Testing the q-Theory of Anomalies

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Abstract

The q-theory explanations of asset pricing anomalies are quantitatively important. We perform a new asset pricing test by using GMM to minimize the difference between average stock returns in the data and average investment returns constructed from observable firm characteristics. Under various specifications, the model-implied average returns display similar magnitudes of dispersion across portfolios sorted on investment-to-asset and on size and book-to-market. But the predicted dispersions in average returns among portfolios sorted on earnings surprises are somewhat smaller in magnitude than those observed in the data.

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1 Introduction

A great deal of research in financial economics has been devoted to documenting stylized facts that are inconsistent with current consumption-based or risk-based asset pricing models—enough research to warrant such extensive surveys as Fama (1998) and Schwert (2003). Dubbed "anomalies," these stylized facts typically link returns to observed firm characteristics or corporate events. A related literature has provided behavioral explanations for these anomalies, arguing that they are strong evidence against efficient markets and rational expectations (e.g., Shleifer, 2000; Barberis and Thaler, 2003).

We propose an alternative explanation for some of these anomalies. To this end, we construct a neoclassical, q-theoretical foundation for explaining time-varying expected returns in terms of corporate policies. The intuition behind this framework is simple. If the firm has constant return to scale technology, its stock return equals its investment return. The firm's optimizing decision over financing and real investment policies then links the investment return to firm characteristics. By signing the partial derivatives of investment returns with respect to these characteristics, we demonstrate analytically that q theory is qualitatively consistent with many anomalies. We then examine empirically the quantitative importance of these q-theory based explanations of anomalies. Our tests are based on estimating a structural model with data on portfolios sorted by the firm characteristics of interest.

The phenomena we examine include the investment-disinvestment anomaly: the investment-to-assets ratio is negatively correlated, but the disinvestment-to-asset ratio is positively correlated with future returns. This anomaly is stronger in firms with high operating income-to-capital. The second important anomaly we study is typically dubbed the value premium: average returns are negatively correlated with the market-to-book ratio, and the magnitude of this correlation decreases with firm market value. The final phenomenon we explore is the post-earnings-announcement drift (earnings momentum) anomaly: firms with high earnings surprises earn higher average returns than firms with low earnings surprises, and this effect is stronger in small firms.

The intuition behind the tests is straightforward. The investment return from time t to $t+1$ equals the ratio of the marginal profit of investment at $t+1$ divided by the marginal cost of investment at t. This definition suggests two economic mechanisms that are potential driving forces behind these anomalies. The first two anomalies can be explained by the connection between optimal investment and time varying expected returns. The intertemporal investment model behind q theory produces a downward-sloping investment-demand function. Therefore, the ratio of investment to assets increases with the net present value of capital, and the net present value decreases with the cost of capital; that is, the expected return. In other words, the investment anomaly occurs because a low cost of capital implies high net present value, which in turn implies high investment demand. The intuition behind the value anomaly is a simple corollary, which is based on the idea that investment is an increasing function of marginal q, which is in turn proportional to the market-to-book ratio. The negative slope of the investment-demand function then implies a negative relation between the expected return and the market-to-book ratio.

The next anomaly can be explained by the marginal product of capital at time $t+1$ in the numerator of the investment return. Specifically, under certain conditions the marginal product is proportional to profitability, a property that implies a positive relation between expected profitability and expected returns. Because profitability is highly positively serially correlated, and because and because earnings surprises and profitability are highly correlated, earnings surprises should be highly correlated with expected returns.

To test this intuition, we proceed in two steps. First, to facilitate empirical tests of the model, we derive new analytical relations between stock and investment returns after incorporating into the q framework flow operating costs, debt financing, and financing costs of external equity. These relations provide a convenient structural framework that allows us to link empirically firm characteristics to expected returns. We then use GMM to minimize the differences between the average stock returns observed in the data and the average stock returns implied from the model.

We find that the mechanisms suggested by q theory are quantitatively important for the asset pricing anomalies we examine. Under various specifications average stock returns and model-generated average investment returns track one another closely across portfolios sorted by size, book-to-market, and investment-to-assets. For example, when we apply the benchmark model with only physical adjustment costs to the Fama-French (1993) 25 portfolios, the average absolute pricing error is only 0.074% per month, and the overidentification test fails to reject the null hypothesis that the average pricing error is zero. In the universe of 25 portfolios, all but one have alphas insignificantly different from zero. Further, more sophisticated models produce a better quantitative fit. Applying the model with costly external finance to the Fama-French 25 portfolios reduces the average absolute pricing error slightly to 0.059% per month. More importantly, all of the 25 portfolios have insignificant alphas.

The model is less successful in generating the pattern of average returns seen in portfolios sorted by standardized unexpected earnings, or SUE. We find significant alphas with magnitudes about 0.40% per month for the two extreme deciles. However, the average returns constructed from the benchmark model for SUE deciles two and nine are 1.12% and 1.72% per month, close to their corresponding average stock returns, 1.00% and 1.73%, respectively. The model does, however, perform better for the nine size and SUE portfolios. Only three of the alphas are significant, and the model-implied average-return dispersion between low and high SUE firms is higher in small firms, consistent with the data.

Our work is closely tied to the empirical implementation of production-based asset pricing models, which starts with Cochrane (1991), who points out that stock returns should equal investment returns in the q model. Restoy and Rockinger (1994) formally establish this equivalence under linear homogeneity. Cochrane (1991) tests this idea using aggregate data, and Cochrane (1996) shows empirically that a factor constructed from firm investment can help explain the cross-section of returns. Our contribution consists of testing a slightly richer model on portfolios sorted on anomaly-related variables.

Our work is also related to the q theory of investment originated by Tobin (1969). For example, the equivalence between stock and investment returns is in essence a restatement of the equivalence between marginal q and average q, a result first proved by Hayashi (1982). Further, the investment Euler equation from q theory has been tested extensively to understand the behavior of capital investment. For early examples see Shapiro (1986) and Whited (1992). We restate the investment Euler equation in terms of returns and test it on cross-sectional return data.

Finally, our paper is also related to models of the real determinants of the cross-section of returns. Examples include Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), and Carlson, Fisher, and Giammarino (2004a, 2004b). Although our model is less rich than those presented in these papers, it stands out in that it is sufficiently simple that it can be taken directly to data without the use of simulation methods. Put differently, we contribute to this literature by implementing a new closed-form and intuitive asset-pricing test, which is motivated by economic theory.

The rest of the trip is organized as follows. Section 2 provides the theoretical framework, and Section 3 discusses the test design. We describe our sample construction in Section 4, and present our empirical results in Section 5. Finally, Section 6 concludes.

2 Theoretical Framework

This section delineates our theoretical framework. We start with the benchmark framework with only physical adjustment costs. We then add more realistic ingredients to the model, including debt financing, costs of outside equity, and multiple capital goods.

2.1 The Benchmark Model

Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as labor, to produce a perishable output. The firm chooses the levels of these inputs each period to maximize its operating profit, defined as its revenue minus the expenditures on these inputs.

Taking the operating profit as given, the firm then chooses optimal investment to maximize its market value. Capital investment involves physical costs of adjustment.

Let $\Pi_t \equiv \Pi(K_t, X_t)$ denote the maximized operating profits at time t, in which K_t is the capital stock at time t, and X_t is a vector of random variables representing exogenous shocks to operating profit, such as aggregate and firm-specific shocks to the production technology, shocks to the prices of costlessly adjusted inputs, or industry- and firm-specific shocks to the demand for firm output. We assume that the operating profit function exhibits constant return to scale.

Firms that stay in production each period must incur a flow operating cost, cK_t, which is proportional to capital stock with $c > 0$. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t. \tag{1}$$

Thus, end-of-period capital equals real investment plus beginning-of-period capital net of depreciation. Capital depreciates at a fixed proportional rate of δ.

When the firm invests, it incurs costs for two reasons: purchase/sale costs and convex costs of physical adjustment. Purchase/sales costs are incurred when the firm buys or sells uninstalled capital. When the firm disinvests, this cost is negative. Convex costs of physical adjustment are nonnegative costs that are zero when $I_t = 0$. These costs are continuous, strictly convex in I_t, non-increasing in K_t, and differentiable with respect to I_t and K_t everywhere. The second-order partial derivative of the convex-cost function with respect to K_t is nonnegative. The total cost of investment represents the sum of purchase/sale costs and convex costs of physical adjustment, and is denoted $\Phi(I_t, K_t)$. The augmented adjustment-cost function $\Phi(I_t, K_t)$ satisfies $\Phi_1(I_t, K_t) \leq 0$, $\Phi_2(I_t, K_t) \leq 0$, and $\Phi_{11}(I_t, K_t) > 0$, where subscript i denotes the first-order partial derivative with respect to the i^{th} argument, and multiple subscripts denote high-order derivatives.

Let q_t be the present-value multiplier associated with equation (1). Firm value, $V(K_t, X_t)$, can be formulated as follows:

$$\max_{\{I_{t+j},K_{t+1+j}\}_{j=0}^{\infty}} \mathrm{E}_t \left[\sum_{j=0}^{\infty} \begin{array}{c} M_{t+j}(\Pi(K_{t+j}, X_{t+j}) - cK_{t+j} - \Phi(I_{t+j}, K_{t+j}) \\ -q_{t+j}[K_{t+j+1} - (1-\delta)K_{t+j} - I_{t+j}]). \end{array} \right] \tag{2}$$

The first-order conditions with respect to I_t and K_{t+1} are, respectively,

$$q_t = \Phi_1(I_t, K_t) \tag{3}$$

$$q_t = \mathrm{E}_t[M_{t+1}[\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)q_{t+1}]]. \tag{4}$$

Combining the first-order conditions in equations (3) and (4) yields:

$$\Phi_1(I_t, K_t) = \mathrm{E}_t[M_{t+1}[\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)\Phi_1(I_{t+1}, K_{t+1})]] \tag{5}$$

Dividing both sides by $\Phi_1(I_t, K_t)$ yields:

$$\mathrm{E}_t[M_{t+1} r^I_{t+1}] = 1, \tag{6}$$

in which r^I_{t+1} denotes the investment return, which can be expressed as

$$r^I_{t+1} = \frac{\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)\Phi_1(I_{t+1}, K_{t+1})}{\Phi_1(I_t, K_t)}. \tag{7}$$

Intuitively, equation (7) says that the investment return is the ratio of the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at time t. The denominator, $\Phi_1(I_t, K_t)$, is the marginal cost of investment. By optimality, it equals marginal q_t—the shadow value of capital, or, equivalently, the expected present value of the marginal profits from investing in capital goods. In the numerator of equation (7), $\Pi_1(K_{t+1}, X_{t+1}) - c$ is the extra operating profits, net of the flow operating costs generated by the extra capital at $t+1$; the effect of extra capital on the augmented adjustment cost is captured by $-\Phi_2(I_{t+1}, K_{t+1})$ captures; and $(1-\delta)\Phi_1(I_{t+1}, K_{t+1})$ is the expected present value of marginal profits evaluated at time $t+1$, net of depreciation.

Proposition 1 *Define the ex-dividend firm value,* P_t*, as*

$$P_t \equiv P(K_t, K_{t+1}, X_t) = V(K_t, X_t) - \Pi(K_t, X_t) + cK_t + \Phi(I_t, K_t),$$

and define stock return as

$$r_{t+1}^S \equiv \frac{P_{t+1} + \Pi(K_{t+1}, X_{t+1}) - cK_{t+1} - \Phi(I_{t+1}, K_{t+1})}{P_t}.$$

If the operating-profit and the augmented adjustment-cost functions are both linear homogeneous, then $P_t = q_t K_{t+1}$ *and* $r_{t+1}^S = r_{t+1}^I$.

Proof. See Appendix A. ■

The equivalence between stock and investment returns is a result of the equivalence between marginal q and average q. The insight that stock and investment returns are equivalent appears first in Cochrane (1991), and is formally established by Restoy and Rockinger (1994).

2.2 Debt Financing

The benchmark model assumes that all firms are entirely equity-financed. This assumption is unrealistic because it ignores debt financing. If the firm finances investment using both equity and debt, then the investment return is a weighted average of equity return and corporate bond return.

For simplicity, we follow Hennessy and Whited (2005) and model only one-period debt.[1] Assume that at the beginning of period t, firms can choose to issue a certain amount of one-period debt, denoted B_{t+1}, that must be repaid at the beginning of next period. Negative B_{t+1} represents cash holdings. The interest rate associated with B_t is $R(X_t)$, and is a function of the exogenous state variable, X_t, and is stochastic. Note that R can be firm-specific because X_t contains both aggregate and firm-specific shocks.

