

Regime Switching and the Shape of the Emission-Income Relationship

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Abstract

We explore the idea of regime switching as a new methodological approach in the analysis of the emission - income relationship. We formalize the idea by using a simple static model of profit maximization where above a threshold income level a more stringent environmental policy induces a decreasing emission-income relationship. At the empirical level we estimate such a regime switching model and we find an inverse-V-shaped emission - income relationship. Our findings are in line with the original papers in this literature. We estimate thresholds which can be viewed as turning points, and which occur at reasonable values.

Keywords: Environmental Kuznets Curve, environmental policy, regime switching, thresholds.

JEL Classification: C2, O1, Q2.

1 Introduction

In the analysis of emission-income relationship,¹ there exists a set of theoretical models which derives inverted “V” shaped curves by having pollution

¹For a recent literature review, see for example Levinson (2002).

increasing with income until some threshold point is passed, after which pollution is reduced. John and Pecchenino (1994) consider an overlapping generations model where economies with low income or high environmental quality are not engaged in environmental investment, that is, pollution abatement. When environmental quality deteriorates with growth, the economy moves to positive abatement, then environment improves with growth and the relationship is inverted “V” shaped. Stokey (1998) generates an inverted “V” shaped curve by considering a static optimization model where below a threshold income level only the dirtiest technologies are used. As economic activity and pollution increase, the threshold level is passed and cleaner activities are used. Jaeger (1998) derives the inverted “V” shaped curve by considering a threshold in consumer preferences. Below the threshold the marginal benefits from improving environmental quality are small, whereas when pollution increases with growth and the threshold is passed, quality may be improved. Jones and Manuelli (2001) develop a different model which relates explicitly to environmental policy. Environmental policy is decided by majority voting and could take the form of either emission taxes or “minimum standards” in technology. In countries with low income, per capita emission taxes are chosen to be zero, and when income increases positive taxes are chosen and an inverted “V” shaped curve is derived. When minimum standards are chosen, the pollution-income relationship is monotonic and converges to a limiting pollution level.

The basic idea underlying all these models is that when some threshold is passed, then the economy moves to another regime, with the emission - income relationship being different between the old and the new regime. In the inverted “V” models, the low income regime corresponds to an increasing emission - income relationship, while in the regime after the threshold the emission - income relationship is decreasing.

The purpose of the present paper is to explore the idea of regime switching as a new methodological approach in the analysis of the emission - income relationship. At a theoretical level we formalize the idea by deriving, using a simple static model of profit maximization, an emission function that depends on income and an environmental policy parameter. For lax or ineffective environmental policy the emission - income relationship is increasing, while for a stringent environmental policy the emission - income relationship is decreasing.

In the real world the environmental policy parameter is not chosen optimally and its stringency depends on the developmental stage of the economy. For example, as noted in Jha and Whalley (2001), a common feature of environmental policy in developing countries, when it exists, is limited compliance and weak enforcement of command and control measures. This however im-

plies that the effectiveness of environmental policy is limited. Thus lax or ineffective environmental policy at low income levels could be associated with the increasing part of the “inverted V”, while stringent or effective policy at high income levels could be associated with the decreasing part of the “inverted V”. Each of part of the “V” corresponds to a different regime, with regime switching at some threshold income level. this threshold level corresponds to a developmental level at which the institutional framework and the public awareness can support the enforcement of a relative more stringent environmental policy. Thus in our model regime switches with respect to the emission - income relationship result from observed behavior of changes in the policy regime, with environmental policy becoming more stringent at relatively higher developmental stages.

At the empirical level we estimate such a regime switching model. In this model regression functions are not identical across all observations in a sample but fall into discrete classes. One class could correspond to a more stringent environmental policy regime, while another class could correspond to a laxer environmental policy regime. By using regime-switching models we manage to find an inverse-V-shaped emission - income relationship. Our findings are in line with the original papers in this literature (Grossman and Krueger, 1995, Shafik and Bandyopadhyay, 1994), which obtained robust inverse-U-shaped relationships. We estimate thresholds which can be viewed as turning points, and which occur at reasonable values. Thus the main contribution of our paper can be regarded as a confirmation and, in a way, a re-establishment of the environmental Kuznets curve by using a different methodological approach.²

