role of equity return differentials.[2] Unlike Tille

[2] Tille (2005) documents that return differentials other than exchange rate movements are quantitatively at least

(2005), we study also situations in which steady-state net foreign assets differ from zero. Blanchard et al (2005) set up a traditional portfolio balance model with imperfect asset substitutability along the lines of Kouri (1982) and discuss valuation effects caused by exchange rate movements. We develop a general equilibrium model which investigates the interaction between valuation driven by equity prices, equities as claims to firm profits, and the transmission of shocks via the terms of trade. Kim (2002) focuses on the consequences of revaluation of nominal asset prices, while changes in nominal prices have no real effect in our model. Devereux and Saito (2005) build a finance model with no production.

Emphasis on macroeconomic dynamics also distinguishes our model from finance models of international equity trading, such as Adler and Dumas (1983). Pavlova and Rigobon (2003) give more prominence to macroeconomic dynamics, but without modeling firm decisions explicitly. Our model differs from earlier DSGE models of international real business cycles (RBCs) in that we incorporate differentiated goods and monopolistic competition to have positive prices of shares in firm profits. Moreover, many international RBC studies do not model equity trading by focusing on complete Arrow-Debreu (or Arrow) asset markets that make all other assets redundant (for instance, Backus, Kehoe, and Kydland, 1992).

A few other papers aim at explaining home bias in portfolios or international financial integration by allowing for international equity trading in standard international macroeconomic models (Engel and Matsumoto, 2005; Heathcote and Perri, 2004; and Kollmann, 2005), but they do not focus on the role of valuation in net foreign asset adjustment.[3] While the focus of this paper and most related literature is positive, Benigno (2006) provides a normative analysis of valuation effects and their consequences for optimal monetary policy, and Ghironi and Lee (2006) explore the role of monetary policy in a sticky-price version of the model developed here.

The rest of the paper is organized as follows. Section 2 spells out the structure of the model. Section 3 studies the valuation channel in two cases that can be solved analytically in log-linear form. Section 4 illustrates the working of the valuation channel by means of numerical examples and discusses the quantitative performance of the model. Section 5 concludes. Technical details are in appendixes.

as important as the latter for valuation effects, and they are more important in 8 of the last 15 years.

[3] Kollmann (2005) studies changes in foreign equity holdings in an effectively complete market, in which equities are defined as claims to country-specific endowments of goods. He finds that changes in net holdings of foreign equity are a key source of current account movements.

2 The Model

There are two countries, home and foreign, populated by infinitely lived, atomistic households. World population equals the continuum $[0,1]$. Home households are indexed by $j \in [0,a)$; foreign households are indexed by $j^* \in [a,1]$.

There is a continuum of monopolistically competitive firms on $[0,1]$, each producing a differentiated good. Home firms are indexed by $z \in [0,a)$; foreign firms are indexed by $z^* \in [a,1]$.

2.1 Households

The representative home household j supplies one unit of labor inelastically in each period in a competitive home labor market for the nominal wage rate W_t. As customary, we denote consumption with C and the consumer price index (CPI) with P. Money serves the sole role of unit of account, since we assume that prices and wages are flexible. Therefore, we adopt a cashless specification following Woodford (2003).

The preferences of the representative home household j are:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(C_s^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{1}$$

with $0 < \beta < 1$ and $\sigma > 0$. The representative foreign household j^* maximizes a similar utility function and supplies one unit of labor inelastically in each period in a competitive foreign labor market.

The consumption basket C aggregates sub-baskets of individual home and foreign goods in a CES fashion:

$$C_t^j = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^j\right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^j\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{2}$$

where $\omega > 0$ is the elasticity of substitution between home and foreign goods. The consumption sub-baskets C_H and C_F aggregate individual home and foreign goods, respectively, in a Dixit-Stiglitz fashion with elasticity of substitution $\theta > 1$:

$$C_{Ht}^j = \left[\left(\frac{1}{a}\right)^{\frac{1}{\theta}} \int_0^a \left(c_t^j(z)\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \quad C_{Ft}^j = \left[\left(\frac{1}{1-a}\right)^{\frac{1}{\theta}} \int_a^1 \left(c_t^j(z^*)\right)^{\frac{\theta-1}{\theta}} dz^*\right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

This structure of consumption preferences implies:

$$P_t = \left[aP_{Ht}^{1-\omega} + (1-a)P_{Ft}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

where P_H (P_F) is the price sub-index for home (foreign)-produced goods – both expressed in units of the home currency. Letting $p_t(z)$ be the home currency price of good z, we have:

$$P_{Ht} = \left(\frac{1}{a} \int_0^a p_t(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}, \quad P_{Ft} = \left(\frac{1}{1-a} \int_a^1 p_t(z^*)^{1-\theta} dz^* \right)^{\frac{1}{1-\theta}}.$$

We assume that the law of one price holds for each individual good: $p_t(z) = \mathcal{E}_t p_t^*(z)$, where $\mathcal{E}_t$ is the nominal exchange rate (the domestic-currency price of a unit of foreign currency) and $p_t^*(z)$ is the foreign currency price of good z.

Consumption preferences are identical across countries. This assumption and the law of one price imply that consumption-based PPP holds: $P_t = \mathcal{E}_t P_t^*$, where P_t^* is the foreign CPI.

Agents in each country can hold domestic, non-contingent bonds denominated in units of the domestic currency, and shares in domestic and foreign firms. Different from most literature, we assume that shares, and not bonds, are traded across countries for international risk sharing and consumption smoothing purposes.

Omitting identifiers for households, firms, and countries, we use x_{t+1} to denote holdings of shares in firms entering period $t+1$, V_t to denote the nominal price of shares during period t, D_t to denote nominal dividends, and B_{t+1} to denote nominal bond holdings entering period $t+1$. Households pay quadratic financial transaction fees to domestic financial intermediaries when they hold share positions that differ from zero. As in models with international trading in uncontingent bonds, these fees are a technical device to pin down equity holdings in the deterministic steady state and ensure reversion to this position after temporary shocks.[4] Table 1 summarizes the details of our notation when agent and country identifiers are taken into account. The budget constraint of home household j is:

$$\begin{aligned} & B_{t+1}^j + \int_0^a V_t^z x_{t+1}^{z,j} dz + \mathcal{E}_t \int_a^1 V_t^{z^*} x_{t+1}^{z^*,j} dz^* + \\ & + \frac{\gamma_x}{2} \int_0^a V_t^z \left(x_{t+1}^{z,j} \right)^2 dz + \mathcal{E}_t \frac{\gamma_{x^*}}{2} \int_a^1 V_t^{z^*} \left(x_{t+1}^{z^*,j} \right)^2 dz^* + P_t C_t^j \\ = \; & (1+i_t) B_t^j + \int_0^a (V_t^z + D_t^z) x_t^{z,j} dz + \mathcal{E}_t \int_a^1 \left(V_t^{z^*} + D_t^{z^*} \right) x_t^{z^*,j} dz^* + W_t + P_t T_t^j, \end{aligned} \tag{4}$$

where i_{t+1} is the nominal interest rate on holdings of bonds between t and $t+1$, T_t^j is a lump-sum transfer from financial intermediaries, and the γ's are positive parameters.

[4] Cooley and Quadrini (1999) model limited participation in financial markets by assuming similar transaction costs. Since aggregate bond holdings are zero in equilibrium in each country, financial fees on bond transactions are omitted without loss of generality.

Table 1. Notation summary
$x_{t+1}^{z,j}$ = share of *home* firm z held by home agent j entering period $t+1$.
$x_{t+1}^{z^*,j}$ = share of *foreign* firm z^* held by home agent j entering period $t+1$.
$x_{*,t+1}^{z,j}$ = share of *home* firm z held by foreign agent j^* entering period $t+1$.
$x_{*,t+1}^{z^*,j}$ = share of *foreign* firm z^* held by foreign agent j^* entering period $t+1$.
V_t^z = price of shares in profits of home firm z starting in period $t+1$.
$V_t^{z^*}$ = price of shares in profits of foreign firm z^* starting in period $t+1$.
D_t^z = dividends paid by home firm z.
$D_t^{z^*}$ = dividends paid by foreign firm z^*.
B_{t+1}^j = stock of *home* bonds held by home agent j entering period $t+1$.
$B_{*,t+1}^{*,j}$ = stock of *foreign* bonds held by foreign agent j^* entering period $t+1$.
$\frac{\gamma_x}{2}\int_0^a \frac{V_t^i}{P_t}\left(x_{t+1}^{z,j}\right)^2 dz$ = *home* intermediation cost of holding shares in home firms.
$\frac{\gamma_{x^*}}{2}\int_a^1 \frac{\mathcal{E}_t V_t^{i^*}}{P_t}\left(x_{t+1}^{z^*,j}\right)^2 dz^*$ = *home* intermediation cost of holding shares in foreign firms.
$\frac{\gamma_x^*}{2}\int_0^a \frac{V_t^i}{P_t}\left(x_{*,t+1}^{z,j}\right)^2 dz$ = *foreign* intermediation cost of holding shares in home firms.
$\frac{\gamma_{x^*}^*}{2}\int_a^1 \frac{\mathcal{E}_t V_t^{i^*}}{P_t}\left(x_{*,t+1}^{z^*,j}\right)^2 dz^*$ = *foreign* intermediation cost of holding shares in foreign firms.

The foreign household's budget constraint is similar. We allow the scaling parameters of financial fees (γ_x, γ_{x^*}, γ_x^*, $\gamma_{x^*}^*$) to differ across countries and across assets. This has implications for the steady state of the model, which we exploit in analytical and numerical exercises below. The financial transaction fees in the budget constraint are rebated to households in equilibrium.[5] Thus, the lump-sum rebate of financial intermediation fees to household j is:

$$T_t^j = \frac{\gamma_x}{2}\int_0^a \frac{V_t^z}{P_t}\left(x_{t+1}^{z,j}\right)^2 dz + \frac{\gamma_{x^*}}{2}\int_a^1 \frac{\mathcal{E}_t V_t^{z^*}}{P_t}\left(x_{t+1}^{z^*,j}\right)^2 dz^*. \tag{5}$$

First-Order Conditions Home household j maximizes (1) subject to (4) taking the financial fee transfer as given. The first-order conditions with respect to B_{t+1}^j the domestic bond), $x_{t+1}^{z,j}$ (share of home firm), and $x_{t+1}^{z^*,j}$ (share of foreign firm) are, respectively:

$$\left(C_t^j\right)^{-\frac{1}{\sigma}} = \beta\left(1+i_{t+1}\right)E_t\left[\frac{P_t}{P_{t+1}}\left(C_{t+1}^j\right)^{-\frac{1}{\sigma}}\right], \tag{6}$$

$$\left(C_t^j\right)^{-\frac{1}{\sigma}} V_t^z\left(1+\gamma_x x_{t+1}^{z,j}\right) = \beta E_t\left[\left(C_{t+1}^j\right)^{-\frac{1}{\sigma}}\left(V_{t+1}^z + D_{t+1}^z\right)\frac{P_t}{P_{t+1}}\right], \tag{7}$$

$$\left(C_t^j\right)^{-\frac{1}{\sigma}} V_t^{z^*}\left(1+\gamma_{x^*} x_{t+1}^{z^*,j}\right) = \beta E_t\left[\left(C_{t+1}^j\right)^{-\frac{1}{\sigma}}\left(V_{t+1}^{z^*} + D_{t+1}^{z^*}\right)\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\frac{P_t}{P_{t+1}}\right]. \tag{8}$$

We omit transversality conditions. Similar Euler equations and transversality conditions hold for the foreign household.

[5] We think about the financial intermediaries in the model as local, perfectly competitive firms owned by home households. There is no cross-border ownership of these firms.

2.2 Firms

The representative, monopolistically competitive, home firm z produces output with linear technology using labor as the only input:

$$Y_t^{Sz} = Z_t L_t^z, \tag{9}$$

where Z_t is the aggregate stochastic home productivity. The assumptions that labor is supplied inelastically and is the only factor of production imply that output of each country's sub-basket of goods is exogenously determined by productivity. Importantly, however, each country's GDP in units of the world consumption basket is endogenous, as it depends on the relative price of the country's output in terms of consumption, which is determined by the pricing decisions of firms.