[1] As shown in Barclay and Smith (1995a, b), the maturity and priority structures of debt are undoubtedly important in the data. However, we leave modeling these realistic features of debt financing for future research.

The market value of equity can be formulated as:

$$V(K_t, B_t, X_t) = \max_{\{I_{t+j}, K_{t+j+1}, B_{t+j+1}\}_{j=0}^{\infty}} \mathrm{E}_t \left[\begin{array}{c} \sum_{j=0}^{\infty} M_{t+j} [\Pi(K_{t+j}, X_{t+j}) - cK_{t+j} \\ -\Phi(I_{t+j}, K_{t+j}) + \ B_{t+j+1} - R\left(X_{t+j}\right) B_{t+j} \\ -q_{t+j}\left(K_{t+j+1} - (1-\delta)K_{t+j} - I_{t+j}\right)] \end{array} \right]. \tag{8}$$

The optimality conditions with respect to I_t, K_{t+1}, and B_{t+1} are, respectively

$$q_t = \Phi_1(I_t, K_t) \tag{9}$$

$$q_t = \mathrm{E}_t \left[M_{t+1} \left[\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta) q_{t+1} \right] \right] \tag{10}$$

$$1 = \mathrm{E}_t \left[M_{t+1} R\left(X_{t+1}\right) \right]. \tag{11}$$

It follows that $\mathrm{E}_t[M_{t+1} r^I_{t+1}] = 1$, and $\mathrm{E}_t[M_{t+1} r^B_{t+1}] = 1$, in which the investment return is

$$r^I_{t+1} \equiv \frac{[\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)\Phi_1(I_{t+1}, K_{t+1})]}{\Phi_1(I_t, K_t)}, \tag{12}$$

and the corporate bond return is

$$r^B_{t+1} \equiv R\left(X_{t+1}\right). \tag{13}$$

Proposition 2 *Define the ex-dividend equity value as*

$$P(K_t, B_t, X_t) \equiv V(K_t, B_t, X_t) - \Pi(K_t, X_t) + cK_t + \Phi(I_t, K_t) - B_{t+1} + R\left(X_t\right) B_t$$

and the stock return as

$$r^S_{t+1} = \frac{P_{t+1} + \Pi(K_{t+1}, X_{t+1}) - cK_{t+1} - \Phi(I_{t+1}, K_{t+1}) + B_{t+2} - R(X_{t+1}) B_{t+1}}{P_t}.$$

Under constant return to scale,

$$q_t K_{t+1} = P(K_t, B_t, X_t) + B_{t+1}. \tag{14}$$

Further, the investment return is the leverage-weighted average of stock and bond returns:

$$r^I_{t+1} = \nu_t r^B_{t+1} + (1-\nu_t) r^S_{t+1}, \tag{15}$$

in which ν_t is the leverage ratio:

$$\nu_t \equiv \frac{B_{t+1}}{P(K_t, B_t, X_t) + B_{t+1}} \tag{16}$$

Proof. See Appendix A. ■

2.3 Costly External Equity

The benchmark framework assumes that firms can finance investment using external equity costlessly. In reality, issuing outside equity is often costly. See, for example, Smith (1977), Lee, Lochhead, Ritter, and Zhao (1996), and Altinkilic and Hansen (2000).

To capture the financing costs of issuing equity, we let $\Psi(O_t, K_t)$ denote the financing-cost function of outside equity, in which O_t is the amount of financing,

$$O_t \equiv [\Phi(I_t, K_t) + cK_t - \Pi(K_t, X_t)]\mathbf{1}^O_t, \tag{17}$$

and in which $\mathbf{1}^O_t \equiv \mathbf{1}_{\{\Phi(I_t,K_t)+cK_t-\Pi(K_t,X_t)\geq 0\}}$ is the indicator function that takes the value of one if the firm uses outside equity and zero otherwise.

We further assume that the financing-cost function is increasing, convex, and has economies of scale, i.e., $\Psi_1 > 0, \Psi_{11} > 0$, and $\Psi_2 \leq 0$. For simplicity, we also assume no fixed costs of financing: $\Psi(0, K_t) = 0$. Finally, Ψ also exhibits constant return to scale,

$$\Psi(O_t, K_t) = \Psi_1(O_t, K_t)O_t + \Psi_2(O_t, K_t)K_t. \tag{18}$$

The market value of equity, $V(K_t, X_t)$, can now be formulated as:

$$\max_{\{I_{t+j}, K_{t+1+j}\}_{j=0}^\infty} \mathrm{E}_t \left[\begin{array}{c} \sum_{j=0}^\infty M_{t+j}(\Pi(K_{t+j}, X_{t+j}) - cK_{t+j} - \Phi(I_{t+j}, K_{t+j}) \\ -\Psi(O_{t+j}, K_{t+j}) - q_{t+j}[K_{t+1+j} - (1-\delta)K_{t+j} - I_{t+j})] \end{array} \right]. \tag{19}$$

The optimality conditions with respect to I_t and K_{t+1} are, respectively,

$$q_t = \Phi_1(I_t, K_t)(1 + \Psi_1(O_t, K_t)\mathbf{1}_t^O)$$

$$q_t = \mathrm{E}_t\left[M_{t+1}\left[\begin{array}{c}(\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}))(1 + \Psi_1(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O) \\ -\Psi_2(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O + (1-\delta)q_{t+1}\end{array}\right]\right].$$

Combining the two equations yields $\mathrm{E}_t[M_{t+1}r_{t+1}^I] = 1$, in which the investment return is

$$r_{t+1}^I = \frac{\left[\begin{array}{c}(\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}))(1 + \Psi_1(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O) \\ -\Psi_2(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O + (1-\delta)\Phi_1(I_{t+1}, K_{t+1})(1 + \Psi_1(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O)\end{array}\right]}{\Phi_1(I_t, K_t)(1 + \Psi_1(O_t, K_t)\mathbf{1}_t^O)}. \quad (20)$$

The investment return in equation (20) can still be interpreted as the ratio of the marginal benefits of investment evaluated at period $t+1$ divided by the marginal costs of investment at period t. Increasing one unit of capital entails marginal purchase/sales and physical adjustment costs that sum up to $\Phi_1(I_t, K_t)$. If this investment is partially financed by outside equity, its marginal financing cost is then $\Psi_1(O_t, K_t)\frac{\partial O_t}{\partial I_t} = \Psi_1(O_t, K_t)\Phi_1(I_t, K_t)$. Adding all three parts of the marginal cost yields the denominator in equation (20). The numerator of equation (20) contains three terms. The interpretation of the first term, $(\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}))$, is the same as that in the benchmark model. If the firm issues outside equity at $t+1$, then the marginal effect of the extra unit of capital on the amount of financing costs is $-\Psi_1(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O\frac{\partial O_{t+1}}{\partial K_{t+1}} = (\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}))\Psi_1(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O$. The extra unit of capital also lowers financing costs because of economies of scale. This benefit is captured by $-\Psi_2(O_{t+1}, K_{t+1})\mathbf{1}_{t+1}^O$. Finally, at the end of period $t+1$, the firm is left with $1-\delta$ units of capital net of depreciation. This capital is worth marginal q evaluated at time $t+1$, which equals the marginal costs of investment at that time.

Proposition 3 *Define the ex-dividend firm value, P_t, as*

$$P_t \equiv P(K_t, K_{t+1}, X_t) = V(K_t, X_t) - \Pi(K_t, X_t) + cK_t + \Phi(I_t, K_t) + \Psi(O_t, K_t),$$

and define stock return as

$$r^S_{t+1} \equiv \frac{P_{t+1} + \Pi(K_{t+1}, X_{t+1}) - cK_{t+1} - \Phi(I_{t+1}, K_{t+1}) - \Psi(O_{t+1}, K_{t+1})}{P_t}.$$

If the operating-profit, the augmented adjustment-cost, and the financing-cost functions are all linear homogeneous, then $P_t = q_t K_{t+1}$ *and* $r^S_{t+1} = r^I_{t+1}$*, in which the investment return is given by equation (20).*

Proof. See Appendix A. ■

It is tempting to incorporate time-to-build into the model, especially for structures. Time-to-build says that multiple periods are required to build new capital projects, instead of the one-period convention embedded in the standard capital accumulation equation (1). Theoretically, several studies have demonstrated the importance of time-to-build in driving business cycle fluctuations; for example, Kydland and Prescott (1982) and Christiano and Todd (1996). Empirically, Lamont (2000) shows that investment plans can predict excess stock returns better than actual aggregate investment, probably because of investment lags. However, the analytical link between the stock and investment returns breaks down with time-to-build, because the investment return measures the trade-off between the marginal benefits and the marginal costs of new investment projects. In contrast, the stock return is the return to the entire firm that derives its market value not only from the new but also from the old, incomplete projects. The details of these derivations are available from the authors upon request.

3 Empirical Design

Our chief goal in this paper is to evaluate how well average investment returns constructed using q theory can quantitatively match average stock returns. Section 3.1 outlines the design for the benchmark test. We then discuss how to include more ingredients into the framework in Sections 3.2.

3.1 The Benchmark Test

Proposition 1 shows that, when the operating-profit and the augmented adjustment-cost functions are both linearly homogeneous, the stock return equals the investment return. Ex-ante, the expected stock returns should then equal the expected investment returns. We use GMM to test this restriction using the following moment conditions:

$$\mathrm{E}\left[\left(r^S_{t+1} - r^I_{t+1}\right) \otimes \mathcal{Z}_t\right] = 0, \tag{21}$$

in which $\mathcal{Z}_t$ is a vector of instrumental variables known at the beginning of period t, and in which r^S_{t+1} are the stock returns of portfolios sorted on various anomaly variables.

The investment literature has tested extensively the investment Euler equation (5) by parameterizing the operating-profit and the augmented adjustment-cost functions.[2] These tests usually assume a constant stochastic discount factor, M_{t+1}. However, a constant M_{t+1} implies that all stocks earn the risk-free rate ex ante, and we therefore do not include the investment Euler equation into our set of moment conditions. Doing so would require us to make strong parametric assumptions on the functional form of M_{t+1}.

Testing Portfolios

We use GMM to estimate the moment conditions given by equation (21). We implement the test at the portfolio level for two reasons. First, simple versions of investment Euler equations are almost always strongly rejected at the firm level as in Whited (1992). The reason is that real investment can be lumpy at the firm level, especially in very small firms. To capture lumpy investment, we must incorporate fixed costs but we lose the differentiability of the augmented adjustment-cost function, $\Phi(I_t, K_t)$, at the point where the investment I_t equals zero. More importantly, anomalies are usually documented at the portfolio level in the literature, it is therefore natural for us to conduct our tests using portfolios.

We use 55 testing portfolios: the Fama-French 25 size and book-to-market portfolios;

[2]Important examples include Whited (1992) and Love (2003). See Hubbard (1998) for a recent survey.

ten portfolios sorted on the investment-to-capital ratio; ten earnings-momentum portfolios sorted on Standardized Unexpected Earnings, or SUE; nine portfolios sorted on size and SUE; and the aggregate stock market portfolio.

Our set of testing portfolios capture a wide array of asset pricing anomalies. We include book-to-market and SUE portfolios, because the value anomaly and post-earnings-announcement drift are two of the most important widely documented anomalies. See, for example, Fama and French (1992, 1993) and Bernard and Thomas (1989, 1990). We also include the investment-to-capital portfolios because the q theory explanation of the value anomaly works though real investment. Finally, we include the aggregate market portfolio to facilitate comparison with previous studies such as Cochrane (1991), who implicitly tests the moment condition in equation (21) by comparing the time series properties of the aggregate stock and investment returns.

Instrumental Variables

We use the following set of instrumental variables,

$$\mathcal{Z}_t \equiv [\iota \ \pi_t \ i_t \ b_t] \tag{22}$$

where ι is a vector of ones, and the others are, respectively, the portfolio profit-to-capital ratio, the investment-to-capital ratio, and the book-to-market ratio. We average all these variables across all the firms in one given portfolio.

We also include into $\mathcal{Z}_t$ a vector of macroeconomic variables such as the dividend yield, default premium, term premium, and short-term interest rate. These variables are often used as common conditioning variables to predict future stock market returns; see, for example, Fama and French (1989) and Ferson and Harvey (1991).

