Portfolio Management: An investigation of the implications of measurement errors in stock prices on the creation, management and evaluation of stock portfolios, using stochastic simulations.

By:
Dikeos Tserkezos, University of Crete
Eleni Thanou, Hellenic Open University

ABSTRACT

In this paper, we investigate the implications of measurement errors in the daily published stock prices on the creation and management of efficient portfolios. Using stochastic simulation techniques and the Markowitz Mean Variance approach in the creation of the weights of the various stocks of a portfolio, we conclude that measurement errors have significant implications on the efficiency of the management of a stock portfolio.

Keywords: Markowitz Mean Variance, Measurement Errors in Returns, Stochastic Simulation.

JEL classification: G45

1. Introduction

An interesting issue for investigation in Portfolio Analysis is the implications of measurement errors in the stock returns that are included in the portfolio's composition. The interesting questions to be investigated are the implications of measurement errors in stock returns both on the portfolio's composition as well as on the calculation of the portfolio's short and longer term returns.[1].

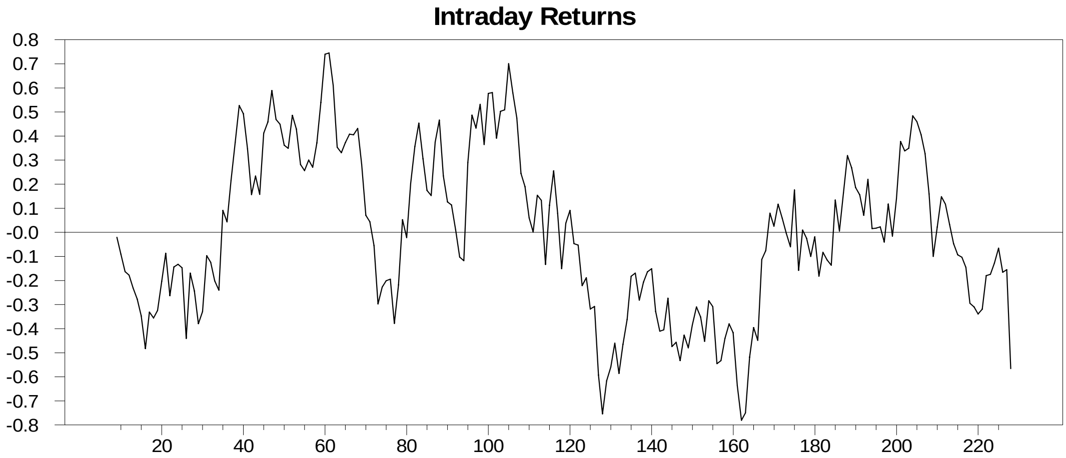
The prices at which stocks are bought and sold during an Exchange's trading session usually vary significantly from the closing price. Each Exchange uses a method to calculate a final, closing price for each stock. This is the price used by investors and fund managers to evaluate their portfolios. In theory, the closing price should reflect in the best possible way the trend of the stock's price as it has been formed during the trading session. If the method of calculation of this price contains "errors", that is it does not accurately reflect all the information contained in intraday prices, then these "measurement errors" could have some implications in stock selection, portfolio management and evaluation etc. Moreover, in practice, the methods of calculation of stocks' closing prices are sometimes prone to manipulation, in other words certain trades are performed during the closing period in view of affecting the official closing price.[2]

In the case we analyze, that of the Athens Stock Exchange, the actual trading hours are from 10:00 in the morning until 5:00 in the afternoon. According to ATHEX's Trading Regulations, the Central Trading System of the ATHEX Exchange can utilize any one of five algorithms to calculate the closing prices. These are: 1. Last traded price 2. Weighted average of the x last trades. 3. Weighted average of x% of the trades, 4. Weighted average of the trades that make up x% of the total volume traded during the session. 4. Weighted average of all the trades of the last x minutes of the session. In all cases, the weights used are the number of stocks in each trade. From our information, the method that is actually used is the last one, where the weighted average of the last 30 minutes is used. In fact, the Board of the Exchange, recognizing the possibility of manipulation, retains the right to use any one of the above methods without formally announcing which one is used.

[1] Aggregate and average returns as well as qualitative portfolio characteristics such as symmetry, kurtosis and deviations.

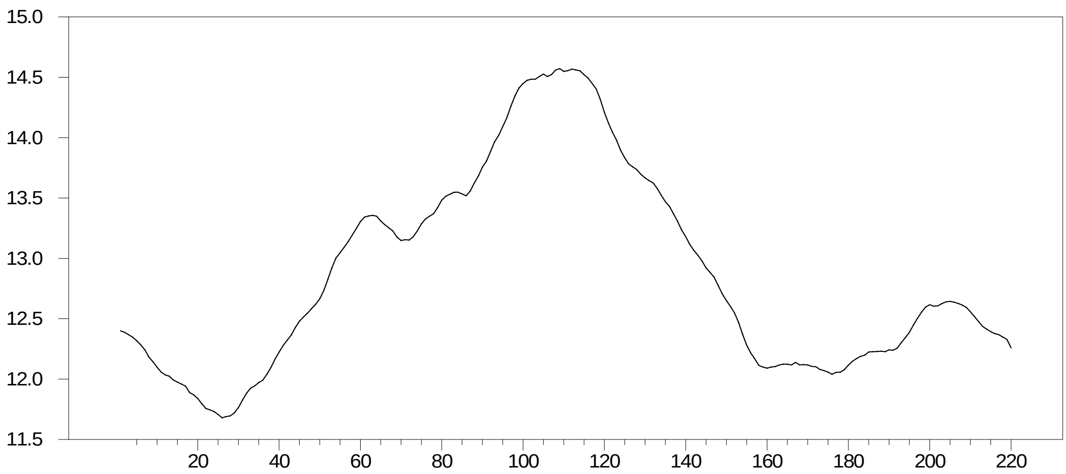
[2] In fact, there have been several references in the Athens daily financial press that there was evidence of manipulation of the closing prices of certain stocks.

To illustrate our point, in graph 1 we present the prices of an ATHEX stock, as they evolved during the trading session, shown in the form of deviations around the closing price (intraday prices).



Graph 1. Intraday variations of an ATHEX stock.

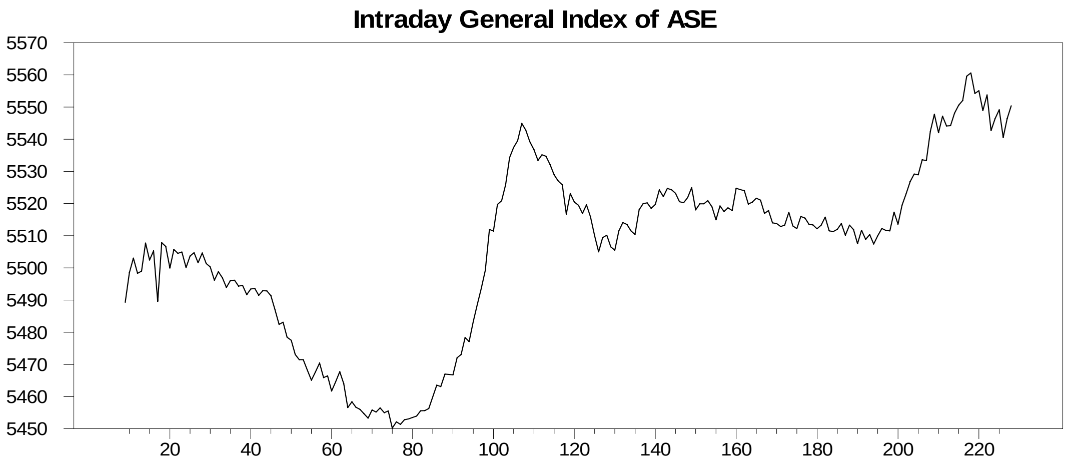
The variability of intraday prices versus the closing price which is generally used in portfolio valuations is evident. In Graph 2, we present the actual intraday prices of the same stock. The stock's opening price was € 12,4, around the middle of the trading session it reached a maximum of €14,5 and finally it closed at €12,3.



