Value-at-Risk for long and short trading positions: The case of the Athens Stock Exchange

by

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Abstract

This paper provides Value-at-Risk estimates for daily stock returns with the application of various parametric univariate models that belong to the class of ARCH models which are based on the skewed Student distribution. We use daily data for three stock indexes of the Athens Stock Exchange (ASE) and three stocks of Greek companies listed in the ASE. We conduct our analysis with the adoption of the methodology suggested by Giot and Laurent (2003). Therefore, we estimate an APARCH model based on the skewed Student distribution to fully take into account the fat left and right tails of the returns distribution. We show that the estimated VaR for traders having both long and short positions in the Athens Stock Exchange is more accurately modeled by a skewed Student APARCH model that by a normal or Student distributions.

Keywords: Value-at-Risk, risk management, APARCH models, skewed Student distribution

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1. Introduction

During the recent years the importance of effective risk management has become extremely crucial. This is the outcome of several significant factors. First, the enormous growth of trading activity that has been taking place in the stock markets, especially those of the emerging economies. Second, the financial disasters that took place in the 1990s that have led to bankruptcy well-known financial institutions. These events have put great emphasis for the development and adoption of accurate measures of market risk by financial institutions. Financial regulators and supervisory committee of banks have favoured quantitative risk techniques which can be used for the evaluation of the potential loss that financial institutions can suffer. Furthermore, given that the nature of these risks changes over time effective risk management measures must be responsive to news such as other forecasts as well as to be easy understood even in complicated cases.

We have observed a substantial increase in financial uncertainty as a result of the increased volatility that was observed in the stock returns of the mature markets but mainly of those of the emerging markets. This was the outcome of the increased flow of portfolio capital from the mature markets to the emerging markets of the South East Asia and the economies of transition of Central and Eastern European countries. Singh and Weisse (1998) report that during the period 1989-1995 the inflow of funds in emerging markets amounted to 107.6 billion US dollars as opposed to a mere 15.1 billion US dollars in the previous period 1983-1988. There are several reasons for these enormous inflow of portfolio funds to the emerging markets but certainly the most important was the fact that during the 1990s the mature markets has reached their limitations with respect to profit opportunities and made portfolio managers and institutional investors to look for new opportunities in these new markets.

Furthermore, the financial crisis of 1997-1998 as well as the bankruptcy of several financial institutions such as the BCCI and Barrings international banks has led to the increased price volatility and financial uncertainty. Such financial uncertainty has increased the likelihood of financial institutions to suffer substantial losses as a result of their exposure to unpredictable market changes. These events have made investors to become more cautious in their investment decisions while it has also led for the increased need for a more careful study of price volatility in stock markets. Indeed, recently we observe an intensive research from academics, financial institutions and regulators of the banking and financial sectors to better understanding the operation of capital markets and to develop sophisticated models to analyze market risk.

Value-at-Risk has become the standard tool used by financial analysts to measure market risk. VaR is defined as a certain amount lost on a portfolio of financial assets with a given probability over a fixed number of days. The confidence level represents 'extreme market conditions' with a probability that is usually taken to be 99% or 95%. This implies that in only 1% (5%) of the cases will lose more than the reported VaR of a specific portfolio. VaR has become a very popular tool among financial analysts which is widely used because of its simplicity. Essentially the VaR provides a single number that represents market risk and therefore it is easily understood.[1]

During the last decade several approaches in estimating the profit and losses distribution of portfolio returns have been developed and a substantial literature of

[1] See also Bank for International Settlements (1988, 1999a,b,c, 2001).

empirical applications have emerged. However, most of these models have focused on the computation of the VaR on the left tail of the distribution which corresponds to the negative returns. This implies that it is assumed that portfolio managers or traders have long trading positions, which means that they bought an asset at a given price and they are concerned with the case that the price of this asset falls resulting in losses.

The present paper deals with modeling VaR for portfolios that includes both long and short positions. Therefore, we consider the modeling and calculation of VaR for portfolio managers who have taken either a long position (bought an asset) or a short position (sold an asset). As it is well known, in the former case the risk of a loss occurs when the price of the traded asset falls, while in the later case the trader will incur a loss when the asset price increases.[2] Therefore, in the first case we model the left tail of the distribution of returns and in the second case we model the right tail of the distribution.

Given the stylized fact that the distribution of asset returns is nonsymmetric, recently, Giot and Laurent (2003) have shown that models which rely on a symmetric density distribution for the error term underperform with respect to skewed density models when the left and right tails of the distribution of returns must be modeled. This implies that VaR for portfolio managers or traders who hold both long and short positions cannot be accurately modeled by the application of the standard normal and Student distributions. Giot and Laurent (2003) also show that similar problems arise when we try to model the distribution with the asymmetric GARCH models which assumes that there is an asymmetry exists between the conditional variance and the lagged squared error term, (see also El Babsiri and Zakoian, 1999).

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[2] Sharpe *et al*. (1999) provide a comprehensive analysis of trading strategies.

To take into account these disadvantages, we apply the univariate Student Asymmetric Power ARCH (APARCH) model introduced by Ding *et al*. (1993) in order to model and calculate the VaR for portfolios defined on long position (long VaR) and short position (short VaR). The performance of this model is compared with those of the standard parametric Riskmetrics and normal and Student APARCH models.

We apply our methodology to portfolios for long and short positions on daily stock indexes (General, Banking, Industrial) and daily stocks of companies which re traded in an emerging stock market the Athens Stock Exchange. VaR models have mainly applied to evaluating positions taken in the mature stock markets. However, the recent enormous trading activity that took place in the emerging markets and the negative effects of the Southeast Asia financial crisis in 1997 have increased the need for a closer look in modeling the volatility of returns of these markets and more importantly to model VaR for portfolios on long and short positions which are mainly constructed from stocks which are traded in emerging markets. Thus, we focus on the joint behaviour of VaR models for long and short trading positions.

The main finding of our analysis is that the skewed Student APARCH improves considerably the forecasts of one-day-ahead VaR for long and short trading positions. Additionally, we evaluate the performance of each model with the calculation of Kupie’s (1995) Likelihood Ratio test on the empirical failure test. Moreover, for the case of the skewed Student APARCH model we compute the expected shortfall and the average multiple of tail event to risk measure. These two measures help us to further assess the information we obtained from the estimation of the empirical failure rates.

The remainder of the paper is organized as follows. Section 2 presents the VaR models used in this analysis. In section 3 we report our empirical results and finally section 4 provides our concluding remarks.