2 Emission-income relationships and environmental policy

We consider an economy where the production sector consists of $j = 1, \dots, J$ firms, each one producing a consumption good q_j using a primary input z_j and a strictly concave production function $q_j = f_j(z_j)$. Output prices $\mathbf{p} = (p_1, \dots, p_J)$ and input price w are given. Firms generate emissions during output production which can be abated using primary input a_j . The emission

²Harbaugh, Levinson and Wilson (2001) argue that after the work by Grossman and Krueger, researchers in this area used sophisticated methods and included multiple control variables with mixed results. Our approach establishes the inverted “V” by using a regime switching model with one control variable, GDP per capita.

function for each firm is defined as

$$e_j = \phi_j(q_j, a_j) = \phi_j(f_j(z_j), a_j) = s_j(z_j, a_j) \quad (1)$$

The emission function is strictly increasing and convex in output for fixed abatement and decreasing in abatement.³

Environmental policy can be introduced into this model by assuming an emission tax $\tau \geq 0$ per unit of output.⁴ Given output prices, input price and the emission tax, a profit maximizing production - emission plan for the firm solves:

$$\max_{z_j \geq 0, a_j \geq 0} p_j f_j(z_j) - w z_j - w a_j - \tau s_j(z_j, a_j) \quad (2)$$

With total factor supply y and assuming interior solutions, equilibrium in the factor market implies

$$p_j \frac{\partial f_j(z_j^0)}{\partial z_j} = w + \tau \frac{\partial s_j(z_j^0, a_j^0)}{\partial z_j} \quad (3)$$

$$w = -\tau \frac{\partial s_j(z_j^0, a_j^0)}{\partial a_j}, j = 1, \dots, J \quad (4)$$

$$\sum_{j=1}^J (z_j^0 + a_j^0) = y \quad (5)$$

Consider now the problem of a social planner seeking to maximize total revenues less environmental damages⁵ which are represented by a strictly increasing and convex damage function $D(E)$, $E = \sum_{j=1}^J e_j = \sum_{j=1}^J s_j(z_j, a_j)$, or

$$\max_{\substack{(z_1, \dots, z_J) \geq 0 \\ (a_1, \dots, a_J) \geq 0}} \sum_{j=1}^J p_j f_j(z_j) - D(E) \quad (6)$$

$$\text{subject to } \sum_{j=1}^J (z_j + a_j) = y \quad (7)$$

Associating the Lagrangian multiplier μ with the resource constraint (7), the

³ \mathbf{z} can be regarded as polluting inputs while \mathbf{a} can be regarded as abatement inputs.

⁴Under the assumptions of this model, similar results can be obtained by tradable emission permits, when product and permit markets are competitive.

⁵This is equivalent to maximizing consumer welfare.

first order conditions for an interior solution for the social optimum imply

$$p_j \frac{\partial f_j(z_j^*)}{\partial z_j} = \mu + D' \tau \frac{\partial s_j(z_j^*, a_j^*)}{\partial z_j} \quad (8)$$

$$\mu = -D' \frac{\partial s_j(z_j^*, a_j^*)}{\partial a_j} \quad (9)$$

$$\sum_{j=1}^J (z_j^* + a_j^*) = y \quad (10)$$

Solving the first order conditions, the equilibrium allocation of pollution and abatement inputs are solutions

$$\begin{aligned} z_{jl}^* &= h_j^*(y) \\ a_{jl}^* &= g_j^*(y) \end{aligned}$$

Then the socially optimal emission income relationship is defined as

$$E^* = \sum_{j=1}^J s_j(h_j^*(y), g_j^*(y)) \quad (11)$$

It is clear from (3)-(5) and (8)-(10) that if we choose $\tau = D'(E^*)$ then the regulated private optimum is equivalent to the social optimum. If however environmental policy is not chosen optimally but the policy parameter is set arbitrarily below the optimal level or $\tau \in [0, D'(E^*))$, then the equilibrium resource allocation, for a *given* choice of environmental policy, is determined by solving (3)-(5) as

$$\begin{aligned} z_j^0 &= h_j^0(y, \tau) \\ a_j^0 &= g_j^0(y, \tau) \end{aligned}$$

in which case the emission - income relationship is defined as:

$$E^0 = \sum_{j=1}^J s_j(h_j^0(y, \tau), g_j^0(y, \tau)) \quad (12)$$

Then (12) can be interpreted as the emission-income relationship for any *given* suboptimal environmental policy. If the environmental policy is chosen optimally in the sense that $\tau = D'(E^*)$, then (12) is identical to (11).

