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Prices vs. Quantities in Cartel Theory with Special Reference to OPEC

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1. Introduction

Because of negotiating costs, it is economical for a cartel to fix its control variables over a period of time without updating them to fully exploit changes in market conditions. Prices and quantities cannot be simultaneously fixed if there are changes in demand. However, prices can be supported by varying production or, alternatively, quantities can be fixed with prices allowed to change to clear the market. The question arises as to whether the cartel should prefer one control strategy to another.

The choice between price-setting and quantity-setting has been studied previously by Weitzman (1974) for the case of a regulated industry and by Young (1979) for the case of a nation choosing between tariffs and quotas as a means of restricting international trade. The problem we envision is one in which a cartel chooses to fix either its prices or quantities through periods of changing demand rather than incur the high costs of internal negotiations which would arise if operating strategies were always updated to reflect current market conditions. Because plans must be formulated before the realization of demand, the problem is one of decision-making under uncertainty. An optimal control strategy is one which provides large expected aggregate profits and which also enhances cartel stability by promoting a stable internal distribution of aggregate profits.

We begin in Section 2 with a model of a cartel that offers a homogeneous product. Such a model has little to say about whether a particular control strategy promotes internal stability. But it is a useful model for determining which of the two strategies, price-setting or quantity-setting, offers greater expected aggregate profits. To keep things simple, we use a single-period model and ignore other cartel problems such as output sharing and cheating.[1/]

Matters are more complicated when the cartel offers more than one economically distinct product. Consider the case of OPEC which supplies crudes of different viscosity. Because members are endowed with different proportions of light and heavy crude, control strategies can differ in the evenness with which changes in market conditions affect member countries. A ranking of control strategies should then involve not only a comparison of the distributions of aggregate profits, but also of the extent to which members' fortunes tend to rise and fall together. Other things being equal, an optimal control strategy should lend stability to the cartel by providing positive covariance between members' profits and low variance in their shares of aggregate profits.

In Section 3 we develop a simple multi-product model to compare the stability properties of various control strategies. The model is structured to depict the decision problem faced by OPEC. There are two producing groups within the cartel--one endowed with heavy crude and the other with light crude. Heavy crude is more difficult than light crude to refine into final products, and the unit cost difference is assumed to rise with the amount of heavy purchased. Randomness appears through the demand for final products.

To abstract from points covered in Section 2, sufficient algebraic structure is imposed on the multi-product model to ensure that maximum expected profits are the same regardless of the control strategy employed. This allows us to easily compare strategies in more subtle dimensions. Section 4 uses the model to compare the effects of price-setting and quantity-setting on the covariance of members' profits and the variance of their shares of aggregate profits. Section 5 uses the same criteria to evaluate the performance of mixed strategies, which involve setting both a price and a quantity.

In Section 6 we draw upon recent events in the world oil market to illustrate some of the theoretical principles developed in Section 4. Evidence is presented to show how, because of OPEC's reliance on price controls, an unexpectedly weak demand for oil products in the 1980s led to disproportionate reductions in production among members and how this, in turn, precipitated a number of unilateral and destabilizing actions within the cartel. The main points of the paper are summarized in Section 7.

2. Comparisons involving expected aggregate profits

The most basic way in which price-setting and quantity-setting could differ is in their capacities to generate aggregate profits. It is natural then to begin by comparing the maximum expected profits available under the two strategies. The simplest model with which to make this comparison is a single-period, single-product model of a joint profit-maximizing cartel. The cartel is assumed to face a demand curve with additive randomness. For the moment, we also ignore any costs of production. The decision problem of the cartel is then to

$$(1) \qquad \text{Max} \quad E(\pi) = E(PQ)$$
$$\text{s.t.} \quad Q = \alpha - f(P)$$

where α is a random variable, P is product price, and Q is quantity supplied. Under a price-setting strategy, price is determined before demand is realized, and quantity is adjusted _ex post_ to clear the market. Under quantity-setting, it is quantity that is predetermined and price that adjusts to actual market conditions.[2/]