2. VaR models

In this section we follow Giot and Laurent (2003) and provide a brief description of the four models used in the analysis. The starting point is the definition of the conditional mean and variance of the disturbance term which is relevant for all alternative VaR specifications. Therefore, we consider a series of daily returns, y_t, with $t = 1...T$. In order to take into account the serial correlation that daily returns exhibit as it is well known we fit an $AR(n)$ model on the y_t series:

$$\Phi(L)(y_t - \mu) = \varepsilon_t \tag{1}$$

where $\Phi(L) = 1 - \phi_1 L - .....\phi_n L^n$ is defined as an AR lag polynomial of order n. Thus, the conditional mean of y_t, i.e. μ_t, is equal to $\mu + \sum_{j=1}^{n} \phi_j (y_{t-j} - \mu)$. The crucial issue in VaR modeling is the specification that the conditional variance takes. As we have already mentioned in the present paper we consider for models with corresponding conditional variance specification, namely, Riskemetrics, Normal APARCH, Student APARCH and skewed Student APARCH.[3] The performance of each model is based on how well it can predict long VaR trading positions (i.e. to model large negative returns) while with respect to the right tail of the distribution of returns the predictive performance of of short VaR is evaluated by its ability to model large positive returns.

[3] Jorion (2000) and Alexander (2003) provide a complete analysis of the VaR methodology and alternative estimation methodologies

2.1. Riskmetrics

J.P. Morgan's Riskmetrics (1996) model combines an econometric model with the assumption of conditional normality for the returns series. Specifically, this model rely on the specification of the variance equation of the portfolio returns and the assumption that the standardized errors are i.i.d.. In this model the autoregressive parameter is pre-specified at given value λ whereas the coefficient of ε_{t-1}^2 equals to $1-\lambda$. For the case of daily data, $\lambda = 0.94$ and we then obtain:

$$\varepsilon_t = \sigma_t z_t \tag{2}$$

where the standardized error z_t is i.i.d $N(0,1)$ and the variance σ^2 is defined as:

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \tag{3}$$

Then the one-step-ahead VaR forecast computed in $t-1$ for the case of long positions is calculated by $\mu_t + z_a\sigma_t$, and for the short position is calculated by $\mu_t + z_{1-a}\sigma_t$, with α chosen to be a standard level of significance.[4] Since $z_\alpha = -z_{1-\alpha}$ the forecasted long and short VaR will be equal.

2.2. *Normal* APARCH

The normal APARCH developed by Ding *et al*. (1993) is an extension of the GARCH model, (Bollerslev; 1986). The advantage of this class of models is its flexibility since it includes a large number of alternative GARCH specifications. The APARCH (1,1) model is given by the following expression:

[4] We note that when calculating the VaR the conditional mean and variance are computed with the replacement of the unknown parameters in equation (1) with their MLE estimates.

$$\sigma^2 = \omega + \alpha_1(|\varepsilon_{t-1}| - \alpha_n\varepsilon_{t-1})^{\delta} + \beta_1\sigma_{t-1}^{\delta} \quad (4)$$

where $\omega, \alpha_1, \alpha_n, \beta_1$ and δ are parameters to be estimated in addition to μ_t and σ_t. The term $\alpha_n(-1 < \alpha_n < 1)$, represents the leverage effect, while the coefficient $\delta(\delta > 0)$ is a Box-Cox transformation of σ_t.[5] He and Terasvista (1999a,b) provide a thorough analysis of the properties of the APARCH model.

The one-step-ahead VaR forecast for the normal APARCH is computed with the same way as for the Riskmetrics model with the only difference that the conditional variance is given by equation (4).[6]

2.3. Student APARCH

It has been well documented in the finance literature that that models which rely on the assumption that the distribution of returns follows the normal one fail to take into account the fat tails of the distribution of results leads to the underestimation of the VaR. This underestimation can be corrected by allowing alternative distributions of the errors such as the Gaussian, *Student's-t* and Generalized Error Distribution. The adoption of the Student APARCH (ST APARCH) is a potential solution to the problem. The specification of errors is given by:

$$\varepsilon_t = \sigma_t z_t \quad (5)$$

where z_t is i.i.d. $t(0,1,\upsilon)$ and σ_t is defined as in equation (4).

[5] Black (1976), French *et al*. (1987) and Pagan and Schwert (1990) among others suggest that the leverage effect means that a positive (negative) value of α_n implies that the past negative (positive) shocks have a deeper impact on current conditional volatility than past positive shocks.

[6] As before σ_t is evaluated at its MLE.

The one-step-ahead VaR for long and short positions is given by $\mu_t + st_{\alpha,\upsilon}\sigma_t$ and $\mu_t + st_{1-\alpha,\upsilon}\sigma_t$, with α chosen to be a standard level of significance.[7]

2.4. Skewed Student APARCH

Recently, Fernandez and Steel (1998) have extended the student distribution with the addition of a skewness parameter to take into consideration the problems of skewness and kurtosis detected in financial databases. This has led to the development of the skewed student APARCH. However, their approach has the disadvantage that this proposed skewness parameter is expressed in terms of the mode and the dispersion. To avoid this deficiency Lambert and Laurent (2001) have re-expressed the skewed student density in terms of the mean and the variance by a re-parameterization of the density so that the innovation process has zero mean and unt variance.[8]

We draw mainly on Giot and Laurent (2003) and we provide a discussion of the statistical properties of the skewed Student APARCH model based on the approach suggested by Lambert and Laurent (2001).

The innovation process z_t is distributed according to the standardized skewed Student distribution if:

[7] As in the case of the normal of distribution, since $st_\alpha = -st_{1-\alpha}$ the forecasted long and short VaR will be equal.

[8] Hansen (1994) argues that this is necessary otherwise we are unable to discriminate between the fluctuations occurred in the mean and variance from the fluctuations occurred in the shape of the conditional density.

$$f(z \mid \xi, \upsilon) = \begin{cases} \dfrac{2}{\xi + \frac{1}{\xi}} sg[sz + m) \mid \upsilon] \text{ if } z < -\dfrac{m}{s} \\ \dfrac{2}{\xi + \frac{1}{\xi}} sg[(sz + m)/\xi \mid \upsilon] \text{ if z} \geq -\dfrac{m}{s} \end{cases} \quad (6)$$

where $g(.|\upsilon)$ is the symmetric (unit variance) Student density and ξ is the asymmetry coefficient. In addition, m and s^2 are respectively the mean and the variance of the non-standardized skewed Student:

$$m = \frac{\Gamma(\frac{\upsilon - 1}{2})\sqrt{\upsilon - 2}}{\sqrt{\pi}\Gamma(\frac{\upsilon}{2})}(\xi - \frac{1}{\xi}) \quad (7)$$

and

$$s^2 = (\xi^2 + \frac{1}{\xi^2} - 1) - m^2 \quad (8)$$

where the density function $f(z_t \mid 1/\xi, \upsilon)$ is the opposite of $f(z_t \mid \xi_, \upsilon)$ with respect to the zero mean. Thus, the sign of $\log(\xi)$ gives an indication of the direction of skeweness, i.e. the skewness factor (m_3) is positive (negative), and the probability density function is skewed to the right (left), if $\log(\xi) > 0 (< 0)$.