Graph 2. Intraday stock prices.

Equivalent and occasionally more pronounced “measurement errors” appear in the various stocks with low turnover, the value of which is calculated towards the end of a trading session using auction techniques.[3].

In Graph 3 we present the intraday prices of the ATHEX General Index from a typical trading day. The Index is calculated every 30 seconds and its closing price is calculated using the closing prices of the participating stocks.



Graph 3. Intraday prices of the ATHEX General Index.

The differences between the intraday prices and the closing price of the index are clearly evident from above graph.

The two examples presented above can be considered as cases that incorporate non-systematic errors in the measured returns of stocks and indexes.

Someone could argue that for stocks, the daily closing price should be an adequate measure of the particular stock’s value on that day, and that this price if used consistently, does not contain errors. On the other hand, the intraday prices contain valuable information on a given stock, which is not reflected in the closing price[4]. Moreover, as mentioned above, it is not uncommon the calculation of a closing price to be based on thin volume or to be the result of manipulation. In such cases, the errors can be significant..

[3] For more info on the calculation of ATHEX closing prices, please visit : www.athex.gr
[4] The closing price could be the highest or lowest point of the intraday trend, the difference is not trivial for a fund manager.

The purpose of this paper is to investigate the implications of using stock prices with measurement errors, as defined above, on the composition, management and evaluation of Mutual Funds. Using stochastic simulation techniques, we reach a series of interesting conclusions that affect significant aspects of a Portfolio's management[5].

This paper consists of four parts. After the introduction, in the second part we present the design of the stochastic process and the optimization procedure used for the calculation of the weights of each stock in the portfolio. In the third part we present our results, while in the last part we draw conclusions and propose ideas for further research.

[5] For the generation of the portfolio weights we use Markowitz's Mean Variance technique.

2. The Design of the simulations and the optimization procedure.

In our simulation experiments we used the following Data Generating Process (DGP): We assume ARCH(p=1) characteristics[6] and autoregressions[7] of the simulated returns:

.

$$y_{jt}^{A} = a_o + a_1 y_{jt-1}^{A} + u_t \qquad \textbf{(1)}$$

$$u_t = v_t \sqrt{(1 + 0.2u_{t-1}^{2})} \qquad \textbf{(2)}$$

$$v_t \approx NID(0,1) \qquad \textbf{(3)}$$

$$y_{jt} = y_{jt}^{A} + \varepsilon_{\mathrm{jt}} \qquad \textbf{(4)}$$

$$\varepsilon_{\mathrm{t}} \sim NI.\mathrm{D}(0, \sigma_{\varepsilon}^{2}) \qquad \textbf{(5)}$$

where y_{jt}^{A} : the simulated actual returns of the j stock for $j = 1,2...,20$

u_t : disturbances with ARCH characteristics.

v_t : disturbances.

y_{jt} : the simulated with error returns of the j stock for j=1,2,…,20.

ε_t : disturbances (Measurement errors).

In graphs 4 and 5 we represent graphically the simulated prices and returns of 20 stocks[8] based on the relationships (1) – (5) above. Graph 5 displays more analytically the measurement error along with the real prices and the corresponding errors for one of the simulated stocks. Moreover, in graph 6 we present simultaneously the returns of a simulated stock along with the evolution over time of the measurement error associated with it.

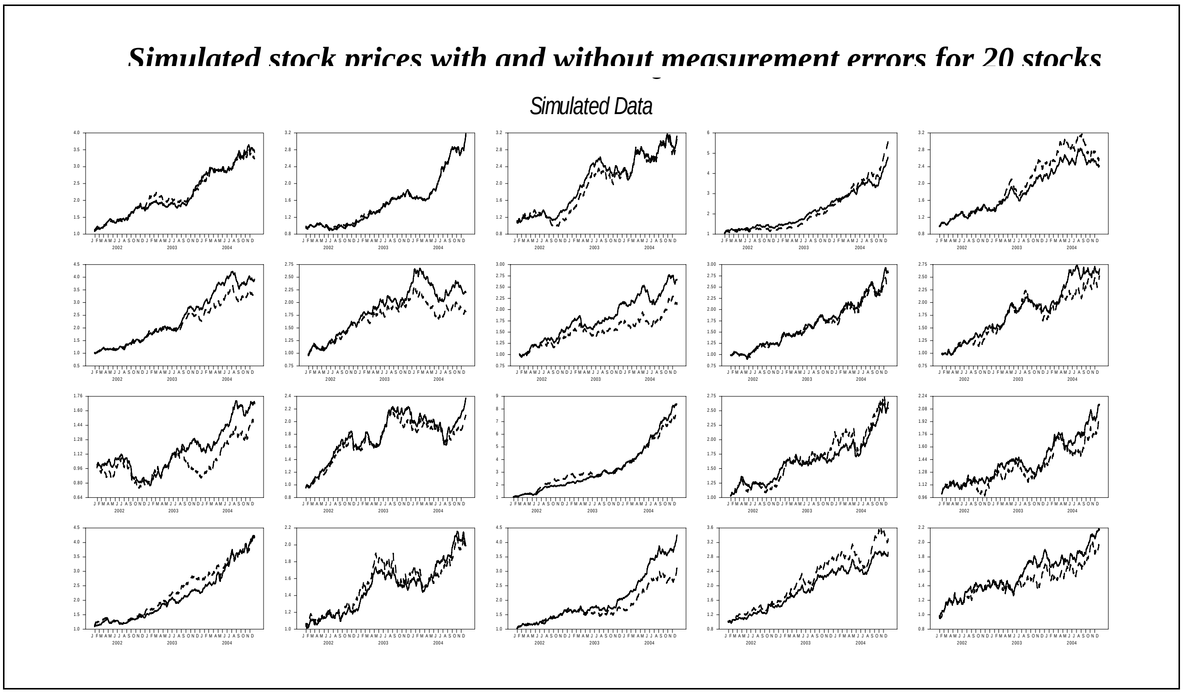
---

[6] More sophisticated models were used in the simulation leading to similar results. More are available from the authors on request.