Functional Forms

We follow the empirical investment Euler equation literature in specifying the marginal product of capital, $\Pi_1(K_t, X_t)$, and the augmented adjustment-cost function, $\Phi(I_t, K_t)$.

We first need to relate the unobservable marginal product of capital to observables. As in Love (2003), if firms have a Cobb-Douglas production function with constant returns to scale, the marginal product of capital of portfolio j is given by:

$$\Pi_1(K_{jt}, X_{jt}) = \frac{\kappa Y_{jt}}{K_{jt}}, \tag{23}$$

where Y_{jt} denotes sales, and κ denotes capital's share. Equation (23) assumes that the shocks to the operating profits, X_{jt}, are reflected in the realizations of sales and variable costs.

To parameterize the augmented adjustment-cost function, we follow Whited (1998) and Whited and Wu (2004) and use a flexible functional form that is linearly homogeneous but allows for nonlinearity in the marginal adjustment-cost function:

$$\Phi(I_{jt}, K_{jt}) = I_{jt} + \left[\sum_{n=2}^{N_\Phi} \frac{1}{n} a_n \left(\frac{I_{jt}}{K_{jt}}\right)^n\right] K_{jt} \tag{24}$$

where $a_n, n = 2, \ldots, N_\Phi$ are coefficients to be estimated, and N_Φ is a truncation parameter that sets the highest power of I_{jt}/K_{jt} in the expansion. If $N_\Phi = 2$, then equation (24) reduces to the standard quadratic adjustment-cost function.

To determine N_Φ, we follow Whited and Wu (2004) and use the test developed by Newey and West (1987). First we choose a high starting value for N_Φ and estimate the model. Then using the same optimal weighting matrix, we estimate a sequence of restricted models for progressively lower values of N_Φ, in which the corresponding coefficient, $a_{N_\Phi+1}$, is set to zero. The final value for N_Φ is then the highest one for which the exclusion restriction on the parameter $a_{N_\Phi+1}$ is not rejected. We start by setting the truncation parameter at six. For most of our portfolios, we find that $N_\Phi = 3$. In what follows we set $N_\Phi = 3$ for all.

Leverage Adjustment

In the presence of debt financing, Proposition 2 shows that the investment return is the leverage-weighted average of the stock return and the corporate bond return. We thus unlever the investment returns in equation (21):

$$\mathrm{E}\left[\left(r^S_{t+1} - \frac{r^I_{t+1} - \nu_t r^B_{t+1}}{1-\nu_t}\right) \otimes \mathcal{Z}_t\right] = 0 \tag{25}$$

Because of the limitations of firm-level corporate bond data, and because few or none of the firms in several of our portfolios have corporate bond ratings, we use the Baa rate for all portfolios. This strategy avoids the use of firm-level bond return data that have a sample size much smaller than that of firm-level stock return data. Also, although unlevering the investment returns with one bond rate for all portfolios introduces potential misspecification into the model, using levered returns results in noticeably poorer model performance. We therefore stick with our unlevering method.

3.2 Costly External Equity

To test the model with financing costs of external equity, we use the moment conditions:

$$\mathrm{E}\left[\left(r^S_{t+1} - \frac{\begin{array}{c}(\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)\Phi_1(I_{t+1}, K_{t+1}))\times \\ (1 + \Psi_1(O_{t+1}, K_{t+1})\mathbf{1}^O_{t+1}) - \Psi_2(O_{t+1}, K_{t+1})\mathbf{1}^O_{t+1}\end{array}}{\Phi_1(I_t, K_t)(1 + \Psi_1(O_t, K_t)\mathbf{1}^O_t)}\right) \otimes \mathcal{Z}_t\right] = 0. \tag{26}$$

The parameterizations of the operating-profit and the augmented adjustment-cost functions are the same as those in the benchmark estimation.

To parameterize the financing-cost function, $\Psi(O_t, K_t)$, we use a flexible functional form similar to that of the adjustment-cost function:

$$\Psi(O_{jt}, K_{jt}) = \left[\sum_{n=2}^{N_\Psi} \frac{1}{n} b_n \left(\frac{O_{jt}}{K_{jt}}\right)^n\right] K_{jt}, \tag{27}$$

in which $b_n, n = 2, \ldots, N_\Psi$ are coefficients to be estimated, and N_Ψ is a truncation parameter that sets the highest power of (O_{jt}/K_{jt}) in the expansion. To determine N_Ψ, we again use the test developed by Newey and West (1987). For most of our portfolios, we find $N_\Psi = 2$. In what follows, we set $N_\Psi = 2$ for all.

4 Data

This section describes our sample construction and descriptive statistics of the data.

4.1 Sample Construction

Our sample of firm-level data is from the annual 2003 Standard and Poor's COMPUSTAT industrial files. We select our sample by first deleting any firm-year observations with missing data or for which total assets, the gross capital stock, or sales are either zero or negative. We also delete any firm that experienced a merger accounting for more than 15% of the book value of its assets. We omit all firms whose primary SIC classification is between 4900 and 4999 or between 6000 and 6999. The reason is that the q theory of investment is inappropriate for regulated or financial firms. The sample period goes from 1972 to 2003.

Our data definitions are as follows: the gross capital stock is COMPUSTAT Item 7; investment is the difference between Items 30 and 107; profits are the sum of Items 18 and 14; output is defined as sales, item 12; total long-term debt is Item 9 plus Item 34; net equity issuance is Item 108 minus Item 115; and the debt-to-assets ratio is defined as the ratio of long-term debt to long-term debt plus the market value of equity, defined as the market value of common equity (Item 199$\times$ Item 25) plus the book value of preferred equity (Item 130).

Our construction of the testing portfolios is standard. We follow Fama and French (1993) in constructing the 25 size and book-to-market portfolios. The portfolios, rebalanced at the end of each June, are the intersections of five portfolios formed on size and five portfolios formed on the ratio of book equity to market equity. The size breakpoints for year t are the NYSE market equity quintiles at the end of June of year t. Book-to-market for June of year t is the book equity for the last fiscal year end in year $t-1$ divided by size for December of year $t-1$. The book-to-market breakpoints are NYSE quintiles. We also sort all stocks at the end of each June into ten portfolios based on the investment-to-asset ratio. Both capital expenditures and assets are dated at the end of previous year.

We follow Chan, Jegadeesh, and Lakonishok (1996) in constructing the earnings momentum portfolios. We rank all stocks by their most recent past standardized unexpected earnings at the beginning of each month and assigned all the stocks to one of ten portfolios. Standardized unexpected earnings is the unexpected earnings defined as the change in quarterly earnings per share from its value four quarters ago divided by the standard deviation of unexpected earnings over the last eight quarters. The breakpoints are based on NYSE stocks only. All stocks are again equally-weighted in a portfolio. We also construct nine size and earnings momentum portfolios based on a double, 3×3 sort on size and SUE.

4.2 Descriptive Statistics

We report in Table 1 descriptive statistics for all of our testing portfolios. We report means and standard deviations of stock returns as well as other key firm-level variables used in constructing the investment returns. These firm-level variables include the investment-to-asset ratio, leverage, the new equity-to-asset ratio, and the sales-to-asset ratio.

From Panel A of Table 1, the value premium exists in our sample. Defined as the average return of high book-to-market or value firms minus the average return of low book-to-market or growth firms, the value premium is stronger among small firms, consistent with Fama and French (1993). Growth firms also invest more than value firms, and have higher sales-to-asset ratios than value firms. This evidence is consistent with Fama and French (1995). Moreover, small-growth firms issue much more equity than firms in other portfolios.

Panel B of Table 1 reports that, consistent with Titman, Wei, and Xie (2004), firms with low investment-to-asset ratios earn on average higher rates of returns than firms with high investment-to-asset ratios.[3] The difference in returns between the high- and low-investment portfolios is about 1.07% per month in our sample. Not surprisingly, high investment firms also have higher profitability than low investing firms, though somewhat lower leverage.

Panel C shows that the earnings-momentum strategy is profitable in our sample. The

[3]See also related evidence in Anderson and Garcia-Feijóo (2005) and Xing (2005). Lyandres, Sun, and Zhang (2005) also document similar evidence and show that this negative investment-return relation is potentially important in driving the underperformance following seasoned equity offerings in the data.

high-SUE portfolio outperforms the low-SUE portfolio by about 0.97% per month. Although investment varies little across these portfolios, sales increases with earnings momentum, and leverage decreases.

5 GMM Estimation and Tests

We now present and discuss the results from our GMM estimation and tests.

5.1 The Benchmark Model

We start with the benchmark model, in which we unlever the investment return in the tests according to equation (25). The sample is monthly from January 1972 to December 2003.

Table 2 reports parameter estimates and overall model performance measures. We report results from both separate estimation and joint estimation. For the separate estimation, each set of portfolios constitutes its own moment conditions, and the estimated parameter values differ across different portfolio sets. In the joint estimation, we pool all the testing portfolios together, and the parameter values are constant across all portfolios. We also report unconditional estimation and conditional estimation separately. Panel A reports the unconditional estimation, where we use a vector of ones as the only instrumental variable. Panel B reports the conditional estimation, where we use our entire list of instrumental variables. In general, the parameter estimates across unconditional and conditional estimation are reasonably close.

From Table 2 the estimated proportional operating costs are all positive and sometimes significant. The capital share, κ, is estimated to be between 0.09 and 0.30, and is often highly significant. The highest estimate occurs in the SUE-sorted portfolios. This result makes sense in that the SUE anomaly is explained primarily by the marginal product of capital, and in that higher estimates of κ produce greater dispersion in the fitted value of the marginal product across portfolios. The estimated adjustment-cost function is increasing and convex, as shown by the positive and significant estimates of a_2. The estimates of a_3

indicate some evidence of higher-order nonlinearity in the adjustment-cost function.

We also report two measures of overall model performance in Table 2. The economic magnitudes of the average absolute pricing errors are reasonable, ranging only from 0.073% to 0.185% per month for the unconditional estimation. We also report the J_T overidentification test. The benchmark model performs quite well when we use unconditional moment conditions to estimate separately the Fama-French 25 size and book-to-market portfolios, the nine size-SUE portfolios, and the ten investment-to-asset portfolios. The J_T tests fail to reject the null hypothesis that the average investment returns equal the average stock returns. By adding more moment conditions, conditional estimation imposes more stringent tests on the model. The average absolute pricing errors generally increase, now ranging from 0.083% to 0.222% per month. Correspondingly, with conditional estimation the J_T test produces rejections of the model overidentifying restrictions for all sets of portfolios.

The average absolute pricing errors and the J_T test only give overall measures of model performance. To provide a more complete picture, we report in Tables 3 and 4 the alphas for all the testing portfolios using unconditional and conditional estimation, respectively. The alpha for one testing portfolio is defined as its average stock return minus its average investment return constructed using estimated parameter values. The alphas are thus the pricing errors from the moment conditions.

From Panel A of Table 3, the benchmark model performs reasonably well in explaining the Fama-French 25 portfolios. In separate estimation, all the alphas are insignificant, and the magnitudes of most of these alphas is small. In joint estimation, all but one of the alphas are insignificant.

To illustrate this quantitative performance of the model, we also plot the average stock returns of the Fama-French 25 portfolios against their average investment returns. From Panels A and B in Figure 1, most of the portfolios are reasonably aligned with the 45-degree line. The implied average investment returns display similar magnitudes of dispersion as the average stock returns across the testing portfolios. The investment returns of high book-to-market firms are on average higher than those of low book-to-market firms. Further, the dispersion in the average investment return between value and growth firms is larger in small firms than in large firms. This evidence shows that the q model can largely account for the value premium. Notably, the benchmark q model captures well the small-growth portfolio that has been notoriously difficult to explain using some of the well-known consumption-based models, for example, Campbell and Vuolteenaho (2004).

Panel B of Table 3 reports that the benchmark model does extremely well in accounting for the average-return dispersion in the ten investment-to-asset deciles. None of the ten alphas is significant, either in the separate or joint estimation. This result is comforting inasmuch as one would hope that an asset pricing model based on investment returns could explain an investment anomaly.