Moreover, Lambert and Laurent (2000) show that the quantile function of $skst^*_{\alpha,\upsilon,\xi}$ of a non standardized skewed Student density is:

$$skst^{*}_{\alpha,\upsilon,\xi} = \begin{cases} \frac{1}{\xi} st_{a,u}[\frac{\alpha}{2}(1+\xi^2) \text{ if } \alpha < \frac{1}{1+\xi^2} \\ -\xi st_{\alpha,\upsilon}[\frac{1-\alpha}{2}(1+\xi^{-2})] \text{ if } \alpha \geq \frac{1}{1+\xi^2} \end{cases} \qquad (9)$$

where $sk_{\alpha,\upsilon}$ is the quantile function of the (unit variance) Student-*t* density. Then we obtain the quantile function of the standardized skewed Student distribution as follows:

$$skst_{\alpha,\upsilon,\xi} = \frac{skst_{\alpha,\upsilon,\xi} - m}{s}$$

Following Ding *et al*. (1993), if it exists, a stationary solution of equation (4) is given by:

$$E(\sigma_t^{\delta}) = \frac{\omega}{1-\alpha_1 E(|z| - \alpha_n z)^{\delta} - \beta_1} \qquad (10)$$

which is a function of the density of z. Such a solution exists if

$$V = \alpha_1 E(|z| - \alpha_n z)^{\delta} + \beta_1 < 1$$

Ding et al. (1993) derived the expression for $E(|z| - \alpha_n z)^{\delta}$ for the Gaussian case. We can also show that for the standardized skewed Student distribution is given as follows:

$$E(|z| - \gamma z)^{\delta} = \{\xi^{-(1+\delta)}(1+\gamma)^{\delta} + \xi^{1+\delta}(1-\gamma)^{\delta}\} \frac{\Gamma(\frac{\delta+1}{2})\Gamma(\frac{\upsilon-\delta}{2})(\upsilon-2)^{\frac{1+\delta}{2}}}{(\xi+\frac{1}{\xi})\sqrt{(\upsilon-2)\pi}\,\Gamma(\frac{\upsilon}{2})} \qquad (11)$$

For the skewed Student APARCH model, the VaR for long and short positions is given by $\mu_t + skst_{\alpha,\upsilon,\xi}\sigma_t$ and $\mu_t + skst_{1-\alpha,\upsilon,\xi},\sigma_t$. $skst_{\alpha,\upsilon,\xi}$ $(skst_{1-\alpha,\upsilon,\xi})$ is the left(right) quantile of the skewed Student distribution at level of significance $\alpha\%$ $(1-\alpha\%)$ with υ degrees of freedom whereas ξ is the asymmetry coefficient. If $\log(\xi)$ is smaller than zero $(or\ \xi < 1)$ then $|skst_{\alpha,\upsilon,\xi}| > |skst_{1-\alpha,\upsilon,\xi}|$ and in this case the VaR for long trading positions will be larger (for the same conditional variance) than the VaR for the short position. When $\log(\xi)$ is positive the opposite situation arises.

3. Empirical results

We apply the alternative Value at Risk model specifications on daily returns. The data set refers to three stock market indexes of the Athens Stock Exchange (ASE), namely, GENERAL, BANKING and INDUSTRIAL (1/1/1988-4/11/2004-4190 observations) and three stocks (blue chips) of Greek companies which are traded in the ASE, namely COCA COLA (2/1/1998-4/11/2004-1707 observations, MIHANIKI(14/1/1997-4/11/2004-1947 observations), and MOUZAKIS (14/1/1997-4/11/2004-1947 observations) and it was obtained from Datastream. We follow this strategy in order to investigate the performance of the VaR measures of market risk for the case of stocks traded in an emerging market. In order to implement our analysis we construct historical portfolios for each case and we choose a specification of the functional form of the distribution of returns. We successively consider the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH. The daily returns are computed as 100 times the difference of the log of the prices, i.e. $y_t = 100[\ln(p_t) - \ln(p_{t-1})]$.

Table 1 reports descriptive statistics for the returns series. We clearly observe that all six return series display similar statistical properties with respect to skewness and kurtosis. Thus, the return series are skewed (either negatively or positively) whereas the large returns (either positive or negative) lead to a large degree of kurtosis. Furthermore, The Lung-Box Q^2 statistics for all returns series are statistically significant, providing evidence of strong second-moment dependencies (conditional heteroskedasticity) in the distribution of the stock price changes.

Figures 1-6 provides descriptive graphs (level of price series, daily returns, density of the daily returns vs. normal and QQ-plots against the normal distribution) for each daily returns series. The density graphs and the QQ-plots the normal distribution show that all the distributions of returns exhibit fat tails. Furthermore, the QQ-plots imply that there is an asymmetry in the fat tails. An additional result of these graphical expositions show that the six return series exhibit volatility clustering, which means that there are periods of large absolute changes tend to cluster together followed by periods of relatively small absolute changes.

Given these salient features of the daily returns for three indexes of ASE as well as three stocks of Greek companies listed in ASE we now move to perform the VaR analysis based on the four chosen models. Table 2 reports the results for the (approximate maximum likelihood) estimation of the skewed Student APARCH model on all six daily return series.[9] The calculated Ljung-Box Q^2-statistic is not significant (except for the Coca Cola stock) and this implies that the skewed Student APARCH model is successful in taking into account the conditional heteroskedasticity exhibited by the data. Furthermore, it is shown that the autoregressive coefficient in the volatility specification β_1 takes values between 0.72

[9] All computations were performed with G@RCH 3.0. procedure on Ox package (see also Laurent and Peters, 2002).

to 0.93 suggesting that there are substantial memory effects. The coefficient α_n is positive and statistically significant for all series, indicating the existence of a leverage effect for negative returns in the conditional variance specification. The next important result concerns the value of $\log(\xi)$, which is positive in all six case and this result implies we were correct in incorporating the asymmetry element in the Student distribution in order to model the distribution of returns in an appropriate way. The final significant result reported in Table 1 refers to the value of δ which takes values from 0.815 and 1.537 statistically significant from 2.[10]

The above results indicate that the skewed Student APARCH model takes into consideration the feature of a negative leverage effect (conditional asymmetry) for the conditional variance as well as with the fact that the existence of an asymmetric distribution for the error term (unconditional asymmetry).

