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**Equilibrium Mode of Competition in Unionized Oligopolies:
Do Unions Act as Commitment Devices to Cournot Outcomes?**

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Equilibrium Mode of Competition in Unionized Oligopolies: Do unions act as commitment devices to Cournot outcomes?

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Abstract

In contrast with previous studies, we postulate that there is no ex-ante commitment over the type of contract (i.e., price or quantity) which a firm offers consumers. In the context of a unionized symmetric duopoly we instead argue that the mode of competition which in equilibrium emerges is the one that entails the most beneficial outcome for both the firm and its labour union, in each firm/union pair, given the choice of the rival pair. Our findings suggest that monopoly unions with risk-averse/neutral members may effectively act as commitment devices driving firms to the symmetric Cournot mode of competition.

JEL Classification: D43; J51; L13

Keywords: Oligopoly; Monopoly unions; Equilibrium mode of competition

1 Introduction

The cornerstones of modern oligopoly theory are the models of Cournot-Nash, where rival firms independently adjust their quantities, and Bertrand-Nash, where the rival firms' strategic variables are their prices. However, and though these alternative hypotheses deliver highly significant implications to the theory and practice of industrial economics (see, among else, Okuguchi, 1987; Qiu, 1997; Amir and Jin, 2001), a full understanding of what induces the mode of competition is still to come. In their seminal paper Singh and Vives (1984) explored this question in the context of a symmetric industry where firms, facing exogenous marginal costs in the upstream input market, compete in a downstream market with differentiated goods. Each firm is assumed to make two types of binding contracts with consumers: the price contract and the quantity

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contract. That is, if a firm chooses the price contract it is committed to supply the amount the consumers demand at a predetermined price, independently of the action of the competitor. Similarly, if a firm chooses the quantity contract, it is committed to supply a predetermined quantity, at the market bearing price. In a two-stage game, where firms simultaneously choose the type of contract to offer consumers, first, and subsequently compete contingent on the chosen types of contracts, the dominant strategy for each firm is found to be the quantity (price) contract, if goods are substitutes (complements). This work has inspired a number of variant studies on the determinants of the different types of imperfect competition (see e.g. Cheng, 1985 ; Klemperer and Meyer, 1986 ; Dastidar, 1997). However, in all those contributions the reason why firms can credibly commit to a particular mode of competition is not explicitly unravelled.¹

More recently, Correa-López and Naylor (2004) extend the analysis of Singh and Vives, by postulating that the input price which each downstream firm faces is the outcome of a first stage-bargain with its upstream input supplier (labour union), rather than being exogenously determined. Given the wages, in the second stage either both firms adjust their quantities, or both firms adjust their prices. The main finding is that, if unions are relatively powerful in the wage bargain and attach high relative importance to wages in their objective functions, equilibrium profits under the Bertrand regime may exceed those under the Cournot regime, in the case of imperfect substitutes. Yet, in line with the reviewed literature, in this study it is also assumed that when firm/union pairs independently bargain over the wage the (ad-hoc) type of contract which firms will symmetrically offer consumers, in the continuation of the game, is credible.

In this paper we refrain from these rather strong assumptions. Our postulate is that there is no ex-ante commitment over the type of contract which each firm will offer consumers, and that each can independently shift from the quantity (price) mode, to the price (quantity) mode, unless a commitment device exists which in equilibrium deters the firm to do so. In the context of a two-tier differentiated duopoly, where firms produce substitute goods and the upstream input (labour) market comprises of two firm-specific unions, we argue that the mode of competition which effectively emerges is the one that entails the most beneficial outcome for both the firm and the union in each firm/union pair, given the choice of the rival pair. This argument reflects the idea that deviating from price (quantity) towards quantity (price) requires an implicit agreement between both parties involved in each pair, since a wage-veto on the part of the union is sufficient for the price (quantity) to be sustained as the firm's mode of competition in the equilibrium. Interestingly, our findings suggest that the quantity (Cournot) mode of competition is always sustained by both firms in equilibrium, so long as unions possess all the power over the wage bargain (monopoly unions), and the average union member is risk averse/neutral. Hence, under rather standard assumptions in the trade unions literature (see e.g., Booth, 1995) we argue that the existence of firm-specific unions can be a decisive factor driving firms to a

¹In a different context, Lambertini (1997) investigates cartel stability in a repeated duopoly game where firms choose non-cooperatively the strategic variable to collude on and finds that they prefer price-setting collusion.

high-profit mode of competition.

