Lumpy Investment in Dynamic General Equilibrium

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Abstract

Microeconomic lumpiness matters for macroeconomics. According to our DSGE model, it explains roughly 60% of the smoothing in the investment response to aggregate shocks. The remaining 40% is explained by general equilibrium forces. The central role played by micro frictions for aggregate dynamics results in important nonlinearities and history dependance in business cycles. In particular, booms feed into themselves. The longer an expansion, the larger the response of investment to an additional positive shock. Conversely, a slowdown after a boom can lead to a long lasting investment slump, which is unresponsive to policy stimuli. Such dynamics are consistent with US investment patterns over the last decade. Furthermore, the reduction in the relative importance of general equilibrium forces for aggregate investment dynamics also facilitates matching conventional RBC moments for consumption and employment.

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1 Introduction

Casual observation suggests that non-convexities in microeconomic capital adjustments is a widespread pattern. Doms and Dunne (1998) corroborate this perception by documenting the lumpy nature of equipment investment in US manufacturing establishments. The question then arises whether or not these microeconomic frictions matter for macroeconomic behavior. In this paper we incorporate lumpy adjustment in an otherwise standard dynamic stochastic general equilibrium (DSGE) model and conclude that they do.

The main impact of microeconomic lumpiness is to generate impulse responses for aggregate investment which are not only more persistent than in the standard RBC model, but also nonlinear and history dependent. In particular, the longer an expansion, the larger the response of investment to further shocks. Booms feed upon themselves. Conversely, a slowdown after a boom can lead to a long lasting investment slump, which is unresponsive to policy stimuli. Such dynamics are consistent with US investment patterns over the last decade.

Underlying our findings is an issue that is of central importance for micro-founded macroeconomics, beyond our particular model. Namely the answer to the question: How much of aggregate smoothing—and impulse responses in general—is accounted for by microeconomic features and how much by general equilibrium forces? The basic RBC model attributes all of the smoothing to the latter. In contrast, our model calibration indicates that microeconomic non-convexities account for an important part of the smoothing in the response of investment to aggregate shocks.

Table 1: FROM AGGREGATE SHOCKS TO AGGREGATE RESPONSE: SOURCES OF SMOOTHING

Aggregate shocks

↓

partial equilibrium smoothing: micro frictions and aggregation

↓

general equilibrium smoothing: supply of labor and funds

↓

Aggregate response

This decomposition is the key to our calibration strategy and explains our starkly different results from recent attempts to embody lumpy adjustment models in a DSGE framework (e.g. Veracierto (2002), Thomas (2002) and Khan and Thomas (2003, 2005)). Table 1 illustrates this decomposition. The objective in any macroeconomic model is to trace the impact of aggregate shocks on aggregate endogenous variables (investment, in our context). The typical response is not one-for-one at impact, as a variety of microeconomic frictions and general equilibrium constraints, smooth and spread over time the response of the endogenous variable. We refer to this process as *smoothing*, and decompose it into partial equilibrium (PE) and general equilibrium (GE) smoothing. In the context of nonlinear lumpy-adjustment models, PE-smoothing includes not only microeconomic frictions but also the effect of aggregation on the mapping from these frictions to aggregate responses. This is key in our class of models, as in many instances aggregation can undo much of the effect of microeconomic frictions (recall the classic Caplin and Spulber (1987) result where price-setters follow (S,s) rules but the aggregate price level behaves as if there were no microeconomic frictions). In a nutshell, our key difference with the previous literature is that through a combination of small adjustment costs and strong offsetting forces from aggregation, previous models left almost no role for PE-smoothing. We argue below that such a conclusion is counterfactual.

Table 2 illustrates our model's decomposition between PE- and GE-smoothing: The upper entry shows the volatility of aggregate investment rates in our model when neither smoothing mechanism is present (in other words, when there are no adjustment costs at the microeconomic level and no price adjustments in the economy). The intermediate entries incorporate PE and GE-smoothing, one at a time, while the lower entry considers both sources of smoothing simultaneously. Of course, both sources of smoothing are not orthogonal, so some care is needed when quantifying their contributions to overall smoothing.

Table 2: CONTRIBUTION OF PE AND GE FORCES TO SMOOTHING OF $\sigma(I/K)$

	No frictions (0.0458)	
↙		↘
Only PE smoothing (0.0093)	↓	Only GE smoothing (0.0134)
↘		↙
	PE and GE smoothing (0.0074)	

Table 2 shows that the reduction of the standard deviation of the aggregate investment rate achieved by PE-smoothing alone amounts to 88.7% of the reduction achieved by the combination of both smoothing mechanisms. Alternatively, the additional smoothing achieved by PE-forces, compared with what GE-smoothing achieves by itself, is 38% of the smoothing achieved by both sources. The 60% mentioned in the abstract—slightly above the average of 63.3% of the above upper and lower bounds—conveniently summarizes the contribution of PE factors to aggregate smoothing.[1]

Given its centrality in differentiating our answer from that of previous models, our calibration strategy is designed to capture the role of PE-smoothing as directly as possible. To this effect, we use sectoral data to calibrate all the micro-frictions and aggregation parameters, *before* general equilibrium forces have a chance to play a smoothing role. Specifically, we argue that the response of semi-aggregated (e.g., 3-digit) investment to corresponding sectoral shocks is less subject to general equilibrium forces, and hence serves to identify the relative importance of PE-smoothing. Once this step is taken, we can use the elasticity of intertemporal substitution as a reduced form parameter to calibrate the extra smoothing given by general equilibrium forces.[2]

Table 3: VOLATILITY AND AGGREGATION

Model	3-digit	Aggregate	3-dig. Agg. Ratio
Data	*0.0186*	*0.0074*	*2.51*
Frictionless:	0.3642	0.0074	49.22
This paper:	0.0186	0.0074	2.51
Khan-Thomas-Lumpy (2005):	0.2524	0.0074	34.11

The first row in Table 3 shows the observed volatility of sectoral and aggregate investment rates, and their ratio. The second row shows the same values for a model with no microeconomic frictions in investment (essentially, the standard RBC model), and the third row does the same for our model. We reserve for later the fourth row, which reports the same statistics for the model in Khan and Thomas (2005). It is apparent from this table that the frictionless RBC model fails to match the disaggregated data (it was never designed to do so). In contrast, by reallocating smoothing from GE forces to PE-forces, the lumpy investment model is able to match both aggregate and sectoral volatility. This pins down our decomposition.

Aside from our main results characterizing the aggregate impact of microeconomic lumpiness, there is an indirect benefit of adding microeconomic lumpiness to the standard model, as it facilitates matching conventional RBC moments for consumption and employment. The reason is that in the standard RBC model, where all the smoothing of the response of quantities to aggregate shocks is done by general equilibrium forces, the volatility of investment relative to that of consumption and employment is too

[1] The exact expressions for the upper and lower bounds for the contribution of PE-smoothing are the following:

$$\begin{aligned} UB &= \log[\sigma(\text{NONE})/\sigma(\text{PE})]/\log[\sigma(\text{NONE})/\sigma(\text{BOTH})], \\ LB &= 1-\log[\sigma(\text{NONE})/\sigma(\text{GE})]/\log[\sigma(\text{NONE})/\sigma(\text{BOTH})] \end{aligned}$$

where NONE refers to the partial equilibrium the model with no microeconomic frictions, PE to the model that only has microeconomic frictions but prices are fixed, GE to the model with only GE constraints, and BOTH to the model with both micro frictions and GE constraints. See Appendix E for more details.

[2] An alternative strategy would be to use plant level data to sort out the different parameter configurations. While much has been learned from such explorations in other contexts, this is not a robust strategy in the case of lumpy adjustment models since the mapping from microeconomic frictions to aggregate data, even before general equilibrium enters, is complex and often not robust. It depends on subtle parameters such as the drift of the (micro) driving forces and, more generally, parameters that affect the cross-section distribution of agents' state variables.

high relative to US data (see, e.g. King and Rebelo, 1999). Thus models that fit the second moments of investment well (such as the standard RBC model), imply consumption and employment that are too smooth. In contrast, in our model lumpy microeconomic frictions smooth investment and hence the strength of general equilibrium forces needed to match investment volatility can be reduced. This results in consumption and employment becoming more volatile, fitting US data.

In our model we control the strength of the general equilibrium forces with the elasticity of intertemporal substitution, which we interpret as a reduced form parameter to capture unmodelled sources of flat quasi-labor supply and capital supply to the primary sector of the economy. We find that the parameter of EIS that matches the data best exceeds 10. Whether one interprets this as a "puzzle" or as a hint that the EIS parameter in these models is not what its microeconomic counterpart purports it to be, as we do, is a matter of taste. However, it is important to stress that our main findings regarding the patterns of aggregate investment survive reducing the EIS parameter to its conventional value of one. Moreover, if one is willing to raise it to Gruber's (2005) recent finding of 2, then our model also improves broader moments-matching by over 40 percent.

Relation to the literature

Our main findings are qualitatively similar to those discussed in the partial equilibrium literature on lumpy investment (see, in particular, Caballero and Engel (1999), Caballero, Engel and Haltiwanger (1995) and Cooper, Haltiwanger and Power (1999)). However, as mentioned above, they are in stark contrast with findings in the first wave of DSGE models, such as Veracierto (2002), Thomas (2002), and Khan and Thomas (2003, 2005), who encountered a sort of "irrelevance" result:[3] Essentially, they found that embedding a model with microeconomic irreversibility and/or lumpiness in an otherwise standard RBC model, makes no difference for macroeconomics (relative to the implications of the frictionless RBC model). The reason for our difference can be seen in the last row of Table 3, which shows that the Khan and Thomas model has a decomposition of smoothing between PE and GE forces similar to that of the frictionless RBC model. That is, their frictions, once filtered by the aggregation process, have almost no effect at the aggregate level *even* in partial equilibrium.

More precisely, Table 4 is analogous to Table 2 but for the Khan-Thomas model.[4] It can be seen that micro frictions imply no additional smoothing after GE forces have set in—they only account for somewhere between 0 and 18% of total smoothing. Thus we view their work as an important methodological contribution on which we build our analysis, but not as an adequate assessment of the equilibrium implications of lumpy microeconomic investment.

The remainder of the paper is organized as follows. In the next section we present our dynamic general equilibrium model. Section 3 discusses the calibration method in detail. Sections 4 and 5 present

[3] More recently, Sim (2006) undoes Veracierto's version of the irrelevance result by relaxing the certainty-equivalence assumption, while Bayer (2006) finds that adjustment costs matter for aggregate investment dynamics in a two-country extension of the Khan and Thomas model.

[4] The difference between the value reported in the last row of this table and the corresponding value in Table 2 reflects differences in calibration strategies. These differences are irrelevant for the substantive implications.

Table 4: CONTRIBUTION OF PE AND GE FORCES TO SMOOTHING OF $\sigma(I/K)$: KHAN AND THOMAS

$$\begin{tikzcd}
 & \begin{array}{c}\text{No frictions}\\ (0.1050)\end{array} \arrow[dl] \arrow[dd] \arrow[dr] & \\
\begin{array}{c}\text{Only PE smoothing}\\ (0.0660)\end{array} \arrow[dr] & & \begin{array}{c}\text{Only GE smoothing}\\ (0.0080)\end{array} \arrow[dl] \\
 & \begin{array}{c}\text{PE and GE smoothing}\\ (0.0080)\end{array} &
\end{tikzcd}$$

the main macroeconomic implications of the model. Section 6 concludes and is followed by several appendices.

2 The Model

In this section we describe our model economy. We start with the problem of the production units, followed by a brief description of the households and the definition of equilibrium. We conclude with a sketch of the equilibrium computation. We follow closely Kahn and Thomas (2005) both in terms of substance and notation. Aside from parametric differences, we have three main departures from Kahn and Thomas (2005). First, production units face persistent sector-specific productivity shocks, in addition to aggregate and idiosyncratic shocks. Second, production units undertake some within-period maintenance investment which is necessary to continue operation (there is fixed proportions and some parts and machines that break down need to be replaced, see McGrattan and Schmitz (1999) for evidence on the importance of maintenance investment). Third, the distribution of aggregate productivity shocks is continuous rather than a Markov discretization.[5]

2.1 Production Units

The economy consists of a large number of sectors, which are each populated by a continuum of production units. Since we do not model entry and exit decisions, the mass of these continua is fixed and normalized to one. There is one commodity in the economy that can be consumed or invested. Each production unit produces this commodity, employing its pre-determined capital stock (k) and labor (n),

[5]This allows us to compute conditional nonlinear impulse response functions, as in Section 4, which are essentially derivatives of an aggregate of interest, such as the investment rate, to the innovation of the aggregate productivity shock.

according to the following Cobb-Douglas decreasing-returns-to-scale production function ($\theta + \nu < 1$):

$$y_t = z_t \epsilon_{S,t} \epsilon_{I,t} k_t^{\theta} n_t^{\nu}, \tag{1}$$

where z_t, ϵ_S and ϵ_I denote aggregate, sectoral and unit-specific (idiosyncratic) productivity shocks. The assumption of decreasing returns captures in reduced form any market power the production unit may have.

