# STRATEGIC BEHAVIOUR AND RISK TAKING IN FOOTBALL

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**Abstract**

This article develops a dynamic game-theoretic model of optimizing strategic behaviour by football teams. Teams choose continuously between defensive and attacking formations and between a non-violent and a violent playing style. Starting from the end of the match and working backwards, the teams' optimal strategies conditional on the current state of the match are determined by solving a series of two-person non-cooperative subgames. Numerical simulations are used to explore the sensitivity of strategic behaviour to variations in the structural parameters. The model is tested empirically, using English football league data. Teams that are trailing are willing to bear an increased risk of a player dismissal in order to increase the probability of scoring. Teams that are leading or level in scores play cautiously. The scoring rates of teams that are trailing are higher than those of teams that are ahead or level. Stochastic simulations are used to obtain probabilities for match results, conditional upon the state of the match at any stage. The article's main theoretical and empirical results constitute novel, non-experimental evidence that the strategic behaviour of football teams can be rationalized in accordance with game-theoretic principles of optimizing strategic behaviour by agents when payoffs are uncertain and interdependent.

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## 1. Introduction

Football (soccer) is a strategic and dynamic game. In every match, the two teams are pitched into direct opposition for a period of play of 90 minutes' notional duration.[1] Each team starts the match with one goalkeeper and ten outfield players. Team managers or coaches are at liberty to deploy their outfield players in any formation of their choosing, and to adjust their team's formation and style of play at any stage of the match. The immediate objectives of the two teams throughout the match are symmetric: each team attempts to score goals and prevent its opponent from scoring.[2] Accordingly there is a high level of interdependence: both teams' strategies have implications for both teams' probabilities of scoring and conceding goals.

Several researchers have noted the usefulness or potential of professional sports for testing economic hypotheses concerning strategic behaviour and risk taking based on non-experimental data (Walker and Wooders, 2001; Chiappori, Levitt and Groseclose, 2002; Palacios-Huerta, 2003). In many sports, only two teams or players are involved in each contest. The contest takes place within a specific and clearly defined time-frame. Strategies and payoffs are relatively simple to identify. Outcomes matter, because large sums of money are at stake for the winners and losers. Large data sets are readily available, and new data are being continually generated.

In this paper, we examine the extent to which the strategic behaviour of football teams throughout the course of matches can be rationalized in accordance with game-theoretic principles of optimizing strategic behaviour by independent agents when payoffs are interdependent. The model that is developed in this paper has two important antecedents in the economics literature. First, in an unpublished working paper, Palomino, Rigotti and Rustichini (2000) (henceforth, PRR) develop a dynamic game-theoretic model, in which the two teams choose continuously between a defensive and an attacking formation. PRR show that teams tend to play more defensively when they are leading than they play when they are either level or trailing, especially during the later stages of matches. Second, a similar model has been used by Banerjee, Swinnen and Weersink (2007) (henceforth, BSW) to examine the impact on team strategies in the National Hockey League of a recent change in the league points scoring system affecting matches that were tied at the end of regular time.

The PRR and BSW models contain several simplifying assumptions that are either controversial or counterfactual, or which limit the generality of their analysis. PRR rely upon an unproven assumption that if one team adopts an attacking strategy and the other adopts a defensive strategy, the goal scoring rate of the attacking team increases by more than that of the defending team. BSW relax this assumption, by introducing a parameter allowing for comparative advantage in either attack or defense. Both PRR and BSW consider the special case in which the attacking and defensive strengths of the two teams are the same; and most of PRR's results are based on a counterfactual assumption that the payoff structure at the end of the match is zero-sum. PRR are unable to account for what appears to be a strong empirical regularity, that after controlling for team quality and duration effects, the scoring rates of both teams when the scores are level tend to be lower than those of teams that are trailing, and not significantly different from those of teams that are leading.

Both PRR and BSW restrict the teams' strategic choices to two options: attack and defense. Neither considers a second strategic dimension: the choice between a violent and non-violent style of play. Teams that play violently commit foul play and other acts of violence or aggression in an attempt to disrupt or sabotage the opposing team. By doing so, however, they incur an increased risk of being subject to disciplinary sanction, which may involve the dismissal of a player from the field of play for the remainder of the match. By allowing the teams to exercise choices concerning the level of violence or foul play, the model that is developed in this paper incorporates the link between strategic behaviour and disciplinary sanction.[3]

In the model that is developed below, we assume that the available strategic choices are discrete: teams choose between 'defensive' and 'attacking' formations, and between 'non-violent' and 'violent' styles of play. These strategic choices influence the probabilities of scoring and conceding goals at the current stage of the match, and the probabilities that players are dismissed. Using numerical simulations based on a dynamic game-theoretic model incorporating a broad spectrum of structural assumptions, we show that the optimal strategic choices at each stage of the match are dependent on the current difference in scores and on the amount of time that has elapsed. The numerical simulations provide a simple and flexible apparatus for exploring the effects on strategic behaviour and outcomes of changes in any of the structural assumptions.

We subject the theoretical model to empirical scrutiny, using data on the timings of goals and player dismissals from more than 12,000 English professional league matches played between 2001 and 2007. We find a high level of consistency between the theoretical model, and observed behaviour and outcomes reflected in the coefficients of a set of empirical hazard functions for the conditional arrival rates of player dismissals and goals.

The remainder of the paper is structured as follows. Section 2 describes the data set, and motivates the development of a theoretical model by identifying a number of key patterns in the data. Section 3 develops the theoretical model. Section 4 presents numerical simulations which identify the theoretical model's predictions for strategic behaviour over a range of structural assumptions. Section 5 describes the specification of the empirical model. Section 6 reports and interprets the estimation results, and uses stochastic simulations to obtain probabilities for match results conditional upon the state of the match at any stage. Finally, Section 7 concludes.

## 2. Player dismissals and goal scoring in English football

The data sample for the empirical analysis that is reported in this paper comprises all 12,216 matches played in the English Premier League (the Premiership) and the three divisions of the English Football League (currently known as the Championship, League One and League Two) during the six football seasons from 2001/02 to 2006/07 (inclusive).[4] Tables 1 and 2 report descriptive statistics, which illustrate several of the key strategic issues that are examined in this paper. Table 1 reports the average rates of player dismissal per minute and the average rates at which goals were scored by the home and away teams per minute, at various durations within matches.[5] Table 2 reports the same averages conditional on the current difference between the scores of the two teams.

Some of the key features of Tables 1 and 2, which we seek to investigate and explain, are as follows. First, there is a pronounced upward trend in the rates of player dismissal over the duration of matches. The rates for $85<t\leq89$ (durations between 85 and 89 minutes) are around 18 times and 9 times those for $0<t\leq10$ for home teams and for away teams, respectively. Second, there is also a clear but less heavily pronounced upward trend in the goal scoring rates. The goal scoring rates for the home and away teams for $85<t\leq89$ are around 1.5 times those for $0<t\leq10$.

Third, except in the case where either team is leading by more than two goals (for which the data are relatively sparse), the probability of having a player dismissed is lowest when the scores are level; is of intermediate value for teams leading by one or two goals ($s=1,2$ for the home team, $s=-1,-2$ for the away team); and is highest for teams trailing by one or two goals ($s=-1,-2$ for the home team, $s=1,2$ for the away team). Fourth and finally, the probability of either team scoring is lowest when the scores are level, and is somewhat increased when the scores are unequal. In the latter case, and provided the current scores difference is not more than two goals, the scoring probabilities of both teams increase in similar proportions (no matter which team is leading and which is trailing). However, a team that is leading by three goals or more appears to experience a large increase in its probability of scoring further goals.

The tendency for scoring rates to increase over the duration of the match (see Table 1) is well known to commentators, pundits and spectators, and is commonly attributed to a tendency for players to become fatigued and commit defensive errors, which lead to goals being scored. Errors due to fatigue could also produce more fouls in last-ditch attempts to prevent goals, which could in turn account for the upwardly-trended rates of player dismissal.

Since goals in football are relatively infrequent events (in comparison with scores in other team sports such as rugby or basketball), observations in Table 2 for level scores tend to be weighted towards the earlier match durations, while observations for a non-zero difference in scores tend to be weighted towards later durations. Therefore player fatigue could also account for some of the variation shown in Table 2. However, the data in Table 2 could also reflect a form of team quality selection effect. Observations of scoring rates when the away team is already leading (for example) are drawn predominantly from matches in which the away team dominates the home team in terms of quality. Therefore we should expect the away team's probability of scoring further goals to be higher than average. This team quality selection effect might explain why teams score more frequently when leading than when scores are level, but it does not explain why teams that are trailing also tend to score more frequently.