Home firm z faces demand for its output given by:

$$Y_t^{Dz} = \left(\frac{p_t(z)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} Y_t^W = \left(RP_t^z\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} Y_t^W, \tag{10}$$

where $RP_t^z \equiv \frac{p_t(z)}{P_t}$ is the price of good z in units of the world consumption basket, $RP_t \equiv \frac{P_{H,t}}{P_t}$ is the price of the home sub-basket of goods in units of the world consumption basket, and Y_t^W is aggregate world demand of the consumption basket.

Firm profit maximization results in the pricing equation:

$$RP_t^z = \frac{\theta}{\theta-1}\frac{w_t}{Z_t}, \tag{11}$$

where $w_t \equiv W_t/P_t$. Since $RP_t^z = RP_t$ at an optimum, labor demand is determined by

$$L_t^z = L_t = RP_t^{-\omega}\frac{Y_t^W}{Z_t}. \tag{12}$$

2.3 Relative Prices, GDP, and Income Distribution

We relegate aggregate equilibrium conditions for household behavior to an appendix and focus here on the determination of some key variables in our model.

Define home aggregate per capita GDP in units of consumption as $y_t \equiv aRP_t Y_t^z/a = RP_t Z_t$ (where we used the equilibrium condition $aL_t^z/a = L_t^z = 1$) and world aggregate per capita GDP as $y_t^W \equiv ay_t + (1-a)\,y_t^* = aRP_t Z_t + (1-a)\,RP_t^* Z_t^*$. Market clearing in aggregate per capita terms requires $aL_t^z/a = L_t = 1 = RP_t^{-\omega} y_t^W/Z_t$, and similarly in the foreign economy. We thus have a system of two equations in two unknowns that pins down home and foreign relative prices:

$$1 = RP_t^{-\omega}\frac{aRP_t Z_t + (1-a)\,RP_t^* Z_t^*}{Z_t}, \tag{13}$$

$$1 = \left(RP_t^*\right)^{-\omega}\frac{aRP_t Z_t + (1-a)\,RP_t^* Z_t^*}{Z_t^*}. \tag{14}$$

PPP implies that the real exchange rate is equal to one in all periods. The terms of trade between representative home and foreign goods, instead, change over time and are given by

$$TOT_t = \frac{p_t(z)}{\mathcal{E}_t p_t^*(z^*)} = \frac{P_{H,t}}{\mathcal{E}_t P_{F,t}^*} = \frac{RP_t}{RP_t^*} = \left(\frac{Z_t^*}{Z_t}\right)^{\frac{1}{\omega}}. \tag{15}$$

A positive productivity shock in the home economy causes the terms of trade to deteriorate as increased supply of home goods lowers their relative price. Note that, when $\omega = 1$, the terms of trade move one-for-one with the productivity differential, as in Cole and Obstfeld (1991) and Corsetti and Pesenti (2001).

Given the path of RP_t implied by the system (13)-(14), the real wage that clears the labor market is in turn determined by:

$$w_t = \frac{(\theta-1) RP_t Z_t}{\theta} = \frac{(\theta-1) y_t}{\theta}. \tag{16}$$

In a perfectly competitive environment in which $\theta \to \infty$, all GDP per capita would be distributed to domestic labor in the form of wage income. In a monopolistically competitive environment with constant markups, a share $1/\theta$ of GDP is distributed as profits:

$$d_t = y_t - w_t = \frac{1}{\theta} w_t. \tag{17}$$

The distribution of GDP between wages and profits will be an important determinant of the properties of our model as we discuss below.

2.4 Net Foreign Assets

Denote aggregate per capita home holdings of home (foreign) equity entering period $t+1$ with x_{t+1} (x_{t+1}^*). Similarly, denote aggregate per capita foreign holdings of home (foreign) equity with x_{*t+1} (x_{*t+1}^*).[6] Equilibrium aggregate per capita real home assets entering $t+1$ are thus given by $v_t x_{t+1} + v_t^* x_{t+1}^*$, where $v_t \equiv V_t/P_t$ and $v_t^* = V_t^*/P_t^*$. Home aggregate per capita net foreign assets entering $t+1$ (nfa_{t+1}) are obtained by netting out the values of home holdings of home shares ($v_t x_{t+1}$) and foreign holdings of home shares (adjusted for the population ratio, $[(1-a)/a] v_t x_{*t+1}$). Thus,

$$nfa_{t+1} \equiv v_t^* x_{t+1}^* - \frac{1-a}{a} v_t x_{*t+1}. \tag{18}$$

Foreign net foreign assets satisfy the market clearing condition:

$$a(nfa_t) + (1-a) nfa_t^* = 0. \tag{19}$$

[6] Details on the computation of these shares are in an appendix.

2.5 Steady-State Net Foreign Assets and Equity Returns

We present the details of the solution for the steady state of the model in appendix. Here, we report the main results on steady-state net foreign assets, and equity returns. We denote steady-state levels of variables by dropping the time subscript.

Steady-state home net foreign assets are:

$$nfa = \frac{\beta(1-a)}{\theta}\left(\frac{\gamma^*_{x^*}}{\Gamma_1} - \frac{\gamma_x}{\Gamma_2}\right),$$

where:

$$\begin{aligned}
\Gamma_1 &= (1-\beta)\left[(1-a)\gamma_{x^*} + a\gamma^*_{x^*}\right] + \gamma_{x^*}\gamma^*_{x^*}, \\
\Gamma_2 &= (1-\beta)\left[(1-a)\gamma_x + a\gamma^*_x\right] + \gamma_x\gamma^*_x.
\end{aligned}$$

To gain intuition on this expression, observe that, for $\beta \to 1$, we have:

$$nfa \to \frac{(1-a)}{\theta}\left(\frac{1}{\gamma_{x^*}} - \frac{1}{\gamma^*_x}\right).$$

Home net foreign assets are lower the higher the intermediation cost faced by home agents in the market for foreign equity, and the lower the intermediation cost faced by foreign agents in the market for home equity. The net foreign asset position is zero if $\gamma_{x^*} = \gamma^*_x$, with gross positions of equal value but opposite sign, proportional to the cost facing a household when buying equity abroad. If $\beta \neq 1$, the net foreign asset position depends on all financial fee scale parameters, reflecting the relative convenience of the two equities for home and foreign households in the two markets.

The difference in steady-state rates of return on equity is:

$$\frac{d}{v} - \frac{d^*}{v^*} = \frac{1}{\beta(1-a)}\left(\frac{\gamma_x\gamma^*_x}{\gamma_x + \gamma^*_x\frac{a}{1-a}} - \frac{\gamma_{x^*}\gamma^*_{x^*}}{\gamma_{x^*} + \gamma^*_{x^*}\frac{a}{1-a}}\right). \tag{20}$$

Even if $\gamma_{x^*} = \gamma^*_x$, the equity return differential may be different from zero. To be zero, it requires equal intermediation costs across home and foreign equity within each country, with potentially different costs across countries for the same equity ($\gamma_x = \gamma_{x^*}$ and $\gamma^*_x = \gamma^*_{x^*}$), or equal costs across countries for the same equity, with potentially different costs across home and foreign equity within each country (if $\gamma_x = \gamma^*_x$ and $\gamma_{x^*} = \gamma^*_{x^*}$).[7] β's) were different across countries. Financial fees would prevent the outcome in which the most patient country owns all the world equity from arising. Details are available on request.

[7] It is possible to verify that the model would feature a unique steady state even if household discount factors (

3 Valuation Changes and the Transmission of Shocks

In this section we provide a decomposition of changes in net foreign assets into valuation changes and the current account, with valuation changes and the current account further decomposed into their components.[8] We then analyze the determinants of valuation changes and the transmission of relative productivity shocks in two special cases of our model that can be solved analytically in log-linear form.

3.1 Valuation Changes and the Current Account

Assume for simplicity that the home and foreign economies have equal size ($a = 1/2$) and the steady state of the model is such that $v = v^*$ and $x^* = x_*$. (Throughout, we assume that structural parameters are such that the symmetry properties we appeal to are satisfied.) Log-linearizing (18) and denoting percent deviations from steady-state levels with a hat yields:

$$\widehat{nfa}_{t+1} = (\hat{v}_t^* - \hat{v}_t) + \left(\hat{x}_{t+1}^* - \hat{x}_{*t+1}\right), \tag{21}$$

where $\widehat{nfa}_{t+1} \equiv \mathsf{d}nfa_{t+1}/vx$, reflecting the fact that $nfa = 0$ when $vx_* = v^*x^*$. The change in net foreign assets is then written as:

$$\widehat{nfa}_{t+1} - \widehat{nfa}_t = \left[\left(\hat{v}_t^* - \hat{v}_{t-1}^*\right) - (\hat{v}_t - \hat{v}_{t-1})\right] + \left[\left(\hat{x}_{t+1}^* - \hat{x}_t^*\right) - (\hat{x}_{*t+1} - \hat{x}_{*t})\right]. \tag{22}$$

The first square bracket on the right-hand side of (22) is the valuation change on the existing stock of net foreign assets due to changes in real equity prices. Changes in real equity values, in turn, can be decomposed into changes in their nominal determinants (nominal equity prices, the price level, and the exchange rate). Specifically, $\hat{v}_t = \hat{V}_t - \hat{P}_t$ and $\hat{v}_t^* = \hat{V}_t^* - \hat{P}_t^* = \hat{V}_t^* - \left(\hat{P}_t - \widehat{\mathcal{E}}_t\right)$. This decomposition allows us to highlight a difference between our model with equity trading and the more familiar framework with international trade in bonds. In our economy, the nominal components of real equity prices have no independent effect on real equity values (and thus net foreign assets) because all prices involved are fully flexible. In an economy with bond trading, the nominal interest rate between $t-1$ and t is predetermined relative to the price level at t, resulting in a valuation effect of nominal price movements on outstanding bond positions via unexpected movements of *ex post* real interest rates under fully flexible goods prices.[9]

[8] As measured in the balance of payments, the current account does not include capital gains on investments, while the international investment position incorporates them. This component of investment income, however, enters the textbook definition of total asset return.

[9] Tille (2005) analyzes the valuation effects arising from this limited flexibility of bond prices. Our model examines the valuation channel when asset prices themselves are fully flexible. Furthermore, in our model, changes in real equity prices are the only source of valuation effects, since the real exchange rate is constant owing to purchasing power parity.

The second square bracket in (22) is the change in net foreign assets due to purchases and sales of assets and liabilities, i.e., portfolio rebalancing. This portfolio rebalancing term corresponds to the current account balance that comprises the income and trade balance. To see this, assume further that the steady state is such that $x = x^* = 1/2$ and $d = d^*$. Log-linearizing the equilibrium budget constraint in aggregate per capita terms and imposing the log-linear asset market equilibrium condition $\hat{x}_{t+1} = -\hat{x}_{*t+1}$, we obtain:

$$\left(\hat{x}^*_{t+1} - \hat{x}^*_t\right) - \left(\hat{x}_{*t+1} - \hat{x}_{*t}\right) = \frac{d}{v}\left[\left(\hat{d}^*_t + \hat{x}^*_t\right) - \left(\hat{d}_t + \hat{x}_{*t}\right)\right] + 2\left(\frac{d}{v}\hat{d}_t + \frac{w}{v}\hat{w}_t - \frac{C}{v}\hat{C}_t\right). \quad (23)$$

The first term on the right-hand side is the dividend income flow from net foreign assets accumulated in the previous period, while the second term is the trade balance.[10] Thus, the portfolio rebalancing term in equation (22) is the current account balance.

As a corollary of the equations above, it is evident that valuation changes play a role in the adjustment of net foreign assets in response to shocks in our model whenever the gross equity positions of a country differ from zero. That is, except in the case where gross (and thus net) foreign asset positions are in zero balance for all countries.