We denote the trend growth rate of aggregate productivity by $(1-\theta)(\gamma-1)$, so that y and k grow at rate $\gamma - 1$ along the balanced growth path. From now on we work with k and y (and later C) in efficiency units. The detrended aggregate productivity level, which we also denote by z, evolves according to an AR(1) process, with normal innovations υ with zero mean and variance σ_A^2:

$$\log z_t = \rho_A \log z_{t-1} + \upsilon_t. \tag{2}$$

The sectoral and idiosyncratic technology processes follow Markov chains, that are approximations to continuous AR(1) processes with Gaussian innovations. The latter have standard deviations σ_S and σ_I, and autocorrelations ρ_S and ρ_I, respectively.[6] Productivity innovations at different aggregation levels are independent. Also, sectoral productivity shocks are independent across sectors and idiosyncratic productivity shocks are independent across productive units.

In each period, each production unit draws from a time-invariant distribution, G, its current cost of capital adjustment, $\xi \geq 0$, which is denominated in units of labor. G is a uniform distribution on $[0, \bar{\xi}]$, common to all units. Draws are independent across units and over time, and employment is freely adjustable.

At the beginning of each period, a production unit is characterized by its pre-determined capital stock, the sector it belongs to and the corresponding sectoral productivity level, its idiosyncratic productivity, and its capital adjustment cost. Given the aggregate state, it decides its employment level, n, production occurs, maintenance is carried out, workers are paid, and investment decisions are made. Then the period ends.

Upon investment the unit incurs a fixed cost of $\omega\xi$, where ω is the current real wage rate. Capital depreciates at a rate δ, but units may find it necessary during the production process to replace certain items.

Define $\bar{\psi} \equiv \frac{\gamma}{1-\delta} > 1$ as the maintenance investment rate needed to compensate depreciation and trend growth. The degree of necessary maintenance, χ, can then be conveniently defined as a fraction of $\bar{\psi}$. If $\chi = 0$, no maintenance investment is needed; if $\chi = 1$, all depreciation and trend growth must be undone for a production unit to continue operation. We can now summarize the evolution of the unit's capital stock (in efficiency units) between two consecutive periods, from k to k' after investment i takes place, as follows:

[6] We use the discretization in Tauchen (1986), see Appendix C for details.

	Fixed cost paid	$\gamma k'$
$i \neq 0$:	$\omega\xi$	$(1-\delta)k+i$
$i = 0$:	0	$\left[(1-\delta)(1-\chi)+\chi\gamma\right]k$

If $\chi = 0$, then $k' = (1-\delta)k/\gamma$ and the table is identical to the one found in Kahn and Thomas (2005), while if $\chi = 100\%$, then $k' = k$. In the paper, we treat χ as a primitive parameter.[7]

Notice that χ is obviously irrelevant for the units that actually adjust at the end of the period. This is not to say that these units do not have to spend on maintenance within the production period, but rather their net capital growth, conditional on incurring the fixed cost and optimal adjustment, is independent of this expenditure. This is essentially a feature of only having fixed adjustment costs, as opposed to more general adjustment technologies that include a component that depends on the magnitude of capital adjustments.

Given the i.i.d. nature of the adjustment costs, it is sufficient to describe differences across production units and their evolution by the distribution of units over $(\epsilon_S, \epsilon_I, k)$. We denote this distribution by μ. Thus, (z, μ) constitutes the current aggregate state and μ evolves according to the law of motion $\mu' = \Gamma(z, \mu)$, which production units take as given.

Next we describe the dynamic programming problem of each production unit. We will take two shortcuts (details can be found in Kahn and Thomas, 2005). First, we state the problem in terms of utils of the representative household (rather than physical units), and denote by $p = p(z, \mu)$ the marginal utility of consumption. This is the relative intertemporal price faced by a production unit. Second, given the i.i.d. nature of the adjustment costs, continuation values can be expressed without explicitly taking into account future adjustment costs.

It will simplify notation to define an additional parameter, $\psi \in [1, \bar{\psi}]$:

$$\psi = 1 + (\bar{\psi} - 1)\chi, \tag{3}$$

and write maintenance investment as:[8]

$$i^M = (\psi - 1)(1-\delta)k. \tag{4}$$

Let $V^1(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$ denote the expected discounted value—in utils—of a unit that is in idiosyncratic state (ϵ_I, k, ξ), and is in a sector with sectoral productivity ϵ_S, given the aggregate state (z, μ). Then the expected value prior to the realization of the adjustment cost draw is given by:

$$V^0(\epsilon_S, \epsilon_I, k; z, \mu) = \int_0^{\bar{\xi}} V^1(\epsilon_S, \epsilon_I, k, \xi; z, \mu) G(d\xi). \tag{5}$$

[7] We note that this maintenance investment is quite different from what Kahn and Thomas (2005) call maintenance investment in their "extended model." For us, maintenance refers to the replacement of parts and machines without which production cannot continue. For them, it is an extra margin of adjustment for small projects.

[8] Note that if $\psi = 1$, then $i^M = 0$, and if $\psi = \bar{\psi}$, then $i^M = (\gamma - 1 + \delta)k$, undoing all trend devaluation of the capital stock.

With this notation the dynamic programming problem is given by:

$$
\begin{aligned}
V^1(\epsilon_S, \epsilon_I, k, \xi; z, \mu) = \max_n \big[& z\epsilon_S\epsilon_I k^\theta n^\nu - \omega(z,\mu)n - i^M + (1-\delta)\psi k \big) p(z,\mu) + \\
& \max \Big\{ -(1-\delta)\psi k p(z,\mu) + \beta \mathrm{E}[V^0(\epsilon_S{}', \epsilon_I{}', \psi \frac{1-\delta}{\gamma} k; z', \mu')] \, , \\
& \max_{k'} \big(-\xi\omega(z,\mu)p(z,\mu) - \gamma k' p(z,\mu) + \beta \mathrm{E}[V^0(\epsilon_S{}', \epsilon_I{}', k'; z', \mu')] \big) \Big\} \Big], \quad (6)
\end{aligned}
$$

where both expectation operators average over next period's realizations of the aggregate, sectoral and idiosyncratic shocks, conditional on this period's values.

The first line represents the flow value of a production unit that optimally adjusts its employment level. The second line is the continuation value, if only necessary maintenance investment has occurred. The third line is the continuation value, if units incur the fixed costs of adjustment and then adjust optimally.

Taking as given intra- and intertemporal prices, $\omega(z,\mu)$ and $p(z,\mu)$, respectively, and the law of motion $\Gamma(z,\mu)$, the production unit chooses optimally labor demand, whether to adjust its capital stock at the end of the period, and the optimal capital stock, conditional on adjustment. This leads to policy functions: $N = N(\epsilon_S, \epsilon_I, k; z, \mu)$ and $K = K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$. Since capital is pre-determined, the optimal employment decision is independent of the current adjustment cost draw.

2.2 Households

We assume a continuum of identical households that have access to a complete set of state-contingent claims. Hence, there is no heterogeneity across households. Moreover, they own shares in the production units and are paid dividends. We do not need to model the household side explicitly, and concentrate instead on the first-order conditions to determine the equilibrium wage and the intertemporal price.

Households have a felicity function in consumption and leisure of the following form:

$$
U(C, N^h) = \begin{cases} \frac{C^{1-\sigma_C}}{1-\sigma_C} - AN^h, & \text{if} \quad \sigma_C \neq 0 \\ \log C - AN^h, & \text{otherwise,} \end{cases} \tag{7}
$$

where C denotes consumption, N^h the household's supply of labor and σ_C is the inverse of the elasticity of intertemporal substitution (EIS). Households maximize the expected present discounted value of the above felicity function. By definition we have:

$$
p(z,\mu) \equiv U_C(C, N^h) = C(z,\mu)^{-\sigma_C}, \tag{8}
$$

and from the intratemporal first-order condition:

$$
\omega(z,\mu) = -\frac{U_N(C, N^h)}{p(z,\mu)} = \frac{A}{p(z,\mu)}. \tag{9}
$$

2.3 Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions

$$\left(\omega, p, V^1, N, K, C, N^h, \Gamma\right),$$

such that

1. *Production unit optimality*: Taking ω, p and Γ as given, $V^1(\epsilon_S, \epsilon_I, k; z, \mu)$ solves (6) and the corresponding policy functions are $N(\epsilon_S, \epsilon_I, k; z, \mu)$ and $K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$.

2. *Household optimality*: Taking ω and p as given, the household's consumption and labor supply satisfy (8) and (9).

3. *Commodity market clearing*:

$$C(z,\mu) = \int z\epsilon_S\epsilon_I k^{\theta} N(\epsilon_S, \epsilon_I, k; z, \mu)^{\nu} d\mu - \int\int_0^{\bar{\xi}} [\gamma K(\epsilon_S, \epsilon_I, k, \xi; z, \mu) - (1-\delta)k] dG d\mu.$$

4. *Labor market clearing*:

$$N^h(z,\mu) = \int N(\epsilon_S, \epsilon_I, k; z, \mu) d\mu + \int\int_0^{\bar{\xi}} \xi \mathscr{J}\left(\psi \frac{1-\delta}{\gamma} k - K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)\right) dG d\mu,$$

   where $\mathscr{J}(x) = 0$, if $x = 0$ and 1, otherwise.

5. *Model consistent dynamics*: The evolution of the cross-section that characterizes the economy, $\mu' = \Gamma(z,\mu)$, is induced by $K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$ and the exogenous processes for z, ϵ_S and ϵ_I.

Conditions 1, 2, 3 and 4 define an equilibrium given Γ, while step 5 specifies the equilibrium condition for Γ.

2.4 Solution

As is well-known, (6) is not computable, since μ is infinite dimensional. Hence, we follow Krusell and Smith (1997, 1998) and approximate the distribution μ by its first moment over capital, and its evolution, Γ, by a simple log-linear rule. In the same vein, we approximate the equilibrium pricing function by a log-linear rule:[9]

$$\log \bar{k}' = a_k + b_k \log \bar{k} + c_k \log z, \tag{10}$$

$$\log p = a_p + b_p \log \bar{k} + c_p \log z, \tag{11}$$

[9] We experimented with an interaction term between $\bar{k}$ and z, but this did not yield any improvement in the fit of the equilibrium rule.

where $\bar{k}$ denotes aggregate capital holdings. Given (9), we do not have to specify an equilibrium rule for the real wage. As usual with this procedure, we posit this form and verify that in equilibrium it yields a good fit to the actual law of motion (see the Appendix C for details).

To implement the computation of sectoral data, we simplify the problem further and impose two additional assumptions: 1) $\rho_S = \rho_I = \rho$ and 2) enough sectors, so that sectoral shocks have no aggregate effects. Both assumptions combined allow us to reduce the state space in the production unit's problem further to a combined technology level $\epsilon \equiv \epsilon_S \epsilon_I$. Now, $\log \epsilon$ follows an AR(1) with first-order autocorrelation ρ and Gaussian innovations $N(0, \sigma^2)$, with $\sigma^2 \equiv \sigma_S^2 + \sigma_I^2$. Since the sectoral technology level has no aggregate consequences by assumption, the production unit cannot use it to extract any more information about the future than it has already from the combined technology level. Finally, it is this combined productivity level that is discretized into a 19-state Markov chain. The second assumption allows us to compute the sectoral problem independently of the aggregate general equilibrium problem.[10]

Combining these assumptions and substituting $\bar{k}$ for μ into (6) and using (**??**) and (10), we get a computable dynamic programming problem:

$$
\begin{aligned}
V^1(\epsilon, k, \xi; z, \bar{k}) = \max_n \big[& z\epsilon k^\theta n^\nu - \omega(z, \bar{k})n - i^M + (1-\delta)\psi k \big) p(z, \bar{k}) + \\
& \max\Big\{ -(1-\delta)\psi k p(z, \bar{k}) + \beta \mathrm{E}[V^0(\epsilon', \psi \frac{1-\delta}{\gamma} k; z', \bar{k}')], \\
& \max_{k'} \big(-\xi \omega(z, \bar{k}) p(z, \bar{k}) - \gamma k' p(z, \bar{k}) + \beta \mathrm{E}[V^0(\epsilon', k'; z', \bar{k}')] \Big\} \Big],
\end{aligned}
\tag{12}
$$

and policy functions $N = N(\epsilon, k; z, \bar{k})$ and $K = K(\epsilon, k, \xi; z, \bar{k})$. We solve this problem via value function iteration on V^0 and Gauss-Hermitian numerical integration over $\log(z)$ (for details, see Appendix C).

