Total Factor Productivity Growth and the Environment: A Case for Green Growth Accounting

Tzouvelekas, E.[*], Vouvaki, D[†] and A. Xepapadeas[‡]

University of Crete, Department of Economics

December 16, 2006

Abstract

We examine whether the use of the environment, proxied by CO_2 emissions, as a factor of production contributes, in addition to conventional factors of production to output growth, and thus it should be accounted for in total factor productivity growth (TFPG) measurement and deducted from the 'residual'. A theoretical framework of growth accounting methodology with environment as a factor of production which is unpaid in the absence of environmental policy is developed. Using data from a panel of 23 OECD countries, we show that emissions' growth have a statistically significant contribution to the growth of output, that emission augmenting technical change is present along with labor augmenting technical change, and that part of output growth which is traditionally attributed to technical change should be attributed to the use of the environment as a not fully compensated factor of production. Our results point towards the need for developing a concept of "Green Growth Accounting".

JEL Classification: O47, Q2

Key Words: Solow Residual, Total Factor Productivity Growth, Growth, Environment, Green Growth Accounting.

1 Introduction

Growth Accounting is the empirical methodology that allows for the breakdown of output growth into its sources which are the factors of production

---

[*]University of Crete, Department of Economics, University Campus, 74 100 Rethymnon, Greece, e-mail: vangelis@econ.soc.uoc.gr

[†]University of Crete, Department of Economics, University Campus, 74 100 Rethymnon, Greece, e-mail: dimitra@econ.soc.uoc.gr. I gratefully acknowledge PENED research programme '03. (University of Crete Secreterial of the Research Commitee PENED '03) for financial support

[‡]University of Crete, Department of Economics, University Campus, 74 100 Rethymnon, Greece, e-mail: xepapad@econ.soc.uoc.gr

and technological progress, and provides estimates of the contribution of each source in output growth. The concept of total factor productivity growth (TFPG) which is central in growth accounting, measures the part of output growth which is attributed to technological progress, and which corresponds to the part of output growth not 'accounted for' by factors of production such as capital or labour. Growth accounting still remains a central concept in growth theory, although there are still conceptual disputes about the subject, and Easterly and Levine (2001) state that "economists need to provide much more shape and substance to the amorphous term TFP". In this paper we try to provide some additional "shape" by considering the use of environment as a source of growth.

It was Solow in the late 1950's, (Solow, 1957) who provided an explicit integration of economic theory into the growth accounting calculations,[1] which imply decomposing total output growth and measuring the contribution to growth of specific factors, including that of technological progress. During the last decades many different approaches have been used to measure TFPG, which include dual approaches using mainly factor prices instead of factor quantities, and approaches which basically involve disaggregations and refinement of inputs in the production function.[2]

In the early 1970's, a new dimension was given to the theory of economic growth with the introduction into growth models of environmental damages created by emissions. This new dimension which has generated a large volume of literature on "Growth and the Environment"[3], implies a new way of looking at TFPG measurement. Brock (1973) stated that "received growth theory is biased because it neglects to take into account the pollution costs of economic growth". This is because in an unregulated market the cost of pollution is not internalized. Pollution in this case is an unpaid factor of production, with production becoming more costly if less pollution is allowed. In this context environment is used as *a factor of production* which is not fully compensated, and its use in the production process can be captured by introducing emissions as an input in an aggregate production function.[4]

Following this methodological approach, the idea developed in this pa-

---

[1] See the historical note by Griliches (1996).

[2] See for example, Barro (1999), Barro and Sala-i-Martin (2005).

[3] See for example Aghion and Howitt (1998) or surveys such as: Brock and Taylor (2005), Xepapadeas (2005).

[4] In this context, the production function has been specified to include the flow of emissions as an input and some times, productivity enhancing environmental quality as a stock variable. This formulation has been used frequently in the theoretical analysis of growth and the environment. In addition to Brock (1973), see for example, Becker (1982), Tahvonen and Kuluvainen (1993), Bovenberg and Smulders (1995), Smulders and Gradus (1996), Mohtadi (1996), Xepapadeas (2005), Brock and Taylor (2005). See also Considine and Larson (2006) for the treatment of environment as a factor of production at the firm level.

per is that when emissions are introduced as an input in the production process and are properly measured, the contribution from the use of environment in total output growth can also be measured. This contribution can be approximated even when emissions is an unpaid factor in the absence of environmental policy. In this sense, emissions can determine, along with other inputs and technological progress output growth in a growth accounting framework. Therefore, the present paper can be regarded as an attempt to explore systematically whether the use of the environment as an input in production contributes to output growth, and how this contribution can be measured.[5],[6]

We develop a growth accounting framework for measuring TFP growth by approximating the use of the environment by carbon dioxide (CO_2) emissions. We argue that environment such as the atmosphere can be regarded as a component of social overhead capital (Uzawa, 2003), and that CO_2 emissions can be thought of as a reduction of this social capital - a form of disinvestment. Thus, we use CO_2 emissions as a proxy for the use of this component of social capital in the production process.[7]. Our purpose is to examine the contribution of CO_2 emissions' growth, as a proxy for the use of environment, on economic growth and to show that since external pollution costs which are created during the production process are not taken into account in the measurement of total factor productivity growth, the current measurements of TFP growth, or the Solow "residual", could provide biased results. Our basic hypothesis, which has been tested empirically, is that environment is basically an *unpaid* source of output growth. If this source is not taken into account into the growth accounting framework, then output growth which should be attributed to the use of the environment will be incorrectly attributed to TFPG. Furthermore, if emissions saving technical change is present this could be another source of growth in addition to the conventional labor augmented technical change. This hypothesis is tested empirically in this paper by using data from a panel of 23 OECD countries.[8]

---

[5] It is important to note that the approach we choose to follow is an aggregate macroeconomic approach that belongs to the Solow tradition of measuring TFP growth from a macroeconomic perspective. This is not the same as TFPG measurement at the micro-level where TFPG is usually measured with the use of distance functions and linear programming approaches. (See for instance, Pitman (1983), Fare, et all, (1989, 1993).

[6] In a recent paper Jeon and Sickles (2004) analyze productivity growth using the directional distance function method and treating CO_2 emissions as a undesirable output. Although they analyze a different time period, some of their results for OECD countries could be comparable to our own.

[7] Strictly speaking CO_2 emissions is not a pollutant but we treat them as such because of their close relation to climate change and the implied environmental damages. See for example the recent Stern Report (2006).

[8] The 23 countries used in our analysis are the following: Canada, U.S.A, Austria, Belgium, Denmark, Finland, France, Greece, Italy, Portugal, Spain, Sweden, Switzerland, U.K., Japan, Iceland, Ireland, Netherlands, Norway, Australia, Mexico, Turkey, Luxembourg.

Our theoretical and empirical analysis seems to suggest that the "unpaid"-due to absence of taxation - environmental factor, proxied by CO_2 emissions could be a source of growth, and an important component in the growth accounting methodology, supporting the case of a "Green Growth Accounting" approach. We feel that this type of analysis could be important, because if the use of the environment is a source of growth, as our results seem to suggest, but environment is used as an unpaid factor, environmental damages remain 'unpaid'. By being 'unpaid' or not 'fully paid' however, they are not kept at a 'socially optimal level' during the growth process and this fact might eventually erode the sustainability of the growth process itself.[9]

Emissions of CO_2 could be related to energy use and energy could have been regarded as an input in the production function with CO_2 emissions as a by product. In the absence of a carbon tax for a given period there still exist an unpaid factor since the full cost of energy - private and social (environmental) - is not fully paid by private markets, and this is a source of potential bias for TFPG measurement. In this paper we choose to use emissions as the 'environmental input' and not energy in order to provide a more direct link between and environment, since CO_2 emissions are related to climate change.[10]

The rest of the paper is structured as follows. Section 2, is a descriptive section that provides some stylized facts related to emissions growth and output growth in per worker terms. Section 3 develops the growth accounting framework and interprets emissions' share in output in the context of optimal growth and competitive market equilibrium. Since in general we don't have taxation for the emissions of CO_2 and therefore the 'environment's share' is not included in National Accounts, estimating TFPG, as it is the most common approach, using data on input shares might provide biased estimates. We try to solve this problem at the empirical level in sections 4 and 5 by: (i) equating the emission's share in total output with the share of environmental damages in total output, using independent estimates for CO_2 damages, and (ii) by estimating directly the emission's

---

[9] In recent papers, Chimeli and Braden (2005) explore the relationship between total factor productivity (TFP) and the Environmental Kuznets Curve, while Kalaitzidakis et al (2006) try to determine an empirical relationship where measured TFP, when capital and labor are used as inputs in the production function, is the dependent variable, and CO_2 emissions are treated as an independent variable. Their results suggest the existence of such a relationship. Our approach differs basically because we provide a net TFPG estimate after all the factors used, including 'uncompensated' environment, have been accounted for. It also allows for the possibility of a 'negative residual' if the cost of using the environment is properly accounted and output growth is not sufficient to cover the true social cost of all inputs used, which will be a strong sign of unsustainable growth.

[10] The long term relationship between energy and CO_2 emissions in USA has been recently explored by Tol et al. (2006). If a stable relationship exits for a given period, then results based on CO_2 emissions can be expressed in terms of energy by appropriate conversion factors.

share from an aggregate production function where CO_2 emissions is an input along with labor and capital. Estimation results suggest that the use of the environment seems to be a statistically significant factor in explaining output growth. This can be interpreted as an indication that the TFPG measurements that do not take the environmental factor into account might be biased in estimating the contribution of technological progress. Our results indicate furthermore, that labor augmenting technological progress, is not the only factor that constitutes the 'true residual' but 'emission augmenting technical change' might also be present. The last section concludes.

2 CO_2 Emissions and Growth: Some Descriptive Results

This section which provides some stylized facts regarding possible links between the growth of CO_2 emissions and output growth for a group of 23 OECD economies[11].

Figure 1, shows gross domestic product (GDP) in per worker terms (GDP/W) for a group of 21 OECD countries[12], relative to the GDP/W in the USA.[13] The years we compare are 1965 and 1990 and it seems that the countries analyzed managed to reduce the growth "distance" from USA in GDP per worker terms and increased their GDP/W from 1965 to 1990, both in absolute terms and relative to the USA.

Figure 1

Figure 2 that follows, makes the same comparisons using emissions of CO_2 per worker (CO_2/W) for the years 1965 and 1990 respectively. It can be noticed that for some countries (6 out of 21),[14] CO_2/W was reduced during these years, while for the rest (15 out of 21)[15], CO_2 emissions per worker increased.

Figure 2

Figure 3, shows CO_2 emissions per unit of GDP (CO_2/GDP) for the years 1965 and 1990. USA is taken as the benchmark country again and the

---

[11]Our data are taken from the Penn Tables v5.6. Real GDP measured in thousands of US$ is the variable (RGDPCH), multiplied by the variable POP in the Penn Tables. Capital stock and employment are retrieved from Real GDP and capital per worker (KAPW) and real GDP per worker (RGDPW). All values are measured in 1985 international prices. CO_2 data are taken from the World Bank and are measured in thousand tons of CO_2 emissions.