2 The model

The structure of our envisaged industrial sector is similar to that of Singh and Vives (1984) and Correa-López and Naylor (2004), thus making our results strictly comparable to those studies. We consider an industry that consists of two firms, and each firm produces a brand $i = 1, 2$ of a good q_i , requiring only labour input, with constant returns to scale. Hence, the production function of firm i is $q_i = L_i$, $i = 1, 2$, where L_i is a firm-specific labour input, whilst the (inverse) demand function for brand i is given by:

$$P_i = a - q_i - \gamma q_j, \quad i, j = 1, 2, i \neq j \quad (1)$$

where, $a > 0$ and $\gamma \in (0, 1)$ is a measure of substitutability among brands; if $\gamma \rightarrow 0$ brands are regarded as (almost) unrelated, whereas $\gamma \rightarrow 1$ corresponds to the case of (almost) homogeneous goods.

The labour market is unionized. Workers are assumed to be organized into two separate, firm-specific, unions and union membership is fixed. Each union is of the utilitarian type, i.e., the union's objective is to maximize the sum of individual workers' utilities:

$$U_i(w_i, L_i) = (w_i)^\varphi L_i \quad (2)$$

Where, w_i is the wage rate of firm i and $\varphi \in (0, 1]$ denotes the representative union member's risk aversion, as measured by the elasticity of substitution between wages and employment. We further consider that each union possesses all the power to set its firm-specific wage rate. In our context this means that, in any firm-union negotiation, the union proposes the firm-specific wage rate, whilst the firm proposes the mode of competition, with employment decisions being left to the firms' discretion.²

The timing of our postulated game is as follows. In the first stage, each union proposes a wage rate to its own firm which is contingent upon the firm's proposal regarding its mode of competition (price or quantity) to be materialized in the second stage. So long as the parties agree on a particular wage/mode of competition scheme, the game proceeds to the second stage.³ In the second

²Although in real life the wage rate and (possibly) the employment level is determined via firm-union negotiations, it is a regular assumption in the union-oligopoly literature that the union has all the power in wage negotiations, while the firm has all the power to set the employment level (see Petrakis and Vlassis, 2004 and the references therein).

³*Agreement* means that a proposed wage/mode of competition scheme is sustained because, neither the union, nor the firm, find a deviation from it on their best interest. If they do, a unanimous consent is as well needed for a new scheme to be sustained. Yet, if no such consent can be found, and since the union possesses all the power to set the firm-specific wage, the chosen mode of competition inevitably is that which is consistent with the union's most preferred one, with the union vetoing any alternative proposal. Effectively, this is done as the union sticks to a wage proposal which is contingent upon its most preferred mode of competition.

stage, given the mode of competition to which each firm has thus been credibly committed, and the wages which have been consistently set, firms compete in the product market by independently adjusting, either their own price, or their own quantity.⁴

3 Equilibrium mode of competition

Solving by backwards induction, we investigate the conditions under which quantity or price emerges as the chosen mode of competition, for either firm, in the *Nash* equilibrium. That is, we propose a *candidate* configuration in the modes of the competition (one for each firm) at the second stage, and subsequently check whether or not it survives *all possible deviations*, at the first stage. If yes, the candidate configuration proposed is the sub-game perfect equilibrium one.

3.1 Symmetric Cournot competition

We begin by proposing as a candidate configuration the one where both firm/union pairs agree over the quantity as the firm's mode of competition. In this case the firm/union pair i may deviate by instead agreeing firm i to set its price as its mode of competition, given that the j firm's mode is its own quantity. Our findings are summarized in Proposition 1.

Proposition 1 *The symmetric Cournot mode of competition is always an equilibrium mode of competition. The reason is that, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, in each firm/union pair i , neither the firm nor its union have an incentive for firm i to deviate from quantity setting to price setting.*

Proof. See Appendix 1 ■

3.2 Symmetric Bertrand competition

We next propose as a candidate configuration the one where both firm/union pairs agree over the price as the firm's mode of competition. In this case the firm/union pair i may deviate by instead agreeing firm i to set its quantity as its mode of competition, given that the j firm's mode is its price. Our findings are summarized in Proposition 2.

Proposition 2 *The symmetric Bertrand mode of competition is never an equilibrium mode of competition. The reason is that, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, in each firm/union pair i , both the firm and its union have an incentive for firm i to deviate from price setting to quantity setting.*

Proof. See Appendix 2 ■

⁴The crucial, yet (due to the symmetric industry) reasonable assumption here is that the firm's and union's agreement upon the mode of competition in each firm/union pair, is not observable by the rival pair, before wage-setting is everywhere completed.

