

# MOSAIC MANAGEMENT IN METAPOPOPULATION MODELS: Optimal Management of Interrelated Species in Patchy Environments

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## 1 Introduction

Optimal management of spatially distributed natural resources that are mobile can rapidly lead to complicated mathematics of the optimal control of objects like partial differential equations, mixed integro-differential equations, stochastic partial differential equations and the like (Brock 2000). To state it more concisely, modelling optimal mosaic management is hard. We propose here to investigate some simple metapopulation models that capture at least some aspects of management of an ecological mosaic. Although our analysis does not correspond to “mosaic management in metapopulations models” in the ecologist’s sense of the word, the models developed in this paper are inspired by actual such work. In their limiting form our models end up looking like Lotka-Volterra models after suitable re-interpretation. Thus our generalized Lotka-Volterra system is mathematically equivalent to versions of spatial models.

Before we begin, we should note that these models are highly stylized and are subject to the same rather devastating criticisms of simple deterministic series models made in Hilborn and Walters (1992). However, we still think it is useful to investigate these models, not only to shed insight on what might happen in more realistic models, but also to prompt appropriate questions to put to those more realistic models.

This paper develops as follows. Section 2 analyzes species competition and coexistence in spatially structured environments, with special attention to interacting species models where there are hierarchical relationships among the species. Although these models can be transformed into multi-species Lotka-Volterra models, the hierarchical relationship structure allows many species to co-exist in equilibrium. This is one way to avoid the limitations on the number

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of species due to competitive exclusion and limiting resources that were present in the models analyzed by Brock and Xepapadeas (2000). The limiting effects of competitive exclusion are mitigated by differential "colonization" rates, hierarchical relationships, and differential birth and death rates. We present results on equilibrium specialization when the system is harvested, and we examine the impact of site destruction on species abundance.

Section 3 analyzes management under three regimes: (i) socially optimal, (ii) rational expectations competitive equilibrium with full property rights, and (iii) rational expectations equilibrium (REE) with no property rights. The case of no (complete) property rights is analytically equivalent to the case of adjoining ...sheries with infinitely fast rates of diffusion of ...sh across each (zero rate of diffusion across each). We obtain equivalence of competitive equilibria under full property rights to social optimum. A main message from this part of the analysis is this. If diffusion rates of the resource are localized enough so the ...shing space can be subdivided into areas large enough that diffusion across them is quite small, but still small enough that Ostrom (1990) type institutions can be constructed to induce individual ...shers within an area to act approximately like the rational expectations optimizers of (ii) above, then approximate efficiency could be achieved. Ostrom's case studies as well as theory suggest that if the number of ...shers is small enough and the interactive relationships are long term, then reasonably efficient common property management institutions can emerge and be sustained.

Let us illustrate by considering the following hypothetical example. Imagine that we have a world market for shrimp, shrimp ...sheries are organized into areas where shrimpers in each area have rights to ...sh that area and to keep others out. (The rights could have been purchased in a public auction so the public itself could capture the rents.) Since shrimp do not diffuse very far, it is plausible that the areas might be constructed so that the number of shrimpers in each area is small enough to be able to organize workable Ostrom-like (cf. Ostrom (1990) collective action social institutions that induce each one to act in the collective long run interest of his/her fellow shrimpers in their jointly controlled area. Of course this institution will require some understanding from regulatory authorities such as anti-trust because the scheme won't work unless "outside" shrimpers can be excluded. An auction institution where shrimpers bid for the rights to be "insiders" would transfer the rent of concern to the antitrust authorities to the public. Anti competitive effects in the shrimp market should be of little concern because the optimal area of a shrimpery will be too small to have an effect on the world market price of shrimp. Of course tariffs and other barriers to outside shrimp sellers on each area's shrimp market must not be allowed to exist.

Under appropriate interpretation, the metapopulation models developed in Sections 2 and 3 of the paper as well as many of the techniques used in the analysis of these models can exploit well known core results of resource economics and bioeconomics (cf. Clark 1990 and Dasgupta and Heal 1979). It is useful to exploit this well known material to build a platform to launch probes into less familiar territory. Thus, given the importance of the socially-optimal solution for regulatory purposes, we develop in section 4 an extension of well known value loss arguments to extend the turnpike theory reviewed in Carlson et al. (1991) to quite general optimal management models of multispecies in patches. Although we work with a finite number of patches and species the

methods we develop to extend value loss arguments suggest the potential of extending of our methods to models of optimal control of partial differential equation models of multiple species on a continuum of patches. Brock (2000) suggests this possibility but the value loss argument developed here does not appear in that paper.

We exhibit examples where optimal decentralized regulation by incentive instruments applied at the individual agent level of open access bionomic equilibria fails due to hysteresis and multiple equilibria. This is familiar from work such as Clark (1990). We show that an appropriate re-examination of the relative speeds of adjustment of economic dynamics to biological dynamics as well as an analysis of appropriate property rights regimes clarifies when hysteretic regulation failure is likely to be a problem for the case of one species. Section 5 concentrates on developing these results for the case of one and many species. For the multi-species case the interaction between demand and biology in this context can lead to complicated patterns of equilibrium and hysteretic relaxation dynamics under open access exploitation. We present a quite general result on decentralized regulation if the biological dynamics is slow enough relative to the economic dynamics. However, if the biological dynamics is fast enough relative to the economic dynamics, the hysteresis traps can re-appear with much more complexity due to the interaction amongst the species on the biological side and the interaction in demands on the economic side.

The information requirements needed to apply conventional propositions of decentralized regulation like those we develop above are huge. This information problem is aggravated by the complexity of interactions and the details about biology and economics that are needed to implement regulation. Even well studied and relatively simple ecosystems present enormous difficulties in practical implementation of regulation (Carpenter, Ludwig and Brock 1999). The setting here may be even more complicated. Furthermore controversies loom large in ecology about which theoretical model is the appropriate baseline. Indeed it is natural, in view of controversies in ecology about models with one stable state or models with multiple stable states (Carpenter, Ludwig and Brock 1999), to consider at least two baseline core models around which one wishes to design a Robust regulatory mechanism that works well for a range of misspecifications centered at each of these baseline models. The analysis in Sections 2 and 3 of this paper represent an analysis centered at one core baseline model that is known exactly.

Section 6 of this paper starts the daunting task of formulating and analyzing frameworks of Robust Regulation. The first part sets up a problem of a Bayesian regulator who puts a posterior probability (presumably after use of all available data and theory) on each of two models,  $M_1$  and  $M_2$ . We abstract away from parameter uncertainty inside each  $M_i$ . The regulator chooses instruments to steer the system towards the maximization of posterior expected social welfare.

But this is just Bayesian regulation and is not Robust regulation in modern parlance. The second part attempts to Robustify your treatment of optimal regulation by introducing a simple rendition or caricature of ideas Epstein and Wang (1994), Liu (1999), and Sargent.<sup>1</sup>

Following Epstein and Wang (1994) and Liu's work (1999) we consider an objective which is a weighted average of posterior expected welfare and the

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<sup>1</sup>See his website for a host of recent working papers on robust control.

minimum of welfare over the two possible models  $M_1$  and  $M_2$ . The idea is this. If regulators are ambiguity averse (cf. Epstein and Wang 1994) with what we shall call here, "degree  $e$ ", then their objective function is  $(1 - e)$  times posterior expected welfare plus  $e$  times the minimum of welfare under  $M_i$ . The idea is that Nature is "mean" and acts to hurt the policy maker with probability  $e$  and is benign with probability  $1 - e$ .

One can view this formulation as an attempt to capture the discounting of ambiguity that has been claimed to occur in experimental situations as well as in the field. It is an example of recent work in decision theory on departures from Bayesian decision making and departures from expected utility. That is, it represents an attempt to conceptualize and measure the notion of ambiguity. Here ambiguity refers to a feature of situations where a notion of subjective probability not only makes no sense but is not consistent with laboratory and field observations of actual decision making.<sup>2</sup>

The evidence suggests that people will pay more to avoid an equal amount of ambiguity than they will pay to avoid an equal amount of quantized and knowable probability risk. Laboratory subjects appear to exhibit some kind of insurance against worst case within known bounds type of behavior. Liu's work (building on that of Epstein and Wang) does a neat job of capturing this feature in a decision making framework that posits the conventional Bayes for  $1 - e$  per cent of the time and minimization of loss over the class of possible misspecifications,  $M$ , for  $e$  per cent of the time. The fraction  $1 - e$  represents confidence on the part of the decision maker that she has the specified class right and can proceed as a conventional Bayesian statistical decision maker. The fraction  $e$  represents her lack of confidence. The class  $M$  of misspecifications represents her ignorance about where the true specification might lie. The world she lives in is more ambiguous the bigger is  $M$ : There is still rather wide latitude in framing decision making frameworks in this class (e.g. Epstein is currently working on conceptualizing Degrees of Ambiguity) but the above framework is rich enough to focus the mind on what human decision making behavior might be like when the class of possible misspecifications  $M$  includes dynamical processes with multiple stable states. Almost all of the ambiguity-decision-theoretic frameworks we have seen produce a type of behavior that maximizes against a worse case scenario within the class  $M$ . In any event, we shall use this version of Robust Decision Making here.

## 2 Competition and Coexistence in Spatially Structured Environments

Tilman and Lehman (Tilman and Kareiva 1997, Chapter 10) put forth the following model of species competition in a subdivided habitat:<sup>3</sup>

$$\dot{x}_i = r_i x_i \left( 1 - D - \sum_{j=1}^n x_j \right) A - m_i x_i - \sum_{j=1}^n r_j x_i x_j ; i = 1; \dots; n \quad (1)$$

<sup>2</sup>See Camerer's chapter in Kagel and Roth (1995).

<sup>3</sup>Tilman and Lehman's notation has been changed for compatibility with Clark (1990). See also Tilman (1994) for a similar model with  $D = 0$ .

The interpretation here is as follows. There are a large number of sites, each of which may contain one adult member of a species. Species 1 is a better competitor than 2,..., species  $n - 1$  is a better competitor than species  $n$ . Here  $x_i$  denotes the fraction of undestroyed sites occupied by species  $i$ ,  $D$  is the fraction of sites destroyed (e.g. by human activities),  $r_i x_i$  is the rate of propagule production by species  $i$ ,  $m_i$  represents the mortality rate of species  $i$ , the other terms represent material balance of occupied and unoccupied sites as well as a hierarchical set of competitive relationships that capture which species can push another out of a site. At an abstract level the ecosystem dynamics represented by (1) can be mapped into the general Lotka-Volterra framework

$$\dot{x}_i = x_i (s_i - \sum_{j=1}^n a_{ij} x_j) = F_i(x); \quad i = 1; 2; \dots; n; \quad \text{or} \quad (2a)$$

$$\dot{x} = x \otimes [s - Tx]; \quad x = (x_1; \dots; x_n); \quad (2)$$

$$s_i = r_i (1 - D) - m_i; \quad a_{ij} = [r_i + r_j] = r_i; \quad j < i - 1; \quad a_{ij} = 0; \quad j > i; \quad a_{ii} = 1;$$

where the  $n \times n$  matrix  $T$  has  $i$ th row,  $\text{Row}_i = (r_1 + r_i; r_2 + r_i; \dots; r_{i-1} + r_i; r_i; 0; 0; \dots; 0)$ : Notice that  $T$  has  $r_i$  on the diagonal of  $\text{Row}_i$  and all zeroes above the diagonal.<sup>4</sup> Hence (1) or (2) represents the ecological dynamics of this competitive hierarchy when there is no harvesting by humans, but there is habitat destruction by humans which is reflected by  $D > 0$ .

Unlike the Tilman type  $R^n$  models used by Brock and Xepapadeas (2001), there can be a large number of co-existing species in steady state equilibrium of (1) or (2). The steady state conditions in Nature can be written using (2) as<sup>5</sup>

$$Tx = s \quad (3)$$

Since the determinant of  $T$  is the product of the diagonal terms and each  $r_i > 0$ , therefore the inverse  $S = T^{-1}$  exists. It is also lower triangular of the same form as  $T$  as can be shown using  $ST = I$ , where  $I$  is the  $n \times n$  identity matrix. Let  $T_n$  denote the  $n \times n$  matrix  $T$  in (2) above. We can use the recursive structure as well as formulae for the inverse of a partitioned matrix to derive a recursive relationship between the inverse of  $T_{n+1}$  in terms of the inverse of  $T_n$ : The matrix  $T_{n+1}$  can be written thus,

$$T_{n+1} = \begin{pmatrix} T_n & 0 \\ g_{n+1} & r_{n+1} \end{pmatrix} \quad (4)$$

where the ...rst  $n$  rows of  $T_{n+1}$  are  $n \times (n + 1)$ , and  $g_{n+1}$  is  $1 \times n$  and is the ...rst  $n$  elements of row  $n + 1$  of  $T_{n+1}$ . The formula for the inverse of a partitioned matrix (4) yields

$$S_{n+1} = \begin{pmatrix} S_n & 0 \\ h_{n+1} & r_{n+1} \end{pmatrix}; \quad h_{n+1} = -\frac{1}{r_{n+1}} g_{n+1} S_n \quad (5)$$

This recursion can be used to rapidly build expressions for the inverse of  $T_n$  for fairly large matrices. This recursive structure can then be used to locate

<sup>4</sup>The notation  $z = x \otimes y$  stands for the vector  $z$  with  $i$ th element,  $z_i = x_i y_i$ . I.e. it is a product which is formed by multiplying each element of the ...rst vector  $x$  into the corresponding element of the second vector  $y$ . When needed, we will form similar products for conformable matrices as well as for conformable vectors.

<sup>5</sup>We would like to thank Shiu-Sheng Chen for help with the results of this section.

sufficient conditions for  $x^a = Ss > 0$ . Here are some examples of such assumptions.

$$\begin{aligned}
 x_1^a &= \frac{S_1}{r_1} = 1 - D - \frac{m_1}{r_1} > 0 \\
 x_2^a &= \frac{1}{r_2} [S_2 - (r_1 + r_2) x_1^a] > 0 \\
 x_3^a &= \frac{1}{r_3} [S_3 - (r_2 + r_3) x_2^a - (r_1 + r_3) x_1^a] > 0 \\
 &\dots\dots\dots \\
 x_k^a &= \frac{1}{r_k} \left[ S_k - \sum_{i=1}^{k-1} (r_k + r_i) x_i^a \right] > 0
 \end{aligned}$$

If  $D = 1 - \frac{m_1}{r_1}$  then the abundance of species 1 goes to zero. Since for  $D = 0$ ;  $x_1^a = 1 - \frac{m_1}{r_1}$ ; species 1 becomes extinct if a proportion of the habitat equal to its occupancy in a virgin environment is destroyed (Tilman and Lehman 1997). For the special case  $r_i = r > 0$  for all  $i$ , we may assume:

$$r > 0; s_i > 0; s_{i+1} > 2s_i ; i = 1; 2; \dots; n - 1$$

It is straightforward to prove that the above assumption implies  $x_i^a > 0$ ,  $i = 1; 2; \dots; n$ :

We introduce harvesting  $H_i$  into the model. Following Clark's (1990) notation, let  $q_i; E_i; p_i; Y_i; c_i$  denote respectively catchability coefficient of species  $i$ , effort spent on capturing species  $i$ , unit price of species  $i$ , total number of units captured of species  $i$ , and cost per unit effort spent on species  $i$ , with  $H_i = Y_i = q_i x_i E_i$ . We first analyze a special case in order to bring into sharp focus the role played by the hierarchical competitive structure of species in optimal harvesting patterns. Let  $U(Y) = \sum_{i=1}^n u_i(Y_i)$  denote benefits from harvesting and  $c(x; E)$  be the general harvesting cost function, and consider the problem

$$\max_0^1 \int_0^1 e^{-\lambda t} (U(Y) - c(E; x)) dt \quad (6)$$

$$\text{s.t. } \dot{x}_i = r_i x_i \left[ 1 - D - \sum_{j=1}^{i-1} x_j \right] - m_i x_i - \sum_{j=1}^{i-1} r_j x_i x_j - H_i \quad (7)$$

Buttel, Durrett, and Levin (2000) (BDL) investigate models that support large numbers of species in equilibrium. Among models they analyze is a special case of (7) above. BDL do this to set the stage for the study of models that are compatible with large numbers of species coexisting. We consider a special case by setting

$$r_i = 1 ; m_i = 1 - \frac{(2i - 1)}{(2n)} \quad i = 1; 2; \dots; n:$$

If all  $H_i = 0$ , one may compute the steady state of (1) or (7) and obtain  $x_i = 1 - 2/n$ . Thus, for  $D = 0$ , as  $n$  increases "to infinity the species are uniformly spread and 1/2 of the sites are occupied..." (Buttel et al., 2000, p. 3). In order to focus on one thing at a time, assume  $u_i = u$  for all  $i$  and embed any harvesting cost in the (net) utility  $u$ . First consider the linear case  $u(H_i) = H_i$

and assume  $\frac{1}{2} = 0$ . We may then find optimal steady states (OSS) by solving the (undistorted by  $\frac{1}{2}$ ) static optimization problem

$$\max_{x} \sum_{i=1}^n H_i \text{ s.t. } H_i = f_i(x) ; i = 1; 2; \dots; n;$$

where  $f_i(x)$  is the right hand side of (7) for each  $i$ :

By solving problems for  $n = 1; 2$ , noticing a pattern in the first-order necessary conditions (FONC) for optimality that holds for all  $n$ , it is easy to show that the optimal thing to do is harvest  $\alpha$  all of the species except species  $n$ . This will be done rigorously below. This result intuitively follows from the survival rate of  $i; 1 - m_i = (2i - 1)/(2n)$  increasing in  $i$ , together with lower numbered species placing negative externalities on productivity of higher numbered species, but each unit having the same net economic value.