[12]Luxembourg, has been excluded for presentation purposes.

[13]USA is used as a benchmark country, so that USA=1.

[14]The countries are: Belgium, Denmark, France, Sweden, UK and Iceland.

[15]The countries are the following: Canada, Austria, Finland, Greece, Italy, Portugal, Spain, Switzerland, Japan, Ireland, Netherlands, Norway, Australia, Mexico, Turkey.

comparisons show that for the majority of countries CO_2 (13 out of 21) [16] emissions per unit of GDP increased whereas in the rest (8 out of 21)[17] emissions per unit of GDP decreased.

Figure 3

In figure 4, the vertical axis measures the average annual growth of GDP per worker and the horizontal axis the corresponding growth of CO_2 per worker between 1965 and 1990. Each point of the scatter diagram represents one of the 23 countries we analyze. There is on the average a positive relationship between the two variables, suggesting that countries with high growth of CO_2 per worker can be associated with a high growth of GDP per worker. This can be regarded as an indication that the growth of CO_2 per worker contributes to the growth of GDP per worker.

figure 4

Figure 5, shows on the horizontal axis GDP per worker relative to the GDP per worker in USA at 1965, and on the vertical axis the average growth of CO_2 per worker for the examined period. Countries with GDP per worker close to the USA GDP per worker in 1965 (which is normalized to 1) had relatively low growth rates of CO_2 per worker. On the other hand, countries that were 'far' (below) in GDP per worker relative to the USA in 1965, show a relatively high rate of growth of CO_2 per worker. An attempt to explain this would be to say that countries with low GDP per worker in 1965 relatively to the USA, were developing relatively fast and during their development processes emitted relatively more carbon dioxide per worker, probably due to the use of 'dirtier' technologies and not sufficiently strong emissions saving technical change.

figure 5

These descriptive data seem to provide some indications that the growth of CO_2 emissions per worker is positively related to the growth of output per worker. This could imply that the use of the environment is a factor that influences the output growth of an economy, and as such it should be taken into account into growth accounting calculations. In the following we are trying to develop a theoretical and empirical framework for testing this hypothesis.

---

[16]The countries are: Austria, Finland, Greece, Italy, Portugal, Spain, Switzerland, Ireland, Netherlands, Norway, Australia, Mexico, Turkey.

[17]These countries are: Canada, Belgium, Denmark, France, Sweden, UK, Japan and Iceland.

3   Primal Growth Accounting with Environmental Considerations

We state first the traditional Solow's residual under environmental considerations. Let,

$$Y = F\left(K, E\right) = F\left(K, AL\right) \tag{1}$$

where Y is aggregate output, K is physical capital, $E = AL$ is effective labour, with L being labour input and A reflecting labour augmenting (Harrod neutral) technical change. The 'Solow residual' is defined (e.g. Romer 1999, Barro and Sala-i-Martin 2004) as:

$$g_S = s_L \left(\frac{\dot{A}}{A}\right) = \frac{\dot{Y}}{Y} - s_K \left(\frac{\dot{K}}{K}\right) - s_L \left(\frac{\dot{L}}{L}\right) \tag{2}$$

where s_K and s_L are the shares of capital and labor in output, with two factors receiving their competitive rewards. Under constant returns of scale, $s_L + s_K = 1$, and we have:

$$g_S = s_L \left(\frac{\dot{A}}{A}\right) = \frac{\dot{y}}{y} - s_K \frac{\dot{k}}{k} \tag{3}$$

where $\dot{y}/y$ is the rate of growth of output per worker ($y = Y/L$) and $\dot{k}/k$ is the rate of growth of capital per worker ($k = K/L$) [18]. The rate of the exogenous labor augmenting technical change $x = \dot{A}/A$ can be directly determined by (3)

By following ideas appeared in Denison (1962), Dasgupta and Mäler (2000), Xepapadeas (2005) which relate environment to growth accounting, we define a standard neoclassical production function that includes human capital and emissions as an input of production and we use it to determine a growth accounting equation. Let

$$Y = F\left(K, H, E, X\right) \tag{4}$$

where in addition to K and E, H is human capital, $X = BZ$ is effective input of emissions, with Z being emissions in physical units and B reflecting emission saving technical change, or input augmenting technical change.

Differentiating (4) with respect to time, and denoting by $\epsilon_j, j = K, H, L, Z$ the elasticity of output with respect to inputs, the basic growth accounting equation is obtained as:

$$\frac{\dot{Y}}{Y} = \epsilon_K \left(\frac{\dot{K}}{K}\right) + \epsilon_H \left(\frac{\dot{H}}{H}\right) + \epsilon_L \left(\frac{\dot{A}}{A}\right) + \epsilon_L \left(\frac{\dot{L}}{L}\right) + \epsilon_Z \left(\frac{\dot{B}}{B}\right) + \epsilon_Z \left(\frac{\dot{Z}}{Z}\right) \tag{5}$$

---

[18] As is the convention in this literature lower case letters denote per worker quantities.

Equation (5) says that the growth rate of GDP can be decomposed into the growth rate of, manufactured capital, human capital, physical labor, emissions in physical units and technical change. To transform equation (5) into a growth accounting equation in factors shares, we use as before, profit maximization in a competitive market set up. We assume that physical and human capital receive there rental rates R_K and R_H, labor receives wage w and emission are taxed at a rate $\tau \geq 0$, since they create external damages. Thus, profits for the representative firm are defined as:

$$\Pi = F(K, H, E, X) - R_K K - R_H H - wL - \tau Z \tag{6}$$

with associated first-order conditions for profit maximization:

$$\frac{\partial F}{\partial K} = R_K \ , \ \frac{\partial F}{\partial H} = R_H \ , \ \frac{\partial F}{\partial E} A = \frac{\partial F}{\partial L} = w \tag{7}$$

$$\frac{\partial F}{\partial X} B = \frac{\partial F}{\partial Z} = \tau \tag{8}$$

Denoting by $s_j, j = K, H, L, Z$ the factors' shares in total output, then under profit maximization the basic growth accounting equation is obtained as:

$$\frac{\dot{Y}}{Y} = s_K \left(\frac{\dot{K}}{K} \right) + s_H \left(\frac{\dot{H}}{H} \right) + s_L \left(\frac{\dot{A}}{A} \right) + s_L \left(\frac{\dot{L}}{L} \right) + s_Z \left(\frac{\dot{B}}{B} \right) + s_Z \left(\frac{\dot{Z}}{Z} \right) \tag{9}$$

where: $s_K = \frac{R_K K}{Y}, s_H = \frac{R_H H}{Y}, s_L = \frac{wL}{Y}, s_Z = \frac{\tau Z}{Y}$.

If we assume that investment in physical and human capital is carried out up to the point where marginal products in each type of capital (physical and human capital) are equated in equilibrium,[19] (see for example Barro and Sala-i-Martin 2004), we have:

$$H = \frac{s_H}{s_K} K \tag{10}$$

Substituting (10) into (9) we obtain:

$$\frac{\dot{Y}}{Y} = s_{KH} \left(\frac{\dot{K}}{K} \right) + s_L \left(\frac{\dot{A}}{A} \right) + s_L \left(\frac{\dot{L}}{L} \right) + s_Z \left(\frac{\dot{B}}{B} \right) + s_Z \left(\frac{\dot{Z}}{Z} \right) \tag{11}$$

$$s_{KH} = s_K + s_H \tag{12}$$

Thus, the Solow residual augmented with human capital and emissions can be defined as:

$$\gamma = s_L \left(\frac{\dot{A}}{A} \right) + s_Z \left(\frac{\dot{B}}{B} \right) = \frac{\dot{Y}}{Y} - s_K \left(\frac{\dot{K}}{K} \right) - s_H \left(\frac{\dot{H}}{H} \right) - s_L \left(\frac{\dot{L}}{L} \right) - s_Z \left(\frac{\dot{Z}}{Z} \right)$$

---

[19]This assumption has been used to justify relatively high estimates of capital's share in empirical growth equations.

or by using the assumption of equality of marginal products between physical and human capital as:

$$\gamma = s_L \left(\frac{\dot{A}}{A} \right) + s_Z \left(\frac{\dot{B}}{B} \right) = \frac{\dot{Y}}{Y} - s_{KH} \left(\frac{\dot{K}}{K} \right) - s_L \left(\frac{\dot{L}}{L} \right) - s_Z \left(\frac{\dot{Z}}{Z} \right) \quad (14)$$

Under constant returns to scale (13) and (14) become:

$$\gamma = \frac{\dot{y}}{y} - s_K \frac{\dot{k}}{k} - s_H \frac{\dot{h}}{h} - s_Z \frac{\dot{z}}{z} \quad (15)$$

$$\gamma = \frac{\dot{y}}{y} - s_{KH} \frac{\dot{k}}{k} - s_Z \frac{\dot{z}}{z} \quad (16)$$

By comparing the new definitions for TFPG, (13)-(14) or (15)-(16) with (2) or (3), it can be seen that the new definitions include the term $s_Z \left(\dot{Z}/Z \right)$. This indicates that there is one more source generating output growth in addition to capital and labour, namely emissions, along with the term $s_Z \left(\dot{B}/B \right)$ which reflect emission augmenting (input saving) technical change in addition to the standard labour augmenting technical change. In order to obtain a "net" estimate of TFPG the environment's contribution should be properly accounted. In the context of our analysis (14) - (16) can be regarded as the *Green Growth Accounting equations*. In order however to provide a meaningful definition of the TFPG when environment is an input, there is a need to clarify what is meant by the share of emissions in output, especially since when it comes to empirical estimations there might be data sets where $\tau = 0$, that is emissions are untaxed and we have one unpaid input in the production function.

3.1 Interpreting the Emissions' Share in Growth Accounting

3.1.1 The Social Planner

To interpret the emissions share even when no environmental taxation is present ($\tau = 0$), we consider the problem of a social planner seeking to optimize a felicity functional defined over consumption and environmental damages and to determine an optimal emission tax, optimal in the sense that if firms pay this tax on their emissions they will emit the socially desirable levels of emissions. An optimal tax would internalize the externalities that the emissions create during the production process.

We assume that emissions (flow variable), accumulate into the ambient environment and that the evolution of the emission stock S, is described by the first order differential equation:

$$\dot{S}(t) = Z(t) - mS(t) , \ S(0) = S_0, m > 0 \quad (17)$$

where m reflects the environment's self cleaning capacity[20]. The stock of emissions generate damages according to a strictly increasing and convex damage function $D(S)$, $D' > 0, D'' \geq 0$.