In contrast, the model does much a poorer job in generating the average-return dispersion across the ten SUE portfolios. Both separate estimation of the model produces significant alphas with magnitudes about 0.40% per month for the two extreme deciles. However, the average returns constructed from the benchmark model for SUE deciles two and nine are 1.12% and 1.72% per month, close to their corresponding average stock returns, 1.00% and 1.73%, respectively. From the scatter plot in Panel F of Figure 1, one can see that the model performance on the SUE deciles from the joint estimation is worse. Although the observations lie around a line with a positive slope, the slope is far from one. Further, the model fails to generate substantial dispersion in average investment returns, despite large dispersion in average stock returns.

The model seems more successful in generating the return patterns for the nine size-SUE portfolios. From Panel D of Table 3, only three of the nine alphas from the separate estimation are significantly different from zero. Further, the average difference in returns between low and high SUE portfolios is about 0.80% per month in small firms, and is only 0.14% in big firms. In other words, our model replicates the stylized fact that the SUE anomaly is more pronounced in small firms than in large firms. However, joint estimation is once again, less successful. The fitted investment returns exhibit substantial dispersion across size terciles, but not within size terciles.

Table 4 reports the alphas for the testing portfolios for the benchmark model but with conditional estimation. Because the use of instrumental variables produces many more moment conditions, the quantitative fit between the average stock returns and the average investment returns in the conditional estimation deteriorates relative to that in the unconditional estimation. The magnitudes of the alphas are generally larger, and they are more often significant. Nonetheless, the basic patterns of the alphas are very similar to those reported in the previous table.

5.2 Costly External Equity

Table 5 reports GMM estimation results for the model with costly external equity finance. The financing-cost function is found to be convex, and the cost parameter, b_2, is positive and often significant. Further, the implied costs of external equity are reasonable. For example, an estimate of b_2 of 0.5 implies that the average marginal flotation cost in our sample is 6.9%, which is quite close to the estimate of 5.1% in Altinkilic and Hansen (2000). The estimates of the other parameters are quantitatively similar to those in the benchmark model. Incorporating financing costs into the model helps reduce somewhat the magnitudes of the average absolute pricing errors, especially for the conditional estimation.

Table 6 reports better quantitative fit between average stock and investment returns as a result of introducing financing costs into the model. From Panel A, most of the alphas for the Fama-French 25 portfolios are reduced relative to the benchmark model. More importantly, none is significant in the separate estimation. In the joint estimation, all the alphas except for one are insignificant. All other aspects of the table are quantitatively similar to Table 3 in the benchmark model.

6 Conclusion

We perform a new asset pricing test by using GMM to minimize the differences between average stock returns and average investment returns constructed from the q model. Under various specifications, we find that the estimated average investment returns display similar magnitudes of dispersion among portfolios sorted on size, book-to-market, and investment-to-assets. However, the model has only limited empirical success in reproducing earnings momentum profits. The difference between these two sets of results indicates that q theory is more successful at explaining asset-pricing anomalies that arise because of variation in the cost of capital than because of variation in profits. Over all, however, our results show that the q theory has substantial power to explain both qualitatively and quantitatively important asset pricing anomalies.

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A Proofs

Proof of Proposition 1 We first expand the value function (2) as follows:

$$\begin{pmatrix} P_t + \Pi(K_t, X_t) \\ -cK_t - \Phi(I_t, K_t) \end{pmatrix} = \begin{pmatrix} \Pi(K_t, X_t) - cK_t - \Phi(I_t, K_t) - q_t(K_{t+1} - (1-\delta)K_t - I_t) \\ + \mathrm{E}_t[M_{t+1}(\Pi(K_{t+1}, X_{t+1}) - cK_{t+1} - \Phi(I_{t+1}, K_{t+1}) \\ -q_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - I_{t+1})) + \ldots] \end{pmatrix} \tag{A1}$$

Recursive substitution using equations (3) and (4) and linear homogeneity of Φ implies that:

$$P_t + \Pi(K_t, X_t) - cK_t - \Phi(I_t, K_t) = q_t(1-\delta)K_t + \Pi(K_t, X_t) - cK_t - \Phi_2(I_t, K_t)K_t$$

Thus, $P_t = q_t(1-\delta)K_t + \Phi_1(I_t, K_t)I_t = q_t((1-\delta)K_t + I_t) = q_t K_{t+1}$. Now using this equation along with equation (1) and linear homogeneity of Π and Φ to rewrite stock return as

$$\begin{aligned} r^S_{t+1} &= \frac{\begin{bmatrix} q_{t+1}(I_{t+1} + (1-\delta)K_{t+1}) + \Pi_1(K_{t+1}, X_{t+1})K_{t+1} \\ -cK_{t+1} - \Phi_1(I_{t+1}, K_{t+1})I_{t+1} - \Phi_2(I_{t+1}, K_{t+1})K_{t+1} \end{bmatrix}}{q_t K_{t+1}} \\ &= \frac{\Pi_1(K_{t+1}, X_{t+1}) - c - \Phi_2(I_{t+1}, K_{t+1}) + (1-\delta)\Phi_1(I_{t+1}, K_{t+1})}{\Phi_1(I_t, K_t)} = r^I_{t+1} \end{aligned}$$

where the second equality follows from equation (3). ■

Proof of Proposition 2 Expanding the value function (8) and using linear homogeneity of Π and Φ yields:

$$\begin{pmatrix} P(K_t, B_t, X_t) + \Pi(K_t, X_t) \\ -cK_t - \Phi(I_t, K_t) \\ +B_{t+1} - R(X_t)B_t \end{pmatrix} = \begin{pmatrix} \Pi(K_t, X_t) - cK_t - \Phi_1(I_t, K_t)I_t - \Phi_2(I_t, K_t)K_t \\ +B_{t+1} - R(X_t)B_t - q_t(K_{t+1} - (1-\delta)K_t - I_t) \\ +\mathrm{E}_t[M_{t+1}(\Pi(K_{t+1}, X_{t+1}) - cK_{t+1} \\ -\Phi_1(I_{t+1}, K_{t+1})I_{t+1} - \Phi_2(I_{t+1}, K_{t+1})K_{t+1} \\ +B_{t+2} - R(X_{t+1})B_{t+1} \\ -q_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - I_{t+1}) + \ldots)] \end{pmatrix} \tag{A2}$$

Substituting the first-order conditions (9)–(11) and simplifying yield

$$\begin{aligned} &P(K_t, B_t, X_t) + \Pi(K_t, X_t) - cK_t - \Phi(I_t, K_t) + B_{t+1} - R(X_t)B_t \\ &= \Pi(K_t, X_t) - cK_t - \Phi_2(I_t, K_t)K_t - R(X_t)B_t + q_t(1-\delta)K_t \end{aligned}$$

Simplifying further and using the linear homogeneity of Φ yield:

$$P(K_t, B_t, X_t) + B_{t+1} = \Phi_1(I_t, K_t)(I_t + (1-\delta)K_t) = q_t K_{t+1}$$

where the last equality follows from equations (1) and (9). Now,

$$\begin{aligned}
\nu_t r_{t+1}^B + (1-\nu_t) r_{t+1}^S &= \frac{1}{q_t K_{t+1}} \begin{bmatrix} R(X_{t+1})\, B_{t+1} + P(K_{t+1}, B_{t+1}, X_{t+1}) + \Pi(K_{t+1}, X_{t+1}) \\ -cK_{t+1} - \Phi(I_{t+1}, K_{t+1}) + B_{t+2} - R(X_{t+1})\, B_{t+1} \end{bmatrix} \\
&= \frac{1}{q_t K_{t+1}} \begin{bmatrix} q_{t+1}(I_{t+1} + (1-\delta)K_{t+1}) + \Pi(K_{t+1}, X_{t+1}) \\ -cK_{t+1} - \Phi(I_{t+1}, K_{t+1}) \end{bmatrix} \\
&= \frac{1}{q_t} \begin{bmatrix} q_{t+1}(1-\delta) + \Pi_1(K_{t+1}, X_{t+1}) \\ -c - \Phi_2(I_{t+1}, K_{t+1}) \end{bmatrix} = r_{t+1}^I
\end{aligned}$$

where the second equality follows from equations (1) and (14) and the third equality follows from equation (9) and the linear homogeneity of Π and Φ. ■

Proof of Proposition 3 Let $\Omega(I_t, K_t)$ denote the sum of the augmented adjustment-cost and financing-cost functions,

$$\Omega(I_t, K_t) \equiv \Phi(I_t, K_t) + \Psi(O_t, K_t) = \Phi(I_t, K_t) + \Psi([\Phi(I_t, K_t) + cK_t - \Pi(K_t, X_t)]\mathbf{1}_t^O, K_t) \quad \text{(A3)}$$

where the second equality follows from equation (17).

Because both Φ and Ψ are assumed to be linearly homogeneous, Ω also satisfies this condition. To see this, using equation (A3) yields

$$\begin{aligned}
&\Omega_1(I_t, K_t)I_t + \Omega_2(I_t, K_t)K_t = \Phi_1(I_t, K_t)I_t + \Psi_1(O_t, K_t)\Phi_1(I_t, K_t)\mathbf{1}_t^O I_t + \Phi_2(I_t, K_t)K_t \\
&\quad + \Psi_1(O_t, K_t)(\Phi_2(I_t, K_t) - \Pi(K_t, X_t))\mathbf{1}_t^O K_t + \Psi_2(O_t, K_t)K_t \\
&\quad = \Phi(I_t, K_t) + \Psi_1(O_t, K_t)(\Phi_1(I_t, K_t)I_t + \Phi_2(I_t, K_t)K_t - \Pi_1(K_t, X_t)K_t)\mathbf{1}_t^O + \Psi_2(O_t, K_t)K_t \\
&\quad = \Phi(I_t, K_t) + \Psi_1(O_t, K_t)O_t + \Psi_2(O_t, K_t)K_t = \Phi(I_t, K_t) + \Psi(O_t, K_t) = \Omega(O_t, K_t)
\end{aligned}$$

Replace $\Phi(I_t, K_t) + \Psi(O_t, K_t)$ by $\Omega(I_t, K_t)$ in the firm's value function (19). The resulting value-maximization problem satisfies the conditions of Proposition 1. The equivalence between stock and investment returns then follows. ■

Table 1 : Descriptive Statistics of Testing Portfolios, January 1972 to December 2003

This table reports, for all testing portfolios. descriptive statistics including means and standard deviations (stds) for stock returns, investment-to-asset, leverage, new equity-to-asset, and sales-to-asset ratios. We choose to report the four firm characteristics in addition to stock returns because they are necessary ingredients in the construction of investment returns. We use data from a sample of 54 portfolios: the Fama-French 25 size and book-to-market portfolios (Panel A), ten portfolios sorted on investment-to-asset (Panel B), ten portfolios sorted on Standardized Unexpected Earnings or SUE (Panel C), and nine portfolios sorted on size and SUE (Panel D). We measure investment-to-capital as capital expenditure divided by the book value of capital stock (Compustat annual item 7), leverage as the book value of debt divided by the sum of the book value of debt and the market value of equity, new equity-to-capital as the change in book equity plus the change in book retained earnings divided by the book value of capital stock (positive numbers indicate net equity issuance and negative numbers indicate net equity repurchase). Finally, we measure sales-to-capital as sales divided by the book value of capital.