When we study real economies the assumption of an optimal environmental policy is very unlikely to hold. What is observed in reality is that

environmental policy is related to the developmental stage of the economy in the sense that the policy parameter can be written as depending on y as follows.

$$\tau_y = \begin{cases} 0 & \text{if } y < y_1 \\ \tau_{y_2} & \text{if } y < y_2 \\ \dots & \\ \tau_{y_n} & \text{if } y > y_n \end{cases} \quad (13)$$

This implies that the emission - income relationship can be written as

$$E^R = \sum_{j=1}^J s_j (h_j^0(y, \tau_y), g_j^0(y, \tau_y)) \quad (14)$$

In (14) τ_y can be regarded as a switch or transition function. When the environmental policy parameter changes in response to income passing a threshold, the emission - income relationship moves to a new regime.

An inverted “V” emission relationship can be derived using the following simple example. Assume only one firm with

$$p = 1, f(z) = z^\beta, s(z, a) = \phi z^\beta - a^\gamma$$

Then conditions (3)-(5) imply

$$Aa^{\frac{\gamma-1}{\beta-1}} + a = y, A = \left(\frac{\tau_y \gamma}{1 - \tau_y \phi} \right)^{\frac{1}{\beta-1}}$$

Assuming $\frac{\gamma-1}{\beta-1} = 1$ we obtain $h^0(y, \tau_y) = y \left(\frac{A}{1+A} \right)$, $g^0(y, \tau_y) = y \left(\frac{1}{1+A} \right)$. A numerical simulation result is presented in figures 1 and 2 for parameter values $\beta = \gamma = 0.8$, $\phi = 0.5$. Initially we set $\tau = 0.01$ and we keep it at this level for $y \in [0, 1000]$. The emission - income relationship is linear and increasing as shown in figure 1.

At the level of $y = 1000$ we introduce a stringent policy with $\tau_{y=1000} = 0.71$. Then the emission - income relationship switches to a new regime and is decreasing as shown in figure 2.

It should be noted that up to $\tau = 0.69$, the emission - income relationship has a positive slope so in our simple example regime switching takes place at $y = 1000$ for $\tau = 0.7$. Thus regime switching takes place at an income level where the environmental policy is sufficiently strong to change substantially the slope of the emission - income relationship.

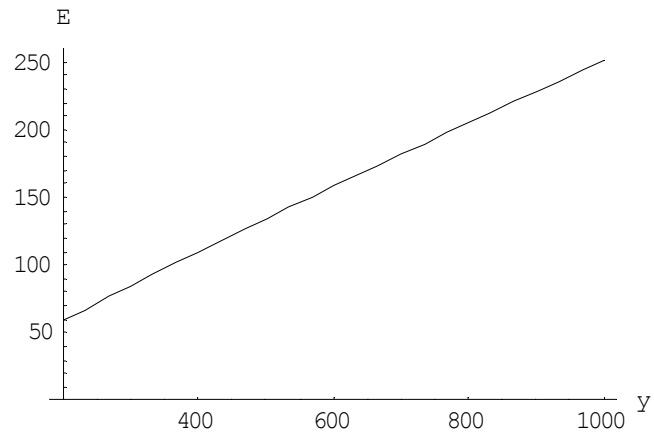


Figure 1: The increasing part of the inverted “V”

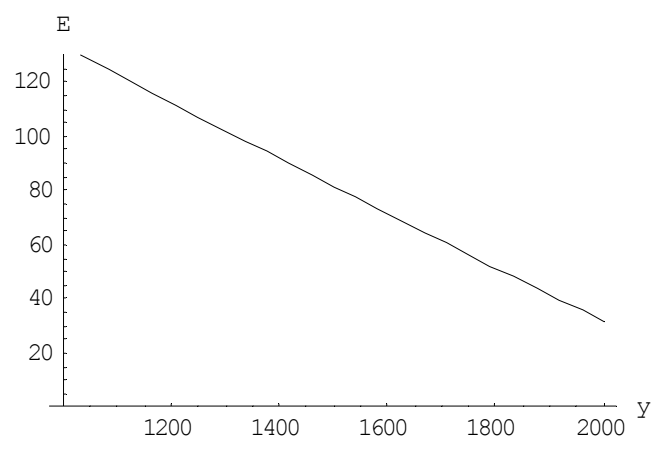


Figure 2: The decreasing part of the inverted “V”