Assume that utility for the "average person" is defined by $U(c(t), S(t))$ where $c(t)$ is consumption per capita, $c(t) = C(t)/N(t)$, with $N(t)$ being population. We assume as usual that $U_c(c, S) > 0, U_S(c, S) < 0 \, U_{cS}(c, S) \leq 0$, that U is concave in c for fixed S, and finally that U is homogeneous in (c, S). Then social utility at time t is defined as $N(t) U(c(t), S(t)) = N_0 e^{nt} U(c(t), S(t))$ where n is the exogenous population growth rate and N_0 can be normalized to one. The objective for the social planner is to choose consumption and emission paths to maximize:

$$\max_{\{c(t), Z(t)\}} \int_0^\infty e^{-(\rho-n)t} U(c, S) \, dt \qquad (18)$$

where, $\rho > 0$ is the utility discount rate, subject to the dynamics of the capital stock and the pollution stock (17). The capital stock dynamics can be described in the following way. Assume a constant returns to scale Cobb-Douglas specification for the production function (4):

$$Y = K^{a_1} H^{a_2} (AL)^{a_3} (BZ)^{a_4} \qquad (19)$$

Expressing output in per worker terms we obtain:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{a_1} \left(\frac{H}{L}\right)^{a_2} \left(\frac{AL}{L}\right)^{a_3} \left(\frac{BZ}{L}\right)^{a_4} , \text{ or}$$
$$y = e^{\zeta t} k^{a_1} h^{a_2} Z^{a_4}, \ \zeta = xa_3 + a_4(b-n)$$

where labor augmenting technical change grows at the constant rate x, input (emission) augmenting technical change grows at a constant rate b, labor grows at the population rate n, and $h = \frac{H}{L}$. Assuming equality of depreciation rates and equality of marginal products between manufactured and human capital in equilibrium, the social planner's problem can be written as:[21]

$$\max_{\{\hat{c}(t), Z(t)\}} \int_0^\infty e^{-\omega t} U(\hat{c}, S) \, dt , \qquad (20)$$
$$\omega = \rho - n - (1-\theta)\xi , \ \xi = \frac{\zeta}{1 - a_1 - a_2}$$
$$\text{subject to:}$$
$$\dot{\hat{k}} = f\left(\hat{k}, Z\right) - \hat{c} - (\eta + \delta + \xi)\hat{k}, \quad f\left(\hat{k}, Z\right) = s\tilde{A}\hat{k}^\beta Z^{a_4} \quad (21)$$
$$\dot{S} = Z - mS \qquad (22)$$

---

[20] We use a very simple pollution accumulation process which has been often used to model global warming. The inclusion of environmental feedbacks and nonlinearities which represent more realistic situations will not change the basic results.

[21] For the derivation see Appendix.

where ˆ indicate variables in efficiency units (see Appendix). The current value Hamiltonian for this problem is:

$$\mathcal{H} = U(\hat{c}, S) + p\left[f\left(\hat{k}, Z\right) - \hat{c} - (\eta + \delta + \xi)\hat{k}\right] + \lambda(Z - mS) \qquad (23)$$

and the optimality conditions implied by the maximum principle are:

$$U_{\hat{c}}(\hat{c}, S) = p \ , \ U_{\hat{c}\hat{c}}(\hat{c}, S)\dot{\hat{c}} + U_{\hat{c}S}(\hat{c}, S)\dot{S} = \dot{p} \qquad (24)$$

$$pf_Z\left(\hat{k}, Z\right) = -\lambda \ or \ Z = g\left(\hat{k}, \lambda, p\right) \qquad (25)$$

$$\dot{p} = \left(\rho + \delta + \theta\xi - f_{\hat{k}}\left(\hat{k}, Z\right)\right)p \ or \qquad (26)$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, g\left(\hat{k}, \lambda, U_{\hat{c}}(\hat{c}, S)\right)\right) - \rho - \delta - \xi\theta\right] - \frac{U_{\hat{c}S}}{U_{\hat{c}\hat{c}}}\dot{S} \qquad (27)$$

$$\dot{\lambda} = (\omega + m)\lambda - U_S(\hat{c}, S) \qquad (28)$$

The system of (27), (28) along with (21) and (22), form a dynamic system, which along with the appropriate transversality conditions at infinity (Arrow and Kurz 1970) characterizes the socially optimal paths of $\left(\hat{c}, \hat{k}, \lambda, S, Z\right)$.

Let the value function of the problem be defined as:

$$J(K_0, S_0) = \max \int_0^\infty e^{-\omega t}U(\hat{c}, S)\,dt \qquad (29)$$

then it holds that (Arrow and Kurz 1970)

$$\frac{\partial J}{\partial S(t)} = \lambda(t) < 0 \qquad (30)$$

Thus the costate variable λ can be interpreted as the shadow cost of the pollution stock. By comparing (25) with (8) and noting (30) it is clear that if a time dependent tax $\tau(t) = -\lambda(t)/p(t)$ is chosen, then firms will choose the socially optimal amount of emissions as input.

Then the emission's share can be written as:

$$s_Z = \frac{\tau Z}{Y} = \frac{\left(-\hat{\lambda}\right)Z}{Y}, \ \hat{\lambda} = \frac{-\lambda}{p} = \frac{-\lambda}{U_{\hat{c}}} \qquad (31)$$

where from (31) $\hat{\lambda}$ can be interpreted as the shadow cost of the pollution stock in terms of marginal utility. Thus the share of emissions in output coincides, under optimal environmental taxation, with the share of environmental damages in total output. It can be further shown that under the emission tax $\tau(t) = \hat{\lambda}(t)$ competitive equilibrium will coincide with the social planners solution.

3.1.2 Competitive Equilibrium

The representative consumer considers the stock of pollution as exogenous and chooses consumption to maximize lifetime utility, or:

$$\max_{c(t)} \int_0^\infty e^{-(\rho-n)t} U\left(c, S\right) dt \tag{32}$$

subject to the budget flow constraint:

$$\dot{a} = w + ra - c - na + \tau z \tag{33}$$

where a is per capita assets, w, r the competitive wage rate and interest rate respectively and τz are per capita transfers due to environmental taxation, $z = Z/L$.

The representative firm maximizes profits given by (6), where by assuming that physical capital, human capital and loans are perfect substitutes as stores of value we have $r = R_K - \delta = R_H - \delta$.

In equilibrium $a = k + h$. Then the following proposition can be stated:

Proposition 1 *Under optimal environmental taxation, that is $\tau\left(t\right) = \frac{-\lambda(t)}{p(t)}$, the paths $\left(\hat{c}\left(t\right), \hat{k}\left(t\right), S\left(t\right), Z\left(t\right)\right)$ of a decentralized competitive equilibrium coincide with the socially-optimal paths.*[22]

For proof see Appendix.

4 TFPG Measurement Issues

As shown above, under optimal taxation the time paths for consumption, capital and pollution at the social optimum coincide with the corresponding optimal paths in a decentralized competitive equilibrium. Our basic problem in measurement is that usually in practice we don't have taxation (optimal or not) for CO_2 emissions, so we need an estimate of damages as a proxy for taxation. The only clear case where CO_2 emissions have a cost for those emitting can be found in the recently created European emission trading scheme. This however is a very recent development and our data set corresponds to the "no regulation" case. Furthermore, since we don't have taxation on emissions and therefore the share of emission taxes are not included in National Accounts, estimating TFPG using data from National Accounts, might provide biased estimates since the share of emissions damages is ignored.

TFP growth estimation involves, most of the times, a direct implementation of growth accounting equations such as (2) using data for Y, K, L, s_K, s_L.

---

[22] It can be shown that a similar result holds if we define the model in terms of energy, and there is a simple proportional relationship between energy and emissions.

There is a difficulty however, as indicated above, if we want to include emissions in the equation. Theory suggests that s_Z is emission damages as a share of GDP. If optimal taxation is applied then s_Z is can be measured as a share of GDP. If however emissions are not taxed, that is environment as an unpaid factor of production, then we need an independent estimate of marginal emission damages. In the absence of such estimate, the implementation of growth accounting equation like (13) or (14) using data on Y, K, L, Z, s_K, s_L, s_z is not possible. Thus, the presence of the environment as an input in the production function and the absence of emission taxation make the non econometric estimations which is usually followed, problematic. In this case, direct adjustments using independent estimates of emission damages, or econometric estimation could be used.

4.1   Direct Adjustment using Marginal Damage Cost Estimates of Carbon Dioxide Emissions

In the absence of environmental policy, but if independent marginal damage cost estimates of CO_2 (MDCCO_2) emissions exist, then adjusted TFPG estimates can be obtained using:

$$\hat{g}_S^i = g_S^i - s_Z^i \left(\frac{\dot{Z}}{Z} \right)_i \tag{34}$$

which can be derived directly from (15) or (16), where g_S^i is the estimate of the traditional Solow residual in country i, $\left(\frac{\dot{Z}}{Z} \right)_i$ the growth of CO_2 emissions, and s_Z^i is the share of CO_2 emissions in GDP defined as

$$s_{zt}^i = \frac{p_z Z_{it}}{GDP_{it}} \tag{35}$$

Since, in the absence of optimal taxation, the share of emissions cannot be obtained from tax data, we use our theoretical result that under optimal taxation the emissions' share in GDP should be equal to the share of damages from carbon dioxide in GDP. Thus, with p_z a proxy for MDCCO_2, (35) can be obtained by using existing MDCCO_2 estimates (e.g. Tol 2005).

4.2   Econometric Estimation

In this case the measurement of TFP growth is based on an aggregate production function which includes CO_2 emissions as an input. This can be regarded as a more appropriate way to estimate input shares and the share of CO_2 emissions which is an unpaid factor in the production process, since it's share in GDP cannot be measured by existing data in the absence of CO_2 emission taxes. An additional advantage of econometric estimation is

that of direct testing the statistical significance of emissions growth as a determinant of output growth.

Using the Cobb-Douglas specification (19), we obtain under constant returns the loglinear specification:[23]

$$\ln y = \alpha_0 + (xa_3 + ba_4)\, t + a_1 \ln k + a_2 \ln h + a_4 \ln z \ , \ \sum_{i=1}^{4} a_i = 1 \quad (36)$$

Equation (36) provides estimates of input elasticities. To have a meaningful interpretation of these elasticities as factors' shares in the absence of optimal environmental policy, we need to consider the choice of emissions in the context of the constraint optimization problem:

$$
\begin{aligned}
\max \Pi \quad &= \quad F\left(K, H, AL, BZ\right) - R_K K - R_H H - wL \qquad (37)\\
&\text{subject to} \quad Z \le \bar{Z}
\end{aligned}
$$

The upper bound on emissions could reflect technical constraints associated with production technologies or emissions. For example, even without CO_2 taxation, general environmental policies on air pollutants (SO_2, NO_x) might introduce technological responses or capacity constraints which eventually generate upper bounds for CO_2 emissions. Associating the Lagrangian multiplier μ with the constraint $Z \le \bar{Z}$ the first order condition for the optimal input choices, including emission choice, which correspond to (37) are:

$$\frac{\partial F}{\partial K} = R_K, \ \frac{\partial F}{\partial H} = R_H, \ \frac{\partial F}{\partial L} = w, \ \frac{\partial F}{\partial Z} = \mu$$

by the envelope theorem μ is the shadow cost of emissions Z, and measures the response of maximum profits to changes in the upper bound $\bar{Z}$. This shadow cost should be distinguished from the shadow cost of the pollution stock, defined in (30), that measures the response of maximum welfare to a change in the stock of pollutants, the stock of CO_2 in our case.

