

COMMON STOCHASTIC TRENDS IN INTERNATIONAL STOCK MARKETS: TESTING IN AN INTEGRATED FRAMEWORK

By

Dimitris A. Georgoutsos (corresponding author)
Department of International and European Economic Studies
Athens University of Economics and Business
GR-10434
Athens, Greece

Email: D.Georgoutsos@aueb.gr

and

Georgios P. Kouretas
Department of Economics
University of Crete
GR-74100
Rethymno, Greece

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Abstract

In this paper we analyze the implications for the identification of common stochastic trends among stock price indices of using data transformed on a "real dollar" basis. By applying a "general" VAR model where all the relevant variables (stock indices, consumer price indices and the exchange rate) are included, we show that the expected results from the cointegration analysis differ substantially. In particular it is shown that if four common stochastic trends drive the system then cointegration between the indices transformed in nominal dollars should be the relevant test while the use of their "real dollars equivalent" is superfluous. In cases where three common stochastic trends exist then a reasonable specification of the model would imply that the *Purchasing Power Parity* condition accounts for one of them while the second one relates to a cointegrating relation between the stock indices in nominal *domestic currency* terms. We apply the testing methodology developed by Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999) to examine the presence of $I(2)$ and $I(1)$ components in a multivariate context using monthly data for the US, UK, Germany and Japan for the period 1980 – 2000. Four possible economic scenarios were considered in a bivariate setting and two of them were found to be statistically supported. By imposing linear restrictions on each cointegrating vector as suggested by Johansen and Juselius (1994), the order and rank conditions for statistical identification are satisfied while the test for economic identification was not significant for each bilateral case, namely US-UK, US-Germany, US-Japan. The main findings suggest that the policy to transform the data into a "real" dollar basis, which is often encountered in the literature, lacks empirical support. Furthermore, the stability results indicate that cointegration was established in the early 1990s which implies that some form of policy coordination between the G-7 countries was implemented in the aftermath of the October 1987 crisis.

Key-words: International stock markets, $I(2)$ cointegration analysis, common trends, identification, purchasing power parity, temporal stability.

JEL classification: G15, G12, G32.

1. Introduction

As an outcome of the stock market crash of 1987 a series of papers was produced for the purpose of examining the possible existence of long-run co-movements among the major stock markets of the world. The results of this examination were of obvious interest to both academic researchers and investment managers. If the evidence were favorable to this co-movement then the diversification of portfolios could not be profitable for investors whose holding period was higher than the time needed for the markets to adjust to their equilibrium path. Moreover, it is well established in the literature that the asset prices from two different efficient markets can not be cointegrated (Granger, 1986, p.218). However, if policy coordination exists among different countries that reduces the number of stochastic trends in the system to some extent, then it is reasonable to find evidence of cointegration.

The main analytical framework within which the investigation of the above problem has been conducted is provided by cointegration theory.¹ The initial evidence supplied by Kasa (1992) was strongly in favor of the presence of a single common stochastic trend that drove the quarterly stock price indices of five countries, namely, the U.S.A., Canada, Germany, Japan and the United Kingdom, for the period 1974:1 to 1990:3. Those findings came under criticism because they had been derived by an arbitrary increase of the number of lags in the VAR model with no adjustment of the critical values of the relevant tables (Richards, 1995). If that had been done then there would have been no evidence of a common stochastic trend. Later studies produced conflicting evidence on the issue, where the number of cointegrating vectors among the stock market indices varied substantially so that no identification of the system was possible. For example, Francis and Leachman's (1998) study produced a single cointegrating vector among the indices of Germany, Japan, the U.K. and the U.S.A. for the period 1974:1 – 1990:8. This result is not easily interpretable since it implies three stochastic trends that are not readily identifiable, given that the data have been converted to their real dollar equivalent. This precludes the identification of one of the trends with the exchange rate or with aggregate demand factors which manifest themselves through the price level. Serletis and King (1997) have studied the problem for the case of ten European Union stock markets for the period 1971:1 – 1992:1. They supply evidence in favor of eight cointegrating vectors instead of nine, as the condition for multi-country convergence would imply. Finally, Richards (1995) fails to reject the null of no cointegration among sixteen

countries when the small sample correction of the critical values is implemented, although these results have been derived from data on stock indices returns instead of their levels.

A common feature of most of the above mentioned studies has been the transformation of the data into their “real dollar” equivalent value using the spot exchange rates and the U.S. consumer price index.² This has been rationalized on the grounds that returns must be “covered” against exchange rate risk or else a model for the pricing of the exchange rate risk would be needed to test the “integration” hypothesis³ (Kasa,1995). However, it is obvious that the use of data on a “real dollar” basis presupposes the satisfaction of certain conditions and in particular that the Purchasing Power Parity, (PPP), hypothesis holds. This in turn implies that failure to reject the null of no cointegration might be attributed to the weak support provided by the data to the PPP hypothesis. In the present paper we provide a systematic way of testing for common trends where we specify the vector autoregressive (VAR) model in its most general form and then we test it down to the specification employed by most researchers in the area. It is shown that the transformation of the indices into their dollar equivalent is superfluous and what is needed is merely transformation to a “*nominal US dollar*” basis. Moreover, it is shown that this is just one of the possible specifications of the model. Another interesting specification, within the $I(1)$ environment, would imply that if the *Purchasing Power Parity* is a valid model for the exchange rate determination then we should test for cointegration among the indices in *domestic currency* terms. The set of possible alternative specifications increases substantially if we allow for the presence of $I(2)$ components in the model.

The analysis is conducted within the context of cointegration and therefore we examine the existence of long-run relationships between the stock price indices, the bilateral exchange rates and the corresponding consumer price indices of the U.S., the U.K., Germany and Japan. Our testing approach is novel in a number of ways. First, we provide a new analysis of the determination of the order of integration of the variables. Although testing for unit roots has become a standard procedure it has been made clear that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely misspecified leading to arbitrary conclusions. Therefore, we employ the recently developed testing methodology suggested by

Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999) which allows us to reveal the existence of $I(2)$ and $I(1)$ components in a multivariate context. This analysis is done by testing successively less and less restricted hypotheses according to the Pantula (1989) principle. Additionally, we apply a recent approach suggested by Juselius (1995) that is based on the roots of the companion matrix and allows us to make firmer conclusions about the rank of the cointegration space. Second, since in a multivariate framework a vector error correction model may contain multiple cointegrating vectors, following Johansen and Juselius (1994) and Johansen (1995b), we impose independent linear restrictions on the coefficients of the accepted cointegrating vectors. Third, given that at least one statistically significant cointegrating vector has been found we examine the stability of the long-run relationships through time. The evidence that two or more stock indices are cointegrated, is exploitable by the investors only if this evidence is sample independent. In the literature this issue has not been treated formally up to now, with the exception of a study by Leachman and Francis (1995) which examines the number of cointegrating vectors before and after the Plaza and Louvre accords. Hansen and Johansen (1993, 1999) propose tests for parameter stability in cointegrated-VAR systems that allow us to provide evidence of the sample independence of the cointegration rank as well as of parameter stability.

There are several interesting findings that stem from our estimation approach. First, for each bilateral case we find evidence of two cointegrating vectors between the domestic and U.S. price indices, the exchange rate and the corresponding stock price indices. Second, three $I(1)$ stochastic trends are established while the hypothesis of an $I(2)$ stochastic trend is rejected. Third, the overidentifying restrictions which associate the first vector with the Purchasing Power Parity and the second one with the proportionality hypothesis among the two nominal stock market indices, are rejected for the US-UK and US-Germany cases while there is weak support for the US-Japan case. Finally, the application of the stability tests shows that cointegration is established in the early 1990s which indicates the existence of some coordination policy among the countries involved in the aftermath of the October 1987 stock market crisis.

The rest of the paper is organized as follows. In section 2 we present the model which distinguishes the implication of the PPP hypothesis from the one implied by the

existence of common stochastic trends among the stock market indices. Section 3 presents the econometric methodology. Section 4 discusses the data and presents the empirical results while our conclusions are given in section 5.