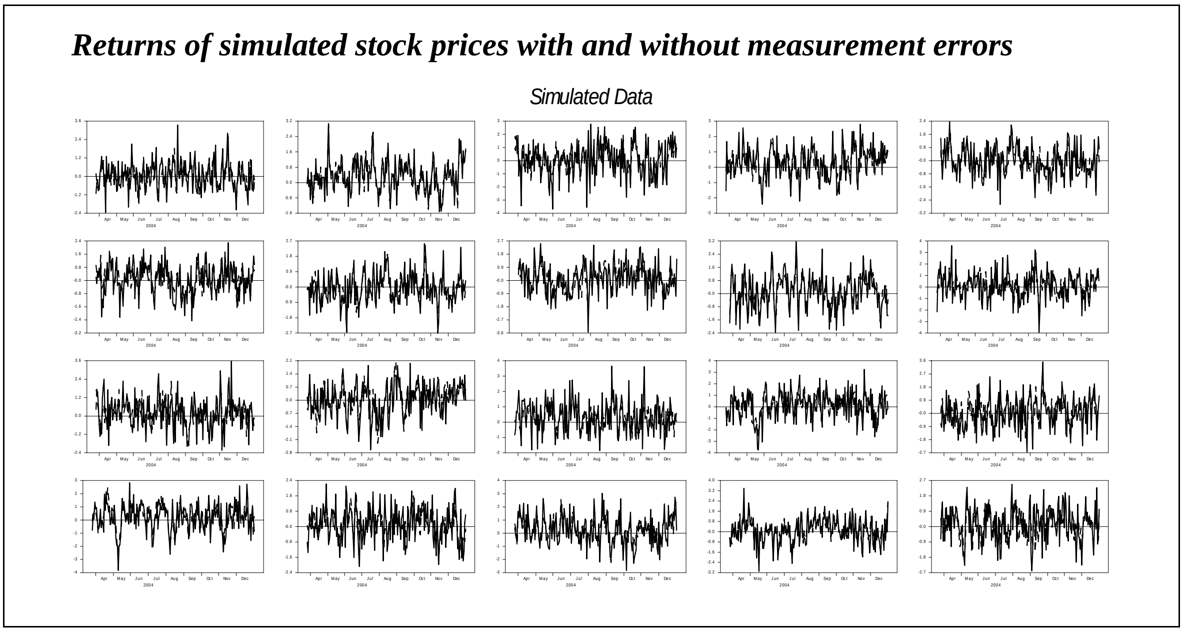
[7] In the simulations the parameters a_o *and* a_1 of (12) were specified as follows:

$a_o = 0.06 \quad a_1 = Uniform\ Distribution(.2,.8)$

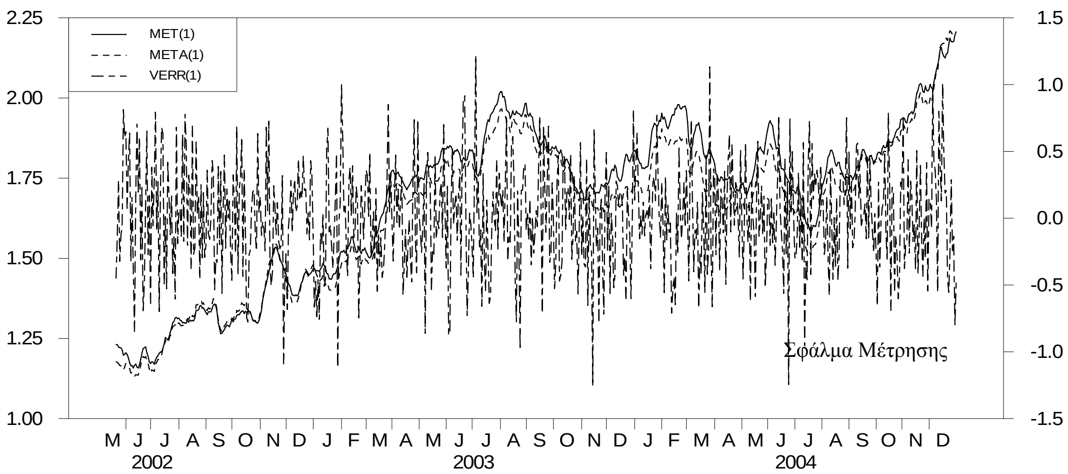
[8] The returns on a stock;s price were calculated on the basis of the relationship $(\frac{P_t - P_{t-1}}{P_{t-1}})100$ where P_t is the price of the stock at time= t.



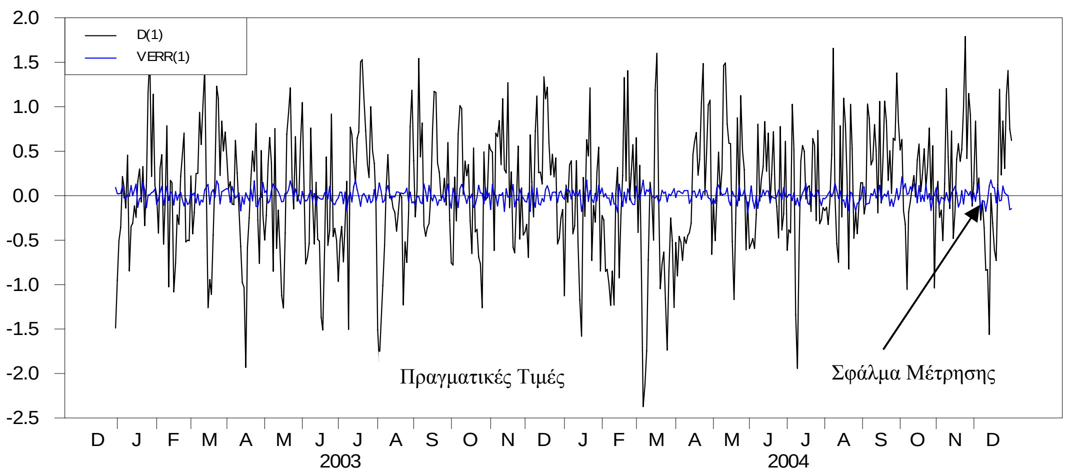
Graph 4, Actual and with error simulated prices of 20 stocks



Graph 5, Stock returns with and without errors for 20 simulated stocks.



Graph 6 Actual and with error simulated prices, measurement error for one of the simulated stocks



Graph 7. Comparison of the actual returns and the measurement error of one of the simulated stocks.

The weights used for each stock's participation in the portfolio of our study are calculated using a procedure that takes the following form[9]:

$$\min_{w.r.t\ \hat{w}_{j=1,2,..N}} \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{w}_j \hat{w}_i \hat{\sigma}_{ij} \quad (6)$$

Under the limitations:

$$\sum_{j=1}^{N=20} \hat{w}_{j,iter} = 1 \qquad (7)$$

$$0 \leq \hat{w}_{j,iter} \leq 1 \qquad (8)$$

[9] Global Mean Variance Portfolio Management (Markowich 1952).

where $w_{j=1,2,\ldots,N}$ are the weights of the stocks that comprise the portfolio used in our study, $\overline{d}_{j=1,2,\ldots,N}$ are the expected average returns of the N stocks and $\sigma_{ij(i=1,2,\ldots,N,i=1,2,\ldots,N)}$ is the ij element of the variance-covariance matrix $\Sigma = Cov(d_{jt}, d_{it})$.

4. The simulation results.

The stochastic experiments were based on the equations of the system[10] (1)-(5). The procedure of conducting the stochastic experiment is the following:

- Using the relationships (1)-(5) we obtain the simulated prices for the actual (y_{jt}^{A}) and with measurement error returns (y_{jt}) of the 20 stocks for a time period of 1200 days.
- Using a different number of historical observations[11] and different time periods of Markowitz style portfolio management[12] (portfolio evaluation[13]) we calculate for each case (6000 times) the portfolio weights using both the actual returns as well as those including measurement errors

The distributions of the weights of some of the 20 stocks[14] obtained by using the actual returns against those containing error are presented in Graphs 8 and 9.