Thus in the absence of environmental policy the share of the unpaid factor in equilibrium is defined as;

$$s_Z = \frac{F_Z Z}{Y} = \frac{\mu Z}{Y} \qquad (38)$$

In general this share will be *different* from the correct share $\left(-\hat{\lambda} Z\right)/Y$, unless $\bar{Z}$ is set at the level corresponding to the social welfare maximization

---

[23]In the empirical analysis we use as proxy for H, an index constructed as $H_{it} = \exp(\phi\left(\epsilon_{jt}\right))$. Where ϵ_{jt} is average years in education in country i at year t, and ϕ is a piecewise linear function with zero intercept and slope 0.134 for $\epsilon_{jt} \le 4$, 0.101 for $4 < \epsilon_{jt} \le 8$, and 0.068 for $\epsilon_{jt} > 8$.(see Hall and Jones (1999); Henderson and Russel (2005)). Data on education were obtained from the World Bank, World Development Indicators (2002).

path for the emissions' flow, in which case $\mu = \hat{\lambda}$. This however, is not the case for the period under investigation.

Therefore the elasticities obtained from the production function can be interpreted as shares associated with the constraint optimization problem (37) but not with the social welfare optimization problem (20). This has certain implications for the interpretation of any estimation results.

Given an estimate of $\hat{s}_Z$, the shadow value of emissions can be obtained as $\hat{\mu} = \hat{s}_Z (Y/Z)$ where Y/Z is the observed ouput-emissions ratio. This not however the 'social shadow cost' of pollution since this 'social shadow cost' is λ, which is based on a social welfare function that incorporates environmental damages.[24]

In the growth accounting exercise the contribution of CO_2 emissions on output growth using elasticities estimated from an aggregate production function, in the absence of CO_2 related environmental policy, can be interpreted in terms of emissions contributions under the existing technological constraints, and not as the 'true' contribution, when environment is properly valued by the welfare cost of using it. On the other hand this is a useful measure since it provides an indication of the impact on aggregate output, from introducing an environmental policy that restricts emissions.

Actually, since in the absence of a CO_2 policy it is expected that emissions constrained only by technological restrictions would be high[25], relative to the case where the socially optimal regulation is followed, the estimate of μ is expected to be low relative to λ.

In this context elasticities can be interpreted as shares, and we can set:

$$a_1 = s_K, a_2 = s_H, a_4 = s_Z \tag{39}$$

By comparing (36) with (13), TFPG can be obtained by estimating $xa_3 + a_4 b$. In this case TFPG is approximated by the contribution of labor augmented technical change and emissions augmented technical change.

There are several ways to further specify the production function.

- With $a_4 \neq 0$, by imposing in (36) $a_2 = 0$, we obtain a production function with emissions but without human capital, or

$$\ln y = (xa_3 + a_4 b) t + a_1 \ln k + a_4 \ln z \tag{40}$$

---

[24] There is a subtle point here associated with the shadow cost of pollutants obtained by productivity studies using mainly micro-data, where emissions or undesirable outputs are included and distance functions or linear programming methodologies are used for estimation purposes. The shadow cost estimates reflect the impact on the objective function associated with emissions, but they do not reflect damages due to emissions. So although these estimates are appropriate for studying the impact of sectoral environmental policies on firms profits or costs, they do not reflect the welfare cost of using the environment, especially if environmental policy is not well defined, or is not present during the sample period.

[25] We have unregulated profit maximization in this case.

- With $a_4 \neq 0$, $a_2 = 0$ and by using, instead of the labour (L) in physical units, the quality adjusted labor input defined as $L_h = LH$ we have:

$$\ln y_h = (xa_3 + a_4 b)\, t + a_1 \ln k_h + a_4 \ln z_h \qquad (41)$$

where all variables are measures in per 'quality adjusted' worker terms.

- Imposing $a_2 \neq 0$ and the assumption of equality of marginal products between human and physical capital, we obtain:

$$\ln y = (xa_3 + a_4 b)\, t + (a_1 + a_2) \ln k + a_4 \ln z \qquad (42)$$

It is clear that for $a_4 = 0$ we have the traditional aggregate production function without emissions as an input.

Each of the production function specifications (40), (41),(42) with the elasticities interpreted as shares by (39), can be associated with a growth accounting equation. Specification (36), which is the most general has as a counterpart the growth accounting equation:

$$\frac{\dot{y}}{y} = \gamma + s_K \frac{\dot{k}}{k} + s_H \frac{\dot{h}}{h} + s_Z \frac{\dot{z}}{z} \qquad (43)$$

$$\gamma = xa_3 + a_4 b \qquad (44)$$

The counterparts of (40), (41) can be easily obtained by imposing appropriate restrictions on elasticities.

Using (36) or (43), TFPG can be estimated econometrically, either from the trend term $xa_3 + a_4 b$ of (36) or the constant term γ of (43). Alternatively, using the estimated shares $\hat{s}_K, \hat{s}_H, \hat{s}_Z$ from (36) and average growth rates of output and inputs per worker, TFPG can be calculated from (43) as

$$\hat{\gamma} = \left(\overline{\frac{\dot{y}}{y}}\right) - \hat{s}_K \left(\overline{\frac{\dot{k}}{k}}\right) - \hat{s}_Z \left(\overline{\frac{\dot{z}}{z}}\right) \qquad (45)$$

The corresponding measures for the other specifications follow directly.

5 Green TFPG Estimates

In this section we provide TFP growth estimates within the framework developed in the previous section by using: (i) independent estimates of MDCCO_2, and (ii) estimates obtained from econometric estimation

5.1 Direct Adjustment of TFPG Estimates

We adjust previous estimates of TFPG using estimates of MDCCO_2 and growth of CO_2 emissions. Tol (2005) reports 103 such estimates gathered by 28 published studies. In order to cover the range of MDCCO_2 estimates, we calculate the emission's share (35) using three point estimates for MDCCO_2, $p_Z = (\$20/tC, \$93/tC, \$350/tC)$. The results are shown in table 1.

Table 1

In table 1, the second column presents traditional TFPG (TTFPG) estimates for 1960–1995 reported in Barro and Sala-i-Martin (2004), the third column shows the corresponding average annual growth of CO_2 emissions, and columns 4-12 show the emission's share in GDP, the 'green' TFPG estimates (GTFPG) and the proportional deviation between the TTFPG and the GTFPG estimates, for the three point estimates of MDCCO_2. It can be noticed that for countries like Canada, USA, Italy and Japan, for which CO_2 emissions grow during the relevant period, total factor productivity growth is overestimated by TTFP relative to GTFPG. The average overestimation ranges from around 4.5% when MDCCO_2 is $20/tC$, to 80% when MDCCO_2 is $350/tC$.[26] This means that when the cost of using the environment is taken into account a certain proportion of what was thought as the contribution of technical change to output growth, is actually the contribution of environment, which was the uncompensated factor because of suboptimal environmental policy. In France where the growth of CO_2 emissions is very small the effect from accounting for the the use of the environment is also very small. For the UK where CO_2 emissions declined during the period under investigation, TTFPG estimates underestimated total factor productivity growth. This is because the recorded output growth corresponds to a decline in the use of the uncompensated factor, therefore there is a larger contribution of technical change to output growth relative to what is captured by TTFPG estimates. So our results suggest that when emissions grow during a given period and policy is not optimal a part of what is interpreted as growth due to technical progress should be attributed to the use of the environment as a factor of production. Negative TFPG estimates in this context could be interpreted as implying that the 'unpaid factor' environment, outweights, as a source of growth, technical change.

5.2 Econometric Estimation of TFPG.

Following the analysis in section 4.2 we estimate the following models:

---

[26]The closest observable proxy for CO_2 'price' is the recent carbon dioxide allowance price in the European Union. For the period March 2005-May 2006, this average price was $26.22 per metric ton, with a maximum of $37 and a minimum of $11.5.

Production Function Equations

$$
\begin{aligned}
PF1 \quad & \ln y = (xa_3 + ba_4)\, t + a_1 \ln k + a_2 \ln h + a_4 \ln z \\
PF2 \quad & \ln y = (xa_3 + a_4 b)\, t + a_1 \ln k + a_4 \ln z \\
PF3 \quad & \ln y_h = (xa_3 + a_4 b)\, t + a_1 \ln k_h + a_4 \ln z_h
\end{aligned}
\tag{46}
$$

Growth Accounting Equations

$$
\begin{aligned}
GA1 \quad & \frac{\dot{y}}{y} = \gamma + a_1 \frac{\dot{k}}{k} + a_2 \frac{\dot{h}}{h} + a_4 \frac{\dot{z}}{z} \\
GA2 \quad & \frac{\dot{y}}{y} = \gamma + a_1 \frac{\dot{k}}{k} + a_4 \frac{\dot{z}}{z} \\
GA3 \quad & \frac{\dot{y}_h}{y_h} = \gamma + a_1 \frac{\dot{k}_h}{k_h} + a_4 \frac{\dot{z}_h}{z_h}
\end{aligned}
\tag{47}
$$

There is a clear correspondence between $PF1 - PF3$ and $GA1 - GA3$. Regarding the estimation of the production function and the growth accounting equations the following observations are in order:

- Estimation of the growth accounting (GA) equations represent estimations of the corresponding production functions in first differences, since we use the approximation $\dot{x}/x = \ln x_t - \ln x_{t-1}$. Thus the GA estimation could address problems associated with the stationarity of the variables in levels.

- The estimation of the production function (PF) models represents estimation of a primal model, that might suffer from endogeneity associated with inputs, implying inconsistency of direct estimators of the production function. However as it has been shown by Mundlak (1996, proposition 3), under constant returns to scale, OLS estimates of a k-input Cobb-Douglas production function, in average productivity form, with regressors in inputs-labour ratio, are consistent. This type of production function is exactly what we have in $PF1$-$PF3$.

- To estimate the PF or the GA models we adopt a panel estimation approach with 'fixed effects' to allow for unobservable 'country effects' (e.g. Islam (1995). As shown by Mundlak (1996) this estimator applied to the primal problem is superior to the dual estimator which is applied to the dual functions. Furthermore the 'fixed effects' estimator addresses the problem of correlation between the constant term γ, which is the TFPG estimator in the GA models, with the regressors.[27]

- GA models can provide individual country TFPG estimates through the 'fixed effects' estimator. They are not however capable of identifying separately the contributions of labour augmenting and input augmenting technical change. Separate identification of the effect of

---

[27] This correlation has been regarded as one of the disadvantages of the regression approach in TFPG measurement (Barro 1999, Barro and Sala-i-Martin 2004).

the two possible sources of technical change is possible in the PF context. It should be noticed first that if both sources of technical change are modeled with the traditional way via a simple time trend, it is impossible to separate these two distinct effects using a single-stage estimation procedure. From $PF1$-$PF3$, it is evident that the parameters a_3 and α_4 cannot all be identified using a single-stage estimation procedure due to the linear dependency among some of the right-hand side variables and the resulting singularity of the variance-covariance matrix. At most either a_3 or a_4 can be identified implying respectively no technical change in conventional or damage abatement inputs (Kumbhakar, Heshmati and Hjalmarsson, 1997)[28].