2. Testing for common trends in an integrated framework.

2.1. Treating the variables as $I(1)$

Consider a 5-dimensional vector autoregressive (VAR) model that in error correction form is given by

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + \gamma D_t + \mu + \varepsilon_t, \quad t=1, \dots, T \quad (1)$$

where, $z_t = \left[p_t, i_t, e_t, p_t^f, i_t^f \right]$ and p (p^f) stands for the logarithm of the consumer price index of the local economy (the country of the base currency, i.e. the US dollar in our case), e is the logarithm of the units of the local currency per one dollar and i (i^f) is the domestic (the base country's, i.e. the U.S.) stock market index expressed in nominal domestic currency terms. Also we consider that z_{k+1}, \dots, z_0 are fixed and $\varepsilon_t \sim Niid_p(0, \Sigma)$. The adjustment of the variables to the values implied by the steady-state relationship is not immediate for a number of reasons like imperfect information or costly arbitrage. Therefore, the correct specification of the dynamic structure of the model, as expressed by the parameters $(\Gamma_1, \dots, \Gamma_{k-1}, \gamma)$, is important in order that the equilibrium relationship be revealed. The matrix $\Pi = \alpha\beta'$ defines the cointegrating relationships, β , and the rate of adjustment, α , of the endogenous variables to their steady-state values. Both matrices, α and β , are of dimension $(5 \times r)$ where r stands for the number of cointegrating vectors. D_t is a vector of non-stochastic variables, such as centered seasonal dummies which sum to zero over a full year by construction and are necessary to account for short-run effects which could otherwise violate the Gaussian assumption, and/or intervention dummies; μ is a drift and T is the sample size.

The model expressed in “real dollar” terms would involve the variables $i_t^T = [i - e - p^f]_t$ and $[i^f - p^f]_t$. If PPP holds then the variable $[p - e - p^f]_t$ will be $I(0)$ and testing for common stochastic trends could be equivalently conducted with the values of the stock price indices expressed in real local currency terms.⁴ On the other hand, if PPP does not hold then failure to statistically identify common stochastic trends with the “real dollar” variables could be consistent with the existence of cointegration between the stock price indices when expressed in real local currency terms.⁵

The interesting problem is to determine the restrictions that model (1) should satisfy in order to derive, if possible, this long-run specification that is common in the literature. Furthermore, in anticipation of the empirical results, those restrictions should comply with interesting economic scenarios that are better understood through the moving average (MA) representation of z_t . This representation of model (1) for the case of $I(1)$ variables is

$$z_t = C \sum_{i=1}^t \varepsilon_i + C \mu t + C \Phi \sum_{i=1}^t D_i + C^*(L)(\varepsilon_t + \mu + \Phi D_t) + Z_0, \quad (2)$$

where $C = \beta_{\perp}(\alpha'_{\perp} \Gamma \beta_{\perp})^{-1} \alpha'_{\perp}$, $C^*(L)$ is a polynomial in the lag operator L , Z_0 is a function of the initial values and α_{\perp} , β_{\perp} are both $(5 \times (5 - r))$ matrices orthogonal to α and β respectively. In the MA representation $(\alpha'_{\perp} \sum \varepsilon)$ determine the $(5 - r)$ stochastic trends while β_{\perp} the variables that are being influenced by them. The realization at time t of the variables in z_t is determined by a stochastic trend component, described by the first term of eq. (2), a deterministic trend component, the cumulated value of the non-stationary variables D_t , a stationary component and the initial values.

Scenario I: One cointegrating vector, four common stochastic trends

This scenario could be associated with the model where there is a co-movement in the long run between the stock market indices in nominal dollar terms. This specification implies a single cointegrating vector $(0, 1, -1, 0, -1)$ among the variables $\left[p_t, i_t, e_t, p_t^f, i_t^f \right]$ which is exactly equivalent from a long-run view to having that $(i - e - p^f)$ and $(i^f - p^f)$

cointegrate, i.e. the indices in real US dollars, since the US price index, (p^f) appears in both variables. In this scenario, when the variables are $I(1)$, the transformation in the real dollar terms of the indices is considered to be superfluous. In this specification the model is driven by four common stochastic trends, $(\alpha'_{\perp} \sum \varepsilon)$, that can be associated with the domestic and the U.S. nominal and real “permanent” disturbances, $(u_{it} = \alpha'_{\perp} \varepsilon_t, i = 1, \dots, 4)$. The system in (2) then can be written as:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \\ c_{51} & c_{52} & c_{53} & c_{54} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^t u_{1k} \\ \sum_{k=1}^t u_{2k} \\ \sum_{k=1}^t u_{3k} \\ \sum_{k=1}^t u_{4k} \end{bmatrix} + \text{stationary components}, \quad (3)$$

The single cointegrating vector of this case is consistent with the satisfaction of the restrictions $c_{21} = c_{31} + c_{51}, c_{22} = c_{32} + c_{52}, c_{23} = c_{33} + c_{53}, c_{24} = c_{34} + c_{54}$.

In order to provide intuition about the consequences this model has on the effect of the various shocks on the stock market indices, let us consider the case of the cumulated aggregate demand shocks of the U.S. economy. The above cointegrating vector implies that those shocks will affect the level of the domestic stock market index by an equal amount after having been corrected for the amount which has been absorbed by changes in the level of the exchange rate (i.e. $c_{23} = c_{33} + c_{53}$). In a similar way an aggregate demand shock in the domestic economy will affect the level of the U.S. stock price index to the same extent after having allowed for the change in the exchange rate (i.e. $c_{21} = c_{31} + c_{51}$).⁶

Another interesting scenario that is often encountered in the literature requires that the stock market indices be cointegrated while the exchange rate is missing from the cointegrating vector, i.e. $(0, 1, 0, 0, -1)$, (Corhay et al. 1993; Arshanapalli and Doukas, 1993). In this case the effect of any shock on the two stock markets will be the same while the exchange rate is not a determinant, in the long run, of this relationship. In this situation the imports and exports of each country are not dependent, in a crucial way, on the other country (Bracker et al. 1999).

Scenario II: Two cointegrating vectors, three common stochastic trends

This model is consistent with a co-movement in the long run, on the one hand between the U.S. dollar equivalent of the domestic price index and the U.S price index, i.e. the PPP holds, and on the other hand between the nominal value of the two stock price indices. Under this specification the two cointegrating vectors are (1,0,-1,-1,0) and (0,1,0,-1,0) among the variables $\left[p_t, i_t, e_t, p_t^f, i_t^f \right]$ and we have three common stochastic trends that drive the system, e.g. two of them may be associated with productivity shocks in the local and the U.S. economy and the other is an aggregate demand shock. In this case a nominal or real shock in the U.S. economy, for example, will have the same effect on the nominal level of the two stock market indices. The law of one-price guarantees that the competitiveness of the companies in the two countries can not change since a positive, for example, productivity shock in the domestic economy which lowers the prices will be accompanied by an equi-proportionate appreciation of its currency vis-à-vis the U.S. dollar. Similarly, a demand shock in the U.S. will leave the international competitiveness of its companies unaltered since either the U.S. dollar will depreciate or the price level of the “domestic” economy will rise or both. The system in (2) can then be written in this case as:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^t u_{1k} \\ \sum_{k=1}^t u_{21k} \\ \sum_{k=1}^t u_{3k} \end{bmatrix} + \text{stationary components}, \quad (4)$$

The two cointegrating vectors in this case are consistent with the satisfaction of the restrictions

$$c_{11} = c_{31} + c_{41}, c_{12} = c_{32} + c_{42}, c_{13} = c_{33} + c_{53} \text{ and } c_{21} = c_{51}, c_{22} = c_{52}, c_{23} = c_{53}.$$

2.2. Treating prices as I(2)

The characterization of the stochastic properties of the data as being integrated of order two, one or zero is an issue that can be settled through empirical investigation rather than on “theoretical” grounds (Juselius, 1999). If the chosen data set spans a short period of time, then it is more likely for some series, like the price level, to be characterized as I(2) processes since there are not enough turning points in the sample. The same series when

studied for its stochastic properties over a longer sample period will be more likely characterized as an $I(1)$ process.

The problem of cointegration within this more general framework was initially studied by Johansen (1997) who has shown that the moving average representation is given by:

$$z_t = C_2 \sum_{s=1}^t \sum_{i=1}^s \varepsilon_i + C_2 \frac{1}{2} \mu t^2 + C_2 \Phi \sum_{s=1}^t \sum_{i=1}^s D_i + \quad (5)$$

$$C_1 \sum_{i=1}^t (\varepsilon_i + \Phi D_i) +$$

$$C^*(L)(\varepsilon_t + \mu + \Phi D_t) + B_0 + A_0 t$$

where $C_2 = \beta_{\perp}^2 \Psi^{-1} \alpha_{\perp}^2$ and Ψ, C_1 are functions of the parameters of the model. α_{\perp}^2 are the coefficients of the common $I(2)$ trends while β_{\perp}^2 define the sensitivity of the variables in z_t to the common trends. The space spanned by the (5×1) vector z_t can be decomposed into r stationary directions, β , and $(5-r)$ nonstationary directions, β_{\perp} , and the latter into the directions $(\beta_{\perp}^1, \beta_{\perp}^2)$, where $\beta_{\perp}^1 = \beta_{\perp} \eta$ is of dimension $(5 \times s_1)$ and $\beta_{\perp}^2 = \beta_{\perp} (\beta_{\perp}^1 \beta_{\perp}^1)^{-1} \eta_{\perp}$ is of dimension $(5 \times s_2)$ and $s_1 + s_2 = 5-r$. If $(r > s_2)$ then the r cointegrating vectors can be decomposed into $(r-s_2)$ directly cointegrating vectors, β_0 , which according to Engle and Granger's (1987) definition are $CI(2,2)$, and s_2 multicointegrating vectors β_1 . The properties of the process are summarized in the table below.