It is obvious from a simple inspection that significant differences can be observed between the distributions[15] of the portfolio weights without and with measurement errors in stock returns.

[10] Our calculations are performed using the software environment RATS 6.02. The code is available to anyone interested and an application is presented in Appendix A.

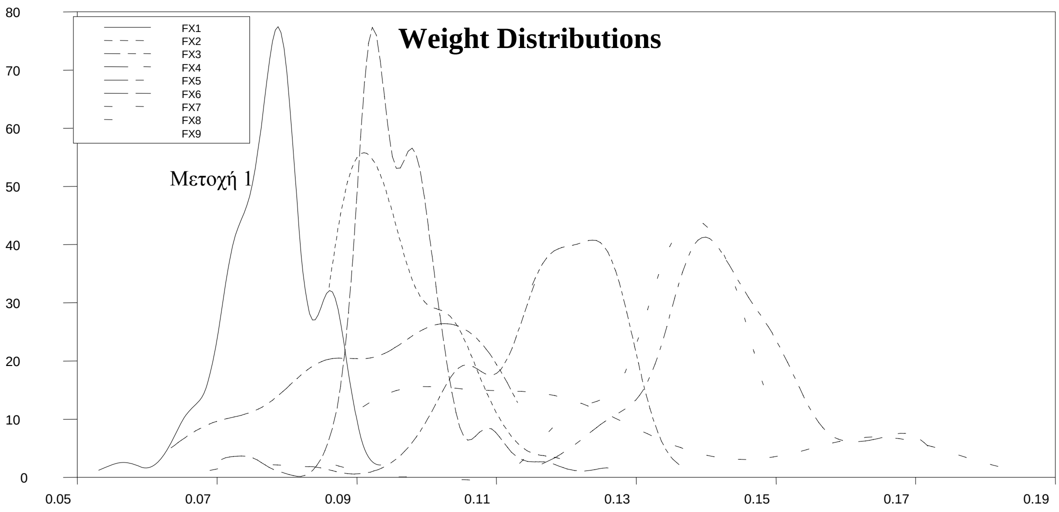
[11] In our experiments we used different number of historical observations in every iteration, in order to explore what are the implications of the number of days in our results through their use in the calculation of the portfolio weights.

[12] The management of the portfolios in our study is passive, we invest in the market and liquidate at predetermined time intervals.

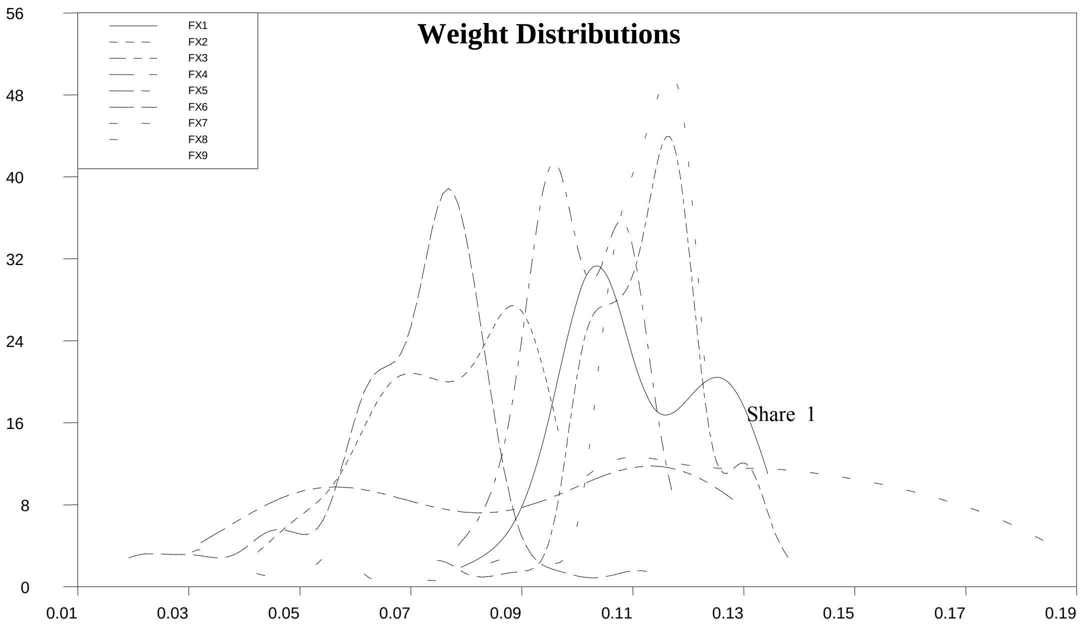
[13] As with the historical data, we experimented with different time lengths of the portfolios implementations. Our results proved that the number of historical observations used as well as the duration of the portfolio's life affected the outcomes of our experiments.

[14] In our experiments we used different number of stocks in some simulations in order to explore as well the effects of the number of shares in the efficiency of the portfolio.

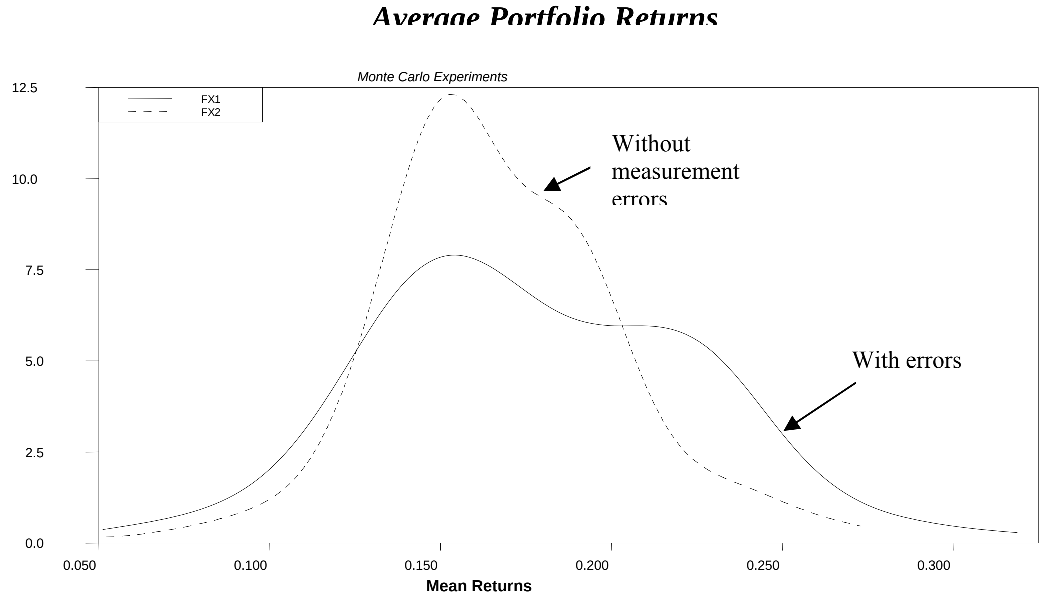
[15] The distributions refer to 6000 iterations. The way the experiment is designed allows us to check the dynamic formulation of the weights in relation to the number of shares that constitute the portfolio and the number of historical observations used for the computation of the weights. More information is available to any interested party.



