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BUILDING AN OPTIMAL PORTFOLIO CONSISTING OF TWO ASSETS AND ITS EFFICIENT FRONTIER

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In modern portfolio theory, it is common practice to first compute the risk-reward efficient frontier and then to support an individual investor in selecting a portfolio that meets his/her preferences for profitability and risk. Potential flaws include (a) the assumption that past data provide sufficient evidence for predicting the future performances of the securities under consideration and (b) the necessity to mathematically determine or approximate the investor's utility function. This paper presents the description of the efficient frontier for a portfolio made of two assets. We use data analysis to obtain two clusters, then, we estimate the risk of each asset corresponding to each class we obtained. Thus, we get the best two assets among the ones we analyzed and for which we will construct the efficient frontier. The originality of our paper consists in the combination of classification theory and risk estimation theory to determine the best assets. To illustrate the efficiency of the method we used, we present a case study which makes reference to the stocks listed at Bucharest Stock Exchange. We consider two stocks with the best features from Bucharest Stock Exchange based on the existent correlation that we obtained by data analyses (for classification), and by the evaluation of the loss repartition (for risk estimation), then we construct the efficient frontier for this portfolio.

Key Words: risk; selection of assets; principal components analysis; optimization; efficient frontier.

JEL Classification: C02, C61, G11.

1. Introduction

Financial assets portfolio optimization is an important area, which developed the theory of Markowitz's mean-variation and the expected utility theory. Mean-dispersion theory has some limitations and can be applied successfully only if the expected returns of financial securities are normally distributed random variables. However, the literature contradicts the hypothesis of normality for the expected returns and it is a strong argument against the use of the mean-variation techniques, which is why they introduced new measures of risk. Value-at-Risk (VaR) is a measure of risk, which plays an important role in investment, risk management and regulatory control of financial institutions. Basel II has incorporated the concept of VaR, and encourages banks to use VaR or the daily risk management. Since in most cases the distribution of random variable risk is not known, a method of evaluation or estimation of VaR is required. The idea to obtain clusters that characterize a set of assets can be found also in Kaski, Onnela and Chakraborti (2003), Ştefănescu, Şerban, Buşu and Ferrara (2010: pp. 93-108) and Mantegna (1999). Methodology based on clustering techniques is a useful tool for understanding and detecting the structure and the hierarchy in the financial data. These methods were successfully applied to analyze the stock markets and exchange. Brida and Risso (2007a, 2007b) applied clustering techniques to classify the stocks of Milan and Frankfurt stock exchanges using the Pearson correlation coefficient.

We propose solving the problem in two stages: selecting assets, risk estimation. The selection of assets is realized

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assets, risk estimation. The selection of assets is realized by applying principal components analysis in order to discover similarities between the assets under consideration. We use PCA to reduce the number of features of assets to be taken into account for each asset. In the second stage, we will present an approach to estimating risk using historical simulation method. At the end we solve a case study for stocks listed on the Bucharest Stock Exchange.

2. Building an Optimal Portfolio Consisting of N Assets

2.1. Stage of selection of assets

In the context of nowadays financial markets it is a huge amount of available financial data. It is therefore very difficult to make use of such an amount of information and to find basic patterns, relationships or trends in data. We apply data analysis techniques in order to discover information relevant to financial data, which will be useful during the selection of assets and decision making. Consider that we have collected information on a number N of assets, each with P features, which represent various financial ratios, still called variables. Denote by x_i^j the j-th variable for action i. Multivariate data set will be represented by a matrix $X=\left(x_i^j\right)_{\substack{i=\overline{1,N}\\ j=1,P}}$ and can be viewed as a set of N points in a P-dimensional space. Principal components analysis (PCA) is a useful technique for analyzing data to find patterns of data in a large-scale data space. PCA involves a mathematical procedure that transforms P variables, usually correlated in a number of $p \leq P$ uncorrelated variables called principal components. After applying the PCA, each asset *i* will be characterized by *p* variables, represented by a set of parameters $x_i^1, x_i^2, \ldots, x_i^p$ therefore, it is possible to form the arrays $X_i=\left(x_i^1, x_i^2, \ldots, x_i^p\right),\ i=\overline{1,N}$, which correspond to a set of S assets. Suppose now that we obtained a data set $X_i=\left(x_i^1, x_i^2, \ldots, x_i^p\right),\ i=\overline{1,N}$. We then use clustering techniques in order to find similarities and differences between the actions under consideration. The idea of clustering is an assignment of the vectors $X_1, X_2, \ldots, X_N$ in T classes $C_1, C_2, \ldots, C_T$. Once completed the selection of activities, we construct the initial portfolio by selecting low-risk asset in each class.