Direction	Dimension	Stationary process
$\beta_0' z_t \sim I(0)$	$r - s_2$, if $(r > s_2)$	
$\beta_1' z_t \sim I(1)$	s_2	$\beta_1' z_t + k' \Delta z_t \sim I(0)$
$\beta_{\perp 1}' z_t \sim I(1)$	s_1	$\beta_{\perp 1}' \Delta z_t \sim I(0)$
$\beta_{\perp 2}' z_{tt} \sim I(2)$	s_2	$\beta_{\perp 2}' \Delta^2 z_{tt} \sim I(0)$

Under this statistical specification two models appear likely to occur. In both of them the aggregate demand shock accumulates twice to form the $I(2)$ common stochastic trend of the prices. As far as the $I(1)$ level is concerned, one expects to find either three stochastic trends in which case there will be two cointegrating vectors one of which will be a multicointegrating one, or four stochastic trends which are consistent with a single multicointegrating relationship.

Scenario I: One cointegrating vector, four $I(1)$ and one $I(2)$ common stochastic trends.

Under the first economic scenario the model in (5) can be written more clearly as follows:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \sum_{s=1}^t \sum_{k=1}^s u_{1k} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \\ c_{51} & c_{52} & c_{53} & c_{54} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^t u_{1k} \\ \sum_{k=1}^t u_{21k} \\ \sum_{k=1}^t u_{3k} \\ \sum_{k=1}^t u_{4k} \end{bmatrix} + \text{stat. and detrm. components}$$

(6)

One plausible multicointegrating vector could be $\{i - i^f - (p - e - p^f) - \beta \Delta p\} \sim I(0)$, which implies that the stock price indices deviate, after the realization of a shock, by an amount equal to the deviation of the consumer prices transformed to a US dollar basis. The presence of the inflation rate can be attributed to the different macro-policies followed by the two countries, as a response to a common rate of inflation, which has a further implication for the stock market. This can be seen more formally through a different Phillips curve the two countries face which connects the inflation rate to output and from there to company profits and to their valuation in the stock market.

Scenario II: Two cointegrating vectors, three $I(1)$ and one $I(2)$ common stochastic trends.

The stochastic trend components of this case can be seen from:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \sum_{s=1}^t \sum_{k=1}^s u_{1k} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^t u_{1k} \\ \sum_{k=1}^t u_{21k} \\ \sum_{k=1}^t u_{3k} \end{bmatrix} + \text{stat. and detrm. components}$$

(7)

A strong candidate for the directly cointegrating vector of this case is the PPP hypothesis $(p - e - p^f) \sim I(0)$. The multicointegrating relationship relates the stock market indices and the rate of inflation, i.e. $(i - i^f - \beta \Delta p)$, and the explanation for the presence of the inflation rate is similar to the one given above. However the inflation rate in this case works through its influence directly on the stock market indices discrepancy and not on the residual that is not “explained” by the difference in the consumer price level.⁷

3. Econometric methodology

We will briefly discuss the estimation procedure starting from the most general case within the framework of Johansen’s (1988, 1991) multivariate cointegration methodology as it was extended in Johansen (1992a, 1995a, 1997) and Paruolo (1996) and Rahbek *et al.* (1999) to take into account the stochastic properties of $I(2)$ variables.

If we allow the parameters of model (1), $\theta = (\Gamma_1, \dots, \Gamma_{k-1}, \Pi, \gamma, \mu, \Sigma)$, to vary unrestrictedly this model corresponds to the $I(0)$ model. The $I(1)$ and $I(2)$ models are obtained if certain restrictions are satisfied. Thus, the higher-order models are nested within the more general $I(0)$ model.

It has been shown (Johansen, 1991) that if $z_t \sim I(1)$, then that matrix Π has reduced rank $r < p$, and there exist $p \times r$ matrices α and β such that $\Pi = \alpha \beta'$. Furthermore, $\Psi = \alpha'_{\perp} (\Gamma) \beta_{\perp}$ has full rank, where $\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$ and α_{\perp} and β_{\perp} are $p \times (p-r)$ matrices orthogonal to α and β , respectively.

Following this parameterization, there are r linearly-independent stationary relations given by the cointegrating vectors β and $p-r$ linearly-independent non-stationary relations. These last relations define the common stochastic trends of the system and the MA

representation shows how they contribute to the various variables. By contrast the AR representation of model (1) is useful for the analysis of the long-run relations of the data.

The $I(2)$ model is defined by the first reduced rank condition of the $I(1)$ model and $\Psi = \alpha_{\perp}' \Gamma \beta_{\perp} = \varphi \eta'$ is of reduced rank s_1 , where φ and η are $(p-r) \times s_1$ matrices and $s_1 < (p-r)$.

Under these conditions we may re-write (1) as

$$\Delta^2 z_t = \Pi z_{t-1} - \Gamma \Delta z_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 z_{t-1} + \gamma D_t + \mu + \varepsilon_t \quad (8)$$

where $\Psi_i = - \sum_{j=i+1}^{k-1} \Gamma_j$, $i=1, \dots, k-2$

Following Rahbek *et al.* (1999) we outline a representation of the restricted VAR (2) which allows the observed process z_t to have (at most) linear deterministic trends and some or all components $I(2)$. In general if $z_t \sim I(2)$ then the unrestricted linear regressor, $\mu_1 t$, allows for cubic trends while the constant regressor, μ_0 , allows for quadratic trends. Rahbek *et al.* (1999) show that to guarantee linear trends in all linear combinations of z_t we must impose restrictions on both μ_1 and μ_0 . Finally, Rahbek *et al.* (1999) provide a likelihood ratio (LR) test, which is asymptotically $\chi^2(r)$ distributed, to test whether the linear trend enters the cointegrating vector significantly.

Johansen (1991) shows how the model can be written in moving average form, while Johansen (1997) derives the FIML solution to the estimation problem for the $I(2)$ model. Furthermore, Johansen (1995a) provides an asymptotically equivalent two-step procedure which computationally is simpler. The two-step estimation procedure consists of first obtaining, by reduced rank regression, the maximum likelihood estimators of α and β and the rank of Π in eq. (8). We first generate the residuals R_{0t} , R_{1t} , R_{2t} by regressing $(\Delta^2 z_t)$, (Δz_{t-1}) , (Δz_{t-2}) on $(\Delta^2 z_{t-i}, i=1, \dots, k-2)$ and then eliminate the unrestricted

parameters Γ by regression. After having formed the residual product moment matrices we solve the usual eigenvalue problem which provides us the rank of Π through a likelihood ratio test and the estimators of α and β . In the second step the rank of the $(p-r) \times (p-r)$ matrix $\Psi = \alpha'_{\perp}(\Gamma)\beta_{\perp}$ is determined as in the case of the $I(1)$ process. By premultiplying eq. (8) with α'_{\perp} the levels disappear and the model is reduced to a $(p-r) \times (p-r)$ dimensional equation system in first and second differences. If we further premultiply Δz_{t-1} by $I = \beta(\beta'\beta)^{-1}\beta' + \beta_{\perp}(\beta'_{\perp}\beta_{\perp})^{-1}\beta'_{\perp}$, eq. (8) is re-parameterised to :

$$\alpha'_{\perp}\Delta^2 z_t = \alpha'_{\perp}\Gamma\beta(\beta'\beta)^{-1}\beta'\Delta z_{t-1} - \varphi\eta(\beta'_{\perp}\beta_{\perp})^{-1}\beta_{\perp}\Delta z_{t-1} + \text{lagged 2}^{nd} \text{ differences} + \text{dummies}$$

where $\alpha'_{\perp}(\Gamma)\beta_{\perp} = \varphi\eta'$ has been imposed. The likelihood analysis of this equation can be performed by reduced rank regression of $\alpha'_{\perp}\Delta^2 z_t$ on $(\beta'_{\perp}\beta_{\perp})^{-1}\beta_{\perp}\Delta z_{t-1}$ corrected for $\beta'\Delta z_{t-1}$ and lagged second differences and dummies. The hypothesis of reduced rank of $\alpha'_{\perp}(\Gamma)\beta_{\perp}$ is tested as in the $I(1)$ case and the test procedure determines the s_1 and $s_2 = p - r - s_1$ directions in which the process is $I(1)$ and $I(2)$ respectively (Johansen, 1995a).