Panel A: Fama-French 25 size and book-to-market portfolios										
	Small	2	3	4	Big	Small	2	3	4	Big
	Stock returns, means, % per month					Stock returns, stds, % per month				
Low	0.95	0.76	0.80	0.91	0.87	8.96	8.66	7.78	7.21	5.76
2	1.40	1.27	1.22	1.15	1.04	7.74	6.87	6.35	5.90	5.24
3	1.66	1.37	1.27	1.30	1.19	6.94	6.44	6.01	5.80	5.46
4	1.74	1.36	1.37	1.42	1.11	6.41	5.99	5.92	5.95	5.42
High	2.05	1.45	1.54	1.40	1.30	6.56	6.54	6.60	7.00	6.00
	Investment-to-capital, means, annualized					Investment-to-capital, stds, annualized				
Low	0.24	0.26	0.24	0.21	0.17	0.06	0.05	0.07	0.05	0.04
2	0.19	0.19	0.17	0.15	0.14	0.04	0.03	0.03	0.04	0.03
3	0.16	0.16	0.14	0.13	0.12	0.03	0.03	0.03	0.02	0.03
4	0.14	0.13	0.13	0.10	0.11	0.04	0.03	0.04	0.02	0.03
High	0.09	0.10	0.10	0.09	0.09	0.02	0.03	0.02	0.03	0.02
	Leverage, means, annualized					Leverage, stds, annualized				
Low	0.21	0.18	0.16	0.13	0.12	0.08	0.06	0.05	0.03	0.04
2	0.28	0.26	0.26	0.22	0.27	0.09	0.07	0.06	0.05	0.08
3	0.34	0.34	0.32	0.31	0.33	0.10	0.11	0.07	0.08	0.10
4	0.41	0.43	0.41	0.40	0.41	0.09	0.12	0.07	0.08	0.08
High	0.55	0.55	0.54	0.50	0.54	0.08	0.11	0.11	0.12	0.17
	New equity-to-capital, means, annualized					New equity-to-capital, stds, annualized				
Low	0.22	0.15	0.08	0.03	-0.02	0.15	0.10	0.07	0.06	0.03
2	0.06	0.03	0.02	-0.00	-0.01	0.05	0.03	0.02	0.02	0.02
3	0.03	0.02	0.00	-0.00	-0.01	0.03	0.02	0.02	0.02	0.02
4	0.01	0.00	0.00	-0.00	-0.00	0.02	0.02	0.01	0.01	0.01
High	0.00	-0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.02
	Sales-to-capital, means, annualized					Sales-to-capital, stds, annualized				
Low	3.17	3.30	3.19	3.16	2.20	0.53	0.54	0.53	0.52	0.23
2	3.27	3.15	2.77	2.50	1.75	0.58	0.50	0.55	0.42	0.38
3	3.31	2.70	2.38	1.99	1.46	0.48	0.43	0.36	0.26	0.43
4	3.07	2.51	2.02	1.59	1.34	0.46	0.40	0.49	0.38	0.29
High	2.46	2.00	1.63	1.35	1.36	0.51	0.42	0.53	0.25	0.41

Panel B: Ten Investment-to-capital deciles

	Low	2	3	4	5	6	7	8	9	High
					Stock returns, % per month					
Means	1.87	1.62	1.65	1.49	1.52	1.45	1.34	1.31	1.08	0.80
Stds	7.04	6.01	5.79	5.74	5.92	6.07	6.47	7.04	7.79	9.21
					Investment-to-capital					
Means	0.07	0.07	0.09	0.11	0.13	0.15	0.17	0.21	0.28	0.36
Stds	0.02	0.01	0.02	0.02	0.02	0.03	0.04	0.05	0.07	0.18
					Leverage					
Means	0.48	0.37	0.31	0.31	0.29	0.27	0.23	0.23	0.19	0.23
Stds	0.10	0.10	0.07	0.08	0.06	0.08	0.10	0.11	0.09	0.13
					New equity-to-capital					
Means	0.01	-0.00	-0.00	-0.01	-0.01	-0.01	0.00	0.01	0.04	0.14
Stds	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.04	0.18
					Sales-to-capital					
Means	1.53	1.38	1.52	1.51	1.80	2.03	2.24	2.49	2.65	3.09
Stds	0.46	0.33	0.34	0.27	0.35	0.31	0.35	0.46	0.62	0.94

Panel C: Ten SUE deciles

	Low	2	3	4	5	6	7	8	9	High
					Stock returns, % per month					
Means	0.76	1.00	1.36	1.42	1.57	1.60	1.61	1.68	1.73	1.53
Stds	7.35	6.58	6.67	6.37	6.17	6.06	6.10	6.14	6.10	5.86
					Investment-to-capital					
Means	0.12	0.12	0.11	0.12	0.12	0.12	0.12	0.12	0.13	0.16
Stds	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
					Leverage					
Means	0.36	0.35	0.32	0.32	0.29	0.27	0.28	0.27	0.24	0.17
Stds	0.11	0.12	0.10	0.10	0.09	0.09	0.10	0.10	0.10	0.06
					New equity-to-capital					
Means	0.00	-0.00	-0.00	-0.00	-0.01	-0.01	-0.00	-0.00	-0.00	-0.02
Stds	0.02	0.02	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03
					Sales-to-capital					
Means	1.43	1.45	1.41	1.49	1.48	1.52	1.58	1.59	1.69	2.05
Stds	0.35	0.32	0.29	0.33	0.27	0.30	0.31	0.36	0.37	0.41

Panel D: Nine Size-SUE Portfolios

	SL	SM	SH	ML	MM	MH	BL	BM	BH
					Stock returns, % per month				
Means	1.20	1.99	2.24	0.93	1.32	1.50	0.96	1.08	1.10
Stds	7.69	7.31	7.25	6.96	6.14	6.26	6.07	5.25	5.32
					Investment-to-capital				
Means	0.11	0.10	0.14	0.11	0.11	0.15	0.12	0.12	0.14
Stds	0.03	0.03	0.05	0.03	0.02	0.03	0.03	0.02	0.03
					Leverage				
Means	0.49	0.44	0.36	0.41	0.36	0.28	0.33	0.27	0.21
Stds	0.12	0.11	0.12	0.09	0.08	0.08	0.10	0.07	0.07
					New equity-to-capital				
Means	0.02	0.02	0.04	0.01	0.01	0.02	-0.00	-0.01	-0.01
Stds	0.02	0.02	0.03	0.01	0.01	0.02	0.01	0.01	0.02
					Sales-to-capital				
Means	2.27	2.48	2.95	1.65	1.77	2.12	1.36	1.43	1.66
Stds	0.54	0.57	0.81	0.35	0.31	0.38	0.25	0.20	0.27

Table 2 : GMM Estimation and Tests, The Benchmark Specification with Costless External Equity

Calculations are based on monthly data from a sample of 54 portfolios: the Fama-French 25 size and book-to-market portfolios, ten portfolios sorted on investment-to-capital, ten portfolios sorted on Standardized Unexpected Earnings (SUE), and nine portfolios sorted on size and SUE. The sample period goes from January 1972 to December 2003. Estimation is done via GMM. The parameters in Panel A are estimated using a vector of ones as the only instrument. The parameters in Panel B are estimated using as instruments the lagged sales-to-capital ratio, the lagged investment-to-capital ratio, the lagged book-to-market ratio, the lagged aggregate term premium, the lagged aggregate dividend yield, and the lagged aggregate default premium. The model is given by equation (25), in which the investment returns are calculated as in equation (25). The cost of external equity is set to zero and the bond return is calculated as the average yield on Baa rated corporate bonds. a_1 and a_2 are adjustment-cost parameters, c is the flow operating cost, and κ is the capital share. To evaluate the overall performance of the model, we also report the average absolute pricing errors (a.a.p.e., in percent per month), the J-test statistics (J_T) for testing the over-identification conditions, and their correspondingly degrees of freedom (d.f.) and p-values.

	Separate Estimation				Joint Estimation
	Fama-French 25	SUE	Size-SUE	Investment-to-capital	
	Panel A: Unconditional Moments				
a_2	1.995	4.005	3.955	2.578	2.487
	(0.831)	(2.177)	(2.084)	(0.922)	(1.239)
a_3	3.082	5.365	9.302	1.104	1.753
	(1.621)	(4.002)	(5.775)	(0.634)	(0.992)
c	0.173	0.376	0.219	0.263	0.192
	(0.135)	(0.141)	(0.077)	(0.097)	(0.082)
κ	0.091	0.300	0.143	0.168	0.149
	(0.066)	(0.136)	(0.081)	(0.083)	(0.051)
a.a.p.e.	0.074	0.185	0.144	0.035	0.128
J_T	23.839	16.975	7.140	8.423	92.058
d.f.	21	6	5	6	51
p-value	(0.301)	(0.009)	(0.210)	(0.209)	(0.000)
	Panel B: Conditional Moments				
a_2	2.545	3.297	3.635	2.586	2.732
	(1.527)	(1.855)	(1.633)	(0.864)	(1.037)
a_3	2.127	5.720	8.099	1.146	2.312
	(0.835)	(3.874)	(6.227)	(0.525)	(1.119)
c	0.169	0.360	0.240	0.285	0.223
	(0.125)	(0.183)	(0.093)	(0.121)	(0.085)
κ	0.087	0.249	0.134	0.178	0.134
	(0.058)	(0.132)	(0.071)	(0.085)	(0.049)
a.a.p.e.	0.083	0.222	0.150	0.051	0.148
J_T	234.698	122.104	114.419	129.568	652.918
d.f.	196	76	68	76	436
p-value	(0.031)	(0.001)	(0.000)	(0.000)	(0.000)

Table 3 : Alphas from the q Model of Expected Returns, The Benchmark Specification with Costless External Equity, Unconditional Estimation

This table reports model-implied average investment return, r^I, alphas defined as the average stock returns minus the average investment returns, and their corresponding t-statistics for testing portfolios. We construct the investment returns with the parameter estimates in Table 2 for the benchmark investment-return equation given by equation (25). The cost of external equity is zero and the bond return is the average yield on Baa rated corporate bonds. There are in total 54 portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten Standardized Unexpected Earnings or SUE portfolios, and nine portfolios sorted on size and SUE. The sample period goes from January 1972 to December 2003. In separate estimation (sepa), we estimate different parameters for different group of testing portfolios. In joint estimate (joint), only one set of parameters is estimated using all the testing portfolios in the moment conditions. Unconditional estimates are obtained using the vector of ones as the only instrument.

Panel A: Fama-French 25 size and book-to-market portfolios

	Small	2	3	4	Big	Small	2	3	4	Big
	Average r^I, separate estimation					Average r^I, joint estimation				
Low	0.81	0.71	0.88	1.01	0.90	0.88	0.78	0.90	1.07	0.98
2	1.28	1.22	1.20	1.22	1.05	1.29	1.26	1.23	1.26	1.05
3	1.60	1.32	1.27	1.24	1.10	1.62	1.36	1.30	1.24	1.05
4	1.70	1.53	1.31	1.28	1.13	1.77	1.54	1.30	1.21	1.06
High	1.87	1.60	1.48	1.26	1.44	1.95	1.60	1.41	1.16	1.33
	α, separate estimation					α, joint estimation				
Low	0.10	0.02	-0.10	-0.11	-0.03	0.04	-0.05	-0.11	-0.17	-0.11
2	0.08	0.03	0.01	-0.08	-0.02	0.07	-0.01	-0.02	-0.12	-0.02
3	0.03	0.04	-0.02	0.05	0.10	0.00	0.00	-0.04	0.06	0.15
4	0.00	-0.19	0.05	0.14	-0.01	-0.07	-0.20	0.05	0.20	0.05
High	0.14	-0.18	0.03	0.12	-0.14	0.06	-0.18	0.10	0.22	-0.04
	t_α, separate estimation					t_α, joint estimation				
Low	0.64	0.23	-0.74	-1.01	-0.33	1.42	-1.62	-0.84	-1.61	-1.19
2	0.76	0.36	0.09	-0.80	-0.22	1.07	-0.25	-1.11	-1.05	-1.18
3	0.43	0.44	-0.17	0.59	1.07	0.01	-0.07	-0.73	0.82	0.97
4	-0.02	-1.74	0.45	1.36	-0.15	-1.20	-2.19	0.57	2.04	1.05
High	1.48	-1.89	0.23	0.86	-0.71	1.78	-1.84	1.07	2.09	-1.20

Panel B: Ten investment-to-capital deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	1.79	1.64	1.54	1.55	1.49	1.43	1.35	1.26	1.07	0.77
	α	0.04	-0.04	0.09	-0.08	0.00	-0.01	-0.03	0.03	-0.01	0.00
	t_α	0.52	-0.84	1.46	-1.34	0.09	-0.15	-0.63	0.62	-0.41	0.35
joint	ave. r^I	1.73	1.57	1.56	1.53	1.58	1.55	1.44	1.27	0.86	0.82
	α	0.10	0.02	0.07	-0.06	-0.08	-0.13	-0.13	0.02	0.20	-0.05
	t_α	1.86	0.48	1.14	-0.89	-0.90	-1.13	-0.26	0.18	2.08	-1.23

Panel C: Ten SUE deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	0.97	1.12	1.14	1.39	1.26	1.28	1.72	1.45	1.72	1.88
	α	-0.21	-0.14	0.20	0.00	0.28	0.30	-0.13	0.22	-0.01	-0.36
	t_α	-2.17	-0.83	2.16	0.34	2.09	2.00	-0.84	1.97	-0.09	-3.54
joint	ave. r^I	1.18	1.24	1.38	1.42	1.38	1.37	1.40	1.40	1.54	1.20
	α	-0.42	-0.26	-0.04	-0.02	0.16	0.21	0.19	0.27	0.17	0.32
	t_α	-3.44	-2.59	-0.45	-0.24	1.61	2.34	2.08	1.91	1.62	2.74