Several features facilitate the solution of the model. First, note that, as mentioned above, the employment decision is static. In particular it is independent of the investment decision at the end of the period. Hence we can use the production unit's first-order condition to maximize out the optimal employment level:

$$N(\epsilon, k; z, \bar{k}) = \Big(\frac{\omega(z, \bar{k})}{\nu z \epsilon k^\theta} \Big)^{1/(\nu-1)}. \tag{13}$$

Next, we examine the production unit's investment decision. Let us denote the gross value of adjusting capital net of the additional wage bill due to adjustment by V_a:

$$V_a(\epsilon; z, \bar{k}) \equiv \max_{k'} \big(-\gamma k' p(z, \bar{k}) + \beta \mathrm{E}[V^0(\epsilon', k'; z', \bar{k}')] \big). \tag{14}$$

From this, it is obvious that neither V_a nor the optimal target capital level, conditional on adjustment, depend on current capital holdings. This reduces the number of optimization problems in the value function iteration considerably. Denote the latter by $k^* = k^*(\epsilon; z, \bar{k})$. Furthermore, let us denote the value of inaction as V_i:

$$V_i(\epsilon, k; z, \bar{k}) \equiv -(1-\delta)\psi k p(z, \bar{k}) + \beta \mathrm{E}[V^0(\epsilon', \psi \frac{1-\delta}{\gamma} k; z', \bar{k}')]. \tag{15}$$

[10] In Appendix C.3 we show that our results are robust to this simplifying assumption.

Comparing (14) with (15) shows that $V_a(\epsilon; z, \bar{k}) \geq V_i(\epsilon, k; z, \bar{k})$,[11] which means that if $\bar{\xi} = 0$, i.e. in a frictionless economy, the necessary maintenance feature is completely irrelevant. With this notation, there exists an adjustment cost factor that would make a production unit just indifferent between adjusting and not adjusting:

$$\hat{\xi}(\epsilon, k; z, \bar{k}) = \frac{V_a(\epsilon; z, \bar{k}) - V_i(\epsilon, k; z, \bar{k})}{\omega(z, \bar{k}) p(z, \bar{k})} \geq 0. \tag{16}$$

We define $\xi^T(\epsilon, k; z, \bar{k}) \equiv \min\big(\bar{\xi}, \hat{\xi}(\epsilon, k; z, \bar{k})\big)$. Production units with $\xi \leq \xi^T(\epsilon, k; z, \bar{k})$ will adjust their capital stock. Thus,

$$k' = K = K(\epsilon, k, \xi; z, \bar{k}) = \begin{cases} k^*(\epsilon; z, \bar{k}), & \text{if} \quad \xi \leq \xi^T(\epsilon, k; z, \bar{k}), \\ \psi \frac{1-\delta}{\gamma} k, & \text{otherwise.} \end{cases} \tag{17}$$

We define *mandated investment* for a unit with current state $(\epsilon, z, \bar{k})$ and current capital k as:

$$x(\epsilon; z, \bar{k}) \equiv \log \gamma k^*(\epsilon; z, \bar{k}) - \log \psi (1-\delta) k.$$

That is, mandated investment is the investment rate the unit would undertake, after maintaining its capital, if its current adjustment cost draw were equal to zero. This concludes the computation of the production unit's decision rules and value function, given the equilibrium pricing and movement rules (**??**) and (10).

The second step of the computational procedure takes the value function $V^0(\epsilon, k; z, \bar{k})$ as given, and pre-specifies a randomly drawn sequence of aggregate technology levels: $\{z_t\}$. We start from an arbitrary distribution μ_0, implying a value $\bar{k}_0$. We then re-compute (12) at every point along the sequence $\{z_t\}$, and the implied sequence of aggregate capital levels $\{\bar{k}_t\}$, *without* imposing the equilibrium pricing rule (10):

$$\begin{aligned} \tilde{V}^1(\epsilon, k, \xi; z_t, \bar{k}_t; p) \equiv \max_n \Big(& z_t \epsilon k^\theta n^\nu - \frac{A}{p} n - i^M + (1-\delta)\psi k \Big) p + \\ & \max\Big\{ -(1-\delta)\psi k p + \beta \mathrm{E}[V^0(\epsilon', \psi \frac{1-\delta}{\gamma} k; z', \bar{k}'(k_t))], \\ & \max_{k'} \big(-\xi A - \gamma k' p + \beta E_{\epsilon'|\epsilon, z'|z_t}[V^0(\epsilon', k'; z', \bar{k}'(k_t))] \big) \Big\}. \end{aligned} \tag{18}$$

This yields new "policy functions"

$$\begin{aligned} \tilde{N} &= \tilde{N}(\epsilon, k; z_t, \bar{k}_t, p) \\ \tilde{K} &= \tilde{K}(\epsilon, k, \xi; z_t, \bar{k}_t, p). \end{aligned}$$

We then search for a p such that, given these new decision rules and after aggregation, the goods market clears (labor market clearing is trivially satisfied). We then use this p to find the new aggregate capital level.

This procedure generates a time series of $\{p_t\}$ and $\{\bar{k}_t\}$ endogenously, with which assumed rules (**??**)

[11] The production unit can always choose $k^* = \psi \frac{1-\delta}{\gamma} k$.

and (10) can be updated via a simple OLS regression. The procedure stops when the updated coefficients a_k, b_k, c_k and a_p, b_p, c_p are sufficiently close to the previous ones. We operationalize this by using an F-test for equality of coefficients. We show in Appendix C that the implied R^2 of these regressions are high for all model specifications, generally well above 0.99, indicating that production units do not make large mistakes by using the rules (**??**) and (10), and that higher moments of μ are unlikely to matter for equilibrium outcomes.

3 Calibration

For most parameters of the model (β, δ, γ, ν, ρ_A and ρ_I) we use the fairly standard values in Kahn and Thomas (2005)—these values can be found in Appendix A. We depart from Kahn and Thomas (2005) with respect to θ, σ_A, σ_I, as well as σ_C and $\bar{\xi}$. The first three are relatively minor departures,[12] the second group is more central to our new calibration procedure. Finally, we determine σ_S by a standard Solow residual calculation, while ρ_S is set equal to ρ_I for computational feasibility (see Appendices A and B for details).

Typically, adjustment cost parameters are calibrated to match establishment level moments. Khan and Thomas (2005), for example, choose $\bar{\xi}$ to match the fraction of LRD plant-level observations with an investment rate above 20%.

There are two problems with using plant level statistics to pin down certain parameters such as those that determine adjustment costs. First, this is usually done assuming that the basic unit in the model corresponds to the units from which the micro investment statistics are calculated (e.g., establishments in the LRD). There is no reason why this correspondence should be correct. Indeed, the stark nature of capital adjustments at the unit level in DSGE models with lumpy investment possibly fits better what is observed within subunits of an establishment, rather than at the establishment level.[13] Second, and more importantly, in nonlinear models such as the lumpy adjustment model, aggregation itself (before general equilibrium) can yield too much power to apparently small changes in microeconomic parameters. For example, anything that changes the drift of mandated investment gaps (such as maintenance investment), changes the mapping from microeconomic adjustment costs to aggregate dynamics (recall Caplin and Spulber's (1987) extreme example where adjustment costs cannot be inferred from the behavior of aggregates).

In Appendix D we present a simple extension of the paper's main model, illustrating that there are too many degrees of freedom for us to use micro-level statistics to pin down the model's parameters. This example shows how, by adding two micro parameters with no macroeconomic consequences, one can obtain a very good fit of observed micro moments. That is, the problems of matching micro moments and matching more aggregate moments are orthogonal in this extension.

[12] Our production function has more curvature than the one considered in Khan and Thomas, yet note that Gourio and Kashyap (2005) consider a much larger curvature than we do and are unable to completely break the irrelevance result. The reason, we conjecture, is that by not having idiosyncratic shocks and maintenance investment, their cross-section distribution remains too close to a self-replicating distribution a la Caplin and Spulber (1987). More on this below.

[13] Abel and Eberly (2002) and Bloom (2005) match a large number—250 or a continuum— of model-micro-units to one observed productive unit (firm or establishment).

Given these concerns, we follow an alternative approach where we replace plant-level data by sectoral data.[14] More precisely, given a value of χ, we choose $\bar{\xi}$ and σ_C to match the volatility of sectoral and aggregate US investment rates, respectively. In this approach we assume that the sectors we consider are sufficiently disaggregated so that general equilibrium effects can be ignored while, at the same time, there are enough micro units to justify the computational simplifications that can be made with a large number of units.

Given a set of parameters, the sequence of sectoral investment rates is generated as follows: the units' optimal policies are determined as described in Section 2, working in general equilibrium. Next, starting at the steady state, the economy is subjected to a sequence of sectoral shocks. Since sectoral shocks are assumed to have no aggregate effects and $\rho_I = \rho_S$, productive units perceive these shocks as part of their idiosyncratic shock and use their optimal policies with a value of the aggregate shock equal to one and the value of the idiosyncratic shock equal to the product of the sectoral and "truly" idiosyncratic shock, i.e. $\log(\epsilon) = \log(\epsilon_S) + \log(\epsilon_I)$.[15]

The remaining parameter values are chosen as follows: θ, the output elasticity of capital, is reduced to 0.18, in order to capture a revenue elasticity of capital, $\frac{\theta}{1-\nu}$, equal to 0.5, while keeping the labor share at its 0.64-value. In reduced form, this allows us to capture the main consequence of imperfect competition for investment decisions. The sectoral TFP calculation results in $\sigma_S = 0.0586$. We fix the combined (idiosyncratic and sectoral) standard deviation, σ, at 0.1, leaving us with a residual σ_I of 0.0812.

The value of sectoral volatility of investment rates we match is 0.0186. As noted in the introduction, this number is one order of magnitude smaller than the one predicted by the frictionless RBC model (or the Khan-Thomas model).[16] This stark difference is immune to working with 4-digit sectors, in which case the average volatility grows only slightly to 0.0254. Yet the assumption of a large enough number of units in every sector is less tenable in the 4-digit case, which is why we work with sectors at the 3-digit level.

Finally, to avoid biasing our comparison against the frictionless model, we recalibrate the standard deviation of aggregate shocks so that this model—the one with higher curvature and $\sigma_C = 1$—matches the volatility of the aggregate investment rate. The corresponding value for σ_A turns out to be 0.0095. In what follows, we refer to this as the "frictionless model" to differentiate it from the "standard RBC model".

[14] Needless to say, an even better approach is to combine data at both levels of aggregation. Moreover, the *time variation* in micro moments contain plenty of useful information for aggregate dynamics. Our general methodological point, however, is to emphasize giving relatively more weight to fairly aggregated data when interested in understanding aggregate phenomena.

[15] The standard deviation of the truly unit specific component of the perceived idiosyncratic shock is set so that the standard deviation of the idiosyncratic component that enters the unit's policy function remains constant and equal to the value used when calculating the policies under GE considerations. Details about the sectoral computation can be found in Appendix C.3. There we also document a robustness exercise where, instead of assuming that sectoral shocks have no general equilibrium effects, we recompute the optimal policies when micro units consider the distribution of sectoral productivity shocks—summarized by its mean—as an additional state variable. The results we obtain confirm the validity of our assumption.

[16] This statement is robust to decreasing the output elasticity of capital as we do in our model: the sectoral standard deviation of investment rates remains well above 0.20 in a frictionless model with the curvature value in Khan and Thomas.

Results

Table 5 presents the parameters we obtain for alternative values of the maintenance parameter χ.[17] The first column in this table depicts the largest adjustment cost units could pay.[18] Of course, the average cost actually paid is much lower, as shown in the second column. Productive units wait for good draws to adjust, and the adjustment cost they pay on average when adjusting is between 6 and 7% of the mean value of the distribution of adjustment costs. Since the average wage in the models is close to one and $N = 0.33$ on average, three times the second column is approximately equal to the average cost paid when adjusting, as a fraction of the wage bill.

Table 5: CALIBRATED PARAMETERS

Model	Largest adj. cost, $\bar{\xi}$	Avge. ξ when adj.	EIS
Frictionless:	0.000	0.0000	1.00
No maintenance:	1.551	0.0478	6.94
25% maintenance:	0.680	0.0225	7.69
50% maintenance:	0.239	0.0083	9.09
100% maintenance:	0.046	0.0014	32.25

The last column in Table 5 shows the estimated value for the elasticity of intertemporal substitution (EIS). It is not surprising that since microeconomic adjustment costs substitute for general equilibrium as a smoothing mechanism, the calibrated EIS are higher in our models. What is noteworthy, nonetheless, is how much higher these are relative to the standard unitary elasticity used in the standard RBC model. Of course, neither in the latter model nor in ours is this parameter likely to represent what it is interpreted to be doing. Rather it is an efficient reduced-form parameter to capture the elasticity of the supply of funds and of the quasi-labor supply. Interpreted in this manner, our calibration suggests that these elasticities are substantially higher at business cycle frequency than conventionally assumed. We return to this issue later in the paper.

Finally, it is useful to highlight at this stage the central role of maintenance investment. Note that as it increases, adjustment costs can be lowered and the EIS raised, and still match sectoral and aggregate investment rates. In other words, it substitutes for both, PE- and GE-smoothing mechanisms. The reason for this role is complex, as it follows from the effect maintenance investment has on the drift of the mandated investment process. As this drift is reduced – which happens as maintenance investment rises – the cross-section distribution of mandated investment becomes less bunched near regions where the probability of adjustment is high, and hence the economy's response to shocks becomes more muted. We return to this issue in the next section, when discussing the aggregate nonlinearities that arise in these models.

[17] In order to avoid computational problems associated with a very extended distribution, when computing the model for $\chi = 1$ we actually work with $\chi = 0.98$.