Note that we can also decompose net foreign asset dynamics between total asset return and trade balance, where the total return comprises valuation change and investment income. This decomposition is less dependent on the accounting convention of the balance of payments statistics or corporate policies on dividend payouts. The assumption of our model is that all profits are distributed as dividends. As noted by Obstfeld (2005), if companies choose to retain profits internally, this can reduce the magnitude of current account variation in (23) and enhance the relative role of valuation changes in external adjustment. Still, the trade balance and investment income need to be combined to calculate portfolio rebalancing.

The analog to (23) in the foreign economy is:

$$-\left(\hat{x}^*_{t+1} - \hat{x}^*_t\right) + \left(\hat{x}_{*t+1} - \hat{x}_{*t}\right) = \frac{d}{v}\left[-\left(\hat{d}^*_t + \hat{x}^*_t\right) + \left(\hat{d}_t + \hat{x}_{*t}\right)\right] + 2\left(\frac{d}{v}\hat{d}^*_t + \frac{w}{v}\hat{w}^*_t - \frac{C}{v}\hat{C}^*_t\right), \quad (24)$$

where we used the fact that $x = x^* = 1/2$ implies $x_* = x^*_* = 1/2$ via market clearing.

Subtracting (24) from (23) and using a superscript D to denote cross-country differences (home minus foreign) yields:

$$\hat{x}^D_{t+1} = \left(1 + \frac{d}{v}\right)\hat{x}^D_t + \frac{w}{v}\hat{w}^D_t - \frac{C}{v}\hat{C}^D_t, \quad (25)$$

where $\hat{x}^D_{t+1} = \hat{x}^*_{t+1} - \hat{x}_{*t+1}$ measures home's net cross-border share holdings. Notice the resemblance between (25) and standard, log-linear laws of motion for net foreign bond holdings in the more

[10] The "2" that normalizes the trade balance originates from the fact that, with equal country size, asset market equilibrium requires $x_{t+1} + x_{*t+1} = 1$. In the symmetric steady state of this example, this implies $x = x_* = 1/2$. Division of both sides by vx in the log-linearization of the budget constraint results in the presence of the 2.

familiar framework. In our model, the steady-state gross return on share holdings replaces the steady-state gross interest rate.

3.2 Valuation and Transmission around a Symmetric Steady State

We now complete the solution of the log-linear model for the case of a fully symmetric steady state in which everything is identical across two equal-sized countries (so that, in particular, $x = x^* = x_* = x_*^* = 1/2$).

Exploiting $\widehat{RP}_t^D = -(1/\omega)\hat{Z}_t^D$ and the definitions of home and foreign GDP's in units of consumption ($y_t = RP_t Z_t$ and $y_t^* = RP_t^* Z_t^*$, respectively) it is immediate to verify that the log-linear GDP differential is proportional to relative productivity:

$$\hat{y}_t^D = \left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D. \tag{26}$$

As expected, there is no GDP differential if $\omega = 1$.

Since dividends and wage income are constant fractions of GDP, it follows immediately that

$$\hat{w}_t^D = \hat{d}_t^D = \left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D. \tag{27}$$

Using the steady-state properties of the model, we may then rewrite the law of motion (25) as:

$$\hat{x}_{t+1}^D = \frac{(1+\gamma)}{\beta}\hat{x}_t^D + \frac{(\theta - 1)(1 - \beta + \gamma)}{\beta}\left(\frac{\omega - 1}{\omega}\right)\hat{Z}_t^D - \frac{\theta(1 - \beta + \gamma)}{\beta}\hat{C}_t^D, \tag{28}$$

where γ is the scaling parameter of financial frictions, common across equities and countries, and β is the household discount factor.

We show in an appendix that no-arbitrage across different equities implies that expected relative consumption growth is tied to net cross-border share holdings, and relative share valuation reflects expected future share prices and dividends:

$$\hat{C}_t^D = E_t\hat{C}_{t+1}^D + \frac{\sigma\gamma}{1+\gamma}\hat{x}_{t+1}^D. \tag{29}$$

$$\hat{v}_t^D = \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1}^D + \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1}^D. \tag{30}$$

Note that, absent financial frictions ($\gamma = 0$), the consumption differential follows a random walk: Any differential at time t is expected to persist at $t+1$. As we show below, consistent with models with bond trading only, the link between expected relative consumption growth and relative cross-border share holdings introduced when $\gamma > 0$ is crucial to deliver stationary responses to temporary shocks.

Equations (27) and (30) allow us to solve for the determinants of relative share prices (and thus the valuation effect). Assuming that home and foreign productivities $\hat{Z}_t$ and $\hat{Z}_t^*$ follow $AR(1)$ processes with common persistence $\phi_Z \in [0, 1)$, we have:

$$\hat{v}_t^D = \eta_{v^D Z^D} \hat{Z}_t^D = \left(\frac{1 - \beta + \gamma}{1 + \gamma - \beta \phi_Z} \right) \left(\frac{\omega - 1}{\omega} \right) \phi_Z \hat{Z}_t^D. \tag{31}$$

The effect of relative productivity shocks on relative share prices depends on the persistence of the shock (ϕ_Z), the elasticity of substitution between home and foreign goods (ω), the size of financial frictions (γ), and the patience of households (β). We assume $0 \leq \gamma < 1$ and $\omega \geq 1$. Combining these assumptions with the restrictions $0 < \beta < 1$ and $0 \leq \phi_Z < 1$, we can conclude that:

$$\frac{\partial \eta_{v^D Z^D}}{\partial \phi_Z} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \omega} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \gamma} \geq 0, \quad \frac{\partial \eta_{v^D Z^D}}{\partial \beta} \leq 0.$$

Relative productivity shocks induce larger changes in relative share valuation the more persistent the shocks, the more substitutable home and foreign goods, the larger financial frictions, and the more impatient households. Notice that purely temporary productivity shocks ($\phi_Z = 0$) have no effect on relative share valuation, because the differential in share prices is determined by its expected future level and expected relative dividends, which are not affected by the shock if this has no persistence.

We note here that some of these results need to be qualified when we abandon the case of a fully symmetric steady-state. Around a non-symmetric steady state, also purely temporary shocks affect relative share valuation, as will be shown in the next sub-section around an extremely non-symmetric steady state. It is also important to keep in mind that the comparative statics above on the effect of changes in the size of financial fees (γ) are performed for unchanged steady state. In other words, changes in γ are such that the symmetry of the steady state appealed to in log-linearization is unaffected. We can then compare shock transmission properties for different size of financial fees around an unchanged steady state. A different, but important question is how changes in the size of financial fees affect dynamics around different steady states when changes in γ's do not leave the steady state unaffected. Since this case becomes algebraically intractable, we will study it by means of numerical exercises in the next section.

No arbitrage around the fully symmetric steady state implies that we can solve for the dynamics of relative consumption and cross-border share holdings independently of the path of $\hat{v}_t^D$, by solving the system of equations (28) and (29). The solution for $\hat{x}_{t+1}^D$ and $\hat{C}_t^D$ takes the form:

$$\hat{x}_{t+1}^D = \eta_{x^D x^D} \hat{x}_t^D + \eta_{x^D Z^D} \hat{Z}_t^D, \tag{32}$$

$$\hat{C}_t^D = \eta_{C^D x^D} \hat{x}_t^D + \eta_{C^D Z^D} \hat{Z}_t^D. \tag{33}$$

Applying the method of undetermined coefficients, the elasticity $\eta_{x^D x^D}$ solves the quadratic equation:

$$\eta_{x^D x^D}^2 - \left[1 + \frac{1+\gamma}{\beta} + \frac{\theta(1-\beta+\gamma)\sigma\gamma}{\beta(1+\gamma)}\right]\eta_{x^D x^D} + \frac{1+\gamma}{\beta} = 0. \quad (34)$$

If $\gamma = 0$, this equation has solutions 1 and $1/\beta$, and – discarding the explosive solution $1/\beta$ – we are left with the familiar unit root for net cross-border share holdings as in models with bond trading only and no stationarity inducing device. When $\gamma > 0$, there is still an explosive solution larger than 1, and the unit root is "pulled" inside the unit circle, between 0 and 1, ensuring stationary net foreign equity dynamics in response to temporary shocks.

Given the stable root $\eta_{x^D x^D}$, the solutions for the other elasticities are:

$$\begin{aligned}
\eta_{C^D x^D} &= \frac{\sigma\gamma}{(1+\gamma)(1-\eta_{x^D x^D})}\eta_{x^D x^D} > 0,\\
\eta_{x^D Z^D} &= \frac{(\theta-1)(1-\beta+\gamma)}{\beta}\left(\frac{\omega-1}{\omega}\right)\left[1 + \frac{\theta(1-\beta+\gamma)}{\beta(1-\phi_Z)}\left(\eta_{C^D x^D} + \frac{\sigma\gamma}{1+\gamma}\right)\right]^{-1} \geq 0,\\
\eta_{C^D Z^D} &= \frac{1}{1-\phi_Z}\left(\eta_{C^D x^D} + \frac{\sigma\gamma}{1+\gamma}\right)\eta_{x^D Z^D} \geq 0.
\end{aligned} \quad (35)$$

Note that our model replicates the result of Cole and Obstfeld (1991) and Corsetti and Pesenti (2001) that the economy mimics complete markets, and there is no movement in net cross-border share holdings nor consumption differential if $\omega = 1$. In that case, the terms of trade move in directly proportional fashion with relative productivity, there is no GDP differential, and $\eta_{x^D Z^D} = \eta_{C^D Z^D} = 0$, ensuring that $\hat{C}_t^D = \hat{x}_{t+1}^D = 0$ in all periods given the initial condition $\hat{x}_t^D = 0$ in the period of a shock.

We are thus in a position to draw conclusions on the determinants of net foreign asset changes. Using the results above yields:

$$\widehat{nfa}_{t+1} - \widehat{nfa}_t = -\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi_Z}\right)\left(\frac{\omega-1}{\omega}\right)\phi_Z\left(\hat{Z}_t^D - \hat{Z}_{t-1}^D\right) - (1-\eta_{x^D x^D})\hat{x}_t^D + \eta_{x^D Z^D}\hat{Z}_t^D. \quad (36)$$

Of course, given the initial condition $\hat{x}_t^D = \hat{Z}_{t-1}^D = 0$, there is no change in net foreign assets if $\omega = 1$, since there is no valuation change and $\eta_{x^D Z^D} = 0$. The relative contributions of valuation and current account to the change in net foreign assets induced by a relative productivity shocks are thus given by:

$$\begin{aligned}
VALShare_t &\equiv \frac{-\left(\hat{v}_t^D - \hat{v}_{t-1}^D\right)}{\widehat{nfa}_{t+1} - \widehat{nfa}_t} = \left(1 - \frac{\hat{x}_{t+1}^D - \hat{x}_t^D}{\hat{v}_t^D - \hat{v}_{t-1}^D}\right)^{-1},\\
CAShare_t &\equiv \frac{\hat{x}_{t+1}^D - \hat{x}_t^D}{\widehat{nfa}_{t+1} - \widehat{nfa}_t} = \left[1 - \left(\frac{\hat{x}_{t+1}^D - \hat{x}_t^D}{\hat{v}_t^D - \hat{v}_{t-1}^D}\right)^{-1}\right]^{-1},
\end{aligned} \quad (37)$$

where the minus sign at the numerator of $VALShare_t$ follows from the fact that an increase in the relative price of home equity contributes negatively to home's net foreign assets. Note that $VALShare_t + CAShare_t = 1$, but $VALShare_t$ and $CAShare_t$ are not individually constrained to being between 0 and 1. For instance, a more than proportional contribution of valuation can offset a negative share of the current account in a given increase in net foreign assets.