3.3 Asymmetric Cournot (Bertrand) – Bertrand (Cournot) competition

Finally, we propose as a candidate configuration the one where firm/union pair i (j) agrees over the quantity (price) as the mode of competition of firm i (j), given that the j (i) firm's mode is its own price (quantity).⁵ In this case, of course, there are two possible deviations: firm/union pair i may deviate by instead agreeing firm i (j) to set its price (quantity) as its mode of competition, given that the j (i) firm's mode is its own price (quantity). Our findings are summarized in Proposition 3.

Proposition 3 *The asymmetric Cournot (Bertrand) – Bertrand (Cournot) mode of competition, is never an equilibrium mode of competition. The reason is that,*

(a) *In firm/union pair i , if $\varphi = 1$ and $0.99 > \gamma > 0.90$, firm i does have an incentive to deviate from quantity setting to price setting. Yet, its union, having the same incentive only if $\varphi = 1$ and $\gamma > 0.99$, will effectively veto such a deviation, by always proposing a wage rate contingent upon the firm's quantity as its mode of competition.*

(b) *In firm/union pair j , for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, both the firm and its union have a unanimous incentive for firm j to deviate from price setting to quantity setting.*

(c) *If $1 > \gamma > 0.99$ and $\varphi = 1$, (a) and (b) imply that a unique stable equilibrium in the asymmetric mode of competition can not be found.*⁶

Proof. See Appendix 3 ■

4 Conclusions

Our main finding is that in unionized oligopolies, with decentralized bargaining, monopoly unions with risk-averse/neutral members may effectively act as commitment devices driving firms to a high-profit mode of competition in quantities. In contrast to Correa-López and Naylor (2004), proposing that, if $\phi > 1$, the ad-hoc Cournot-Bertrand profit differential will take negative values, we propose that monopoly unions, always driven by the risk aversion of their average member (e.g., $\phi \leq 1$), will always set wages sufficiently high so that their firms will not be marginalized in terms of output/employment.⁷ Hence, in line with Singh and Vives (1984), we suggest that the union's rents, along with the firm's profit, will be maximised only when the firm may achieve sufficient mark-up to pay back a high labour bill. Our argument is made more clear by recalling that, only if $\phi = 1$ and $\gamma > 0.99$ (see e.g., Proposition 3 (a)), the union of firm i

⁵Of course, due to the symmetric industry structure, the reverse configuration is as well (implicitly) proposed as the candidate configuration.

⁶Note that a unique stable equilibrium of this type may however exist in mixed strategies. Yet, such a consideration is beyond the scope of our present analysis.

⁷Note that $\phi > 1$ effectively implies that the average union's member's marginal utility of income is increasing. It further entails that the union's objective's indifference curves may be horizontal in the (w, L) space. This, in turn, implies that the "efficient bargains" and the "right-to-manage" hypotheses are empirically indistinguishable.

will (also) have an incentive for firm i to deviate from quantity setting to price setting, given that firm j also adjusts its price in the downstream market. The reason is that, since products will in this case be almost perfect substitutes, firms will be driven to set prices equal to marginal costs, under the emerging Bertrand competition in homogenous products. Therefore, it is only when union members place an equal weight to wage and employment (i.e., $\phi = 1$) that the adverse effect on union rents, brought about by the lower wage charged, can be overcompensated by the ensuing higher output/employment.

Appendix

Candidate Equilibria

Symmetric Cournot competition: Given the wages, in the second stage, firms simultaneously set quantities so as to maximize profits

$$\Pi_i^C = (a - q_i - \gamma q_j - w_i) L_i \quad (\text{A1})$$

The first order condition (foc) of eq. (A1) provides firm i 's reaction function

$$q_i^C(q_j) = \frac{1}{2}(a - \gamma q_j - w_i) \quad (\text{A2})$$

Solving the system of foc, we obtain the Cournot-Nash equilibrium quantities

$$q_i^C(w_i, w_j) = \frac{a(\gamma - 2) + 2w_i - \gamma w_j}{\gamma^2 - 4} \quad (\text{A3})$$

Given (w_i, w_j) , firm i 's Cournot-Nash equilibrium profits are

$$\Pi_i^C(w_i, w_j) = \left[\frac{a(\gamma - 2) + 2w_i - \gamma w_j}{\gamma^2 - 4} \right]^2 \quad (\text{A4})$$