Following Weitzman, we assume that the randomness in demand is sufficiently small as to justify a second-order approximation to the quantity-setting profit function. It is shown in the appendix that the optimal quantity under quantity-setting (Q^*) then equals the expected quantity under optimal price-setting ($\overline{Q}$). Further, the difference between maximal expected profits in the two cases can be written as

$$(2) \qquad E(\pi)\big|_{P^*} - E(\pi)\big|_{Q^*} \approx \frac{\tfrac{1}{2}\overline{Q}f''(P^*)\sigma^2}{[f'(P^*)]^3}$$

where σ^2 is the variance of α. If demand is nonrandom, there is no difference between the expected profits of the two strategies. With randomness in demand, however, the comparison hinges upon the nature of the curvature in the demand function. Specifically, the maximum expected profits available through price-setting will exceed (fall short of) the maximum expected profits available through quantity-setting if demand is concave (convex) in price.

These results must be qualified if there are nonlinearities in the cartel's cost function. This is easy to see because costs are assumed to be free of stochastic elements. From Jenson's inequality and the (approximate) equality between $\overline{Q}$ and Q^*, we have

$$(3) \qquad E[C(Q)]|_{P*} \leq (\geq) \; C(Q^*) \quad \text{as } C \text{ is concave (convex) in } Q$$

where C denotes the cartel's total costs. Thus, optimal expected costs will be smaller (larger) under price-setting than under quantity-setting when costs are concave (convex) in output. Accounting for both revenues and costs, price-setting unambiguously provides larger expected profits if demand is concave in price and costs are concave in output. If both the demand and cost functions are convex in their arguments, maximal expected profits will be larger under quantity-setting. Comparisons in other cases involve the relative degrees of curvature in the two functions.

3. A simple multi-product model

If cartel members produce different products, the internal distribution of aggregate profits may be more sensitive to fluctuations in demand under some control strategies than others. If so, a control strategy should be evaluated not only on the basis of its expected aggregate profits, but also on its ability to reduce the variability in the internal distribution of profits.

Our analysis of this issue will be specialized in two respects. First, the structure of demand is fashioned after the world oil market. While some generality is inevitably lost, we are able to more specifically identify those control strategies that would be relatively successful in insulating OPEC from large swings in its internal distribution of profits. Second, an algebraic structure is imposed on the model to ensure that expected aggregate profits are invariant to the control strategy selected. This allows us to abstract from points already covered in Section 2, and it results in simple expressions for higher-order moments of the relevant profit distributions.

An oil-producing cartel is assumed to consist of two groups--one endowed with heavy crude and another endowed with light crude.[3/] Because of the short time horizon implicit throughout the paper (a horizon equal to the time between cartel negotiations), we ignore all complications arising from resource exhaustion and the finiteness of reserves. The model is once again a single-period model. We shall also assume that there are no external sources of oil.[4/] The market demands for heavy and light crude are then the same as those faced by the cartel.

Crude oil demands are derived from a demand function for final products and from representations of refinery costs. The demand for final products is taken to be

(4) $Q = \alpha - bP.$

The term α is a random variable, and it is the only source of randomness in the model. Linearity is assumed in (4) in order to eliminate the effect of strategy choice on expected aggregate profits.

Our description of refinery costs is intended to capture two characteristics of refining technology. First, heavy crude is more difficult than light crude to refine into final products. Second, while refineries designed for processing heavy crudes can also accommodate light crudes, light refineries are incapable of processing heavy crudes. Refinery capacity is then more quickly approached for heavy crudes than for light crudes. To model these characteristics in as simple terms as possible, suppose that each unit of final product can be refined from either one unit of heavy crude or one unit of light crude. We will abstract from any possible losses of material through processing. Also suppose that the costs of refining crude oil into final products can be represented by

(5) $$C_L = P_L L$$

(6) $$C_H = P_H H + \tfrac{1}{2} c H^2$$

where P_L and P_H denote the prices, and L and H the quantities, of light and heavy crude, respectively. Thus, for equal prices of heavy and light, the marginal (and average) cost of processing heavy crude exceeds the marginal (average) cost of processing light. The difference in costs rises with the amount of heavy refined, as heavy refineries approach capacity.