[18] We also choose the parameter A that captures the relative importance of leisure in the household's utility by matching the fraction of time worked to 1/3. The resulting value varies between 2.20 (frictionless case) and 0.968 ($\chi = 1$).

4 Aggregate Investment Dynamics

Our model calibration indicates that microeconomic non-convexities account for an important part of the smoothing in the response of investment to aggregate shocks. In this section we characterize in more detail the rich aggregate features, beyond smoothing, that emerge from lumpy microeconomic adjustment. In fact, many of the investment features highlighted in the partial equilibrium literature also appear in our DSGE setting. In particular, here we show that, as in Caballero and Engel (1999), lumpy adjustment models have the potential to generate aggregate impulse responses that are nonlinear and history dependent.

Let us illustrate these features with the help of a particular sample path that is roughly designed to mimic the boom-bust investment episode in the US during the last decade. For this, we simulate the paths of the frictionless and lumpy (with $\chi = 0.5$) economies that result from a sequence of five consecutive two-standard deviations positive aggregate productivity shocks, followed by a long period where the innovations are equal to zero. Both economies start from their respective steady states.

Figure 1: Investment boom-bust episode

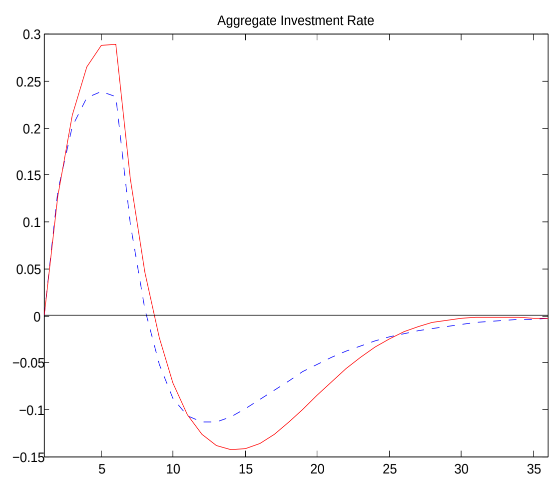
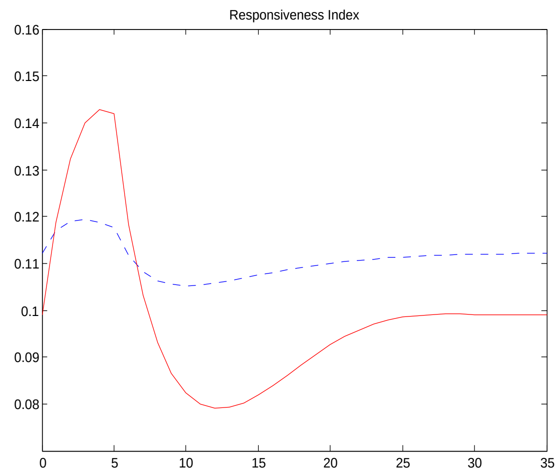


Figure 1 shows the evolution of the aggregate investment rates (as log-deviations from their steady state values) for these two economies. There are important difference between them: While at the outset of the boom phase their response is similar, eventually the investment rate in the lumpy economy reacts by more than the frictionless economy to further positive shocks. The flip side of the lumpy economy's larger boom is a more protracted decline in investment during the bust phase. Let us discuss these two phases in turn.

Figure 2 plots the evolution of the responsiveness index defined in Caballero and Engel (1993b), both for the lumpy model and for the frictionless model. This index captures the response of the aggregate investment rate to an increase in the current aggregate shock. At each point in time, this index is calculated conditional on the history of shocks, summarized by the current distribution of capital across units

Figure 2: Responsiveness Index



(see Appendix F for the formal definition).[19]

Note first that the index fluctuates much less in the frictionless economy than in the lumpy economy. Recall also that the frictionless economy only has general equilibrium forces to move this index around. Moreover, since the general equilibrium forces are much stronger in the frictionless model than in the lumpy economy, we can safely conclude that the contribution of the general equilibrium forces to the volatility of the index in the lumpy economy is minor.

It then follows from this figure that it is the decline in the strength of the PE-smoothing mechanism that is responsible for the rise in the index during the boom phase. As a result, eventually the index of responsiveness in the lumpy economy vastly exceeds that of the frictionless economy, which explains the larger investment boom observed in the lumpy economy after a history of positive shocks.[20]

The reason why PE-smoothing falls as the boom progresses, and hence the index of responsiveness rises, can be understood in relation to the Caplin and Spulber (1987) result. In that economy there is no aggregate price (the equivalent of our investment) smoothing regardless of the extent of micro frictions. That is, there is *no* partial equilibrium smoothing mechanism, despite the presence of micro frictions (lumpy price adjustment, in their case). This disconnect between micro and macro frictions is due to the fact that while few agents adjust to the most recent aggregate shock, the price increase each adjuster chooses is inversely proportional to the fraction of adjusters, so that the aggregate responds one-for-one with the shock. More precisely, Caplin and Spulber assume a simple (S,s) model and, crucially, also assume that the cross section distribution of price deviations from a common target is uniform in the (S,s)-interval.[21] In this context, an infinitesimal (positive) shock Δm implies that a fraction $\Delta m/(S-s)$

[19]The index is normalized by $c \equiv 1/(1-\alpha-\theta)$ so that in the absence of adjustment costs, equilibrium forces and aggregate productivity shocks the index takes the value one, see Appendix F for details.

[20]Note that while the frictionless economy has a a higher responsiveness index at the outset, this gap is short-lived so while the investment rate in the frictionless economy exceeds that of the lumpy economy early on, this difference is not noticeable in the scale of the figure.

[21]See Caballero and Engel (1991) for conditions under which the economy converges to the uniform distribution in Caplin

of the agents adjust by $(S-s)$, where S is the trigger threshold and s is the target level of the (S,s) policies followed by agents. It follows that the aggregate price response is:

$$\frac{\Delta m}{S-s} \times (S-s) = \Delta m,$$

and micro frictions have no aggregate implications.

In our lumpy model, the economy is not in such a limit: The product of the fraction of adjusters and the average size of their adjustment is much less than the aggregate shock, and hence there is substantial PE-smoothing. However, while not at the limit, the lumpy economy does move in the direction of Caplin and Spulber's "frictionless" limit as further positive shocks accumulate (and away from this limit as these shocks cease and the investment overhang is undone).

Figure 3: Investment boom-bust episode: Cross-section and hazard

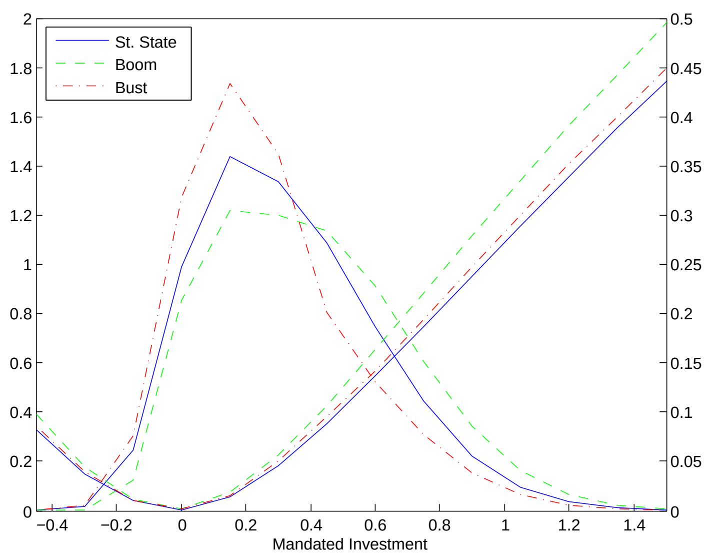


Figure 3 illustrates the mechanism described in the previous paragraphs. It shows the cross-section of mandated investment (and the probability of adjusting, conditional on mandated investment) at three points in time: the beginning of the episode with the economy at its steady state (solid line), the peak of the boom (dashed line) and the trough of the cycle (dash-doted line).[22] It is apparent that during the boom the cross-section of mandated investment moves toward regions where the probability of adjustment is higher. The fraction of units with mandated investment close to zero decreases considerably during the boom, while the fraction of micro units with mandated investment rates above 40% increases significantly. Also note that the fraction of units in the region where mandated investment is negative decreases during the boom, since the sequence of positive shocks moves units away from this region.

and Spulber (1987).

[22] See Section 2 for the formal definition of mandated investment. Also note that the scale on the left of the figure is for the mandated investment densities, while the scale on the right is for the adjustment hazards.

The convex curves in Figure 3 depict the adjustment hazard, that is, the probability of adjusting conditional on the corresponding value of mandated investment. It is clear that the probability of adjusting increases with the (absolute) value of mandated investment. This is the 'increasing hazard property' described in Caballero and Engel (1993a). We also note that as the boom proceeds, the adjustment hazard shifts upward, so that aggregate investment becomes more responsive to positive and negative shocks (see Figure 2) not only because units concentrate in a region where they adjust by more, but also because the probability of adjusting in this region is higher.

In summary, the decline in the strength of PE-smoothing during the boom (and hence the larger response to shocks) results mainly from the rise in the share of agents that adjust to further shocks. This is in contrast with the frictionless (and Calvo style models) where the only margin of adjustment is the average size of these adjustments. This is shown in Figure 4, which decomposes the responsiveness index into two components: one that reflects the response of the fraction of adjusters and another that captures the response of average adjustments of those who adjust. Of course, both series add up to the overall responsiveness index in Figure 2. It is apparent that most of the smoothing—approximately 70% in this metric—is done by variations in the fraction of adjusters.

Figure 4: Decomposition of Responsiveness Index: Intensive and Extensive Margins

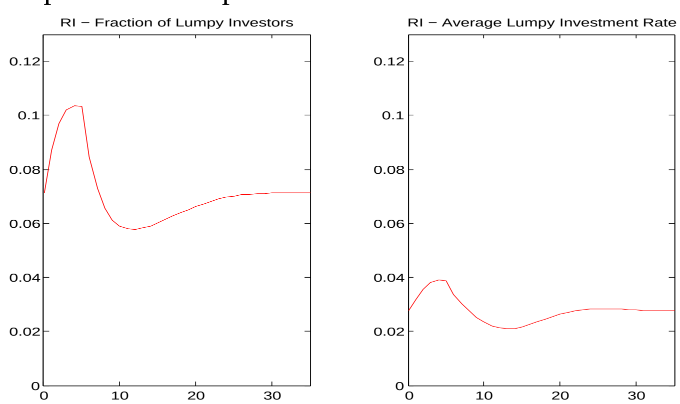
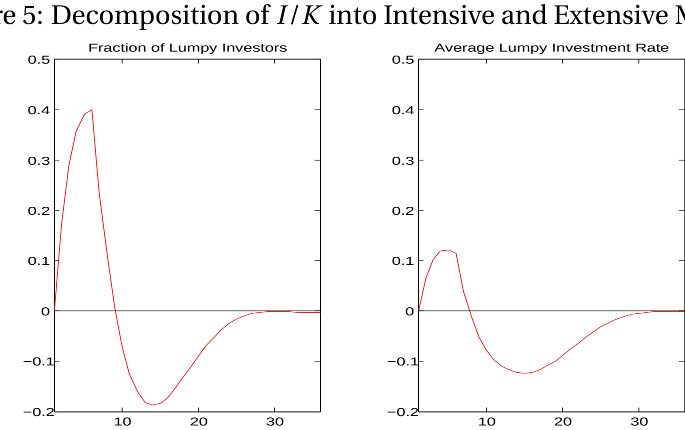
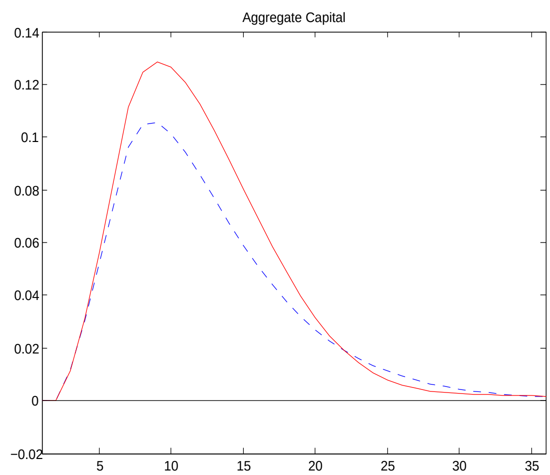


Figure 5: Decomposition of I/K into Intensive and Extensive Margins



The importance of fluctuations in the fraction of adjusters is even more pronounced if we decompose the path of the aggregate investment rate into the contributions from the fluctuation of the fraction of adjusters and the fluctuation of the average size of adjustments, as shown in Figure 5. Both series are in log-deviations from their steady state values. This is consistent with what Doms and Dunne (1998) documented for establishment level investment in the US, where the fraction of units undergoing major investment episodes accounts for a much higher share of aggregate (manufacturing) investment than the average size of their investment.