Turn now to the case  $\frac{1}{2} > 0$ : If  $u_i(H_i) = a_i H_i$ , we have a Most Rapid Approach Problem (MRAP)<sup>6</sup>. Hence the solution is to apply controls  $\{H_i\}$  to move the state vector  $x(t)$  to

$$x^* = \operatorname{argmax}_x \left\{ \sum_{i=1}^n \{a_i (f_i(x) - \frac{1}{2}x_i)\} \right\} \quad (8)$$

as rapidly as possible. Thus if  $a_i = a$  for all  $i$ , we obtain the same form of solution as above: Harvest down to extinction all species except species  $n$  as rapidly as possible. Let us state this result as

Theorem 1 Under the assumptions above,  $x^* = (0; 0; \dots; 0; x_n^*)$ ;  $x_n^* = \frac{s_n}{2}$ , where  $s_i = \frac{(2i - 1)}{(2n)} - D - \frac{1}{2}$ .

For proof see Appendix.

Theorem 1 reflects a polar case where specialization is optimal. It is useful to have the opposite polar case where it is optimal to equate the  $x_j$  for contrast. Put  $\frac{1}{2} = 0$  and consider the Leontief utility,

$$U(H) = \min\{H_i; i = 1; 2; \dots; n\} \quad (9)$$

Here the object is to achieve the highest common value of  $f_i(x)$ . Since  $s_1 < s_2 < \dots < s_n$  it is natural to look for solutions where species 1 constrains all the others and optimize steady state harvest over species 1 subject to these constraints. We have

Theorem 2 The optimal  $H$  is found by solving the problem

$$\begin{aligned} \max x_1 (s_1 - x_1) \text{ s.t.} \\ f_1(x) &= x_1 (s_1 - x_1) = f_2(x) = x_2 (s_2 - 2x_1 - x_2) = \dots = \\ f_n(x) &= x_n (s_n - 2[x_1 + \dots + x_{n-1}] - x_n) \end{aligned}$$

and under Assumption A below:

$$\text{Assumption A: } \frac{\mu s_1}{2} > \text{maximum} \left( \frac{\mu (s_k - s_{k-1})}{2} \right) ; k \neq 1$$

<sup>6</sup>See Clark (1990, Section 2.7 and elsewhere).

the optimal solution to this problem is given by

$$x_1^a = \frac{\hat{s}_1}{2}; x_2^a = \operatorname{argmax}\{x_2(\hat{s}_2 - x_1^a - x_2)\} = \frac{(\hat{s}_2 - \hat{s}_1)}{2}; \dots;$$

$$x_n^a = \operatorname{argmax}\{x_n(\hat{s}_n - 2[x_1^a + \dots + x_{n-1}^a] - x_n)\} = \frac{(\hat{s}_n - \hat{s}_{n-1})}{2}$$

For proof see Appendix.

Let us compute  $x_k^a$  for the special case  $m_k = 1 - (2k - 1) = (2n) - 2k + 1$ ;  $k = 1; 2; \dots; n$ . Here  $\hat{s}_1 = 1 = (2n) - D$ ;  $\hat{s}_k - \hat{s}_{k-1} = 1 = (2n) - 2k + 1$ .

For the system to be viable we must have  $\hat{s}_1 > 0$ . Clearly an increase in  $D$  (or an increase in  $n$ ) crashes the entire system. If  $D$  is shocked by Nature on a slower time scale than harvesting and  $D$  is badly measured by management, then optimal steady state management may be in for a lot of unpleasant surprises.

As we have seen before, if  $D = 0$ , Nature's equilibrium for this system is to set each  $x_k = 1 = (2n)$ . Abuse notation and let  $\hat{s}_i = 1 - m_i$ ;  $S_i = \hat{s}_i - D$ . Direct computation for  $D > 0$  shows that Nature's equilibrium is given by

$$x_1 = S_1; x_2 = S_2 - 2S_1; x_3 = S_3 - 2(S_2 - S_1)$$

$$x_4 = S_4 - 2(S_1 + S_3 - S_2); x_5 = S_5 - 2(S_2 + S_4 - S_1 - S_3)$$

We see right away that odd numbered species are injured by  $D > 0$  whereas even numbered species are helped. For the special case  $\hat{s}_i = (2i - 1) = (2n) - 2i + 1$ , direct computation shows that

$$x_i = 1 = (2n) + D \text{ for } i \text{ even};$$

$$x_i = 1 = (2n) - D \text{ for } i \text{ odd}$$

Notice that  $D > 1 = (2n)$  implies all odd numbered species go extinct. Compare Nature's equilibrium with the optimal economic steady state equilibrium under zero discounting for Leontief utility (9) above which is given by  $x_1^a = 1 = (2n) - D$ ;  $x_i^a = 1 = 2n$  for all  $i$  not equal to one. Economic management undoes Nature's threat of extinction of all odd numbered species except the ...rst. The lunch is not free however. Species 1 is still threatened and the even numbered species are less abundant under economic management than in Nature. This ...nding is reminiscent of alternating equilibrium abundances in hierarchical trophic cascade models in ecology. That is, if 1 eats 2 eats 3, ..., eats  $n$ , then high abundance of 1 leads to low abundance of 2 which leads to high abundance of 3, ..., etc. In such settings economic management can take pressure off of species at risk by harvesting down the over abundant species in trophic cascades.

If one lets  $D$  be driven by a slow variable one can get interesting dynamics out of this model. For example if  $D$  moves slowly up then slowly down in a sinusoidal pattern, we would see recurring waves of extinctions of large numbers of species if  $n$  is large. Hence if  $D$  proxies for habitat quality this kind of model can be suggestive of possible ways to investigate economic trade-offs between the costs of maintaining or enhancing habitat quality (including the opportunity costs of alternative uses for the habitat such as economic development) and the impact of habitat quality on revenue generated by natural resource capture industries.

For example, if slow periodic trend movements (including stochastic fluctuations about such trends) in climate drive the magnitude of  $D$ , these movements alone could generate patterns that look like human-induced over harvesting.



However, abundance distributions like these described above do not look like abundance distributions observed in Nature. But BDL (2000) show how to ...nd parameters (e.g. by varying the colonization rates  $c_i$  as well as the mortality rates) of this model to produce abundance distributions that are more realistic. It would be worthwhile to work out optimal economic equilibria as well as Clark (1990) bionomic equilibria for more realistic abundance distributions. We still think, however, that this model is enough to reveal how small changes in the habitat destruction parameter can lead to large and surprising reverberations (even mass extinctions) in abundance distributions.

### 3 Welfare Optimum, Rational Expectations and Bionomic Equilibria in Spatially Structured Environments

Having examined the impact of harvesting and habitat destruction in the special case above, we now analyze in more detail the economic characteristics of our model. We start by analyzing a simple Clark (1990, Section 5.2) static bionomic equilibrium to this setting. After this is done, we extend Clark's (1990) treatment of dynamics to this setting.

The flow of economic rent, i.e. profit, generated by species  $i$  is given by

$$R_i = (p_i q_i x_i - c_i) E_i \quad (10)$$

which implies that the steady state interior bionomic equilibrium is given by

$$p_i q_i x_i - c_i = 0; p_i = P_i(Y_1; \dots; Y_n); \quad (11)$$

$$Y_i = q_i E_i x_i = \text{RHS of (1)}; i = 1; 2; \dots; n; \quad (12)$$

where  $P_i$  denotes the demand function for species  $i$ <sup>7</sup>.

For the case  $n = 1$ , the system (10)-(12) is thoroughly analyzed by Clark (1990), not only for the linear Schaefer type of cost structure, but also for various generalizations of the cost structure<sup>8</sup>.

#### 3.1 Equilibria for a Single Species Case

Let us ...rst do the case  $n = 1$ . We investigate three regimes. First, we investigate the socially-optimal management problem (SOMP). Second, we show that if we have  $N$  (without loss of generality put  $N = 1$  for a "stand-in" ...shery) identical ...sheries with the same dynamics and there is no mixing or diffusion of ...sh across any of the  $N$  ...sheries, then a version of the standard Equivalence Theorem of capital theory (Becker and Boyd 1996) applies for rational point expectations equilibrium. That is, the solution of the REE Problem is the same as SOMP.

<sup>7</sup>Bionomic equilibria may be non-interior and have some species with zero price, if the unit cost of effort is too high relative to demand and if the catchability coefficient is too small. To allow for these boundary equilibria, replace (11) by

$$\begin{aligned} Y_i &< \text{RHS of (1)} \Rightarrow p_i = 0 \\ p_i &> 0 \Rightarrow Y_i = \text{RHS of (1)} \end{aligned}$$

<sup>8</sup>See Clark (1990, Figures 5.9, 5.10, 5.11, and surrounding text).

Hence this form of intertemporal competitive equilibrium is efficient. This is so because each of the  $N$  fisheries is operated to internalize any spillovers in  $x$ . To put it another way, each fishery is operated taking the path of price as parametric to maximize capitalized profits but the dynamics of  $x$  are taken into account. Third, we study the problem where there is perfect mixing or perfect diffusion of fish across the boundaries of the  $N$  fisheries. In this case we assume each fishery is operated without regard to the effect on the dynamics of  $x$ . We shall see that this case is inefficient for the same reasons discussed by Clark (1990). That is to say, each fishery indulges in "scramble" competition to get their fish first before someone else does.

### 3.1.1 The Welfare Optimum

For the SOMP we define welfare derived by catch  $Y = qEx$ ; by the sum of consumer and producer surplus or  $U(Y) = S(Y) - cE$ : Where  $S(Y) = \int_0^Y P(u) du$  is the area under the demand curve  $p = P(Q)$ ; up to  $Q = Y = qEx$  and  $S^0(Y) = P(Y) = p$ : The socially-optimal management problem is defined as:

$$\max_{f^E(t)} \int_0^{\infty} e^{-\rho t} [S(qEx) - cE] dt \quad (13)$$

$$\text{subject to } \dot{x} = x(s - rx) - qEx; \quad x(0) = x_0 > 0 \quad (14)$$

$$s = r(1 - D) - m$$

The current value Hamiltonian for this problem is given by

$$H = S(qEx) - cE + \lambda x(s - rx - qE)$$

The FONC for optimality are given by:

$$\frac{\partial H}{\partial E} = 0; \text{ or } \lambda = P(Y) - \frac{c}{qx} \quad (15)$$

$$\dot{\lambda} = -\rho \lambda - \frac{\partial F}{\partial x} + qE \lambda - qEP(Y); \quad F(x) = x(s - rx) \quad (16)$$

$$\dot{x} = x(s - rx - qE) \quad (17)$$

### 3.1.2 Rational Expectations Equilibrium

Let us now consider REE. Here the stand-in fishery takes the price function  $p(t)$ ,  $t \geq 0$  as parametric, and chooses  $E$  to maximize discounted profits  $pqEx - cE$  subject to the biomass dynamics (14). So the REE solves

$$\max_{f^E(t)} \int_0^{\infty} e^{-\rho t} [pqEx - cE] dt \quad (18)$$

$$\text{subject to (10) and } 0 \leq E \leq E_{\max} \quad (19)$$

The current value Hamiltonian for this problem<sup>9</sup> is given by

$$H = pqEx - cE + \lambda x(s - rx - qE) \quad (20)$$

<sup>9</sup>The additional constraint  $0 \leq E \leq E_{\max}$  is required in order to make the REE problem well posed, given its linear structure.

The FONC for a singular solution are given by:

$$\dot{p} = P'(Y) - \frac{c}{qx}$$

where  $p(t) = P(Y(t))$ ; along with (16) and (17). Let  $\{E^s; x^s\}$  be optimal solutions and let  $p^s = P(qE^s x^s)$ : Then REE is defined by the requirement that  $p = p^s$  for all dates  $t$ : By comparing the optimality conditions of the SOMP and the REE problems it is clear that the FONC are the same for both problems. Hence if there is only one solution for the socially-optimal problem, REE replicates it and the equivalence between the two problems holds.

### 3.1.3 Bionomic Equilibrium

Now consider the third problem. Here each fishery takes  $p(t)$  as parametric, but it also takes  $x(t)$  as parametric because of the perfect mixing i.e. perfect diffusion of fish across boundaries. Of course, in the real world, fish mix or diffuse at a finite rate so each fishery would partially take into account the effect of their harvesting on the stock of fish, but it is instructive to analyze the polar case of perfect mixing (infinitely fast diffusion). In this case each fisher solves the problem

$$\max_E \{pqEx - cE\} \quad (21)$$

taking both  $p$  and  $x$  as parametric.<sup>10</sup> Optimality implies:

$$p = P(qEx); P(qEx)qx = c; \text{ with} \quad (22)$$

$$\dot{x} = x(s - rx - qE) \quad (23)$$

At steady state we have, for steady state  $x > 0$ , from (22), and (23).

$$p = P(qEx); P \frac{q}{r} E(s - qE) - \frac{q}{r} (s - qE) = c \quad (24)$$

Conditions (24) are a steady state version of Clark's bionomic equilibrium for one species. Since the curve  $P \frac{q}{r} E(s - qE) - \frac{q}{r} (s - qE)$  need not be monotonic in  $c$ ; there could be multiple bionomic equilibria.<sup>11</sup>

Turn now to a comparison of Clark's bionomic equilibrium with the social optimum for OSSs. It will turn out that the conditions used here for OSS are the same as for zero discount rate OSS for the SOMP.

For  $\delta = 0$ , by rewriting the OSS conditions above for the SOMP with  $\delta = 0$ , we see that the OSS effort is given by

$$\max \{S(qEx) - cE\} \text{ s.t: } 0 = x(s - rx) - qEx \quad (25)$$

For  $x > 0$ ,  $rx = s - qE$ , rewrite (21) thus,

$$\max_E \left\{ S \frac{q}{r} E(s - qE) - cE \right\} = \max J(E) \quad (26)$$

<sup>10</sup>As in the REE problem above in order to ensure that the problem is well posed for each  $p; x$  we place an upper bound  $E_{max}$  on  $E$  and a lower bound of zero on  $E$ .

<sup>11</sup>Multiple equilibria and their implications for regulation will be analyzed in detail in section 5.

The first and second-order necessary conditions for an optimum are  $J^0 = 0$ ;  $J^0 \leq 0$ . Thus, we obtain

$$p \frac{q}{r} (s - 2qE) = c; p = P(qEx); rx = s - qE \quad (27)$$

We compare the SOMP equilibrium (27) with interior bionomic equilibrium as in Clark (1990, p. 131), which is given by:

$$p \frac{q}{r} (s - qE) = c; p = P(qEx); rx = s - qE \quad (28)$$

Theorem 3 The bionomic equilibrium defined by (28) is inferior in terms of welfare relative to the socially-optimal equilibrium defined by (27).

For proof see Appendix.

The main economic conclusion for the case  $n = 1$  is the inefficiency of the open access equilibrium which of course is obvious from Clark (1990). More interesting to study are the different regulatory attempts to correct the problem.

### 3.2 The Multi Species Case

The socially-optimal problem using general matrix notation can be written as

$$\begin{aligned} \max_{f \in (t)g} \int_0^{\infty} e^{-\rho t} S(q \otimes E \otimes x) - \sum_{i=1}^n c_i E_i dt \quad (29) \\ \text{s. t. } \dot{x} = x \otimes (s - Tx) - q \otimes E \otimes x; s = r \otimes (I - D) - m \\ S(q \otimes E \otimes x) = S(q_1 E_1 x_1; \dots; q_n E_n x_n); Y_i = q_i E_i x_i \\ \frac{\partial S}{\partial Y_i} = p_i; p_i = P_i(Y_1; \dots; Y_n) = P_i(Y) \end{aligned}$$

where  $D$  is the fraction of sites destroyed (e.g. by human activities).