3 Empirical Analysis

3.1 Methodology

A natural approach to modeling economic variables seems to be to define different states of the world or regimes, and to allow for the possibility that the behavior of economic variables depends on the regime that occurs at any given observation. By ‘regime-switching behavior’ it is meant that regression functions are not identical across all observations in a sample or fall into discrete classes. One of the most prominent among the regime-switching models in the macroeconometrics area has been the threshold class of models (Tong, 1983; Tong and Lim, 1980) and its smooth transition generalization (STAR models) promoted by Teräsvirta and his co-authors (Teräsvirta and Anderson, 1992; Granger and Teräsvirta, 1993; Teräsvirta, 1994). Regime-switching models are flexible enough to allow several different types of effects that could be observed in the relation between pollution and income. The structural equation of interest is the one-threshold smooth transition regression (STR) model given by

$$E_{it} = \beta_{00} + \beta_{01}y_{it} + (\beta_{10} + \beta_{11}y_{it}) F(y_{it}) + u_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T \quad (15)$$

where E_{it} is a measure of air pollution in monitoring station i in year t , y_{it} is per capita GDP in year t in the country in which station i is located, $\beta \equiv (\beta_{00}, \beta_{01}, \beta_{10}, \beta_{11})'$ is a parameter vector, and u_{it} is an error term. The function $F(y_{it})$ is the transition function, which is continuous and bounded by zero and unity and y_{it} is assumed to act as the transition variable. That is, in terms of the theoretical model developed in the previous section, the $F(y_{it})$ is the switch function τ_y . Values of zero by the transition function identify the one regime, say the “no or very lax environmental policy regime”, and values of unity identify the alternative “strict environmental policy regime”. An alternative intuitive way of writing (15), which can be compared to (13), is:

$$E_{it} = \begin{cases} \beta_{00} + \beta_{01}y_{it} + u_{it} & \text{if } F(y_{it}) = 0 \\ (\beta_{00} + \beta_{01}y_{it}) + (\beta_{10} + \beta_{11}y_{it})y_{it} + u_{it} & \text{if } F(y_{it}) = 1 \end{cases} \quad (16)$$

Obviously, a weighted mixture of these two regressions applies if $0 < F(y_{it}) < 1$. If $F(y_{it})$ smoothly changes from 0 to 1 (or vice versa), then the coefficients themselves change smoothly between the two extremes.

The practical applicability of the above specification depends on how $F(y_{it})$ is defined. One form of transition function used in the literature is the logistic function

$$F(y_{it}; \gamma, c) = [1 + \exp(-\gamma(y_{it} - c))]^{-1} \quad \gamma > 0 \quad (17)$$

where the parameter c is the threshold between the two regimes or the location of the transition function, and the parameter γ determines the smoothness of the change in the value of the logistic function and thus the speed of the transition from one regime to the other. When $\gamma \rightarrow \infty$, then $F(y_{it})$ becomes a step function ($F = 0$ if $y_{it} \leq c$ and $F = 1$ if $y_{it} > c$), and the transition between the regimes is abrupt. In that case, the model approaches a threshold model. Hence, the STR model nests the threshold model as a special case.

To estimate the STR model it is computationally convenient to first concentrate on the transition function parameters. Note that giving fixed values to the parameters in the transition function makes the STR model linear in parameters. That is, conditional on the transition function, the parameters of the STR can be estimated by OLS. We first carried out a two-dimensional grid search procedure using 150 values of γ (1 to 150) and at least 100 equally spaced values of c within the observed range of the transition variable. Essentially, y_{it} is ordered by value, extremes are ignored by omitting the most extreme 20 values at each end and the 100 values are specified over the range of the remaining values. This procedure attempts to guarantee that the values of the transition function contains enough sample variation for each choice of γ and c . The model with the minimum RSS value from the grid search is used to provide γ and c . Following Teräsvirta (1994) the exponent of the transition function is standardized by the sample standard deviation of the transition variable. This makes γ scale-free and helps in determining a useful set of grid values for this parameter.