Distinct from the player fatigue and team quality selection effects, an alternative explanation for the variations in scoring rates shown in Tables 1 and 2 relates to rational strategic behaviour on the part of the teams throughout the duration of the match. In the dynamic game-theoretic model that is developed below, a team that switches from a defensive to an attacking formation increases both its own and its opponent's probabilities of scoring. By leaving itself open to a counter-attack and the possibility of having to commit a last-ditch foul in order to prevent an opponent from scoring, it also increases its own probability of having a player dismissed. A team that switches from a non-violent to a violent style of play disrupts the pattern of play, in a manner that increases both teams' scoring probabilities.[6]

The dismissal of a player imposes two separate costs upon his own team. First, the team must complete the match at a numerical disadvantage in players (unless an opposing player is also dismissed). Table 3 indicates that there is a large reduction in the scoring probability for a team that is playing at a numerical disadvantage, and a still larger increase in the probability of conceding a goal. This portion of the cost of a player dismissal is highest for dismissals that occur at the early stages of matches, because the team that loses a player is exposed to these adverse probabilities for longer. Second, a dismissal results in the suspension of the player concerned from one or more future matches, reducing his team's prospects of obtaining league points because the team manager or coach has to select from a weaker pool of players for those matches. This portion of the cost is independent of the stage of the (current) match at which the dismissal occurs. Therefore the overall cost of a dismissal in the current match, expressed in terms of expected league points foregone, is higher for dismissals during the early stages of matches. A tendency for dismissal rates to increase over the duration of matches (see Table 1) seems consistent with rational strategic behaviour in the light of this cost structure. In the following section, we develop a game-theoretic model that seeks to capture the dynamic aspects of the interplay between interdependence, risk and outcomes.

## 3. A dynamic game-theoretic model of strategic behaviour and risk taking

In the theoretical model, we assume that each match consists of a number of discrete unit time intervals, such that durations within the match can be represented by t=0 ,..., T where T is the complete match duration. In the numerical simulations that follow in the next section, we let each time interval represent one minute, and T=90.

The payoff for each team at the end of the match is dependent on the number of league points gained and the cost of future player suspensions arising from any dismissals incurred during the current match. To allow for the possibility of risk-averse behaviour on the part of the teams, the payoffs from the league points gained are determined through a utility function, which may be either linear in points gained (risk-neutrality) or concave (risk-aversion). The available league points are 3 for a win, 1 for a draw (tie), and 0 for a loss. Without any loss of generality, the utility function is specified as follows: U(0)=0, U(1)=1, U(3)=1+2$\lambda$, where $\lambda$=1 represents risk neutrality and 0<$\lambda$<1 represents risk aversion. For simplicity the utility cost of a dismissal (arising from future player suspensions) is a fixed utility deduction per player dismissed, denoted $\omega$.

Let s denote the difference in scores at duration t, and let $d_h$ and $d_a$ denote the numbers of dismissals incurred by the home and away teams prior to t, respectively. For notational simplicity, the t-subscripts are suppressed from s, $d_h$ and $d_a$. Let the value functions $\bar{h}_t(s, d_h, d_a)$ and $\bar{a}_t(s, d_h, d_a)$ denote the final payoffs to the home and away teams at the end of the match (for t=T), and the expectations of these final payoffs at earlier durations (for t=0,...,T–1). At the end of the match, $\bar{h}_T(s, d_h, d_a)$ = 1+2$\lambda$–$\omega d_h$ for s>0 (and for any $d_a$); 1–$\omega d_h$ for s=0; and –$\omega d_h$ for s<0. Similarly, $\bar{a}_T(s, d_h, d_a)$ = –$\omega d_a$ for s>0 (and for any $d_h$); 1–$\omega d_a$ for s=0; and 1+2$\lambda$–$\omega d_a$ for s<0.

For any time interval within the match denoted (t,t+1) for t=0, ..., T–1, each team manager or coach can select either a non-violent or a violent style of play, and either a defensive or an attacking team formation. The home and away teams' strategic choices are represented by i and j, respectively: i,j=1 denotes (non-violent, defend); i,j=2 denotes (violent, defend); i,j=3 denotes (non-violent, attack); and i,j=4 denotes (violent, attack). Let $p_{i,j,x,t}$ denote the probability that the home team scores a goal during (t,t+1), conditional on x = $d_h$–$d_a$ and on i and j; and let $q_{i,j,x,t}$ denote the corresponding probability for the away team. Let $u_{i,t}$ denote the probability that the home team has a player dismissed during time interval (t,t+1) conditional on i; and let $v_{j,t}$ denote the corresponding probability for the away team conditional on j. For simplicity, we assume that only one goal may be scored and one player dismissed within each time interval.

Starting at time interval (T–1,T) and working backwards from the end to the start of the match, the team managers' optimal tactical selections can be solved as a series of two-person non-cooperative subgames. Let $h_t^{i,j}(s, d_h, d_a)$ denote the expectation at duration t of the home team's final payoff, as a function of the score differential and the numbers of players already dismissed at t, conditional on the teams' choices of i and j over (t, t+1). Similarly let $a_t^{i,j}(s, d_h, d_a)$ denote the expectation at t of the away team's final payoff. The following expressions govern the relations between these expectations at t conditional on i and j, and the expectations at t+1 unconditional on i and j:

$$
\begin{aligned}
h_t^{i,j}(s, d_h, d_a) = {} & q_{i,j,x,t}\,[v_{j,t}\bar{h}_{t+1}(s-1, d_h, d_a) + \{1-u_{i,t}-v_{j,t}\}\,\bar{h}_{t+1}(s-1, d_h, d_a) + u_{i,t}\bar{h}_{t+1}(s-1, d_h+1, d_a)] \\
& + \{1 - p_{i,j,x,t} - q_{i,j,x,t}\}[v_{j,t}\bar{h}_{t+1}(s, d_h, d_a+1) + \{1-u_{i,t}-v_{j,t}\}\,\bar{h}_{t+1}(s, d_h, d_a) + u_{i,t}\bar{h}_{t+1}(s, d_h+1, d_a)] \\
& + p_{i,j,x,t}\,[v_{j,t}\bar{h}_{t+1}(s+1, d_h, d_a+1) + \{1-u_{i,t}-v_{j,t}\}\,\bar{h}_{t+1}(s+1, d_h, d_a) + u_{i,t}\bar{h}_{t+1}(s+1, d_h+1, d_a)]
\end{aligned}
$$

$$
\begin{aligned}
a_t^{i,j}(s, d_h, d_a) = {} & q_{i,j,x,t}\,[v_{j,t}\bar{a}_{t+1}(s-1, d_h, d_a+1) + \{1-u_{i,t}-v_{j,t}\}\,\bar{a}_{t+1}(s-1, d_h, d_a) + u_{i,t}\bar{a}_{t+1}(s-1, d_h, d_a)] \\
& + \{1 - p_{i,j,x,t} - q_{i,j,x,t}\}[v_{j,t}\bar{a}_{t+1}(s, d_h, d_a+1) + \{1-u_{i,t}-v_{j,t}\}\,\bar{a}_{t+1}(s, d_h, d_a) + u_{i,t}\bar{a}_{t+1}(s, d_h+1, d_a)] \\
& + p_{i,j,x,t}\,[v_{j,t}\bar{a}_{t+1}(s+1, d_h, d_a+1) + \{1-u_{i,t}-v_{j,t}\}\,\bar{a}_{t+1}(s+1, d_h, d_a) + u_{i,t}\bar{a}_{t+1}(s+1, d_h+1, d_a)]
\end{aligned}
$$

Starting at duration t=T–1, the teams' optimal values of i and j over (T–1,T) can be determined, since $\bar{h}_T()$ and $\bar{a}_T()$ are already known. This establishes $\bar{h}_{T-1}()$ and $\bar{a}_{T-1}()$. The optimal choices of i and j over (T–2,T–1) are then similarly determined. By iterating backwards, the optimal choices of i and j at any stage of the match are determined.

## 4. Numerical simulations

In this section, we illustrate the behavioural properties of the theoretical model by means of numerical simulation. Some of the model's parameters can be calibrated accurately from inspection of the data that are summarized in Tables 1, 2 and 3. Some parameters, however, cannot be calibrated in this manner. In the case of the latter, simulations of the model are run over ranges of plausible parameter values.