The ratio $\left(\hat{x}^D_{t+1} - \hat{x}^D_t\right) / \left(\hat{v}^D_t - \hat{v}^D_{t-1}\right)$ has solution:

$$\frac{\hat{x}^D_{t+1} - \hat{x}^D_t}{\hat{v}^D_t - \hat{v}^D_{t-1}} = \frac{-\left(1 - \eta_{x^D x^D}\right)\hat{x}^D_t + \eta_{x^D Z^D}\hat{Z}^D_t}{\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi_Z}\right)\left(\frac{\omega-1}{\omega}\right)\phi_Z\left(\hat{Z}^D_t - \hat{Z}^D_{t-1}\right)}. \tag{38}$$

The elasticity $\eta_{x^D x^D}$ from (34) does not depend on substitutability between home and foreign goods (ω). Thus, when evaluating the effect of ω on the relative share of valuation in net foreign asset changes, we may restrict attention to the ratio

$$\frac{\eta_{x^D Z^D}\hat{Z}^D_t}{\left(\frac{1-\beta+\gamma}{1+\gamma-\beta\phi_Z}\right)\left(\frac{\omega-1}{\omega}\right)\phi_Z\left(\hat{Z}^D_t - \hat{Z}^D_{t-1}\right)}.$$

Inspection of the solution for $\eta_{x^D Z^D}$ in (35) shows that this ratio is independent of ω (because $(\omega-1)/\omega$ appears at both numerator and denominator).[11]ω applies also to the alternative decomposition of net foreign asset dynamics between the movements in trade balance and total rate of return. Therefore, the degree of substitutability between home and foreign goods has no effect on the relative shares of valuation and the current account in net foreign asset changes. The effect of other parameters – specifically, of the size of financial frictions, γ – on the relative share of valuation versus the current account in net foreign asset changes cannot be disentangled analytically in such simple fashion. Thus, we evaluate it by means of numerical examples in the next section. However, before turning to a different special case that can be tackled analytically, we address the consequences of completely removing financial frictions.

Trade in Risky Assets Revisited The issue of what happens with $\gamma = 0$ is of interest because the textbook intuition is that frictionless trade in two equities in an environment with only two shocks – such as the one we are exploring – should reproduce the full insurance allocation of complete asset markets. So the question we address here is whether our model delivers the complete markets equilibrium if $\gamma = 0$ (and $\omega \neq 1$) owing to the ability to trade equity at no cost in the presence of productivity shocks only.

With $\gamma = 0$, the symmetric steady state around which the model has been log-linearized is only one of infinitely many possible, chosen as a matter of convenience. It is $\eta_{x^D x^D} = 1$ and the

[11] This independence from

solution of the model takes the form:

$$\hat{x}^D_{t+1} = \hat{x}^D_t + \eta_{x^D Z^D} \hat{Z}^D_t, \quad (39)$$

$$\hat{C}^D_t = \eta_{C^D x^D} \hat{x}^D_t + \eta_{C^D Z^D} \hat{Z}^D_t. \quad (40)$$

For the solution to replicate complete markets, it must be $\hat{C}^D_t = 0$. In other words, (given the initial condition $\hat{x}^D_t = 0$ at the time of a shock) it must be $\eta_{C^D Z^D} = \eta_{x^D Z^D} = 0$. The conjecture (39)-(40) must now be substituted in the system:[12]

$$\begin{aligned} \hat{x}^D_{t+1} &= \frac{1}{\beta}\hat{x}^D_t + \frac{(\theta-1)(1-\beta)}{\beta}\left(\frac{\omega-1}{\omega}\right)\hat{Z}^D_t - \frac{\theta(1-\beta)}{\beta}\hat{C}^D_t, \\ \hat{C}^D_t &= E_t \hat{C}^D_{t+1}. \end{aligned}$$

Doing this and applying the method of undetermined coefficients yields:

$$\eta_{C^D x^D} = \frac{1}{\theta}, \quad \eta_{x^D Z^D} = \frac{(\theta-1)(1-\beta)(1-\phi_Z)}{1-\beta\phi_Z}\left(\frac{\omega-1}{\omega}\right), \quad \eta_{C^D Z^D} = \frac{(\theta-1)(1-\beta)}{\theta(1-\beta\phi_Z)}\left(\frac{\omega-1}{\omega}\right).$$

Therefore, the solution does not coincide with the full insurance outcome in which $\hat{C}^D_t = 0$. Relative productivity shocks case a consumption differential on impact, and the consumption differential persists as a consequence of the unit root in net cross-border share holding dynamics.[13]

This result highlights an important property of our model with equity trading. It is well known that if the world economy consists of two countries consuming the same good, with country-specific stochastic endowments of the good, CRRA preferences, and the ability to trade equity in the form of shares in the endowments of the good, frictionless trade in these equities will lead to the complete markets equilibrium. (For instance, see the discussion of this case in Obstfeld and Rogoff's, 1996, textbook.) The same mechanism carries through to the case of two goods and a CES aggregator. But the crucial difference is that our model does not allow trade in equity claims on endowments. Our equity provides claims to profits, with the rest of a country's income going to wages. To put it differently, even with symmetric equity holdings, only part of GDP gets to be shared between home and foreign residents. The wage portion is kept wholly by the residents of each country. As a result, even with $\gamma = 0$ (and thus frictionless trade in two equities in a world with only two shocks), the equilibrium does not converge to complete risk sharing.

This reasoning is confirmed by the results above. For equity trading to result in full insurance around the fully symmetric steady state, all of a country's GDP should be distributed as profit,

[12] The solution for the case $\gamma = 0$ cannot be obtained simply by setting $\gamma = 0$ in (35). Note that the implied expression for $\eta_{C^D x^D}$ would not be defined, as $\eta_{x^D x^D} = 1$ would imply division by 0.

[13] It is easy to verify that one obtains the same solution for the case $\gamma = 0$ even if the conjecture for $\hat{x}^D_{t+1}$ is written without imposing the restriction $\eta_{x^D x^D} = 1$. In this case, applying the method of undetermined coefficients simply yields $\eta_{x^D x^D} = 1$ along with the elasticities above.

leaving nothing for wages. The share of dividends in GDP is $1/\theta$, implying that all of GDP goes to shareholders in the limiting case in which $\theta \to 1$ (the maximum possible degree of monopoly power). As one can see from the expressions above, $\eta_{C^D Z^D} \to 0$ in this case, and so does $\eta_{x^D Z^D}$. There is full risk sharing under the initial, symmetric equity allocation, and (given the initial condition $\hat{x}_t^D = 0$ at the time of a shock) the equilibrium is such that $\hat{C}_t^D = \hat{x}_{t+1}^D = 0$ in all periods.

Under this interpretation, we can conclude that a proper definition of equity in a production economy (claims to profit rather than whole output) is sufficient to disturb completeness of the market in the "conventional" case. The deviation from full consumption risk sharing around the symmetric steady state is smaller the higher the degree of monopoly power along the two dimensions that are commonly explored in international macroeconomics: the higher monopoly power of individual producers within a country (the closer θ to 1) and the higher monopoly power of a country over its sub-basket of goods (the closer ω to 1). This result points to a difference between economies with bond trading only and our model with equity trading. In the economy with bond trading, $\omega = 1$ is the only scenario in which incomplete markets reproduce the full consumption insurance of complete markets. Once we allow for international trade in shares issued by firms with monopoly power, full consumption insurance across countries arises also with $\omega \neq 1$ if firms' monopoly power is extreme and long-run equity positions are fully symmetric.

It is important to remark at this point that the risk sharing implications of extreme firm-level monopoly power in our model are conditional on the assumption of a symmetric steady state in which each country owns fifty percent of the other country's equity. As we show below, a different distribution of income between wages and profits, associated with less-than-extreme firm-level monopoly power, is required for equity trade to deliver full consumption insurance with $\omega \neq 1$ when the steady state asset position is different. The results below, combined with those of this sub-section, prove the more general result that the risk sharing properties of international equity trading are crucially affected by the distribution of income between profits and labor income when equity is defined as claims to firm profits.[14]

As for responses to productivity shocks, the elasticities with $\gamma = 0$ imply that the qualitative direction of responses starting from the symmetric steady state is the same as with $\gamma > 0$. Responses when $\gamma = 0$ will feature a permanent change in net cross-border share holdings and consumption differential, but quantitative differences will depend on parameter values, as we discuss in the next section.[15]

[14] See Cass and Pavlova (2004) for additional findings on the fragility of the welfare properties of the Lucas Trees model. Ghironi and Lee (2006) explore the normative implications of this result and its consequences for optimal monetary policy in a sticky-price version of the model.

[15] It is impossible to pin down the response of holdings of individual equities when $\gamma = 0$, but this does not limit our ability to solve for all variables of interest, including the paths of net cross-border share holdings, net foreign assets, and the current account.

Finally, one more question deserves our attention before we turn to a different case: What is the share of valuation in net foreign asset adjustment when $\gamma = 0$? It is possible to verify that:

$$VALShare_0 = \frac{\phi_Z}{1 - \theta(1 - \phi_Z)}, \quad VALShare_{t \geq 1} = \frac{1}{\theta},$$

with $CAShare_t = 1 - VALShare_t$ and t=0 denoting the time of a shock. When $\gamma = 0$, the unit root in net cross-border share holdings implies that the share of valuation in external adjustment is constant in all periods after the initial one, and is determined by the share of income distributed to profits. If $\theta \to 1$, the share of valuation in net foreign asset changes tends to 1 in all periods, consistent with the fact that there is full risk sharing and no change in net cross-border share holdings.

3.3 A Non-Symmetric Steady State: The Case of Full Cross-Shareholding

Consider now a steady state in which equities issued by each country are wholly owned by residents of the other country (we call this full cross-shareholding). In terms of our notation: $x = 0$, $x_* = 1$, $x_*^* = 0$, and $x^* = 1$. This portfolio allocation arises endogenously by assuming that investing abroad is costless in both the home and the foreign economy (with common friction of size γ for domestic investment). Under this steady-state configuration, equity prices are $v = v^* = \beta / [\theta(1 - \beta)]$, but steady-state levels of wages, dividends, consumption, and relative prices are the same as in the previous case.

We show in an appendix that the following system now determines the dynamics of relative equity and the consumption differential:

$$\hat{x}_{t+1}^D = \frac{1}{\beta}\hat{x}_t^D + \frac{(1-\beta)(\theta-2)}{2\beta}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_t^D - \frac{(1-\beta)\theta}{2\beta}\hat{C}_t^D, \tag{41}$$

$$\hat{C}_t^D = E_t\hat{C}_{t+1}^D + \sigma\gamma\hat{x}_{t+1}^D, \tag{42}$$

where we have used $\hat{x}_{t+1}^D = -2\hat{x}_{*t+1}$ that holds under full cross-shareholding.[16]

The solution of this system has the same form as (32)-(33). The elasticity $\eta_{x^D x^D}$ now satisfies:

$$\beta\eta_{x^D x^D}^2 - \left[1 + \beta + \frac{(1-\beta)\sigma\gamma\theta}{2}\right]\eta_{x^D x^D} + 1 = 0. \tag{43}$$

As before, we select the stable root between 0 and 1 when $\gamma > 0$. The other elasticities are

[16] In this scenario, for analytical tractability, we allow countries to go short in their aggregate equity positions.

determined by:

$$\begin{aligned}
\eta_{C^D x^D} &= \frac{\sigma\gamma\eta_{x^D x^D}}{1-\eta_{x^D x^D}}, \\
\eta_{x^D Z^D} &= \frac{(1-\beta)(\theta-2)}{2\beta}\left(\frac{\omega-1}{\omega}\right)\left[1+\frac{\theta(1-\beta)\sigma\gamma}{2\beta(1-\eta_{x^D x^D})(1-\phi_Z)}\right]^{-1}, \\
\eta_{C^D Z^D} &= \frac{\sigma\gamma\eta_{x^D Z^D}}{(1-\eta_{x^D x^D})(1-\phi_Z)}. \qquad (44)
\end{aligned}$$

The equity price differential now obeys (see the appendix for details):

$$\hat{v}_t^D = \beta E_t \hat{v}_{t+1}^D + (1-\beta) E_t \hat{d}_{t+1}^D - \gamma \hat{x}_{t+1}^D, \qquad (45)$$

where the difference from the symmetric case is due to the non-symmetric steady-state equity holdings. Notice that the dynamics of share holdings now affect relative share valuation. This has the consequence of making relative valuation sensitive to zero-persistence productivity shocks via their effect on share holdings entering the following period.