For the REE each stand-in fishery takes the price  $p(t)$ ,  $t \geq 0$  as parametric, and chooses the effort vector  $E$  to maximize joint discounted profits  $\sum_{i=1}^n (p_i q_i E_i x_i - c_i E_i)$  subject to the biomass dynamics. So the REE solves

$$\begin{aligned} \max_{f \in (t)g} \int_0^{\infty} e^{-\rho t} \sum_{i=1}^n (p_i q_i E_i x_i - c_i E_i) dt \\ \text{subject to (37) and } 0 \leq E \leq E_{\max} \end{aligned}$$

where  $p_i(t) = P_i(Y(t))$ : By comparing the optimality conditions of the SOMP and the REE problems it is clear that the FONC are the same for both problems. Hence if there is only one solution for the socially-optimal problem, REE replicates it and the equivalence between the two problems holds. Assume that the conditions for positive biomasses at Nature's steady state described in section 2 are satisfied. Thus, when harvesting takes place the steady state of the ecological system is defined using (2) as:

$$s + Tx = q \otimes E \otimes x; \text{ or } x = S[q \otimes E - s]; S = T^{-1}$$

Thus bionomic equilibrium is defined as:

$$\begin{aligned} p &= P(Y) = P(q \otimes E \otimes x); x = S[q \otimes E - s] \\ \text{and } p \otimes q \otimes x &= c \Rightarrow p \otimes q \otimes [S[q \otimes E - s]] = c \quad (30) \end{aligned}$$

From (44) we have  $n$  equations in  $n$  unknowns,  $E$ , as follows:

$$p(q \otimes E \otimes S[q \otimes E - s]) \otimes q \otimes [S[q \otimes E - s]] = c$$

As in the simpler case of  $n = 1$ ; this system of  $n$  equations in  $n$  unknown  $E$ s may have more than one solution.

The characterization of the SOMP and the identified deviations between the SOMP solution and the bionomic equilibrium suggests that the SOMP solution should be used as a yardstick for regulation purposes. Given however the complexity of the dynamic model describing the ecosystem, which turns out not to satisfy the usual concavity requirements, it is of interest to examine under what conditions the SOMP for the general model converges to a steady state.

## 4 Sufficient Conditions for Convergence of the SOMP

Consider the problem (29) with  $\frac{1}{2} = 0$ ; written, using (2a) for resource dynamics as:

$$\max \int_0^1 S(H)dt; \text{ s.t. } \dot{x}_i = F_i(x) - H_i; i = 1; 2; \dots; n$$

where  $S(H)$  denotes net current benefits from harvesting  $H = (H_1; \dots; H_n)$ ; It is defined as  $S(H) = S_1(H) - TC(H)$ ; where  $TC(H)$  denotes the total cost of capturing the harvest  $H$ ; and maximize is in the overtaking sense (Carlson et al. 1991). The standard value loss arguments (cf. Carlson et al. 1991) may be adapted to produce the new value loss argument for the following SOMP model. Let  $H^a; x^a$  solve the OSS SOMP problem

$$\max S(H); \text{ s.t. } H = F(x) \tag{31}$$

Take the integral of (31) up to horizon  $T$  and write:<sup>12</sup>

$$\int_0^T [S(H) - S(H^a)]dt = \int_0^T [\partial_H S(H^a)(H - H^a)]dt - D_1 = \int_0^T p^a [F(x) - \dot{x} - F(x^a)]dt - D_1$$

Assumption A1:  $S(H)$  is strictly concave and differentiable in  $H$ .

Assumption A2:  $x^a = \text{argmax } p^a F(x)$ :

Notice that the FONC for a maximizer of  $p^a F(x)$  are the same as for a maximizer of the OSS objective,  $S(F(x))$ . Suppose A1 holds for our problem. If so we may write

$$\int_0^T p^a [F(x) - \dot{x} - F(x^a)]dt - D_1 = p^a(x_0 - x(T)) - D_1 - D_2$$

where

$$D_2 = \int_0^T p^a [F(x) - F(x^a)]dt$$

<sup>12</sup> $\partial_x$  denotes the first derivative operator on a vector valued function  $f(x)$ :

The key thing to note is that under A1, the term  $D_1$  is an integral of non-negative terms, each of which is zero if and only if  $H = H^*$ . Under A2, each term of  $-D_2$  is non-negative. Hence under modest regularity conditions the standard value loss arguments reviewed in Carlson et al. (1991) may be adapted to prove:

**Theorem 4** Assume A1, A2 and regularity conditions. Let  $x^*(t; x_0)$ ;  $H^*(t; x_0)$  solve the SOMP with initial condition  $x(0) = x_0$ . Then for any initial condition, we have

$$x^*(t; x_0) \rightarrow x^*; H^*(t; x_0) \rightarrow H^*; t \rightarrow \infty$$

There is a new element contained in this theorem. Standard value loss arguments assume concavity of  $F$  and make one more support argument like that used to obtain term  $D_1$  to obtain term  $D_2$ . We use Assumption A2 here. At this level of generality the theorem holds for any  $F(x)$ . We must investigate the plausibility of Assumption A2. The first-order conditions for a maximum of  $p^*F(x)$  are linear in  $x$ , since  $F(x)$  is quadratic in  $x$ . Hence  $p^*_{@x}F$  can be written in the form.

$$p^*_{@x}F = L - Jx$$

where  $L$  is an  $n \times 1$  vector and  $J$  is an  $n \times n$  matrix which will be non-singular, generically. Hence  $x$  that solves

$$0 = L - Jx \tag{32}$$

will be generically unique. Since  $x^*$  solves (32), Assumption A2 is automatically satisfied for this case provided that  $x^*$  is non-negative.

The main theorem above suggests that we should expect convergence of the SOMP to OSS for small  $\frac{1}{2}$  by a continuity argument. This kind of argument is formalized in discrete time by Scheinkman's Visit Lemma argument (cf. Becker and Boyd 1997). We expect a similar result to hold in continuous time. More importantly, many generalizations of the above argument suggest themselves. For example suppose systems of the form  $\dot{x} = F(x)$  are placed upon sites and are coupled by diffusion across such sites as in Clark (1990). For example let us couple two such systems as follows

$$\begin{aligned} \dot{x}_1 &= F(x_1) - c_1 + z(x_2 - x_1) \\ \dot{x}_2 &= F(x_2) - c_2 + z(x_1 - x_2) \end{aligned}$$

The coupling only adds linear terms, therefore the same argument can be applied as above to prove that the SOMP converges to a unique OSS under zero discounting of the future. Hence, no matter how complicated the coupling and no matter how many sites, so long as the diffusion terms only add linear terms, the same argument applies to obtain convergence.

#### 4.1 Convergence of SOMP for Clark/Schaefer Models

The Value Loss arguments that extend methods reviewed in Carlson et al. (1991) that we made above are not directly applicable to Clark/Schaefer (cf. Clark 1990) where harvest  $H_i = q_i E_i x_i$ . To do this, first replace the transition equations of the section above using (2a) by

$$\dot{x}_i = x_i(K_i - \sum_{j=1}^n a_{ij} x_j) - q_i E_i x_i = x_i f_i(x) - q_i E_i x_i; i = 1; 2; \dots; n \tag{33}$$

Let  $S(H)$  be social benefit,  $P(H) = \partial_H S(H)$  and assume  $S(H)$  is concave nondecreasing in the harvest vector  $H$ . Net benefit is given by  $S(H) - cE$ . The OSS problem for the Clark/Schaefer model can be written as:

$$\max S(H) - cE \text{ s.t. } 0 = F(x) - H; F(x) = x \otimes f(x) \quad (34)$$

Suppose we have aggregated species in the model to the extent that it is sensible to assume that all  $x_i^* > 0$ . Let  $c=q$  denote the vector with  $i$ th element  $c_i=q_i$ . Notice that the social optimum problem can then be written thus: Maximize  $S(F(x)) - (c=q)f(x)$  over  $x$ . Let

$$x^* = \arg \max S(F(x)) - (c=q)f(x) \quad (35)$$

and assume  $x^*$  is unique. Notice that for the case  $n = 1$ , problem (34) is a concave problem because  $S(H)$  is concave and nondecreasing in  $H$ ;  $F(x)$  is concave in  $x$  and  $f(x)$  is linear in  $x$ . In the general model, however, because of species interactions coefficients  $a_{ij}$  in (33), problem (34) will not be a concave programming problem even though  $F(x)$  is quadratic and  $f(x)$  is linear. However for  $a_{ij} = 0$  for  $i$  not equal to  $j$ , it is a concave programming problem. Hence there will be an open set of  $A$  matrices for which (34) is a concave problem. Look at the  $i$ th equation of (34). We may solve it for  $E_i$  and write:

$$E_i = (1-q_i)f_i(x) - \left[ \frac{dx_i}{dt} \right] = (1-q_i)f_i(x) - \left[ \frac{d \ln(x_i)}{dt} \right]$$

Notice the appearance of the derivative of the natural logarithm of  $x_i$  in this equation. This will be important in what follows. Now consider the following

$$W(T) - W^*(T) = \int_0^T [S(H) - cE - S(H^*) + cE^*] dt = \int_0^T \{p^*[F(x) - x - F(x^*)] - c(E - E^*)\} dt - D_1$$

The term  $\int_0^T -c(E - E^*) dt$  can be written

$$\int_0^T -c(E - E^*) dt = \int_0^T -(c=q)[f - f^*] dt + \int_0^T (c=q) \frac{d \ln(x_i)}{dt} dt$$

Putting it all together we finally obtain

$$W(T) - W^*(T) = p^*:[x_0 - x(T)] + (c=q)[\ln(x(T)) - \ln(x_0)] - D_1 + \int_0^T [p^*[F(x) - (c=q)f(x)] - [p^*:F(x^*) - (c=q):f(x^*)]] dt \quad (36)$$

Finally recall that  $F(x)$  is quadratic and  $f(x)$  is linear. Therefore the programming problem

$$\max [p^*F(x) - (c=q)f(x)] \quad (37)$$

is a quadratic programming problem with FONC for optimal  $x$  which are linear in  $x$ . Furthermore the FONC for optimal  $x = x^*$  that solve (34) are given by

$$\partial_x S \partial_x F - (c=q) \partial_x f = 0 \quad (38)$$

The FONC for (37) are given by

$$p^* \nabla_x F - (c-q) \nabla_x f = 0 \quad (39)$$

Since  $p^* = S^0(F^*)$  then  $x^*$  solves (39) since it solves (38) with  $\nabla S = p^*$ .

Since (39) is a linear set of equations, generically  $x^*$  is the one and only solution to (39). Consequently if the optimum to (37) is interior, which we shall assume, therefore the optimum is  $x^*$  by the argument above. Hence we have the value loss statement

$$p^* F(x) - (c-q)f(x) \leq p^* F(x^*) - (c-q)f(x^*); \forall x \geq 0:$$

Thus we may write

$$-D_2 = \int_0^T [p^* F(x) - (c-q)f(x)] - [p^* F(x^*) - (c-q)f(x^*)] dt$$

Now that this preliminary work is done an obvious adaptation of Value Loss arguments reviewed in Carlson et al. (1991) gives us

Theorem 5 Let  $x^*(t|x_0); E^*(t|x_0)$  solve the generalized Clark/Schaeffer model

$$\max \int_0^T [S(H) - cE] dt \text{ subject to (33)}$$

Then if the OSS  $x^*, E^*$  is unique, we have

$$x^*(t|x_0); E^*(t|x_0) \xrightarrow{c} (x^*; E^*); t \rightarrow \infty$$

## 5 Regulatory Approaches

Having characterized the steady state and its stability properties at the SOMP we turn now to an analysis of regulatory approaches that would direct a harvested spatially structured system towards the socially-optimal steady state.

### 5.1 The Single Species Case

We start again by considering the special case of a single species model because of the insight that it provides for the more general models. Suppose that under open access harvesting, the industry expands effort when profits are positive and shrinks when profits are negative. That is, suppose the adjustment mechanism

$$\dot{E} = \lambda [(J_1(E) - c) E]; \lambda > 0 \quad (40)$$

$$J_1(E) = P \frac{aE(b-E)}{a + \frac{q^2}{r} E}; a = \frac{q^2}{r}; b = \frac{S}{q} \quad (41)$$

$$\text{where } J_1^0(E) = aP \frac{P_0}{P} - 1 + (b-E) \frac{P_0}{P} a(b-2E) \quad (42)$$

describes the industry dynamics.<sup>13</sup> Since  $(J_1(E) - c) E$  is short term equilibrium profits when the industry is putting out effort  $E$ , one would expect positive

<sup>13</sup>This is ad hoc to be sure. But we doubt that a rational expectations dynamic like that of Brock (1972) will change the point we wish to make here. Any dynamic mechanism where fishermen increase (decrease) efforts when net profits per unit effort are positive (negative) will yield the results we exhibit here. Even under farsighted rational expectations, in an adjustment cost type model, "scramble" competition is still likely to give us similar results.



profits to attract more effort into the industry and vice versa for negative profits.

Thus

$$E : \begin{matrix} \delta & \eta \\ < & > \\ = & = \\ > & ; \end{matrix} 0 \text{ as } J_1(E) : \begin{matrix} \delta & \eta \\ < & > \\ = & = \\ > & ; \end{matrix} c \quad (43)$$

It is natural to assume that demand price is higher than  $c$  for small  $Y = qEx$ : Thus demand price is higher than  $c$  for small  $E$  and large  $E$  slightly below  $s=q$ . Therefore the smallest steady state of (29) will be locally stable under (29). If there are only two steady states, only the smallest will be stable. If there are three steady states the middle one will be unstable.

To analyze regulatory approaches consider first a tax per unit effort. Suppose the fishermen themselves own catch quotas that are transferable so they want to jointly maximize the value of the fishery and achieve the social optimum. Suppose they impose upon themselves a tax on  $E$  of  $\zeta$  per unit  $E$  which is to be redistributed lump sum back to themselves by an authority that they, themselves, construct.

Let  $E^*$  denote the socially-optimal effort, which is defined from (27) as the solution  $J_2(E) = c$ ;  $J_2(E) = P(aE(b-E)) / a(b-2E)$ : An optimal tax per unit effort  $\zeta^*$  should be such that the bionomic equilibrium effort for the regulated fishery is  $E^*$ :

Since however bionomic equilibrium is determined by the steady states of (25), while the socially-optimal equilibrium is determined by  $E^* : J_2(E^*) = c$ , we characterize the optimal tax per unit effort  $\zeta^*$  in terms of the structure of bionomic and socially-optimal equilibria.

We examine first the curve

$$J_1(E) = P(aE(b-E)) / a(b-E); \quad a = q^2=r; \quad b = s=q; \quad E \in (0; b) :$$

Two cases can be distinguished:

1.  $J_1(E)$  is downward sloping on  $(0; b)$
2.  $J_1(E)$  initially decreases, then increases, then decreases on  $(0; b)$ ; so that multiple bionomic equilibria exist

**Proposition 1** For  $J_1(E)$  to initially decrease, then increase, then decrease again on  $(0; b)$ ; so that multiple bionomic equilibria exist, it is necessary that  $|\epsilon_P| < 1$  somewhere in the domain of  $E$ ; where  $\epsilon_P$  is the price elasticity of demand.

For proof see Appendix.

Thus we see that demand elasticity must be small enough relative to biological and economic parameters for a solution of  $J_1^0(E) = 0$  to exist in  $(0; b)$ . When such a solution exists  $J_1$  initially decreases, then increases, then decreases again on  $(0; b)$ : Furthermore, as can be seen from the definition of  $J_1^0$  above,  $J_1^0 < 0$  for  $E \in [0; b/2]$ ; since  $\frac{P^0}{P} < 0$ : Thus the  $J_1$  curve has the shape presented in figures 2-4. In general we could have one or an odd number of equilibria for (29). With the direction of the flow as shown in the figures, a locally unstable equilibrium is between a low effort locally stable equilibrium and a high effort locally stable equilibrium.