An alternative model capable to overcome the aforementioned identification problem can be applied by altering the specification of technical change in the production function. More specifically, it is possible to separate these effects by employing Baltagi and Griffin (1988) general index to model technical change in conventional inputs and traditional simple time-trend to account for changes in the productivity of damage abatement input (Karagiannis et al., 2002). In particular relation $PF1$ may take the form[29]:

$$
\begin{aligned}
\ln y_{it} &= \zeta t + A(t) + a_1 \ln k_{it} + a_2 \ln h_{it} + a_4 \ln z_{it} \\
A(t) &= \sum_{t=1}^{T} (ba_4)_t \, D_t \\
\zeta &= xa_3
\end{aligned}
$$

and D_t is a time dummy for year t. All the relevant parameters in the above relation can be identified by imposing the restriction that as initially was suggested by Baltagi and Griffin (1988). The above specification, apart of enabling the identification of the two technical change effects is flexible as $A(t)$ is not constrained to obey any functional form, it is capable of describing complex and sometime erratic patterns of technical change consisting of rapid bursts of rapid changes and periods of stagnation, which might be relevant when we study the emission, that is, the input augmenting technical change.

---

[28]Hypothetically the Cobb-Douglas production function in relations (47) can be estimated including only the technical change in conventional inputs under a fixed or a random effects formulation and then in a second-stage individual country effects can be regressed separately against time to identify the technical change in damage abatement inputs. However, this consists only an artificial way to separate these two effects and in general is unsatisfactory solution to aforementioned identification problem. Moreover, in econometric grounds, arguments related to the efficiency of the estimated parameters surely apply compared to a single-stage estimation procedure.

[29]Relations ($PF2$) and ($PF3$) can be adjusted accordingly.

- All different specifications PF and GA were estimated using weighted least squares (WLS) in order to take into account both cross-section heteroscedasticity and contemporaneous correlation among countries in the sample. The estimation is carried out in two steps. In the first step the model is estimated via simple OLS. Using the obtained residuals the conditional country specific variance is calculated and it is used to transform both the dependent and independent variables of the second-stage regression. Specifically for each country, y_i and each element of x_i (independent variables) are divided by the estimate of the conditional standard deviation obtained from the first-stage. Then a simple OLS is performed to the transformed observations expressed as deviations of their means. This results in a feasible generalized least square estimator described by Wooldridge (2000, Ch. 8) and Greene (2003, Ch. 11)

  Estimation results are summarized in tables 4-6.

Tables 2a, 2b show estimates of the shares s_k, s_h, s_z for models $PF1 - PF3$ and $GA1 - GA3$ respectively[30].

Table 2a

Table 2b

The estimates of the input shares from the PF estimation, suggest a value for capital's share between 32% and 49.6%, a share for CO_2 emissions between 7.8% and 3.3% and a share for education in the only equation which is used as a proxy for human capital, of 4.3%.[31] When we use the GA equations, the share of capital goes down by approximately 10% while the share of emissions goes up to around 15%. The higher value of the capital share both in PF and GA estimations occur in the equation where labor input is adjusted for education with the use of the variable $L_h = LH$. In all estimations

---

[30] PF models were also estimated by using as regressors the original regressors lagged, one period, and by instrumental variables estimation using as instruments the original regressors lagged one period. There was no substantial change in the results.

[31] Capital's share increases and emissions' share decreases as we move from a model where labour is measured in physical units, to a model where labour is measured in 'quality adjusted terms' as $L_h = LH$. This can be explained in the context of an argument put forward by Griliches (1957) for a Cobb-Douglas production function. Consider the two production functions, disregarding technical change to simplify notation, $Y = K^{b_1} L^{b_2} Z^{b_3}$ and $Y = K^{a_1} (HL)^{a_2} Z^{a_3}$ and the 'auxiliary' equation $H = K^{p_1} L^{p_2} Z^{p_3}$. If the true production function is the one where labour is measured in 'quality adjusted terms', then input elasticities will be $\epsilon_1 = a_1 + p_1 a_2$, $\epsilon_2 = a_2 + p_2 a_2$, $\epsilon_3 = a_3 + p_3 a_2$. If there is a positive relationship between labour quality and capital, since higher quality of labor increases the marginal productivity of capital, and a negative relationship between labour quality and emissions (because higher quality of labor could imply high-tech and relatively clean production process), then $p_1 > 0$ and $p_3 < 0$, and capital's elasticity increases, while emissions elasticity decreases when we use quality adjusted labour input.

where labour is measured in physical units, the sum of capital's share and emissions' share is between 35% and 39%., an estimate within the expected range. The estimates for the CO_2 share in all estimated regressions, with the interpretation given in (38), are highly significant and in a sense this suggests a significant contribution of CO_2 emissions in output. This result seems to justify empirically the introduction of emissions as an input in the production function. Furthermore, by using (38), we can obtain the shadow cost of emissions as, $\mu = \hat{s}_z (Y/Z)$. Using the average values for GDP and CO_2 for the 23 OECD countries, the average shadow value of emissions μ for the sample period is between 32\$ and 76\$ per ton of CO_2. This value, which reflects the private costs in terms of profits related to CO_2 emissions, should be contrasted with the value of λ that reflects the social cost of the accumulated CO_2.

Table 3a, provides overall average estimates of labor augmenting technical change x, emission augmenting technical change b, and estimates of average TFPG obtained as $xa_3 + ba_4$. For the models that includes human capital (approximated by years of education) or does not include human capital at all, average TFPG is around 1%. When we use quality adjusted labor as input, the TFPG estimate drops to 0.4%. It should be noticed here, that our methodology allows to distinguish between two different types of technical change and identifies positive emissions augmenting technology. This result can be also regarded as an empirical verification for introducing input augmenting technical change in the production function, through the term $X = BZ$.

$Table$ 3a

Table 3b provides individual country TFPG estimates from the GA models. The estimates are obtained by adding to the overall constant of each regression the estimate of individual country fixed effect.

$Table$ 3b

As shown in table 3b the average TFPG estimates are very close to the estimates obtained from the production function in table 3a.

Table 4 uses the growth accounting equations (45) and the estimated shares from the production function to obtain TFPG estimates for individual countries.

$Table$ 4

It should be noticed that the average estimates of TFPG in table 4,, are very close to those obtained directly from the regressions using $xa_3 + ba_4$, and the GA estimates This can be regarded as providing a confirmation of the robustness of our estimations. Negative estimates of TFPG correspond

to the case where we use quality adjusted labor as input. These numbers seem to suggest that for these specific countries, the contribution of physical capital, capital quality adjusted labor and emissions to output per worker growth, exceeds the growth of output per worker.

6   Concluding Remarks

This paper aimed at formulating a new approach to Total Factor Productivity Growth measurement methodology, at a macroeconomic level, which would take into account the use of environment in the traditional TFPG measurement. We approximate the use of environment by CO_2 emissions. Our contribution at the theoretical level lies in deriving growth accounting equations with the input space of the aggregate production function augmented to include emissions and emission augmenting technical change, and interpreting the emissions share in output, in the context of a completive equilibrium under optimal taxation, as well as in the contrasting case where emissions is an unpaid factor, that is when emissions are not taxed. At the empirical level we provide (i) adjustments of existing TFPG estimates when CO_2 damages are taken into account, (ii) direct estimates of TFPG from an aggregate production function, and (iii) decomposition of technical change to labour augmenting and emissions augmenting technical change. Our approach can be regarded as a Green TFPG measurement methodology.

Our results suggest that when emissions grow, that is environment is used in production, traditional estimates overestimate TFPG relative to our estimates, by attributing part of environment's contribution to output growth, to technical change. The opposite happens when emissions decline, that is, when there are savings of environment as a factor of production, then traditional estimates underestimate TFPG. The size of deviation depends on size of damage estimates of CO_2 emissions. Direct econometric estimation of TFPG, suggests an average TFPG which for the period 1965-1990 and for the countries under examination is around 1%, or less. It also suggests that emissions in the form of CO_2 is a statistically significant input in the aggregate production function and that emission augmenting technical change coexist with labour augmenting technical change. This implies that the use of the environment approximated by CO_2 emissions, which is an unpaid factor, contributes to the growth of output along with physical capital, human capital, and labour, and its contribution should be accounted for in TFPG measurements. It should be also noted that the environment's contribution we estimated through the production function analysis might underestimate or overestimate the "socially optimal contribution", which is associated with an optimal tax determined by marginal environmental damages along the optimal path. If marginal damages are relatively high the socially optimal use of the environment in the growth process, should be relatively small,

while the opposite holds for low marginal damages. If, in the absence of optimal environmental policy, this contribution is sizable, and our results suggest that the CO_2 emissions contribution is statistically significant with a share in output which could be as high as 14%, then excess use of the environment as an input might question the eventual sustainability of the current growth process. For example if, after solving the social planner's problem, we have an estimate of λ, the true shadow value of the CO_2, and calculate emissions' share, s_Z as $\left(-\hat{\lambda}Z\right)/Y$, then the growth accounting equation (13) might produce a negative result. This result can be interpreted as an indication that total use of resources, including the "unpaid" environment properly valued, exceeds the output growth generated by these resources. In this case development that uses "unpaid" factors may be considered as not sustainable.[32] This observation provides a link between direct adjustments and econometric estimations, which approach the problem from different directions. The two approaches will coincide only along a socially optimal path.

Areas of future research include TFPG estimates by approximating environment's use not just by CO_2 emissions, but by a more general index that will include additional environmental variables; introduction of stock variables into the aggregate production function; use of our production function estimates along with damage functions for CO_2 to solve the social planners problem and define the structure and the parameters of the corresponding value functions; reformulating, at a more general level, some of the recent empirical approaches to growth to take into account possible unpaid and damage generating factors of production. We hope that this approach will enhance growth empirics by incorporating the environmental dimension in a meaningful way.

---

[32] Along the socially-optimal path the use of the "unpaid" factor the environment, will be determined by its true social shadow cost.