We next move to examine whether the skewed Student APARCH model provides better VaR estimates and forecasting performance than the other three models, Riskemetrics, normal APARCH and Student APARCH. To this end we move on to provide in-sample VaR computations and this is accomplished by computing the one-step-ahead VaR for all models. This procedure is equivalent to backtesting the model on the estimation sample. We test all models with a VaR level of significance, (α), that takes values from 0.25% to 5% and we then evaluate their performance by calculating the failure rate for the returns series y_t. The failure rate is defined as the number of times returns exceed the forecasted VaR. Following Giot and Laurent (2003) we define a failure rate f_l for the long trading positions, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions. In

[10] The fact that for all six series the value of δ is not statistically significant different from 1 suggest that instead of modeling the conditional variance is better to model the conditional standard deviation.

a similar manner, we define f_s as the failure rate for short positions as the percentage of positive returns larger than the one-step-ahead VaR for short position.[11]

To evaluate the in-sample forecasting ability of the alternative VaR measures we employ the unconditional backtesting criterion developed by Kupiec (1995). This criterion tests the hypothesis that the proportion of violations (failures) is equal to the expected one.[12] Under the null hypothesis Kupiec (1995) developed a likelihood ratio statistic given as follows:

$$LR_{uc} = 2\ln[1-\hat{f}]^{T-N} fN] - 2\ln[(1-f)^{T-N} f^{N}\} \sim \chi_1^2 \tag{12}$$

where $f = N/T$ is the failure rate, $\hat{f}$ is the empirical (estimated) failure rate, N is the number of days over a period T that a violation has occurred. Giot and Laurent (2003) suggest that the computation of the empirical failure rate defines a sequence of yes/no, under this testable hypothesis. Table 3 reports the corresponding p-values for the four VaR models and for given significance levels.

Table 4 reports the full results for the three indices as well as the three individual stocks. These results clearly lead to the conclusion that the models which assume the normal distribution for the returns, i.e. RiskMetrics and normal APARCH, exhibit a poor performance in modelling large positive and negative returns. Moreover, we see that the use of the symmetric Student APARCH certainly leads to better results than the models based on the normality assumption but we definitely obtain the best results when the skewed Student APARCH model is applied. This

[11] When the VaR model is correctly specified then the failure rate should be equal to the pre-specified VaR level.
[12] A violation is defined as the case where the predicted VaR is unable to cover the realized loss (or to foresee the realized profit)

model improves substantially on all other specifications for both negative and positive returns.

The picture that emerges from Table 4 further reinforces the superiority of the skewed Student APARCH model over the alternative specifications. Indeed, this specification successfully models almost all VaR levels for either long or short trading positions since in only one case (Coca Cola) we get a value which is away from the 100 target. Moreover, we note that the skewed Student APARCH performs better that the student APARCH since it corrects a number of deficiencies that the latter model has inherited as a result of its conservatism.

We further assess the performance of the competing models by computing the out-of-sample VaR forecasts. This is considered as the 'true' test for any VaR model. Out-of-sample evaluation of a specific model requires the estimation of the model for the known data points and then based on the estimated equation and we then provide forecasts for a specific time horizon. This testing procedure is implemented to provide one-day-ahead VaR forecasts.[13] Following Giot and Laurent (2003) we apply an iterative procedure in which the estimated model for the whole sample is estimated and we then compare the predicted one-day-ahead VaR for both the long and short positions with the actual return. This procedure is repeated for all known observations and every time the estimation sample includes one more day and we forecast the corresponding VaR. These forecasts are saved and they are used for the evaluation of the out-of-sample predictive performance of the models.[14] The iteration procedure ends when, as it is the common practice, we have included the $t-1$ days in the

[13] Christoffersen and Diebold (2000) document that the ARCH-class of models exhibit good volatility forecastability for short horizon their performance is poor when it comes to long horizon prediction. Although the latter may be more important for portfolio managers we only provide short run analysis of predictive performance.

[14] To conduct our out-of-sample forecasting analysis we employ the last five years (1260 obs.) of our sample. We also use a 'stability window' of 50 days to update the model parameters.

estimation of the model. The predictive performance of the skewed Student APARCH model is the evaluated using the Kupiec (1995) likelihood ratio test as in the in-sample case. However, this time the failure rate was calculated for both the long and short positions by comparing the corresponding forecasted VaR_{t+1} with the observed return y_{t+1}.

The results for the six return series are given in Table 5. Like Table 3 we report the calculated p-values for alternative level of significance for both long and short trading positions. The overall conclusion is that the skewed Student APARCH model performs well for out-of-sample VaR prediction. However, a comparison with the in-sample results given in Table 3 reveals that the out-of-sample predictive ability of the model appears to be inferior which is a rather expected finding. Furthermore, we note that the combined (i.e. long and short VaR) success rate is equal to 100% for the three stock indexes and the three stocks of the ASE and this finding further reinforces the suitability of the skewed Student APARCH model in measuring market risk in this emerging market.

We complete our econometric analysis we further analyze the characteristics of normal, symmetric and the skewed Student APARCH model. This analysis refers to the calculation of two additional measures relevant to any VaR analysis, the expected shortfall and the average multiple of tail event to risk measure. First, Scaillet (2000) defines the expected shortfall measure as the expected value of the losses conditional on the loss to be larger that the calculated VaR. Second, Hendricks (1996) considers the average multiple of tail event to risk as being the degree to which events in the tail of the distribution of returns commonly exceed the VaR measure. This is accomplished with the calculation of the average multiple of these events with respect to the VaR measures.

The results for the expected shortfall measure for the three stock indexes are summarized in Table 6. Following Scaillet (2000) and Giot and Laurent (2003) we calculate this measure for the in-sample estimation. For the case of the long trading position this measure is calculated as the average of the actual returns which are smaller than the long VaR while for the case of the short trading position the expected shortfall is calculated as the average of the actual returns which are larger than the short VaR. With respect to the general, banking and industrial indexes we observe that the expected shortfall is overall smaller for the models based on the normal distribution as compared to the models based on the student distribution. A possible explanation for this outcome can be given if we consider the difference between frequencies of failures with their size. Thus, although the latter models fail less frequently the size of their failure is usually larger.

Table 7 reports the corresponding calculated values of the average multiple of tail event to risk for the three indexes of the ASE. This measure is calculated in a similar manner as the expected shortfall measure. These figures show the calculated average loss/predicted loss when the VaR model fails. Therefore, with respect to the general index and the skewed Student APARCH model the reported value of 1.44 implies that at the 5% level of significance an investor is expected to lose 1.44 the amount given by the VaR when the returns are smaller for the long VaR. In the case of the short trading position the calculate average multiple measure the corresponding figure of 1.43 implies that at the 5% level of significance one expects to gain 1.43 the amount given by the VaR when the returns are larger than the short VaR..

4. Summary and conclusions

During the last decade we have observed a substantial change in the way financial institutions evaluate risk. Faced with increased volatility of stock returns as well as with the heavy losses that banks and securities houses have experienced portfolio managers and supervising committees of financial markets have sought for a continuous improvement to potential measures of market risk. Value at Risk is one of the major tools for measuring market risk on a daily basis and is recommended by the Basel Committee on Banking Supervision and this is documented in the Basel Accord Amendment of 1996.

These models of risk management have become the standard tool for measuring internal risk management as well as for external regulatory purposes. However, most of the recent applications of evaluating VaRs for a wide range of markets have mostly applied for the case of negative returns, i.e. for the negative tail of the distribution of returns. The present paper deals with modeling VaR for portfolios that includes both long and short positions. Therefore, we consider the modeling and calculation of VaR for portfolio managers who have taken either a long position (bought an asset) or a short position (sold an asset).