[Figures 1- 4]

For example with a linear demand function  $P(Y) = A - BY$ ;  $J_1$  is given by  $J_1(E) = A(1 - \frac{c}{A} - a(b - E)E)a(b - E)$ ;  $\frac{c}{A} = B/A$ ; assuming that the market is large enough so that  $1 - \frac{c}{A} > \max\{a(b - E)E\}$ : In this case  $J_1(E)$  is a cubic function, that in general provides three equilibria as solutions to the cubic equation  $(1 - \frac{c}{A} - Ba(b - E)E)a(b - E) - c^0 = 0$ ;  $c^0 = c/A$ :<sup>14</sup> For the three equilibria linear demand model, the curve  $J_1(E)$  is decreasing in  $[0; b/2]$  and has a local minimum and a local maximum in  $(b/2; b)$ : The local extrema can be determined as follows. The first and second derivatives of  $J_1(E)$  are

$$\frac{dJ_1(E)}{dE} = 4\frac{c}{A}a^2Eb - 3\frac{c}{A}a^2E^2 - \frac{c}{A}a^2b^2 - a$$

$$\frac{d^2J_1(E)}{dE^2} = 4\frac{c}{A}a^2b - 6\frac{c}{A}a^2E$$

From  $\frac{dJ_1(E)}{dE} = 0$  we obtain for  $\frac{c}{A}ab^2 - 3 > 0$  the two real roots  $E^{\max} = \frac{1}{3} \frac{2\frac{c}{A}ab + \sqrt{\frac{c}{A}a^2(b^2 - 3)}}{\frac{c}{A}a}$ ;  $E^{\min} = \frac{1}{3} \frac{2\frac{c}{A}ab - \sqrt{\frac{c}{A}a^2(b^2 - 3)}}{\frac{c}{A}a}$ : It can be easily seen that  $\frac{d^2J_1(E^{\max})}{dE^2} < 0$ ;  $\frac{d^2J_1(E^{\min})}{dE^2} > 0$ : Thus  $E^{\max}$  is the local maximum and  $E^{\min}$  is the local minimum. In the  $(a; b)$  space the graph of the function  $b = \frac{3}{\frac{c}{A}a}$  is a bifurcation curve. When the biological parameters cross the curve in the sense that they move to the subspace where  $b > \frac{3}{\frac{c}{A}a}$ ; then one equilibrium (the single root of  $J_1(E) = c$ ) becomes three equilibria (the three roots of  $J_1(E) = c$ ): Furthermore the depth of the bump in  $J_1(E)$  is

$$J_1(E^{\max}) - J_1(E^{\min}) = \frac{4}{27\sqrt{3}} \sqrt{\frac{c}{A}a} (b^2 - 3)^{\frac{3}{2}}$$

Therefore multiple equilibria will occur if unit harvesting cost  $c$  and market size  $A$  are such that  $J_1(E^{\min}) < c < J_1(E^{\max})$ : The width  $E^{\max} - E^{\min}$  is determined as  $\frac{2}{\frac{c}{A}a} \sqrt{\frac{c}{A}a(b^2 - 3)}$ :

On the other hand since the  $J_2$  curve is downward sloping on  $(0; b/2)$  as shown below, a unique socially-optimal effort level exists.

Proposition 2 When demand price is higher than  $c$  for small  $E$  a unique socially-optimal effort level exists on  $(0; b/2)$ :

For proof see Appendix.

Thus while under inelastic demand the open access bionomic equilibrium could produce three equilibria, the social optimum results in a unique stable equilibrium. Given these discrepancies we examine specific regulatory approaches.

<sup>14</sup>The roots of this equation are given by

$$E_1 = \frac{1}{6} \frac{K_1^2 + 4\frac{c}{A}a(-ab^2i - 3 + bK_1)}{aK_1}$$

$$E_2 = -\frac{1}{12} \frac{K_1^2 + 4\frac{c}{A}a(-ab^2i - 3 + 2bK_1) + i\sqrt{3}(K_1^2 + 4\frac{c}{A}a^2b^2i - 12\frac{c}{A}a)}{aK_1}$$

$$E_3 = -\frac{1}{12} \frac{K_1^2 + 4\frac{c}{A}a(-ab^2i - 3 + 2bK_1) - i\sqrt{3}(K_1^2 + 4\frac{c}{A}a^2b^2i - 12\frac{c}{A}a)}{aK_1}$$

where

$$K_1 = \sqrt[3]{-4\frac{c}{A}a^2b^3 - a^2 - 9ab + 27c^0 - 3\sqrt{3} \sqrt{\frac{c}{A}a^2b^2 + 4a + 4b^3 - 2a^2c^0i - 18\frac{c}{A}abc^0 + 27\frac{c}{A}(c^0)^2} - 2a}$$

### 5.1.1 Taxation on unit effort

To determine the tax per unit effort that can attain the socially-optimal effort  $E^s$  we consider the following two cases:

CASE I : Both  $J_1$  and  $J_2$  are downward sloping on  $(0; b)$ ;  $(0; b=2)$  respectively.

In this case if  $E^s$  is the unique solution of  $J_2(E) = c$ , we can find  $\zeta^s$  such that  $J_1(E) = c + \zeta^s$  implies  $E = E^s$ : This case is depicted in Figure 1.

CASE II: Function  $J_1$  initially decreases, then increases, then decreases on  $(0; b)$ ;  $J_2$  decreases on  $(0; b=2)$ .

Let  $J_1(E^{\min})$  denote the local minimum and  $J_1(E^{\max})$  denote the local maximum of  $J_1$  on  $(0; b)$ . The following cases can be considered

1.  $J_2(E^s) \in (J_1(E^{\min}), J_1(E^{\max}))$  as shown in Figure 2. There are three bionomic equilibria, two locally stable ( $E_1^1$  and  $E_3^1$ ); and one ( $E_2^1$ ) which is locally unstable. Bionomic equilibrium depends on initial conditions.
  - (a) Bionomic equilibrium is at  $E_1^1$ : Then a tax  $\zeta^s$  can be defined as in CASE I above to attain the socially-optimal effort  $E^s$ ;
  - (b) Bionomic equilibrium is at  $E_3^1$ : Application of the same tax will not bring down effort to  $E^s$  since the locally stable equilibria are located on the right side on the graph of  $J_1(E)$  against  $E$ : As  $\zeta$  continues to increase there will be a hard loss of stability at  $c + \zeta$  equal to the local maximum of  $J_1$  on  $(0; b)$ , which moves the system to the left side of the graph at  $E_0 < E^s$ . This may be viewed as too harsh so the tax should be lowered in an attempt to implement  $E^s$ .<sup>15</sup>
2.  $J_2(E^s) > J_1(E^{\max})$  as shown in Figure 3. This case is equivalent to CASE I where both  $J_1$  and  $J_2$  are downward sloping and  $E^s$  can be implemented by a tax per unit effort.
3.  $J_2(E^s) < J_1(E^{\min})$ ;  $c < J_1(E^{\min})$  as shown in Figure 4. There is a unique bionomic equilibrium which is globally stable at the high effort level  $E_3^1$ : Increasing the tax at the local maximum of  $J_1$  will move the system to the left side of the graph at  $E_0 < E^s$ : But then the fishery would stay undeveloped relative to the desired target  $E^s$ . The tax would be lowered in an attempt to implement  $E^s$  as in case 1b, but there would be another hard loss of stability at  $c + \zeta$  equal to the local minimum of  $J_1$  on  $(0; b)$ , which moves the system to the right side of the graph at  $E_0 > E^s$ : Then the fishery would be overexploited. This is a hysteresis trap where the classic instrument of a tax on unit effort cannot produce the socially-optimal outcome.

The above results indicate that if demand is elastic enough  $J_1^0 < 0$  on  $(0; b)$  then we may control a bionomic open access equilibrium with taxation of effort.<sup>16</sup> On the other hand if demand is inelastic so that multiple open access

<sup>15</sup> This is a hysteresis effect. It is related to work on lake management problems by Carpenter et al. (1999), Brock and Starrett (1999), Dechert and Brock (1999), Maler, de Zeeuw and Xepapadeas (2000).

<sup>16</sup> Of course this conclusion ignores other problems with taxation of effort such as monitoring, measurement, and policing of other mechanisms of evasion.

bionomic equilibria emerge, then there are cases where the social optimum cannot be implemented by a linear tax per unit effort, and the regulated bionomic equilibrium could be trapped either in a high effort region of overexploitation, or in a low effort region of underexploitation. Another possibility, depending on the parameters of the problem, is that the implementation of the socially-optimal effort requires first increasing and then decreasing the tax on effort, in order to counterbalance the effects of hysteresis.

### 5.1.2 Transferable Effort Quotas

When transferable effort quotas are considered as a regulatory instrument, the representative boat considers  $p$  and  $x$  as parameters and solves the constraint optimization problem

$$\max_E \{pqEx - cE\} \text{ s.t. } E \leq \bar{E}$$

where  $\bar{E}$  is the effort quota. The boat can buy or sell quota units. Assume that a competitive quota market exists and that quota can be bought or sold at price  $v$ : The net benefit from an additional quota unit is given by

$$pqx - c = J_1(E) - c$$

Quotas will be demanded or supplied according to whether  $J_1(E) - c > v$  respectively. Thus in equilibrium

$$J_1(E) - c = v \tag{44}$$

Suppose that  $J_1(E)$  is downward sloping on  $(0; b)$ ; then (44) determines the representative boat demand for quotas. The total demand for quotas is determined as the sum of individual demand as  $G(v; a; b; c)$ : If the total quota is  $\bar{E} = \sum E$  then the equilibrium quota price is determined by

$$\bar{E} = G(v; a; b; c)$$

It is clear that if the total quota is chosen to be equal to the socially-optimal effort level or  $\bar{E} = E^*$ ; then quotas are equivalent to taxation, since (44) implies that for  $E = E^*$ ;  $v = z^*$ :

If however  $J_1(E)$  initially decreases, then increases, then decreases on  $(0; b)$ ; then (44) implies that a downward sloping demand function for quotas for the representative boat can not be defined. That is, for a given quota price there will be three equilibrium quota quantities, two stable and one unstable. The analysis is similar to the case of taxation and indicates that under inelastic demand regulation of the bionomic equilibrium of the open access fishery through transferrable quotas faces the same difficulties as regulation through taxation of unit effort.

### 5.1.3 Limits on effort

When non transferable limits (or quotas) on effort are set, the representative boat considers again  $p$  and  $x$  as parameters and solves the constrained optimization problem

$$\max_E \{pqEx - cE\} \text{ s.t. } E \leq E^m$$

where  $E^m$  is the maximum allowed effort. The Lagrangian for this problem is given by

$$L = [pa(b - E) - c]E + \lambda(E^m - E)$$

The Kuhn-Tucker conditions for an interior solution  $E^0$  imply

$$\begin{aligned} pa \frac{\partial}{\partial E} [b - E] - c &= \lambda \\ \lambda \frac{\partial}{\partial \lambda} [E^m - E] &= 0; \lambda \geq 0 \end{aligned} \quad (45)$$

where  $pa \frac{\partial}{\partial E} [b - E] = J_1 \frac{\partial}{\partial E} [E^0]$ : If we set  $E^m = E^a$ ; then since  $E^a < E^1$  and  $J_1(E^1) = c$  in open access bionomic equilibrium, then for any locally stable bionomic equilibrium we have  $J_1 \frac{\partial}{\partial E} [E^0] - c = \lambda > 0$ : But then (45) implies that  $E^m = E^0 = E^a$ : So regulation through limits on effort attains the socially-optimal effort level. In Figure 4 for example this type of regulation implies that the representative boat restricts its effort to  $E^a$ : At this level however the representative boat earns positive profits since  $J_1(E^a) - c > 0$ , which implies that there are incentives for new entries into the open access fishery which would expand total effort beyond the socially-optimal level. Thus we need an entry fee to deter new entrants. This fee should be set such that

$$pa(b - E^a)E^a - cE^a - F = 0; p = P(a(b - E^a))$$

#### 5.1.4 Non linear taxation on effort

The socially-optimal effort level can also be implemented by a nonlinear costing of effort at the margin rather than the constant costing of effort at the margin by the tax  $\lambda$ . That is, we must find a schedule  $\lambda(E)$ , and an entry fee  $F$  such that

1. The representative boat maximizes profits facing  $p$ ;  $x$  as parametric, and paying a tax  $\lambda(E)$  per unit effort or

$$\max_E \{(pqx - c)E - \lambda(E)E\} \quad (46)$$

which implies that effort should be chosen so that

$$pqx - c - \lambda(E) - \lambda'(E)E = 0$$

2. At the profit-maximizing effort  $E^0$  excess entry or exit is prevented by a fee  $F$  ( $\geq 0$ ) given by

$$\begin{aligned} pqE^0x - cE^0 - \lambda(E^0)E^0 - F &= 0 \\ p &= P(qE^0x); rx = s - qE^0 \end{aligned}$$

The problem then is to find  $T(E) = E\lambda(E)$  in (44) to mimic the FONC of the socially-optimal problem. To determine  $T(E)$  we consider that the representative boat treats  $p$  and  $x$  as parameters and solves

$$\max_E \{(pqx - c)E - T(E)\}$$

with FONC

$$pqx - c = T^0(E) \text{ or } pa(b - E) - c = T^0(E); \text{ where}$$

$$x = \frac{s - qE}{r}; a = \frac{q^2}{r}; b = \frac{s}{q}$$

Since the FONC for the SOMP are given by

$$pa(b - 2E) - c = 0$$

by equating coefficients we obtain  $T^0(E) = paE$ : Then

$$T(E) = \frac{paE^2}{2} \text{ or } \lambda(E) = \frac{paE}{2}$$

## 5.2 The Multiple Species Case $i = 1; \dots; N$

For clarity purposes we focus on two species, that is  $N = 2$ : In this case biomass dynamics are determined by:

$$\begin{aligned} \dot{x}_1 &= x_1(s_1 - r_1x_1) - q_1E_1x_1 = F_1(x_1) - q_1E_1x_1 \\ \dot{x}_2 &= x_2(s_2 - (r_1 + r_2)x_2 - r_2x_1) - q_2E_2x_2 = F_2(x_1; x_2) - q_2E_2x_2 \\ s_i &= r_i(1 - D) - m_i; i = 1; 2 \end{aligned}$$

### 5.2.1 Full property rights for each species

We assume that there are two groups of price takers and each group has full property rights on each species. Each group takes as fixed the biomass of the other species and solves

$$\begin{aligned} &\max_{E_i} \int_0^{\infty} [p_i q_i E_i x_i - c_i E_i] dt \\ \text{s.t. } &\dot{x}_i = F_i(x_i) - q_i E_i x_i, i = 1; 2 \end{aligned}$$

The current value Hamiltonian is defined as

$$H = p_i q_i E_i x_i - c_i E_i + \lambda_i [F_i(x_i) - q_i E_i x_i]$$

and the FONC imply that

$$\begin{aligned} \lambda_i &= p_i - \frac{c_i}{q_i x_i} \text{ singular solution} \\ \lambda_i &= \frac{1}{2} \left[ \frac{\partial F_i}{\partial x_i} - q_i E_i \right] - p_i q_i E_i \end{aligned}$$

On the other hand the FONC for the SOMP are given by

$$\begin{aligned} \lambda_i &= p_i - \frac{c_i}{q_i x_i} \text{ singular solution} \\ \lambda_1 &= \frac{1}{2} \left[ \frac{\partial F_1}{\partial x_1} - \frac{\partial F_2}{\partial x_1} - q_1 E_1 \right] - p_1 q_1 E_1 \\ \lambda_2 &= \frac{1}{2} \left[ \frac{\partial F_2}{\partial x_2} - q_2 E_2 \right] - p_2 q_2 E_2 \end{aligned}$$

Comparing SOMP with the full property rights equilibrium it is clear that the full property rights equilibrium ignores the effects of species 1 on the biomass of species 2. Thus fishery 1 fails to internalize an external effect on fishery 2 reflected in the term  $\frac{\partial F_2}{\partial x_1}$ . That is, fishery 2 benefits when fishery 1 aggressively harvests or  $x_1$ . An appropriate dynamic subsidy per unit effort from two to one can be designed to align the incentives so that the necessary conditions for optimization are the same as the necessary conditions for the SOMP. Since it is straightforward to do this, we concentrate on the question of whether it is possible to design taxes on effort to steer the open access bionomic equilibria to the social optimum.