The rates of goal scoring within each time interval for the home and away teams are observable at an aggregate level, but the rates conditional on the teams' strategic choices i and j are unobservable, because i and j are themselves unobservable. Let $I_i$ and $J_j$ denote indicator variables for strategic choices i (home team) and j (away team), respectively. The parameterization of the scoring rates per minute is as follows:

Home team:

$p_{i,j,0,t} = 0.011 + 0.0015\theta + 0.00004t + 0.005\phi[2(I_2+I_4)+J_2+J_4] + 0.005\psi[(I_3+I_4)\varphi+(J_3+J_4)(2–\varphi)]$

$p_{i,j,x,t} = (1–0.25x_t)p_{i,j,0,t}$ for x>0, $p_{i,j,x,t} = (1+0.75x_t)p_{i,j,0,t}$ for x<0

Away team:

$q_{i,j,0,t} = 0.008 – 0.0015\theta + 0.00004t + 0.005\phi[I_2+I_4+2(J_2+J_4)] + 0.005\psi[(I_3+I_4)\varphi+(J_3+J_4)(2–\varphi)]$

$q_{i,j,x,t} = (1–0.75x_t)q_{i,j,0,t}$ for x>0, $q_{i,j,x,t} = (1+0.25x_t)q_{i,j,0,t}$ for x<0

Naturally the scoring rates of both teams depend on relative team quality, represented by the parameter θ. Positive values of θ represent matches in which the home team is stronger than the away team, θ=0 represents a match between two evenly balanced teams, and negative values of θ represent matches in which the away team is stronger than the home team. When θ=0, the home team's scoring rate is higher than that of the away team due to home-field advantage. θ=–1 represents a match in which the home and away teams have identical scoring rates, because the away team's team quality advantage is just sufficient to offset the home-field advantage. In the numerical simulations θ=0 is the baseline value, and we examine variations in the range θ=–2,–1,0,1,2.

It is apparent from the raw data that the scoring rates increase with a near-linear trend over the duration of the match, and it appears unlikely that the existence of this trend is explained in its entirety by changes in strategic behaviour during the match. In the simulations the scoring rates are assumed to follow a linear trend. For simplicity, the trend component is the same for home and way teams, and it does not vary with either relative team quality or the teams' strategic choices.

The assumed impact of the teams' strategic choices on their scoring rates is as follows. First, if either or both teams play violently (i,j=2 or 4), the effect on the scoring probabilities is determined by the parameter $\phi$. If one team is violent and the other is non-violent, the violent team's scoring rate increases by $0.01\phi$ and the non-violent team's by $0.005\phi$. If both teams are violent, both scoring rates increase by $0.015\phi$. In the numerical simulations $\phi=1$ is the baseline value, and we examine variations in the range $\phi=0.5,1,1.5$.

Second, by attacking (i,j=3 or 4), a team increases its own and its opponent's scoring rates by amounts that are proportional the parameter $\psi$. In the case where both teams attack, both teams' scoring rates increase by the same amount. In the case where one team defends and the other attacks, the teams' scoring rates increase by smaller amounts. The relativities depend upon which team (if either) has a comparative advantage when one team attacks and the other defends. The parameter $\varphi$ controls for comparative advantage: $\varphi=1$ implies there is no comparative advantage; $\varphi>1$ implies the home team has a comparative advantage when it attacks and the away team defends; and $\varphi<1$ implies the home team has a comparative advantage when it defends and the away team attacks. In the numerical simulations the baseline values are $\psi=1$ and $\varphi=1$, and we examine variations in the ranges $\psi=0.5,1,1.5$ and $\varphi=1.33,1,0.67$.

The impact on the scoring rates if either team is currently playing with a numerical disadvantage in players is observable in the raw data. For simplicity we assume that the team with a numerical disadvantage experiences a reduction in its own scoring rate and an increase in its opponent's scoring rate, both of which are proportionate to the size of the numerical disadvantage. In practice, it is unusual for the numerical disparity in players to exceed one player, and almost unknown for the disparity to exceed two.

In common with the scoring rates, the rates of player dismissal within each time interval are observable at an aggregate level. The assumed expressions are as follows:

Home team: $u_{i,t} = [0.0004 + 0.00001t + 0.0002(I_3+I_4)][1+\beta(I_2+I_4)]$

Away team: $v_{j,t} = [0.0004 + 0.000012t + 0.0002(J_3+J_4)][1+\beta(J_2+J_4)]$

The player dismissal rates appear to be trended over the duration of the match, and the trend appears to be more pronounced for away teams. For simplicity we ignore any relationship between relative team quality and the dismissal rates, but we do allow for a home-field advantage effect through the coefficient on the trend. As before, the impact of strategic behaviour on the dismissal rates is unobservable. In the simulations, the parameter β controls the amounts by which the dismissal rate increases when a team switches from non-violence (i,j=1 or 3) to violence (i,j=2 or 4). In the numerical simulations β=2 is the baseline value, and we examine variations in the range β=1,2,3. We also assume that a switch from defense (i,j =1 or 2) to attack (i,j =3 or 4) produces an increase in the team's own dismissal rate.[7]

The utility cost of future player suspensions arising from dismissals is unobservable. In the numerical simulations ω=0.5 is the baseline value, and we examine variations in the range ω=0.25,0.5,1. The risk-aversion parameter λ is also unobservable. At the baseline value of λ=0.5, the difference in utility between a win and a draw is the same as the difference between a draw and a loss, even though the league points difference between a win and a draw is twice the difference between a draw and a loss.[8] In the numerical simulations we examine variations in the range λ=0.25,0.5,0.75,1. At the maximum value of λ=1, the relationship between league points and utility values is linear.

Table 4 presents a full analysis of the two-person non-cooperative subgame at one arbitrarily selected stage of a match, the 75th minute, with the variable parameters set to their baseline values as detailed above. The analysis is presented conditional on the difference in scores at the end of the 74th minute, for s=–1 (away team leading by one goal), 0 (scores level) and 1 (home team leading by one goal). It is assumed that no player from either team has been dismissed prior to t=74.

The left-hand panel of Table 4 shows the expected payoffs to the home and away teams at t=74 for s=–1, conditional on the teams' strategic choices for the 75th minute, and assuming that optimal choices will be made in each subsequent minute. The home team needs to score at least one goal to collect any league points from the match. Its expected payoffs from i=4 (violent, attack) are higher than those from i=1,2,3, because i=4 maximizes its probability of scoring. Therefore i=4 is the home team's dominant strategy. In contrast, the away team's interests are best served by keeping the scoring rates as low as possible, and j=1 (non-violent, defend) is the away team's dominant strategy. Accordingly the outcome in the 75th minute is (i=4,j=1).

The central panel of Table 4 shows the expected payoffs for s=0. In this case, i=2, j=2 (violent, defend) is the dominant strategy for both teams. In comparison with this non-cooperative solution, both teams would be made better off by cooperating to achieve either i=1,j=1 (non-violent, defend) or i=3,j=3 (non-violent, attack). This subgame has a prisoner's dilemma structure. Finally, the right-hand panel of Table 4 shows the expected payoffs for s=1. In this case, the priorities of the home and away teams are the reverse of those in the case s=–1, and the dominant strategies are i=1 (non-violent, defend) and j=4 (violent, attack).

Table 5 summarizes the outcomes of the two-person non-cooperative subgames at three different stages of the match: the 25th, 50th and 75th minutes, allowing for variations in each of the parameters $\theta$, $\phi$, $\psi$, $\varphi$, $\beta$, $\omega$ and $\lambda$ in turn, while the other parameters are set to their baseline values. The main findings from this numerical simulation exercise can be summarized as follows:

1. In general teams that are trailing tend to attack, and teams that are trailing tend to play violently for as long as there is a realistic chance of salvaging the match. Teams that are trailing by three goals tend not to play violently. The stage at which a team that is trailing by two goals ceases to play violently, implicitly conceding that the match is beyond salvation, depends upon relative team quality and upon home-field advantage: stronger teams when trailing play violently for longer than weaker teams, and home teams when trailing play violently for longer than away teams. Towards the closing stages of matches in which scores are level, when the impact of a dismissal on the match outcome is relatively small, there is a tendency for both teams to defect from non-violence to violence. This feature of the theoretical model seems consistent with the empirical tendency for the number of dismissals to increase significantly in the later stages of matches (see Table 1).

2. The propensity for teams that are either trailing or level to play violently varies positively with the effect of violent play on the scoring probabilities. The propensity for teams that are either trailing or level to attack varies positively with the effect of attacking play on the scoring

probabilities. In a few situations, the existence of a comparative advantage in attack or defense influences the choice of strategy, but in most situations the current state of the match seems to be the decisive factor. A team that is trailing is very likely to choose to attack, even if by so doing it increases its opponent's scoring probability by more than it increases its own scoring probability.

3. The propensity for teams that are either trailing or level to play violently varies negatively with the effect of violent play on the probability of a dismissal, and negatively with the utility cost of a dismissal in terms of league points foregone in future matches.