2.2. Phase estimation risk

We evaluate the performance of an asset using expected future income, an indicator widely used in financial analysis. Denote by $S_j(t)$ the closing price for an asset j at time t. Expected future income attached to the time horizon $[t, t+1]$ is given by: $R_j(t)=\ln P_j(t+1)-\ln P_j(t),\ j\in\overline{1,N}$. Similarly, we define the loss random variable, the variable L_j, for asset j for $[t, t+1]$ as: $L_j(t)=-R_j(t)=\ln P_j(t)-\ln P_j(t+1),\ j\in\overline{1,N}$.

Using Rockafellar & Uryasev (2002), define the risk measure VaR corresponding loss random variable L_j. Probability of L_j not to exceed a threshold $z\in \mathrm{R}$ is $G_{L_j}(z)=P\left(L_j \leq z\right)$..

Value at risk of loss random variable L_j associated with the value of asset j income and corresponding probability level $\alpha\in(0,1)$ is:

$VaR_\alpha\left(L_j\right)=\min\left\{z\in \mathbf{R} \,\middle|\, G_{L_j}(z)\geq 1-\alpha\right\}$ or $P(X>VaR)=\alpha$.

If G_{L_j} is strictly increasing and continuous, $VaR_\alpha\left(L_j\right)$ is the unique solution of equation $G_{L_j}(z)=1-\alpha$ then $VaR_\alpha\left(L_j\right)=G_{L_j}^{-1}(1-\alpha)$.

One of the most frequently used methods for estimating the risk is the *historical simulation method.*

This risk assessment method is useful if empirical evidence indicates that the random variables in question may not be well approximated by normal distribution or if we are not able to make assumptions on the distribution. Historical simulation method calculates the value of a hypothetical change in the current portfolio, according to historical changes in risk factors (Fulga & Dedu, 2009; Fulga, Dedu & Şerban, 2009). The great advantage of this method is that it makes no assumption of probability distribution, using the empirical distribution obtained from analysis of past data. Disadvantage of this method is that it predicts the future development based on historical data, which could lead to inaccurate estimates if the trend of the past no longer corresponds.

If L_j is the loss random variable and $\hat{G}_n$ is empirical distribution function of L_j and $\alpha\in(0,1)$ a fixed level of probability, then $\hat{G}_n(z)=\frac{1}{n}\sum_{i=1}^{n}F_{\{L_j\leq z\}}$. We can prove that

$$VaR(L_j)=\min\left\{z\in\mathbf{R}\,|\,\frac{1}{n}\sum_{i=1}^{n}F_{\{L_j^i\leq z\}}\geq 1-\alpha\right\}.$$

3. The Efficient Frontier of a Portfolio Composed From Two Assets

Assume that on the market there are 2 risky actives. The active „*i*" has percentage of rentability r_i, where:

- the average $M(r_i)=\mu_i$;
- standard deviation of r_i = σ_i.

Definition 1: A portfolio is called "profitable" if among all the portfolio with the same standard deviation of the rentability, has the best average (Best et al, 2000).

Definition 2 : A portfolio is called „efficient" if , starting from a set of shares and taking all the linear combination of the titles within the portfolio, we are looking for those titles which dominates other (profitable) titles.

Definition 3: The set of profitable portfolios is called „the frontier of rentability"(Markowitz).

Consider that the sale of the live actives is not permitted (this means that the weights $X_{1,2}$ of the actives within the portfolio are strictly positive) and that the risks $r_{1,2}$ of the actives have the averages $\mu_{1,2}$, with $\mu_1<\mu_2$ and deviations $\sigma_{1,2}$, with $\sigma_1<\sigma_2$.