In the first stage of the game, firm-level unions simultaneously set their firm-specific wage rates, so as to maximize

$$U_i^C(w_i, w_j) = \frac{(w_i)^\varphi [a(\gamma - 2) + 2w_i - \gamma w_j]}{\gamma^2 - 4} \quad (\text{A5})$$

The union i 's wage response function in wages under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market is given by

$$w_i^C(w_j) = \frac{2a\varphi - a\gamma\varphi + \gamma\varphi w_j}{2(1 + \varphi)} \quad (\text{A6})$$

From the system of equations (A6), we get a unique stable solution for the wages

$$w_i^C = \frac{a\varphi(\gamma - 2)}{\varphi(\gamma - 2) - 2} \quad (\text{A7})$$

Substituting w_i^C into (A3), (1), (A1) and (A5) gives the corresponding equilibrium market outcomes

$$q_i^C = \frac{2a}{4(1+\varphi) + \gamma(2-\varphi\gamma)} \quad (\text{A8})$$

$$p_i^C = \frac{a [\varphi(\gamma^2 - 4) - 2]}{[\varphi(\gamma - 2) - 2](\gamma + 2)} \quad (\text{A9})$$

$$\Pi_i^C = \left[\frac{2a}{4(1+\varphi) + \gamma(2-\varphi\gamma)} \right]^2 \quad (\text{A10})$$

$$U_i^C = \frac{2a \left[\frac{a\varphi(\gamma-2)}{\varphi(\gamma-2)-2} \right]^\varphi}{4(1+\varphi) + \gamma(2-\varphi\gamma)} \quad (\text{A11})$$

Symmetric Bertrand competition: Solving the system of inverse demand functions given by eq. (1), gives the direct demand function for brand i

$$q_i^B = \frac{a(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2} \quad (\text{A12})$$

Given the wages, in the second stage, firms simultaneously set prices so as to maximize profits

$$\Pi_i^B = (p_i - w_i) \frac{a(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2} \quad (\text{A13})$$

The first order condition (foc) of eq. (A13) provides firm i 's reaction function

$$p_i^B(p_j) = \frac{1}{2} (a - a\gamma + \gamma p_j + w_i) \quad (\text{A14})$$

Solving the system of foc, we obtain the Bertrand-Nash equilibrium prices

$$p_i^B(w_i, w_j) = \frac{a(\gamma^2 + \gamma - 2) - 2w_i - \gamma w_j}{\gamma^2 - 4} \quad (\text{A15})$$

Given (w_i, w_j) , firm i 's Bertrand-Nash equilibrium profits are

$$\Pi_i^B(w_i, w_j) = \left[\frac{(\gamma^2 - 2)w_i + \gamma w_j - a(\gamma^2 + \gamma - 2)}{(\gamma^2 - 4)^2(\gamma^2 - 1)} \right]^2 \quad (\text{A16})$$

In the first stage of the game, firm-level unions simultaneously set their firm-specific wage rates, so as to maximize

$$U_i^B(w_i, w_j) = \frac{(w_i)^\varphi [(\gamma^2 - 2)w_i + \gamma w_j - a(\gamma^2 + \gamma - 2)]}{\gamma^4 - 5\gamma^2 + 4} \quad (\text{A17})$$

The union i 's wage response function in wages under the assumption of a non-cooperative Bertrand-Nash equilibrium in the product market is given by

$$w_i^B(w_j) = \frac{\varphi [a(\gamma^2 + \gamma - 2) - \gamma w_j]}{(1 + \varphi)(\gamma^2 - 2)} \quad (\text{A18})$$

From the system of equations (A18), equilibrium wages are given by

$$w_i^B = \frac{a\varphi(\gamma^2 + \gamma - 2)}{\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2} \quad (\text{A19})$$

Substituting w_i^B into (A12), (1), (A16) and (A17) gives the corresponding equilibrium market outcomes

$$q_i^B = \frac{a(2 - \gamma^2)}{(\gamma^2 - \gamma - 2)[\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2]} \quad (\text{A20})$$

$$p_i^B = \frac{a(\gamma - 1)[\varphi(\gamma^2 - 4) + \gamma^2 - 2]}{(\gamma - 2)[\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2]} \quad (\text{A21})$$

$$\Pi_i^B = \frac{a^2(1 - \gamma)(\gamma^2 - 2)^2}{(\gamma - 2)^2(\gamma + 1)[\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2]^2} \quad (\text{A22})$$

$$U_i^B = \frac{(2 - \gamma^2) \left[\frac{a\varphi(\gamma^2 + \gamma - 2)}{\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2} \right]^{\varphi+1}}{\varphi(\gamma^4 - 5\gamma^2 + 4)} \quad (\text{A23})$$