In a multivariate context a vector error correction model may contain multiple cointegrating vectors, and in such a case the individual cointegrating vectors are *underidentified* in the absence of sufficient linear restrictions on *each* of the vectors. The issue of identification in cointegrated systems has recently been addressed by Johansen and Juselius (1994) and Johansen (1995b).

Consider again the long-run matrix $\Pi = \alpha\beta'$ and let Φ be any $r \times r$ matrix of full rank. Then $\Pi = \alpha\Phi^{-1}\Phi\beta' = \alpha^*\beta'^*$, where $\alpha^* = \alpha\Phi^{-1}$ and $\beta^* = \Phi\beta'$ and without imposing restrictions on α and β so that to limit the admissible matrices, Φ , the cointegrating vectors are not unique. In fact given the normalization under which both α and β are calculated, only the space spanned by the β vectors is uniquely determined. Thus,

we need to impose restrictions implied by economic theory, for example homogeneity and zero restrictions, so that we are able to discriminate between them.

The necessary and sufficient conditions for identification in a cointegrated system in terms of linear restrictions on the columns of β are analogous to the classical identification problem that we face in the simultaneous equations problem. Thus, the order condition for identification of each of the r cointegrating vectors is that we can impose at least $r - 1$, just identifying restrictions and one normalization on each vector without changing the likelihood function. This is a necessary condition. The necessary and sufficient condition for identification of the i th cointegration vector, *the Rank condition*, is that the rank $(R_i' H_1, \dots, R_i' H_k) \geq k$, where i and $k = 1, \dots, r - 1$ and $k \neq i$ (Johansen and Juselius, 1994). The linear restrictions of the model are of the form $R_i \beta_i = 0$, where R_i is a $(p \times k_i)$ matrix, or equivalently by $R_i' H_i = 0, i = 1, \dots, r$, where H_i is a known $(p \times s_i)$ design matrix which satisfies $\beta_i = H_i' \tau_i$ and τ_i is a $(s_i \times 1)$ vector of freely varying parameters $(k_i + s_i = p)$.

An equally important issue, along with the existence of at least one cointegration vector, is the issue of the stability of such a relationship through time as well as the stability of the estimated coefficients of such a relationship. Hansen and Johansen (1993, 1999) have suggested methods for the evaluation of parameter constancy in cointegrated VAR models, formally using estimates obtained from the Johansen FIML technique. Three tests have been constructed under the two VAR representations. In the “Z-representation” all the parameters of model (2) are re-estimated during the recursions while under the “R-representation” the short-run parameters $\Gamma_i = 1, \dots, k - 1$ are fixed to their full sample values and only the long-run parameters α and β are re-estimated.

The first test is called the *Rank test* and it examines the null hypothesis of sample independence of the cointegration rank of the system. The trace test statistics, scaled by the corresponding 95% critical values, are plotted against time and calculated for each subsample during the recursive analysis. An interesting result in the recursive analysis is that the rank test as a function of time will be upward sloping for chosen rank $r < r^*$ and

approximately constant for $r > r^*$ (Johansen, 1988, Hansen and Johansen, 1993). The second test for the constancy of the cointegration space considers the hypothesis $H : sp(b) = sp(\beta)$ where b is a known $p \times r$ matrix and is chosen so that $b = \hat{\beta}(T)$, i.e. the full - sample estimate of β . In the recursive analysis we perform a sequence of likelihood ratio tests

$$-2\ln(Q(H/\hat{\beta}(t))) = t \sum_{i=1}^r \ln \left[\frac{1 - \hat{\rho}_i(t)}{1 - \hat{\lambda}_i(t)} \right], t = T_0, \dots, T.$$

where λ are the roots of the unrestricted problem and ρ of the restricted one. For each estimation period the LR test has the same form and Johansen and Juselius (1992) have shown that it is asymptotically χ^2 distributed with $(p - r) \times r$ degrees of freedom. In the third test we examine the constancy of the eigenvalues, λ , which are related to the cointegration vectors, β , and the loadings, α , through the relationship

$\hat{\beta}' S_{10} S_{00}^{-1} S_{01} \hat{\beta} = \hat{\alpha}' S_{00}^{-1} \alpha = \text{diag}(\hat{\lambda}_1, \dots, \hat{\lambda}_r)$, under the normalization $\hat{\beta}' S_{00}^{-1} \hat{\beta} = I$ for the eigenvectors (S_{01} , S_{00} are product moment matrices, Johansen, 1988). Thus, an evaluation of the time path for λ_i , ($i = 1, \dots, r$) can be seen as an evaluation of the i^{th} column of β or α , and structural changes in α and β will be reflected in the estimated eigenvalues. The asymptotic distribution of the estimator for λ has been derived by Hansen and Johansen (1993, 1999).

4. Empirical evidence

In this paper we study four stock markets, those of the U.S., Germany, the U.K. and Japan. The time period of the analysis extends from January 1980 through May 2000. The price data are the *Capital International* indices constructed by Morgan Stanley. These are end-of-month value-weighted indices of a large sample of firms in each market and they are taken from *Datastream*. The *Capital International* indices correspond quite closely to the standard published indices, such as the NYSE index, the Topix, Commerzbank and FT-

industrial 'Ordinary', but they have the advantages of being constructed on a consistent basis across countries and of netting out cross-listed securities. The data for the consumer price indices as well as the exchange rate series of pound sterling/U.S. dollar, Deutschemark/U.S. dollar and Yen/U.S.dollar have been obtained from the *International Financial Statistics* of the International Monetary Fund. Figure 1, which provides plots of the price indices, shows that for the U.S., U.K. and Germany the biggest drop occurred, as expected, in October 1987, while for Japan it is related to the stock price deflation at the end of 1989.

Table 1 provides another interesting feature of the stock price data, which is the contemporaneous correlation between monthly changes of the various markets. It is clear that the U.S. and U.K. are much more correlated with each other than they are with Germany and Japan.

4.1 Determination of the cointegration rank and the order of integration

The first step in the analysis is the determination of the cointegration rank index, r , and the order of integration of the variables. We begin by considering the three bilateral cases, those of U.S.-U.K, U.S.-Germany and U.S.-Japan. As a first check for the statistical adequacy of model (1) we report some univariate misspecification tests in Table 2, in order to ascertain that the estimated residuals do not deviate from being Gaussian white noise errors.

A structure of four lags for each case was chosen based on these misspecification tests. We note that our conditional VAR model is well specified for any country, except for the presence of non-normality. Normality can be rejected as a result of skewness (third moment) or excess kurtosis (fourth moment). Since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis we report the univariate Jarque-Bera test statistics together with the third and fourth moment around the mean. It turns out that the rejection of normality is essentially due to excess kurtosis, and hence not so serious for the estimation results. The ARCH(4) tests for fourth order autoregressive heteroscedasticity and is rejected for all equations. Again cointegration estimates are not very sensitive to ARCH effects.⁸ The R^2 measures the improvement in explanatory power relative to the random walk (with drift) hypothesis, i.e. $\Delta x_t = \mu + \varepsilon_t$. They

show that with this information set we can explain quite a large proportion of the variation in the inflation rates, but to a much lesser extent the variation in the bilateral exchange rates and the stock price indices.

The Johansen - Juselius multivariate cointegration technique, as explained in section 3, is applicable only in the presence of variables that are realizations of $I(1)$ processes and/or a mixture of $I(1)$ and $I(0)$ processes, in systems used for testing for the order of cointegration rank. Until recently the order of integration of each series was determined via the standard unit root tests. However, it has been made clear by now that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely misspecified leading to arbitrary conclusions. Thus, in the presence of $I(1)$ series, Johansen and Juselius (1990) developed a multivariate stationarity test which has become the standard tool for determining the order of integration of the series within the multivariate context.

Additionally, when the data are $I(2)$, one also has to determine the number of $I(2)$ trends, s_2 , among the $p-r$ common trends. The two-step procedure discussed in section 3 is used to determine the order of integration and the rank of the two matrices. The hypothesis that the number of $I(1)$ trends = s_1 and the rank = r is tested against the unrestricted H_0 model based on a likelihood ratio test procedure discussed in Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999).

Table 3 reports the trace test statistics for all possible values of r and $s_1 = p - r - s_2$, under the assumption that the data contain linear but no quadratic trends. The 95% critical test values reported in italics below the calculated test values are taken from the asymptotic distributions reported in Rahbek *et al.* (1999, Table 1). Starting from the most restricted hypothesis $\{r=0, s_1 = 0, s_2 = 5\}$ we test successively less and less restricted hypotheses according to the Pantula (1989) principle. The last column of Table 3(a) reports the standard Johansen trace test, Q_r . Therefore, the first hypothesis that we were unable to reject was $\{r = 1, s_1 = 4, s_2 = 0\}$ for the US / UK and US / Japan cases, which implies that there are no $I(2)$ components, there is one linear cointegrating relation and four $I(1)$ common trends in the multivariate framework.^{9,10} For the US / Germany case the first hypothesis we were unable to

reject was $\{r = 2, s_1 = 3, s_2 = 0\}$ which indicates the presence of no $I(2)$ roots in the system, two cointegrating vectors and three $I(1)$ roots.