Oil refining is assumed to be competitive. The market demands for heavy and light crude are derived by first using (5) and (6) to obtain refinery demands, and then using (4) to integrate the market for final products. The resulting demand functions are given by

(7) $H = (P_L - P_H)/c$

(8) $L = \alpha - bP_L - (P_L - P_H)/c.$

Heavy and light crude are substitutes, of course. Also note that the randomness from the demand for final products appears only in the demand for light crude.[5/]

It is well known that optimal depletion is sequential, with the better quality deposits mined first.[6/] This result is also a property of our model. An unconstrained maximization of expected aggregate profits requires that heavy crude be entirely withheld from the market. Side payments should be made to provide heavy producers with incentives to remain inactive. Such arrangements are not practiced, of course, neither in OPEC nor in other cartels. Thus, to ensure that some heavy production is undertaken, while at the same time avoiding the question of how the sharing agreements are made, we assume that the selection of control variables is constrained by

(9) $E(H) = \theta E(L).$

Eq. (9) requires that expected outputs lie in a predetermined proportion to one another. Through a suitable choice of θ, it is also possible to achieve any desired ratio of expected profits.

As a final simplification, we shall assume that there are no costs of production.[7/] The decision-problem of the cartel can then be written as

$$\text{(10)} \qquad \text{Max} \quad E(\pi) = E(P_H H + P_L L)$$
$$\text{s.t. } (7)\text{-}(9).$$

Under a price-setting strategy, the prices of light and heavy crude are chosen to maximize expected aggregate profits. Outputs are adjusted when demands are realized so as to support these prices. Under a quantity-setting strategy, it is the quantities of light and heavy that are determined <u>ex ante</u>, with prices allowed to adjust to actual market conditions.

Analytic solutions for the control variables are provided in Table 1. Because of the simple algebraic structure used in (10), the optimal expected values of these variables are independent of the particular strategy employed. For example, the optimal price of light crude under a price-setting strategy is the same as the expected price of light under the optimal quantity-setting strategy. This implies that maximum expected profits are the same regardless of the strategy used. The strategies differ, however, in higher-order moments of members' profit distributions. It is to these comparisons that we now turn.

4. Comparisons involving variation in the internal distribution of profits

Issues of stability in cartel theory are generally of a long-term nature, with emphasis on equitable arrangements for reducing output and effective means of enforcing price agreements. For OPEC in particular, stability is often judged by the willingness of nations with the largest reserves and lowest immediate revenue needs to make the bulk of the production cuts that are necessary if monopoly power is to be exercised.[8/] In contrast, the concept of stability explored here deals with the extent to which fluctuations in demand are absorbed evenly by cartel members. The more equitable the arrangements for sharing risk, the more stable the cartel.

The basic differences between price-setting and quantity-setting can be seen in Figure 1. The line labelled $\overline{DD}$ represents the mean demand for final products. The optimal expected price of final products is assumed to be P^*_L, which is the same as the expected price of light crude given the simple way in which we have described the light oil refinery process. The expected quantity of final products is (H^*+L^*), the sum of the mean quantities of heavy and light crude. The breakdown between expected quantities is determined by the parameter θ. The optimal expected price of heavy crude equates the marginal cost of heavy refining with the marginal cost of light refining. This requires that the expected price of heavy be P^*_H.

Figure 1 can now be used to show the effects of a deviation in the demand for final products. Suppose that the actual demand for final products is D'D' rather than $\overline{DD}$. Under a price-setting strategy, the prices of light and heavy remain fixed at P_L^* and P_H^*. Marginal costs must be equalized for refiners to be in equilibrium. This implies that the quantity of heavy actually sold must be H*, the same as its expected value. The quantity of light sold, however, will be (Q'-H*). All of the unanticipated increase in final demand is produced with light crude. This occurs because of our assumption that light refining is a constant cost operation, while heavy refining is subject to increasing costs. A similar, though less extreme, result would emerge if all refining exhibited increasing costs, but capacity constraints were more rapidly approached for heavy crude than for light crude.