Appendix
Derivation of the Social Planner's Problem

Capital accumulation in per worker terms, assuming that the two capital goods depreciate at the same constant rate (Barro and Sala-i-Martin, 2004), is given by:

$$\dot{k} + \dot{h} = y - c - (\eta + \delta)(k + h) \tag{48}$$

Define in efficiency units $k = \hat{k}e^{\xi t}$, and $h = \hat{h}e^{\xi t}$, $c = \hat{c}e^{\xi t}$ so that $\dot{k} = \dot{\hat{k}}e^{\xi t} + \xi\hat{k}e^{\xi t}$ and $\dot{h} = \dot{\hat{h}}e^{\xi t} + \xi\hat{h}e^{\xi t}$. Substituting $\dot{k}$ and $\dot{h}$ in (48) we obtain:

$$\dot{\hat{k}}e^{\xi t} + \xi\hat{k}e^{\xi t} + \dot{\hat{h}}e^{\xi t} + \xi\hat{h}e^{\xi t} = e^{\zeta t}(\hat{k}e^{\xi t})^{a_1}(\hat{h}e^{\xi t})^{a_2}Z^{a_4} - \hat{c}e^{\xi t} - (\eta+\delta)(\hat{k}e^{\xi t} + \hat{h}e^{\xi t})$$

dividing by $e^{\xi t}$:

$$\dot{\hat{k}} + \dot{\hat{h}} = e^{-\xi t}\left(e^{\zeta t}\hat{k}^{a_1}e^{\xi t a_1}\hat{h}^{a_2}e^{a_2\xi t}Z^{a_4}\right) - \hat{c} - (\eta + \delta + \xi)(\hat{k} + \hat{h}), \text{ or}$$

$$\dot{\hat{k}} + \dot{\hat{h}} = e^{(\zeta - \xi + a_1\xi + a_2\xi)t}\hat{k}^{a_1}\hat{h}^{a_2}Z^{a_4} - \hat{c} - (\eta + \delta + \xi)(\hat{k} + \hat{h})$$

to make the above equation time independent we choose ξ such that $\zeta - \xi + a_1\xi + a_2\xi = 0$ or $\xi = \frac{\zeta}{1-a_1-a_2} = \frac{xa_3+a_4(b-n)}{1-a_1-a_2}$

$$\dot{\hat{k}} + \dot{\hat{h}} = \hat{k}^{a_1}\hat{h}^{a_2}Z^{a_4} - \hat{c} - (\eta + \delta + \xi)(\hat{k} + \hat{h}) \tag{49}$$

Assuming as above that the allocation between physical and human capital is such that the marginal products for each type of capital are equated in equilibrium if we use both forms of investment, we have that[33]:

$$a_1\frac{\hat{y}_t}{\hat{k}_t} - \delta = a_2\frac{\hat{y}_t}{\hat{h}_t} - \delta \tag{50}$$

The equality between marginal products implies a one to one relationship between physical and human capital, or:

$$\hat{h} = \frac{a_2}{a_1}\hat{k}, \ \dot{\hat{h}} = \frac{a_2}{a_1}\dot{\hat{k}} \tag{51}$$

Using (51) in (49) we obtain:

$$\dot{\hat{k}} + \frac{a_2}{a_1}\dot{\hat{k}} = \hat{k}^{a_1}\left(\frac{a_2}{a_1}\hat{k}\right)^{a_2}Z^{a_4} - \hat{c} - (\eta + \delta + \xi)\left(\hat{k} + \frac{a_2}{a_1}\hat{k}\right)$$

---

[33]This substitution is convenient since by adopting it we do not need a seperate state equation for human capital. It does not however affect the basic results of this section regarding the interpretation of the emissions share in output.

$$\dot{\hat{k}} = \tilde{A}\hat{k}^{\beta}Z^{a_4} - \hat{c} - (\eta + \delta + \xi)\hat{k}, \tag{52}$$

$$\tilde{A} = \left(\frac{a_2^{a_2}a_1}{a_1^{a_2}(a_1 + a_2)}\right), \ \beta = a_1 + a_2$$

Considering a utility function $U(c, S) = \frac{1}{1-\theta}c^{1-\theta}S^{-\gamma} \quad \theta, \gamma > 0$ we obtain using the substitution $c = \hat{c}e^{\xi t}$.

$$U(c, S) = \frac{1}{1-\theta}c^{1-\theta}S^{-\gamma} = \frac{1}{1-\theta}\left(\hat{c}e^{\xi t}\right)^{1-\theta}S^{-\gamma} = \tag{53}$$

$$= e^{(1-\theta)\xi t}\frac{1}{1-\theta}\hat{c}^{1-\theta}S^{-\gamma} = e^{(1-\theta)\xi t}U(\hat{c}, S)$$

Using (18), (53), (17), and (52) the social planners problem can be written as (20) ∎

Proof of Proposition 1:. *Consumers:* Defining the current value Hamiltonian for the representative consumer as:

$$H = U(c, S) + \pi(w + ra - c + na + \tau z) \tag{54}$$

standard optimality conditions imply:

$$U_c(c, S) = \pi, \ U_{cc}(c, S)\dot{c} + U_{cS} = \dot{\pi} \tag{55}$$

$$\dot{\pi} = (\rho - r)\pi \text{ or} \tag{56}$$

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) - \frac{U_{cS}}{U_{cc}}\dot{S} \tag{57}$$

*Firms:*The profit function for the firm can be written in per worker terms, using the Cobb-Douglas specification and setting $k = \hat{k}e^{\xi t}$, $h = \hat{h}e^{\xi t}$, and $\zeta - \xi + a_1\xi + a_2\xi = 0$, $\xi = \zeta - a_1\xi - a_2\xi$ as:

$$\Pi = F(K, H, E, X) - R_K K - R_H H - wL - \tau Z \tag{58}$$

or

$$\frac{\Pi}{L} = e^{\zeta t}k^{a_1}h^{a_2}Z^{a_4} - R_K k - R_H h - w - \tau z$$

$$\frac{\Pi}{L} = e^{\xi t}\hat{k}^{a_1}\hat{h}^{a_2}Z^{a_4} - R_K\hat{k} - R_H\hat{h} - w - \tau z$$

$$\tilde{\pi} \equiv \frac{\Pi}{L} = e^{\xi t}\left[f\left(\hat{k}, \hat{h}, Z\right) - R_K\hat{k} - R_H\hat{h} - we^{-\xi t} - \tau z e^{-\xi t}\right], \ z = \frac{Z}{L} \tag{59}$$

In equilibrium firms take R_K, R_H, w, and τ as given and maximize for any given level $\hat{l} = Le^{\xi t}$ by setting:

$$f_{\hat{k}} = R_K = r + \delta \tag{60}$$

$$f_{\hat{h}} = R_H = r + \delta \tag{61}$$

$$f_Z = \frac{\tau}{\hat{l}} \Rightarrow f_Z\hat{l} = \tau, \tag{62}$$

$$f_Z = \tau\frac{1}{L}e^{-\xi t}, \ z = \frac{Z}{L}, \ \hat{l} = Le^{\xi t}, \ L = \hat{l}e^{-\xi t}, \ f_Z = \frac{\tau}{\hat{l}}. \tag{63}$$

$$e^{\xi t}\left[f\left(\hat{k}, \hat{h}, Z\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \left(f_Z\hat{l}\right)ze^{-\xi t}\right] = w \tag{64}$$

The wage w equals the marginal value of labor and ensures that profits are zero in equilibrium, since by substituting (60)-(64) into (59) we obtain:

$$f\left(\hat{k}, \hat{h}, Z\right) - R_K\hat{k} - R_H\hat{h} - e^{\xi t}\left[f\left(\hat{k}, \hat{h}, Z\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \tau z e^{-\xi t}\right]e^{-\xi t} - \tau z e^{-\xi t} =$$
$$f\left(\hat{k}, \hat{h}, Z\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - f\left(\hat{k}, \hat{h}, Z\right) + f_{\hat{k}}\hat{k} + f_{\hat{h}}\hat{h} + \left(f_Z\hat{l}\right)ze^{-\xi t} - \left(f_Z\hat{l}\right)ze^{-\xi t} = 0$$

Equilibrium: In equilibrium $a = k + h$ so $\hat{a} = \hat{k} + \hat{h}$, then the flow budget constraint :

$$\dot{a} = w + ra - c - na + \tau z \qquad (65)$$

can be written as:

$$\dot{k} + \dot{h} = w + r(k + h) - c - n(k + h) + \tau z \qquad (66)$$

Setting as before $k = \hat{k}e^{\xi t}$ and $h = \hat{h}e^{\xi t}$, $c = \hat{c}e^{\xi t}$, and taking the time derivatives of k and h we obtain:

$$\dot{\hat{k}}e^{\xi t} + \xi\hat{k}e^{\xi t} + \dot{\hat{h}}e^{\xi t} + \xi\hat{h}e^{\xi t} = \qquad (67)$$
$$w + r\left(\hat{k}e^{\xi t} + \hat{h}e^{\xi t}\right) - \hat{c}e^{\xi t} - n\left(\hat{k}e^{\xi t} + \hat{h}e^{\xi t}\right) + \tau z \qquad (68)$$

substituting (60)-(64) into (66), and using in equilibrium $r = f_{\hat{k}} - \delta = f_{\hat{h}} - \delta$, $f_Z\hat{l} = \tau$, $\hat{l} = Le^{\xi t}$ we obtain:

$$\dot{\hat{k}}e^{\xi t} + \xi\hat{k}e^{\xi t} + \dot{\hat{h}}e^{\xi t} + \xi\hat{h}e^{\xi t} =$$
$$e^{\xi t}\left[f\left(\hat{k}, \hat{h}, Z\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \left(f_Z\hat{l}\right)ze^{-\xi t}\right] + \left(f_{\hat{k}} - \delta\right)\hat{k}e^{\xi t} +$$
$$\left(f_{\hat{h}} - \delta\right)\hat{h}e^{\xi t} - \hat{c}e^{\xi t} - n\left(\hat{k}e^{\xi t} + \hat{h}e^{\xi t}\right) + \left(f_Z\hat{l}\right)z \qquad (69)$$

Dividing by $e^{\xi t}$ we obtain under the Cobb-Douglas assumption:

$$\dot{\hat{k}} + \dot{\hat{h}} = \hat{k}^{a_1}\hat{h}^{a_2}Z^{a_4} - \hat{c} - (\eta + \delta + \xi)(\hat{k} + \hat{h}) \qquad (70)$$

Using as above the assumption that in equilibrium the allocation between physical and human capital is such that the marginal products for each type of capital are equated if we use both forms of investment, we have as before $a_1\frac{\hat{y}_t}{\hat{k}_t} - \delta = a_2\frac{\hat{y}_t}{\hat{h}_t} - \delta$ and $\hat{h} = \frac{a_2}{a_1}\hat{k}$, $\dot{\hat{h}} = \frac{a_2}{a_1}\dot{\hat{k}}$. Then (70) becomes

$$\dot{\hat{k}} = f\left(\hat{k}, Z\right) - \hat{c} - (\eta + \delta + \xi)\hat{k} \ , \quad f\left(\hat{k}, Z\right) = s\tilde{A}\hat{k}^{\beta}Z^{a_4} \qquad (71)$$

which is the social planners transition equation.