Panel D: Nine Size-SUE Portfolios

		SL	SM	SH	ML	MM	MH	BL	BM	BH
sepa	ave. r^I	1.49	1.67	2.29	1.03	1.10	1.34	1.00	1.00	1.14
	α	-0.32	0.28	-0.08	-0.11	0.19	0.15	-0.05	0.07	-0.04
	t_α	-3.04	3.08	-1.09	-1.64	2.47	1.08	-0.48	0.95	-0.49
joint	ave. r^I	1.71	1.77	1.70	1.22	1.25	1.22	1.02	1.03	1.05
	α	-0.53	0.18	0.51	-0.30	0.04	0.27	-0.06	0.04	0.05
	t_α	-4.40	2.18	3.95	-3.74	0.72	3.07	-0.27	0.46	0.19

Table 4 : Alphas from the q Model of Expected Returns, The Benchmark Specification with Costless External Equity, Conditional Estimation

This table reports model-implied average investment returns, r^I, alphas defined as the average stock returns minus the average investment returns, and their corresponding t-statistics for testing portfolios. We construct the investment returns with the parameter estimates in Table 2 for the benchmark model with costless external equity given by equation (25). The cost of external equity is set to zero and the bond return is calculated as the average yield on Baa rated corporate bonds. There are in total 54 portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten Standardized Unexpected Earnings or SUE portfolios, and nine portfolios sorted on size and SUE. The sample period is from January 1972 to December 2003. In separate estimation (sepa), we estimate different parameters for different group of testing portfolios. In joint estimate (joint), only one set of parameter values is estimated using all the testing portfolios in the moment conditions. Conditional estimates are obtained using instruments including lagged profit-to-capital, lagged investment-to-capital, lagged book-to-market, lagged aggregate term premium, lagged aggregate dividend yield, and lagged aggregate default premium.

Panel A: Fama-French 25 size and book-to-market portfolios

	Small	2	3	4	Big	Small	2	3	4	Big
	Average r^I, separate estimation					Average r^I, joint estimation				
Low	0.72	0.58	0.76	1.01	0.90	0.82	0.71	0.85	1.05	0.94
2	1.31	1.25	1.22	1.23	1.03	1.29	1.25	1.22	1.25	1.01
3	1.65	1.34	1.27	1.22	1.05	1.66	1.35	1.28	1.20	0.98
4	1.73	1.54	1.28	1.23	1.07	1.80	1.53	1.25	1.13	0.96
High	1.86	1.56	1.43	1.19	1.36	1.91	1.51	1.29	1.01	1.18
	α, separate estimation					α, joint estimation				
Low	0.19	0.14	0.03	-0.11	-0.02	0.09	0.02	-0.07	-0.14	-0.07
2	0.05	0.00	-0.01	-0.09	0.00	0.07	0.00	-0.01	-0.11	0.02
3	-0.02	0.02	-0.02	0.07	0.14	-0.03	0.01	-0.02	0.09	0.22
4	-0.02	-0.19	0.08	0.19	0.04	-0.10	-0.19	0.11	0.28	0.15
High	0.15	-0.14	0.08	0.20	-0.07	0.10	-0.08	0.22	0.37	0.12
	t_α, separate estimation					t_α, joint estimation				
Low	1.84	1.62	0.23	-1.04	-0.25	0.96	0.20	-0.86	-1.07	-1.68
2	0.57	0.01	-0.17	-0.94	0.01	0.11	-0.02	-0.19	-1.58	0.97
3	-0.35	0.21	-0.20	0.87	1.50	-0.34	0.10	-0.71	0.15	7.02
4	-0.34	-2.01	0.81	1.89	0.45	-1.59	-4.87	1.13	0.13	2.50
High	1.43	-1.52	0.76	1.70	-0.35	1.34	-1.04	3.38	3.59	1.63

Panel B: Ten investment-to-capital deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	1.86	1.70	1.59	1.60	1.53	1.45	1.36	1.26	1.08	0.83
	α	-0.02	-0.10	0.04	-0.14	-0.03	-0.03	-0.04	0.03 -	0.03	-0.06
	t_α	-0.28	-1.72	0.65	-2.05	-0.53	-0.56	-0.80	0.50 -	0.43	-0.63
joint	ave. r^I	1.63	1.52	1.57	1.54	1.62	1.61	1.50	1.31	0.84	0.84
	α	0.21	0.07	0.07	-0.08	-0.13	-0.19	-0.18	-0.02	0.21	-0.07
	t_α	1.99	1.03	0.87	-1.06	-1.53	-3.42	-2.24	-0.20	2.31	-0.43

Panel C: Ten SUE deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	0.96	1.41	1.19	1.25	1.27	1.31	1.79	1.44	1.73	1.83
	α	-0.20	-0.43	0.15	0.14	0.27	0.27	-0.20	0.23	-0.02	-0.31
	t_α	-2.36	-3.35	1.22	1.63	2.43	2.37	-1.55	1.82	-0.17	-4.75
joint	ave. r^I	1.39	1.46	1.40	1.44	1.40	1.40	1.43	1.43	1.39	1.39
	α	-0.63	-0.47	-0.06	-0.04	0.14	0.19	0.16	0.24	0.32	0.13
	t_α	-4.38	-2.03	-0.32	-0.31	0.95	2.21	1.78	4.18	3.23	0.37

Panel D: Nine Size-SUE Portfolios

		SL	SM	SH	ML	MM	MH	BL	BM	BH
sepa	ave. r^I	1.49	1.68	2.36	1.01	1.09	1.35	1.01	1.00	1.15
	α	-0.31	0.28	-0.15	-0.10	0.20	0.14	-0.05	0.07	-0.05
	t_α	-2.26	3.47	-1.07	-1.14	3.21	1.42	-0.65	1.13	-0.75
joint	ave. r^I	1.73	1.82	1.79	1.20	1.25	1.25	1.00	1.03	1.06
	α	-0.56	0.14	0.42	-0.29	0.04	0.24	-0.04	0.04	0.04
	t_α	-4.04	0.77	3.95	-3.39	0.54	2.94	-0.47	0.57	0.56

Table 5 : GMM Estimation and Tests, The Costly-External-Equity Model

Calculations are based on monthly data from a sample of 54 portfolios: the Fama-French 25 size and book-to-market portfolios, ten portfolios sorted on investment-to-capital, ten earnings-momentum portfolios sorted on Standardized Unexpected Earnings (SUE), and nine portfolios sorted on size and SUE. The sample period goes from January 1972 to December 2003. Estimation is done via GMM. The parameters in Panel A are estimated using a vector of ones as an instrument. The parameters in Panel B are estimated using lagged the lagged profit-to-capital ratio, the lagged investment-to-capital ratio, the lagged book-to-market ratio, lagged SUE, the lagged aggregate term premium, the lagged aggregate dividend yield, and the lagged aggregate default premium. The moment conditions under costly external finance is given by equation (26), in which the investment returns are given by (20). We calculate the bond return as the average yield on Baa rated corporate bonds. b_2 is the financing-cost parameter, a_2 and a_3 are the adjustment-cost parameters, c is the flow operating cost, and κ is the capital share. To evaluate the overall performance of the model, we also report the average absolute pricing errors (a.a.p.e., in percent per month), the J-test statistics (J_T) for testing the over-identification conditions, and their correspondingly degrees of freedom (d.f.) and p-values.

	Separate Estimation				Joint Estimation
	Fama-French 25	SUE	Size-SUE	Investment-to-capital	
			Panel A: Unconditional Moments		
a_2	1.512	1.663	2.203	4.286	1.664
	(0.663)	(1.001)	(1.169)	(1.837)	(0.766)
a_3	4.788	6.330	4.670	-11.186	5.619
	(2.043)	(4.964)	(2.857)	(6.178)	(2.801)
c	0.219	0.182	0.113	0.203	0.231
	(0.139)	(0.089)	(0.053)	(0.068)	(0.108)
κ	0.191	0.333	0.164	0.179	0.175
	(0.076)	(0.167)	(0.096)	(0.085)	(0.052)
b_2	0.481	0.561	0.477	0.537	0.406
	(0.194)	(0.230)	(0.241)	(0.231)	(0.236)
a.a.p.e.	0.059	0.135	0.142	0.031	0.124
J_T	21.452	3.455	7.123	8.310	163.034
d.f.	20	5	4	5	50
p-value	(0.371)	(0.630)	(0.130)	(0.140)	(0.000)
			Panel B: Conditional Moments		
a_2	2.562	1.923	2.342	3.698	1.437
	(1.008)	(0.833)	(0.980)	(1.254)	(0.725)
a_3	3.039	5.988	4.141	-9.323	4.564
	(1.675)	(3.247)	(2.326)	(6.836)	(2.581)
c	0.211	0.205	0.111	0.235	0.222
	(0.125)	(0.095)	(0.066)	(0.063)	(0.093)
κ	0.187	0.350	0.192	0.176	0.151
	(0.069)	(0.176)	(0.089)	(0.086)	(0.062)
b_2	0.468	0.645	0.450	0.521	0.482
	(0.211)	(0.256)	(0.212)	(0.222)	(0.246)
a.a.p.e.	0.082	0.143	0.142	0.041	0.145
J_T	221.381	107.070	104.819	128.954	574.983
d.f.	195	75	67	75	435.000
p-value	(0.095)	(0.009)	(0.002)	(0.000)	(0.000)

Table 6 : Alphas from the q Model of Expected Returns, The Costly-External-Finance Model, Unconditional Estimation

This table reports alphas, defined as the average stock returns minus the average investment returns, as well as their corresponding t-statistics for all the testing portfolios. We construct the investment returns with the parameter estimates reported in Table 5 for the model with costly external finance. The moment conditions are given by equation (26), in which the investment returns are calculated as in equation (20). The bond return is calculated as the average yield on Baa rated corporate bonds. There are in total 54 portfolios: the Fama-French 25 size and book-to-market portfolios (Panel A), ten portfolios sorted on investment-to-capital (Panel B), ten portfolios sorted on Standardized Unexpected Earnings or SUE (Panel C), and nine portfolios sorted on size and SUE (Panel D). The sample period is from January 1972 to December 2003. In separate estimation (sepa), we estimate different parameters for different group of testing portfolios. In joint estimate (joint), only one set of parameters is estimated using all the testing portfolios in the moment conditions. Unconditional estimates are obtained using the vector of ones as the only instrument.

Panel A: Fama-French 25 size and book-to-market portfolios

	Small	2	3	4	Big	Small	2	3	4	Big
	Average r^I, separate estimation					Average r^I, joint estimation				
Low	0.93	0.69	0.81	0.97	0.88	1.04	0.76	0.86	1.01	0.96
2	1.28	1.21	1.19	1.21	1.05	1.33	1.23	1.20	1.25	1.05
3	1.61	1.32	1.27	1.25	1.11	1.64	1.38	1.30	1.24	1.05
4	1.70	1.52	1.31	1.29	1.13	1.78	1.55	1.36	1.22	1.07
High	1.89	1.59	1.49	1.27	1.42	1.92	1.59	1.39	1.14	1.32
	α, separate estimation					α, joint estimation				
Low	-0.02	0.04	-0.02	-0.07	-0.01	-0.13	-0.03	-0.07	-0.11	-0.09
2	0.08	0.04	0.01	-0.07	-0.01	0.03	0.02	0.01	-0.11	-0.02
3	0.02	0.03	-0.01	0.05	0.09	-0.01	-0.02	-0.05	0.06	0.15
4	0.00	-0.18	0.04	0.13	-0.01	-0.08	-0.21	-0.01	0.20	0.04
High	0.12	-0.16	0.02	0.11	-0.13	0.09	-0.17	0.12	0.24	-0.03
	t_α, separate estimation					t_α, joint estimation				
Low	-0.60	0.47	-0.32	-0.76	-0.13	-0.74	-0.40	-0.76	-0.85	-0.73
2	0.70	0.43	0.14	-0.71	-0.19	0.23	0.30	0.10	-1.02	-0.23
3	0.29	0.38	-0.15	0.50	0.95	-0.13	-0.24	-0.53	0.68	1.59
4	0.01	-1.56	0.40	1.21	-0.13	-1.34	-2.44	-0.10	2.14	0.44
High	1.30	-1.73	0.18	0.77	-0.65	1.51	-1.90	1.13	1.87	-0.13