Graph 8. Distributions of the portfolio weights of different stocks using error free returns.



Graph 9. Distributions of the portfolio weights of different stocks using returns with measurement errors.



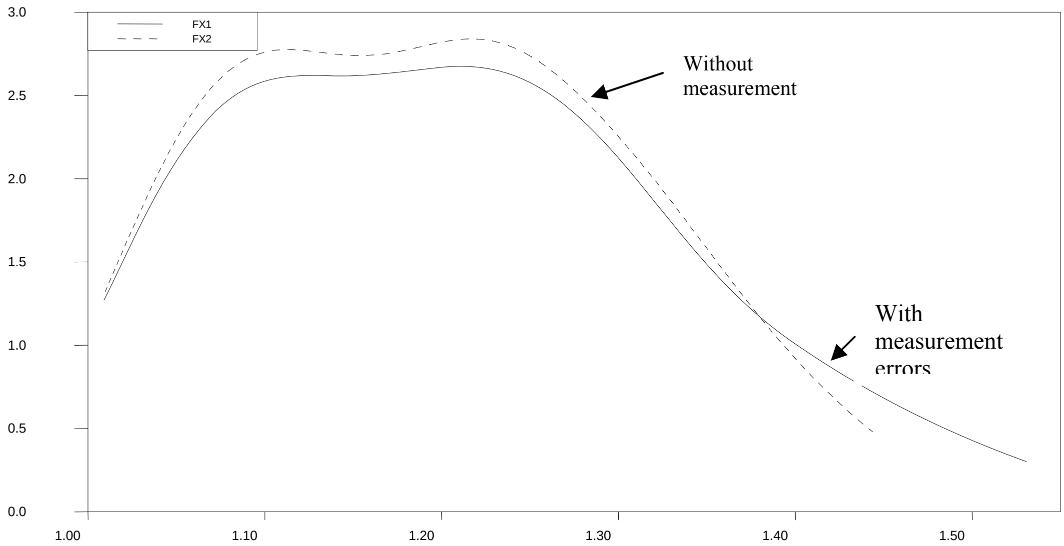
Graph 10 Distributions of average portfolio returns with and without measurement errors in stock prices.

4.1. Effects on portfolio average and total returns

Using every time the different portfolio weights, we calculate the average and total returns[16] of the two portfolios. In graph 10 we present a comparison of the average portfolio returns when the stock prices contain measurement errors and with out them. It is evident that measurement errors in stock prices result in lower average portfolio returns compared with the returns obtained with error free stock prices. Moreover the distribution of average returns with measurement errors is more spread out and asymmetrical.

[16] Total returns are calculated using the relationship $\prod_{t=1}^{T}(1+r_t)$.

Aggregate Portfolio Returns



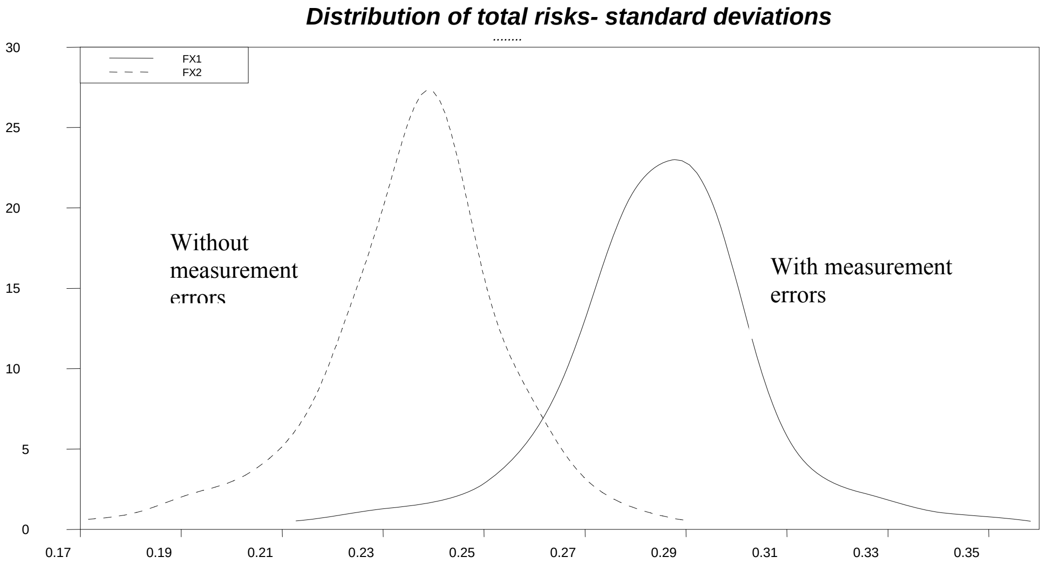
Graph 11. Distributions of aggregate portfolio returns with and without measurement errors.

The results are analogous when we look at the aggregate returns of the two groups of portfolios, as can bee seen in Graph 11. The introduction of measurement errors negatively affects the returns.

4.2. Effects on portfolio risks

In Graph 12, we perform a graphical comparison of aggregate portfolio risks[17] , using stock prices with and without measurement errors. It is obvious that the portfolios constructed with stocks containing measurement errors gives both lower returns and higher risks, measured by higher standard deviation.

[17] Aggregate portfolio risk is calculated as follows: $\frac{1}{T}\sum(d_j - \overline{d}_j)$ με $\overline{d}_j = \frac{1}{T}\sum d_j$.



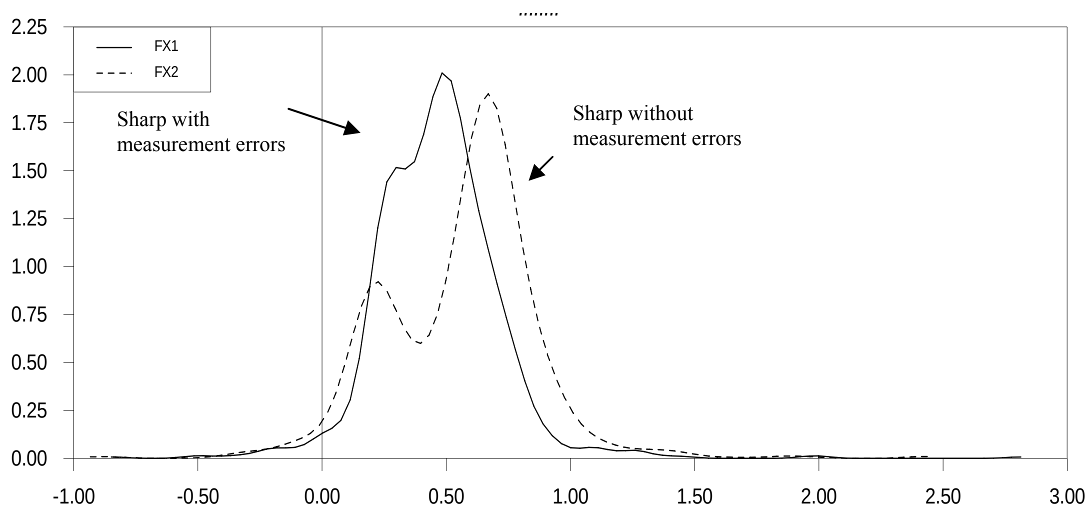
Graph 12. Distributions of aggregate risks in portfolios created with and without measurement errors.