We can obtain the following algebraic expression for the relative contribution of the current account and valuation changes following shocks with no persistence:

$$\frac{\hat{x}_{t+1}^D - \hat{x}_t^D}{\hat{v}_t^D - \hat{v}_{t-1}^D} = -\frac{1-\beta\eta_{x^D x^D}}{\gamma} \quad \forall t \geq 0, \qquad (46)$$

where 0 is the time of the shock. The relative contribution of the valuation change in (46) is higher the higher γ (the common financial intermediation cost on domestic shares).[17] When financial intermediation is more costly (larger γ), valuation changes play a bigger role around the steady state, as portfolio rebalancing entails larger costs. The relative contribution of valuation changes also increases with $\eta_{x^D x^D}$, which is larger when σ and/or θ become smaller. Lower values of these parameters lead to lower intertemporal substitution (more consumption smoothing) and weaker competition (higher profits and equity prices). As in the symmetric case, the elasticity of substitution between home and foreign goods, ω, does not affect the relative contribution of valuation change and current account, but it plays a critical role in determining the extent of adjustment via terms of trade movements.

Trade in Risky Assets Revisited (II) Does the asymmetry of the steady state affect the conclusion we reached above on the inability of trade in equity to mimic complete markets? The answer is no, but with an interesting difference. In this case, it is possible to verify that $\gamma = 0$

[17] A larger γ also causes $\eta_{x^D x^D}$ to decrease, but the net effect on the relative share of valuation in net foreign asset changes is positive.

yields the solution:

$$\hat{x}_{t+1}^{D} = \hat{x}_{t}^{D} + \frac{(1-\beta)(\theta-2)}{2}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_{t}^{D},$$

$$\hat{C}_{t}^{D} = \frac{2}{\theta}\hat{x}_{t}^{D} + \frac{(1-\beta)(\theta-2)}{\theta(1-\phi_{Z})}\left(\frac{\omega-1}{\omega}\right)\hat{Z}_{t}^{D}.$$

As before, the equilibrium does not mimic complete markets, and the intuition is the same – sharing is limited only to a portion of GDP. However, assuming $\omega \neq 1$, it is no longer the case that $\theta = 1$ (and thus complete distribution of GDP to profits) is required for full consumption insurance to arise. This now happens when $\theta = 2$, i.e., with a share of dividends in GDP equal to 1/2. With full cross-border shareholding, complete consumption insurance arises when half of GDP is allocated to profits, and the remainder goes to wages. The wage income portion is now necessary to compensate for the effect of full initial cross-border equity ownership on income sharing. Recalling the findings and discussion in the previous sub-section, we have thus established the general result that, once equities are defined as claims to firm profits in a production economy, the risk sharing properties of international equity trading are tied to the distribution of income between profits and wages determined by substitutability across individual product varieties in consumer preferences.

4 Quantitative Analysis

In this section we compare the relative importance of alternative channels of risk sharing in the model under two different degrees of international financial integration. International financial integration is represented by the different size of the gross foreign asset positions in 1990, the beginning of the most recent and rapid period of international integration, and the last year for which we have the data, 2004 (Figure 2). In the first scenario, which we call "home bias in equity", gross foreign assets and liabilities are about 40 percent of annual GDP, as approximately in the data in 1990. In the second scenario, which we call "international financial integration," gross foreign assets and liabilities are about 100 percent of annual GDP, again, approximately as in the data. Consistent with section (3), the comparison assumes zero net foreign asset positions in both scenarios, but additional results assuming a net position different from zero (at about 25 percent of GDP) are available on request from the authors.

The rest of this section describes and discusses the model parametrization and its performance against the data and the results of the comparison.

4.1 Parameter Values and Model Evaluation

The only parameter values that differ across scenarios are those characterizing the financial intermediation technology. In the home bias scenario, the assumption is that intermediation costs for home (foreign) agents on foreign (home) shares are larger than costs on home (foreign) shares—i.e., $\gamma_x = \gamma^*_{x^*} = 0.01$ and $\gamma_{x^*} = \gamma^*_x = 0.03$. In the integration scenario, the assumption is that intermediation costs for home (foreign) agents on foreign (home) shares are the same as the costs on home (foreign) shares—i.e., $\gamma^*_{x^*} = \gamma_x = \gamma_{x^*} = \gamma^*_x = 0.01$. This cost assumptions induce the steady state portfolio shares and returns summarized in Table 2 and described above. As we assumed in section (3), bonds are not traded internationally in both scenarios.

In the integration scenario with negative net foreign assets, the assumption is that the cost for foreign agents on home shares is lower than the cost on foreign shares (i.e., $\gamma^*_{x^*} = 0.01$ and $\gamma^*_x = 0.006$), while the cost for home agents on foreign shares is higher than the cost on home shares (i.e., $\gamma_x = 0.006$ and $\gamma_{x^*} = 0.01$). Thus, interpreting home as the U.S. economy, U.S. shares are cheaper for both U.S. and foreign agents, who consequently have a stronger preference for them. As a result, the steady state price of U.S. equity shares is higher than the price of foreign equity shares, while the distribution of equity holdings remains symmetric as in the integration case with zero net foreign assets.[18] This assumption induces a net foreign asset position of about 25 percent of annual GDP.

All other model parameters are constant across the two scenarios and the specific values chosen are standard. The model is perfectly symmetric and, as in section (3), countries have equal size ($a = 1/2$). We set relative risk aversion to the standard value of 2 ($\sigma = 0.5$) and rate of time preference so that the annual real interest rate is about 4 percent in steady state ($\beta = \beta^* = .99$).

The elasticity of substitution among individual good varieties within each economy, θ, determines the constant degree of market power, the profit share of income, and the dividend ratio in our model. This parameter thus determines the extent of risk sharing through international equity trading when there is no financial intermediation cost, for a given amount of risk sharing through terms of trade changes. We set $\theta = 6$ to imply a 20 percent markup of prices over marginal cost, like Rotemberg and Woodford (1992).

The elasticity of substitution between the home and foreign basket of goods determines the extent of risk sharing through terms of trade changes in the model. Estimates of this elasticity range from values close to 1, in the macro literature, to values as high as 12 in the micro, trade literature. We set $\omega = 2$. Higher values deliver more realistic correlations between home and foreign consumption and home and foreign output, but would be less conventional. Lower values, closer to structural estimates based on richer models, would imply completely unrealistic terms of trade dynamics and risk sharing properties (results not reported).[19] Note however that this parameter does not affect the share of valuation change in net foreign asset change at any time horizon. So

[18] Note that the configuration of parameters that generates a given net foreign asset position is not unique.

[19] As most international real business cycle models, our model predicts consumption correlations across countries always higher than output correlations regardless of the value of ω.

we can safely condition on any specific value.

Finally, we assume that the exogenous (log) productivity in the two countries follow $AR(1)$ processes with no cross-border spillover, a standard autoregressive parameter of 0.9, and innovation variance of one percent per quarter. If persistence of the productivity process is higher, say 0.95, the model matches better the unconditional moments of the U.S. net foreign assets, even assuming zero net foreign assets in steady state, but this results in excessively volatile output and consumption compared to U.S. data (results not reported). So we set persistence to a standard value.

Data-based business cycle moments for the U.S. economy as well as values from our benchmark economy and some of the alternatives we consider are reported in Table 1.[20] Looking at the data for the U.S. economy first, three features of the change in net foreign assets stands out. First, this measure of external balance is much more volatile than the trade balance or the current account balance. Second, while the trade balance and the current account are countercyclical, the change in net foreign assets is a-cyclical. Third, the change in net foreign assets is slightly less persistent than the trade balance and current account.[21] From Table 1, we also can see that home equity prices are highly volatile, highly correlated across countries, procyclical, as well as relatively persistent.

The model parametrization described above ($a = 1/2$, $\sigma = 0.5$, $\beta = .99$, $\theta = 6$, $\omega = 2$, $\phi = 0.9$), with financial integration and a net foreign assets different from zero, matches qualitatively the volatility, comovement with output, and persistence of changes in the U.S. net foreign asset position well, albeit not perfectly. The model can generate changes in net foreign assets that are more volatile, are much less correlated with output and are less persistent than the trade balance and the current account. The model, however, underpredics equity price volatility and overpredicts equity price comovement across countries. As a result, the matching of the moments of the changes in U.S. net foreign assets is less than fully satisfactory from a quantitative standpoint if the net foreign asset position is zero. The absence of investment in the model also generates a pro-cyclical trade balance and current account.

4.2 Results

To illustrate the relative importance of alternative risk sharing channels in the model, under alternative degrees of international financial integration, we report selected impulse responses to a productivity shock originating in the home economy (Figure 3). We consider a temporary but persistent one-percent productivity shock in the home economy (i.e., a one-standard-deviation innovation).

[20] Theoretical moments are exact and computed with DYNARE. Data-based moments are computed as we describe in appendix.

[21] The persistence of the change in net foreign assets is much smaller than that of the current account or the trade balance at annual frequency (see Kollmann, 2005, for instance.)

Panel A and B compare, in the case of home bias in equity and international financial integration, respectively, the response of changes in net foreign assets (DNFA1) and its three components: (i) the valuation change (VAH1), the trade balance (TBH); and the income balance (IBH) (in percent of GDP absolute deviation from steady state).[22] Panel C and D compare the response of the terms of trade and world consumption, as well as the cross-country consumption differentials between scenarios (in percent deviation from steady state).

Under both scenarios, a favorable relative productivity shock to the home economy causes the relative price of shares in home equity to increase, and home households to increase their holdings of foreign equity relative to foreign holdings of home equity (i.e., to run a current account surplus) to smooth consumption. The initial current account surplus, however, is smaller than the trade balance surplus, as the income balance goes in deficit in response to the shock. Thus, initially, both the relative increase in the value of home equity and the income balance contributes negatively to the change in home net foreign assets, and thus positively to risk sharing. Thus adjustment to the shock through total equity returns (capital gains and dividends) more than offsets adjustment through the trade balance on impact. Since the valuation effect depends on the change in the equity price differential, and this is a function of the productivity differential, given the time profile of the productivity shock, the valuation change is negative in all periods after the initial one, while the income (and trade balance) swing into surplus (deficit) more gradually. As a result, unlike in a standard bond-only model, changes in net foreign assets are negative on impact, and relatively less positive during the adjustment path.

Two considerations are in order here. First, note that the wealth transfer through equity price changes does not take place through violation of any arbitrage condition in the model at the time of the shock, or thereafter. So valuation effects in our model reflect the forward looking nature of equity prices, which jump on impact to restore equilibrium in the asset market and then return gradually to steady state. So they are an equilibrium phenomenon in our set up. Second, note that the value of ω is the same across the two scenarios, and the responses to the shock of world consumption (and thus income) and the terms of trade are identical. So the amount of risk sharing through terms of trade changes is also the same across scenarios. Further, we know from the theoretical analysis that the share of valuation change in net foreign asset changes is not affected by the particular value of ω we assumed, although this does affect the absolute magnitude of the valuation change. So the only determinant of any change in the overall amount of risk sharing in the model, or in the relative importance of different financial channels of risk sharing is the different degree of financial integration.

Interestingly, as Figure 4 shows, the features of the transmission of a productivity shock we

[22] Steady state values for these variables are zero.

described thus far are robust to the removal of any financial intermediation cost. Figure 4 compares the response of the model to the same shock around the same steady state level of gross foreign asset positions, with and without financial intermediation costs (Panel B and A, respectively).[23] As we can see, the dynamic response of net foreign assets, the current account, and the valuation change are qualitatively the same in the two cases.

Returning to the comparison between home bias in equity and international financial integration. Figure 3 also shows that inducing larger gross foreign asset positions, while keeping the degree of risk sharing through terms of trade changes and the income distribution constant (i.e., for given ω and θ), increases the overall amount of risk sharing through asset markets, and affects its split between the current account (or portfolio rebalancing) and valuation changes. The higher the degree of international financial integration the higher the overall amount of risk sharing allowed for in the model through asset markets. Valuation effects are also larger on impact (enhancing risk sharing through this channel) the higher the degree of integration. However, their share of the total change in net foreign assets is smaller than the case of home bias in equity. This suggests that the relative importance of valuation effects in net foreign asset dynamics decreases as we move from a less to a more integrated economy. The consumption differentials (Figure 3, Panel C and D) are smaller under financial integration, and the correlation between home and foreign consumptions is larger, and the volatility of consumption is lower (Table 1). These effects are not large in absolute terms though, but increase with higher values of omega and may have a significant impact on welfare. Interestingly, however, going from home bias in equity to international financial integration, a larger share of net foreign asset change takes place through a larger trade imbalance and less destabilizing income balance dynamics, rather than through a larger role for valuation changes. This is easily seen on impact from Figure 3. The valuation change at the time of the shock, is larger in absolute value under integration but its share of the change in net foreign assets is smaller than under home bias. Even cumulating the response of net foreign assets and its components over the first 40 quarterly periods, we find that the income balance contributes to a smaller build up of net foreign assets (liabilities) in the home (foreign) economy under integration than home bias (Table 3). The income balance contributes to a smaller accumulation of total net foreign asset change because larger equity cross share holdings permit financing of the trade balance through larger capital income flows from abroad.