On the other hand, suppose that the cartel had adopted a strategy of quantity-setting. The quantities of heavy and light would be invariant to deviations in final demand. However, with actual demand at D'D', the price of final products, and hence the price of light crude, would equal P'. And since the quantity of heavy crude is fixed at H*, there could be no variation in the price differential. Thus the price of heavy would also deviate from its mean by the amount $(P'-P_L^*)$. With a quantity-setting strategy, unanticipated changes in demand have the same qualitative impact on the profits of all cartel members.

To more formally compare the two strategies, Table 2 provides expressions for the second-order moments of the individual and aggregate profit distributions. Our particular interest is in the evenness with which random fluctuations in demand affect member's profits. There are at least two ways of approaching this question. One could simply compare the covariances of the profits of the two groups.[9/] According to this measure, quantity-setting would clearly be preferred to price-setting. When quantities are fixed, prices and hence profits correlate positively. Under price-setting, the profits of heavy producers are nonrandom, while the profits of light producers rise or fall with demand.

Alternatively, the stability of each strategy might be measured by the variances of members' shares of aggregate profits. Since there are only two groups in the model, this would be equivalent to simply comparing the variances of the ratio of group profits, $Var(\pi_L/\pi_H)$. The variance of a ratio of random variables will tend to be smaller the smaller are the individual variances and the greater is the covariance between the variables. As can be seen from Table 2, the quantity-setting strategy offers a smaller variance in π_L and a greater covariance. Each tends to reduce the variance of π_L/π_H. However, the variance of π_H is clearly lower under price-setting, since the profits of heavy producers are independent of random fluctuations in demand. This effect can dominate if the expected profits of the heavy-producing group are sufficiently small. Thus it is not generally possible to rank the variances of π_L/π_H.

Something conclusive can be said, however, if the output-sharing parameter θ is selected so as to equate the expected profits of the two groups. In this case, the variance of π_L/π_H can be approximated by[10/]

$$(11) \qquad Var(\pi_L/\pi_H) \approx (\bar{\pi}_L)^{-2}\{Var(\pi) - 4Cov(\pi_H,\pi_L)\}.$$

Which strategy yields a smaller variance of profit shares, or equivalently a smaller variance of π_L/π_H, depends only upon the variance of aggregate profits and the covariance of group profits. The quantity-setting strategy provides a smaller variance of aggregate profits. Therefore, it offers a smaller variance of profit shares. This can be demonstrated with the formulas in Table 2. Or return to Figure 1 and consider the effects of a deviation in demand on aggregate profits. Under price-setting, profits increase by the area "abcd". Under quantity-setting, profits increase by the area "aefg". A ranking of the two areas depends upon the elasticity of demand for final products. It is easy to show that, in a neighborhood of the mean equilibrium, the elasticity of final demand is greater than one. Thus, the deviation in final demand produces a greater disturbance in aggregate profits under price-setting than under quantity-setting.

In summary, control strategies can differ in the evenness with which fluctuations in demand affect members' profits. In our model of an oil-producing cartel, a greater covariance of group profits is achieved when quantities are fixed than when prices are fixed. Which of the two strategies yields a smaller variance of profit shares depends upon agreements concerning the internal distribution of expected aggregate profits. However, if the expected profits of producer groups are not widely disparate, a quantity-setting strategy also yields a smaller variance of profit shares.

5. Mixed strategies

A mixed strategy is one whose control vector contains both a price and a quantity. There are four mixed strategies in our two-product model. But because there is no randomness in the demand for heavy crude, two of these are redundant. Both strategies that involve fixing the quantity of heavy crude are equivalent to a pure price-setting strategy. The strategy involving the price and quantity of light crude is also uninteresting since all of the randomness in final demand would be borne by the heavy producers. Our analysis will then be confined to the mixed strategy whose control variables are the price of heavy and the quantity of light.