### 5.2.2 Open access harvesting

We define again gross surplus as  $S(Y_1; Y_2)$ ;  $Y_i = q_i E_i x_i$ ;  $i = 1; 2$ ; with  $\frac{\partial S}{\partial Y_i} = p_i$ . For  $\lambda = 0$  the OSS conditions are obtained by solving the problem

$$\begin{aligned} \max \quad & S(q_1 E_1 x_1; q_2 E_2 x_2) - c_1 E_1 - c_2 E_2 \\ \text{s.t. } 0 = \quad & x_1 (s_1 - r_1 x_1) - q_1 E_1 x_1 \\ 0 = \quad & x_2 (s_2 - (r_1 + r_2) x_1 - r_2 x_2) - q_2 E_2 x_2 \\ s_i = \quad & r_i (1 - D) - m_i; \quad i = 1; 2 \end{aligned}$$

For  $x_i > 0$  we obtain from the biomass equations

$$x_1 = \frac{s_1 - q_1 E_1}{r_1} \quad (47)$$

$$x_2 = \frac{s_2 - q_2 E_2}{r_2} - \frac{(r_1 + r_2)(s_1 - q_1 E_1)}{r_1 r_2} \quad (48)$$

Then we define

$$\begin{aligned} Y_1 &= q_1 E_1 x_1 = E_1 a_1 (b_1 - E_1) \\ Y_2 &= q_2 E_2 x_2 = E_2 [a_2 (b_2 - E_1) - \theta (b_1 - E_1)] \\ \theta &= \frac{q_1 q_2 (r_1 + r_2)}{r_1 r_2} \end{aligned}$$

Thus the SOMP can be written as

$$\begin{aligned} \max_{E_1; E_2} \quad & S(E_1 a_1 (b_1 - E_1); E_2 [a_2 (b_2 - E_1) - \theta (b_1 - E_1)]) - c_1 E_1 - c_2 E_2 \\ \text{or } \max_{E_1; E_2} \quad & J(E_1; E_2) - c_1 E_1 - c_2 E_2 \end{aligned}$$

Assuming that  $J(E_1; E_2)$  has a negative definite Hessian the socially-optimal effort levels  $(E_1^s; E_2^s)$  are determined by the solution of the FONC for the social optimum:

$$\begin{aligned} p_1 a_1 (b_1 - 2E_1) + p_2 \theta - c_1 &= 0 \text{ or } J_1^1(E_1; E_2) - c_1 = 0 \\ p_2 a_2 (b_2 - 2E_2) - c_2 &= 0 \text{ or } J_2^2(E_1; E_2) - c_2 = 0 \end{aligned}$$

The open access bionomic equilibria are defined on the other hand by

$$\begin{aligned} p_1 a_1 (b_1 - E_1) - c_1 &= 0 \text{ or } J_1^1(E_1) - c_1 = 0 \\ p_2 [a_2 (b_2 - E_2) - \theta (b_1 - E_1)] - c_2 &= 0 \text{ or } J_1^2(E_1; E_2) - c_2 = 0 \end{aligned}$$

For the open access bionomic equilibria the effort for species 1  $E_1^1$  is determined by the solution of  $J_1^1(E_1) - c_1 = 0$ ; while the effort for species 2 is determined by the solution of  $J_1^2(E_1^1; E_2) - c_2 = 0$ :

Suppose that as in the case of one species, industry dynamics are characterized by the adjustment mechanism

$$\begin{aligned} \dot{E}_1 &= \lambda_1 [J_1^1(E_1) - c_1] E_1^\alpha; \lambda_1 > 0 \\ \dot{E}_2 &= \lambda_2 [J_1^2(E_1; E_2) - c_2] E_2^\alpha; \lambda_2 > 0 \end{aligned}$$

Equilibria are determined by a recursive solution. Thus for  $E_1$  we could have three equilibria as in the case of one species, under the appropriate restrictions on the elasticity of demand for this species. For each equilibrium for species 1 the adjustment mechanism for species 2 determines a new set of equilibria. So we could have a maximum of nine equilibria for species 2, three locally unstable and six locally stable. Thus a hard loss of stability due to hysteresis in species 1, which creates a jump to another isolated locally stable equilibrium, could produce a consequent hard loss of stability in species 2.

To compare SOMP with open access bionomic equilibrium we start by comparing species 1, using

$$\begin{aligned} J_2^1(E_1; E_2) &= p_1 a_1 (b_1 - 2E_1) + p_2^\circ \\ J_1^1(E_1) &= p_1 a_1 (b_1 - E_1) \end{aligned}$$

With  $p_2^\circ > 0$  for  $0 \leq E_2 \leq \bar{E}_2$  the  $J_2^1(E_1; E_2)$  curve shifts to the right of the corresponding curve for a single species. On the other hand  $J_1^1(E_1)$  remains the same as in the case of a single species. This comparison indicates that the fishery for species 1 could be underexploited in bionomic equilibrium relative to the social optimum, because bionomic equilibrium does not internalize the impact of species 1 on species 2 (Figure 5).

[Figure 5]

A regulatory approach with taxes per unit effort can be used to determine taxes (or subsidies) per unit effort  $\lambda_1^\alpha; \lambda_2^\alpha$  such that

$$\begin{aligned} J_1^1(E_1) &= c_1 + \lambda_1^\alpha \implies E_1 = E_1^\alpha \\ J_1^1(E_1^\alpha; E_2) &= c_2 + \lambda_2^\alpha \implies E_2 = E_2^\alpha \end{aligned}$$

Taxes per unit effort will, however, face the same difficulties regarding the attainment of the social optimum as in the case of a single species because of hysteresis. The problem could be even worse given the increase in the number of possible equilibria for species 2.

On the other hand, non transferable limits on total effort along with entrance fees to restrict entry, as in the case of a single species, could be a more feasible regulation method.

A non linear tax as in the case of a single species could also be used. The FONC for the private optimum under tax schedules  $T_i(E_i) = \lambda_i(E_i) E_i$ ;  $i = 1; 2$  are

$$p_i q_i x_i - c_i = T_i^0(E_i)$$



Combining with the FONC for the social optimum and equating coefficients we obtain

$$\begin{aligned} \lambda_1(E_1) &= \frac{\mu_1 a_1 E_1}{2} - p_2^\circ \\ \lambda_2(E_2) &= \frac{p_2 a_2 E_2}{2} \end{aligned}$$

Entrance fees to restrict entry are also required as in the case of a single species.

For the general multi-species case, we seek solutions that are locally stable under the industry dynamics

$$\dot{E} = A \{ p(q \otimes E \otimes S[q \otimes E - s]) \otimes q \otimes [S[q \otimes E - s]] - c \}$$

The regulatory framework will be of the same structure as the one analyzed for the cases of  $n = 1$  and  $n = 2$ : The potential emergence of multiple equilibria and traps of hysteresis creates the difficulties in the use of linear effort taxes discussed for the cases of  $n = 1$  and  $n = 2$ .

### 5.3 A Role of Fast and Slow Time Scales in Decentralization Possibilities

We pointed out in the sections above a difficulty in using effort taxes to steer an open access fishery to the social optimum. However, we assumed that the biomass relaxes fast to the steady state relative to the economic variables in that treatment. Let us bring issues into bold relief by treating the polar opposite case here. Let  $x^*$  denote the socially-optimal solution, i.e. of problem (12).

At date  $t$ , let profits to an individual representative fishery with output  $Y_t$  taxed at  $\tau_t^*$  per unit be given by

$$p_t Y_t - c E_t - \tau_t^* Y_t$$

where  $\tau_t^*$  is the socially-optimal solution for the costate variable associated with the Hamiltonian function of problem (12). Recall that  $Y = qEx$ , so assume that effort expands so rapidly relative to the speed of change of  $x$  that temporary economic equilibrium given  $x_t$  is given at date  $t$  (dropping  $t$  subscripts to ease notation) by:

$$pqx - c - \tau^*(qx) = 0 \tag{49}$$

$$\dot{x} = x(s - rx) - qEx; \quad x(0) = x_0 \text{ given}; \tag{50}$$

$$p = P(Y); \quad Y = qEx \tag{51}$$

If it is assumed that economic equilibration is fast relative to the rate of change of biomass  $x$ , then (49)-(51) is a reasonable abstraction.

Here is the key question for decentralized regulation: Will the function  $\tau^*(\cdot)$  cause the system (49)-(51) to produce the socially-optimal path  $\{x^*(\cdot); E^*(\cdot); p^*(\cdot)\}$ ; where  $p^* = P(Y^*); Y^* = qE^*x^*$ ?

Clearly the starred solution is a solution of (49)-(51) as can be seen by inspection of the FONC (15)-(17) for the socially-optimal problem. Assume the demand function  $D(\cdot)$  maps the positive real line onto itself. Then, equations (49) and (51) can be solved to give the solution

$$Y_t = qE_t x_t = E[x_t; \tau_t^*] = P^{-1}\left[\frac{c}{qx_t} + \tau_t^*\right] \tag{52}$$

at each date  $t$ . Hence we may rewrite (50) as a time dependent differential equation as follows:

$$\dot{x} = x(s - rx) - P_i^{-1} \left[ \frac{c}{qx_t} + 1_t^a \right] = g(x; t); \quad x(0) = x_0 \text{ given} \quad (53)$$

The key observation is this. The function  $x^a(t)$  solves the same differential equation. Hence, since  $x^a(0) = x(0) = x_0$ , the basic uniqueness theorem of solutions of ordinary differential equations delivers (under modest regularity conditions) the result that  $x^a(t) = x(t)$  for all non-negative  $t$ . This argument is generalized to the case of  $N$  species below.

At date  $t$ , let  $1_{it}^a$  be the tax per unit output of species  $i$ ; where  $1_{it}^a$  is the socially-optimal solution for the costate variables associated with the Hamiltonian function of problem (29). The equilibrium conditions are given, after dropping  $t$ ; by:

$$p_i q_i x_i - c_i - 1_i^a (q_i x_i) = 0 \quad (54)$$

$$\dot{x} = x \otimes (s - Tx) - q \otimes E \otimes x; \quad x(0) = x_0 \text{ given} \quad (55)$$

$$p_i = P_i(Y_1; \dots; Y_n) = P_i(Y); \quad Y_i = q_i E_i x_i \quad (56)$$

As before the socially-optimal solution is a solution of (54)-(56) as can be seen by inspection of the FONC for the socially-optimal problem.(29). Assume that the system of demand functions (56) is defined in an open set  $\mathcal{U} \subset \mathcal{R}_+^n$  and that all demand functions are smooth. Let  $\partial_Y P(Y)$  be the Jacobian matrix associated with system (56) and assume that this Jacobian matrix is non singular. Then there is an open set  $\mathcal{V} \subset \mathcal{R}_+^n$ ,  $P_i(Y) \in \mathcal{V}$  and a function  $P_i^{-1}: \mathcal{V} \rightarrow \mathcal{R}_+^n$ . Thus if the Jacobian of the demand system is non singular<sup>17</sup> the demand system is invertible in some open set. Then, using (54)

$$q_i E_{it} x_{it} = E[x_t; 1_t^a] = P_i^{-1} \left[ \frac{c}{qx_t} + 1_t^a \right]; \quad i = 1; \dots; n$$

and (55) can be written as

$$\dot{x} = x \otimes (s - Tx) - P_i^{-1} \left[ \frac{c}{qx_t} + 1_t^a \right] \quad (57)$$

The socially-optimal function  $x^a(t)$  solves the same differential equation. Hence, since  $x^a(0) = x(0) = x_0$ , the basic uniqueness theorem of solutions of ordinary differential equations delivers the result that  $x^a(t) = x(t)$

This is a powerful result. It says that all we must do is impose an output tax equal to the social shadow price of a unit of biomass,  $1^a(t)$ , at each date  $t$  and let free entry of effort and market equilibration deliver the social optimum. Of course this particular lunch is not free. It relies on the assumption that biomass dynamics move slowly relative to economic dynamics. Our previous result on the potential impossibility of decentralized implementation of social optimum by output taxation assumed the polar opposite assumption on relative dynamical adjustment speeds. That is, that result assumed biomass dynamics were fast relative to economic dynamics. Surely the real world is somewhere in between. In any event, our brief treatment of the role of fast/slow dynamical speeds

<sup>17</sup>For  $n = 2$ ; this implies  $\frac{\partial P_1}{\partial Y_1} \frac{\partial P_2}{\partial Y_2} \neq \frac{\partial P_1}{\partial Y_2} \frac{\partial P_2}{\partial Y_1}$ :

of adjustment of economic dynamics and biological dynamics has exposed an important consideration in the practical design of decentralized implementation of social objectives. That is to say, in systems where there may be multiple stable states in the biological dynamics, it may not be possible to steer the joint dynamics into desirable parts of the state space using decentralized regulatory instruments such as catch taxes.

### 5.3.1 Regulation of capital accumulation

Let us now turn to a brief discussion of decentralized regulation of capital accumulation into a fishery. The socially-optimal problem is given by

$$\begin{aligned} \max \int_0^{\infty} e^{-\rho t} [S(NqEx) - NcE - A(I) - QI] dt \\ \text{s.t. } \dot{x} = x(s - rx) - NqEx; \quad x(0) = x_0; \text{ given} \\ \dot{N} = I - nN; \quad N(0) = N_0; \text{ given} \end{aligned}$$

Here the extra terms are the number of boat units,  $N$ , a convex cost of adjustment function,  $A(I)$  for investment  $I$ ;  $Q$  is the price of a unit of investment, and  $n$  is the rate of depreciation. Let  $\lambda^x, \lambda^N$  be the socially-optimal current value costates (i.e. shadow prices) of biomass  $x$  and capital stock  $N$ . An analog of the argument above shows that the social optimum can be implemented by imposing catch taxes on  $Y$  at rate  $\lambda^x$  under rational point expectations equilibrium as follows.

Let the privately-optimal problem be given by having the representative stand-in fishery take price function  $\{p(t)\}$  and biomass function  $\{x(t)\}$  as given and choose  $\{E; N\}$  to maximize

$$\begin{aligned} \max \int_0^{\infty} e^{-\rho t} [pNqEx - NcE - A(I) - QI - \lambda^x NqEx] dt \\ \text{s.t. } \dot{N} = I - nN; \quad N(0) = N_0; \text{ given} \end{aligned}$$

Let  $\{N^0; E^0; I^0\}$  be the optimal solutions. Let  $\{x^0\}$  solve

$$\dot{x} = x(s - rx) - N^0 q E^0 x; \quad x(0) = x_0; \text{ given}$$

Let  $p^0$  be given by

$$p^0 = D[N^0 q E^0 x^0]$$

REE is defined by the requirement

$$\begin{aligned} p &= p^0 \\ x &= x^0 \end{aligned}$$

for all dates  $t$ . That is, this is the concept of rational point expectations (That is, self consistent perfect foresight).<sup>18</sup> This proposition is proved by showing that quantities in the FONC match. It is a special case of ideas contained in the general equivalence theorem (cf. Becker and Boyd 1997).

<sup>18</sup>It can easily be generalized to stochastic settings (cf. Stokey and Lucas 1989, Sargent 1987).