Asymmetric quantity (price) - price (quantity) competition: Assume that firm 1 sets the quantity while firm 2 the price. In that case, firm 1 sets q_1 to maximize its profits p_1q_1 , subject to $p_1 = a(1 - \gamma) + \gamma p_2 - (1 - \gamma^2)q_1$, taking p_2 as given. Thus, firm 1's profit function is given by

$$\Pi_1^{QP} = q_1 [a(1 - \gamma) + \gamma p_2 - (1 - \gamma^2)q_1 - w_1] \quad (\text{A24})$$

The corresponding reaction function is

$$q_1^{QP}(p_2^{QP}) = \frac{a(1 - \gamma) + \gamma p_2 - w_1}{2(1 - \gamma)^2} \quad (\text{A25})$$

Similarly, firm 2 sets p_2 to maximize its profits p_2q_2 , subject to $q_2 = a - \gamma q_1 - p_2$, taking p_1 as given. Thus, firm 2's profit function is given by

$$\Pi_2^{QP} = (p_1 - w_2)(a - \gamma q_1 - p_2) \quad (\text{A26})$$

The corresponding reaction function is

$$p_2^{QP}(q_1^{QP}) = \frac{1}{2}(a - \gamma q_1 + w_2) \quad (\text{A27})$$

The intersection of the above reaction functions yields the Nash equilibrium

$$q_1^{QP}(w_1, w_2) = \frac{a(\gamma - 2) + 2w_1 - \gamma w_2}{3\gamma^2 - 4} \quad (\text{A28})$$

$$p_2^{QP}(w_1, w_2) = \frac{a(\gamma^2 + \gamma - 2) - \gamma w_1 + (2\gamma^2 - 2)w_2}{3\gamma^2 - 4} \quad (\text{A29})$$

Given (w_1, w_2) , firms' equilibrium profits are

$$\Pi_1^{QP}(w_1, w_2) = \frac{(1 - \gamma^2) [a(\gamma - 2) + 2w_1 - \gamma w_2]^2}{(3\gamma^2 - 4)^2} \quad (\text{A30})$$

$$\Pi_2^{QP}(w_1, w_2) = \left[\frac{a(\gamma^2 + \gamma - 2) - \gamma w_1 - (\gamma^2 - 2)w_2}{3\gamma^2 - 4} \right]^2 \quad (\text{A31})$$

In the first stage of the game, firm-level unions simultaneously set their firm-specific wage rates, so as to maximize

$$U_1^{QP}(w_1, w_2) = \frac{(w_1)^\varphi [a(\gamma - 2) + 2w_1 - \gamma w_2]^2}{3\gamma^2 - 4} \quad (\text{A32})$$

$$U_2^{QP}(w_1, w_2) = \frac{(w_2)^\varphi [a(\gamma^2 + \gamma - 2) - \gamma w_1 - (\gamma^2 - 2)w_2]}{3\gamma^2 - 4} \quad (\text{A33})$$

The corresponding unions' wage response functions in wages are given by

$$w_1^{QP}(w_2) = \frac{\varphi [a(\gamma - 2) + \gamma w_2]}{2(1 + \varphi)} \quad (\text{A34})$$

$$w_2^{QP}(w_1) = \frac{\varphi [a(\gamma^2 + \gamma - 2) - \gamma w_1]}{(1 + \varphi)(\gamma^2 - 2)} \quad (\text{A35})$$

From the system of the above equations, we get a unique stable solution for the wages

$$w_1^{QP} = \frac{a\varphi [\varphi(3\gamma^2 - 4) - (\gamma^2 - 2)(\gamma - 2)]}{\gamma^2 [\varphi(3\varphi + 4) + 2] - 4(1 + \varphi)^2} \quad (\text{A36})$$

$$w_2^{QP} = \frac{a\varphi [\gamma^2(3\varphi + 2) + 2\gamma - 4(1 + \varphi)]}{\gamma^2 [\varphi(3\varphi + 4) + 2] - 4(1 + \varphi)^2} \quad (\text{A37})$$