Figure 6: Aggregate Capital



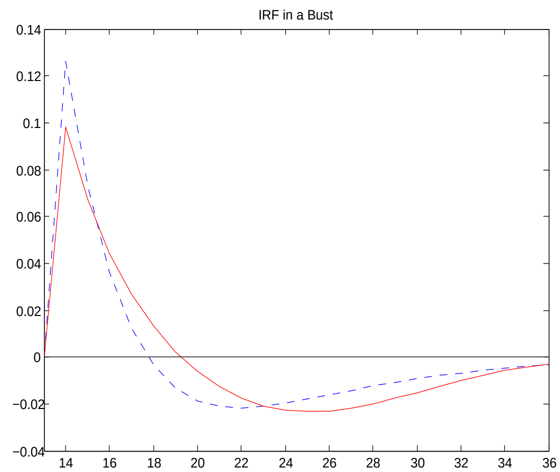
Let us now turn to the bust phase. Figure 6 illustrates the "overaccumulation" of capital resulting from the large investment boom in the lumpy economy. As a result of this boom, once the positive shocks subside, the economy experiences an "overhang" that leads to the protracted investment slump shown in Figure 1.

Returning to Figure 3, we see the large capital accumulation during the boom leaves an unusually large fraction of units in the region close to zero mandated investment, where units are very unlikely to respond to a shock, due to the low values of the adjustment hazard in this region. This explains the sharp drop in the responsiveness index shown in Figure 7.

The observation of the slump in the responsiveness index has important implications for the economy's ability to return to its steady state investment rate, as the latter becomes unresponsive to positive stimuli, such as a positive aggregate shock or policy intervention (e.g., an investment tax credit). Figure 7, illustrates this mechanism by plotting the impulse responses of the frictionless and lumpy economies following a positive aggregate shock that takes place in period 14, when the gap between index of responsiveness of the frictionless and lumpy economy is maximal.[23] The more sluggish response of investment

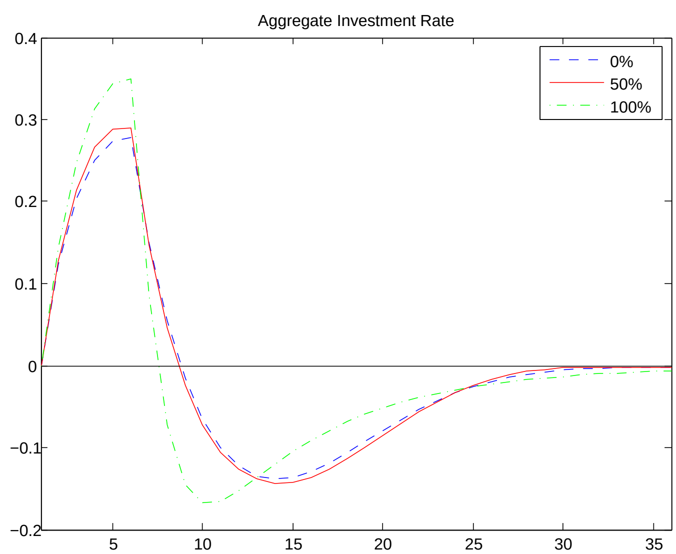
[23] These impulse responses are plotted in deviation from the paths without the new shock, and—like the responsiveness index—normalized by the standard deviation of the aggregate shock and $c \equiv 1/(1-\alpha-\theta)$. See footnote 19 and Appendix F for the rationale underlying the latter normalization constant.

Figure 7: Impulse Responses at the Trough



in the lumpy economy is apparent.

Figure 8: Boom-bust episode and maintenance: Investment rate



Let us conclude this section by returning to the role of the maintenance parameter. Figure 8 illustrates the boom-bust cycle for different values of this parameter. It is apparent that the size of the boom-bust cycle increases with the importance of maintenance. The reason, again, is linked to the mechanism discussed above. When maintenance investment is large, the drift of the processes for microeconomic mandated investment (defined as the investment rate if the unit draws a zero adjustment cost) is small, since maintenance investment offsets depreciation and trend growth. This is important in these models, as it implies that the cross section distributions of such investment are far from the Caplin-Spulber limit, and hence there is plenty of space for them to vary in response to shocks. As before, this variation translates into countercyclical fluctuations in the degree of PE-smoothing, which exacerbates the magnitude of an aggregate investment boom in the face of an unusually long string of positive aggregate shocks.

5 Indirect Effects: Improved Conventional RBC Moment-Matching

While the frictionless RBC model fits the volatility of investment well, it falls short in terms of the volatility of consumption, output and employment (King and Rebelo, 1999). Since in our model microeconomic frictions smooth aggregate investment, they simultaneously improve the fit of the relative volatility of investment to that of other aggregates and create space to raise the volatility of investment through a reduction in GE-smoothing mechanisms. However, the latter reduction also raises the volatility of consumption and employment. While we did not use information on consumption and employment volatility in our calibration, the tables below show that an indirect benefit of our procedure is a significant improvement in the fit of the model along these dimensions as well.

We also use this section to show that our results on aggregate investment dynamics survive maintaining the degree of GE-smoothing at conventional levels (EIS around one).

5.1 Volatility and Persistence

Table 6: VOLATILITY OF AGGREGATES

	St.dev.				St.dev. rel. to $\sigma(Y)$		
	Y	C	I	N	C	I	N
Data:	2.00	1.73	5.94	2.00	0.86	2.97	1.00
Frictionless:	1.40	0.65	6.61	0.85	0.47	4.71	0.61
King-Rebelo:	1.39	0.61	4.09	0.67	0.44	2.95	0.48
This paper:	2.15	1.60	5.85	2.01	0.75	2.73	0.94

Table 6 reports the observed volatility of U.S. aggregates, and those implied by the frictionless model, by the standard RBC model (from King and Rebelo (1999), which differs from frictionless in the curvature parameter and its quarterly frequency), and by our model, both in absolute terms (percentages) and relative to the standard deviation of output. For our model we assume 50% maintenance yet the results that follow are valid for all values of the parameter χ (for other values of χ, ranging from 0 to 100%, see Appendix G).[24] It is apparent from this table that our model is successful in fitting the volatility of aggregate consumption, investment, employment and capital, which we did *not* use in the calibration stage (recall that we calibrated the volatility of sectoral and aggregate investment rates). In fact, the lumpy model does substantially better than the frictionless and standard RBC models. Table 7 shows that our

[24] Since, by construction, our models match the volatility of the aggregate investment rate, we do not include this aggregate. As usual, but with the exception for the aggregate investment rate, the series are log-HP-filtered with a smoothness parameter of 100. Also, for obvious reasons, our model's counterpart of output is $C+I$.

model also provides a better match for four of the five observed persistence (first-order autocorrelation) measures.[25]

Table 7: PERSISTENCE OF AGGREGATES

	Y	C	I	N	I/K
Data:	0.53	0.58	0.47	0.52	0.71
Frictionless:	0.42	0.61	0.36	0.35	0.57
This paper:	0.47	0.52	0.43	0.47	0.69

Figure 9 exhibits the impulse response function for consumption, employment, and the investment rate, for the frictionless and our model ($\chi = 0.50$).[26] They corroborate the findings reported in the previous tables. It is apparent that there are significant differences between the lumpy model and the frictionless model for consumption and employment. Even for small shocks and an economy that starts off at its steady state (this is what the impulse response function reports), there are clear differences in the dynamic response of aggregate quantities. More importantly, these differences constitute an improvement over the frictionless model in terms of the fit of US aggregate data. The differences for the investment rate are smaller, which is not surprising since we imposed that both models have the same volatility. Yet even in this case, the fact that our model exhibits higher persistence than the RBC model, brings it closer to the data (see the last column in Table 7).

Given the success of the lumpy-high EIS model, we went further and tested formally whether it is rejected. Column (1) in Table 8 considers the variance and autocovariances of C, I, N and Y when calculating the standard chi-square-statistic (see Ingram and Lee, 1991).[27] Since the resulting weighting matrix is very close to singular, we exclude both moments involving Y in column (2).[28] It is clear that our model also outperforms the frictionless model using this formal approach. Furthermore, if we avoid a poorly conditioned weighting matrix by excluding one of the moments, our model is not rejected by the data, which is a rarity for this kind of highly over-identified structural models.

Table 8: CHI-SQUARE-STATISTICS

	(1)	(2)
Frictionless:	53.3	44.7
This paper:	30.5	1.9
Critical value:	11.1	7.8

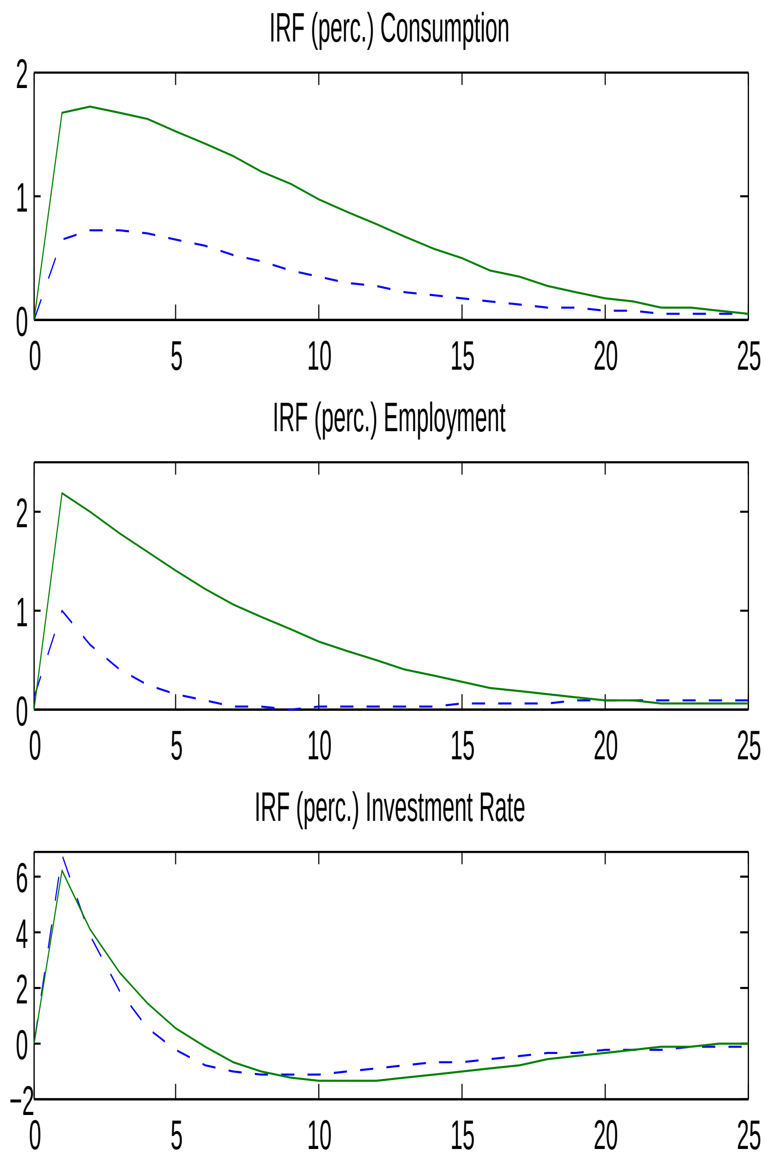
[25] For $\chi = 0.75$ the fit is better for all persistence measures, see Appendix G.

[26] The impulse responses are the log-deviations from the steady-state to a one-standard deviation innovation in the aggregate productivity shock.

[27] The chi-square statistics we obtain vary little with whether we consider the standard or the autocorrelation robust weighting matrix. The results we report are for the latter.

[28] Recall that: $Y = C + I$. The conditioning number for the weighting matrix falls by a factor of 20.

Figure 9: Impulse response of C, N and I/K



5.2 On general equilibrium smoothing

The main reason for the gain in matching conventional RBC moments for consumption, employment and output, is that microeconomic lumpiness generates substantial smoothing of aggregate investment, thereby reducing the burden on general equilibrium smoothing to match investment volatility. Once the relative importance of general equilibrium smoothing is reduced, aggregate consumption and employment can react more aggressively to aggregate shocks.

The only parameter to control the strength of general equilibrium forces in our model is the EIS, which needs to be raised substantially to match aggregate moments. If interpreted literally as a microeconomic preferences parameter, our numbers for the EIS are much higher than the standard estimates found in the literature. The most recent analysis of this matter is Gruber (2005), who uses a careful identification strategy based on households responses to tax movements. He finds an EIS of two, which is on the high end of previous estimates.[29] Table 9 below reports the moments from our lumpy adjustment model, both when we impose the conventional EIS value of one (which is used mainly for analytical convenience) and when we use Gruber's estimate.

Table 9: RELATIVE VOLATILITY AND PERSISTENCE OF AGGREGATES

	St.dev. rel. to $\sigma(Y)$			Persistence				
	C	I	N	Y	C	I	N	I/K
Data:	0.86	2.97	1.00	0.53	0.58	0.47	0.52	0.71
Frictionless:	0.47	4.71	0.61	0.42	0.61	0.36	0.35	0.57
This paper:	0.75	2.73	0.94	0.47	0.52	0.43	0.47	0.69
EIS = 1:	0.60	3.54	0.44	0.44	0.48	0.42	0.42	0.69
EIS = 2:	0.69	3.03	0.68	0.45	0.49	0.42	0.44	0.69
Frictionless with high EIS:	0.72	5.72	0.95	0.53	0.80	0.28	0.50	0.44

The volatility results are reported normalized by the standard deviation of output, since the overall volatility of quantities is too low now that we add more sizeable GE-smoothing to PE-smoothing.[30,31]

It is apparent from this table that the lumpy model with more conventional EIS values still does substantially better than the frictionless model in terms of relative volatility and persistence. When the EIS is set to one, the lumpy model does better in six out of the eight statistics reported in the table, and when the EIS is raised to two (as in Gruber), it does better for seven out of eight statistics.