Model (15) has a single threshold. An obvious extension could be to permit more multiple thresholds. For example, the double threshold or three-regime model takes the form

$$E_{it} = \beta_{00} + \beta_{01}y_{it} + (\beta_{10} + \beta_{11}y_{it}) F_1(y_{it}) + (\beta_{20} + \beta_{21}y_{it}) F_2(y_{it}) u_{it}$$

where y_{it} determines both transitions, and the second transition function is defined analogously to (17). If it is assumed that $c_1 < c_2$, the parameters of this model change smoothly from $\beta_0 \equiv (\beta_{00}, \beta_{01})'$ via $\beta_1 \equiv (\beta_{10}, \beta_{11})'$ to $\beta_2 \equiv (\beta_{20}, \beta_{21})'$ for increasing values of y_{it} . Specification of the double threshold model involves a modeling procedure analogous to the single transition case. Here, a four dimensional grid search is performed over $\gamma_1, \gamma_2 = 1, \dots, 150$ and 50 values of c_1, c_2 over the range of the transition variable.⁶

⁶Essentially, the first threshold is considered to be over the left part of the observed range of GDP series whereas the second threshold is over the right part.

3.2 Empirical results

To analyze common air pollutants we use the data for sulphur dioxide (SO₂), smoke and total suspended particulates (TSP) studied by Harbaugh, Levinson and Wilson (2001). For SO₂ there are 2381 annual observations from 45 countries, 102 cities and 285 sites over the period 1971 to 1992. The available sample for the other two pollutants is relatively small. For smoke, for example, there are 687 annual observations from 21 countries, 32 cities and 96 sites over the same period. National income is measured by real per capita GDP, in 1985 dollars, from the Penn World Tables as described in Summers and Heston (1991).⁷

The estimated STR models for SO₂ and smoke are presented in Table 1. In the first panel, the single-equation model for SO₂ gives a threshold at per capita GDP of \$8,779, which is a mid-point in the distribution of the GDP variable. In view of that, the implication of the coefficients is that the effect of income on pollution is negative, though smaller in (absolute) magnitude in countries with ‘low-to-middle’ income. Furthermore, ‘middle-to-high’ income countries are associated with an intercept of 108.67, while the model for ‘low-to-middle’ income countries implies an intercept of 67.93. This indicates that as income increases, the negative effect on pollution is relatively larger for ‘middle-to-high’ income countries. On the other hand, the double-threshold model is more intuitive. The threshold estimates are \$5,472 per capita and \$10,220 implying three classes of countries, those with ‘low’ income, ‘middle’ income and ‘high’ income. What is more interesting, however, is that pollution increases with economic growth in ‘low’ and ‘middle’ income countries, whereas it eventually begins to decline in ‘high’ income ones. It is also interesting to notice that when comparing ‘low’ income with ‘middle’ income countries, the model implies stronger (positive) GDP effects in the latter group, though GDP in the first STR component (concerning middle income countries) is not statistically significant (t-ratio is 1.451). In most cases, the income variables and constants are significant at the 10%, 5% and even 1% level. As to the slope (smoothness) parameters in both models the estimated values are large, implying abrupt regime-switch and therefore threshold specifications.

The same results can also be drawn from the model for smoke reported in the third panel of Table 1. The estimates show that pollution is initially increasing and peaks at per capita GDP of \$7,511, but after that point increases in income are associated with an improvement in environmental quality. As before, the regime-switch here is also instantaneous. We also tried to fit a two-threshold model but it seemed spurious since the second

⁷See Harbaugh, Levinson and Wilson (2001), for more details on the data.

threshold was too extreme to represent ‘low’ income countries.

These findings are in line with the original papers in this literature (Grossman and Krueger, 1995; Shafik and Bandyopadhyay, 1994), which find robust inverse-U-shaped relationships. Essentially, the main contribution of our study is that it confirms and in a way re-establishes the environmental Kuznets curve from another angle. By using regime-switching models we manage to find an inverse-V-shaped emission-income relationship. The thresholds, which can be viewed as turning points, occur at reasonable values. Concentrations of sulphur dioxide and smoke are found to peak at a relatively early stage in national development (before a country reaches a per capita income of \$10,220), and then decrease at high levels of income.