Setting $c = \hat{c}e^{\xi t}$ and $\dot{c} = \xi\hat{c}e^{\xi t} + \dot{\hat{c}}e^{\xi t}$ into (57) and using (60) we obtain

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, Z\right) - \rho - \delta - \xi\theta\right] - \frac{U_{\hat{c}S}}{U_{\hat{c}\hat{c}}}\dot{S} \tag{72}$$

Under optimal taxation we have from the social planner's problem that $f_Z\left(\hat{k}, Z\right) = -\lambda/p = \tau$, with $p = U_{\hat{c}}\left(\hat{c}, S\right)$, then $Z = g\left(\hat{k}, \lambda, p\right)$. Substituting Z into the equation above and into (17) we obtain

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, g\left(\hat{k}, \lambda, p\right)\right) - \rho - \delta - \xi\theta\right] - \frac{U_{\hat{c}S}}{U_{\hat{c}\hat{c}}}\dot{S}, \tag{73}$$

$$\dot{S} = g\left(\hat{k}, \lambda\right) - mS \tag{74}$$

The dynamic system (71), (73) and (74) determines the evolution of $\left(\hat{c}, \hat{k}, S\right)$ in a decentralized competitive equilibrium under optimal emission taxation. By comparing them with (21), (27), (28) it is clear that the path of the decentralized competitive equilibrium under optimal emission taxation coincides with the socially optimal path. ∎

References

1. P. Aghion and P. Howitt, "Endogenous Growth Theory," The MIT Press, Cambridge Massachusetts, (1998).

2. K. J. Arrow and M. Kurz, "Public Investment, the Rate of Return and Optimal Fiscal Policy in Baltimore", Johns Hopkins University Press for Resources for the Future, (1970).

3. B.H. Baltagi and J.M. Griffin, A General Index of Technical Change, *Journal of Political Economy* **96,** 20-41 (1988).

4. R. Barro, Notes on Growth Accounting, *J. Economic Growth* **4,** 119–137 (1999).

5. R.J Barro and X. Sala-i-Martin, "Economic Growth", McGraw Hill, New York (2004).

6. R. Becker, Intergenerational Equity: The capital Environment Trade-off, *J. Environ Econom Management* **9,** 165-185 (1982).

7. A. L. Bovenberg and S. Smulders, Environmental Quality and pollution augmenting technological change in a two sector endogenous growth model, *J. Public Econom.* **57,** 369-391 (1995)

8. W.A. Brock, A Polluted Golden Age, in "Economics of Natural and Environmental Resources" (V.L. Smith, Ed.), New York: Gordon & Breach (1973).

9. W. Brock and M. Scott Taylor, Economic Growth and the Environment: A Review of Theory and Empirics in "The Handbook of Economic Growth", (P. Aghion and S. Durlauf, Eds), a volume in the series Handbooks in Economics, edited by, Kenneth Arrow and Michael D. Intrilligator, Elsevier Publisher (2005).

10. A.B. Chimelia, J.B. Braden, Total factor productivity and the environmental Kuznets curve, J. Environ. Econom. Managem. **49** 366–380 (2005).

11. T. J. Considine, D.F. Larson, The Environment as a Factor of Production, *J. Environmen. Econom. Managem.* vol.52 **3**, 645-662 (2006).

12. P.S. Dasgupta and K.-G. Mäler, Net national product, wealth and social well-being, *Environm. Develop. Econom* **5**, 69-93 (2000).

13. E. F. Denison, "The Sources of Economic Growth in the United States and the Alternatives Before, Us", Washington DC: Committee for Economic Development (1962).

14. W. Easterly and R. Levine, What have we learned from a decade of empirical research on growth? It's not factor accumulation: Stylized facts and growth models, *The World Bank Economic Review* vol 15, **2**, 177-219 (2001).

15. R. Fare, S. Grosskopf, C.A.K. Lovell and Pasurka, Multilateral Productivity Comparisons when some Outputs are Undesirable: A Nonparametric Approach, *Review Econom.Statist.* **71**, 90-98 (1898).

16. R. Fare, S. Grosskopf, C.A.K. Lovell and S. Yaisawarng, Derivation of shadow prices for Undesirable Outputs: A Distance Function Approach, *Review Econom Statist.* **75**, 374-380 (1993).

17. W.H Greene, "Econometric Analysis", New Jersey: Pearson Education Inc. (2003).

18. Z. Griliches, Specification bias in estimates of production functions, *Journal of Farm Econom.* **39**, 8-20 (1957).

19. Z. Griliches, The Discovery of the Residual: A Historical Note, *J. Econom. Literat.* vol. XXXIV, 1324-133 (1996).

20. R. E. Hall, C. Jones, Why Do Some Countries Produce so Much More Output than Others, *Quarter J. Econom*, vol. **114**, 83-116 (1999).

21. C. Henderson, R. Russel, Human Capital and Convergence: A Production Frontier Approach, *Intern. Econom. Review*, 1167-1205 (2005).

22. N. Islam, Growth Empirics: A panel Data Approach, *Quart. J. Econom*, vol. 110, no.4, 1127-1170 (1995).

23. B. Jeon, R. Sickles, The role of environmental factors in Growth Accounting, *J. Applied Econometrics.* **19,** 567–591 (2004).

24. P. Kalaitzidakis T. Mamuneas, T. Stengos, The Contribution of Pollution to Productivity Growth, Mimeo University of Crete, (2006).

25. G. Karagiannis, P. Midmore, V. Tzouvelekas, Separating Technical Change from Time-Varying Technical Inefficiency in the Absence of Distributional Assumptions, *J. Productivity Analysis.* **18,** 23-38 (2002).

26. H. Mohtadi, Environment, growth and Optimal Policy design, *J. Public Econom.* **63**, 119-140 (1996).

27. Y. Mundlak, Production Function Estimation: Reviving the Primal, *Econometrica.* **64**, 431-438 (1996).

28. R. W. Pittman, Multilateral Comparisons with Undesirable Outputs, *Economic J.* **93**, 883-891, (1983).

29. S. Smulders, R. Gradus, Pollution Abatement and Long term Growth, Europ. J. Political Econ. **12**, 505-532 (1996).

30. R. Solow, Technical Change and the Aggregate Function, *Review Econom. Statist.* **39**, 312-20 (1957).

31. N. Stern, Stern Review on the Economics of Climate Change, *HM Treasury*, Cabinet Office (2006).

32. O.Tahvonen J.Kuuluvainen, Economic Growth, Pollution and Renewable Resources, *J. Environm. Econom. Manag.* **24**, 101-118 (1993).

33. R.S.J. Tol, The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties, *Energy Policy.* **33**, 2064–2074 (2005).

34. H. Uzawa, "Economic Theory and Global Warming", Cambridge University Press (2003).

35. J.M. Wooldridge, "Introductory Econometrics: A Modern Approach", Cincinati: South Western (2000).

36. World Bank, *World Development Indicators* (2002).

37. A. Xepapadeas, Economic Growth and the Environment in the "Handbook of Environmental Economics", Economy wide and International Environmental Issues, (Mäler K.-G., and J. Vincent, Eds), a volume in the series Handbooks in Economics, edited by, Kenneth Arrow and Michael D. Intrilligator, Elsevier Publishers (2005).
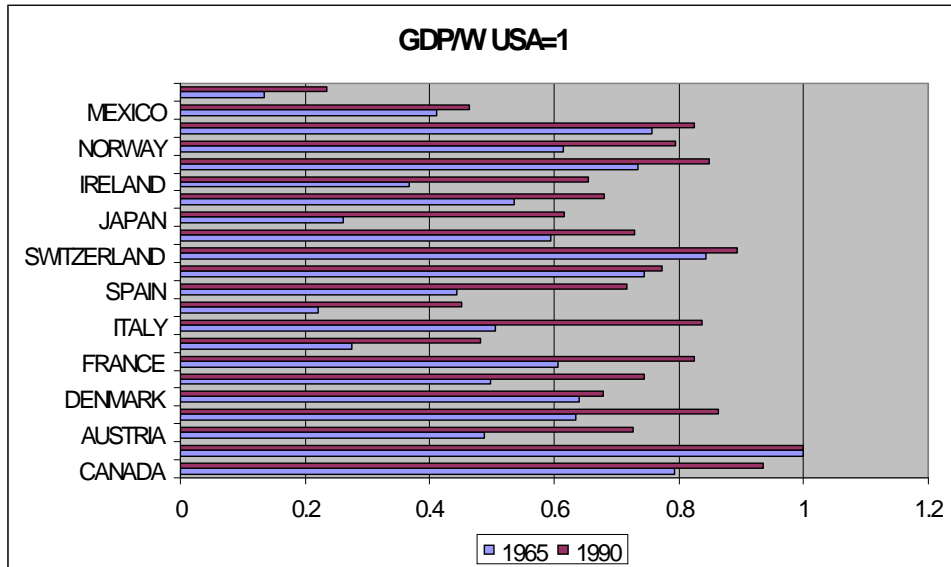
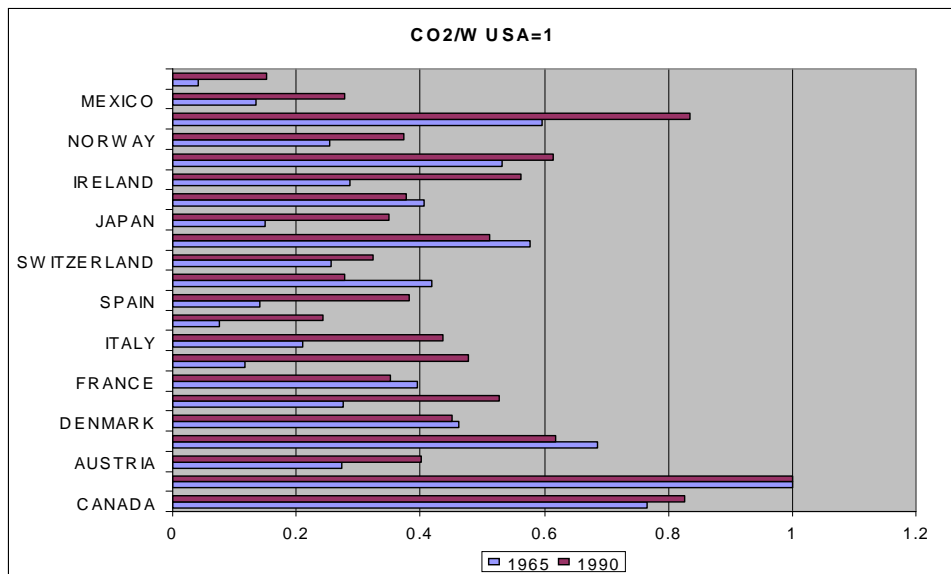Figure 1: GDP per worker in 1965 and 1990 (USA=1)