Substituting w_1^{QP} and w_2^{QP} into (A28), (A29), (1), (A30), (A31), (A32) and (A33) gives the corresponding equilibrium market outcomes

$$q_1^{QP} = \frac{2a [\varphi(4 - 3\gamma^2) + (\gamma^2 - 2)(\gamma - 2)]}{\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4)} \quad (\text{A38})$$

$$q_2^{QP} = \frac{a(\gamma^2 - 2) [\gamma^2(3\varphi + 2) + 2\gamma - 4(1 + \varphi)]}{\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4)} \quad (\text{A39})$$

$$p_1^{QP} = \frac{a [\varphi(3\gamma^2 - 4) - (\gamma^2 - 2)(\gamma - 2)] (3\varphi\gamma^2 + 2\gamma^2 - 4\varphi - 2)}{\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4)} \quad (\text{A40})$$

$$p_2^{QP} = \frac{a [\gamma^2(3\varphi + 2) + 2\gamma - 4(1 + \varphi)] [\varphi(3\gamma^2 - 4) + \gamma^2 - 2]}{\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4)} \quad (\text{A41})$$

$$\Pi_1^{QP} = \frac{4a^2(1 - \gamma^2) [\varphi(4 - 3\gamma^2) + (\gamma^2 - 2)(\gamma - 2)]^2}{\left[\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4) \right]^2} \quad (\text{A42})$$

$$\Pi_2^{QP} = \frac{a^2(\gamma^2 - 2)^2 [\gamma^2(3\varphi + 2) + 2\gamma - 4(1 + \varphi)]^2}{\left[\left[[2 + \varphi(4 + 3\varphi)]\gamma^2 - 4(1 + \varphi)^2 \right] (3\gamma^2 - 4) \right]^2} \quad (\text{A43})$$

$$U_1^{QP} = \frac{2a [\varphi(4 - 3\gamma^2) + (\gamma^2 - 2)(\gamma - 2)] \left[\frac{a\varphi[\varphi(3\gamma^2 - 4) - (\gamma^2 - 2)(\gamma - 2)]}{\gamma^2[\varphi(3\varphi + 4) + 2] - 4(1 + \varphi)^2} \right]^\varphi}{(3\gamma^2 - 4) \left[2 + \varphi(3\varphi + 4)\gamma^2 - 4(1 + \varphi)^2 \right]} \quad (\text{A44})$$

$$U_2^{QP} = \frac{(\gamma^2 - 2) \left[\frac{a\varphi[\gamma^2(3\varphi + 2) + 2\gamma - 4(1 + \varphi)]}{\gamma^2[\varphi(3\varphi + 4) + 2] - 4(1 + \varphi)^2} \right]^\varphi}{\varphi(3\gamma^2 - 4)} \quad (\text{A45})$$

Proof of Proposition 1

Suppose that firm/union pair $j \equiv 2$ sticks to the quantity, as the firm 2's mode of competition, while firm/union pair $i \equiv 1$ decides to deviate towards setting the price. In this case, in the first stage union 2 sets the wage w_2^C that corresponds to the symmetric Cournot competition, while union 1 uses its wage response function $w_1^{PQ}(w_2)$, to optimally adjust its wage. Thus, union 1 sets

$$w_{1d}^C = \frac{a\varphi \left[\gamma^2 + \gamma - 2 - \frac{\varphi\gamma(\gamma - 2)}{\varphi(\gamma - 2) - 2} \right]}{(1 + \varphi)(\gamma^2 - 2)}$$

where $w_{1d}^C < w_1^C$. The ensuing utility for the union 1 will be $U_{1d}^C = U_1^{PQ}(w_1, w_2)$, where: $w_1 = w_{1d}^C$ and $w_2 = w_2^C$, with

$$U_{1d}^C = \frac{a \left[\frac{\alpha\varphi[\gamma^2 + \gamma - 2 - \frac{\varphi\gamma(\gamma-2)}{\varphi(\gamma-2)-2}]}{(1+\varphi)(\gamma^2-2)} \right]^\varphi [\varphi(\gamma^2-2)(\gamma-2) - 2(\gamma^2 + \gamma - 2)]}{(1+\varphi)[\varphi(\gamma-2) - 2](4-3\gamma^2)}$$

where $U_{1d}^C < U_1^C$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that the union 1 does not have an incentive for its firm to deviate towards setting its own price as its mode of competition.