In addition to the formal test, Juselius (1995) offers further insight into the $I(2)$ and $I(1)$ analysis as well as the correct cointegration rank. She argues that the results of the trace and maximum eigenvalue test statistics of the $I(1)$ analysis, i.e. from the estimation of the model without allowing for $I(2)$ trends, should be interpreted with some caution for two reasons. First, the conditioning on intervention dummies and weakly exogenous variables is likely to change the asymptotic distributions to some (unknown) extent. Second, the asymptotic critical values may not be very close approximations in small samples. Juselius (1995) suggests the use of the additional information contained in the roots of the characteristic polynomial. Table 3 also provides the five first, in size, pxk roots of the companion matrix. If there are $I(2)$ components in the vector process, then the number of unit roots in the characteristic polynomial is $s_1 + 2s_2$. Hence, if $r = 1$, implying one cointegrating vector, there should be four unit roots in the process, all of which are $I(1)$ components, and if $r = 2$ there should also be four unit roots if one of them was an $I(2)$ process. The evidence from the estimated roots of the companion matrix is consistent with that provided by the formal tests for the cointegration rank for the US/UK and US/Japan cases under which $\{r = 1, s_1 = 4, s_2 = 0\}$ while for the US/Germany case there is a disagreement. The formal test indicates two cointegrating vectors and three unit roots while the companion matrix is also consistent with one cointegrating vector and four unit roots.

A property of the models which allow for an $I(2)$ common stochastic trend is that the two price indices share this trend and therefore the variable $(p - p^*)$ should be integrated of order one. The hypothesis of long-run proportionality between the domestic and the US price indices is a testable implication of both scenarios in the $I(2)$ case where the test statistic is asymptotically distributed as $\chi^2(4)$ or $\chi^2(8)$ depending on whether we allow for one or two cointegrating vectors. In all the cases we examined the estimated marginal significance level was never different from zero, indicating rejection of the hypothesis. This led us to concentrate on the $I(1)$ case under which the preferred model is $\{r = 1, s_1 = 4, s_2 = 0\}$. However, considering the sensitivity of the critical values to alternative maintained hypotheses

we also chose to examine scenario II of the $I(1)$ case under which the characteristics of the model are given by $\{r = 2, s_1 = 3, s_2 = 0\}$ and that is also statistically supported according to the evidence in Table 3.

Finally, we allow for the presence of a linear trend. Dornbusch (1989) has suggested that due both to differing productivity trends in the tradeable and non-tradeable goods sectors and to inter-country differences in consumption patterns, a decline in domestic prices relative to foreign prices could appear as a linear trend in the purchasing power parity relationship underlying the model. We tested for the significance of the deterministic trend in the multicointegrating relation by applying the likelihood ratio statistic discussed in (7). The test statistic in the U.S. – U.K. case is 24.53 with a p-value (0.00), in the U.S. - Germany case is 11.29 with a p-value (0.00) and in the U.S. – Japan case is 9.30 with a p-value (0.00). Therefore we reject the null hypothesis that the linear trend does not enter significantly in any one of the estimated cointegrating spaces.

The next stage of the cointegration analysis involves the stability analysis of our cointegration results. The overall conclusion drawn from the three tests is mixed. Specifically, for all three cases it is shown that the rank of the cointegration space is dependent on the sample size from which it has been estimated, since the null hypothesis of a constant rank, one or two in our case, is rejected. It is worth noticing however that this evidence is consistent with previous findings in the literature according to which cointegration is established on samples extending after 1990. This has been attributed to the coordination policies pursued by the G-7 countries in the aftermath of the 1987 crisis (Arshanapalli and Doukas, 1993; Leachman and Francis, 1995). From the second test we obtain overwhelming evidence in favor of constancy of the estimated coefficients, since we are unable to reject the null hypothesis for the sample independence of the cointegration space for a given cointegration rank. Finally, the last test provides substantial evidence against the constancy of the cointegrating vectors since a substantial drift was detected on the time paths of the two largest in size eigenvalues. The exception appears to be the U.S.-Germany case where both eigenvalues are relatively constant especially after 1987 and this again manifests the presence of some form of intervention in the market. It is also interesting to remark that the

eigenvalues appear to have sustained a structural break around 1987 which is further evidence of the effect that coordination among the G-7 countries had on the stock markets.

To assess the statistical properties of the chosen variables the test statistics reported in Table 4 are useful. The test of long-run exclusion is a check of the adequacy of the chosen measurements and shows that none of the variables can be excluded from the cointegration space. The test for stationarity indicates that none of the variables can be considered stationary under any reasonable choice of r . Finally, the test of weak exogeneity shows that the U.K. stock price index and the U.K. consumer price for the U.K./U.S. case, the U.S. stock price index for the Germany/U.S. case and the Yen/dollar exchange rate and the Japanese consumer price index can be considered weakly exogenous for the long-run parameters β . All three tests are χ^2 distributed and are constructed following Johansen and Juselius (1990, 1992). Furthermore, Table 4 presents diagnostics on the residuals from the cointegrated VAR model which indicate that they are *i.i.d.* processes since no evidence of serial correlation was detected in each bilateral case. This provides further support for the hypothesis of a correctly specified model.

The final stage of our analysis deals with the issue of the economic identification of our system. Under the first scenario of the $I(1)$ case, in section 2.1., the system has one cointegrating vector which can be identified with the long-run co-movement of the stock price indices if their levels in either domestic currency or nominal US dollars terms cointegrate.

Imposing the corresponding restrictions on the vector $z_t = [p_t, i_t, e_t, p_t^f, i_t^f]'$, the matrix of the linear and homogeneous restrictions is given by:

$$\beta' = [0, 1, 0, 0, -1], \text{ or } \beta' = [0, 1, -1, 0, -1], \quad (9)$$

Under the second scenario of the $I(1)$ case we allow for two cointegrating vectors among the five variables. On the first cointegrating vector we impose four restrictions, namely proportionality between the exchange rate and the consumer price indices and exclusion of the two stock price indices (i.e. the PPP hypothesis). In fact the imposition of these four restrictions overidentifies this relationship. Identification of the second cointegrating vector

requires a set of restrictions that is independent of the one on the first vector. This implies that from the accepted cointegrated vectors only one can possibly describe the long-run purchasing power parity. Thus, on the second vector we impose three exclusion restrictions, on the prices and the exchange rate (i.e. the real stock price indices have a common stochastic trend). Imposing the above restrictions on the vector $z_t = \begin{bmatrix} p_t, i_t, e_t, p_t^f, i_t^f \end{bmatrix}$, the matrix of the linear and homogeneous restrictions in its most general form is the following :

$$\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \quad (10)$$

The results of the estimated restricted vectors along with the likelihood ratio test for the acceptance of the overidentifying restrictions, for the U.S./U.K., the U.S./Germany and the U.S./Japan case, are given in Table 5. According to the evidence we reject the joint restrictions on the single cointegrating vector which is contradictory to the specification and results derived in studies like the ones by Arshanapalli and Doukas (1993) and Richards (1995). Negative results are also derived under the two cointegrating vectors specification where the first one describes the long-run purchasing power parity and the second one a long-run relationship between the respective stock price indices for all cases. The only exception is the U.S.-Japan case where at the 1% significance level we fail to reject the overidentifying restrictions. These results explain the failure of a number of previous studies to establish a long-run relationship among international stock markets when they use data in “real U.S. dollar” terms. Moreover, it brings on the forefront the issue of identification in the cointegration analysis; having found a number of cointegrating vectors has little implication for the statistical determination of co-movements between two or more stock market indices if it is not identified with the theoretical structure.

5. Conclusions

In this paper we analyze the implications for the identification of common stochastic trends among stock price indices by using a model where the transformation of the data to a domestic or US dollar basis is decided statistically and not imposed a priori. By applying a

general VAR model where all the relevant variables (stock indices, consumer price indices and the exchange rate) are included, we show that the expected results from the cointegration analysis differ substantially. In particular, the use of the “transformed” data presupposes that the Purchasing Power Parity condition has been imposed. If this is not the case then the adoption of the “transformed” data leads to an entirely different economic identification of the model. Also, by allowing for the presence of $I(2)$ variables we enrich the set of possible specifications of the long-run co-movements among the stock price indices.