Panel B: Ten investment-to-capital deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	1.81	1.64	1.53	1.53	1.48	1.42	1.35	1.27	1.06	0.77
	α	0.02	-0.04	0.10	-0.07	0.02	-0.00	-0.04	0.02	-0.00	0.00
	t_α	0.66	-0.97	1.90	-1.91	0.45	-0.06	-0.99	0.41	-0.09	0.47
joint	ave. r^I	1.80	1.60	1.57	1.53	1.56	1.52	1.40	1.21	0.78	0.89
	α	0.04	-0.01	0.06	-0.06	-0.06	-0.10	-0.08	0.08	0.27	-0.12
	t_α	0.47	-0.11	0.86	-1.11	-1.26	-1.66	-1.30	0.96	2.68	-1.95

Panel C: Ten SUE deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	0.82	1.06	1.30	1.52	1.80	1.12	1.44	1.63	1.83	1.52
	α	-0.06	-0.07	0.04	-0.13	-0.26	0.46	0.15	0.04	-0.12	-0.00
	t_α	-0.39	-0.39	0.38	-1.03	-2.17	3.31	1.35	0.32	-1.13	-0.69
joint	ave. r^I	1.14	1.25	1.39	1.42	1.38	1.37	1.40	1.40	1.45	1.31
	α	-0.39	-0.27	-0.05	-0.02	0.16	0.21	0.19	0.27	0.26	0.20
	t_α	-3.63	-3.60	-0.69	-0.46	1.94	4.01	3.60	4.23	3.34	1.47

Panel D: Nine Size-SUE Portfolios

		SL	SM	SH	ML	MM	MH	BL	BM	BH
sepa	ave. r^I	0.90	2.16	2.28	1.16	1.22	1.30	1.04	0.95	1.14
	α	0.27	-0.20	-0.06	-0.24	0.07	0.19	-0.08	0.12	-0.03
	t_α	1.75	-2.23	-2.31	-2.67	0.86	2.14	-1.12	1.44	-1.25
joint	ave. r^I	1.51	1.73	1.76	1.25	1.27	1.21	1.04	1.05	1.05
	α	-0.34	0.23	0.45	-0.34	0.02	0.28	-0.08	0.02	0.05
	t_α	-3.75	3.99	4.62	-3.30	0.35	3.37	-1.05	0.36	0.50

Table 7 : Alphas from the q Model of Expected Returns, The Costly-External-Finance Model, Conditional Estimation

This table reports alphas (the average stock returns minus the average investment returns) and their t-statistics for all the testing portfolios. We construct the investment returns with the parameters reported in Panel B (Conditional Estimates) of Table 5 for the model with costly external finance. We calculate the bond return as the average yield on Baa rated corporate bonds. There are in total 54 portfolios: the Fama-French 25 size and book-to-market portfolios, ten portfolios sorted on investment-to-capital, ten portfolios sorted on Standardized Unexpected Earnings or SUE, and nine portfolios sorted on size and SUE. The sample period is from January 1972 to December 2003. In separate estimation (sepa), we estimate different parameters for different group of testing portfolios. In joint estimate (joint), only one set of parameters is estimated using all the testing portfolios in the moment conditions. All alphas are based on the conditional estimates using as instruments the lagged profit-to-capital, the lagged investment-to-capital, the lagged book-to-market, the lagged SUE, the lagged aggregate term premium, the lagged aggregate dividend yield, and the lagged aggregate default premium. All portfolio-specific instruments are lagged by 12 months to avoid look-ahead bias.

Panel A: Fama-French 25 size and book-to-market portfolios

	Small	2	3	4	Big	Small	2	3	4	Big
	Average r^I, separate estimation					Average r^I, joint estimation				
Low	0.82	0.54	0.68	0.98	0.87	1.24	0.73	0.79	0.90	0.83
2	1.30	1.23	1.21	1.22	1.03	1.15	1.10	1.10	1.21	0.96
3	1.65	1.33	1.27	1.22	1.05	1.58	1.28	1.25	1.21	0.97
4	1.72	1.54	1.28	1.24	1.07	1.85	1.55	1.28	1.18	0.98
High	1.88	1.57	1.43	1.19	1.37	2.05	1.57	1.35	1.03	1.18
	α, separate estimation					α, joint estimation				
Low	0.09	0.19	0.11	-0.07	0.00	-0.32	0.00	0.00	0.00	0.04
2	0.06	0.02	0.00	-0.08	0.00	0.21	0.15	0.10	-0.07	0.08
3	-0.03	0.02	-0.02	0.07	0.14	0.05	0.08	0.01	0.09	0.23
4	-0.02	-0.20	0.07	0.18	0.04	-0.15	-0.20	0.08	0.24	0.14
High	0.14	-0.15	0.08	0.19	-0.08	-0.04	-0.15	0.16	0.35	0.11
	t_α, separate estimation					t_α, joint estimation				
Low	1.47	1.64	1.44	-0.91	0.01	-2.13	0.00	0.01	-0.02	0.30
2	0.62	0.19	-0.05	-0.90	0.07	2.10	1.66	1.21	-0.69	0.97
3	-0.45	0.24	-0.19	0.84	1.49	0.69	0.94	0.10	1.08	2.48
4	-0.32	-1.99	0.79	1.83	0.44	-1.82	-2.58	0.88	2.67	1.31
High	1.38	-1.61	0.69	1.30	-0.38	-0.43	-1.72	1.43	2.74	0.52

Panel B: Ten investment-to-capital deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	1.85	1.70	1.59	1.50	1.53	1.45	1.36	1.26	1.08	0.84
	α	-0.02	-0.10	0.04	-0.04	-0.03	-0.03	-0.04	0.03	-0.03	-0.07
	t_α	-0.21	-1.74	0.65	-0.53	-0.52	-0.56	-0.80	0.56	-0.42	-0.68
joint	ave. r^I	1.63	1.46	1.61	1.59	1.65	1.57	1.38	1.16	0.86	0.88
	α	0.21	0.13	0.03	-0.13	-0.15	-0.15	-0.06	0.12	0.20	-0.12
	t_α	1.49	1.66	0.28	-1.51	-2.09	-2.65	-1.05	1.51	1.95	-0.92

Panel C: Ten SUE deciles

		Low	2	3	4	5	6	7	8	9	High
sepa	ave. r^I	0.63	0.86	1.46	1.46	1.76	1.18	1.69	1.52	1.70	1.62
	α	0.13	0.12	-0.12	-0.07	-0.23	0.40	-0.10	0.15	0.01	-0.11
	t_α	0.33	1.20	-0.79	-0.73	-1.69	2.67	-0.65	1.27	0.05	-1.42
joint	ave. r^I	0.76	0.98	1.34	1.40	1.54	1.58	1.59	1.67	1.71	1.52
	α	-0.43	-0.31	-0.08	-0.07	0.11	0.16	0.14	0.22	0.33	0.15
	t_α	-4.22	-3.71	-1.12	-1.14	1.86	2.83	2.37	3.39	4.19	1.19

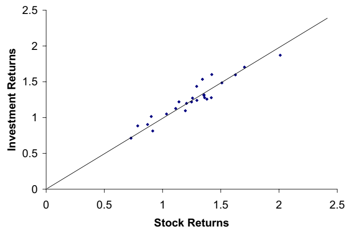
Panel D: Nine Size-SUE Portfolios

		SL	SM	SH	ML	MM	MH	BL	BM	BH
sepa	ave. r^I	1.38	1.83	2.50	0.91	0.99	1.49	1.13	1.04	1.26
	α	-0.20	0.12	-0.29	0.00	0.30	-0.01	-0.17	0.03	-0.16
	t_α	-2.03	1.29	-2.05	-0.02	2.48	-0.05	-1.40	0.31	-1.53
joint	ave. r^I	1.55	1.86	1.83	1.18	1.25	1.21	0.95	0.99	1.00
	α	-0.37	0.09	0.38	-0.27	0.04	0.28	0.01	0.08	0.10
	t_α	-3.06	0.96	3.48	-2.44	0.64	3.42	0.09	1.27	1.15

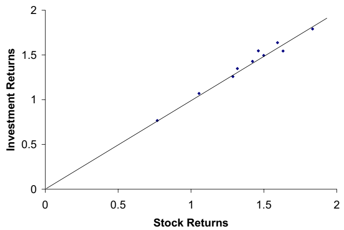
Figure 1 : Pricing Errors, The Benchmark Model with Costless External Equity, Unconditional Estimation

This figure plots the pricing errors associated with the unconditional moment conditions estimated from the benchmark model with costless external equity. In unconditional estimation, we use a vector of ones as the only instrument. We perform GMM estimation on monthly data of 55 testing portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten portfolios sorted on Standardized Unexpected Earnings (SUE), nine portfolios sorted on size and SUE, and the market portfolio. In separate estimation (sepa) reported in Panels A to D, we use the moment conditions formed by one group of portfolios separately in the GMM estimation. In joint estimation (joint) reported in Panels E to H, we use the moment conditions formed by all the testing portfolios as well as the market portfolio jointly in the GMM estimation. The sample period is from January 1972 to December 2003. The benchmark investment-return equation is given by equation (25). The cost of external equity is set to zero, and the bond return is calculated as the average yield on Baa-rated corporate bonds.

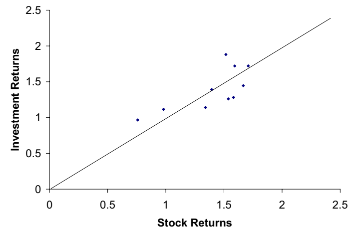
Panel A: Fama-French 25 Portfolios (sepa)



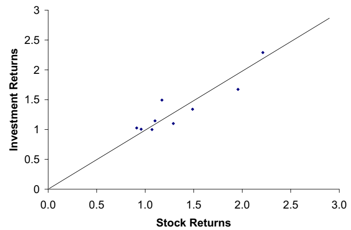
Panel B: Ten Investment-to-Capital Deciles (sepa)



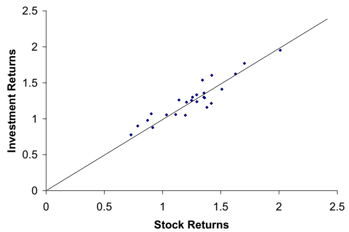
Panel C: Ten SUE Deciles (sepa)



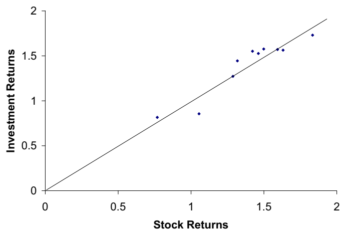
Panel D: Nine Size-SUE Portfolios (sepa)



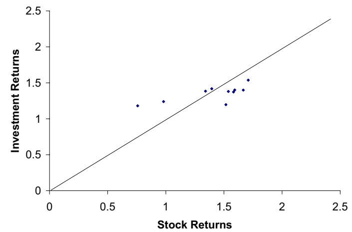
Panel E: Fama-French 25 Portfolios (joint)



Panel F: Ten Investment-to-Capital Deciles (joint)



Panel G: Ten SUE Deciles (joint)



Panel H: Nine Size-SUE Portfolios (joint)

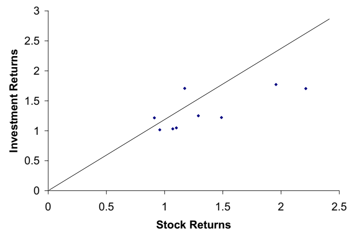


Figure 2 : Pricing Errors, The Benchmark Model with Costless External Equity, Conditional Estimation

This figure plots the pricing errors associated with the unconditional moment conditions estimated from the benchmark model with costless external equity. In conditional estimation, we use as instruments the lagged profit-to-capital, the lagged investment-to-capital, the lagged book-to-market, the lagged SUE, the lagged aggregate term premium, the lagged aggregate dividend yield, and the lagged aggregate default premium. All portfolio-specific instruments are lagged by 12 months to avoid look-ahead bias. We perform GMM estimation on monthly data of 55 testing portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten portfolios sorted on Standardized Unexpected Earnings (SUE), nine portfolios sorted on size and SUE, and the market portfolio. In separate estimation (sepa) reported in Panels A to D, we use the moment conditions formed by one group of portfolios separately in the GMM estimation. In joint estimation (joint) reported in Panels E to H, we use the moment conditions formed by all the testing portfolios as well as the market portfolio jointly in the GMM estimation. The sample period is from January 1972 to December 2003. The benchmark investment-return equation is given by equation (25). The cost of external equity is set to zero, and the bond return is calculated as the average yield on Baa-rated corporate bonds.