Similar conclusions are reached by using Sharp's criterion[18], as shown in Graph 13. We observe that the average of the Sharp's criterion distribution is higher in portfolios without measurement errors compared to those with errors..

[18] *The Sharp Ratio(1966) is a traditional total performance measure used to measure the expected return of the two portfolios per unit of risk: Sharp Ratio*$_j$= $\frac{\sum_{s=1}^{T} d_{js} - r^f}{\sigma_j}$ *for* $j = 1,2,...,4$ *with* d_j = *Returns of the j index in the portfolio evaluation period and* $r_r^{\ f}$ = *is the risk free return. In our analysis we assumed a risk free return equal to 3.5%.*

Distributions of Sharp's criterion of simulation portfolios

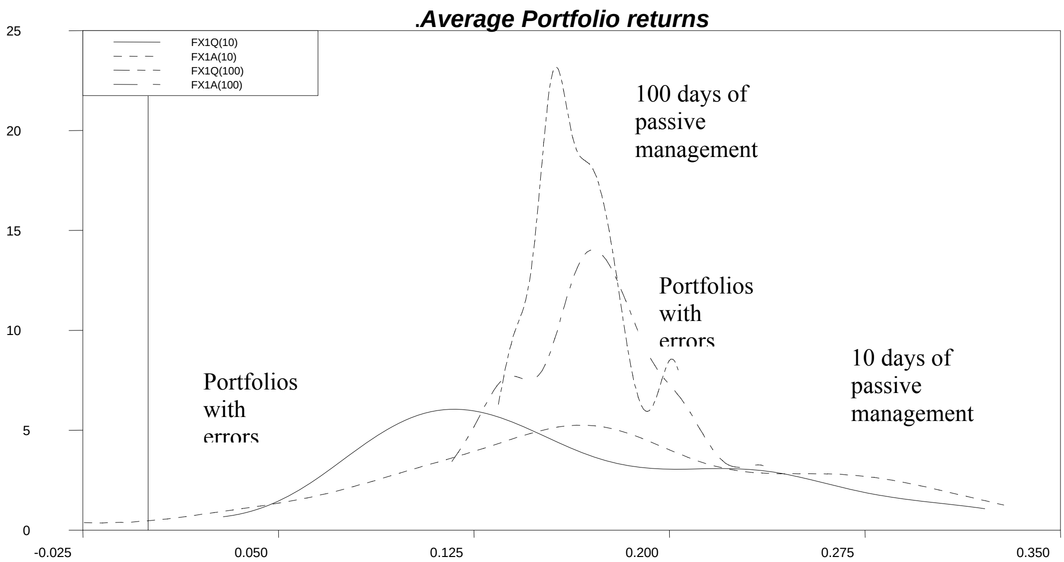
....



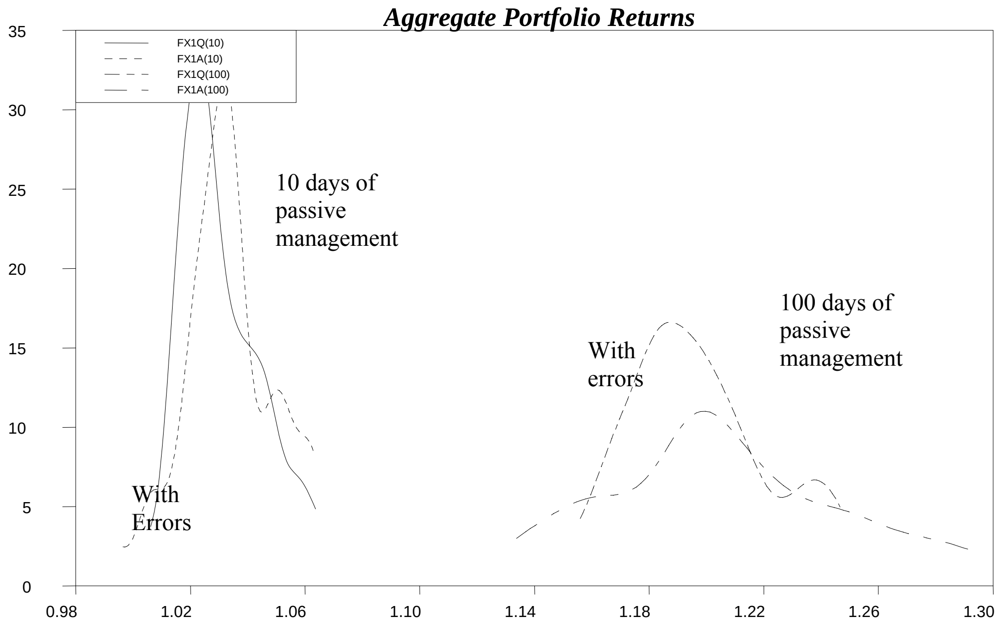
Graph 13. Distributions of the Sharp ratios of the portfolios created with and without measurement errors.

4.3 Portfolio Management results in relation to the number of days of the portfolios' retention.

In general, the performance of portfolios is related to length of time of the portfolio's life. To investigate the robustness of our results in relation to the time parameter, we compare groups of portfolios retained for 10 days, constructed with and without management errors, to groups of portfolios with and without errors, retained and passively managed for 100 days. The results are presented in graph 14, where the distributions of the average returns of the two groups of portfolios are shown. As expected, the longer a portfolio is retained, the higher the average returns and the narrower the distributions. Moreover, in both portfolio durations, the returns of the portfolios constructed without error are superior to those of the portfolios containing measurement errors in stock prices.



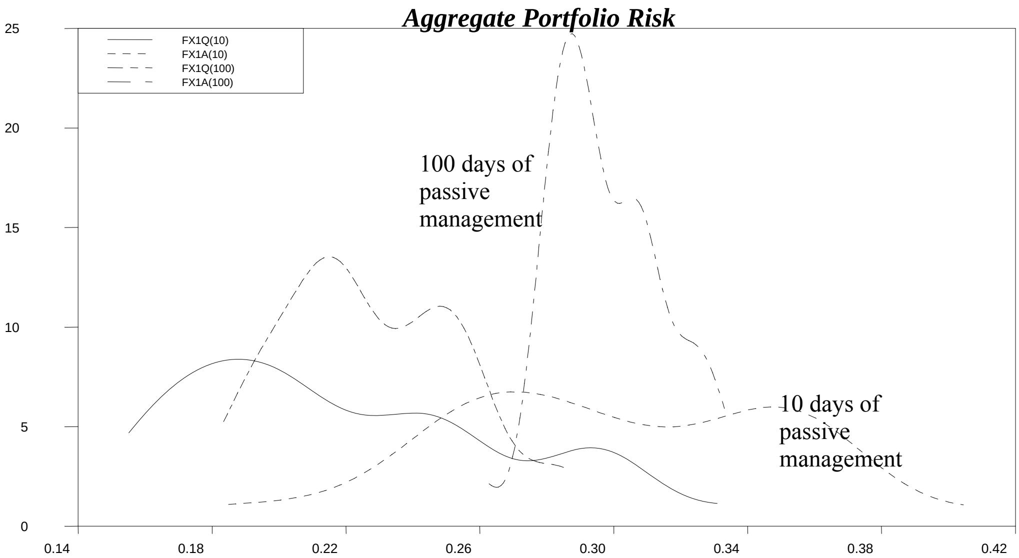
Graph 14. Distribution of the average returns of two portfolio groups with and without measurement errors and with different retention periods.