This paper focused on the comparison of four alternative models for the estimation of one-step-ahead VaR for long and short trading positions. We have applied a battery of univariate tests on four parametric VaR models namely, RiskMetrics, normal APARCH, Student APARCH and skewed Student APARCH. Contrary to most of the recent applications which provide evidence on VaR evaluation for several mature markets we focus on an emerging market that of the Athens Stock Exchange. Recently, emerging markets have attracted the attention of portfolio managers and investors for their higher returns but at the same time these returns exhibit higher volatility. Therefore, modelling volatility and evaluating VaR measures for these markets is very important for market participants. Our overall results lead to the overwhelming conclusion that the skewed Student APARCH model outperforms all other specification modelling VaR for either long or short positions.

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Table 1. Descriptive statistics

	Stock indexes			Stocks		
	General	Bank	Industry	CocaCola	Mihaniki	Mouzakis
Annual s.d.	22.49	20.92	26.29	23.89	37.71	40.28
Skewness	0.14	0.39	28.63	0.06	-0.84	-2.22
Excess Kurtosis	5.32	5.86	14.12	3.83	14.79	40.72
Minimum	-10.57	-12.54	-16.51	-15.59	-42.42	-65.85
Maximum	13.75	16.58	129.78	15.03	28.45	30.11
$Q^2(10)$	803.16	826.88	2986.00	304.84	80.45	8.99

Notes: Descriptive statistics for the daily returns of the corresponding financial asset (stock index or individual stock) expressed in %. All values are computed using PcGive. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 on the squared series.

Table 2. Skewed Student APARCH

	Stock indexes			Stocks		
	General	Bank	Industry	CocaCola	Mihaniki	Mouzakis
ω	0.086(0.016)	0.148(0.031)	0.070(0.014)	0.028(0.018)	0.362(0.133)	0.268(0.118)
α_1	0.243(0.024)	0.249(0.025)	0.213(0.023)	0.088(0.023)	0.249(0.039)	0.184(0.035)
α_n	0.055(0.033)	0.022(0.034)	0.076(0.048)	0.172(0.103)	0.026(0.062)	-0.036(0.035)
β_1	0.770(0.022)	0.752(0.025)	0.799(0.022)	0.926(0.021)	0.725(0.047)	0.815(0.037)
δ	1.498(0.183)	1.537(0.196)	0.815(0.105)	1.263(0.291)	1.057(0.192)	1.350(0.225)
$\log(\xi)$	0.035(0.022)	0.050(0.022)	0.036(0.029)	0.023(0.030)	0.139(0.031)	0.018(0.025)
ν	5.322(0.423)	5.026(0.385)	4.225(0.276)	4.400(0.506)	4.329(0.506)	0.884(0.553)
V	0.966	0.956	0.952	0.991	0.909	0.958
$Q^2(10)$	5.641(0.687)	23.679(0.003)	31.948(0.022)	57.548(0.000)	0.499(0.078)	0.953(0.998)

Notes: Estimation results for the validity specification of the Skewed Student APARCH model. Standard errors are reported in parenthesis. $V = \alpha_1 E(|z| - \alpha_n z)^{\delta} + \beta_1$ while $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 on the squared series.

Table 3(a). VaR results for GENERAL, BANKING and INDUSTRIAL (in-sample)

α	5%	2.5%	1%	0.5%	0.25%
	VaR for long positions (GENERAL)				
RiskMetrics	0.862	0.096	0	0	0
N APARCH	0	0.381	0.097	0	0
ST APARCH	0.047	0.199	0.439	0.258	0.871
SKST APARCH	0.210	0.501	0.890	0.509	0.871
	VaR for long positions (BANKING)				
RiskMetrics	0.969	0.041	0	0	0
N APARCH	0	0.439	0.530	0.096	0
ST APARCH	0.066	0.165	0	0.105	0.253
SKST APARCH	0.862	0.787	0.152	0.509	0.425
	VaR for long positions (INDUSTRIAL)				
RiskMetrics	0	0	0	0	0
N APARCH	0	0.215	0.049	0	0
ST APARCH	0.022	0.011	0.052	0.508	0.640
SKST APARCH	0.054	0.108	0.474	0.868	0.847
	VaR for short positions (GENERAL)				
RiskMetrics	0.034	0	0	0	0
N APARCH	0.268	0.078	0	0	0
ST APARCH	0.219	0.116	0.889	0.990	0.299
SKST APARCH	0.857	0.365	0.438	0.835	0.871
	VaR for short positions (BANKING)				
RiskMetrics	0.116	0	0	0	0
N APARCH	0.806	0.063	0	0	0
ST APARCH	0.034	0.419	0.222	0.513	0.451
SKST APARCH	0.417	0.638	0.745	0.990	0.883
	VaR for short positions (INDUSTRIAL)				
RiskMetrics	0	0	0.037	0.040	0
N APARCH	0.340	0.114	0	0	0
ST APARCH	0.219	0.198	0.649	0.819	0.641
SKST APARCH	0.645	0.748	0.086	0.868	0.265

Table 3(b). VaR Results for COCA-COLA, MIHANIKI and MOUZAKIS (in-sample)

α	5%	2.5%	1%	0.5%	0.25%
	VaR for long positions (COCA-COLA)				
RiskMetrics	0.241	0.089	0.002	0	0
N APARCH	0.482	0.504	0.641	0.010	0
ST APARCH	0.201	0.465	0.796	0.189	0.898
SKST APARCH	0.089	0.832	0.990	0.189	0.898
	VaR for long positions (MIHANIKI)				
RiskMetrics	0.193	0.443	0.009	0	0
N APARCH	0	0.148	0.918	0.317	0.006
ST APARCH	0.012	0.037	0.068	0.356	0.686
SKST APARCH	0.814	0.812	0.568	0.688	0.618
	VaR for long positions (MOUZAKIS)				
RiskMetrics	0.776	0.443	0.156	0.065	0
N APARCH	0.233	0.325	0.569	0.317	0.363
ST APARCH	0.979	0.148	0.007	0.197	0.363
SKST APARCH	0.553	0.812	0.568	0.813	0.686
	VaR for short positions (COCA-COLA)				
RiskMetrics	0.761	0.016	0.001	0	0
N APARCH	0.414	0.207	0.004	0	0
ST APARCH	0.338	0.334	0.610	0.350	0.220
SKST APARCH	0.765	0.605	0.304	0.359	0.220
	VaR for short positions (MIHANIKH)				
RiskMetrics	0.230	0.033	0.026	0	0
N APARCH	0.328	0.364	0.042	0.008	0.002
ST APARCH	0.013	0.011	0.156	0.567	0.363
SKST APARCH	0.624	0.495	0.291	0.011	0.140
	VaR for short positions (MOUZAKIS)				
RiskMetrics	0.814	0.842	0.005	0.002	0
N APARCH	0.511	0.531	0.104	0.035	0
ST APARCH	0.230	0.957	0.738	0.813	0.950
SKST APARCH	0.856	0.196	0.007	0.197	0.362