4. The propensity for teams that are either trailing or level to play violently and/or to attack is increasing in $\lambda$, the risk-aversion parameter. For $\lambda$=0.75 or 1 (mild risk-aversion or risk-neutrality), teams are likely to accept additional risk when the scores are level, because scoring and proceeding to win the match yields twice as many league points as are lost by conceding and proceeding to lose. However, for $\lambda$=0.25 or 0.5 (severe risk-aversion), both teams tend to opt for caution when the scores are level.

## 5. An empirical model for the arrival rates of player dismissals and goals

In this section we develop an empirical model for the arrival rates of player dismissals and goals. Following a modelling approach similar to that of Dixon and Robinson (1998), we assume that the arrival rates can be represented as Poisson processes, such that the probability that a new arrival occurs is independent of the time that has elapsed since the previous arrival. We also simplify slightly by assuming that arrivals of player dismissals and goals are independent of each other.[9]

Let $\lambda_k$ denote the arrival rate for event k, where k=1 denotes a home team player dismissal, k=2 denotes an away team player dismissal, k=3 denotes a goal scored by the home team, and k=4 denotes a goal scored by the away team. Let $m_k$ denote the number of minutes that elapse before the next occurrence of event k. If player dismissals and goals are Poisson processes, $m_k$ follows an exponential distribution, with distribution function $F_k(t) = \text{prob}(m_k \leq t) = 1 - \exp(-\lambda_k t)$ and density function $f_k(t) = F_k'(t) = \lambda_k \exp(-\lambda_k t)$.

We estimate a competing risks model comprising hazard functions for the four events (k=1,...,4), in which $\lambda_k$ are assumed to be linear in a set of covariates that are time-varying over the duration of the current match. Spells of continuous play that end in the occurrence of event k are treated as right-censored in the likelihood functions for the events other than k.

The covariate definitions for the conditional hazard functions for the player dismissals ($\lambda_1$ and $\lambda_2$) are as follows. DUR is the duration (number of minutes elapsed prior to the start of the current minute) within the current match, measured from 0 to 89. M45 and M90 are 0-1 dummy variables that allow for step changes in the rates of player dismissal recorded for the 45th and 90th minutes of the match, due to stoppage time. Team quality effects enter the hazard functions through the covariates RELQUAL = HWINPR+ 0.5×DRAWPR, and UNCERT = RELQUAL×(1–RELQUAL). HWINPR and DRAWPR are *ex ante* probabilities for the current match to end in a home win and a draw, computed by generating out-of-sample fitted values from a bivariate Poisson regression forecasting model for goals scored by the two teams, described in full by Goddard (2005).[10] Measured on a scale from 0 to 1, RELQUAL measures home team quality relative to away team quality, also taking account of home-field advantage. RELQUAL appears in the player dismissal hazard functions because previous empirical evidence suggests that players from lower-quality teams and players from away teams are at greater risk of disciplinary sanction (Dawson et al., 2007).

UNCERT = RELQUAL×(1–RELQUAL), a standard measure of uncertainty of match outcome, also appears in the player dismissal hazard functions. In accordance with the theoretical model and simulation results, and with empirical evidence reported by Dawson et al. (2007), violent play is more likely when the two teams are closely balanced in terms of quality than when they are unbalanced. Therefore positive coefficients on UNCERT are expected in both the home team and away team hazard functions.

DIFFm (for m=–3,–2,–1,1,2,3) are a set of 0-1 dummy variables indicating the goal difference between the home and away teams at the start of the current minute. DIFF–3 = 1 if the away team is leading by three goals or more at the start of the current minute, and 0 otherwise. DIFF–2 = 1 and DIFF–1 =1 if the away team is leading by two goals or one goal, respectively. DIFF+1, DIFF+2 and

DIFF+3 indicate the home team leading by one goal, by two goals, or by three goals or more, respectively.

The covariate definitions for the conditional hazard functions for goal scoring ($\lambda_3$ and $\lambda_4$) are as follows. MINUTE, M45, M90 and DIFFm (for m=–3,–2,–1,1,2,3) are as defined above. We expect scoring rates to be relatively low in the 1st minute (the start of the match), the 46th minute (the resumption of the match following the half-time interval), and in the minute immediately after a goal is scored by either team. In each case, play starts or resumes from a kick-off at the halfway line, and the teams regroup into balanced defensive formations when play kicks off. Goals are unlikely to be scored during the first few seconds following kick-off, and we expect the scoring rates for those minutes to be depressed accordingly. We define KICKOFF = 1 for the 1st and 46th minutes, and for any minute when a goal has been scored by either team in the preceding minute, and 0 otherwise.

The covariates EXPHG and EXPAG, the *ex ante* expected numbers of goals scored by the home and away teams in the current match, control for the relationship between relative team quality and the scoring rates of the home and away teams. These covariates are non-time-varying across all of the observations pertaining to the current match. EXPHG and EXPAG are computed in the same manner as HWINPR and AWINPR (see above), using Goddard's (2005) forecasting model.

Finally, HOFF and AOFF are 0-1 dummy variables indicating whether either team is currently experiencing a numerical disadvantage due to one or more players having previously been dismissed during the current match. HOFF = 1 if the home team is currently at a disadvantage in player numbers, and 0 otherwise, and AOFF is defined similarly for the away team.

## 6. Estimation results and interpretation

*Empirical hazard functions for player dismissals*

Table 6 reports the estimated hazard functions for home team player dismissals (equations [1]-[3]) and away team player dismissals ([4]-[6]). Three alternative specifications are reported. [1] and [4] exclude any effects related to the current difference in scores. [2] and [5] include dummy variables for the current difference in scores. Finally, [3] and [6] also include a quadratic trend in duration, and interaction terms allowing for variations in the linear and quadratic trends for teams that are currently trailing by one goal.

In all of the player dismissal hazard estimations, the coefficients on M45 and M90 are positively signed and significant, and require no further comment. The coefficients on DUR, the linear trend in duration, are positive and significant. The coefficients on RELQUAL are insignificant in the home team player dismissal hazard, but significant in the away team hazard. The coefficients on UNCERT are positively signed throughout, but significant in the away team hazard only.

When DIFFm are added to the specification (comparing [2] with [1], and [5] with [4]), the coefficients and z-statistics on DUR are reduced, but only marginally. The coefficients on DIFF–2 and DIFF–1 are positive and significant in the home team player dismissal hazard [2], while the coefficient on DIFF–3 is positive but insignificant. The coefficients on DIFF+1 and DIFF+2 are negative and insignificant, while the coefficient on DIFF+3 is negative and significant. The pattern for the away team player dismissal hazard [5] is almost the mirror image of the home team hazard. The coefficient on DIFF+1 is positive and significant; the coefficients on DIFF+2 and DIFF+3 are positive but insignificant; the coefficients on DIFF–1 and DIFF–2 are negative and insignificant; and the coefficient on DIFF–3 is negative and significant.

These findings are consistent with the results of the numerical simulation exercise reported in Table 5. The probability of incurring a player dismissal tends to be higher for a team that is trailing than for the same team either when it is leading or when the scores are level. Teams that are currently leading tend to play cautiously (non-violent, defense) in order to minimize both the probability of conceding a goal and the probability of losing a player through dismissal.[11] Teams that are currently trailing tend to take risks (violent, attack), because it is worthwhile bearing an increased probability of a dismissal in order to increase the probability of scoring. However, this willingness to bear additional risk is conditional on the prospects of salvaging the match. Home teams trailing by either one or two goals, and away teams trailing by one goal, are more inclined to bear additional risk than teams that are trailing by larger margins; and home teams when trailing are likely to persist with risky strategies for longer than away teams.

This argument, together with several of the results reported in Table 5, suggests that the incentive for a team trailing by only one goal to take risks in an effort to salvage the match increases as the match approaches its conclusion; but for a team trailing by two goals this incentive tails off as any realistic prospect of salvaging the match disappears. In other words, some of the coefficients on DIFFm may themselves depend on DUR, in a non-linear fashion. In [3] and [6], we attempt to incorporate such effects by adding a quadratic term in duration denoted DUR^2; and by interacting the terms in DUR and DUR^2 with DIFF–1 (home team trailing by one goal) in [3], and with DIFF+1 (away team trailing by one goal) in [6].

In [3] and [6], neither of the estimated coefficients on the quadratic term in DUR^2 is significant (although the coefficient in [6] falls only narrowly short). However, the interaction terms are jointly significant in both [3] and [6]. In both cases, the estimated coefficients suggest that for a team trailing by one goal, the player dismissal hazard increases with duration at an increasing rate; in other words, the player dismissal hazard is convex in duration.