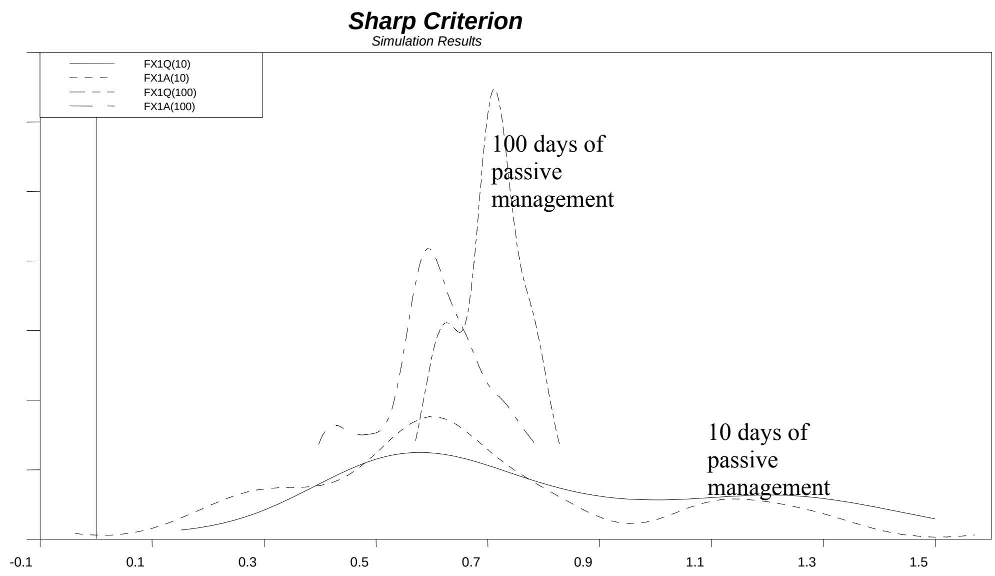
Graph 15 Distribution of aggregate returns of two portfolio groups with and without measurement errors and with different retention periods.

In graph 15 we present the distributions of aggregate returns of the two types of portfolios retained for 10 and 100 days respectively. In all cases, the performances of the portfolios that use error free stock prices are superior to those containing errors.

Lastly, concerning the measurement of portfolio risks, in relation to the length of time of a portfolio's retention, the relevant distributions are shown in graphs 16 and 17. It is again evident that in both portfolio retention periods, the associated risks are higher in the portfolios constructed using prices with measurement errors.



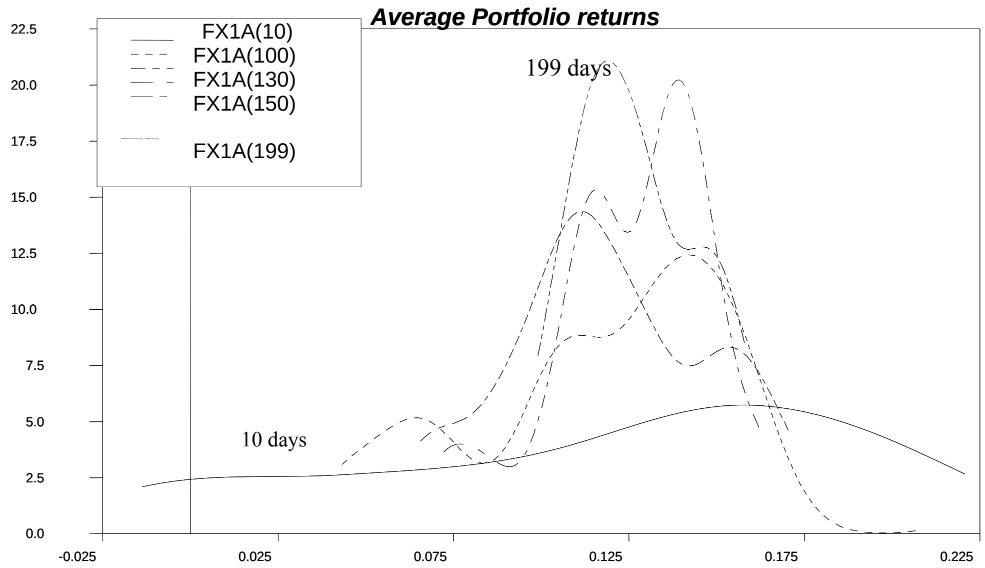
Graph 16. Frequency Distribution of portfolio risks of the two types of portfolios constructed and retained for different lengths of time.



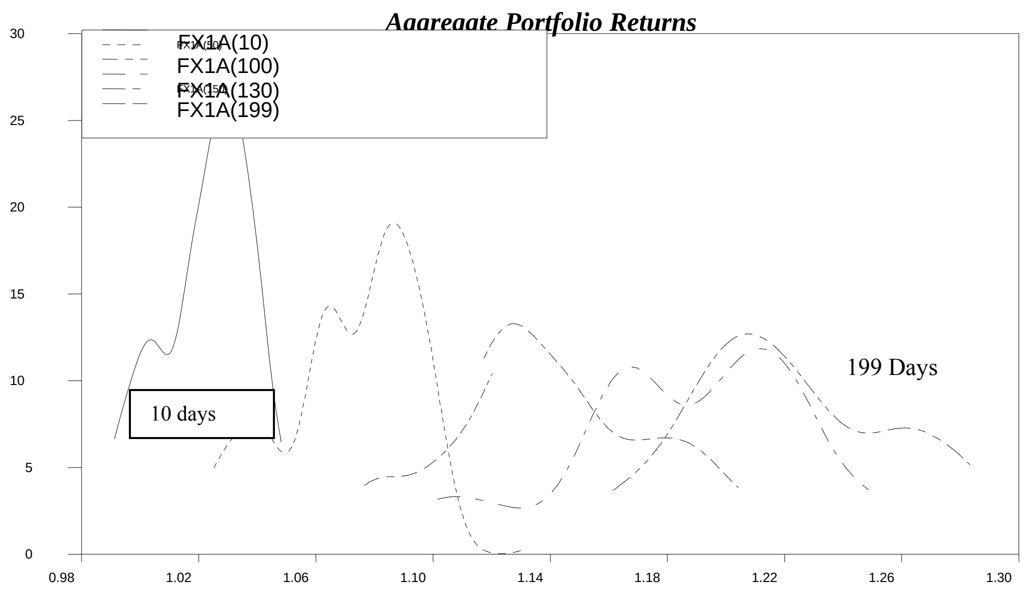
Graph 17. Frequency distribution of Sharp's criterion for two types of portfolios, constructed and retained for different lengths of time.

4.4. Portfolio management results in relation to the length of historical data used in stock selection for the portfolio composition

The number of days of historical data utilized for the calculation the weights of each stock in the portfolio's composition plays an important role in the portfolio's performance. This aspect of portfolio management is also affected by errors in the measurement of individual stock prices and returns. In graphs 18 and 19 we present the frequency distributions of portfolio returns with measurement errors as the number of historical days increases from 10 to 199.