Notes: P -values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α, top of the table) and $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α, bottom of the table). α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 4. VaR results for all stock indexes and individual stocks (in-sample)

	VaR for long positions					
	Stock Indexes			Stocks		
	GEN	BANK	IND	COCA	MIH	MOUZ
RiskMetrics	40	40	0	40	40	80
N APARCH	40	60	40	60	60	80
ST APARCH	100	80	80	100	80	100
SKST APARCH	100	100	100	80	100	100

	VaR for short positions					
	Stock Indexes			Stocks		
	GEN	BANK	IND	COCA	MIH	MOUZ
RiskMetrics	20	20	0	20	40	40
N APARCH	40	40	40	40	40	60
ST APARCH	100	100	100	100	60	100
SKST APARCH	100	100	100	100	100	80

Notes: Number of times (out of 100) that the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α, top of the table) is not rejected and $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α, bottom of the table) is not rejected for the combined five possible values of α (the level of significance is 5%). The models are successively the Riskmetrics, normal APARCH, Student APARCH and skewed Student APARCH.

Table 5. VaR results (Skewed Student APARCH, out-of-sample)

α	5%	2.5%	1%	0.5%	0.25%
		VaR for long positions			
GENERAL	0.523	0.256	0.237	0.784	0.337
BANKING	0.080	0.256	0.643	0.515	0.487
INDUSTRIAL	0.166	0.788	0.356	0.311	0.932
COCACOLA	0.377	0.863	0.986	0.733	0.543
MIHANIKI	0.504	0.279	0.121	0.738	0.544
MOUZAKIS	0.377	0.279	0.629	0.769	0.543
		VaR for short positions			
GENERAL	0.523	0.256	0.237	0.784	0.337
BANKING	0.601	0.928	0.697	0.904	0.487
INDUSTRIAL	0.431	0.535	0.283	0.325	0.487
COCACOLA	0.805	0.279	0.121	0.274	0.810
MIHANIKI	0.504	0.640	0.323	0.274	0.810
MOUZAKIS	0.340	0.863	0.325	0.276	0.940

Notes: P -values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading position is equal to α, top of the table) and $f_s = \alpha$ (i.e. failure rate for the short trading position is equal to α, bottom of the table). α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The failure rates are computed for the skewed Student APARCH model (out-of-sample estimation).

Table 6. Expected shortfall for GENERAL, BANKING and INDUSTRIAL (in sample)

α	5%	2.5%	1%	0.5%	0.25%
	Expected short-fall for long positions (GENERAL)				
RiskMetrics	-3.327	-4.036	-4.725	-5.182	-5.308
N APARCH	-3.671	-4.211	-4.948	-5.492	-5.792
ST APARCH	-3.501	-4.207	-5.441	-6.385	-6.857
SKST APARCH	-3.510	-4.143	-5.339	-6.544	-6.857
	Expected short-fall for long positions (BANKING)				
RiskMetrics	-3.915	-4.606	-5.369	-5.464	-5.650
N APARCH	-4.384	-4.962	-5.789	-6.639	-6.685
ST APARCH	-4.161	-4.981	-6.518	-6.894	-8.507
SKST APARCH	-3.976	-4.889	-6.168	-6.712	-8.338
	Expected short-fall for long positions (INDUSTRIAL)				
RiskMetrics	-7.942	-8.407	-10.357	-12.968	-12.968
N APARCH	-3.898	-4.466	-5.194	-5.257	-6.256
ST APARCH	-3.556	-4.611	-5.826	-6.907	-7.666
SKST APARCH	-3.562	-4.390	-5.422	-6.668	-7.395
	Expected short-fall for short positions (GENERAL)				
RiskMetrics	3.377	3.808	5.404	4,661	5.024
N APARCH	3.602	4.045	4.593	5.160	5.732
ST APARCH	3.377	4.035	5.108	5.649	6.552
SKST APARCH	3.461	4.115	5.216	5.805	7.415
	Expected short-fall for short positions (BANKING)				
RiskMetrics	4.316	4.887	5.404	5.832	6.199
N APARCH	4.552	5.151	6.042	6.502	7.241
ST APARCH	4.283	5.131	6.424	7.726	9.103
SKST APARCH	4.379	5.238	6.457	7.937	9.963
	Expected short-fall for short positions (INDUSTRIAL)				
RiskMetrics	3.663	4.977	7.436	9.993	12.862
N APARCH	3.623	3.973	4.627	5.143	5.418
ST APARCH	4.015	5.005	8.133	11.585	20.991
SKST APARCH	4.112	5.217	9.287	12.332	25.212

Notes: Expected shortfall (in-sample evaluation) for the long and short VaR (at level α) given by the normal APARCH, Student APARCH, Riskmetrics and skewed Student APARCH. α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%.

Table 7. Average multiple of tail event to risk measure for GENERAL, BANKING and INDUSTRIAL (in sample)

α	5%	2.5%	1%	0.5%	0.25%
	AMTERM for long positions (GENERAL)				
RiskMetrics	1.412	1.373	1.356	1.299	1.283
N APARCH	1.403	1.368	1.346	1.330	1.304
ST APARCH	1.437	1.376	1.340	1.400	1.306
SKST APARCH	1.444	1.387	1.344	1.398	1.352
	AMTERM for long positions (BANKING)				
RiskMetrics	1.422	1.366	1.314	1.306	1.339
N APARCH	1.400	1.346	1.368	1.400	1.408
ST APARCH	1.417	1.349	1.494	1.470	1.583
SKST APARCH	1.414	1.359	1.413	1.423	1.585
	AMTERM for long positions (INDUSTRIAL)				
RiskMetrics	1.262	1.168	1.231	1.327	1.237
N APARCH	1.485	1.457	1.439	1.367	1.458
ST APARCH	1.504	1.494	1.511	1.436	1.450
SKST APARCH	1.529	1.490	1.462	1.445	1.425
	AMTERM for short positions (GENERAL)				
RiskMetrics	1.432	1.377	1.302	1.271	1.287
N APARCH	1.418	1.349	1.310	1.379	1.425
ST APARCH	1.429	1.349	1.371	1.394	1.313
SKST APARCH	1.431	1.340	1.376	1.377	1.357
	AMTERM for short positions (BANKING)				
RiskMetrics	1.467	1.390	1.335	1.311	1.316
N APARCH	1.410	1.365	1.360	1.321	1.339
ST APARCH	1.425	1.373	1.376	1.408	1.396
SKST APARCH	1.430	1.409	1.334	1.346	1.379
	AMTERM for short positions (INDUSTRIAL)				
RiskMetrics	1.748	1.696	1.983	2.324	2.671
N APARCH	1.435	1.373	1.376	1.408	1.396
ST APARCH	1.440	1.405	1.338	1.351	1.341
SKST APARCH	1.856	1.980	1.289	3.277	6.044

Notes: Average multiple of tail event to risk measure (AMTERM, in-sample evaluation) for the long and short VaR (at level α) given by the normal APARCH, Student APARCH, Riskmetrics and skewed Student APARCH. α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%.