## 6 Rational Routes to Robust Regulation

Most economists and ecologists argue that in view of the difficulty of actually estimating the parameters of coupled ecological/economic systems, regulatory design should reflect our ignorance especially in the presence of irreversible acts. This concern underlies arguments for Precautionary Principles and related ideas. We attempt here to sketch an approach based on recent work on Robust Control (Hansen and Sargent (2001) and Knightian Uncertainty (Epstein and Wang 1994, Brock and Durlauf 2001). In particular Hansen and Sargent (2001) review approaches to macroeconomic management in the face of ignorance and uncertainty that are similar in spirit to what we do here. To be specific, return to the problem of designing taxes on effort to regulate an open access fishery in bionomic equilibrium. This problem has the following structure. There is a welfare function,  $W(E(t; \mu); t; \mu)$ ; where  $t$  is the policy maker's instrument, and  $E(t; \mu)$  is the system's equilibrium response for each value of the instrument  $t$ . A Bayesian would face this problem by forming a prior on  $\mu$ ;  $p(\mu)$ , collecting a dataset (DS), forming the posterior  $p(\mu|DS)$  and choosing  $t$  to maximize the posterior mean of  $W$ . As treated by Epstein and Wang (1994) a Knightian might proceed as follows. She has a baseline prior  $p^\pi(\mu)$  and a family FP of perturbed priors around  $p^\pi(\cdot)$ . She might form a posterior using her dataset DS for each of these to get a family (abuse notation and call it FP) of posteriors with baseline  $p^\pi(\mu|DS)$ . She might then choose  $t$  to maximize

$$(1 - e)\mathcal{E}^\pi W(E(t; x); t; x) + e \ln f \mathcal{E}_p W(E(t; x); t; x) \quad (58)$$

where  $\mathcal{E}^\pi$  is expectation w.r.t. the baseline posterior  $p^\pi(\mu|DS)$ ,  $\mathcal{E}_p$  is expectation w.r.t. posterior  $p$ , where the  $\ln f$  ("lnf" stands for "In...imum") is taken over the family of posteriors FP. We will make this expression precise below. One can think of this procedure as follows. There is a probability  $e$  that Nature is mean to the policy maker. If  $e = 0$ , the policy maker is a conventional Bayesian decision maker. But the policy maker believes that Nature is mean with probability  $e$ . Furthermore she has beliefs about the set FP of alternative posteriors that Nature could play against her to minimize her welfare. To hedge against such a Nature, the Knightian policy maker chooses  $t$  to solve problem (67) above. This kind of behavior of a policy maker could be justified by ambiguity aversion. For example the experimental literature has shown that subjects will pay more to avoid an ambiguous urn than a Bayesian would. Another justification is that forms of extreme bounds approaches to robust econometrics can be justified as solving decision problems of the form (67). See Hansen and Sargent (2001) and Epstein and Wang (1994) for a discussion of rationales for studying such behavior. However, Sims (2001) raises several questions about this approach including: (i) the possibility of Dutch Books arising from irrational behavior, (ii) appropriateness of assuming minimax behavior of social planners and regulators in contrast to the individual agents, and (iii) the tendency to focus on deviations about a central model. Sims argues that compromises made in order to achieve analytical tractability may end up causing the researcher to focus "on relatively unimportant sources of model uncertainty. In particular he argues that by focusing the minimaxing on "a narrow, technically convenient, uncontroversial range of deviations from a central model....the danger is that we will be misled by the rhetoric of robustness into devoting less attention than we should to technically inconvenient, controversial deviations from the central

model.” In the first part of this section we investigate an “e-contaminated” version of Knightian Uncertainty studied by Epstein and Wang (1994). Later on we investigate a caricature of a controversial deviation from a central model with no hysteresis possibilities in the form of another model equally consistent with the data (and possibly the theory too) that displays hysteresis possibilities. For example, Hansen and Sargent (2001) argue that the class of possible misspecifications one wishes to robustify against should be disciplined by the data in the sense that the class should only contain members consistent with available data. Of course, we oversimplify their argument here.<sup>19</sup> In order to have a precise context return to the definitions of  $J_1(E)$ ;  $J_2(E)$  which we copy here for convenience.

$$J_1(E) = P[aE(b - E)]a(b - E); J_2(E) = P[aE(b - E)]a(b - 2E) \quad (59)$$

Recall that we wish to implement  $E^*$  such that  $J_2(E^*) = c$  by attempting to find a  $t^*$  such that

$$J_1(E) = P[aE(b - E)]a(b - E) = c + t^* \text{ implies } E = E^* \quad (60)$$

We showed in the case where  $J_1$  has two zeroes  $E_1 < E_2$  and  $E^* \in (E_1; E_2)$  that there were problems in implementation of  $E^*$ . Here the parameter vector  $\mu = (a; b; c; \rho)$ ;  $W(E; \mu) = S(Ea(b - E)) - cE$ ; and  $S^0(Y) = P(Y) = p$ . Here  $\rho$  denotes parameters of demand. We assume the policy maker is trying to choose  $t$  to implement  $E$  via (60) to maximize  $W$ , but, like a policy maker in real world, she does not know  $\mu$ . In order to be specific suppose the uncertainty resides in her knowledge of  $b$ . Let  $W(E; b) = S(aE(b - E)) - cE$  and suppose the policy maker’s ignorance is of the e-contaminated form

$$P(e) = (1 - e)b + em; m \in M(B)$$

where  $b$  represents a point mass of unity at  $b$  and  $M$  represents the entire set of probability measures with support  $[b - B; b + B]$ . Following Epstein and Wang (1994, p. 288, equations (2.3.1) and (2.3.2)) we shall assume the policy maker wishes to choose action  $E$  to maximize

$$\int W dP(e) = (1 - e)W(E; b) + e[\int W(E; w)dm(w)] \quad (61)$$

Since  $W(E; b)$  is increasing in  $b$ , and  $M$  contains all probability measures over  $b$  values with support  $[b - B; b + B]$ , (61) boils down to

$$\int W dP(e) = (1 - e)W(E; b) + eW(E; b - B) \quad (62)$$

Let  $E^*(b) = \text{argmax}_E W(E; b)$ . If we did a mean preserving spread of  $b$  by taking  $\mathcal{E}W(E; b + ez)$  where  $z$  is a random variable with zero mean and finite variance, we would find that  $dE^* = de = 0$  when evaluated at  $e = 0$ . This is the classic second order effect of risk analysis.

Let us compute the analog for Knightian Uncertainty of the e-contaminated form. The first-order necessary condition for

$$E^*(b; B) = \text{argmax}_E (1 - e)W(E; b) + eW(E; b - B) \quad (63)$$

<sup>19</sup>See Hansen and Sargent (2001) for the full details.

is given by

$$(1 - e)W_E(E^*; b) + eW_E(E^*; b - B) = 0 \quad (64)$$

Differentiate (64) w.r.t.  $e$  and evaluate at  $e = 0$  to obtain

$$W_{EE}(E^*; b) \frac{dE^*}{de} = W_E(E^*; b) - W_E(E^*; b - B) > 0 \quad (65)$$

Since  $W_{EE} < 0$  by concavity of  $W$  in  $E$ , we have  $dE^*/de < 0$  by (65). Compare this result with an increase in  $e$  for  $\mathcal{E}W(E; b + ez)$ . The first-order necessary condition for  $E^*$  is given by

$$\mathcal{E}W_E(E^*; b + ez) = 0 \quad (66)$$

Differentiate (66) w.r.t.  $e$  to obtain

$$\frac{1}{2} \mathcal{E} W_{EE}(E^*; b + ez) \frac{dE^*}{de} + W_{Eb}(E^*; b + ez)z = 0 \quad (67)$$

Evaluate (67) at  $e = 0$  to obtain

$$W_{EE}(E^*; b) \frac{dE^*}{de} = 0 \quad (68)$$

Hence  $dE^*/de = 0$ , in contrast to the Knightian Uncertainty case. Even though this is simple it serves as one way to formalize the notion of a Precautionary Principle for ecological management in the face of ambiguity and ignorance of the true parameters of the system. For example the parameter  $b$  could serve as a measure of the width of the uncertainty and the parameter  $e$  could measure the degree of ignorance. The case  $e = 1$  represents the case of complete ignorance. However Sims's (2001) critique prompts several questions about the objectives (58) and (62) as well as interpretation of the results. For example the welfare function  $W(E) = S(Y) - cE$  can be written as a sum of consumers' surplus and producers' surplus as follows,

$$W(E) = S(Y) - cE = S(Y) - D(Y)Y + D(Y)Y - cE$$

where the first term is consumers' surplus and the second term is producers' surplus. It can be seen that both are positive at the social optimum  $E^*$  depicted in Figure 1. Of course under open access equilibrium producers' surplus is driven down to zero at  $E^1$  depicted in Figure 1. It is well known that quasi-linear utility functions rationalize consumers' surplus expressions like the above. Hence in order for the regulator's preferences to represent a faithful aggregation of the preferences of the consumers and producers in the economy, the consumers must be quasi-linear Knightians and the producers must be linear utility of dollars Knightians. Notice that standard arguments for Bayesian posterior utility maximization and expected utility theory may produce Dutch Books against consumers and producers in this economy. When such potential inconsistencies in their choice rules are pointed out to them, they may drop their Knightian behavior. However, assuming that both consumers and producers are appropriately sheltered from Dutch Book arbitrageurs and the data is vague enough so that producers and consumers are not able to differentiate across the class of

potential misspecifications, the same arguments used by Gilboa and Scheidler (as discussed by Hansen and Sargent (2001)) as well as by Hansen and Sargent (2001) could be used by these agents to act Knightian as formulated here. Alternatively, the regulator herself could be Knightian when she aggregates the preferences of the inhabitants of the economy that she represents. It does not seem difficult to generate plausible arguments for a regulator's behavior that does some insuring against worst case scenarios, especially when one looks at the incentives that regulators themselves face. For example they tend to be paid on government salaries that reward them much less when things turn out right than the punishment they receive when things go wrong, especially if things go badly wrong. In any event, the reader is warned that we are working in a controversial area here. Nevertheless it seems appropriate, in the face of the huge uncertainties faced by both regulators and the regulated when it comes to ecosystem management, to start exploiting recent work on robustness. We did so above for a simple steady state non-dynamic example that centered at a baseline model and entertained misspecifications in the form of perturbations around that particular model. Turn now to cases where there is more than one baseline model. Let us first study a Bayesian regulator facing the problem of opening a new fishery (e.g. this fishery may have been closed for some time to achieve restoration). Assume the regulator only has the effort tax instrument available. Suppose a Bayesian analysis has placed posterior probabilities  $p_1$  and  $1 - p_1$  on  $\mu_1$  and  $\mu_2$  respectively. This Bayesian expected utility maximizer chooses the tax  $t$  to solve

$$W(E(t; \mu_1); t; \mu_1)p_1 + (1 - p_1)W(E(t; \mu_2); t; \mu_2) \quad (69)$$

Expression (69) contains an ambiguity. We need to specify how  $E(t; \mu_2)$  is chosen when there are two possibilities. See Figure 2 for the zone of values of  $c; c + t$  where there are two possible locally stable rest point solutions  $E$  to the differential equation (40). It seems natural that in the case where the fishery is being opened after having been closed, that effort would expand from a low level so the left most solution in Figure 2 would be chosen.

Suppose the initial  $E_0$  is small so the purpose of regulation is to restore a recovered fishery or to open up a relatively pristine area or fishery. We look for a rare but large shock (a Holling surprise) that could frustrate our attempts to use the instrument  $t$  to keep the system near the desired target  $E^*$ .

We construct Figure 4a where  $b/2$  and  $b$  are quite large relative to  $P(0)$  and this makes it easier to find a  $c$ -line such that an uncertainty band of width  $d$  around this  $c$ -line puts  $\mu$ -instrument regulator in a more precarious position if there is a rare but large enough tailed shock to  $E^*$  that could push it out of the attraction basin of  $E^*$ , past the root  $E_1$  of  $J_1^0 = 0$  that separates the two basins of attraction:

[Figure 4a]

In equation (69) let  $\mu$  be replaced by  $c$  and in Figure 4a consider a  $c$ -band of width  $d$ . Also let us represent mathematically the possibility of a large but rare shock to  $E$  chosen by Nature (it could be a technical surprise or an oversight by the regulator of current technical possibilities that enables effort to be increased by fishers more than the regulator expected) by  $f$  = probability there is such a large enough shock to  $E_0$  that puts the system into a bad  $E$ -basin (if parameters

are such that a bad basin exists). Assume that given  $t$  and shock (if there is one), the system “slaves” onto the stable branch of Figure 4a corresponding to the basin that the system is shocked (if there is a shock) into. Hence if the band  $[c + t^* - d; c + t^* + d]$  is wide enough to include parts of the rightmost stable branch of  $J_1$  on Figure 4a, where  $t^*$  is chosen to implement  $E^*$  when  $d = 0; f = 0$ , the Bayes/Blinder (Blinder 1998, Onatski 2000) regulator would choose  $t$  to maximize

$$(B/B) \quad (1 - f)\mathcal{E}W(E(t; z; E_0); c + z) + f\mathcal{E}W(E(t; z; E_0 + S); c + z):$$

We need to define the objects in equation (B/B). To keep things simple suppose the distribution of the shock  $S$  is a point mass at a number  $S$  large enough that for all  $z \in [c - d; c + d]$  the system is pushed into the Bad Basin area of  $E$ . Assume  $t$  moves first  $z$  moves second, and the system relaxes to the stable branch of  $J_1$  in Figure 4a where  $E_0 + S$  is. Notice that we can construct graphs in which for some  $z$ s such that the horizontal line  $c + t + z$  doesn't cut the graph of  $J_1$  on the right side (only the left side is cut), we will still go to the good side of  $J_1$  even though the initial condition is  $E_0 + S$ . We now have a mathematically well defined rule that selects  $E(t; z; E_0 + S)$ .

Now turn to our Bayesian in (69). Call this kind of regulation, indirect control via instrument  $t$ . The simplest version is to assume that the variance of  $z$  is zero so all attention is placed upon  $(f; S)$ . Since  $z = 0$  and  $c$  is fixed throughout the analysis, suppress them in the notation for (B/B) and put

$$B(t; f) = (1 - f)W(E_1(t + c)) + fW(E_3(t + c)): \quad (70)$$

In the case where  $J_1$  is not monotone decreasing but is first decreasing, then increasing, then decreasing, let  $J_1^{\min}, J_1^{\max}$  denote the local minimum and the local maximum values of  $J_1$ . Let  $E^{\min}, E^{\max}$  denote the local minimizer and local maximizer of  $J_1$ . Let  $J_{1i}$  denote the restriction of  $J_1$  to  $[0; E^{\min}]; [E^{\min}; E^{\max}]; [E^{\max}; \infty)$ . Each of these functions is invertible on its domain. Let  $J_{1i}^{-1}$  denote the inverse function. Put

$$J_1(E_i^{-1}(t + c)) = t + c \quad (71)$$

where  $i = 1; 3$  denote the smallest and largest locally stable solutions and  $i = 2$  is the middle (locally unstable solution). Let  $t^*(f)$  maximize (70) s.t. (71).

Theorem 6  $dt^*/df > 0$

For Proof see Appendix.

Notice that this Bayesian facing a “Holling nasty surprise” with probability  $f$ , facing slaving of  $E$  to  $E_1^{-1}$  or  $E_3^{-1}$  depending on whether the surprise appears to shock the small initial condition  $E_0$  into the Bad Basin of  $E_3$  acts observationally (to us scientists) rather like a Knightian! In fact we could probably formulate a Knightian problem (without Holling surprises) that gives the same  $t^*$  response of the regulator as does this Bayesian regulator. That is, this Bayesian reacts first order to increasing  $f$  at  $f = 0$  in contrast to the usual second-order reaction of a Bayesian to a mean preserving spread.

Now we can ask what happens to the optimal  $t^*$  in a Knightian indirect via instrument  $t$  regulatory control problem facing both Holling surprise shocks and



mean preserving spread  $z$ . Let the timing be as follows. Regulator chooses  $t$ , Nature then draws  $z$  from density  $f_Z(z)$ ; She flips a device with Bad occurring with probability  $f$ ; and if Bad happens the shock  $S$  is added to  $E_0$ . The actual  $E$  is then determined by:

$J_1(E_1^{-1}(t + c + z)) = t + c + z$ , root  $E_1^{-1}$  chosen if Good. If Bad and  $E_0 + S$  is in the basin of the bad root  $E_3^{-1}$ , root  $E_3^{-1}$  is chosen. Otherwise root  $E_1^{-1}$  is chosen.

Define  $B(t; f; d)$  by

$$B(t; f; d) = (1 - f)\mathcal{E}W(E_1^{-1}(t + c + z)) + f\mathcal{E}W(E_3^{-1}(t + c + z))$$

Assume the regulator chooses  $t^*$  to maximize

$$(1 - e)B(t; f; d) + e \ln f B(t; f; z)$$

where the support of  $z$  is  $[-d; d]$ . The Infoperator is defined by

$$\text{Inf} B(t; f; z) = \text{Inf}((1 - f)W(E_1(t + c + z)) + fW(E_3(t + c + z))) \quad (72)$$

where the Infimum of RHS (72) is taken over all  $z$  in  $[-d; d]$ .

With this setup we may now compute  $t^*(e; f)$ , compute  $t_f^*(0; 0)$  and see if it increases with  $e$ . That is, we can also compute the cross partial  $t_{fe}^*(0; 0)$  and see if it is positive. We expect it to be positive because we would expect the Knightian to be even more cautious.