*Empirical hazard functions for goals scored*

Table 7 reports the estimated hazard functions for home team goals (equations [7]-[9]) and away team goals (equations [10]-[12]). In each case the progression between the three specifications is the same as for the player dismissal hazards. The coefficients on KICKOFF, M45, M90, EXPHG, EXPAG, HOFF and AOFF in the goals hazard functions are all correctly signed and significant, and require no further comment. The coefficients on DUR are positive and significant. As before, there is a small reduction in the magnitudes of the coefficients on DUR when DIFFm are added to the specification (comparing [8] with [7], and [11] with [10]).

The coefficients on DIFF–3, DIFF–2 and DIFF–1 are positive and significant in the home team goals hazard [8]. The coefficients on DIFF+1, DIFF+2 and DIFF+3 are all insignificant in [8]. The coefficient on DIFF+1 is positive and significant in the away team goals hazard [11], while the coefficients on DIFF+2 and DIFF+3, although positive fall short of being significant. The coefficients on DIFF–3, DIFF–2 and DIFF–1 are insignificant in [11].

Table 7 suggests that the (conditional) goal scoring rates of teams that are leading are not significantly different from those when the scores are level. In contrast, Table 1 indicates that the (unconditional) scoring rates of teams that are leading are higher than those of teams that are level, and are similar to the (unconditional) scoring rates of teams that are trailing. We now attribute this pattern in Table 1 to a team quality selection effect (see section 2). After controlling for relative team quality in [7]-[12], the differences that are apparent in the raw data, between the scoring rates of teams that are leading and of teams that are level, disappear.

As before, [9] and [12] allow for non-linear duration-dependent effects on the incentive to take risks (violent, attack) for the case where the team concerned is trailing by one goal. The estimated coefficients on DUR^2 are negative and significant, indicating that the goals hazard is typically concave in match duration. However, the coefficients on the interaction terms between DUR and DUR^2 and DIFF–1 in [9] and DIFF+1 in [12] imply that for teams trailing by one goal the hazard becomes convex in duration. As before, and again in accordance with the numerical simulations, the probability that a team trailing by one goal will score tends to increase at an increasing rate over the closing minutes of the match.

*Stochastic simulations for match result probabilities conditional on the current state of the match*

Finally, we develop a practical application of equations [3], [6], [9] and [12] in Tables 6 and 7, for the use of team managers, bettors and others who may wish to obtain probabilities for the final outcome of a match conditional upon the current state of the match at any stage, defined by the difference in scores and any numerical disparity in players, as well as the relative quality of the two teams. For the purposes of reporting this exercise, we define a modified value function for the home team $\tilde{h}_t(s, x; h_0) = \pi_H + 0.5\pi_D$. $\pi_H$ and $\pi_D$ are the probabilities of a home win and a draw conditional on the state of the match at duration t. These probabilities are conditional on t, but for notational simplicity the t-subscripts are suppressed. Using the same notation as before, the state of the match at t is defined by s (difference in scores) and $x = d_h – d_a$ (numerical disparity in players). $h_0$ is the value function at the start of the match, interpreted as a measure of the relative strengths of the home and away teams. In defining the home team value function in this manner, we ignore the future cost of player dismissals. The away team value function is $1-\tilde{h}_t(s,x;h_0)$.

Given $h_0$, s and x and starting from match duration t, $\pi_H$ and $\pi_D$ can be calculated from stochastic simulations of player dismissals and goals over the remainder of the match (t+1,...,T). In accordance with the underlying probability model, we simulate occurrences of each of the relevant events (k=1,2 for player dismissals, k=3,4 for goals scored) by means of random drawings from an exponential distribution with conditional mean $\lambda_k$, which varies over the remaining duration of the simulated match in accordance with [3], [6], [9] and [12]. The probabilities $\pi_H$ and $\pi_D$ are the proportions of simulated home wins and draws obtained over 10,000 replications of this procedure.

The stochastic simulation procedure is repeated over a range of values for $h_0$=.483,.550,.593, .630 and .694. These values correspond approximately to the 10th, 25th, 50th, 75th and 90th percentiles of the distribution of the covariate RELQUAL in the empirical player dismissal hazard functions. In each case Table 8 reports values of $\tilde{h}_t(s,x;h_0)$ for t=15,30,45,60 and 75 minutes, s=–2, –1, 0,1,2 and x=1,0,–1. For example, referring to the probabilities shaded grey in the central panels of Table 8 ($h_0$=.593), the value of an opening goal scored by the home team in the 15th minute is $\tilde{h}_{15}(1,0;.593)-\tilde{h}_{15}(0,0;.593)=.801-.572=.229$. The value of an opening goal scored in the 75th minute is $\tilde{h}_{75}(1,0;.593)-\tilde{h}_{75}(0,0;.593)=.904-.531=.373$. Naturally, the later in the match the opening goal (or any goal that establishes a lead) is scored, the greater is the impact on the match outcome and the greater is the value of the goal to the scoring team.

Similarly, Table 8 can be used to determine the cost to either team (in terms of the outcome of the current match) of incurring a player dismissal. Using the same example (and with the relevant values in Table 8 also shaded grey), the cost to the home team of a dismissal after 15 minutes is $\tilde{h}_{15}(0,0;.593)-\tilde{h}_{15}(0,1;.593)=.572-.360=.212$. The cost of a dismissal after 75 minutes, $\tilde{h}_{75}(0,0;.593)-\tilde{h}_{75}(0,1;.593)=.531-.452=.079$, is less, because the exposure to a numerical disadvantage in players is for a shorter remaining match duration. Naturally, the data shown in Table

8 closely reflect the payoff structure for subgames that is embedded in the game-theoretic model that has been developed in this paper.

## 7. Conclusion

In this paper, we develop a dynamic game-theoretic model of optimizing strategic behaviour by football teams. At any stage of the match, teams choose between defensive and attacking formations and between a non-violent and a violent style of play, so as to maximize their expected payoffs at the end of the match. Their strategic choices determine the probabilities of scoring and conceding goals at the current stage the match, and the probabilities of having a player dismissed.

In the theoretical model, optimal strategic behaviour at any stage of the match is found to be dependent on the current difference in scores, and on the amount of playing time that has elapsed. Teams that are trailing tend to attack, and teams that are trailing tend to play violently for as long as there is a realistic chance of salvaging the match. Stronger teams when trailing play violently for longer than weaker teams, and home teams when trailing play violently for longer than away teams. Towards the end of matches in which scores are level, the subgames have a prisoner's dilemma structure, and there is a tendency for the teams to defect from non-violence to violence. This feature of the model is consistent with an observed tendency for the number of dismissals to increase markedly during the closing stages of matches.

We subject the theoretical model to empirical scrutiny, using data on the timings of goals and player dismissals from a sample of more than 12,000 English professional league matches. We also use stochastic simulations to obtain probabilities for match results, conditional upon the current state of the match at any stage, defined by an underlying relative team quality measure, the difference in scores, and any numerical disparity in players.

Several features of the empirical hazard functions for the conditional arrival rates of player dismissals and goals are found to be consistent with the theoretical model. Teams that are currently trailing are willing to bear an increased probability of losing a player through dismissal, in order to increase the probability of scoring a goal quickly. In contrast, teams that are currently leading tend to play defensively and non-violently. The incentive for teams trailing by only one goal to bear additional risk increases at an increasing rate as the match approaches its conclusion.

Similar patterns are also evident in the empirical hazard functions for the arrival rates of goals. Scoring rates for home teams that are trailing by any margin, and for away teams that are trailing by one goal, are significantly higher than they are for teams that are leading or level in scores. The scoring rates of teams that are trailing by one goal increase at an increasing rate as the match approaches its conclusion. We interpret the main theoretical and empirical findings of the paper as novel, non-experimental evidence that the strategic behaviour of football teams can be rationalized in accordance with game-theoretic principles of optimizing strategic decision-making by agents when payoffs are uncertain and interdependent.

**Notes**

1. The 90-minute duration is notional, because there are frequent discontinuities in play for various reasons: play is interrupted when the ball travels outside the confines of the pitch, when a foul is committed or a player is penalized under the offside law, when a goal is scored, and when a player requires treatment on the pitch for injury. Although a few minutes of stoppage time are usually added at the end of each 45-minute period of play, the time added is invariably less than the time lost through routine stoppages.