The values of P_H and L that maximize expected aggregate profits are the same as those in Table 1. Maximal expected profits under this strategy are then identical to those under the pure price and quantity strategies. The higher-order moments of the profit distributions are different, however. Our interest is in the stability properties of the strategy, as measured by the covariance of group profits and the variance of profit shares.

Under the mixed strategy, the profits of the two producing groups will exhibit positive covariance. This is clear from eqs. (7)-(8). With the price of heavy and the quantity of light fixed, a positive deviation in final demand raises the price of light. This, in turn, increases the quantity of heavy. The revenues of both groups rise. As measured by the covariance of group profits, short-term stability is more effectively promoted by both the mixed strategy and the pure quantity-setting strategy than by the pure price-setting strategy.

A ranking of the covariances of the mixed and quantity strategies is less straightforward. The formal condition can be derived from Table 2 as

$$(12) \qquad \mathrm{Cov}(\pi_H,\pi_L)|_{H^*,L^*} \geq \mathrm{Cov}(\pi_H,\pi_L)|_{P_H^*,L^*} \text{ iff.}$$

$$\theta(1+\theta) + bc\{\theta^2-1+2\theta bc\} \geq 0.$$

Which of the strategies yields a greater covariance depends, among other things, upon the output-sharing parameter θ. If heavy producers receive at least one-half of expected output ($\theta \geq 1$), then the covariance under quantity-setting will exceed the covariance under the mixed strategy. The ranking is reversed, however, for sufficiently small values of θ.

In order to compare strategies in terms of their implied variances of profit shares, we must once again restrict ourselves to cases where the expected profits of the cartel are equally distributed between producer groups. As may be recalled from eq.(11), the comparisons then involve only the variances of aggregate profits and the covariances of group profits. It is easy to show from Table 2 that the variance of aggregate profits is smaller under the mixed strategy than under the pure quantity strategy. This gives us the following ranking:

$$(13) \qquad \mathrm{Var}(\pi)|_{P_H^*,L^*} < \mathrm{Var}(\pi)|_{H^*,L^*} < \mathrm{Var}(\pi)|_{P_H^*,P_L^*}.$$

By providing both a greater covariance between members' profits and a smaller variance in aggregate profits, each of the mixed and quantity strategies yields a variance of profit shares that is smaller than the variance of shares implicit in a pure price strategy.

A ranking of the variances of profit shares between the mixed and quantity strategies is less clear. The mixed strategy offers a smaller variance of aggregate profits. But when θ is greater than one, as is assumed in the derivation of eq.(11), the quantity strategy provides a greater covariance of group profits.

6. OPEC's recent experience with price controls

Since the oil embargo of 1973-74, OPEC's operating strategy has been generally one of pegging prices for different types of crude and supporting these prices with the necessary adjustments in production. Most of its oil has been purchased on long-term contracts at official prices. Differences in grades of oil and distances to markets have been accounted for by a schedule of official price differentials which are adjusted periodically in response to fundamental changes in market conditions. Up until 1982, OPEC's experience with price controls was a tranquil one. The period was characterized by increasing demand and, although the extra revenues of the cartel were not always shared evenly, no member suffered a decline in revenues that was not a consequence of reduced productive capacity. There was little conflict within the organization.

The destabilizing feature of a price-setting strategy became evident after the major price increases of 1979-1980. These price increases resulted in an unexpectedly strong reduction in the demand for OPEC oil, as external sources of oil were developed and as the world began to substitute away from oil and and oil-related products. Substantial cuts in production were necessary if prices were to remain at official levels. Moreover, with rigid price differentials, the required production cuts were unevenly distributed across producing groups. This was particularly evident during the period immediately preceeding the price reductions in March 1983. As shown in Table 3, producers of lighter oils (those with a weight greater than 31 degrees API) made significant cuts in their production from June 1982 to February 1983. The only exceptions were Iran and Iraq, whose behavior was distorted by the war. Producers of heavy crudes, on the other hand, actually raised their production levels during the period.