The analysis was conducted using monthly data for the U.S., the U.K., Germany and Japan for the period January 1980 to May 2000. Several recent developments in the econometrics of non-stationarity and cointegration were applied and a number of novel results stem from our analysis. First, this paper makes use of the recently developed testing methodology suggested by Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999) to test for the existence of $I(2)$ and $I(1)$ components in a multivariate context. The joint hypothesis of either one or two cointegrating vectors and the presence of a significant deterministic trend in the cointegrating vector could not be rejected although the hypothesis of at least one $I(2)$ component was rejected in all three cases. Second, we tested for parameter stability and it was shown that both the dimension of the cointegration rank and the estimated cointegrating vectors with their associated loadings were sample dependent. Furthermore, the stability analysis revealed that in all three cases cointegration was established during the early 1990s. This finding provides support for the argument that some degree of policy coordination between the G-7 countries was implemented to avoid widespread financial crises like the one in October 1987. Finally, for a given cointegration rank we formally imposed independent and linear restrictions on each vector in order to identify our system. Based on a likelihood ratio test for overidentifying restrictions (Johansen and Juselius, 1984) we rejected the joint restriction that the system represents the long-run purchasing power parity and a long-run relationship between the relevant stock price indices.

Footnotes

* An earlier version of this paper was presented at the 4th International Conference on Macroeconomic Policy and International Finance, Rethymno, 25-28 May 2000, the 9th European Financial Management Association Meetings, Athens, 28 June – 1 July 2000, the 51st International Atlantic Economic Conference, Athens, 13-19 March 2001, the *IEFS* Conference on Finance, Trade and Factor Mobility Issues in the Global Economy, Thessaloniki, 16-19 May 2001, the 8th Multinational Finance Society Conference, lake Garda, Italy, 23-27 June 2001, and at the 56th European Meeting of the Econometric Society, Luzanne, 25-29 August 2001 and thanks are due to conference participants for many helpful comments and discussions. This paper has also benefited from comments and discussions by seminar participants at Athens University of Economics and Business and the University of Crete. We would also like to thank without implicating Angelos Antzoulatos, Keith Cuthbertson, Soren Johansen, Katarina Juselius, Angelos Kanas, Costas Karfakis, Anthony Richards, Mike Wickens and Mark Taylor for many helpful comments and discussions.

1. Another branch of the literature focuses on the short-run inter-dependence of prices and / or price volatility across national equity markets (Longin and Solnik; 1995, King and Wadhvani; 1990, Eun and Shin, 1989).

2. Serletis and King (1997) have transformed their data into "real deutschemark" units, Richards (1995) has employed the excess return of the indices in nominal US dollars while Arshanapalli and Doukas (1993) work with nominal stock market indices.

3. The general tendency is to draw a distinction between tests for capital integration and those for the existence of common trends. The important implication of integrated capital markets is the equalization among countries of marginal rates of substitution in consumption both inter-temporally and across states of nature (Lucas, 1982; Kasa, 1995).

4. Recall that the variable i_t can be written as $[i - p + p]_t$.

5. We present the model for the case of two countries but the results can easily be generalized for any number of countries.

6. In this case it might be more realistic to restrict the impact on the U.S. price index and have the exchange rate absorb all the shock (i.e. $c_{51} = 0, c_{21} = c_{31}$). As a matter of fact the common trends in (2) are overidentified and up to three restrictions can be imposed without changing the likelihood function.

7. The restrictions that should be satisfied among the coefficients in the MA representation of the processes for the long run relationships to exist can be derived in a similar way to those in the $I(1)$ case.

8. Gonzalo (1994) shows that the performance of the maximum likelihood estimator of the cointegrating vectors is little affected by non-normal errors. Lee and Tse (1996) have shown similar results when conditional heteroskedasticity is present.

9. The calculations of all tests as well as the estimation of the eigenvectors have been performed using the program CATS 1.1 in RATS 4.20 developed by Katarina Juselius and Henrik Hansen, Estima Inc. Illinois, 1995.

10. A small sample adjustment has been made to the Trace test statistics, Q_r , for the $I(1)$ analysis

$$-2\ln Q = -(T - kp) \sum_{i=r_0+1}^k \ln(1 - \hat{\lambda}_i) \text{ as suggested by Reimers (1992).}$$

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Table 1. Contemporaneous correlations between monthly changes

	USA	UK	GER	JAP
USA	1.0			
UK	0.673	1.0		
GER	0.498	0.520	1.0	
JAP	0.351	0.374	0.335	1.0

Notes: The stock price indices are in nominal terms in domestic currency.

Table 2. Residual misspecification tests of the model with $k = 4$

U.S.-U.K.

Eq.	σ_ε	LB(36)	ARCH(4)	η_3	η_4	NORM(4)	R^2
Δp	0.002	22.11	50.58*	0.36	1.16	12.40*	0.696
Δi	0.045	25.72	11.06	-0.83	3.81	45.07*	0.161
Δe	0.028	33.65	12.84*	-0.01	0.62	5.18	0.189
Δp^f	0.001	29.00	10.04	-0.21	1.70	23.43*	0.574
Δi^f	0.040	27.67	3.14	-0.65	2.94	37.16*	0.131

U.S.-Germany

Eq.	σ_ε	LB(36)	ARCH(4)	η_3	η_4	NORM(4)	R^2
Δp	0.002	33.62	0.56	-1.08	18.88	451.81*	0.332
Δi	0.052	39.41	5.28	-0.40	1.45	16.40*	0.179
Δe	0.027	38.55	4.26	0.05	0.09	0.56	0.272
Δp^f	0.001	29.53	9.72	-0.09	1.29	16.02*	0.569
Δi^f	0.039	26.83	4.69	-0.71	3.07	37.02*	0.165

U.S.-Japan

Eq.	σ_ε	LB(36)	ARCH(4)	η_3	η_4	NORM(4)	R^2
Δp	0.002	19.00	2.57	0.53	1.13	12.72*	0.642
Δi	0.052	34.40	11.58*	-0.04	0.73	6.66	0.176
Δe	0.032	33.89	3.99	-0.35	0.29	5.22	0.143
Δp^f	0.001	21.05	10.86	-0.02	1.49	20.16*	0.538
Δi^f	0.039	26.02	3.96	-0.75	3.62	45.56*	0.170

Notes: σ_ε is the standard error of the residuals, η_3 and η_4 are the skewness and kurtosis statistics. LB is the Ljung-Box test statistic for residual autocorrelation, ARCH is the test for heteroscedastic residuals, and NORM the Jarque-Bera test for normality. The ARCH and NORM statistics are distributed as χ^2 with 4 and 2 degrees of freedom, respectively and the LB statistic is distributed as χ^2 with 36 degrees of freedom. *(**) denotes significance at the 5% (1%) level.

Table 3. Testing the Rank of the I(2) and the I(1) Model

Testing the joint hypothesis $H(s_1 \cap r)$

U.S. – U.K.

p-r	r	$Q(s_1 \cap r / H_0)$					Q_r
5	0	471.98 <i>198.2</i>	349.74 <i>167.9</i>	264.52 <i>142.2</i>	181.75 <i>119.8</i>	131.24 <i>101.5</i>	102.68 <i>87.2</i>
4	1		357.49 <i>137.0</i>	241.89 <i>113.0</i>	157.19 <i>92.2</i>	91.28 <i>75.3</i>	60.74 <i>62.8</i>
3	2			233.15 <i>86.7</i>	121.10 <i>68.2</i>	48.59 <i>53.2</i>	29.13 <i>42.7</i>
2	3				100.28 <i>47.6</i>	44.57 <i>34.4</i>	16.78 <i>25.4</i>
1	4					36.07 <i>19.9</i>	7.45 <i>12.5</i>
s_2		5	4	3	2	1	0

U.S. - Germany

p-r	r	$Q(s_1 \cap r / H_0)$					Q_r
5	0	436.23 <i>198.2</i>	350.24 <i>167.9</i>	281.54 <i>142.2</i>	216.49 <i>119.8</i>	162.43 <i>101.5</i>	127.61 <i>87.2</i>
4	1		313.05 <i>137.0</i>	246.10 <i>113.0</i>	181.06 <i>92.2</i>	124.86 <i>75.3</i>	77.41 <i>62.8</i>
3	2			177.56 <i>86.7</i>	116.81 <i>68.2</i>	64.97 <i>53.2</i>	32.41 <i>42.7</i>
2	3				105.38 <i>47.6</i>	50.39 <i>34.4</i>	13.78 <i>25.4</i>
1	4					15.92 <i>19.9</i>	5.19 <i>12.5</i>
s_2		5	4	3	2	1	0

U.S. - Japan

p-r	r	$Q(s_1 \cap r / H_0)$					Q_r
5	0	452.10 <i>198.2</i>	338.79 <i>167.9</i>	261.15 <i>142.2</i>	200.58 <i>119.8</i>	143.38 <i>101.5</i>	107.51 <i>87.2</i>
4	1		327.01 <i>137.0</i>	216.85 <i>113.0</i>	142.79 <i>92.2</i>	85.67 <i>75.3</i>	60.84 <i>62.8</i>
3	2			231.08 <i>86.7</i>	130.95 <i>68.2</i>	66.00 <i>53.2</i>	33.28 <i>42.7</i>
2	3				115.34 <i>47.6</i>	51.71 <i>34.4</i>	15.15 <i>25.4</i>
1	4					30.44 <i>19.9</i>	2.45 <i>12.5</i>
s_2		5	4	3	2	1	0

Notes: p is the number of variables, r is the rank of the cointegration space, s_1 is the number of $I(1)$ components and s_2 is the number of $I(2)$ components. For each case a structure of four lags was chosen according to a likelihood ratio test, corrected for the degrees of freedom (Sims, 1980) and the Ljung-Box Q statistic for detecting serial correlation in the residuals of the equations of the VAR. A model with an unrestricted constant in the VAR equation and a linear trend restricted in the cointegration space is estimated for all three cases according to the Johansen (1992a,b) testing methodology. The numbers in italics are the 95% critical values (Rahbek *et al.*, 1999, Table 1).