2. The payoff structure, however, is other than zero-sum, for two reasons. First, if the scores at the end of the match are unequal, three league points are awarded to the winning team and none to the losing team. If the scores are level, one point is awarded to each team. In our data set, 27.4% of matches were drawn having finished level after 90 minutes' play (including stoppage time). In English league football, match durations are never extended beyond 90 minutes (plus stoppage time), and penalty shootouts are never used to settle drawn matches. Second, a player who commits serious foul play may be dismissed while the match is in progress. No replacement is permitted, and play resumes with the dismissed player's team at a numerical disadvantage in players. A dismissal also results in the player's suspension from up to three future matches; in this case, replacements are permitted. However, a suspension still imposes a cost upon the team concerned, which is obliged to select from a smaller or weaker pool of players. The corresponding gain accrues to the team's future opponents and not its current opponent.

3. In previous empirical studies of the incidence of foul play and disciplinary sanction in football, Garicano and Palacios-Huerta (2005) find that following a change in the league points scoring system in Spain's *La Liga* from 2-1-0 to 3-1-0, introduced in 1995, indicators of both attacking intensity and foul play increased overall. However, after taking the lead, teams tended to play more defensively in an attempt to prevent their opponents from equalizing. Dawson et al. (2007) find that the incidence of disciplinary sanction is inversely related to team quality (measured by *ex ante* win probabilities for the two teams) and is higher for away teams than for home teams. However,

the tendency for away teams to be penalized more frequently than home teams also appears to reflect a refereeing bias favouring the home team.

4. The data source is **www.4thegame.com**. Match reports posted on this website record the match durations (in minutes) at which player dismissals occurred and goals were scored. The data set was compiled by transcribing these details for each match individually by hand.

5. In Table 1 the data for the 45th and the 90th minutes are displayed separately from those for other durations. These cells include all player dismissals and goals scored during the 45th and 90th minutes, and during stoppage time (played immediately after the 45th and 90th minutes have elapsed). The data source does not record the amount of stoppage time played, which is variable but typically of between 2 and 5 minutes' duration at the end of each 45-minute period. Accordingly the rates of player dismissal and goal scoring recorded in these cells are several times the magnitudes of those in adjacent cells.

6. Garicano and Palacios-Huerta (2005) assume that an increase in foul play by one team results in a reduction in its opponent's probability of scoring. However, this assumption seems difficult to reconcile with Tables 1 and 2, which suggest that a team that is leading by one goal, whose main priority is to prevent its opponent from scoring, tends to play less violently (and not more so) than a team that is trailing by one goal.

7. Teams that attack are vulnerable to counterattack, and run an increased risk of being obliged to commit a last-ditch foul in order to prevent an opposing player from scoring. The punishment for a player who denies an opponent a goal scoring opportunity in this manner is dismissal.

8. Until the 1981/82 season the league points scoring scheme in the English league was 2-1-0 (win-draw-lose). Subsequently, the scheme was changed to 3-1-0, in an attempt to encourage attacking play.

9. This is a simplification because a player who commits a foul in his own team's penalty area, and by so doing denies the opposing team a goal scoring opportunity, is liable to be dismissed, while the opposing team has an opportunity to score from the resulting penalty kick. However, relatively small numbers of dismissals and goals arrive in this manner.

10. This model is estimated using league match results data from the ten football seasons immediately preceding the season in which the current match is played. *Ex ante* probabilities for the result of the current match are obtained by substituting into the fitted model covariate values that relate to the current match, all of which are calculated using data that are available prior to the start of the current match (predominantly based on the results of previous matches involving the home and away teams).

11. The probability of incurring a dismissal is particularly low when either team is leading by three goals or more, suggesting that in reality a third tactical selection may be available, which tends to be adopted only when the match result is already beyond reasonable doubt, and which involves a particularly low level of foul play.

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Table 1 Rates of player dismissal and goal scoring conditional on current duration

| Match duration, t (minutes) | Players dismissed per minute | | Goals scored per minute | |
|---|---|---|---|---|
| | Home team | Away team | Home team | Away team |
| $0 < t \leq 10$ | .00007 | .00028 | .0121 | .0089 |
| $10 < t \leq 20$ | .00039 | .00043 | .0136 | .0101 |
| $20 < t \leq 30$ | .00041 | .00061 | .0146 | .0103 |
| $30 < t \leq 40$ | .00049 | .00083 | .0145 | .0111 |
| $40 < t \leq 44$ | .00043 | .00111 | .0149 | .0105 |
| $44 < t \leq 45$ | .00360 | .00434 | .0558 | .0380 |
| $45 < t \leq 55$ | .00064 | .00088 | .0157 | .0122 |
| $55 < t \leq 65$ | .00102 | .00151 | .0162 | .0124 |
| $65 < t \leq 75$ | .00110 | .00195 | .0166 | .0126 |
| $75 < t \leq 85$ | .00130 | .00215 | .0172 | .0130 |
| $85 < t \leq 89$ | .00129 | .00250 | .0174 | .0127 |
| $89 < t \leq 90$ | .00950 | .01441 | .0842 | .0625 |

Note:
Based on data for all 12,216 matches played in the English professional leagues (Premiership, Championship, League One and League Two) suring seasons 2001/02 to 2006/07 (inclusive).

Table 2 Rates of player dismissal and goal scoring conditional on current difference in scores

| Difference in scores, s (home team goals – away team goals) | Players dismissed per minute | | Goals scored per minute | |
|---|---|---|---|---|
| | Home team | Away team | Home team | Away team |
| $s \leq -3$ | .00139 | .00052 | .0177 | .0163 |
| $s = -2$ | .00214 | .00135 | .0178 | .0136 |
| $s = -1$ | .00129 | .00132 | .0172 | .0122 |
| $s = 0$ | .00060 | .00096 | .0154 | .0115 |
| $s = 1$ | .00078 | .00192 | .0166 | .0130 |
| $s = 2$ | .00083 | .00207 | .0177 | .0134 |
| $s \geq 3$ | .00066 | .00214 | .0205 | .0130 |

Note:
Based on data for all 12,216 matches played in the English professional leagues (Premiership, Championship, League One and League Two) suring seasons 2001/02 to 2006/07 (inclusive).

Table 3 Rates of goal scoring conditional on numerical disparity in players

| | Goals scored per minute | |
|---|---|---|
| | Home team | Away team |
| Away team with disadvantage in player numbers | .0285 | .0090 |
| Neither team with disadvantage in player numbers | .0160 | .0121 |
| Home team with disadvantage in player numbers | .0119 | .0229 |

Note:
Based on data for all 12,216 matches played in the English professional leagues (Premiership, Championship, League One and League Two) suring seasons 2001/02 to 2006/07 (inclusive).

Table 4 Analysis of two-person non-cooperative subgame in the 75th minute

| | s = –1 | | | | s = 0 | | | | s = 1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | j=1 | j=2 | j=3 | j=4 | j=1 | j=2 | j=3 | j=4 | j=1 | j=2 | j=3 | j=4 |
| Home team payoffs | | | | | | | | | | | | |
| i=1 | .2599 | .2612 | .2626 | .2640 | 1.0122 | 1.0109 | 1.0122 | 1.0110 | **1.7478** | **1.7448** | **1.7452** | **1.7423** |
| i=2 | .2618 | .2631 | .2645 | .2659 | **1.0124** | **1.0111** | **1.0124** | **1.0112** | 1.7454 | 1.7425 | 1.7428 | 1.7399 |
| i=3 | .2624 | .2638 | .2651 | .2665 | 1.0120 | 1.0107 | 1.0120 | 1.0108 | 1.7450 | 1.7421 | 1.7424 | 1.7395 |
| i=4 | **.2641** | **.2654** | **.2668** | **.2682** | 1.0119 | 1.0106 | 1.0119 | 1.0107 | 1.7423 | 1.7394 | 1.7397 | 1.7368 |
| Away team payoffs | | | | | | | | | | | | |
| i=1 | **1.6981** | 1.6956 | 1.6954 | 1.6925 | .9336 | **.9337** | .9336 | .9335 | .2093 | .2111 | .2119 | **.2134** |
| i=2 | **1.6951** | 1.6926 | 1.6924 | 1.6895 | .9324 | **.9326** | .9324 | .9323 | .2107 | .2124 | .2132 | **.2147** |
| i=3 | **1.6955** | 1.6930 | 1.6928 | 1.6899 | .9338 | **.9340** | .9339 | .9337 | .2121 | .2138 | .2147 | **.2161** |
| i=4 | **1.6926** | 1.6900 | 1.6898 | 1.6870 | .9327 | **.9329** | .9328 | .9327 | .2135 | .2152 | .2161 | **.2176** |

Note:
s denotes (home team score *minus* away team score) at the end of the 74th minute.
It is assumed no players have been dismissed previously during the match ($d_h$=$d_a$=0).
Home team strategies are i=1 (defend, non-violent), i=2 (defend, violent), i=3 (attack, non-violent), i=4 (attack, violent).
Away team strategies are j=1,2,3,4, defined as for home team.
Table 4 shows expected payoffs for the home and away teams, denoted $h_{74}^{i,j}(s, d_h, d_a)$ and $a_{74}^{i,j}(s, d_h, d_a)$ in the text, respectively.
Payoffs for dominant strategies are shown in **bold** font. The solution to each two-person non-cooperative subgame is shaded grey.