These changes in the distribution of revenues, both within and outside of OPEC, put destabilizing pressure on the cartel. In response to slumping sales, the U.K., a light oil producer, unilaterally reduced its official prices.[11/] Nigeria, a direct competitor with the U.K., followed with price cuts of its own. These actions resulted in an emergency agreement to cut the marker price (Arab Light) by $5/barrel and to realign the price differentials. The agreement was reached after threats of a complete collapse of the organization.

More recently, OPEC has begun to move toward greater use of quantity controls. There is an increasing acceptance of sales on the spot market by members. Before 1982, nearly all of the oil sold was through long-term contracts at official prices. At the present time, however, nearly half is sold on a spot market basis, making the official price structure increasingly irrelevant. The organization has also hired an accounting firm to provide a detailed monitoring of production to make information on cheating by individual members more readily available. Although the accounting firm is eventually to monitor prices, its current emphasis is on production.

7. Conclusions

In order to reduce the costs of internal negotiations, a cartel may choose to fix either its prices or quantities through periods of changing demand. This paper has explored the ways in which the distributions of members' profits are affected by the choice between variables that are to be predetermined and variables that are to adjust to actual market conditions. We have specifically focused on the effect of the strategy choice on the expected aggregate profits of the cartel and the variation in its internal distribution of profits, as measured by the covariance of members' profits and the variance of profit shares.

Differences in maximum expected profits can arise if there are nonlinearities in either market demand or in the cost function of the cartel. A strategy of setting prices will produce larger expected profits if demand is concave in price and costs are concave in output. When demand and costs are convex in their arguments, however, larger expected profits can be obtained by setting quantities.

If cartel members produce different products, control strategies may also differ in the evenness with which fluctuations in demand affect members' profits. This would be particularly true for OPEC, since members are endowed with different proportions of heavy and light crude. In a simple model designed to capture the salient aspects of the world oil market, quantity-setting was shown to yield a larger covariance of members' profits than price-setting. Which of the two strategies provides a smaller variance of profit shares depends upon agreements concerning the distribution of expected aggregate profits between members. However, if the expected profits of producer groups are not widely disparate, the quantity-setting strategy also yields a smaller variance of profit shares.

As demonstrated by the events leading up to the March 1983 price cut, OPEC's choice of a price-setting strategy resulted in light oil producers having to bear the bulk of the production cuts necessitated by a weakness in oil demand which developed during the early 1980s. This, in turn, threatened the stability of the cartel. It would be erroneous to suggest that, by setting quantities rather than prices, all problems could have been avoided. Profits would have fallen in any case. However, with profit shares more stable, it might have been easier to negotiate an agreement on prices and quantities that were more consistent with the longer-term structure of oil demand.

Footnotes

1. See Osborne (1976) for a useful discussion of these problems.

2. This differs from the method of analysis used by Weitzman. In the Weitzman model, it is in effect marginal cost rather than price that is set and defended through variations in production. When a value for marginal cost is announced, output is selected after costs are realized so as to be consistent with that level of marginal cost. Price is then determined after demand is realized as the highest price which can be charged for that output. If costs are nonrandom, this kind of control strategy is equivalent to quantity-setting.

3. Some OPEC countries produce more than one grade of oil. Saudi Arabia, for example, produces large amounts of both heavy and light oil. For most members, however, it is not possible to make major substitutions between grades.

4. There is little loss of generality in making this assumption. See n.5.

5. Had we recognized alternate sources of oil, the price coefficients in (7) and (8) would embody the production responses of external suppliers, as well as characteristics of refining technology and the price responsiveness of final demand. However, the randomness in final demand would continue to express itself only through the demand for light crude, and this is what is crucial for our results.

6. For example, see Herfindahl (1967) and Dasgupta and Heal (1979, pp. 172-175).

7. The principal influence of production costs on the choice between control strategies is as in Section 2. Convexity (concavity) in the cartel's cost function tends to favor quantity-setting (price-setting).

8. For example, see Danielson (1980) and Mohammad (1984).

9. Since expected profits are the same, it would not be necessary to standardize the covariances before comparing them across control strategies.