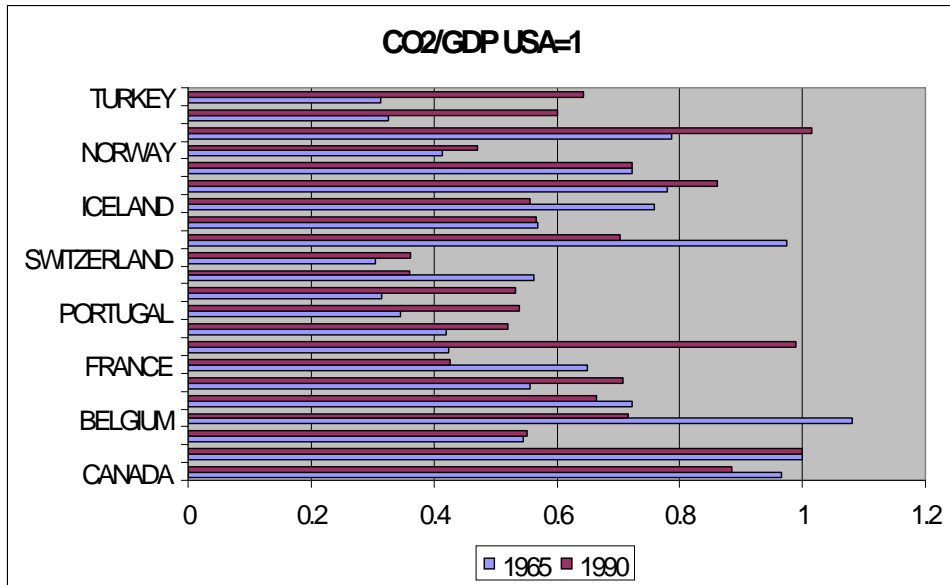Figure 2: CO_2 per worker in 1965 and 1990 (USA=1)

1

Figure 3: CO_2/GDP in 1965 and 1990 (USA=1)
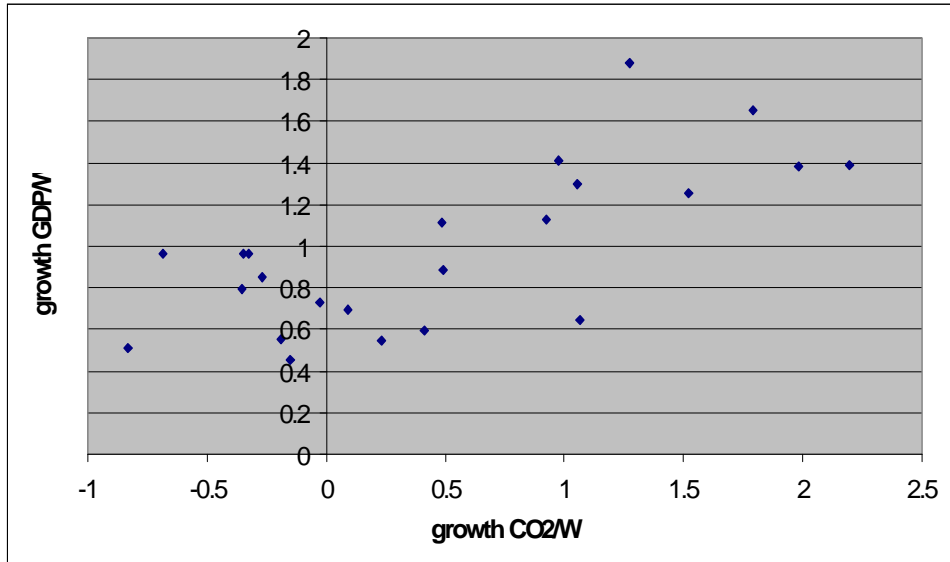

Figure 4: Growth of GDP per worker vs growth of CO_2 per worker.
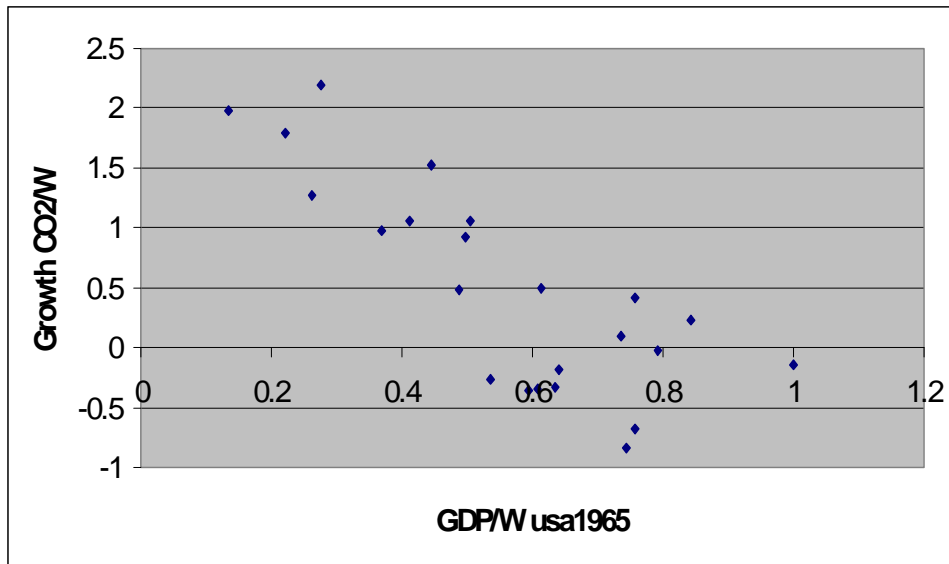
2

Figure 5: Growth CO_2/W vs GDP/W (GDP/W for USA=1 at 1965)

Table1− *Direct adjustments of traditional TFPG estimates*

Countries	*T.TFPG* (%)	*Gr.CO$_2$* (%)	MDCCO$_2$ $20/tC$			*MDCCO$_2$* $93/tC$			*MDCCO$_2$* $350/tC$		
			s_{iz}	*GTFPG* (%)	*%Dev.*	s_{iz}	*GTFPG* (%)	*%Dev.*	s_{iz}	*GTFPG* (%)	*%Dev.*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(6)	(9)	(10)	(11)	(12)
CANADA	0.57	2.12	0.0241	0.52	−9.01	0.1123	0.33	−41.9	0.4226	−0.33	−157
U.S.A.	0.76	1.32	0.0266	0.72	−4.62	0.1237	0.60	−21.5	0.4657	0.14	−80.9
FRANCE	1.3	0.04	0.0146	1.30	−0.05	0.0679	1.30	−0.22	0.2556	1.29	−0.84
ITALY	1.53	2.97	0.0128	1.49	−2.50	0.0597	1.35	−11.6	0.2248	0.86	−43.7
U.K.	0.8	−0.35	0.0213	0.81	0.94	0.0991	0.83	4.39	0.3729	0.93	16.5
JAPAN	2.65	4.07	0.0156	2.59	−2.39	0.0724	2.35	−11.1	0.2724	1.54	−41.9

Column (2): Traditional TFPG estimates. Source: Barro and Sala-i-Martin (2004)

Column (3): Average annual growth of CO_2 emissions

Columns (4,7,10): Emissions share in GDP using the corresponding MDCCO_2 estimate.

Columns (5,8,11): Green TFPG estimates

Columns (6,9,12): Proportional deviation between traditional TFPG and GTFPG estimates.

Table2a— *Production Function Estimation for the three PF models*

	$PF1$	$PF2$	$PF3$
c	-0.25711	-0.20460	-0.08791
$a_1 = s_k$	0.32199	0.32597	0.49580
$a_4 = s_z$	0.07603	0.07774	0.03294
$a_2 = s_h$	0.04256	$-$	$-$
ba_4	0.002059	0.002064	0.0028012
xa_3	0.009169	0.008611	0.000593
R^2	0.99	0.99	0.99
DW	2.00875	2.02950	2.00932

All coefficients are significant at 1% level

Table2b— *Growth Accounting Estimation for the three GA models*[*]

	$GA1$	$GA2$	$GA3$
$a_1 = s_k$	0.21494	0.21485	0.44633
$a_4 = s_z$	0.14407	0.14448	0.15488
$a_2 = s_h$	0.02405	$-$	$-$
R^2	0.89	0.89	0.97
DW	2.05828	2.05849	2.06371

All coefficients are significant at 1% level

(*) We do not report the constant term since the overall constant plus the fixed effect estimator for each county defines the TFPG for this country. These estimates are reported in table 3b.

1

Table 3a:— *TFPG and technical change estimates using the production function*

	xa_3	ba_4	x	b	$TFPG$
$PF1$	0.00917	0.00206	0.01639	0.02708	0.01122
$PF2$	0.00861	0.00206	0.01444	0.02656	0.01067
$PF3$	0.00059	0.00280	0.00126	0.08504	0.00339

Table 3b: *TFPG estimates using the growth accounting equations*

Countries	$GA1$	$GA2$	$GA3$
CANADA	0.009825	0.009452	−0.00057
U.S.A.	0.005149	0.004922	−0.003864
AUSTRIA	0.011807	0.011726	−0.00208
BELGIUM	0.017204	0.01691	0.01179
DENMARK	0.007932	0.007759	−0.000514
FINLAND	0.017121	0.016993	0.007033
FRANCE	0.014472	0.014404	0.002705
GREECE	0.014883	0.015025	−0.001442
ITALY	0.018542	0.018566	0.007159
LUXEMBOURG	0.021252	0.021199	0.013261
PORTUGAL	0.023597	0.023525	0.009182
SPAIN	0.010792	0.010754	−0.00578
SWEDEN	0.009019	0.00885	0.000109
SWITZERLAND	0.005414	0.005204	−0.002261
U.K	0.01332	0.013055	0.007811
JAPAN	0.023158	0.022758	0.007299
ICELAND	0.011533	0.010966	0.002536
IRELAND	0.022938	0.022673	0.013404
NETHERLANDS	0.010253	0.00991	0.003105
NORWAY	0.018458	0.018253	0.01395
AUSTRALIA	0.007183	0.006713	0.000304
MEXICO	0.005397	0.004921	−0.006345
TURKEY	0.014218	0.013845	0.000786
AVERAGES	0.013629	0.013408	0.003373

Table 4: *TFPG calculations using factor shares estimates from the production function*

Countries	PF1	PF2	PF3
CANADA	0.00657	0.005774	−0.002668
U.S.A.	0.00221	0.001713	−0.006308
AUSTRIA	0.00659	0.006204	−0.003676
BELGIUM	0.01329	0.012640	0.009634
DENMARK	0.00399	0.003563	−0.003396
FINLAND	0.01424	0.013842	0.0070440
FRANCE	0.00959	0.009315	−0.000681
GREECE	0.01306	0.013059	0.001541
ITALY	0.01589	0.015753	0.007459
LUXEMBOURG	0.01761	0.017429	0.009761
PORTUGAL	0.02146	0.021091	0.011636
SPAIN	0.00682	0.006479	−0.004975
SWEDEN	0.00356	0.003142	−0.004999
SWITZERLAND	0.00230	0.001786	−0.003274
U.K	0.00931	0.008716	0.005570
JAPAN	0.01724	0.016188	0.007519
ICELAND	0.00705	0.005885	−0.000350
IRELAND	0.01981	0.019137	0.013902
NETHERLANDS	0.00739	0.006656	0.001841
NORWAY	0.01816	0.017760	0.013792
AUSTRALIA	0.00578	0.004844	0.000414
MEXICO	0.00441	0.003427	−0.00555
TURKEY	0.01253	0.011634	0.00356
Averages	0.010385	0.0098277	0.002513