Conversely, the last row of the table shows that if one runs the frictionless model with our estimate of the EIS for the $\chi = 0.5$ case, which is around 9, the volatility of investment rises too much and its

[29]Also, see Hansen and Singleton (1996), who find a slightly higher value for the EIS.

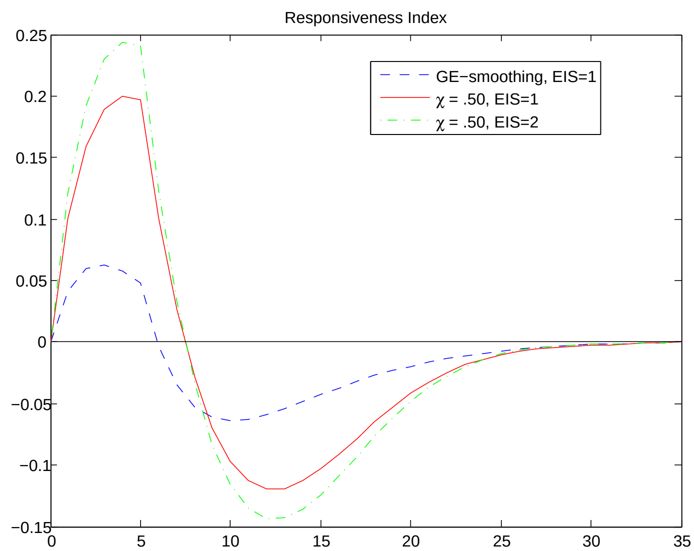
[30]Alternatively, we could recalibrate σ_A so as to match the aggregate investment rate. The moments reported in the table do not vary if we take this approach—nor do the persistence measures—and overall volatility is in the right ballpark now. The values of σ_A obtained this way are 0.0133 for EIS= 1 and 0.0122 for EIS= 2.

[31]The standard deviation of aggregate investment rates declines from 0.0074 to 0.0053 and 0.0058, respectively, in the models with EIS= 1 and EIS= 2. The upper bound on the fraction of overall smoothing accounted for by the partial equilibrium are 74% and 78%, respectively, while the lower bounds are 16 and 20%. These numbers are in the same ballpark as those reported in Table 2. The resulting percentage standard deviations of output are 1.18% and 1.52%.

persistence drops too much relative to US data.[32]

Finally, a note on the robustness of our main results. Figure 10 reports the path of the index of responsiveness for the same experiment as in the previous section for conventional levels of the EIS (and the frictionless model).[33] Again, it is apparent that the source of nonlinearities reported in the previous section survives the increase in GE-smoothing brought about by the reduction in the EIS. This conclusion is confirmed by Figure 11 which shows that, for values of the EIS equal to 1 and 2, the path of the aggregate investment rate in the model with lumpy investment differs substantially from the corresponding trajectory for the model where GE forces are the only source of smoothing. As before, the boom is more pronounced and the overhang period more protracted.

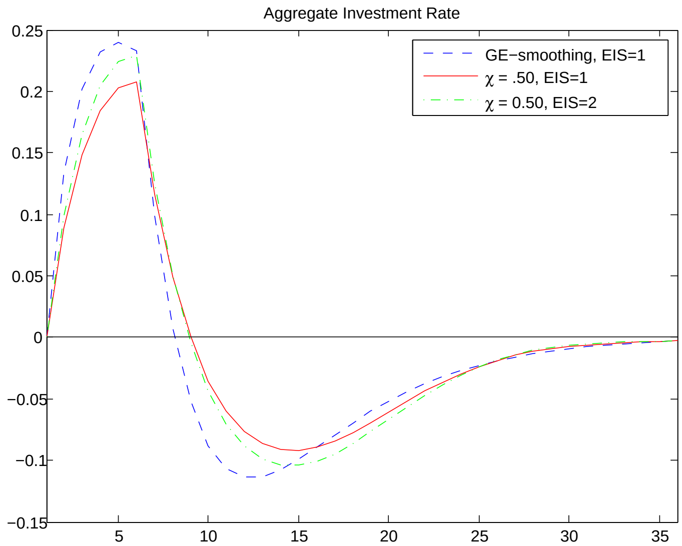
Figure 10: Responsiveness Index and Boom-bust Episode: Robustness



[32] We re-calibrate the standard deviation of aggregate technology, so that the standard deviation of aggregate investment rates is exactly matched. This results in $\sigma_A = 0.0048$, and a percentage standard deviation for output of 1.25%.

[33] Since now the models overall display less volatility compared to the frictionless model, we depict log-deviations of the sensitivity index from the steady state.

Figure 11: Aggregate Investment Rate and Boom-bust Episode: Robustness



6 Final Remarks

We have shown that adding realistic lumpy capital adjustment at the microeconomic level to an otherwise standard RBC model has important macroeconomic implications. In particular, the impulse response functions of aggregate investment become history dependent. Relative to the standard DSGE model, booms feed into themselves and can lead to significantly larger capital accumulation following a string of positive shocks. Busts, on the other hand, can lead to protracted periods of depressed investment, which are largely unresponsive to policy stimuli.

That is, the differences introduced by the lumpy model are most significant at the times that matter the most: during pronounced booms and recessions. Furthermore, the smoothing of aggregate investment stemming from the microeconomic frictions reduces the burden of smoothing that is typically borne by general equilibrium forces. This shift in the smoothing mechanism has the important side effect of significantly improving the fit of consumption and employment volatility as well.

Roughly, we calibrated the strength of the partial equilibrium smoothing mechanism by fitting sectoral data and used the elasticity of intertemporal substitution to control the additional smoothing that takes place from sectoral to aggregate data. It is apparent that in this logic, or in that of the standard RBC model, the EIS as is not a structural parameter but a reduced form way of capturing more complex labor and capital market specifications. Thus, the substantial gains obtained from increasing the EIS point in the direction of finding flatter labor and capital supplies than implied by the standard model. On the capital supply side, there are many good reasons why even with a true EIS around one, the effective capital supply is substantially flatter. Most prominently, the US economy is open and receives massive capital flows. Also, capital can be reallocated across sectors which are not perfectly synchronized in their cyclical responses. On the labor supply side, there is a large number of theories and evidence of flat quasi-labor supplies. These are old themes, which our model and findings only help making a stronger case for.

Either way, whether one interprets the EIS parameter structurally or not, or whether one is married to an EIS of one or not, this papers has shown that contrary to previous claims, the lumpy model enriches the dynamic responses of DSGE models in important dimensions.

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A Parameter Appendix

The following table summarizes the common parameters of all the model specifications explored in the paper:

ρ_A	σ_A	ρ	σ_S	σ_I	δ	γ	β	θ	ν
0.8254	0.00953	0.53	0.0583	0.0812	0.0690	1.0160	0.9770	0.1800	0.6400

The parameters $\rho_A, \delta, \gamma, \rho, \nu$ and β are taken from Kahn and Thomas (2005). They are standard values. The calibration of the other parameters is explained in Section 3.

B Data Appendix

B.1 Aggregate Data

We use yearly U.S. data on consumption, investment, employment and capital, from 1960-1996. Since our model is a closed economy without government, we look at $C + I$ rather than GDP data. The standard moments, however, do not differ much. The data on investment and capital include equipment and structures. They stem from the BEA: Stock of net nonresidential fixed assets and real cost investment.[34] These series are in 1996 chained dollars. Consumption data are from the yearly "Personal real consumption expenditures - billions of chained 2000"-series (PCECCA), from St. Louis FED. Employment data are from the "Total private employment"-series (CES0500000001), from the Bureau of Labor Statistics. They exclude farm employment, and are based on payroll data. The key statistics for aggregate investment rates are a standard deviation of 0.0074 and a persistence of 0.71

Throughout the paper, for both real data and simulated data, we take the raw series for investment rates, since they do not exhibit an obvious trend in our time frame, but we do follow the RBC convention of log-hp-filtering, with bandwidth parameter 100, the series for consumption, investment, employment and "output".

B.2 Sectoral Data

For lack of good industry data outside of manufacturing, the data source here is the NBER manufacturing data set, publicly available on the NBER website. It contains yearly 4-digit industry data for the manufacturing sector, according to the SIC-87 classification. We look at the years 1960-1996, later years are not available. We take out industry 3292, the asbestos products, because this sector essentially dies out in the nineties. This leaves us with 458 industries altogether.

Since the sectoral model analysis has to satisfy two requirements: 1) isolation from general equilibrium effects, 2) contain a large number of production units, we think that the 3-digit level is the best

[34] (http://www.bea.gov/bea/dn/faweb/details/).

compromise aggregation level. This leaves us with 140 industries. Hence, we sum employment levels, real capital, nominal investment and nominal value added onto the 3-digit level. The deflators for investment and shipments are a weighted sum (weighted by investment and value added, respectively). This allows us to compute series of real investment and real value added. Since the data base does not contain separate deflators for value added (as opposed to shipments), we use the one for shipments to compute a real value added series. Moreover, since the data base does not contain implicit deflators for capital, we just sum real capital. The deflators on the 4-digit level are generally identical or very close to each other, so that this is a justifiable procedure.

TFP-Calculation: Since our model is essentially about value added production as opposed to output production—we do not model utilization of materials and energy—we do not use the TFP-series in the data set, which are based on a production function for output. Rather, we use a production function for real value added in employment and real capital with payroll as a fraction of value added as the employment share, and the residual as capital share, and perform a standard Solow residual calculation for each industry separately.

Next, in order to extract the residual industry-specific and uncorrelated-with-the-aggregate component for each industry, we regress each industry time series of logged Solow residuals on the time series of the cross-sectional average of logged Solow residuals and a constant. The residuals of this regression are then taken as the pure sectoral Solow residual series, by construction, they are uncorrelated with the cross-sectional average series. We then compute an AR(1)-specification for each of these series. Finally, the value-added-weighted average of these coefficients is then taken to be $\rho_S = 0.70$ and $\sigma_S = 0.0583$.

Since this computation is subject to substantial measurement error and somewhat arbitrary choices, we perform a number of robustness checks: 1) We fix the employment share and capital share to $\nu = 0.64$ and $\theta = 0.18$, as in our model parametrization for all industries. 2) We study a production function that distinguishes between production workers and non-production workers. 3) We look at raw industry-specific Solow residual series, and a series, where we simply subtract the time series of the cross-sectional average. 4) We look at non-weighted averages to get the final AR(1) coefficients. 5) We look at medians instead of averages. The results for σ_S are fairly robust. Finally, to check how much the results are influenced by using 3-digit data, the corresponding values for the 4-digit data are: $\rho_S = 0.69$ and $\sigma_S = 0.0762$.

Calculation of I/K-Moments: To extract a pure sectoral component of the time series of the industry investment rate, we perform the same regression that was used for TFP-calculation. We do not log or filter the investment rate series. The common component is now a capital-weighted average of the industry investment rates. Again, we perform robustness checks 3)-5) from above, with fairly stable results. The resulting sectoral time series moments for the 3- and 4-digit level are given in the following table:

	Persistence	STD
3-digit	0.65	0.0186
4-digit	0.49	0.0299

For calibration, we use the 3-digit level standard deviation. Similar results would obtain if we use the 4-digit standard deviation instead, since the standard deviation of sectoral investment rates in the frictionless model are one order of magnitude higher than the numbers above (see footnote 16).

C Numerical Appendix

In this appendix, we describe in detail the numerical implementation of the model computation. All codes were computed in Matlab 6.5R13.

C.1 Decision Problem

Given the assumptions we made in the main paper: 1) $\rho_S = \rho_I = \rho$, and 2) approximating the distribution μ by the aggregate capital stock, $\bar{k}$, the dynamic programming problem has a 4-dimensional state space: $(k, \bar{k}, z, \epsilon)$. Since the employment problem has an analytical solution, there is essentially just one continuous control, k'. We discretize the state space in the following ways:

1. k: $n_k = 30$ grid points from $[0, 5]$, with a lower grid width at low capital levels, where the curvature of the value function is highest. In general, the value function is fairly linear.

2. $\bar{k}: n_{\bar{k}} = 11$ grid points in $[0.60, 1.10]$, equi-spaced.

3. $z : n_z = 10$ grid points in $[0.93, 1.075]$ with closer grid points around unity. For the Gauss-Hermitian integration (see Judd, 1998) we use 7 integration nodes.

4. ϵ: $n_\epsilon = 19$. The grid points are equi-spaced (in logs) and the total grid width is given by $3 \times \sqrt{\frac{\sigma^2}{1-\rho^2}}$, the unconditional variance of the combined technology process. For the transition matrix we use the procedure proposed in Tauchen (1986). The large state space here slows down computation considerably, but we need it for a meaningful sectoral simulation.