## 6.1 General Treatment of Bayesian Facing Alternative Stable States with Initial Condition Shocks

We formulate the general problem first, then turn to a special case. Consider the following set of differential equations

$$\begin{aligned} dp/dt &= I_p(P^{-1}(p) - qEx); p_0 \text{ given;} \\ dx/dt &= I_x(x(s - rx) - qEx); x_0 \text{ given} \\ dE/dt &= I_E(pqEx - (c + t)Ex); E_0 \text{ given; } E_0^0 = E_0 + S \end{aligned}$$

Let the regulator set  $t$  to maximize

$$\mathcal{E} \text{Limsup}_T \int_0^T [S(qEx) - cE] dt \quad (73)$$

Here the integral is from 0 to  $T$ , and we take the Limsup above because the time average may not converge whereas the Limsup is always defined. The Limsup is taken as  $T$  tends to infinity. The expectation is taken over the distribution of initial  $E$ -state shocks  $S$ . Consider the infinite horizon present value of a benefit stream discounted at rate  $r$ , call this  $PV(r)$ . There are theorems that relate the limiting value of  $PV(r)=r$  to time averages like (73). Hence we use (73) as an analytically tractable "first cut" approximation to an objective like  $PV(r)=r$  for small values of  $r$ . Now let  $I_p$  tend to infinity relative to the other  $I$ s. This amounts to assuming that price formation is fast relative to the other dynamics. We also assume (which is automatic for the simple one

dimensional case above) that the price dynamics has only one stable state. Thus we assume

$$P^{-1}(p) - qEx = 0; \text{ i.e. } p = P(qEx)$$

Now suppose that  $I_x$  is also infinite. We must also assume that  $s - qE > 0$ . Thus

$$x[(s - rx) - qE] = 0; \text{ i.e. } rx = s - qE; \text{ for } x > 0$$

It is easy to see that the solution  $rx = s - qE > 0$  is globally stable for all initial  $x_0 \in (0; \infty)$ . Finally we assume that  $I_E$  is also infinite. Our problem (73) now reduces to the following problem. Choose  $t$  to maximize

$$\mathcal{E}W = F_S(E_2(c + t))[W(E_1(c + t))] + (1 - F_S(E_2(c + t)))[W(E_3(c + t))] = K(t):$$

That is, choose  $t$  to maximize  $K(t)$ . Here  $F_S(x) = \Pr[S < x]$  is the cumulative distribution function of random shock  $S$ . Root  $E_1$  of  $J_1(E) = c + t$  is chosen if  $E_0 + S < E_2(c + t)$  where  $E_2(c + t)$  is the middle root of  $J_1(E) = c + t$  and root  $E_3$  of  $J_1(E) = c + t$  is chosen if  $E_0 + S > E_2(c + t)$ . Obvious choices are made for the case where  $J_1(E) = c + t$  has only one root. The function  $K(t)$  is now well defined.

Notice that Nature is mean with probability  $1 - F_S(E_2(c + t))$ . We now have a justification for the appearance of  $e$  in the Knightian formulation! To put it another way, we have endogenized by hierarchic time scales and alternative stable states together with possibilities of large shocks to the state of the system. That is, if Nature chooses a shock to the initial condition of your managed ecosystem big enough to put you in a bad basin for that ecosystem and regulatory institutions are such that there is a cost to changing the instrument's value rapidly enough to undo the damage caused by this large initial-state shock then the regulator should act like a Knightian with

$$e = 1 - F_S(E_2(c + t))$$

Notice that  $W(E_3(c + t))$  is the worst steady state value that Nature can play against the regulator! In the original formulation of Knightians and of Robust controllers there is a set of priors that one minimizes over a fraction of the time. If we replace that set with the worst steady states and replace  $e$  with the probability that the shock to the initial state condition throws the system into the basin of attraction of the worst steady states, we are close mathematically to the original Knightian setup! But now the parameter  $e$  as well as the worst possible outcome are derived from the underlying structure of the scientific problem rather than being imposed in a somewhat ad hoc manner. In general if a regulator is choosing instruments to regulate a system that is complex enough to possess multiple time scales, and we take the time average of the welfare generated for each fixed value of the regulatory instrument  $t$ , we get the general problem above. We believe that this particular analytical regulatory framework may be more appropriate for the analysis of regulation of complex ecosystems with a hierarchy of time scales and potentially complex dynamics with multiple attractors than conventional analytical frameworks. We have already seen how our framework exposed a new consideration in designing regulatory instruments. That is, we posed the question of how one should deal with potential alternative stable states and potential surprise shocks to the state of the system.

## 7 Concluding Remarks

In this paper we have studied the optimal economic management of multispecies ecosystems where the underlying ecosystem was modeled by a series of differential equations. While the underlying differential equation dynamics are mathematically equivalent to a generalized Lotka-Volterra multispecies system, we discussed alternative interpretations which included hierarchical metapopulation models with an underlying spatial mosaic. Different contexts give different underlying generalized interaction matrices.

We studied optimal regulation of these systems using decentralized instruments such as taxes. Novelty arose from dealing with problems caused by multiple equilibria and hierarchical time scales. For example we showed that implementation of social optimum by decentralized taxes on effort in open access institutions was possible if biomass dynamics are slow enough relative to economic dynamics. But implementation was not possible if biomass dynamics were fast relative to economic dynamics. We also located sufficient conditions for infinite horizon management to drive the system to a unique steady state. This was a nontrivial generalization of received value-loss arguments in the turnpike theory literature.

Perhaps most importantly, in Section 6 we extended the received theory of regulation of ecosystem management to the case of Robust Regulation when there is not only uncertainty, but also ambiguity about the dynamics. We uncovered a new justification for recently received mathematical models of Robust Regulation. Our approach arises from two time scales and shocks to initial conditions after regulatory actions are taken. We believe that potential extensions of our result may help produce new models of Robust Regulation that overcome some of the criticisms of the recently received theory.

## Appendix

Proof of Theorem 1: Put  $a = 1$ , W.L.O.G. Write  $U = P \{(f_i - \frac{1}{2}x_i)\}$  thus, using  $s_i$  notation,

$$U = x_1(s_1 - x_1) + x_2(s_2 - 2x_1 - x_2) + \dots + x_n(s_n - 2[x_1 + \dots + x_{n-1}] - x_n)$$

$$s_1 < s_2 < \dots < s_n$$

Observe that if  $x_i > 0$  and  $x_j > 0$  solve the FONC for an interior optimum we must have

$$s_i = 2[x_1 + \dots + x_n] = s_j$$

which is a contradiction to  $s_i < s_j$  for  $i < j$ . Hence only one  $x_i$  can be positive at an optimum. Check that it must be  $x_n = s_n/2$  by showing that this particular specialization yields maximum economic value. Q.E.D.

Proof of Theorem 2: Clearly  $x_1^a = \frac{s_1}{2}$ ;  $x_2^a = \frac{(s_2 - s_1)}{2}$ ;  $\dots$ ;  $x_n^a = \frac{(s_n - s_{n-1})}{2}$ . Since the optimal harvest  $H^a$  is the minimum of the maximal  $f_k(x)$ , therefore under Assumption  $f_1^a > \max\{f_k^a; k \neq 1\}$ . Q.E.D.

Proof of Proposition 1: Define the curves  $J_1(E)$  for the bionomic equilibrium and  $J_2(E)$  for the SOMP optimum as follows

$$J_1(E) = P \frac{q}{r} E(s - qE) - \frac{q}{r} (s - qE)$$

$$J_2(E) = P \frac{q}{r} E(s - qE) - \frac{q}{r} (s - 2qE)$$

We see that  $J_1$  lies above  $J_2$ ; that  $J_2 < 0$  for  $E > s/(2q)$ , and that  $J_1 < 0$  for  $E > s/q$ . Thus the smallest root of  $J_2(E) = c$  is achieved for a smaller value of  $E$  than the smallest root of  $J_1(E) = c$ . Put  $a = q^2/r$ ;  $b = s/q$  and rewrite the equations above as follows

$$J_1(E) = P (aE(b - E)) - a(b - E)$$

$$J_2(E) = P (aE(b - E)) - a(b - 2E)$$

Since  $J_1(0) = J_2(0) = P(0)ab > 0$ ,  $J_1 < 0$  for  $E > b$ ; and  $J_2 < 0$  for  $E > b/2$ ; it follows that  $J_1$  lies above  $J_2$ , for  $E \in (0; b)$ .<sup>20</sup> Thus we see that at the conditions  $J_1(E^1) = c$  for bionomic equilibrium and  $J_2(E^2) = c$  for social optimal, bionomic equilibrium loses welfare relative to the optimum, since  $J_1(E^2) \neq c$ . The welfare loss results from the taking of too many fish. Q.E.D.

Proof of Proposition 1: Since the demand price is higher than  $c$  for small  $E$ , it follows that  $J_1$  initially decreases for small  $E$ . Then  $J_1$  becomes negative for  $E > b$ . Therefore if  $J_1$  fails to be monotone decreasing on  $(0; b)$ ; its first derivative,  $J_1^0$ ; must be zero at some point  $\hat{E}$  in  $(0; b)$ . This gives the necessary condition

$$J_1^0 = aP \left[ -1 + (b - E) \frac{P^0}{P} - a(b - 2E) \right] = 0$$

Since  $P^0 = P = (QP^0(Q))$ , we have  $\frac{P^0}{P} = \frac{1}{P} \frac{1}{Q}$ : Substituting for  $Q = aE(b - E)$  and inserting into the necessary condition above gives us

$$(b - E) \frac{1}{P} \frac{1}{aE(b - E)} a(b - 2E) = 1$$

<sup>20</sup>Notice that the only difference between  $J_1$  and  $J_2$  is the "2" in  $J_2$ .

or  $\frac{1}{P} \frac{(b-2E)}{E} = 1$  i.e.  $\hat{E} = \frac{b}{2+P}$ : Hence for  $\hat{E} < b$ , we must have  $b > \frac{b}{2+P}$ ; i.e.  $|P| < 1$  Q.E.D.

Proof of Proposition 2: The first derivative of  $J_2$  is:

$$J_2^0 = 2aP \left[ -1 + \frac{1}{2} \frac{\tilde{A} P^0}{P} \right] a(b-2E)^2 < 0$$

since  $P > 0$ ;  $P^0 < 0$ : Therefore  $J_2$  is monotone decreasing on  $(0; b/2)$  and a unique socially-optimal effort level exists. Q.E.D.

Proof of Theorem 6:  $B_t(t^*(f); f) = 0$ ;  $B_{tt}[t^*] + B_{tf} = 0$ ;  $B_{tt} < 0$  (by SONC). So we must show that  $-B_{tf} < 0$ : Now at  $f = 0$ ;  $W_E(E_1^1) = 0$  ( $E_1^1 = E^*$  at  $f = 0$ ). Since

$$B_t = (1-f)W_E(E_1^1)E_1^{1^0} + fW_E(E_3^1)E_3^{1^0}$$

Therefore,

$$B_{tf} = -W_E(E_1^1)E_1^{1^0} + W_E(E_3^1)E_3^{1^0} = W_E(E_3^1)E_3^{1^0} > 0$$

since  $E_3^{1^0} < 0$ ;  $W_E < 0$

Q.E.D.

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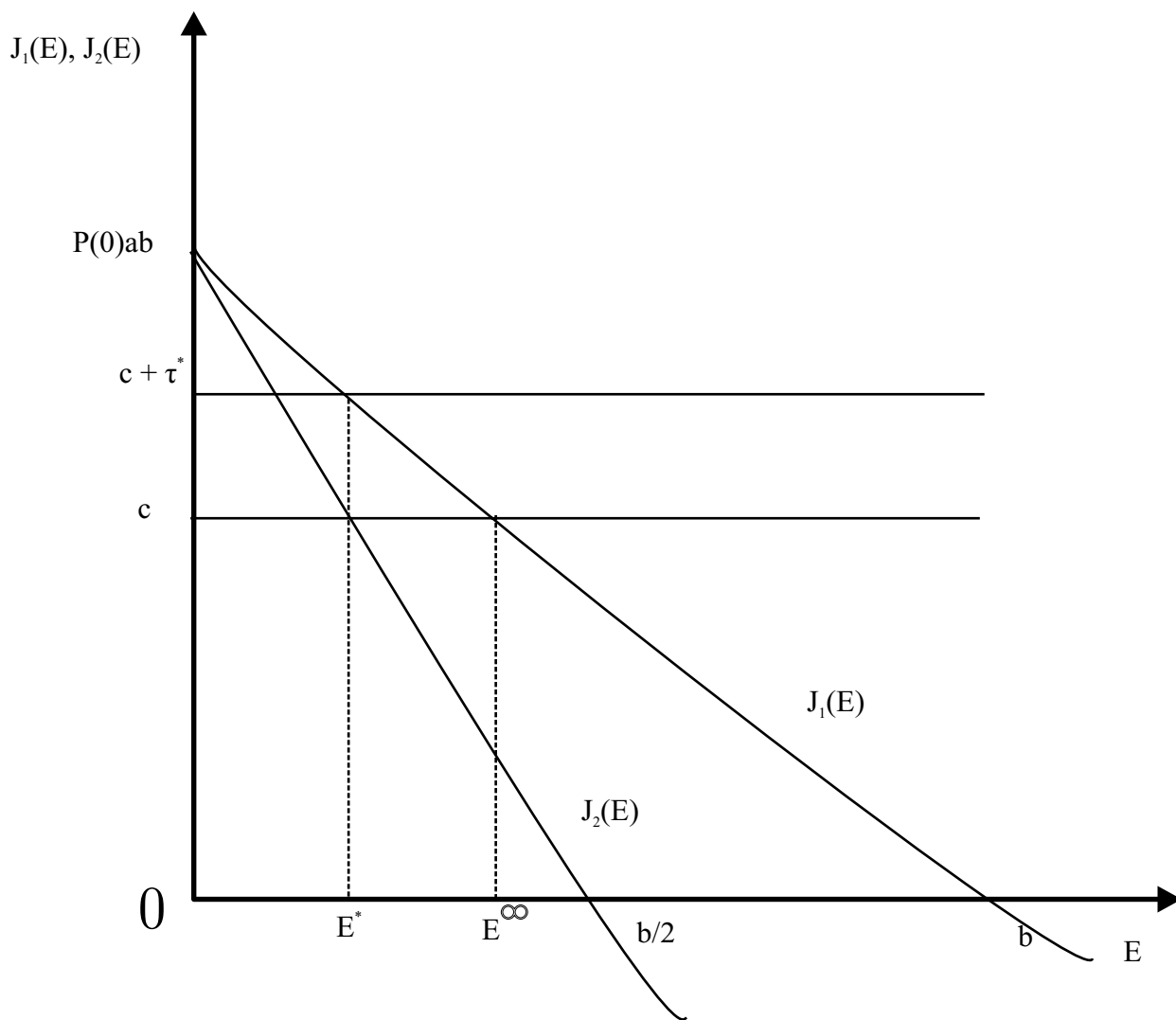


Figure 1: Social optimum and bionomic equilibrium for a single species, one equilibrium



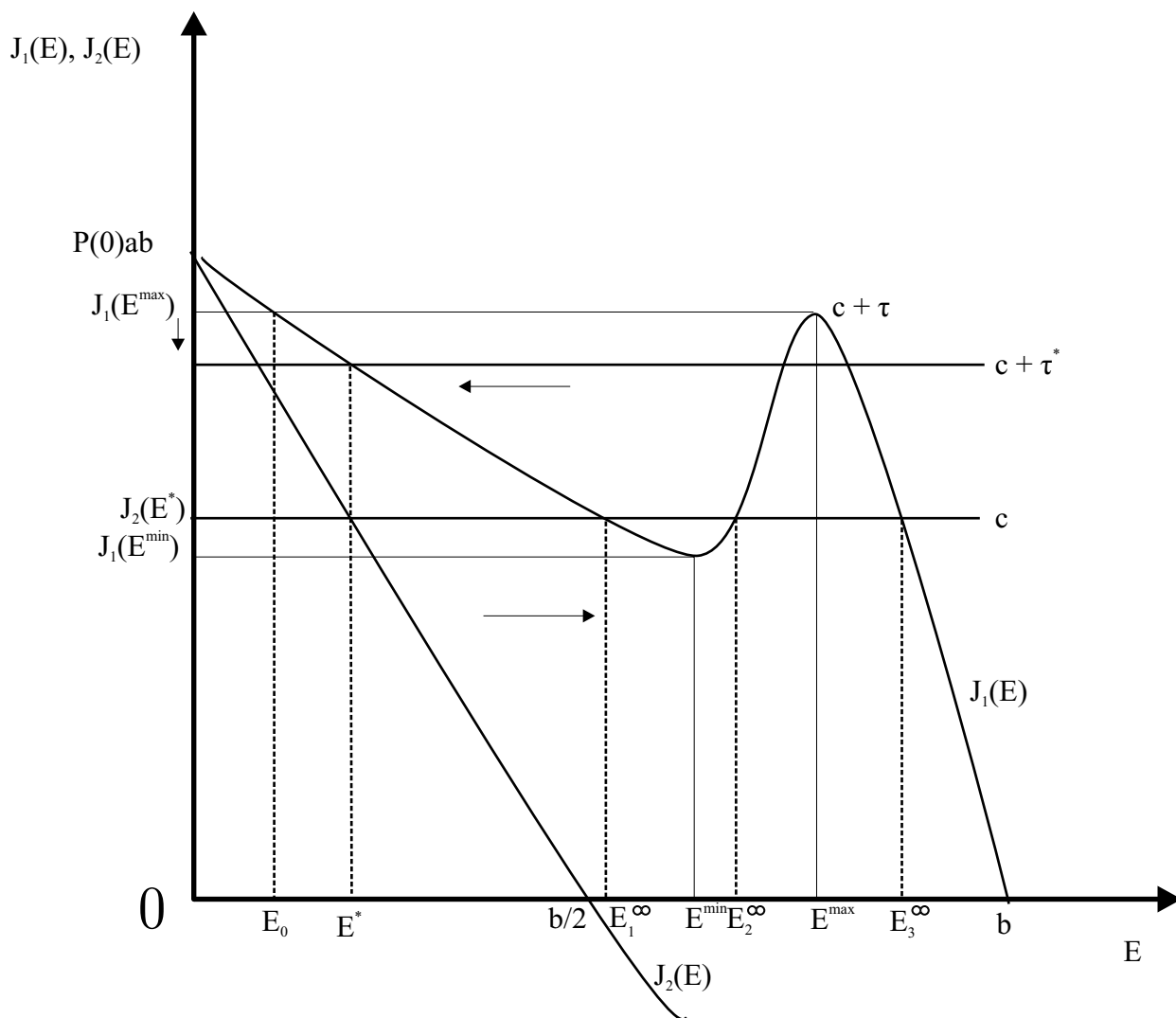


Figure 2: Social optimum and bionomic equilibrium for a single species, three equilibria and hysteresis