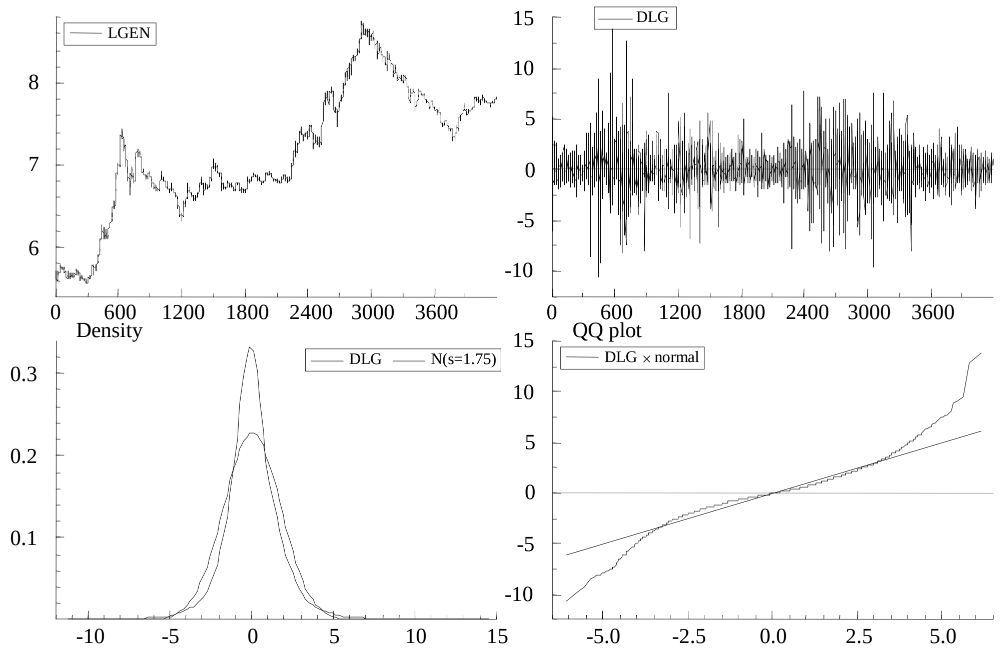

Figure 1: GENERAL/ASE stock index in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 04/01/1988 -01/11/2004.

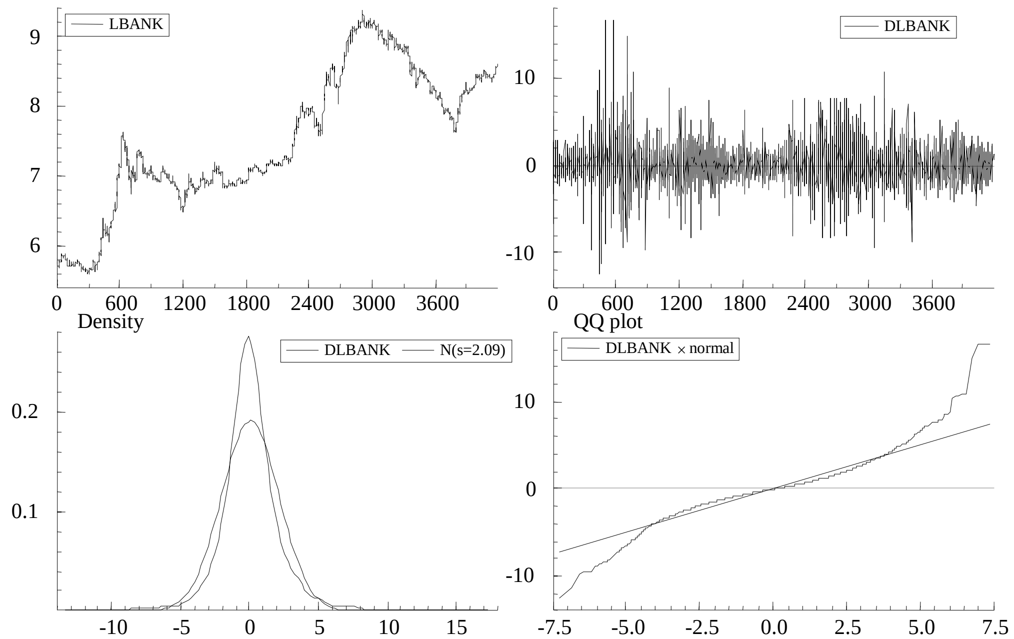

Figure 2: BANKING/ASE stock index in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 04/01/1988 -01/11/2004.

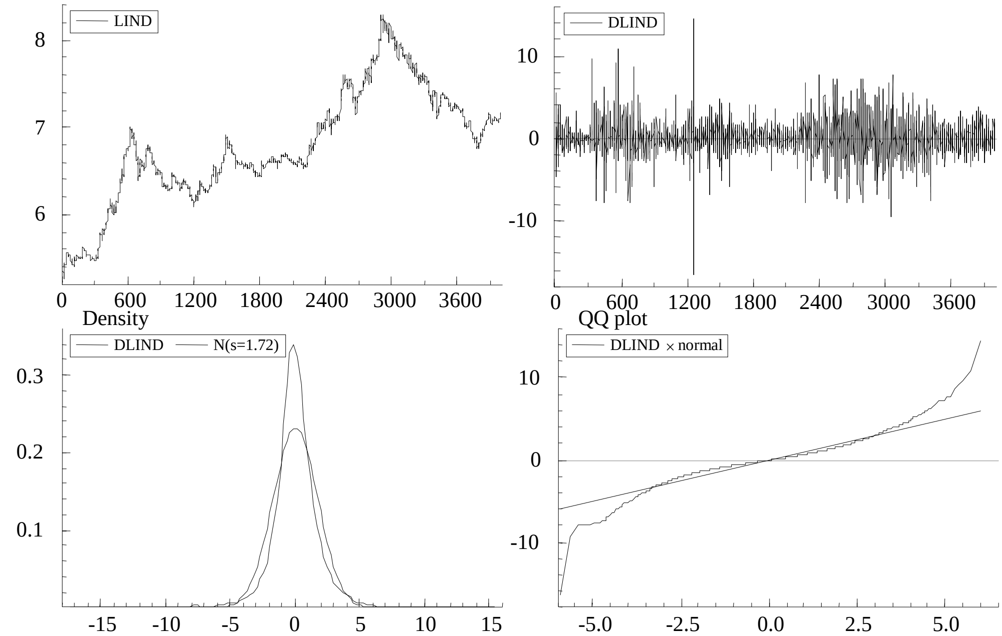

Figure 3: INDUSTRIAL/ASE stock index in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 04/01/1988 -01/11/2004.