We check the robustness of our computations by varying the number of grid points and Gauss-Hermitian integration nodes.

We note that for all partial equilibrium computations the dimension of the state space collapses to three, $\bar{k}$ is no longer needed to compute prices and aggregate movements. Instead, we follow Kahn and Thomas (2005) in fixing the intertemporal price and the real wage at their average levels from the general equilibrium simulations.

Since we allow for a continuous control, k, and $\bar{k}$ and z can take on any value continuously, we can only compute the value function exactly at the grid points above and interpolate for in-between values. This is done by using a multidimensional cubic splines procedure, with a so-called "not-a-knot"-condition to address the large number of degrees of freedom problem, when using splines (see Judd, 1998). We compute the solution by value function iteration, using 20 steps of policy improvement after each actual optimization procedure. The optimum is found by using a golden section search, which is fast and robust. Due to the nature of the non-convexity, the optimal return level does not depend on k, which reduces the number of optimization problems to be solved at each iteration to $n_{\bar{k}} \times n_z \times n_\epsilon$. Upon convergence, we check single-peakedness of the objective function, to guarantee that the golden section search is reasonable.

C.2 Equilibrium Simulation

For the calibration of the general equilibrium models we draw one random series for the aggregate technology level and fix it across models. For calibration purposes we use $T = 600$ and discard the first 100 observations. The statistics we report are then based on a series of $T = 2600$, with the first 600 identical to those in the calibration process. We find that, generally, the statistics are robust to T. We start from an arbitrary individual capital distribution and the stationary distribution for the combined productivity level. The model economies typically settle fast into their stochastic steady state. Since with idiosyncratic shocks, adjustment costs and necessary maintenance some production unit may not adjust for a very long time, we take out any individual capital stock in the distribution that has a marginal weight below 10^{-10}, in order to save on memory. We re-scale the remaining distribution proportionally.

As in the production unit's decision problem, we use a golden section search to find the optimal target capital level, given p. We find the market clearing intertemporal price, using the Matlab built-in function fzero, which uses a combination of bisection, secant and inverse quadratic interpolation methods. Precision of the market-clearing outcome is generally below 10^{-5} for the frictionless models, and below 10^{-7} for the lumpy models (these numbers are maxima, not averages).

To further assess the quality of the assumed log-linear equilibrium rules, we perform the following simulation: for each point in the $T = 2600$ time series, we iterate for a time series of $\tilde{T} = 100$ aggregate capital and the intertemporal price forward, using only the equilibrium rules. We then compare the aggregate capital and p after $\tilde{T}$ steps with the actually simulated ones, when the equilibrium price was updated at each step. We then compute maximum absolute percentage deviations, mean squared percentage deviations, and the correlation between the simulated values and the out-of-sample forecasts. The following two tables summarize the numerical results for each model. The rows contain: the coefficients of the log-linear regression, its R^2 and standard error, the R^2 of a regression that includes the log of the standard deviation of the capital distribution to assess the room for improvement by using higher moments,[35] the F-value for equality of coefficients in the equilibrium loop, and the three above measures that assess the out-of-sample quality of the equilibrium rules. First for aggregate capital:

[35] Note that the standard deviation was not actually used in the equilibrium calculation.

	FL	0-maint.	0.25-maint.	0.5-maint.	1.00-maint.
$a_{\bar{k}}$	-0.0534	-0.0631	-0.0563	-0.0514	-0.0622
$b_{\bar{k}}$	0.7361	0.7967	0.7971	0.7991	0.7546
$c_{\bar{k}}$	0.6143	0.5795	0.5820	0.5796	0.5908
R^2	1.0000	0.9991	0.9988	0.9987	0.9981
SE	0.0001	0.0010	0.0012	0.0013	0.0013
R^2_{std}	1.0000	0.9999	0.9998	0.9997	0.9983
F	$7.5e{-}5$	$1.05e{-}10$	0	0	0
MAD(%)	0.11	0.87	0.99	1.04	1.25
MSE(%)	0.02	0.35	0.40	0.42	0.31
Correl.	0.9999	0.9954	0.9942	0.9937	0.9955

Then for p:

	FL	0-maint.	0.25-maint.	0.5-maint.	1.00-maint.
a_p	0.7947	0.1056	0.0948	0.0801	0.0215
b_p	-0.3044	-0.0918	-0.0841	-0.0728	-0.0263
c_p	-0.6622	-0.2299	-0.2133	-0.1884	-0.0610
R^2	1.000	0.9967	0.9971	0.9978	0.9983
SE	0.0000	0.0004	0.0003	0.0002	0.0000
R^2_{std}	1.0000	0.9997	0.9997	0.9997	0.9987
F	0.0002	$5.5e{-}12$	0	0	$1.1e{-}11$
MAD(%)	0.03	0.11	0.10	0.09	0.05
MSE(%)	0.01	0.03	0.03	0.02	0.01
Correl.	0.9999	0.9991	0.9990	0.9990	0.9989

Figure 12: Assessing the approximate forecasting rules

It can be seen from Figure 12 and the table above that the equilibrium rules generally perform very well. Inclusion of the second moment of the capital distribution may yield a mild improvement of the approximation quality, but it is doubtful, whether aggregate statistics can be changed in any significant way. As can be expected, in models I-V, with low σ_C, the influence of aggregate variables on prices drastically decreases, these scenarios are close to partial equilibrium.

C.3 Sectoral Simulation

Underlying the sectoral simulation are four assumptions: first, a large enough number of sectors and, secondly, that σ_S is large enough relative to σ_A, so that we can compute the sectoral implications of our model independently of the aggregate general equilibrium calculations. This is also reflected in our treatment of the sectoral data as residual values, which are uncorrelated with aggregate components. Thirdly, we make use of the assumption that a sector is large enough to comprise a large number of production units by invoking a law of large numbers now for the true idiosyncratic productivity. Finally, $\rho_S = \rho_I$, and the independence of sectoral and the idiosyncratic productivity, so that we can treat sectoral and truly idiosyncratic uncertainty as one state variable in the general equilibrium problem.

We start by fixing the aggregate technology level at its average level: $z^{SS} = 1$. The converged equilibrium law of motion for aggregate capital can then be used to compute the steady state aggregate capital level that belongs to this aggregate productivity. It is the fix point of the aggregate low of motion, evaluated at z^{SS}:

$$\bar{k}^{SS} \equiv \exp \frac{a_{\bar{k}}}{1 - b_{\bar{k}}}. \tag{C3.1}$$

This, in turn, leads to the steady state $p^{SS} \equiv \exp(a_p + b_p \log(\bar{k}^{SS}))$.

Then we specify a separate grid for idiosyncratic and sectoral productivity in such a way that all new grid points and any product of them will lie on the original 19-state grid for the combined productivity, used in the general equilibrium problem. Recall that this was specified for $(\rho = 0.53, \sigma = 0.1)$. Given the equi-spaced (in logs) nature of the combined grid this is obviously possible. Thus, the idiosyncratic grid comprises 11 grid points, and the sectoral grid 9 grid points, both equi-spaced and centered around unity. This naturally reflects $\sigma_I > \sigma_S$. The implied grid width for the idiosyncratic grid is 2.0514 times the unconditional standard deviation, and 2.2870 times the unconditional standard deviation for the sectoral grid. Both values are well within commonly used ranges. We then use Tauchen's method to compute transition matrices for the Markov chain, given by the sectoral and the truly idiosyncratic grid. Parameters used are $(\rho = 0.53, \sigma_S = 0.0586)$ and $(\rho = 0.53, \sigma_I = \sqrt{\sigma^2 - \sigma_S^2} = 0.0812)$, respectively.

We then recompute optimal target capital levels as well as gross values of investment at $z^{SS}, \bar{k}^{SS}$, once and for all at the 19 values for ϵ. By construction, these are then also the values for any (ϵ_S, ϵ_I)-combination. Note that we use the value functions computed from the general equilibrium case. We draw a random series of $T = 2600$ for ϵ_S, which remains fixed across all models, start from an arbitrary capital distribution and the stationary distribution for the idiosyncratic technology level, and follow the behavior of this representative sector, using the sectoral policy rules. The details are similar to the one used in the equilibrium simulation.

We finally test our first two assumptions in the sectoral computation: a continuum of sectors, so that sectoral shocks do not have aggregate consequences; and that the fact that $\sigma_S \gg \sigma_A$ allows us to fix the aggregate environment to its steady state level, when computing sectoral data and working with sectoral data that have been purified of their aggregate components.

To this end, we compute the equilibrium with a finite number of sectors. Also, the sectoral data are computed subjecting it to the moving aggregate environment, now explicitly allowing for full GE-smoothing. This requires introducing another aggregate state variable, given by: $\bar{\epsilon}_{S,t} \equiv \sum_{i=1,...,N_S} \log(\epsilon_{S,t}(i))$, where N_S is the finite number of sectors. Obviously, $\bar{\epsilon}_{S,t} = 0, \forall\ t$, as $N_S \to \infty$, by the law of large numbers and assuming sectoral independence.

We fix a set of N_S independent draws of ϵ_S of length $T = 600$. We then fit an AR(1)-process to the resulting $\bar{\epsilon}_{S,t}$-process. Not surprisingly, $\rho_{\bar{\epsilon}_S} \approx \rho_I = \rho_S$. We fix it at the latter value. This additional aggregate state is then integrated over by Gauss-Hermitian integration, which is facilitated by the fact that the $\bar{\epsilon}_{S,t}$-process is independent of the aggregate technology process (by assumption).

We choose two different values for N_S. First, 400, which roughly equals the number of 3-digit SIC-87 sectors in the U.S. (395). Since, however, sectors are of very different size and overall importance, and also often correlated, we decrease, secondly, N_S to 100 for robustness reasons. The resulting $\sigma_{\bar{\epsilon}_S}$ are 0.0030 and 0.0060, respectively. Notice that in both cases $\sigma_{\bar{\epsilon}_S}$ is considerably smaller than σ_A, so that we should not expect too large an effect from this additional aggregate uncertainty.

In order to make the computation viable, we have to scale down the numerical specification of the computation, in particular the grid lengths: $n_k = 20$, $n_{\bar{k}} = 7$, and $n_z = 7$. The grid length for the additional aggregate shock is also 7, equi-spaced, between $[-0.03, 0.03]$ for $N_S = 100$, and $[-0.015, 0.015]$ for $N_S = 400$. We use 3 nodes for both continuous aggregate shocks in the Gauss Hermitian integration. We also check that these numerical changes as such do not affect the results too much in the original simplified computations.

The following table shows the aggregate and sectoral standard deviations for investment rates for the frictionless model and the model for $\chi = 0.5$. The sectoral standard deviations are shown as a weighted average (the unweighted averages are only insignificantly different) both for the raw sectoral investment rates and the residual sectoral investment rates (see section B.2).

Frictions:	GE	GE	GE and micro	GE and micro
Number of sectors:	100	400	100	400
Aggr. St.dev.	0.0095	0.0079	0.0078	0.0075
Sect. St.dev. - raw	0.2037	0.2050	0.0196	0.0196
Sect. St.dev. - res.	0.2033	0.2047	0.0180	0.0180

The first important observation is that the numbers obtained here are not much different from what we have obtained in the simplified computation, which is in particular true for the $\chi = 0.5$-model. Specifically, the frictionless model continues to fail to match observed sectoral volatility by an order of magnitude. Secondly, the numbers deviate in the expected direction: the aggregate standard deviation increases, because there is an additional aggregate shock, but only slightly so; the sectoral standard deviations decrease a little bit, because now general equilibrium forces contribute also to sectoral smoothing.

And, most importantly, the numbers show that the results obtained in the main part of the paper are biased in favor of the frictionless model, in particular if we look at the $N_S = 100$ case. Following our original calibration, in this case σ_A would have to be decreased below its current value to match observed aggregate volatility of investment rates, but then in the $\chi = 0.5$-model, the calibrated σ_C would have to be even lower, thus placing an even lower weight on general equilibrium forces.

D Matching Establishment Statistics

One argument we used in the main text to justify the use of sectoral rather than plant level data to calibrate micro frictions was that in nonlinear models small changes at the micro level can lead to large differences after aggregating across micro units, before general equilibrium forces set in, and hence it is better to calibrate at a more robust level of aggregation. We provided the Caplin and Spulber (1987) model as an example of the complex mapping from micro frictions to aggregate smoothing due to them.

The other, closely related, argument we used is that there are many determinants of plant level moments which are irrelevant for the macro dimensions we are concerned with, and hence do not seem to be fruitful moments to base a macro model on. In this appendix we provide support to this claim by showing in a model that matches sectoral and aggregate moments, that minor modifications of the micro underpinnings of the model can lead to a satisfactory match of establishment level moments as well. Furthermore, in the simple extension we propose, the initial match of sectoral and aggregate moments is unaffected by the extension.