On the other hand, the deviant firm 1's profits are given by $\Pi_{1d}^C = \Pi_1^{PQ}(w_1, w_2)$, where: $w_1 = w_{1d}^C$ and $w_2 = w_2^C$. Thus,

$$\Pi_{1d}^C = \frac{a^2 [\varphi(\gamma^2-2)(\gamma-2) - 2(\gamma^2 + \gamma - 2)]^2}{(1+\varphi)^2 [\varphi(\gamma-2) - 2]^2 (4-3\gamma^2)^2}$$

It proves that $\Pi_{1d}^C < \Pi_1^C$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that neither firm 1 has an incentive to deviate from the symmetric Cournot mode of competition.

Proof of Proposition 2

Suppose that firm/union pair $j \equiv 2$ sticks to the price as the firm 2's mode of competition, while firm/union pair $i \equiv 1$ decides to deviate towards setting the quantity. In this case, in the first stage union 2 sets the wage w_2^B that corresponds to the symmetric Bertrand competition, while union 1 uses its wage response function $w_1^{QP}(w_2)$, to optimally adjust its wage. Thus, union 1 sets

$$w_{1d}^B = \frac{a\varphi \left[2 - \frac{\gamma(\gamma^2-2)}{\gamma^2 + \varphi(\gamma^2 + \gamma - 2) - 2} \right]}{2(1+\varphi)}$$

where $w_{1d}^B > w_1^B$. The ensuing utility for the union 1 will be $U_{1d}^B = U_1^{QP}(w_1, w_2)$, where: $w_1 = w_{1d}^B$ and $w_2 = w_2^B$, with

$$U_{1d}^B = \frac{\left[\frac{a\varphi \left[2 - \frac{\gamma(\gamma^2-2)}{\gamma^2 + \varphi(\gamma^2 + \gamma - 2) - 2} \right]}{1+\varphi} \right]^\varphi [(\gamma^2-2)(\gamma-2) - 2\varphi(\gamma^2 + \gamma - 2)]}{2\varphi(1+\varphi)(3\gamma^2-4)[\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2]}$$

where $U_{1d}^B > U_1^B$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that the union 1 does have an incentive for its firm to deviate towards setting its own quantity as its mode of competition.

On the other hand, the deviant firm 1's profits are given by $\Pi_{1d}^B = \Pi_1^{QP}(w_1, w_2)$, where: $w_1 = w_{1d}^B$ and $w_2 = w_2^B$. Thus,

$$\Pi_{1d}^B = \frac{a^2 [\varphi(\gamma^2-2)(\gamma-2) - 2(\gamma^2 + \gamma - 2)]^2}{(1+\varphi)^2 [\varphi(\gamma-2) - 2]^2 (4-3\gamma^2)^2}$$

It proves that $\Pi_{1d}^B > \Pi_1^B$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that also firm 1 has always an incentive to deviate from the symmetric Bertrand competition, towards setting its own quantity as its mode of competition.

Proof of Proposition 3

Firm/union 1 deviate

Suppose, that firm/union pair 2 sticks to the price as the firm 2's mode of competition, while firm/union pair 1 decides to deviate towards setting the price too. In this case, in the first stage union 2 sets the wage w_2^{QP} that corresponds to the asymmetric equilibrium (Quantity - Price), while union 1 uses its wage response function $w_1^B(w_2)$, to optimally adjust its wage. Thus, union 1 sets

$$w_{1d}^{QP} = \frac{a\varphi \left[\gamma^2 + \gamma - 2 - \frac{[\gamma^2(3\varphi+2)+2\gamma-4(1+\varphi)]}{[\varphi(3\varphi+4)+2]\gamma^2} \right]}{(\gamma^2 - 2)(1 + \varphi)}$$

where $w_{1d}^{QP} > w_2^{QP}$. The ensuing utility for the union 1 will be $U_{1d}^{QP} = U_1^B(w_1, w_2)$, where: $w_1 = w_{1d}^{QP}$ and $w_2 = w_2^{QP}$, with

$$U_{1d}^{QP} = \frac{A \left[\frac{a\varphi \left[\gamma^2 + \gamma - 2 - \frac{[\gamma^2(3\varphi+2)+2\gamma-4(1+\varphi)]}{[\varphi(3\varphi+4)+2]\gamma^2} \right]}{(\gamma^2 - 2)(1 + \varphi)} \right]^\varphi}{2^\varphi(1 + \varphi)(\gamma^2 - 4) \left[\gamma^2 \left[2 + \varphi(4 + 3\varphi) - 4(1 + \varphi)^2 \right] \right]}$$

$$A = a \left[(1 + \varphi) [4\gamma + 2(4 + 5\varphi)\gamma^2] - (1 + \varphi)[8 + 2\gamma^3] - [2 + \varphi(3\varphi + 6)] \gamma^5 \right]$$

where $U_{1d}^{QP} > U_1^{QP}$ only if $\varphi = 1$ and $\gamma > 0.99$. This implies that the union 1 has an incentive for its firm to deviate towards setting the price as its mode of competition only if $\varphi = 1$ and $\gamma > 0.99$.