In fact, equity price differentials are smaller under financial integration and equity prices are more correlated across countries (Table 1), consistent with our theoretical analysis, although gross positions are larger in this case. One intuition is that, with higher integration, gross positions

[23] All γ parameters are zero in the case of no intermediation cost. In this case, the net and the gross foreign asset positions depends on initial conditions and no longer revert to their initial level after the shock. As we discussed in section (3), the consumption differentials and net foreign assets become unit root processes.

are larger (Table 2), but not large enough to offset the fall in the equity price differential in response to same shock when we switch from one steady state to the other. A second intuition is that with portfolio quantities less costly to rebalance, asset prices need to do a lesser job in transmission. Agents therefore are more willing to engage in international trade in equity to smooth their consumption, and asset prices play a smaller role in the transmission of productivity shocks.

Of course, this analysis depends on the presence of the financial intermediation cost. It is therefore interesting to compare the results in the case in which these costs are set to zero. When we remove the financial intermediation cost, while leaving the gross foreign asset positions unchanged at "integration" scenario, some differences emerges. The relative importance of the valuation share of net foreign asset change is larger than the current account share on impact. This is because lowering the intermediation cost to zero reduces the equity price differential on impact and hence the valuation effect. However, the portfolio rebalancing term is also smaller if intermediation costs are zero because of the smaller deterioration in the income balance in this case, in turn due to the smaller gross equity returns. As a result the overall change in net foreign asset is smaller, and the valuation share of this is larger. In the long run, the valuation share is slightly larger than the portfolio rebalancing share if the intermediation cost is zero, but this is because there are permanent effects of the shocks on these shares (result not reported).

5 Conclusions

Ongoing financial integration has greatly increased gross foreign asset holdings, enhancing the scope for a "valuation channel" of external adjustment. We examine this channel of adjustment in a dynamic stochastic general equilibrium model with international equity trading in incomplete asset markets. We show that the risk-sharing properties of international equity trading are tied to the distribution of income between labor income and profits when equities are defined as claims to firm profits in a production economy. We also find that, for a given level of gross foreign asset positions, the relative importance of the valuation channel increases with the degree of substitutability across goods, the size of financial frictions, and the persistence of shocks. Increasing the size of gross asset positions, the overall amount of risk sharing going through financial markets increases, but the relative importance of the valuation channel decreases in net foreign asset dynamics.

A Appendix

A.1 Aggregation and Equilibrium Household Behavior

We present here aggregate equilibrium conditions for household behavior, focusing on the home economy. Before doing that, we first define the following notation for equity holdings:

$\int_0^a x_{t+1}^{z,j} dz = a x_{t+1}^{z,j} \equiv x_{t+1}$: share of home equity held by the representative home household;

$\int_a^1 x_{t+1}^{z^*,j} dz^* = (1-a)\, x_{t+1}^{z^*,j} \equiv x_{t+1}^*$: share of foreign equity held by the representative home household;

$\int_0^a x_{*t+1}^{z,j} dz = a x_{*t+1}^{z,j} = x_{*,t+1}$ = share of home equity held by the representative foreign household;

$\int_a^1 x_{*t+1}^{z^*,j} dz^* = (1-a)\, x_{*t+1}^{z^*,j} = x_{*,t+1}^*$ = share of foreign equity held by the representative foreign household.

Households Equilibrium in bond markets implies that aggregate per capita bond holdings are zero in each country, since bonds are not traded internationally. Given the notation above, equilibrium in the international market for equities requires:

$$\begin{aligned} a x_{t+1} + (1-a) x_{*t+1} &= a, \\ a x_{t+1}^* + (1-a) x_{*t+1}^* &= 1-a. \end{aligned} \tag{47}$$

Equilibrium versions of household budget constraint and Euler equations in aggregate per capita terms are thus given by:

$$v_t x_{t+1} + v_t^* x_{t+1}^* + C_t = (v_t + d_t)\, x_t + (v_t^* + d_t^*)\, x_t^* + w_t, \tag{48}$$

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} \frac{1 + i_{t+1}}{1 + \pi_{t+1}^{CPI}} \right], \tag{49}$$

$$C_t^{-\frac{1}{\sigma}} v_t \left(a + \gamma_x x_{t+1}\right) = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} a \left(v_{t+1} + d_{t+1}\right) \right], \tag{50}$$

$$C_t^{-\frac{1}{\sigma}} v_t^* \left(1 - a + \gamma_{x^*} x_{t+1}^*\right) = \beta E_t \left[(C_{t+1})^{-\frac{1}{\sigma}} (1-a) \left(v_{t+1}^* + d_{t+1}^*\right) \right], \tag{51}$$

where $v_t \equiv V_t/P_t$, $v_t^* \equiv V_t^*/P_t^*$, $d_t = D_t/P_t$, $d_t^* = D_t^*/P_t^*$, $1 + \pi_{t+1}^{CPI} \equiv P_{t+1}/P_t$, and we used PPP. Similar budget constraint and Euler equations hold abroad.

A.2 Solving for the Steady State

Steady-State Equilibrium Conditions Denoting a product with "·" where necessary for clarity, steady state equilibrium conditions are as follows:

Relative prices:

$$RP^{\omega}Z = aRP \cdot Z + (1-a)\, RP^{*}Z^{*}, \tag{52}$$

$$(RP^{*})^{\omega}Z^{*} = aRP \cdot Z + (1-a)\, RP^{*}Z^{*}, \tag{53}$$

GDPs:

$$y = RP \cdot Z, \quad y^{*} = RP^{*}Z^{*}, \tag{54}$$

Real wages:

$$w = \frac{\theta-1}{\theta}RP \cdot Z, \quad w^{*} = \frac{\theta-1}{\theta}RP^{*}Z^{*}. \tag{55}$$

Real dividends:

$$d = y - w, \quad d^{*} = y^{*} - w^{*}. \tag{56}$$

Budget constraints:

$$vx + v^{*}x^{*} = (v+d)\, x + (v^{*}+d^{*})\, x^{*} + w - C. \tag{57}$$

$$vx_{*} + v^{*}x_{*}^{*} = (v+d)\, x_{*} + (v^{*}+d^{*})\, x_{*}^{*} + w^{*} - C^{*}, \tag{58}$$

Equity market equilibrium:

$$\begin{aligned} ax + (1-a)x_{*} &= a, && (59)\\ ax^{*} + (1-a)x_{*}^{*} &= 1-a. && (60) \end{aligned}$$

Households' first-order conditions for bond and equity choices (with r denoting the steady-state real interest rate):

$$\begin{aligned} 1 &= \beta\,(1+r)\,, && (61)\\ v\,(a+\gamma_{x}x) &= \beta a\,(v+d)\,, && (62)\\ v^{*}\,(1-a+\gamma_{x^{*}}x^{*}) &= \beta\,(1-a)\,(v^{*}+d^{*})\,, && (63)\\ v\,(a+\gamma_{x}^{*}x_{*}) &= \beta a\,(v+d)\,, && (64)\\ v^{*}\,(1-a+\gamma_{x^{*}}^{*}x_{*}^{*}) &= \beta\,(1-a)\,(v^{*}+d^{*})\,. && (65) \end{aligned}$$

Solution Consider the case in which $Z = Z^{*} = 1$. Then:

$$RP^{\omega} = aRP + (1-a)\, RP^{*} = (RP^{*})^{\omega}\,, \tag{66}$$

implying

$$RP = RP^{*} = 1. \tag{67}$$

It follows that

$$y = y^* = 1, \quad w = w^* = \frac{\theta - 1}{\theta}, \quad d = d^* = \frac{1}{\theta}. \tag{68}$$

From the budget constraints,

$$C = \frac{1}{\theta}(x + x^*) + \frac{\theta - 1}{\theta}, \quad C^* = \frac{1}{\theta}(x_* + x_*^*) + \frac{\theta - 1}{\theta}. \tag{69}$$

In steady states with $x + x^* = 1$ ($x_* + x_*^* = 1$), with complete home bias or symmetric equity holdings, this simplifies to $C = C^* = 1$.

From the steady-state Euler equations (62)-(65), equity prices, gross returns on equity holdings, and home and foreign equity positions are, respectively:

$$v = \frac{1}{\theta}\frac{\beta a}{a(1-\beta) + \gamma_x x}, \quad v^* = \frac{1}{\theta}\frac{\beta(1-a)}{(1-a)(1-\beta) + \gamma_{x^*} x^*}, \tag{70}$$

$$1 + \frac{d}{v} = \frac{1}{\beta a}(a + \gamma_x x), \quad 1 + \frac{d^*}{v^*} = \frac{1}{\beta(1-a)}(1 - a + \gamma_{x^*} x^*), \tag{71}$$

$$x = \frac{\gamma_x^* \frac{a}{1-a}}{\gamma_x + \gamma_x^* \frac{a}{1-a}}, \quad \text{with } x_* = \frac{a}{1-a}(1 - x), \tag{72}$$

$$x^* = \frac{\gamma_{x^*}^*}{\gamma_{x^*} + \gamma_{x^*}^* \frac{a}{1-a}}, \quad \text{with } x_*^* = 1 - \frac{a}{1-a}x^*. \tag{73}$$

A.3 No-Arbitrage in the Symmetric Case

Consider the equilibrium Euler equation for home holdings of foreign equity. In log-linear terms:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) + \frac{\gamma}{1+\gamma}\hat{x}_{t+1}^* = -\hat{v}_t^* + \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1}^* + \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1}^*.$$

Similarly, the equilibrium, log-linear Euler equation for foreign holdings of home equity is

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}^* - \hat{C}_t^*\right) + \frac{\gamma}{1+\gamma}\hat{x}_{*t+1} = -\hat{v}_t + \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1} + \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1},$$

and the difference between these equations yields:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}^D - \hat{C}_t^D\right) + \frac{\gamma}{1+\gamma}\hat{x}_{t+1}^D = \hat{v}_t^D - \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1}^D - \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1}^D. \tag{74}$$

Similarly, the log-linear Euler equations for home holdings of home equity and foreign holdings of foreign equity may be written:

$$\begin{aligned}
\frac{1}{\sigma}E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) - \frac{\gamma}{1+\gamma}\hat{x}_{*t+1} &= -\hat{v}_t + \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1} + \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1}, \\
\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}^* - \hat{C}_t^*\right) - \frac{\gamma}{1+\gamma}\hat{x}_{t+1}^* &= -\hat{v}_t^* + \frac{\beta}{1+\gamma}E_t\hat{v}_{t+1}^* + \frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}_{t+1}^*,
\end{aligned}$$

and their difference implies:

$$\frac{1}{\sigma}E_t\left(\hat{C}^D_{t+1}-\hat{C}^D_t\right)+\frac{\gamma}{1+\gamma}\hat{x}^D_{t+1}=-\hat{v}^D_t+\frac{\beta}{1+\gamma}E_t\hat{v}^D_{t+1}+\frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}^D_{t+1}. \tag{75}$$

Inspection of equations (74) and (75) makes it possible to conclude that, for both equations to hold at the same time, it must be:

$$\frac{1}{\sigma}E_t\left(\hat{C}^D_{t+1}-\hat{C}^D_t\right)+\frac{\gamma}{1+\gamma}\hat{x}^D_{t+1} = 0, \tag{76}$$

$$-\hat{v}^D_t+\frac{\beta}{1+\gamma}E_t\hat{v}^D_{t+1}+\frac{1-\beta+\gamma}{1+\gamma}E_t\hat{d}^D_{t+1} = 0. \tag{77}$$

A.4 No-Arbitrage under Full Cross-Shareholding

Assume $\gamma_{x*}=\gamma^*_x=0$ and $\gamma^*_{x*}=\gamma_x>0$. In the implied asymmetric steady state with full cross-shareholding, we have:

$$v=v^*=\frac{1}{\theta}\frac{\beta}{1-\beta},\quad x=0=x^*_*,\quad x_*=1=x^*.$$

The equilibrium Euler equation for home holdings of home equity is:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}-\hat{C}_t\right)=\beta E_t(\hat{v}_{t+1})-\hat{v}_t+2\gamma\hat{x}_{*t+1}+(1-\beta)E_t(\hat{d}_{t+1}). \tag{78}$$

To write this equation in log-linear form, we used the implication of the equilibrium condition: $\mathsf{d}x_{t+1}=-\hat{x}_{*t+1}x_*=-\hat{x}_{*t+1}$.