Table 3. continues

The roots of the companion matrix

Modulus of 5 largest roots						
	U.S. – U.K.					
Unrestricted model	0.99	0.95	0.95	0.93	0.76	
r = 2	1.00	1.00	1.00	0.95	0.78	
	U.S. - Germany					
Unrestricted model	0.99	0.99	0.95	0.95	0.82	
r = 2	1.00	1.00	1.00	0.92	0.73	
	U.S. - Japan					
Unrestricted model	0.99	0.99	0.95	0.95	0.78	
r = 2	1.00	1.00	1.00	0.94	0.79	

Notes: The table shows the modulus of the estimated $p \times k$ roots of the companion matrix from the VAR system, p is the number of variables and k is the number of lags of the VAR. We report the first five roots which are of interest to us.

Table 4. Statistical Properties and Misspecification Tests of the Model

(a) Tests for long-run exclusion, stationarity, and weak exogeneity

	Long-run exclusion			Stationarity			Weak exogeneity		
	US/UK	US/GE	US/JP	US/UK	US/GE	US/JP	US/UK	US/GE	US/JP
p	15.64*	12.12*	10.71*	13.42*	32.89*	11.71*	5.04	8.15*	0.62
i	7.34*	26.37*	6.63*	13.94*	36.16*	24.11*	5.10	6.54*	14.78*
e	10.45*	14.12*	14.02*	11.43*	38.71*	15.52*	14.31*	22.89*	1.52
p^f	22.24*	16.71*	7.22*	13.50*	35.32*	12.72*	20.13*	28.17*	24.27*
i^f	9.00*	11.75*	9.29*	18.12*	40.47*	14.73*	9.84*	2.54	8.72*

Notes: The long-run exclusion restriction and the weak exogeneity tests are χ^2 distributed with two degrees of freedom and the 5% critical level is 5.99, and the stationarity test is a χ^2 distributed with four degrees of freedom and the 5% critical level is 9.49. An (*) denotes statistical significance at the 5 percent critical level.

(b) Multivariate Residuals Diagnostics

Case	L-B(60)	LM(1)	LM(4)	χ^2 (10)
U.S - U.K	1517.67(0.01)	17.46(0.86)	30.09(0.22)	81.03(0.00)
U.S.- Germany	1520.41(0.01)	25.55(0.43)	34.24(0.10)	528.89(0.00)
U.S. - Japan	1432.45(0.27)	24.53(0.49)	28.93(0.27)	84.96(0.00)

Notes: L-B is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto- and cross- correlations of the first [T/4=60] lags distributed as a χ^2 with 1400 degrees of freedom. LM(1) and LM(4) are the tests for first- and fourth-order autocorrelation distributed as χ^2 with 25 degrees of freedom and χ^2 is a normality test which is a multivariate version of the Shenton-Bowman test. Numbers in parentheses refer to marginal significance levels.

Table 5. Tests of overidentified restrictions

Scenario I : one cointegrating vector, four $I(1)$ common stochastic trends

U.S.-U.K.

(a) $\beta' = [0 \ 1 \ 0 \ 0 \ -1 \ 0.05]$ $Q(4)=20.61(0.00)$

(b) $\beta' = [0 \ 1 \ -1 \ 0 \ -1 \ -0.05]$ $Q(4)=24.18(0.00)$

U.S. – Germany

(a) $\beta' = [0 \ 1 \ 0 \ 0 \ -1 \ 0.002]$ $Q(4)=12.15 (0.02)$

(b) $\beta' = [0 \ 1 \ -1 \ 0 \ -1 \ -0.002]$ $Q(4)=23.01 (0.00)$

U.S. – Japan

(a) $\beta' = [0 \ 1 \ 0 \ 0 \ -1 \ 0.05]$ $Q(4)=23.03(0.00)$

(b) $\beta' = [0 \ 1 \ -1 \ 0 \ -1 \ -0.17]$ $Q(4)=23.43(0.00)$

Scenario II : two cointegrating vectors, three $I(1)$ common stochastic trends

U.S – U.K.

(a) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0.036 \\ 0 & 1 & 0 & 0 & -1 & -0.095 \end{bmatrix}$ $Q(8) = 26.45(0.00)$

(b) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0.018 \\ 0 & 1 & 0 & 0 & -0.63 & -0.033 \end{bmatrix}$ $Q(7) = 24.98(0.00)$

(c) $\beta' = \begin{bmatrix} 0.57 & 0 & -1 & -0.57 & 0 & -0.002 \\ 0 & 1 & 0 & 0 & -1 & 0.015 \end{bmatrix}$ $Q(7) = 25.28(0.00)$

(d) $\beta' = \begin{bmatrix} 1.39 & 0 & -1 & -1.39 & 0 & 0.001 \\ 0 & 1 & 0 & 0 & -1 & -0.021 \end{bmatrix}$ $Q(6) = 23.60(0.00)$

U.S. – Germany

(a) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0.003 \\ 0 & 1 & 0 & 0 & -1 & 0.002 \end{bmatrix}$ Q (8) = 31.63(0.00)

(b) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0.003 \\ 0 & 1 & 0 & 0 & -0.85 & 0.002 \end{bmatrix}$ Q(7) = 31.50(0.00)

(c) $\beta' = \begin{bmatrix} 4.95 & 0 & -1 & -4.95 & 0 & 0.000 \\ 0 & 1 & 0 & 0 & -1 & 0.002 \end{bmatrix}$ Q(7) = 27.66(0.00)

(d) $\beta' = \begin{bmatrix} 4.37 & 0 & -1 & -4.37 & 0 & 0.000 \\ 0 & 1 & 0 & 0 & -0.84 & 0.001 \end{bmatrix}$ Q(6) = 27.11(0.00)

U.S. - Japan

(a) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 2.295 \\ 0 & 1 & 0 & 0 & -1 & 7.130 \end{bmatrix}$ Q (8) = 27.28(0.00)

(b) $\beta' = \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 3.347 \\ 0 & 1 & 0 & 0 & -4.8 & -16.69 \end{bmatrix}$ Q(7) = 15.07(0.04)

(c) $\beta' = \begin{bmatrix} 15.15 & 0 & -1 & -15.15 & 0 & 0.015 \\ 0 & 1 & 0 & 0 & -1 & -0.065 \end{bmatrix}$ Q(7) = 17.16(0.02)

(d) $\beta' = \begin{bmatrix} 2.30 & 0 & 1 & -2.30 & 0 & 0.026 \\ 0 & 1 & 0 & 0 & -5.36 & -0.076 \end{bmatrix}$ Q(6) = 14.98(0.02)

Notes : Q denotes a likelihood ratio test for overidentifying restrictions as suggested by Johansen and Juselius (1994) and is distributed as a χ^2 with the corresponding degrees of freedom given in parentheses. The last column refers to the estimate of the trend. Numbers in brackets denote marginal significance levels.