On the other hand, if firm 1 deviates towards setting the price, its profits are given by $\Pi_{1d}^{QP} = \Pi_1^B(w_1, w_2)$, where: $w_1 = w_{1d}^{QP}$ and $w_2 = w_2^{QP}$, with

$$\Pi_{1d}^{QP} = \frac{a^2 \left[8(1 + \varphi)^2 - (1 + \varphi) [4\gamma + 2(4 + 5\varphi)\gamma^2 + 2\gamma^3] + [2 + \varphi(4 + 3\varphi)\gamma^4] \right]^2}{(\gamma - 2)^2(\gamma + 1) [\varphi(\gamma^2 + \gamma - 2) + \gamma^2 - 2]^2}$$

where $\Pi_{1d}^{QP} > \Pi_1^{QP}$ only if $\varphi = 1$ and $\gamma > 0.90$, implying that, given $\varphi = 1$, the firm 1 has an incentive to deviate from the asymmetric Cournot-Bertrand competition towards setting the price for a lower (than its union) γ value.

Firm/union 2 deviate

Suppose, that firm/union pair 1 sticks to the quantity as the firm 1's mode of competition, while firm/union pair 2 decides to deviate towards setting the quantity too.

In this case, in the first stage union 1 sets the wage w_1^{QP} that corresponds to the asymmetric equilibrium (Quantity - Price), while union 2 uses its wage response function $w_2^C(w_2)$, to optimally adjust its wage. Thus, union 2 sets

$$w_{2d}^{QP} = \frac{a\varphi \left[2 - \frac{\gamma(\gamma^2-2)[\varphi(\gamma+2)+2]}{\gamma^2[2+\varphi(3\varphi+4)]-4(1+\varphi)^2} \right]}{2(1+\varphi)}$$

where $w_{2d}^{QP} > w_2^{QP}$. The ensuing utility for the union 2 will be $U_{2d}^{QP} = U_2^C(w_1, w_2)$, where: $w_1 = w_1^{QP}$ and $w_2 = w_{2d}^{QP}$, with

$$U_{2d}^{QP} = \frac{B \left[\frac{a\varphi \left[2 - \frac{\gamma(\gamma^2-2)[\varphi(\gamma+2)+2]}{\gamma^2[2+\varphi(3\varphi+4)]-4(1+\varphi)^2} \right]}{2(1+\varphi)} \right]^\varphi}{2\varphi(1+\varphi)(\gamma^2-4) \left[\gamma^2 \left[2 + \varphi(4+3\varphi) - 4(1+\varphi)^2 \right] \right]}$$

$$B = [\varphi^2(8-6\gamma^2) + [2+\varphi(\gamma+4)](\gamma^2-2)(\gamma-2)]$$

where $U_{2d}^{QP} > U_2^{QP}$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that union 2 has always an incentive for its firm to deviate towards setting the quantity as its mode of competition.

On the other hand, the deviant firm 2's profits are given by $\Pi_{2d}^{QP} = U_2^C(w_1, w_2)$, where: $w_1 = w_1^{QP}$ and $w_2 = w_{2d}^{QP}$, with

$$\Pi_{2d}^{QP} = \frac{a^2 [\varphi^2(8-6\gamma^2) + 2(\gamma^2-2)(\gamma-2) + \varphi(\gamma-2)(\gamma+4)(\gamma^2-2)]^2}{(1+\varphi)^2(\gamma-2)^2(\gamma+2)^2 [\gamma^2[2+\varphi(4+3\varphi)]-4(1+\varphi)^2]^2}$$

where $\Pi_{2d}^{QP} > \Pi_2^{QP}$, for all $\gamma \in (0, 1)$ and $\varphi \in (0, 1]$, implying that firm 2 has always an incentive for its firm to deviate towards setting the quantity as its mode of competition.

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