The equilibrium Euler equation for home holdings of foreign equity is:

$$\frac{1}{\sigma}E_t\left(\hat{C}_{t+1}-\hat{C}_t\right)=\beta E_t(\hat{v}^*_{t+1})-\hat{v}^*_t+(1-\beta)E_t(\hat{d}^*_{t+1}) \tag{79}$$

This equation has no portfolio choice term because the corresponding equity investment costs (γ's) were assumed to be zero.

Subtracting equations (78) and (79) and using $\hat{x}^D_{t+1}=-2\hat{x}_{*t+1}$ yields equation (45).

A.5 Data

The U.S. variables used in the quantitative analysis are defined and constructed as follows.[24]

CPI: Consumer Price Index, IMF IFS, series code 62064...ZF.

PPI: Producer Price Index, IMF IFS, series code 62063...ZF.

NEER: Nominal Broad Trade-Weighted Exchange Value of the U.S. dollar, Jan 1997=100, Haver Analytics, series FXTWB@USECON.

[24] SAAR means Seasonally adjusted, quoted at annual rates.

REER: Real Broad Trade-Weighted Exchange Value of the U.S. dollar, Jan 1997=100, Haver Analytics, series FXTWBC@USECON.

VUS: MSCI US Equity Price Index (in U.S. dollar), Bloomberg, series MXUS.

VEXUS: MSCI World Index through 1987Q4 and MCI All Country World Index from 1988Q1 to 2004Q4 (in U.S. dollar), Bloomberg, series MXWDU and MXWOU.

GDPB$: Gross Domestic Product, seasonally adjusted quoted at annual rates (SAAR), Billion of Dollar, Haver Analytics, series code GDP@USECON.

CAB$: Balance on current account, SAAR, Billion of Dollar, Haver Analytics, series CAB@USECON.

NXB$: Net Exports of Goods and Services, SAAR, Billion of Dollar, Haver Analytics, series XNET@USECON.

NFAB$: Net Foreign Assets, Interpolated linearly from annual data, Billion of Dollar, Lane and Milesi-Ferretti (2006).

Output (GDPH): Real GDP, SAAR, Chained 2000 dollar, Haver Analytics, series GDPH@USECON.

Consumption: (CH): Real Personal Consumption Expenditures, SAAR, Chained 2000$, Haver Analytics, series CH@USECON.

Trabe balance/Output (NX/GDP): NXB$/ GDPB$.

Current account/Output (CA/GDP): CAB$/GDPB$.

NFA change/Output (DNFA/GDP): First difference of NFAB$/GDPB$.

Current transfer/Output (CT/GDP): Current transfer/GDP$

Income Balance/Output (IB/GDP): Income balance /GDP$

Valuation Change/Output (VC/GDP): (CAB$-DNFA)/GDP$.

Foreign Equity Price (VF): (VEXUS*REER)/CPI.

Home Equity Prices (VH): VUS/CPI.

Foreign Equity Prices (Return):

Home Equity Prices (Return):

All variables are percent deviations from HP-filtered trend (with smoothing parameter equal 1600). Variables are transformed in natural logarithm whenever possible. All indices are rebased so that 2000 is 100. All series except NFA, which is interpolated, are quarterly.

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Figure 1. U.S. Cumulative current account balance and net foreign asset position

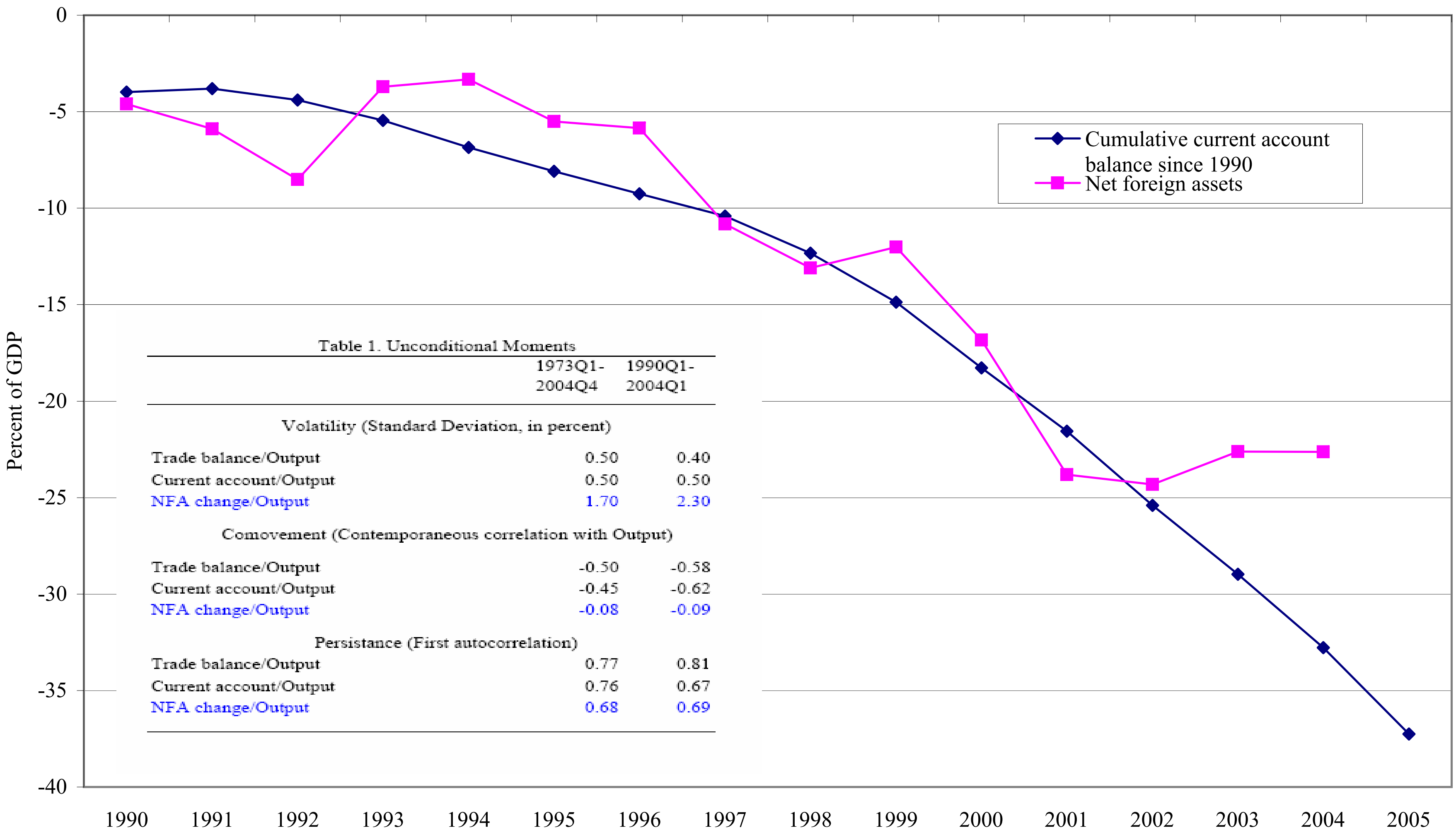
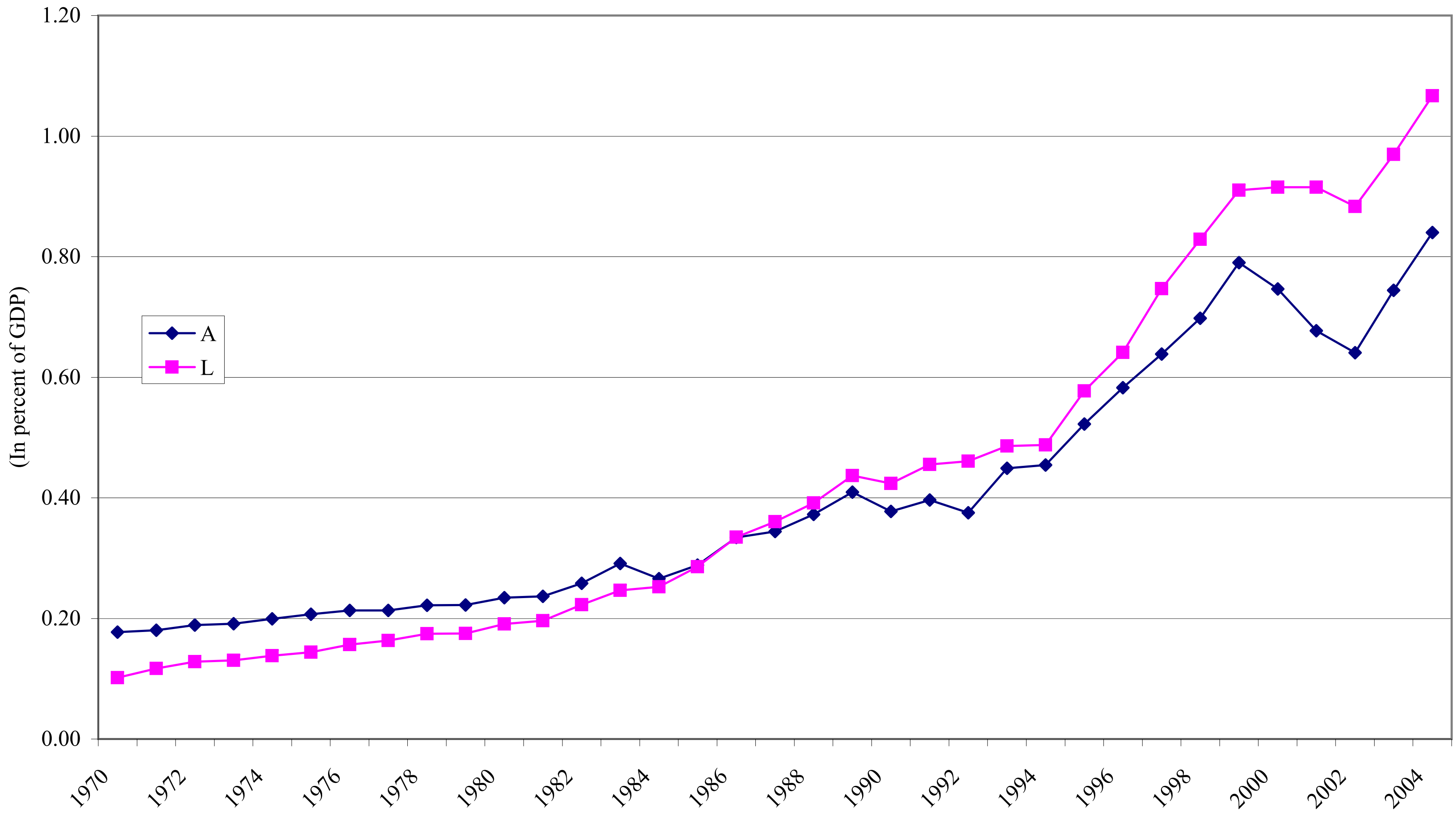


Table 1. Unconditional Moments

	1973Q1-2004Q4	1990Q1-2004Q1
Volatility (Standard Deviation, in percent)		
Trade balance/Output	0.50	0.40
Current account/Output	0.50	0.50
NFA change/Output	1.70	2.30
Comovement (Contemporaneous correlation with Output)		
Trade balance/Output	-0.50	-0.58
Current account/Output	-0.45	-0.62
NFA change/Output	-0.08	-0.09
Persistance (First autocorrelation)		
Trade balance/Output	0.77	0.81
Current account/Output	0.76	0.67
NFA change/Output	0.68	0.69

Source: see data appendix.