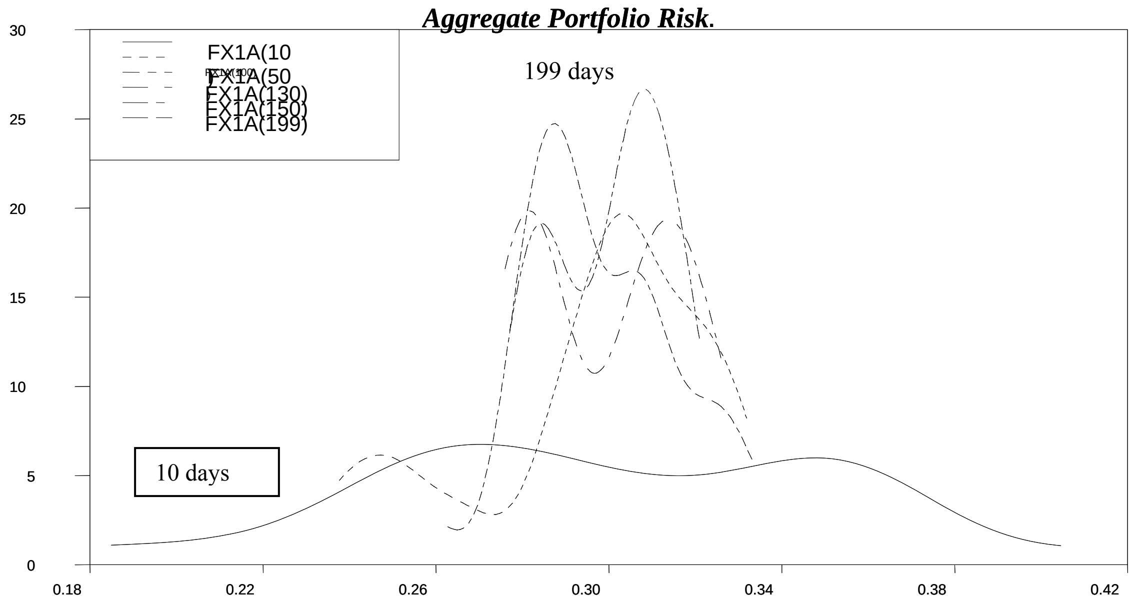


Graph 18 Frequency distributions of average portfolio returns with measurement errors for different periods of historical data used in portfolio composition**.**

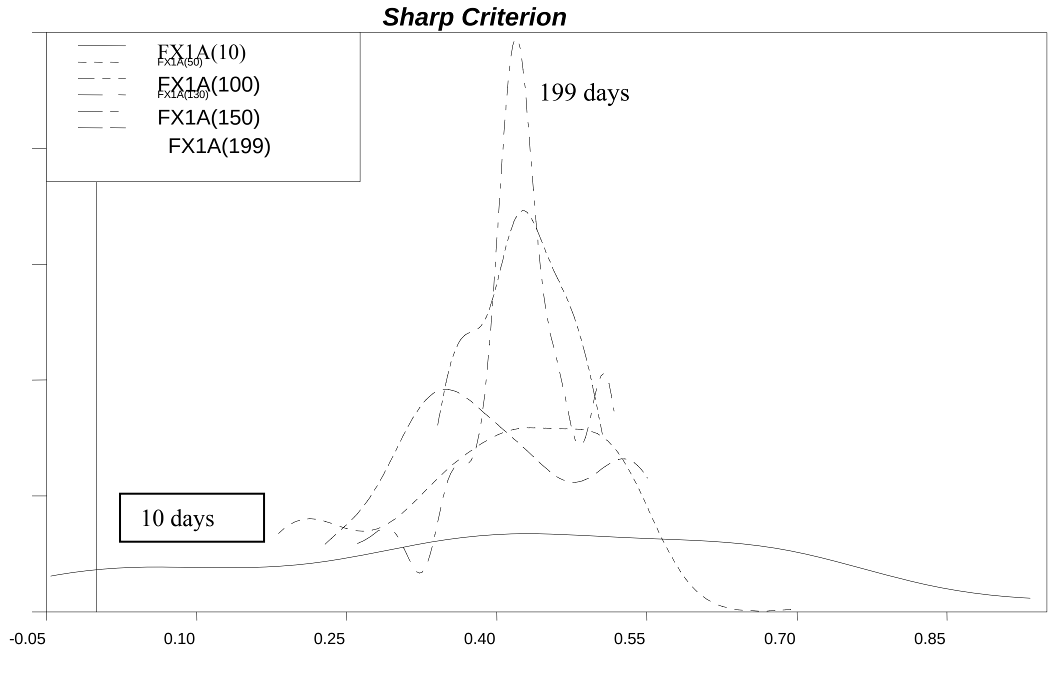


Graph 19 . Frequency distributions of aggregate portfolio returns with measurement errors for different periods of historical data used in portfolio composition

A careful observation of the above graphs leads to the conclusion that as the number of historical observations used in the calculation of portfolio weights grows, average returns are "normalized" and aggregate returns are significantly higher. The results are in the same direction with respect to the portfolio risks, measured either in standard deviation terms or using Sharp's criterion. The longer the period of historical data used in portfolio construction, the lower the risks and the more "normal" the frequency distributions.



Graph 20 Frequency distributions of portfolio risks with measurement errors for different periods of historical data used in portfolio composition.



Graph 21. Frequency distributions of Sharp's criterion with measurement errors for different periods of historical data used in portfolio composition.

5. Conclusions.

In this paper we investigate the implications of measurement errors in individual stock returns on the creation and management of a Portfolio, based on the Mean Variance Technique proposed by Markowich. Using stochastic simulation techniques, we create portfolios with varying retention periods, varying the length of historical observations used in constructing the individual stock weights as well as varying the start dates of each portfolio. In this way, we reach a series of interesting and intuitively plausible conclusions that affect important aspects of portfolio management.

First, average weights in portfolio compositions are strongly biased by measurement errors, thus resulting in lower average and aggregate returns, coupled with higher risks compared to the portfolios constructed and managed using error free stock data.

Thus, using stock return data with measurement errors in portfolio management, all the quantitative and qualitative portfolio performance measures are negatively affected, in the short, medium and longer portfolio retention periods. The negative implications are less pronounced in the longer retention periods, however the measurement errors in all cases introduce distortions in portfolio management[19] .

[19] The qualitative measures refer to the kurtosis and symmetry of the frequency distributions.

5. Bibliography

Markowitz, H. M. (1952), "Portfolio Selection," Journal of Finance, 7(1),77–91.

Markowitz, H.M., 1959, Portfolio selection: Efficient diversification of investments, (John Wiley and Sons, Inc., New York).

Markowitz, H.M., (1956): "The Optimization of a Quadratic Function Subject to Linear Constraints," Naval Research Logistics Quarterly, III, 111–133.

Markowitz, H. M., and P. Todd (2000): Mean-Variance Analysis in Portfolio Choice and Capital Markets. Frank J. Fabozzi Associates, New Hope, Pennsylvania.

Sharpe, W.F.,1963, A simplified model for portfolio analysis, Management Science 9(2), 277-293.

Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance 19(3), 425-442.

Sharpe, W.F., 1966, Mutual fund performance, Journal of Business 39(1), Part II, 119-138.

Sharpe, W.F. and G. Alexander, 1990, Investments, 4th Ed. (Prentice Hall, Englewood Clirs, NJ).

Treynor, J.L., 1965, How to rate management investment funds, Harvard Business Review 43(1), 63-75.