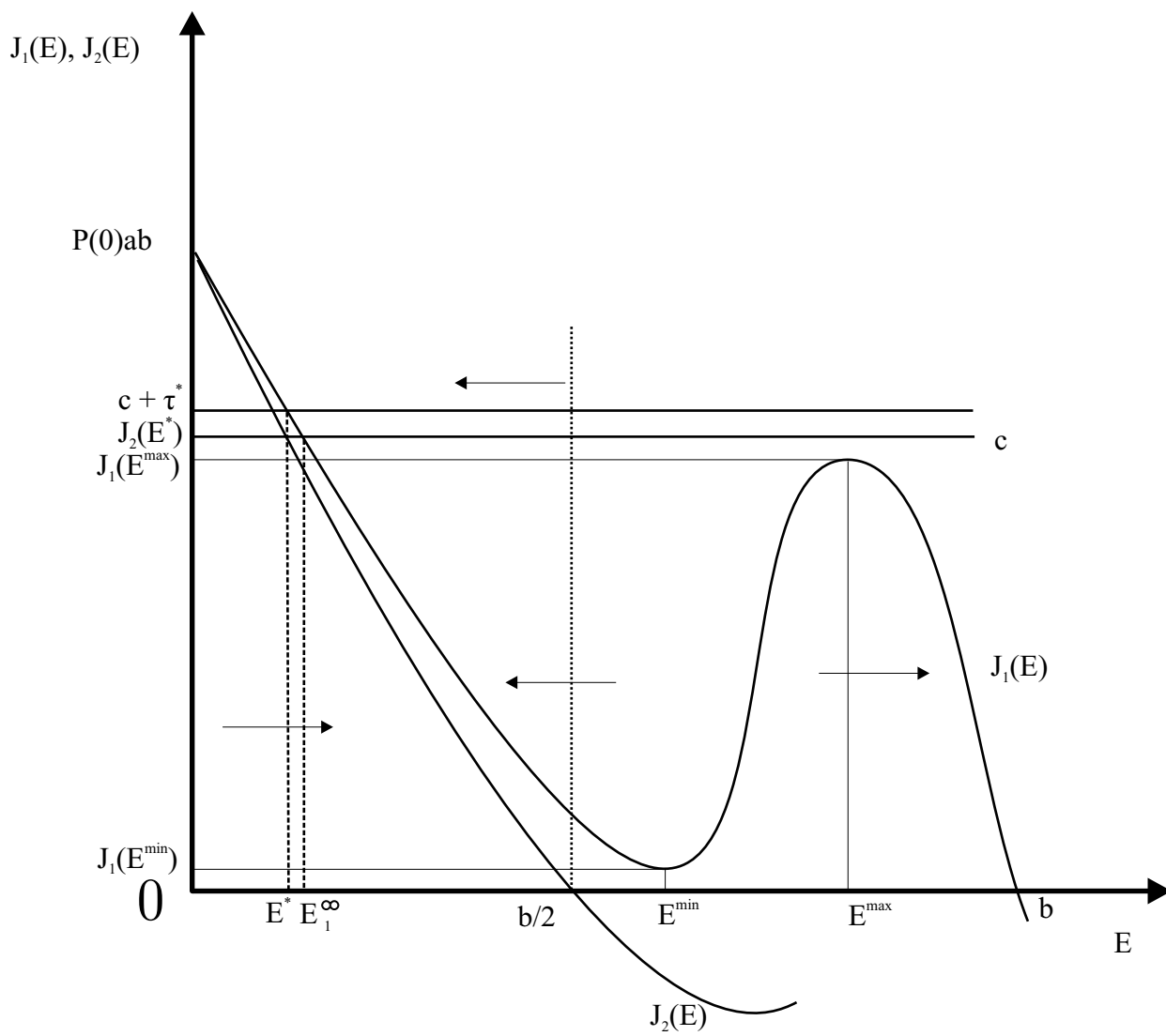


Figure 3: Social optimum and bionomic equilibrium for a single species, three equilibria without hysteresis trap

$|\varepsilon_P| \ll 1$  three equilibria without hysteresis trap

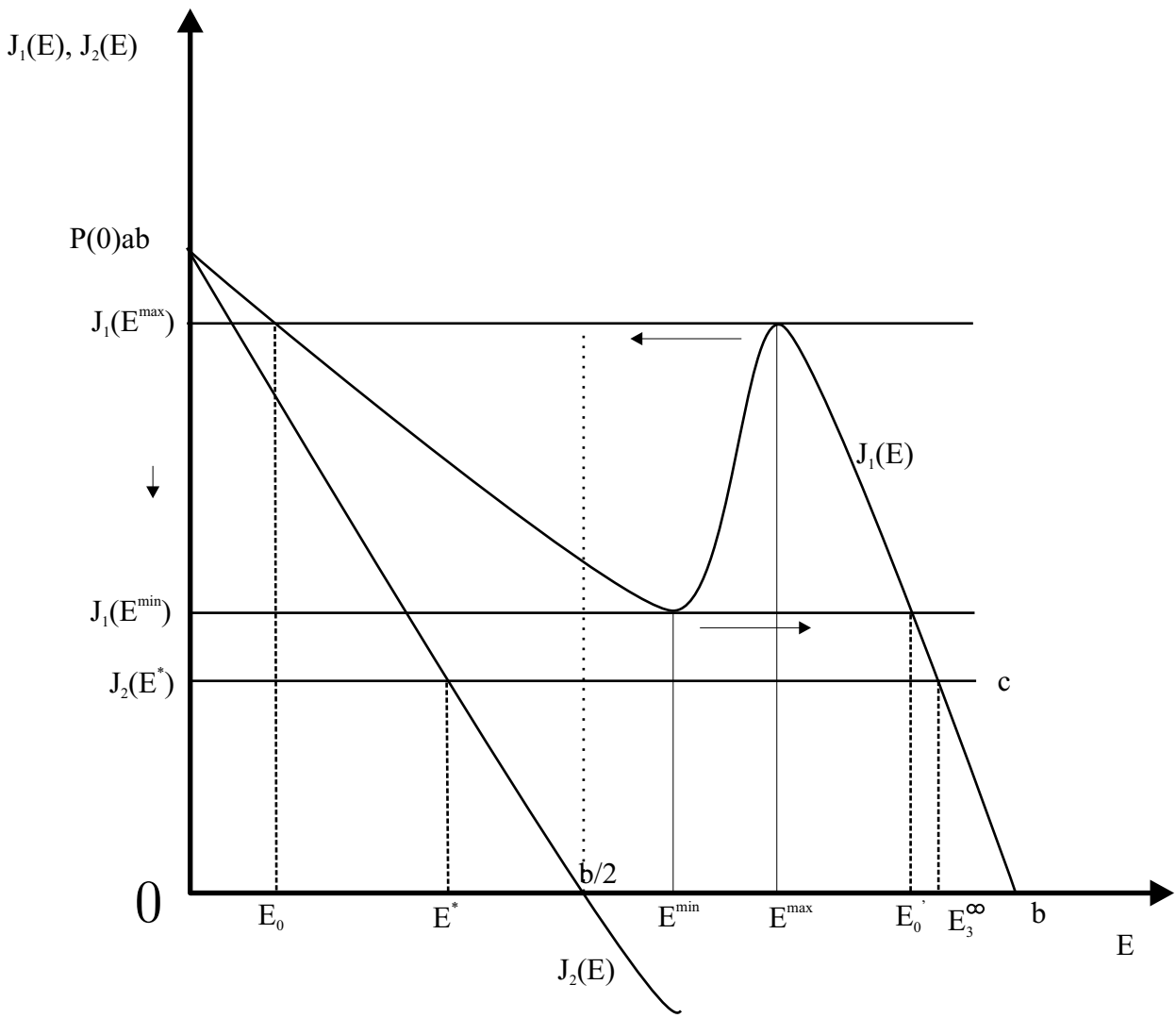


Figure 4: Social optimum and bionomic equilibrium for a single species, three equilibria with hysteresis trap

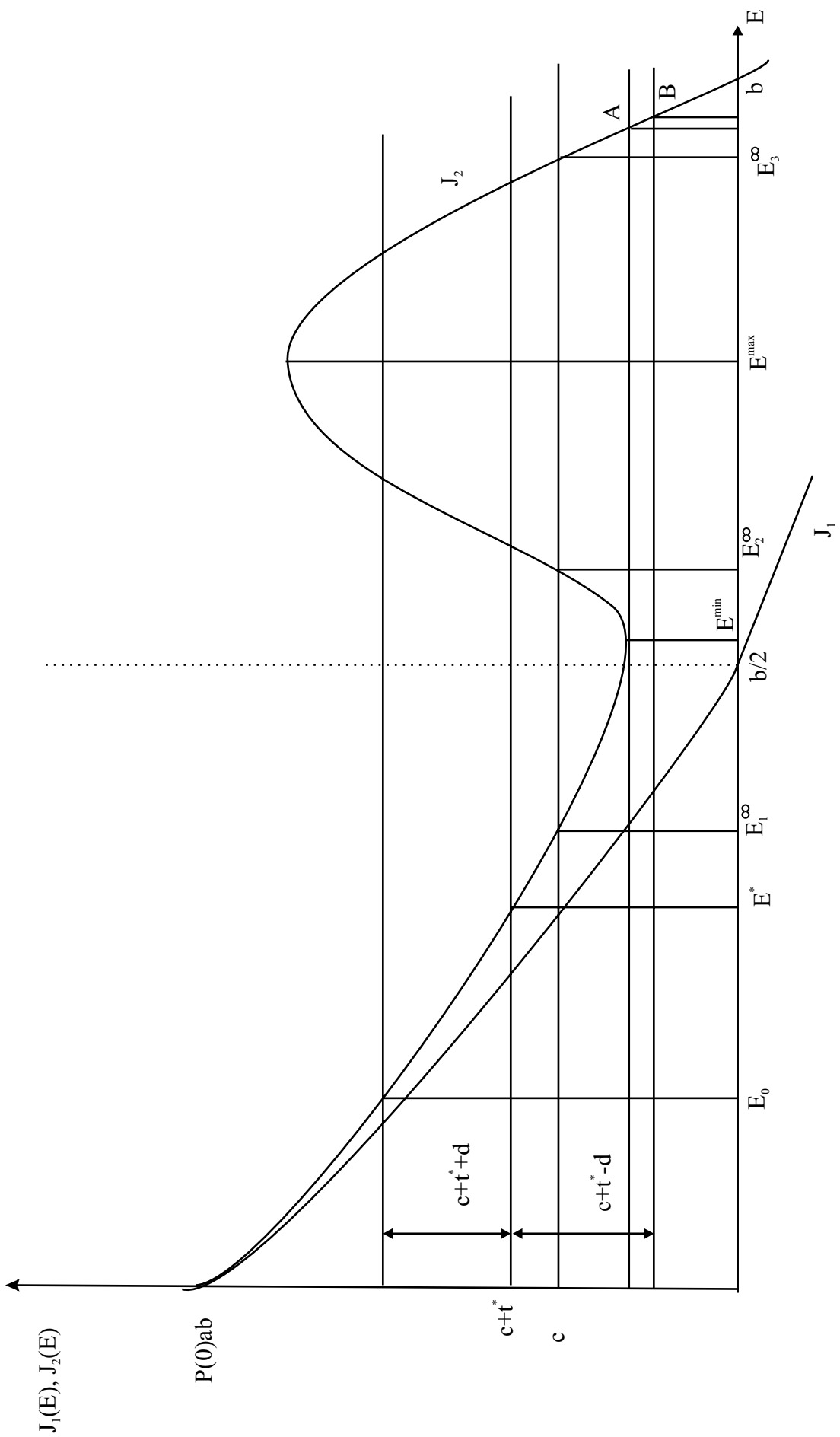


Figure 4a: Robust regulation

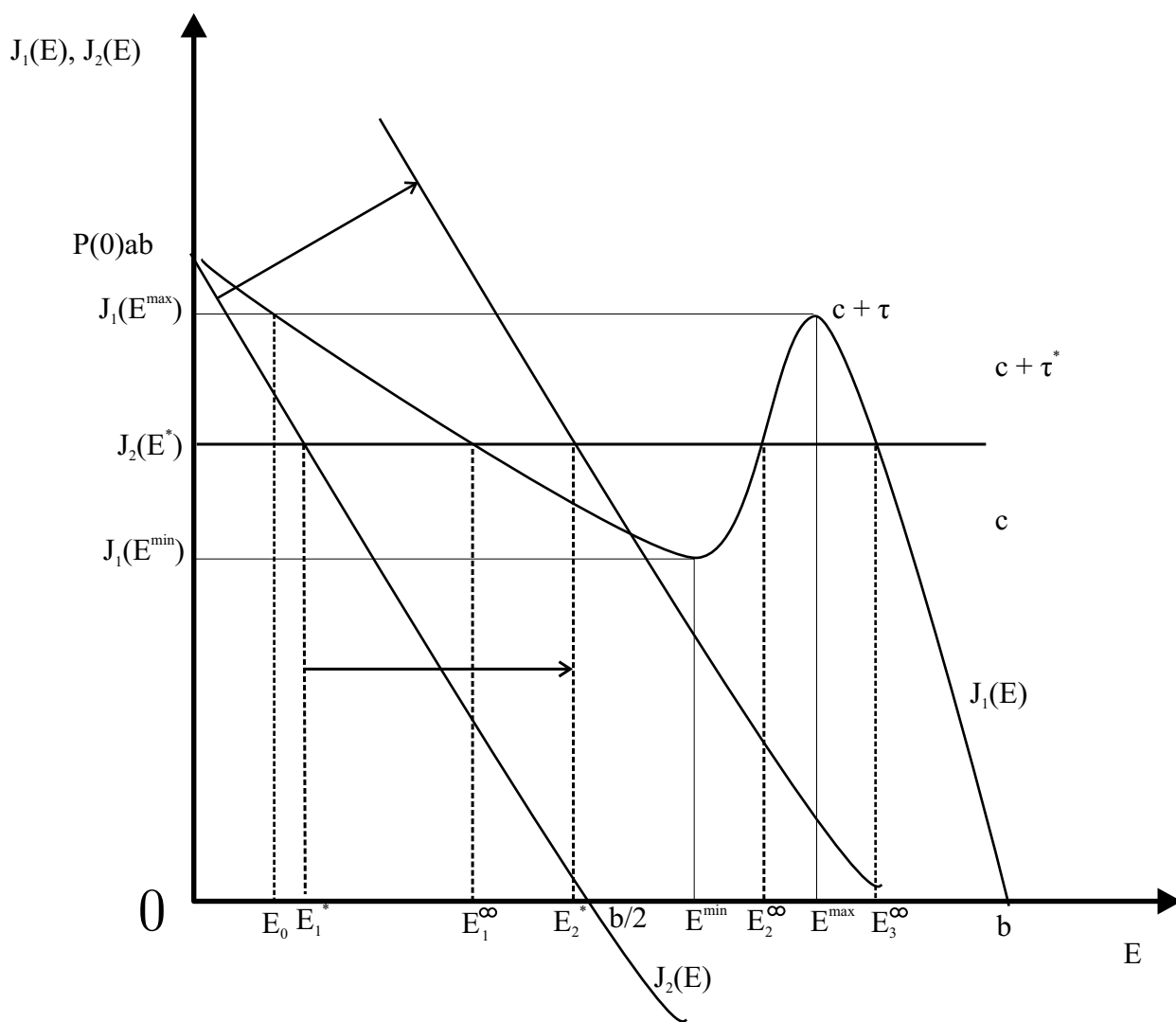


Figure 5: Social optimum and bionomic equilibrium for two species