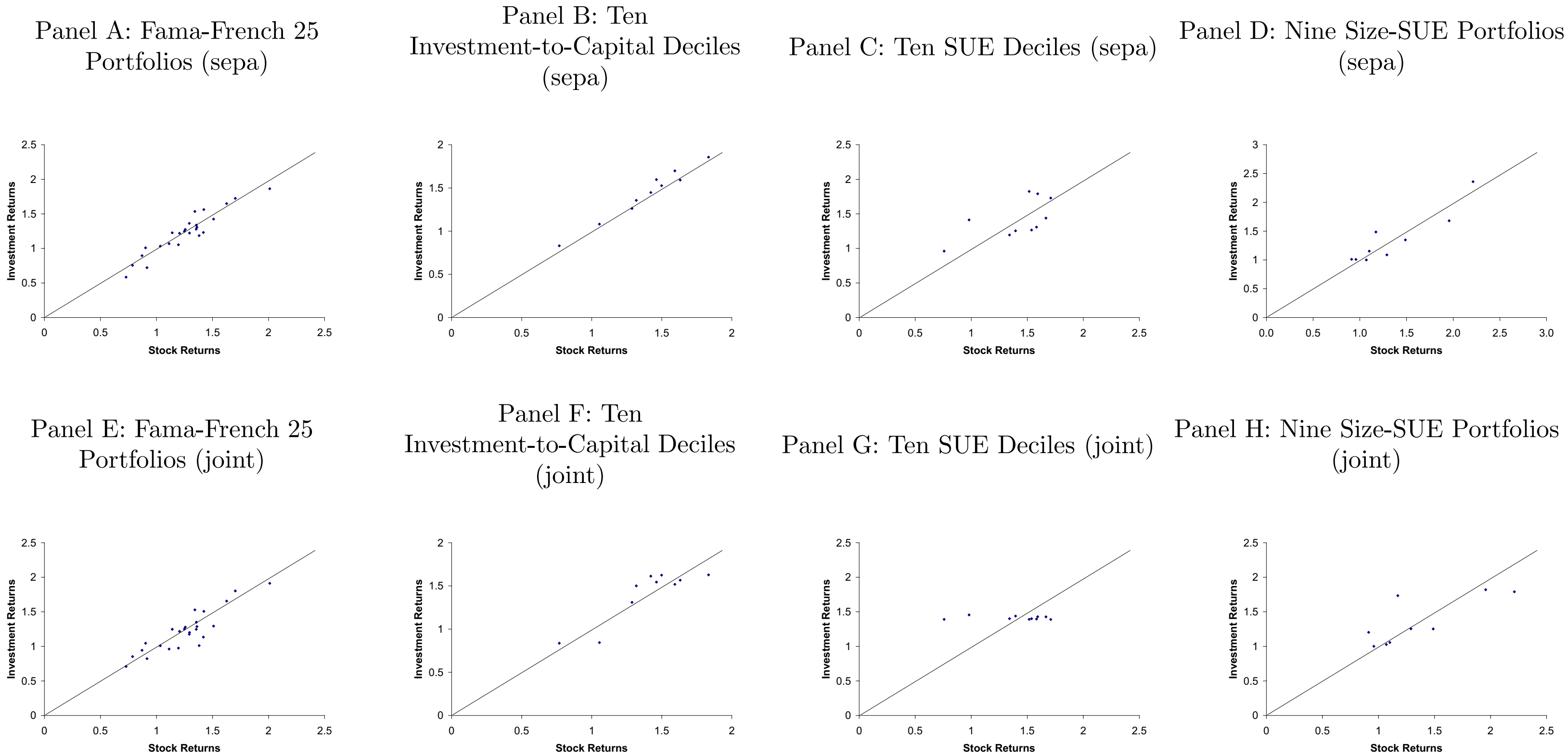


Figure 3 : Pricing Errors, The Benchmark Model with Costly External Equity, Unconditional Estimation

This figure plots the pricing errors associated with the unconditional moment conditions estimated from the model with costly external equity. In unconditional estimation, we use a vector of ones as the only instrument. We perform GMM estimation on monthly data of 55 testing portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten portfolios sorted on Standardized Unexpected Earnings (SUE), nine portfolios sorted on size and SUE, and the market portfolio. In separate estimation (sepa) reported in Panels A to D, we use the moment conditions formed by one group of portfolios separately in the GMM estimation. In joint estimation (joint) reported in Panels E to H, we use the moment conditions formed by all the testing portfolios as well as the market portfolio jointly in the GMM estimation. The sample period is from January 1972 to December 2003. The moment conditions under costly external finance is given by equation (26), in which the investment returns are given by (20). The bond return is calculated as the average yield on Baa-rated corporate bonds.

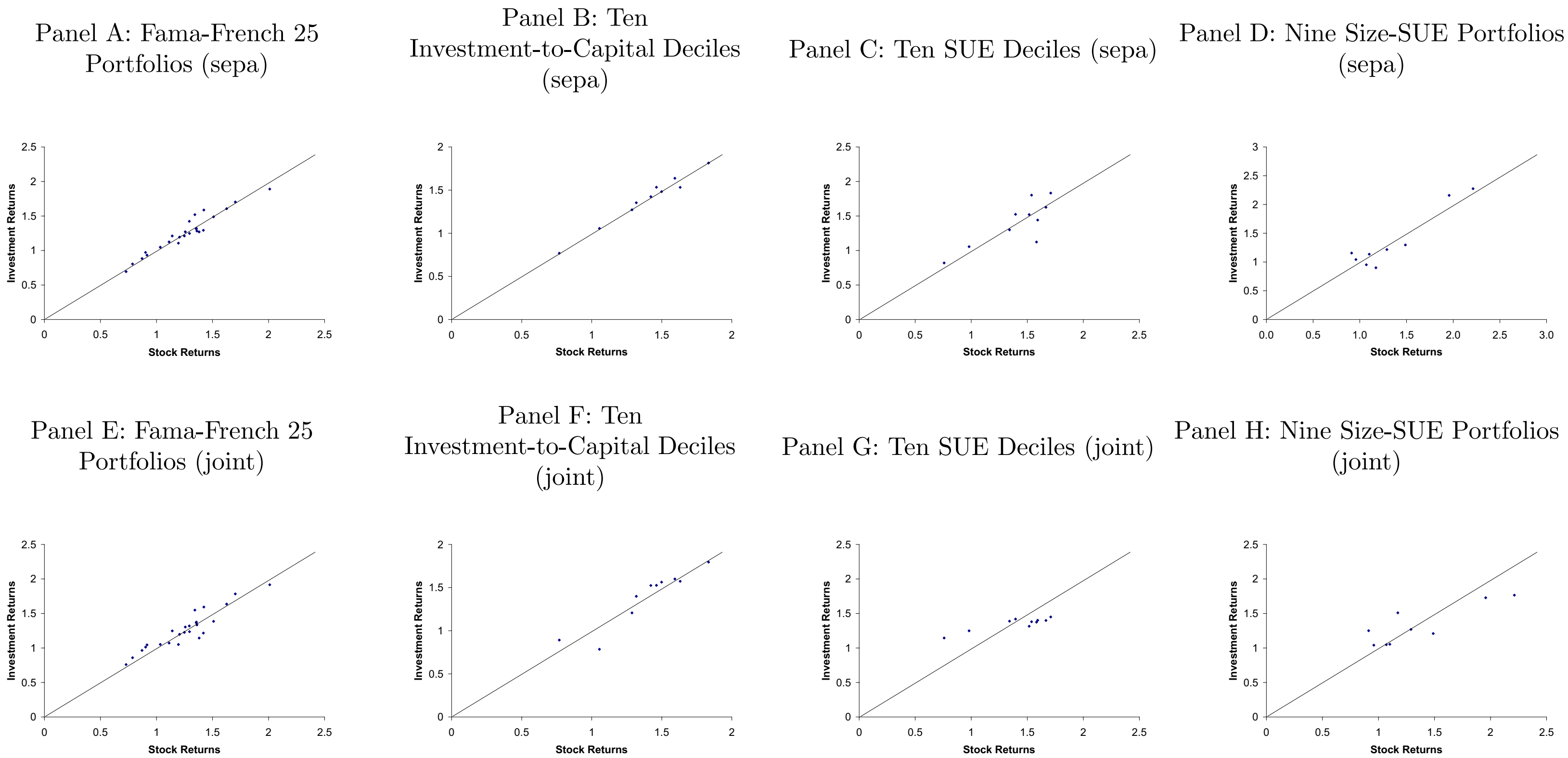
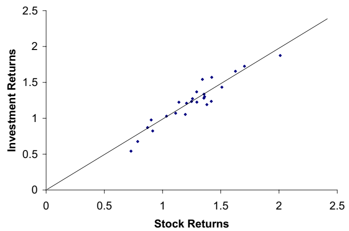


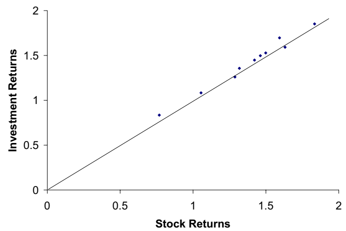
Figure 4 : Pricing Errors, The Benchmark Model with Costly External Equity, Conditional Estimation

This figure plots the pricing errors associated with the unconditional moment conditions estimated from the benchmark model with costless external equity. In conditional estimation, we use as instruments the lagged profit-to-capital, the lagged investment-to-capital, the lagged book-to-market, the lagged SUE, the lagged aggregate term premium, the lagged aggregate dividend yield, and the lagged aggregate default premium. All portfolio-specific instruments are lagged by 12 months to avoid look-ahead bias. We perform GMM estimation on monthly data of 55 testing portfolios: the Fama-French 25 size and book-to-market portfolios, ten investment-to-capital portfolios, ten portfolios sorted on Standardized Unexpected Earnings (SUE), nine portfolios sorted on size and SUE, and the market portfolio. In separate estimation (sepa) reported in Panels A to D, we use the moment conditions formed by one group of portfolios separately in the GMM estimation. In joint estimation (joint) reported in Panels E to H, we use the moment conditions formed by all the testing portfolios as well as the market portfolio jointly in the GMM estimation. The sample period is from January 1972 to December 2003. The moment conditions under costly external finance is given by equation (26), in which the investment returns are given by (20). The bond return is calculated as the average yield on Baa-rated corporate bonds.

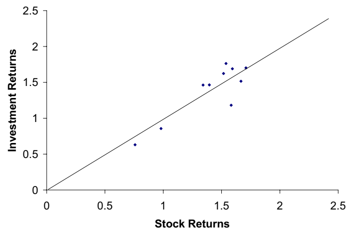
Panel A: Fama-French 25 Portfolios (sepa)



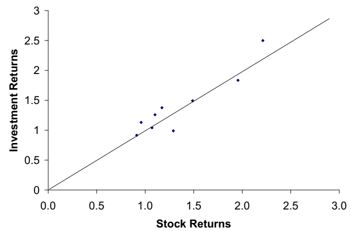
Panel B: Ten Investment-to-Capital Deciles (sepa)



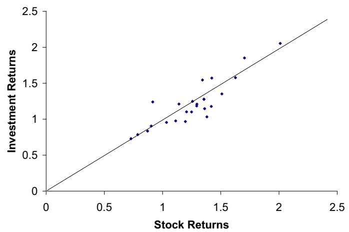
Panel C: Ten SUE Deciles (sepa)



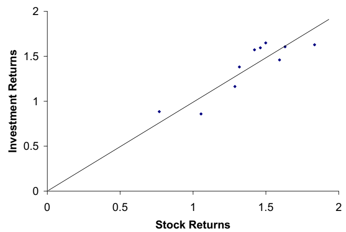
Panel D: Nine Size-SUE Portfolios (sepa)



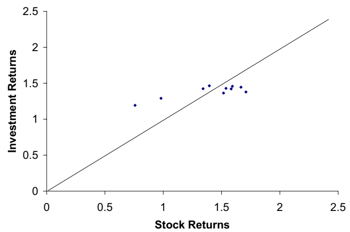
Panel E: Fama-French 25 Portfolios (joint)



Panel F: Ten Investment-to-Capital Deciles (joint)



Panel G: Ten SUE Deciles (joint)



Panel H: Nine Size-SUE Portfolios (joint)