Table 5 Outcomes of two-person non-cooperative subgame in the 25th, 50th and 75th minutes over various parameter values

| | | | | | | | 25th minute | | | | | | | 50th minute | | | | | | | 75th minute | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | s → | –3 | –2 | –1 | 0 | 1 | 2 | 3 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $\theta$ | $\phi$ | $\psi$ | $\varphi$ | $\beta$ | $\omega$ | $\lambda$ | | | | | | | | | | | | | | | | | | | | | |
| 1. Variation in $\theta$ (relative team quality) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| –2 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | v / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | a / - | va/ - | v / - | - /va | - / a | - / a |
| –1 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 1 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / v | - /va | - /va | - / a | a / - | va/ - | va/ - | - / a | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / - |
| 2 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | v / - | - / a | - /va | - / a | - / a | a / - | va/ - | va/ - | - / a | - /va | - / a | - / a | a / - | a / - | va/ - | - /va | - /va | - / a | - / - |
| 2. Variation in $\phi$ (effect of violent play on scoring probabilities) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0.5 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | a / - | a / - | - / a | - / a | - / a | - / a | a / - | a / - | a / - | - / a | - / a | - / a | - / a | a / - | a / - | va/ - | - / - | - /va | - / a | - / - |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.5 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | va/ - | va/ - | va/ - | v / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v / v | - /va | - /va | - / a |
| 3. Variation in $\psi$ (effect of attacking play on scoring probabilities) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1.0 | 0.5 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | v / - | - / v | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | - / - | a / - | va/ - | v / v | - /va | - / a | - / - |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.5 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / a | - /va | - / a | - / a | a / - | a / - | va/ - | v / a | - /va | - / a | - / a |
| 4. Variation in $\varphi$ (attack/defense comparative advantage) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1.0 | 1.0 | 1.33 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / - |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 0.67 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / a | - /va | - /va | - / a | a / - | va/ - | va/ - | - / a | - /va | - / a | - / a | - / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 5. Variation in $\beta$ (effect of violent play on probability of dismissal) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1.0 | 1.0 | 1.0 | 1 | 0.5 | 0.5 | va/ - | va/ - | va/ - | v / v | - /va | - /va | - /va | va/ - | va/ - | va/ - | v / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v / v | - /va | - /va | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 3 | 0.5 | 0.5 | a / - | a / - | a / - | - / a | - / a | - / a | - / a | a / - | a / - | a / - | - / - | - / a | - / a | - / a | a / - | a / - | va/ - | - / - | - /va | - / a | - / - |
| 6. Variation in $\omega$ (utility cost of dismissals, incurred in future matches) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.25 | 0.5 | va/ - | va/ - | va/ - | v / v | - /va | - /va | - /va | a / - | va/ - | va/ - | v / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v / v | - /va | - /va | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 1.0 | 0.5 | a / - | a / - | a / - | - / - | - / a | - / a | - / a | a / - | a / - | va/ - | - / - | - / a | - / a | - / a | - / - | a / - | va/ - | - / - | - /va | - / a | - / - |
| 7. Variation in $\lambda$ (risk-aversion or risk-neutrality) | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.25 | a / - | va/ - | a / - | - / - | - /va | - / a | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | - / - | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.5 | a / - | va/ - | va/ - | - / - | - /va | - /va | - / a | a / - | va/ - | va/ - | - / - | - /va | - / a | - / a | a / - | a / - | va/ - | v / v | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 0.75 | a / - | va/ - | va/ - | - / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v / a | - /va | - /va | - / a | a / - | a / - | va/ - | va/va | - /va | - / a | - / a |
| 0 | 1.0 | 1.0 | 1.0 | 2 | 0.5 | 1.0 | va/ - | va/ - | va/ - | v / v | - /va | - /va | - / a | a / - | va/ - | va/ - | v/va | - /va | - /va | - / a | a / - | a / - | va/ - | va/va | - /va | - / a | - / a |

<u>Note</u>:

s denotes (home team score *minus* away team score) at the end of the 24th, 49th or 74th minute.

It is assumed no players have been dismissed previously during the match ($d_h$=$d_a$=0).

Subgame outcomes are reported in the following format: ‘home team strategic choice’ / ‘away team strategic choice’. Strategic choices are reported as follows:
- denotes (non-violent, defend); v denotes (violent, defend); a denotes (non-violent, attack); va denotes (violent, attack).

Table 6 Estimation results: player dismissal hazard functions

| | Home team player dismissal hazard | | | Away team player dismissal hazard | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| DUR | $.015\times10^{-3}$ | $.014\times10^{-3}$ | $.015\times10^{-3}$ | $.023\times10^{-3}$ | $.022\times10^{-3}$ | $.013\times10^{-3}$ |
| | (17.09) | (13.75) | (4.32) | (24.16) | (17.83) | (3.42) |
| DUR^2 | - | - | $-.020\times10^{-6}$ | - | - | $.087\times10^{-6}$ |
| | | | (-0.49) | | | (1.82) |
| M45 | .00289 | .00285 | .00289 | .00320 | .00318 | .00331 |
| | (5.32) | (5.25) | (5.32) | (5.35) | (5.32) | (5.55) |
| M90 | .00813 | .00808 | .00789 | .01223 | .01222 | .01166 |
| | (9.20) | (9.15) | (8.89) | (11.34) | (11.23) | (10.68) |
| RELQUAL | $.043\times10^{-3}$ | $.166\times10^{-3}$ | $.184\times10^{-3}$ | .00175 | .00151 | .00156 |
| | (0.10) | (0.42) | (0.46) | (3.67) | (3.07) | (3.18) |
| UNCERT | .00231 | .00267 | .00272 | .00826 | .00760 | .00916 |
| | (1.38) | (1.64) | (1.67) | (4.20) | (3.88) | (4.61) |
| DIFF–3 | - | $.501\times10^{-3}$ | $.528\times10^{-3}$ | - | $-.780\times10^{-3}$ | $-.672\times10^{-3}$ |
| | | (1.40) | (1.47) | | (-4.65) | (-4.01) |
| DIFF–2 | - | $.985\times10^{-3}$ | .00101 | - | $-.090\times10^{-3}$ | $-.028\times10^{-6}$ |
| | | (4.37) | (4.47) | | (-0.54) | (-0.00) |
| DIFF–1 | - | $.273\times10^{-3}$ | $.721\times10^{-3}$ | - | $.103\times10^{-3}$ | $-.014\times10^{-3}$ |
| | | (3.26) | (2.01) | | (-1.22) | (-0.16) |
| DIFF+1 | - | $-.019\times10^{-3}$ | $-.015\times10^{-3}$ | - | $.249\times10^{-3}$ | $.217\times10^{-3}$ |
| | | (-0.33) | (-0.25) | | (2.51) | (0.80) |
| DIFF+2 | - | $-.149\times10^{-3}$ | $-.137\times10^{-3}$ | - | $.210\times10^{-3}$ | $.326\times10^{-3}$ |
| | | (-1.53) | (-1.37) | | (1.26) | (1.97) |
| DIFF+3 | - | $-.300\times10^{-3}$ | $-.288\times10^{-3}$ | - | $.374\times10^{-3}$ | $.487\times10^{-3}$ |
| | | (-2.22) | (-2.07) | | (1.37) | (1.78) |
| DIFF–1×DUR | - | - | $-.032\times10^{-3}$ | - | - | - |
| | | | (-1.83) | | | |
| DIFF–1×DUR^2 | - | - | $.040\times10^{-6}$ | - | - | - |
| | | | (2.16) | | | |
| DIFF+1×DUR | - | - | - | - | - | $-.024\times10^{-3}$ |
| | | | | | | (-1.67) |
| DIFF+1×DUR^2 | - | - | - | - | - | $.482\times10^{-6}$ |
| | | | | | | (3.02) |

Note:
z-statistics for the significance of the estimated coefficients are reported in parentheses.