D.1 A Simple Extension

A first choice we need to make when matching the model to micro data is to decide how many micro units in the model correspond to one establishment. Choices by other authors have covered a wide range, going from one to a number large enough—sometimes a continuum—so that adding additional units makes no difference.[36]

Two additional issues arise if we choose to model an establishment as the aggregation of many micro units. First, we must address the extent to which shocks—both productivity and adjustment costs—are correlated across units within an establishment.[37] Second, we must take a stance on the fact that establishments sell off and buy what in our model corresponds to one or more micro units.

Next we present a simple model that incorporates the issues mentioned above. The economy is composed of sectors (indexed by s), which are composed of establishments (indexed by e), which are composed of units (indexed by u). The log-productivity shock faced by unit u in establishment e in sector s at time t is decomposed into aggregate, sectoral, establishment and unit level shocks as follows:

$$\log z_{uest} = \log \varepsilon_t^A + \log \varepsilon_{st}^S + \log \varepsilon_{est}^F + \log \varepsilon_{uest}^U,$$

[36] Cooper and Haltiwanger (2005) and Khan and Thomas (2005) are examples of the former; Abel and Eberly (2002) and Bloom (2005) of the latter.

[37] For tractability, all models assume that decisions are made at the micro-unit level, not the establishment level.

where $\log\varepsilon_t^A \sim \text{AR}(1;\rho_A,\sigma_A)$, $\log\varepsilon_{st}^S \sim \text{AR}(1;\rho_S,\sigma_S)$, $\log\varepsilon_{est}^F \sim \text{AR}(1;\rho_E,\sigma_E)$ and $\log\varepsilon_{uest}^U \sim \text{AR}(1;\rho_U,\sigma_U)$.[38,39] Consistent with the assumptions we made in the paper, we assume $\rho_S = \rho_E = \rho_U$ and denote the common value by ρ.

An establishment is composed of a large number (continuum) of units. The extent to which the behavior of units within an establishment is correlated will depend on the relative importance of σ_U and σ_E. The larger σ_E, the larger the correlation of productivity shocks across units within an establishment and the more coordinated their investment decisions will be. For simplicity we assume that the adjustment costs drawn by units belonging to an establishment are independent, so that even if units' productivity shocks are perfectly correlated, there is some heterogeneity in units' behavior.

The sectoral and aggregate investment series generated by this model will be the same as those generated by the model we developed in the main text as long as $\sigma_E^2 + \sigma_U^2 = \sigma_I^2$, since all we are doing in this extension is grouping micro units into groups we call "establishments" which has no implication for sectoral aggregates.[40] We therefore can decompose σ_I^2 into the sum of σ_U^2 and σ_E^2 as we please, without affecting sectoral and aggregate statistics of our model. We define $\zeta \in [0,1]$ via $\sigma_U^2 = \zeta\sigma_I^2$, so that $\sigma_E^2 = (1-\zeta)\sigma_I^2$. Productivity shocks are the same across units within a establishment when $\zeta = 0$, their correlation decreases as ζ increases.

Regarding the sale and purchase of micro units, we assume that in every period an establishment with capital K_{est} suffers a sales/purchase shock τ_{est}, so that its capital becomes $(1+\tau_{est})K_{est}$. The τ's are i.i.d. draws from a zero mean normal distribution with standard deviation σ_τ. Since the sectors in our model are composed of a continuum of establishments, our choice of a distribution with zero mean for purchase/sales shocks ensures that sectoral and aggregate statistics are unaffected by this extension as well. We choose a normal distribution because it incorporates only one additional parameter (parsimony) and it is symmetric (thus any asymmetries in the histogram of investment rates cannot be attributed to this choice).

We denote by $\tilde{i}_{est}$ the investment rate for a given establishment according to our model, and by i_{est} the corresponding investment rate recorded by the LRD. The latter differs from the former in that it includes the sale/purchase of units from other establishments, which is ignored in our original model. We then have:

$$i_{est} = (1-\tau_{est})\tilde{i}_{est} - \tau_{est}(1-\delta). \tag{19}$$

Summing up, our (admittedly simple) extension introduces two parameters over which we can optimize to fit establishment level moments without affecting the match of sectoral and aggregate statistics. These parameters are the degree to which productivity shocks are correlated across units within an es-

[38] $x_t \sim \text{AR}(1;\rho,\sigma)$ means that the process x_t follows an AR(1) with first order autocorrelation ρ and standard deviation of innovations equal to σ.

[39] Sectoral innovations are independent across sectors and independent from the innovations of the aggregate shock. Establishment level innovations are independent across establishments and independent from the innovations of the aggregate and sectoral shocks. Finally, unit level innovations are independent across units and independent from the innovations of the aggregate, sectoral and establishment-level shocks.

[40] The assumption that investment decisions are made at the unit level—and not at the establishment level—is important here. Remember that our objective here is not to add realism to our original model, it is to show that matching micro moments isn't a robust way of pinning down microeconomic parameters.

tablishment, and the average magnitude of sales and purchases of micro units across establishments.

D.2 Matching Establishment Level Statistics

We work with $\chi = 0.50$. For a fixed value of ζ, we generate a histogram with 2,500 realizations of establishment level I/K using our model.[41]

Denote by f_i, $i = 1,\ldots,5$ the fraction of LRD establishments that adjusted less than −20%, between −20 and −1%, between −1% and 1%, between 1 and 20% and above 20%, respectively. And denote by $\pi_i(\sigma_\tau)$ the fraction of units with adjustment in the previous bins after applying the transformation described in (19). We choose the value of σ_τ that minimizes $\sum_i |f_i - \pi_i(\sigma_\tau)|/f_i$, that is, we minimize the absolute relative error.

Table 10 presents our results. We present the estimated values for the five statistics considered in the extension Khan and Thomas (2005) introduce to obtain a better match of LRD moments. As can be seen, our model does a reasonable job matching the micro statistics which have been considered earlier in the literature. In fact, our fit is similar to the one Khan and Thomas (2005) obtain when they extend their model to fit the micro statistics. Also, the statistics we obtain vary rather little with ζ, as long as ζ is larger than zero (say, above 0.1). We report our estimates for $\zeta = 1$ and $\zeta = 0.5$ (the corresponding values for σ_τ are 0.134 and 0.133, respectively).

Table 10: MATCHING LRD MOMENTS

Model	$\lvert I/K\rvert < 1\%$	$I/K > 20\%$	$I/K < -20\%$	$I/K \geq 1\%$	$I/K \leq -1\%$
Data	*8.2*	*18.7*	*1.9*	*80.9*	*10.9*
Khan-Thomas extension:	4.8	18.0	1.5	72.0	23.2
Our model extension ($\zeta = 1$):	4.8	20.1	1.9	70.7	24.5
Our model extension ($\zeta = 0.5$):	4.8	20.3	1.9	70.6	24.6

E Decomposing PE- vs. GE-smoothing

This section describes how we decompose the relative contributions of smoothing by PE- and GE-forces.

We first remove both smoothing from adjustment costs as well as GE-smoothing from the model, by fixing the intertemporal price and the real wage at their average values, the resulting model has no sources of smoothing (NONE). Next, we introduce micro frictions and aggregate across units (PE), and then also include GE-smoothing through market prices (BOTH). We also consider the case with general equilibrium smoothing without micro frictions (GE).

The first four columns in Table 11 report the standard deviation of aggregate investment rates for all possible combinations of sources of smoothing. The last column reports upper and lower bounds,

[41] We compute these investment rates using the approximation described in Appendix C.3 with $\sigma_S^2 + \sigma_E^2$ in the role of σ_S^2, and $\sigma_I^2 - \sigma_E^2$ in the role of σ_I^2.

UB, LB, on the relative importance of PE-smoothing in the various models, as measured by:

$$UB = \log[\sigma(\text{NONE})/\sigma(\text{PE})]/\log[\sigma(\text{NONE})/\sigma(\text{BOTH})],$$
$$LB = 1-\log[\sigma(\text{NONE})/\sigma(\text{GE})]/\log[\sigma(\text{NONE})/\sigma(\text{BOTH})]$$

In the case of our model, the importance of PE-smoothing increases with χ. This is consistent with our discussion in Section 4, since the cross-section of mandated investment for $\chi = 0$ is closest to that in the Caplin and Spulber (1987) setting with no smoothing via partial equilibrium aggregation. Yet even for $\chi = 0$ we have that the midpoint of the interval defined by the lower and upper bound for the fraction explained by micro smoothing is almost 60%.

Table 11: SMOOTHING DECOMPOSITION

Model	Sources of smoothing				PE/total smoothing	
	None	PE	GE	PE + GE	Lower bd.	Upper bd.
Khan-Thomas-Lumpy (2005):	0.1050	0.0660	0.0080	0.0080	0.0%	18.0%
Our model (0 maint.):	0.0458	0.0096	0.0133	0.0074	32.2%	85.7%
Our model (25% maint.):	0.0458	0.0094	0.0138	0.0074	34.2%	86.9%
Our model (50% maint.):	0.0458	0.0091	0.0148	0.0074	38.0%	88.7%
Our model (75% maint.):	0.0458	0.0089	0.0159	0.0074	42.0%	89.9%
Our model (100% maint.):	0.0458	0.0083	0.0236	0.0074	63.6%	93.7%

It becomes apparent that in this metric the lumpy model put forth by Khan and Thomas (2005) is very close to a frictionless model already in its PE set up, so that their irrelevance result for the aggregate becomes somewhat less surprising.

F The Responsiveness Index

Given an economy characterized by μ_t and aggregate productivity level z_t we denote the resulting aggregate investment rate by $\frac{I}{K}(\mu_t, \log z_t)$ and define

$$\mathscr{I}^+(\mu_t, \log z_t) \equiv \left[\frac{I}{K}(\mu_t, \log z_t + \sigma_A) - \frac{I}{K}(\mu_t, \log z_t)\right]/\sigma_A,$$
$$\mathscr{I}^-(\mu_t, \log z_t) \equiv \left[\frac{I}{K}(\mu_t, \log z_t - \sigma_A) - \frac{I}{K}(\mu_t, \log z_t)\right]/(-\sigma_A),$$

where σ_A is the standard deviation of the aggregate innovation.

Following Caballero and Engel (1993) we define the Responsiveness Index $F(\mu_t, \log z_t)$ for $\frac{I}{K}$ as:

$$F_{k,t} \equiv 0.5(1-\theta-\nu)\left[\mathscr{I}^+(\mu_t, \log z_t) + \mathscr{I}^-(\mu_t, \log z_t)\right]. \tag{20}$$

The factor $(1-\theta-\nu)$ is included so that the index is approximately one when no sources of smoothing are present. More precisely, in a static, partial equilibrium setting, with no time-to-build, micro units solve:[42]

$$\max_{k,n} \; zk^{\theta}n^{\nu} - \omega n - k.$$

Solving this problem leads to the following optimal capital target level as a function of aggregate technology:

$$k^* = Cz^{1/(1-\theta-\nu)},$$

where C is a constant that depends on the wage and the technology parameters. Taking logs and first differences leads to

$$\Delta \log k^* = \frac{1}{1-\theta-\nu}\Delta \log z,$$

thereby justifying the normalization constant.

G Robustness to Variations in the Maintenance Parameter

In this appendix we show that the results reported for our model in Section 5 vary little with the choice of the maintenance parameter χ. Hence our conclusions are robust to having considered only the case $\chi = 0.50$ in that section. The tables below present the volatility measures, persistence measures, and J-statistic for values of χ between 0 and 100%.

Table 12: VOLATILITY OF AGGREGATES AND χ

	St.dev.				St.dev. rel. to $\sigma(Y)$		
	Y	C	I	N	C	I	N
Data:	2.00	1.73	5.94	2.00	0.86	2.97	1.00
Frictionless:	1.40	0.65	6.61	0.85	0.47	4.71	0.61
King-Rebelo:	1.39	0.61	4.09	0.67	0.44	2.95	0.48
0% maint.:	2.06	1.51	5.91	1.91	0.73	2.87	0.93
25% maint.:	2.09	1.54	5.90	1.95	0.73	2.82	0.93
50% maint.:	2.15	1.60	5.85	2.01	0.75	2.73	0.94
75% maint.:	2.22	1.67	6.10	2.09	0.75	2.75	0.94
100%-maint.:	2.42	1.93	6.81	2.37	0.80	2.81	0.98

[42] For notational simplicity we leave out idiosyncratic and sectoral shocks.

Table 13: PERSISTENCE OF AGGREGATES AND χ

	Y	C	I	N	I/K
Data:	0.53	0.58	0.47	0.52	0.71
Frictionless:	0.42	0.61	0.36	0.35	0.57
0% maint.:	0.46	0.51	0.43	0.45	0.68
25% maint.:	0.46	0.51	0.43	0.46	0.69
50% maint.:	0.47	0.52	0.43	0.47	0.69
75% maint.:	0.48	0.56	0.40	0.47	0.65
100%-maint.:	0.50	0.62	0.36	0.50	0.56

Table 14: J-STATISTICS AND χ

	(1)	(2)
Frictionless:	53.3	44.7
0% maint.:	22.2	2.0
25% maint.:	25.6	1.8
50% maint.:	30.5	1.9
75% maint.:	28.9	1.3
100%-maint.:	28.2	9.7
Critical value:	11.1	7.8