10. For a derivation of eq.(11), see Mood, Graybill, and Boes (1974, p.181).

11. The U.K. is not a member of OPEC, but it tends to set official prices in line with OPEC.

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Appendix: Derivation of Eq.(2)

To derive eq.(2), first consider a second-order Taylor approximation of the quantity-setting profit function, $\pi = Qf^{-1}(\alpha - Q)$. Expand about the point $(\bar{\alpha}, \bar{Q})$, where $\bar{\alpha}$ is the mean of the random variable α and $\bar{Q}$ is the expected quantity under the optimal price-setting strategy. Now take the expected value of the approximated function. All terms involving $(\alpha - \bar{\alpha})$ will vanish. The resulting expression can be written as

$$\text{(A1)} \qquad E(\pi) \simeq \bar{Q}f^{-1}(\bar{\alpha} - \bar{Q}) + \frac{\partial \pi}{\partial Q}(Q - \bar{Q}) + \frac{1}{2}\frac{\partial^2 \pi}{\partial \alpha^2}\sigma^2 + \frac{1}{2}\frac{\partial^2 \pi}{\partial Q^2}(Q - \bar{Q})^2.$$

A number of simplifications are now possible. The first term on the r.h.s. of (A1) is the maximal value of expected profits under price-setting. By using the first-order condition to the price-setting problem, it is also easy to show that the first-order partial derivative in (A1) vanishes when evaluated at $(\bar{\alpha}, \bar{Q})$. Finally, it can be shown from the second-order condition to the price-setting problem that the coefficient on $(Q - \bar{Q})^2$ must be negative. It then follows that, to a second-order approximation, the optimal quantity-setting strategy is to set quantity equal to its expected value under the optimal price-setting strategy. Eq.(2) in the text follows from this result.

Table 1: Optimal Prices and Quantities

$P_H^* = D^{-1}(\overline{\alpha}/b)\{(1+\theta)^2 + \theta(\theta-1)bc\}$

$P_L^* = D^{-1}(\overline{\alpha}/b)\{(1+\theta)^2 + 2\theta^2 bc\}$

$H^* = D^{-1}\theta(1+\theta)\overline{\alpha}$

$L^* = D^{-1}(1+\theta)\overline{\alpha}$

where $D = 2(1+\theta)^2 + 2\theta^2 bc$

Table 2: Moments of Profit Distributions

strategy	prices	quantities	mixed
$Var(\pi)$	$(P_L^*)^2\sigma^2$	$(1+\theta)^2(L^*/b)^2\sigma^2$	$(1+bc)^{-2}\{P_H^*+cL^*\}^2\sigma^2$
$Var(\pi_H)$	0	$\theta^2(L^*/b)^2\sigma^2$	$(1+bc)^{-2}(P_H^*)^2\sigma^2$
$Var(\pi_L)$	$(P_L^*)^2\sigma^2$	$(L^*/b)^2\sigma^2$	$(1+bc)^{-2}(cL^*)^2\sigma^2$
$Cov(\pi_H,\pi_L)$	0	$\theta(L^*/b)^2\sigma^2$	$(1+bc)^{-2}(cP_H^*L^*)\sigma^2$

Table 3:
OPEC capacity utilization in period preceeding March 1983 price cut

Country	Weight*	Available Capacity** (1000 bbl/day)	Capacity Utilization June 1982 (Percent)	Capacity Utilization February 1983 (Percent)
Algeria	42	900	72.22	66.67
Libya	40	1,750	68.57	51.43
Qatar	40	600	68.00	28.50
UAE	39	1,245	92.93	83.05
Indonesia	35	1,650	79.76	61.58
Iraq	35	900	88.89	94.44
Iran	34	3,200	68.75	75.00
Nigeria	34	2,200	74.81	41.18
Saudi Arabia	34	7,000	95.24	50.14
Kuwait	31	1,250	71.44	71.52
Gabon	29	150	101.80	103.33
Ecuador	28	225	95.56	102.67
Venezuela	26	2,200	68.63	94.55

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* The weight reported in the table indicates the most predominant weight traded on an official basis. Most countries have multiple grades ranging from light to heavy. PIW reports prices for over 100 "major" grades traded by OPEC.