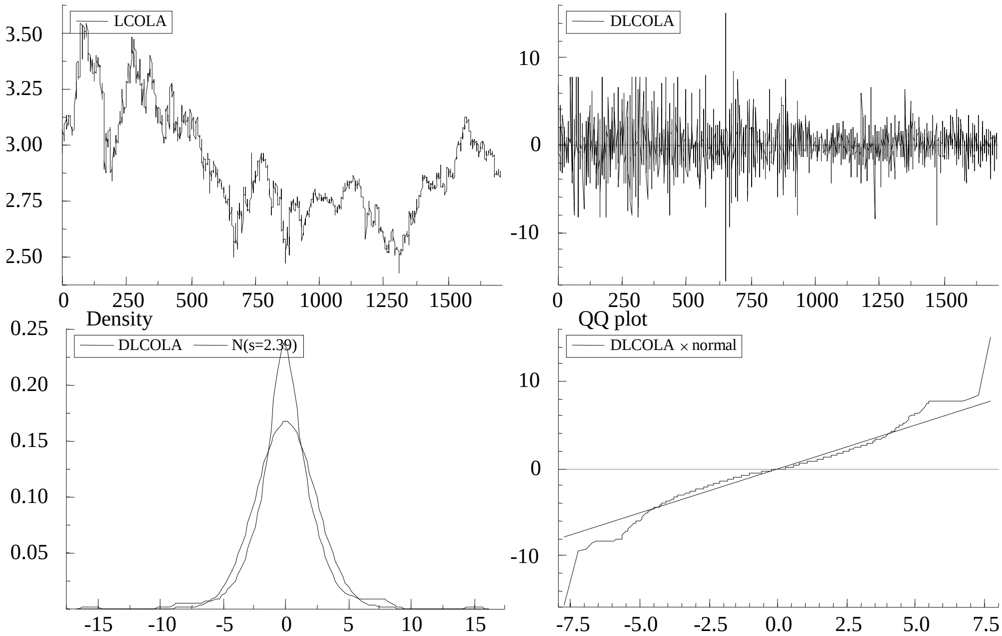

Figure 4: COCA-COLA in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 02/01/1998 - 04/11/2004.

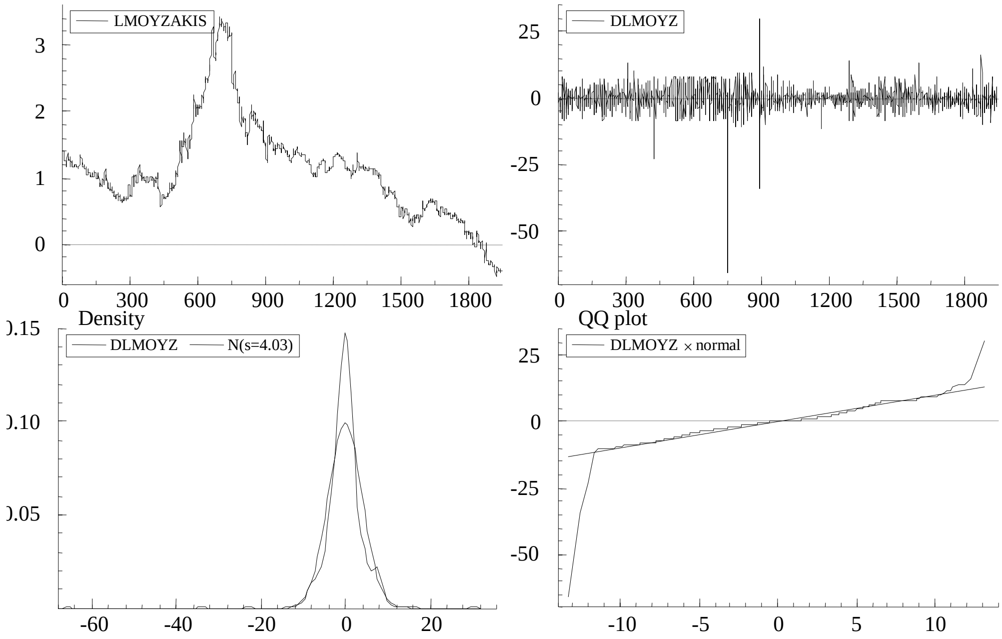

Figure 5: MOUZAKIS in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 14/01/1997-04/11/2004

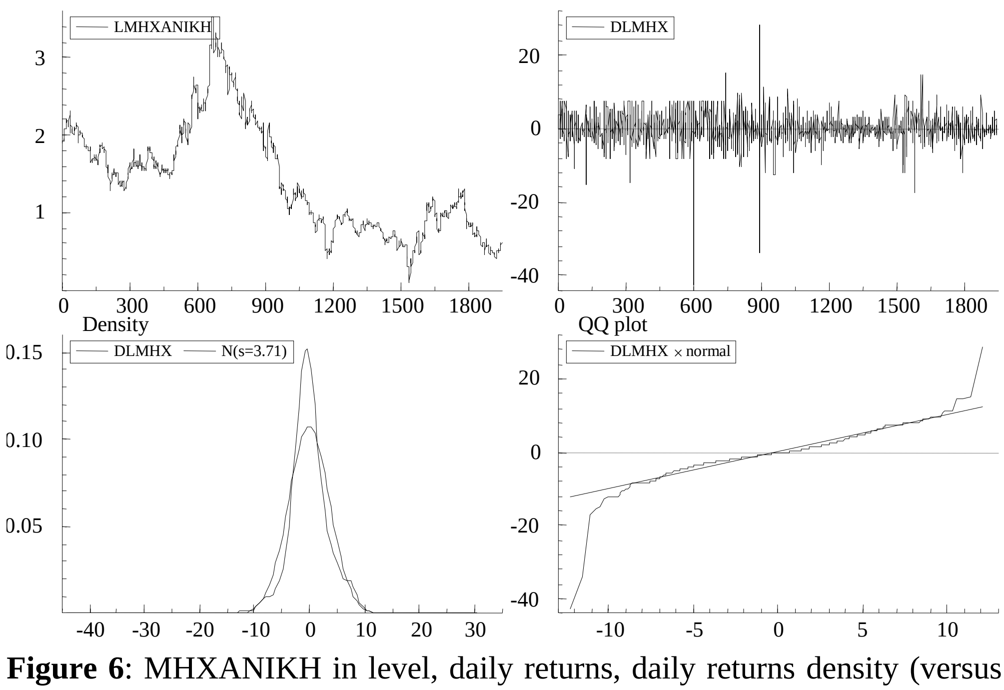

Figure 6: MHXANIKH in level, daily returns, daily returns density (versus normal) and QQ-plot against the normal distribution. The time period is 14/01/1997 -04/11/2004