Table 7 Estimation results: goal scoring hazard functions

| | Home team goal hazard | | | Away team goal hazard | | |
|---|---|---|---|---|---|---|
| | [7] | [8] | [9] | [10] | [11] | [12] |
| DUR | $.049\times10^{-3}$ | $.043\times10^{-3}$ | $.083\times10^{-3}$ | $.048\times10^{-3}$ | $.044\times10^{-3}$ | $.080\times10^{-3}$ |
| | (10.74) | (8.60) | (4.57) | (12.05) | (9.91) | (4.84) |
| DUR^2 | - | - | $-.453\times10^{-6}$ | - | - | $-.428\times10^{-6}$ |
| | | | (-2.23) | | | (-2.32) |
| KICKOFF | -.00725 | -.00738 | -.00729 | -.00373 | -.00384 | -.00373 |
| | (-19.31) | (-19.60) | (-19.30) | (-10.04) | (-10.33) | (-10.00) |
| M45 | .04028 | .04022 | .04003 | .02648 | .02643 | .02627 |
| | (18.84) | (18.81) | (18.69) | (15.02) | (14.99) | (14.87) |
| M90 | .06583 | .06587 | .06626 | .04849 | .04853 | .04881 |
| | (25.00) | (25.01) | (25.02) | (21.37) | (21.39) | (21.39) |
| EXPHG | .00644 | .00653 | .00652 | - | - | - |
| | (15.34) | (15.49) | (15.55) | | | |
| EXPAG | - | - | - | .00598 | .00609 | .00608 |
| | | | | (14.02) | (14.19) | (14.17) |
| HOFF | -.00486 | -.00518 | -.00518 | .00897 | .00900 | .00904 |
| | (-6.94) | (-7.30) | (-7.19) | (8.48) | (8.51) | (8.54) |
| AOFF | .00998 | .01004 | .01010 | -.00415 | -.00428 | -.00424 |
| | (11.06) | (11.09) | (11.15) | (-8.24) | (-8.44) | (-8.35) |
| DIFF–3 | - | .00238 | .00237 | - | .00126 | .00132 |
| | | (2.00) | (1.97) | | (1.07) | (1.13) |
| DIFF–2 | - | .00136 | .00134 | - | $.088\times10^{-3}$ | $.054\times10^{-3}$ |
| | | (2.04) | (2.01) | | (0.15) | (0.09) |
| DIFF–1 | - | .00114 | .00316 | - | $-.274\times10^{-3}$ | $-.350\times10^{-3}$ |
| | | (3.25) | (2.37) | | (-0.91) | (-1.14) |
| DIFF+1 | - | $.278\times10^{-3}$ | $.146\times10^{-3}$ | - | .00102 | .00262 |
| | | (0.88) | (0.46) | | (3.64) | (2.42) |
| DIFF+2 | - | $.045\times10^{-3}$ | $-.025\times10^{-3}$ | - | $.707\times10^{-3}$ | $.684\times10^{-3}$ |
| | | (0.09) | (-0.05) | | (1.62) | (1.54) |
| DIFF+3 | - | .00103 | .00106 | - | $.771\times10^{-3}$ | $.850\times10^{-3}$ |
| | | (1.23) | (1.27) | | (1.17) | (1.27) |
| DIFF–1×DUR | - | - | $-.110\times10^{-3}$ | - | - | - |
| | | | (-1.79) | | | |
| DIFF–1×DUR^2 | - | - | $.001\times10^{-3}$ | - | - | - |
| | | | (1.78) | | | |
| DIFF+1×DUR | - | - | - | - | - | $-.099\times10^{-3}$ |
| | | | | | | (-1.99) |
| DIFF+1×DUR^2 | - | - | - | - | - | $.001\times10^{-3}$ |
| | | | | | | (2.15) |

Note:
z-statistics for the significance of the estimated coefficients are reported in parentheses.

Table 8 Weighted sum of home win and draw probabilities, conditional on relative team strengths and the state of the match at various durations

| | x → | +1 | | | | | 0 | | | | | –1 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $h_0$ ↓ | t ↓ s → | –2 | –1 | 0 | 1 | 2 | –2 | –1 | 0 | 1 | 2 | –2 | –1 | 0 | 1 | 2 |
| .483 | 15 | .036 | .105 | .272 | .506 | .718 | .095 | .243 | .488 | .727 | .892 | .260 | .467 | .709 | .873 | .962 |
| | 30 | .031 | .109 | .304 | .552 | .786 | .084 | .233 | .482 | .750 | .909 | .210 | .426 | .680 | .875 | .962 |
| | 45 | .028 | .112 | .336 | .622 | .845 | .064 | .207 | .489 | .783 | .934 | .147 | .355 | .646 | .873 | .968 |
| | 60 | .016 | .092 | .362 | .702 | .908 | .034 | .159 | .487 | .820 | .961 | .085 | .277 | .619 | .892 | .981 |
| | 75 | .009 | .076 | .420 | .822 | .970 | .013 | .111 | .496 | .882 | .984 | .033 | .179 | .570 | .920 | .991 |
| .550 | 15 | .050 | .143 | .321 | .552 | .767 | .130 | .299 | .544 | .768 | .912 | .295 | .522 | .741 | .901 | .969 |
| | 30 | .046 | .137 | .343 | .612 | .809 | .103 | .272 | .541 | .789 | .934 | .237 | .470 | .720 | .895 | .975 |
| | 45 | .038 | .140 | .369 | .664 | .870 | .079 | .242 | .535 | .810 | .947 | .171 | .403 | .686 | .896 | .977 |
| | 60 | .018 | .109 | .397 | .735 | .925 | .040 | .182 | .516 | .846 | .971 | .101 | .304 | .646 | .910 | .987 |
| | 75 | .007 | .086 | .437 | .835 | .970 | .016 | .122 | .518 | .892 | .987 | .038 | .189 | .592 | .930 | .995 |
| .593 | 15 | .059 | .163 | .360 | .587 | .787 | .148 | .332 | .572 | .801 | .926 | .328 | .555 | .768 | .915 | .977 |
| | 30 | .055 | .161 | .373 | .638 | .837 | .121 | .303 | .568 | .815 | .937 | .271 | .498 | .747 | .915 | .978 |
| | 45 | .048 | .151 | .398 | .692 | .881 | .089 | .260 | .554 | .832 | .952 | .184 | .420 | .707 | .906 | .979 |
| | 60 | .022 | .128 | .421 | .748 | .929 | .049 | .204 | .547 | .863 | .974 | .106 | .324 | .671 | .919 | .989 |
| | 75 | .010 | .089 | .452 | .844 | .975 | .018 | .132 | .531 | .904 | .989 | .042 | .199 | .609 | .935 | .995 |
| .630 | 15 | .076 | .192 | .387 | .621 | .811 | .176 | .368 | .624 | .826 | .941 | .358 | .594 | .808 | .929 | .984 |
| | 30 | .061 | .183 | .406 | .663 | .849 | .140 | .333 | .601 | .828 | .947 | .287 | .528 | .763 | .921 | .984 |
| | 45 | .052 | .170 | .422 | .708 | .891 | .102 | .282 | .584 | .842 | .959 | .210 | .445 | .729 | .917 | .984 |
| | 60 | .027 | .135 | .438 | .766 | .933 | .055 | .217 | .566 | .876 | .978 | .118 | .339 | .681 | .927 | .991 |
| | 75 | .011 | .098 | .453 | .850 | .977 | .022 | .142 | .542 | .913 | .991 | .043 | .207 | .619 | .940 | .995 |
| .694 | 15 | .096 | .225 | .444 | .669 | .843 | .220 | .426 | .674 | .861 | .958 | .418 | .660 | .838 | .951 | .989 |
| | 30 | .080 | .218 | .442 | .700 | .875 | .168 | .378 | .648 | .865 | .963 | .331 | .573 | .805 | .945 | .986 |
| | 45 | .067 | .194 | .454 | .735 | .909 | .125 | .324 | .618 | .864 | .969 | .232 | .483 | .762 | .935 | .987 |
| | 60 | .035 | .159 | .458 | .792 | .943 | .068 | .235 | .599 | .890 | .981 | .136 | .369 | .707 | .940 | .994 |
| | 75 | .013 | .110 | .480 | .865 | .979 | .024 | .154 | .557 | .919 | .992 | .047 | .224 | .634 | .948 | .997 |

Notes:

Table 8 shows the value function $\tilde{h}_t(s, x; h_0)$ enumerated at various values of t, s, x and $h_0$ (values shaded grey are those referred to in the text).

$\tilde{h}_t(s, x; h_0) = \pi_H + 0.5\pi_D$ is the weighted sum of the probabilities of a home win (weight=1) and draw (weight=0.5) at match duration t.

$h_0$ is the weighted sum of probabilities at the start of the match, used as a measure of the relative strengths of the home and away teams.

$x = d_h – d_a$ is the number of home team players dismissed *minus* the number of away team players dismissed at duration t.

s is the difference between the home team score and away team score at duration t.