** Available capacity is based on existing infrastructure and includes announced production ceilings by some members. It represents the maximum production that could be sustained without new investment in production or transportation facilities. The available capacity figure used is for March 1983.

Sources: Petroleum Intelligence Weekly
International Energy Statistical Review, U.S. Central Intelligence Agency

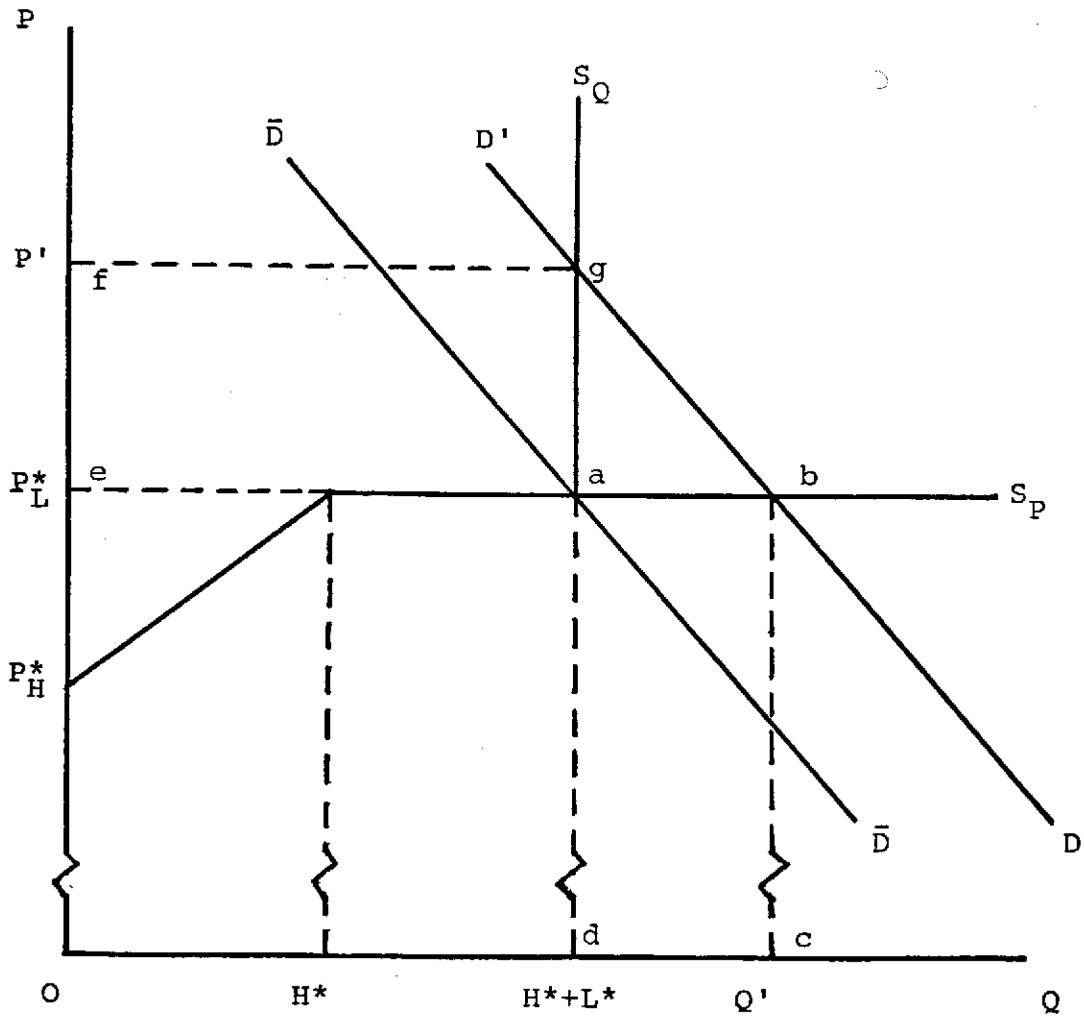

Figure 1: Effects of a Change in Demand under Price-Setting and Quantity-Setting