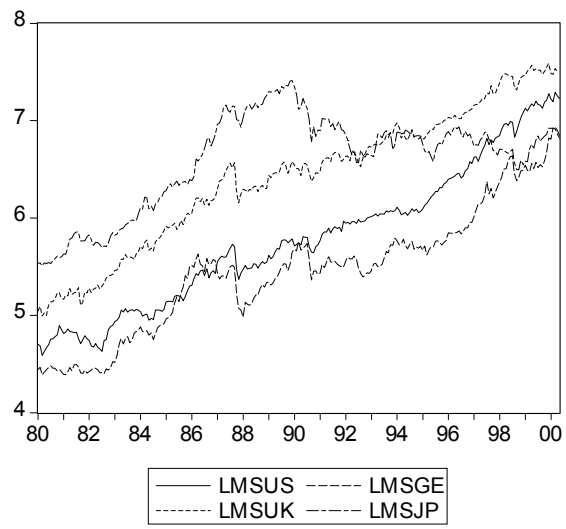


Figure 1. Stock market indices (nominal domestic terms/logs)

The Trace Test

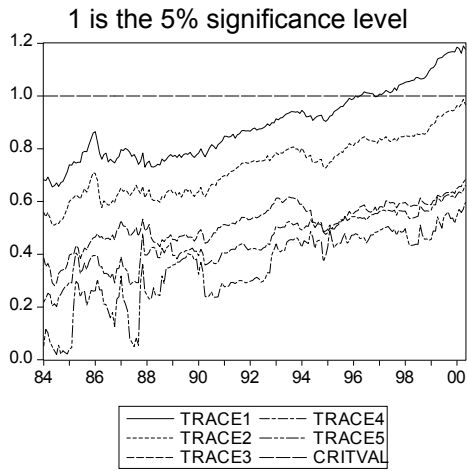


Fig. 2(a). U.K. – U.S. case

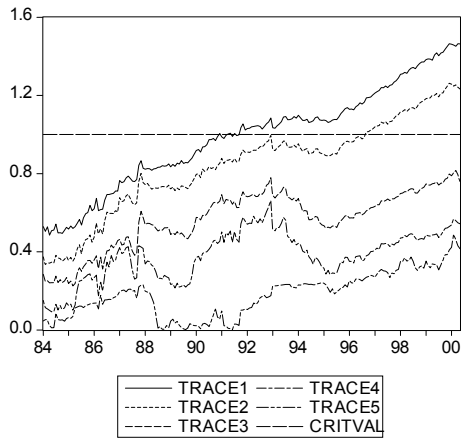


Fig. 2(b). Germany – U.S. case

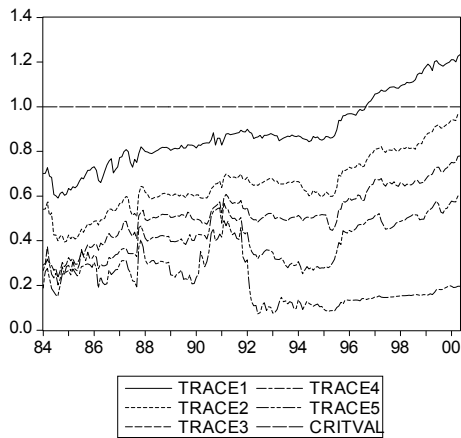


Fig. 2 (c). Japan – U.S. case

The Test of Constancy of Beta

1 is the 5% significance level

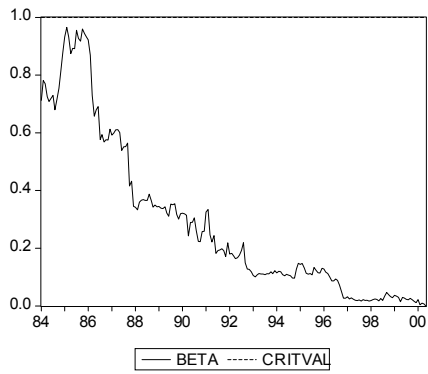


Fig. 3(a). The U.K. – U.S. case

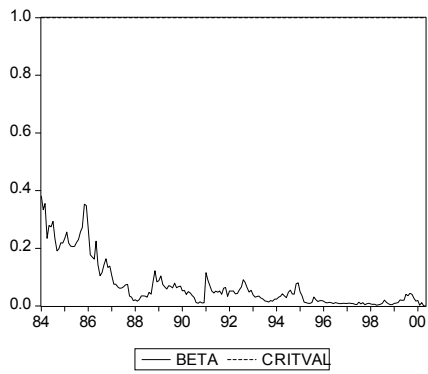


Fig. 3(b). Germany – U.S. case

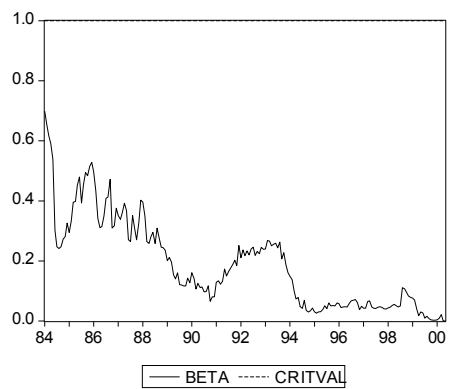


Fig. 3(c). Japan – U.S. case

The Eigenvalue Test

U.K. – U.S. case

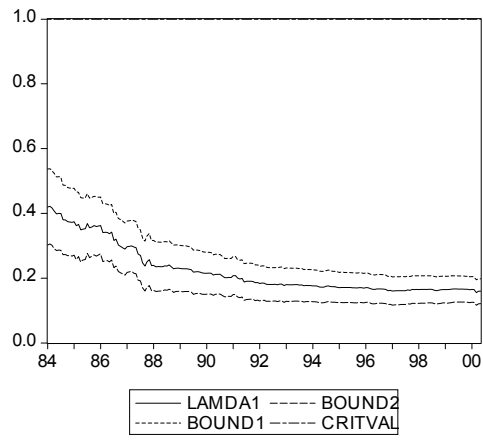


Fig. 4(a). Test for lambda 1

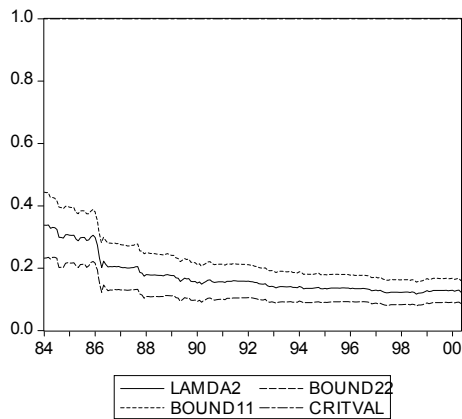


Fig. 4(b). Test for lambda 2

The Eigenvalue Test Germany – U.S. case

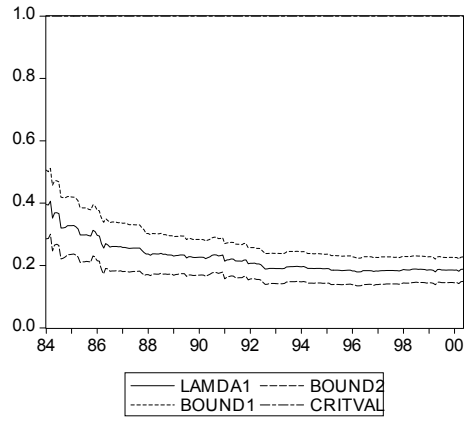


Fig. 5(a). Test for lambda 1

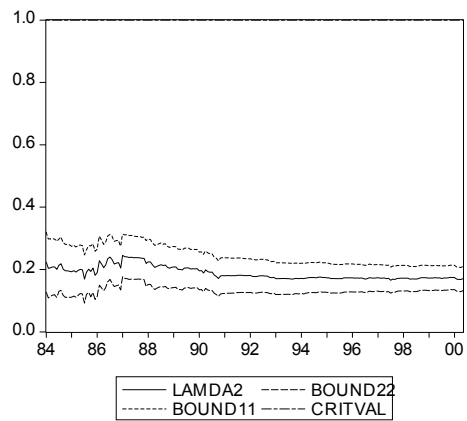


Fig. 5(b). Test for lamda 2

The Eigenvalue Test

Japan – U.S. case

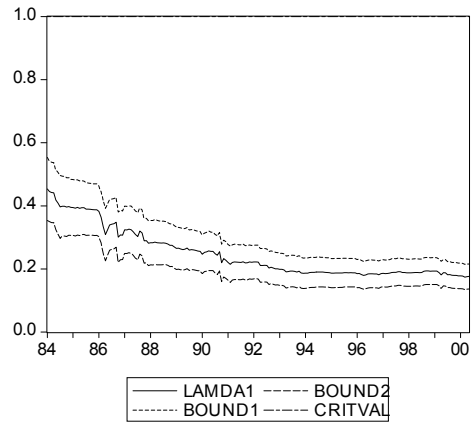


Fig. 6(a). Test for lambda 1

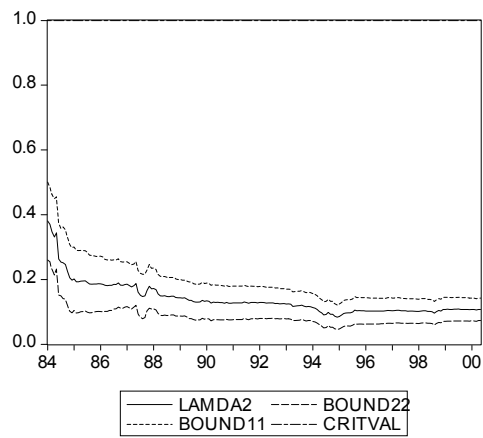


Fig. 6(b). Test for lambda 2