Figure 2. U.S. Gross Foreign Assets and Liabilities



Source: see appendix.

Figure 3. Selected impulse responses
One percent increase in home productivity

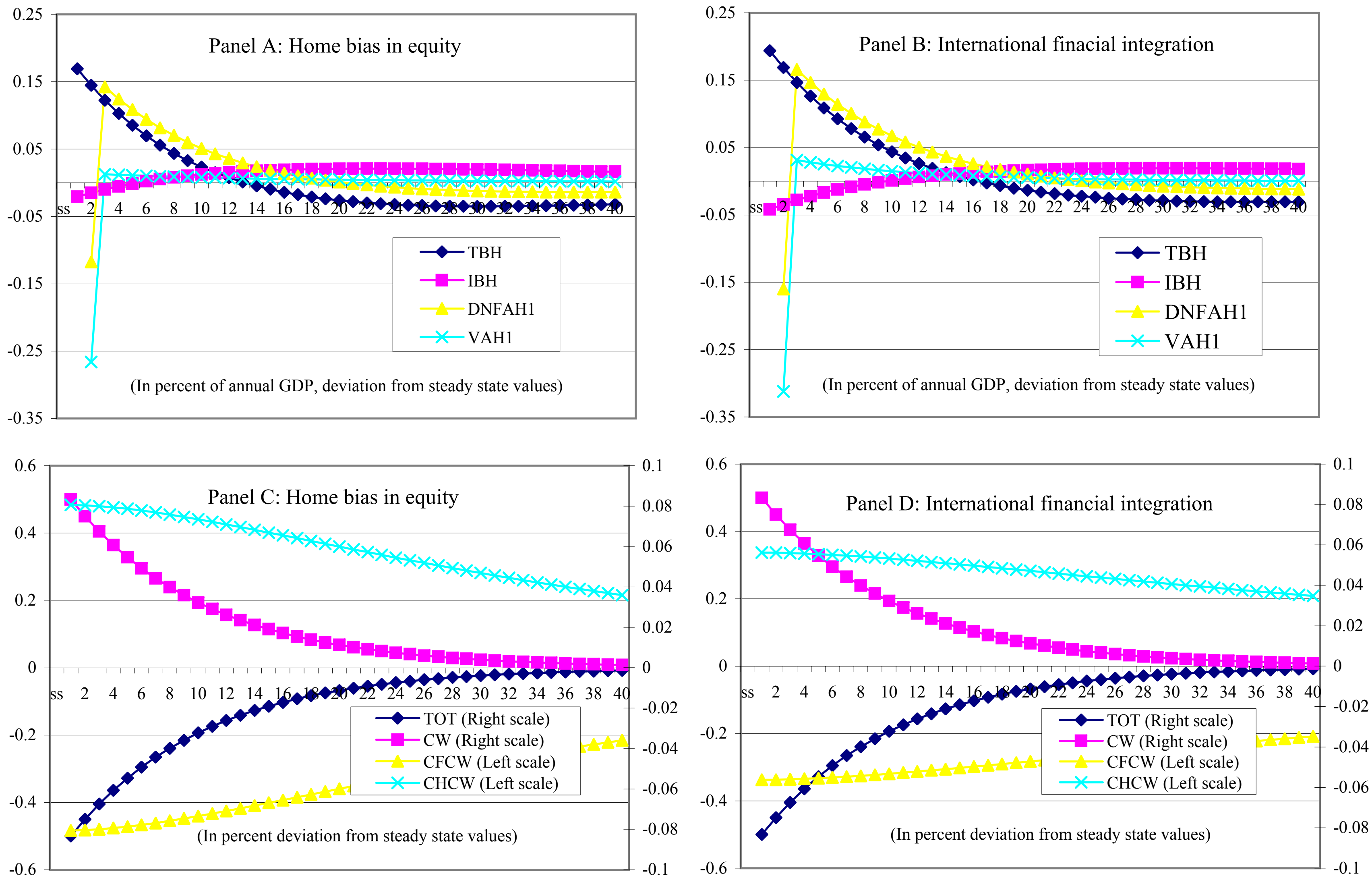


Figure 4. Selected impulse responses

One percent increase in home productivity

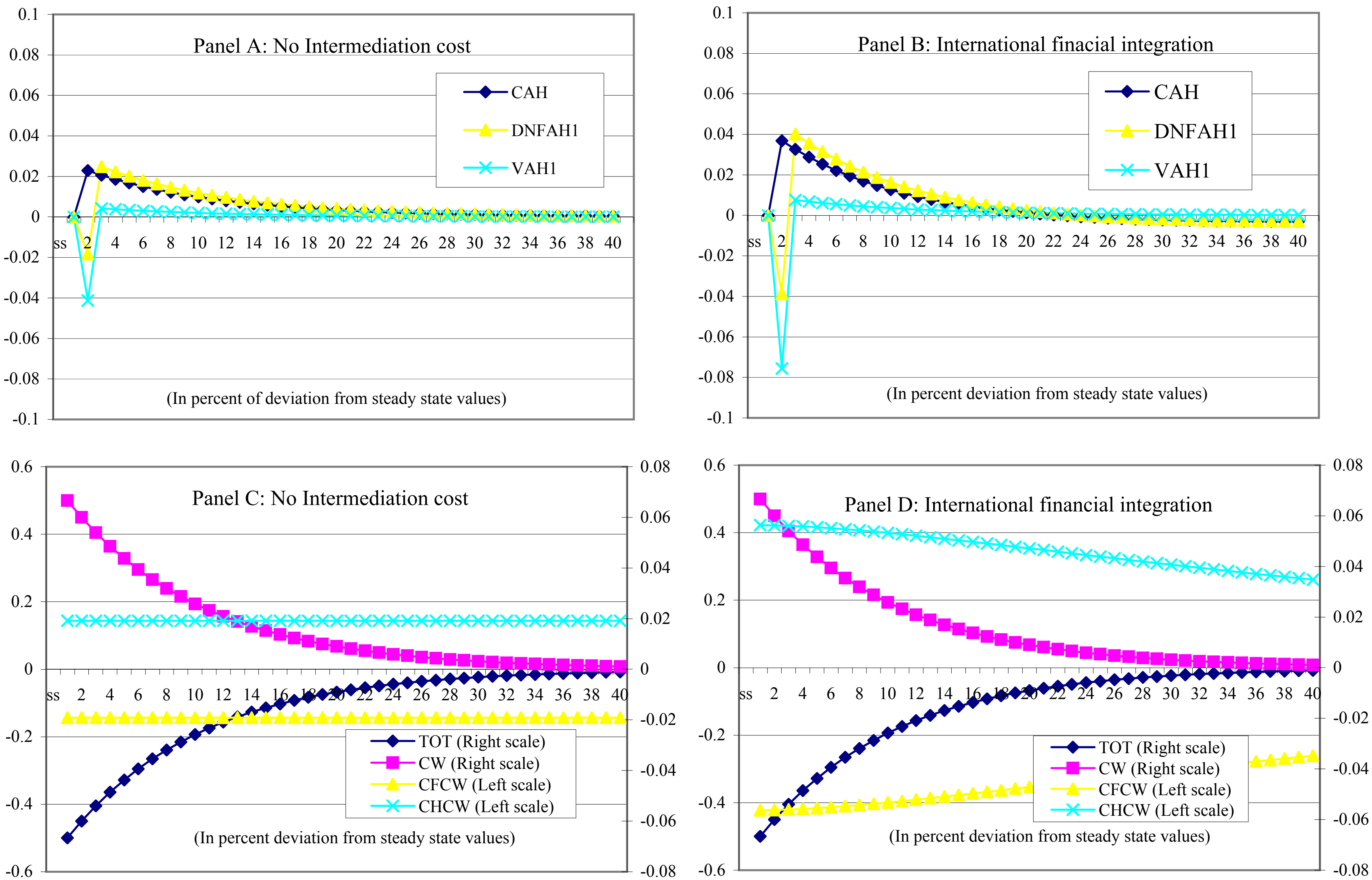


Table 1. Unconditional Moments: Data and Alternative Models

	Data 1/	Home bias in portfolio	International financial integration	International financial integration
		Zero NFA	Zero NFA	Negative NFA
	1973Q1-2004Q4	Rho=0.9, Omega=2, Theta=6, Sigma=0.5, Gamma on home shares=0.01; Gamma on foreign shares=0.03. NFA=0.	Rho=0.9, Omega=2, Theta=6, Sigma=0.5, Gammaon home shares=0.01; Gamma on foreign shares=0.01. NFA=0.	Rho=0.9, Omega=2, Theta=6, Sigma=0.5, Gamma on home shares=0.006; Gamma on foreign shares=0.01. NFA=-.25 percent of GDP.
Volatility (Standard Deviation, in percent)				
Output	1.60	1.81	1.81	1.81
Consumption	1.20	1.72	1.70	1.69
Trade balance/Output	0.40	0.53	0.62	0.65
Current account/Output	0.50	0.43	0.46	0.49
NFA change/Output	1.70	0.45	0.55	1.55
Home equity price	10.10	2.98	3.00	3.04
Comovement (Contemporaneous correlation with Output)				
Consumption	0.85	0.96	0.94	0.93
Trade balance/Output	-0.50	0.31	0.35	0.36
Current account/Output	-0.45	0.40	0.42	0.42
NFA change/Output	-0.08	0.27	0.27	-0.08
Home equity price	0.41	0.94	0.91	0.88
Comovement (Contemporaneous cross-correlation)				
Home and Foreign output	0.28 2/	0.60	0.60	0.60
Home and Foreign consumption	0.15 2/	0.77	0.82	0.84
Home and Foreign equity price	0.69	0.96	0.99	1.00
Persistance (First autocorrelation)				
Output	0.88	0.90	0.90	0.90
Consumption	0.88	0.91	0.91	0.91
Trade balance/Output	0.77	0.89	0.89	0.90
Current account/Output	0.76	0.87	0.88	0.89
NFA change/Output	0.68	0.59	0.56	0.04
Home equity price	0.82	0.90	0.90	0.90

1/ See Appendix for details.

2/ Ambler, Cardia, and Zimmermann (2004, Table 1). Sample period 1973Q1-2000Q4.

Table 2: Steady State Portfolios 1/

	Home bias in portfolio with zero-NFA	International financial integration with zero-NFA	International financial integration with non zero-NFA
Home holding of home shares	0.75	0.5	0.5
Home holding of foreign shares	0.25	0.5	0.5
Home equity price	6.6	8.3	10.3
Foreign equity price	6.6	8.3	8.3
Home stock market capitalization (in percent of annual GDP)	165	206.3	256.3
Home gross foreign assets (in percent of annual GDP)	41	103	103
Home net foreign assets (in percent of annual GDP)	0	0	-25.0
Home total assets (in percent of annual GDP)	165	206.3	231.3
Home equity return on home shares (annual, gross of intermediation cost, in percent)	9.7	7.8	6.2
Home equity return on foreign shares (annual, gross of intermediation cost, in percent)	9.7	7.8	7.8
Home real interest rate (annual, in percent)	4.0	4.0	4.0
Home GDP (quarterly)	1	1	1

1/ See table 1 for definition of the scenarios.

Table 3. Cumulative impulse responses--net foreign assets and its components 1/

	Trade Balance	Total return 2/	Income Balance	Valuation change	NFA
Home bias in equity					
As share of GDP	0.12	0.47	0.53	-0.07	0.59
As share of NFA	0.21	0.79	0.91	-0.11	1
International financial integration					
As share of GDP	0.63	0.29	0.30	-0.01	0.92
As share of NFA	0.69	0.31	0.32	-0.01	1

1/ Over the first 40 quarterly periods
2/ Sum of Income balance and valuation change