Table 1: Regime-switching STR models for SO₂ and Smoke

<i>Single-threshold model for SO₂</i>
$SO_2 = 67.93 - 2.069 * GDP + (40.73 - 2.773 * GDP) * F(GDP)$ (24.14) (-3.270) (5.510) (-3.504)
Classification of regimes
$SO_2 = 67.93 - 2.069 * GDP, GDP < 8,779$ 1066obs
$SO_2 = 108.7 - 4.842 * GDP, GDP \geq 8,779$ 1315obs
$R^2 = 0.0791$
<i>Double-threshold model for SO₂</i>
$SO_2 = 60.37 + 6.114 * GDP + (-103.5 + 4.914 * GDP) * F_1(GDP)$ (13.25) (1.737) (-2.189) (1.451)
$+ (107.9 - 13.3 * GDP) * F_2(GDP)$ (3.102) (-2.863)
Classification of regimes
$SO_2 = 60.37 + 6.114 * GDP, GDP < 5,472$ 801obs
$SO_2 = -43.1 + 11.03 * GDP, 5,472 \leq GDP < 10,220$ 402obs
$SO_2 = 64.88 - 2.271 * GDP, GDP \geq 10,220$ 1178obs
$R^2 = 0.0764$
<i>Single-threshold model for Smoke</i>
$SM = 55.38 + 4.683 * GDP + (46.07 - 11.56 * GDP) * F(GDP)$ (8.315) (3.277) (1.951) (-4.425)
Classification of regimes
$SM = 55.38 + 4.683 * GDP, GDP < 7,511$ 405obs
$SM = 101.4 - 6.881 * GDP, GDP \geq 7,511$ 282obs
$R^2 = 0.2097$

Notes: All estimated slope parameters are large, implying threshold models; values in parentheses are *t*-ratios.

It is worth mentioning that two extensions were also considered in the

process of the empirical analysis before we settled on the model proposed here. First, we expand our specification by estimating a STR model where the relationship between pollution and income was assumed to be a quadratic or/and cubic polynomial in GDP in line with specifications used in the literature. In fact, only the coefficients in the quadratic polynomial model were significant and, therefore, we only extended the model to a quadratic STR specification. However, the estimated threshold we obtained was very extreme which was an indication of an inadequate model.⁸ Second, we considered the threshold model for balanced panels with individual-specific fixed effects introduced by Hansen (1999). However, it was not possible to estimate this model since our dataset is highly unbalanced.⁹ While for developed countries continuous time series observations from many different sites and cities are available for long periods, for developing countries, the sample size is small¹⁰.

4 Concluding Remarks

In this paper we explore the possibility of modelling the EKC through a threshold model. The underlying assumption is that as income goes through a certain threshold, a more stringent environmental policy or possible output composition effects introduce a new emission - income regime. In the new regime the emission - income relationship is decreasing, while in the old regime, say with lax environmental policy, the emission - income relationship is increasing. We develop a simple theoretical model that generates this result. We empirically estimate the EKC using threshold models. We argue that the composition and technology effects¹¹ imply increasing per capita income, so we focus solely on the relationship between pollution and income by motivating the use of regime-switching models. Our results confirm the early literature (e.g. Grossman and Krueger 1995) regarding inverted “U” shapes for the EKC. A message that can be drawn from our results is that the empirical literature has concentrated on postulating sophisticated models and incorporating too many explanatory variables to answer the fundamental question: does environmental quality deteriorate with economic growth? Since, however, a possible explanation of the inverted “U” is that it is caused by switching to a new regime where factors such as stringency of environ-

⁸Results are available from the authors upon request.

⁹When we restrict the sample size to make it balanced, we are left with at most 450 observations.

¹⁰The US and Canada together account for almost one-third of the SO₂ observations.

¹¹For a detailed explanation of these effects, see Grossman and Krueger (1995).

mental policy, or output composition, which are correlated with per capita GDP, determine the shape of the emission-income relationship, the regime switching model with only per capita GDP as an explanatory variable could be a more promising approach.

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