Therefore,

- covariance $\sigma_{12}=\rho\cdot\sigma_1\cdot\sigma_2$, where „$\rho$" is the correlation coefficient.
- the weights $X_{1,2}$ of the assets of the portfolio are $X_1=X$, respectively $X_2=1-X$, $X\in(0,1)$.
- the profitability rate of the portfolio is a random variable r_p: $r_p=X\cdot r_1+(1-X)\cdot r_2$
- the average of the profitability rate is:

$$\mu_p=X\cdot\mu_1+(1-X)\cdot\mu_2 \quad (1)$$

- - the variance of the profitability, assimilated with a risk index, is:

$$\sigma_p^2=X^2\cdot\sigma_1^2+(1-X)^2\cdot\sigma_2^2+2X\cdot(1-X)\cdot\rho\cdot\sigma_1\sigma_2 \quad (2)$$

In what follows, we will highlight the contribution of the correlation coefficient „ρ":

A) Case $\rho=1$:

Relation (2) becomes $\sigma_p^2=[X\cdot\sigma_1+(1-X)\cdot\sigma_2]^2$, therefore:

$$\sigma_p=X\cdot\sigma_1+(1-X)\cdot\sigma_2 \quad (3)$$

$$\left.\begin{array}{l} From\ (1)\Rightarrow X=\dfrac{\mu_p-\mu_2}{\mu_1-\mu_2} \\ From\ (3)\Rightarrow X=\dfrac{\sigma_p-\sigma_2}{\sigma_1-\sigma_2}\end{array}\right\}\Rightarrow \mu_p=\frac{\mu_2\sigma_1-\mu_1\sigma_2}{\sigma_1-\sigma_2}+ +\frac{\mu_1-\mu_2}{\sigma_1-\sigma_2}\cdot\sigma_p \quad (4)$$

Relation (4) represents the equation of a line in the (σ,μ)-plane (*Figure 1*).

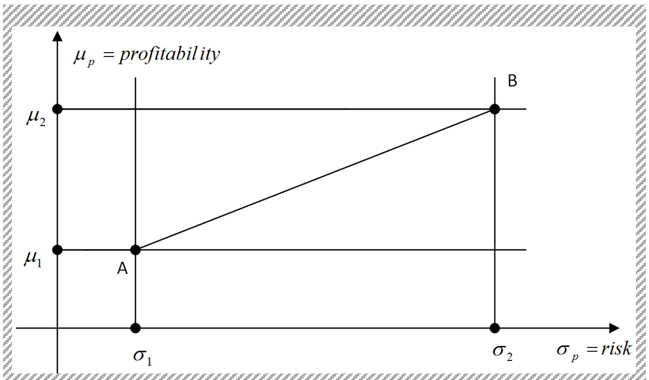

Figure 1. Relationship risk – profitability for $\rho=1$

Comments for the case $\rho=1$:

- One cannot expect any gain from diversifying the portfolio: The frontier of efficiency AB coincide with the set of all portfolios possible to achieve with the given assets by linear combination;
- Any raise in the average of profitability (i.e. μ) will be accompanied by a corresponding growth of the risk (i.e. σ).

B) Case $\rho=-1$:

From relation (2) we deduce: $\sigma_p^2=[X\cdot\sigma_1+(1-X)\cdot\sigma_2]^2$; since $\sigma_p>0$, we get

$$\sigma_p=|X\cdot\sigma_1-(1-X)\cdot\sigma_2| \quad (5)$$

From relations (1), (5), we deduce:

$$\mu_p=\frac{\mu_2\sigma_1+\mu_1\sigma_2}{\sigma_1+\sigma_2}\pm\frac{\mu_1-\mu_2}{\sigma_1+\sigma_2}\cdot\sigma_p \quad (6)$$

Graph of the relation (6) is given by the line (ACB) in *Figure 2*.

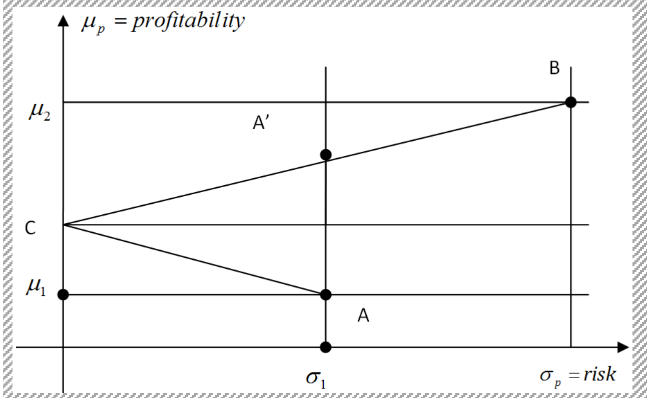

Figure 2. Relationship risk – profitability for $\rho = -1$

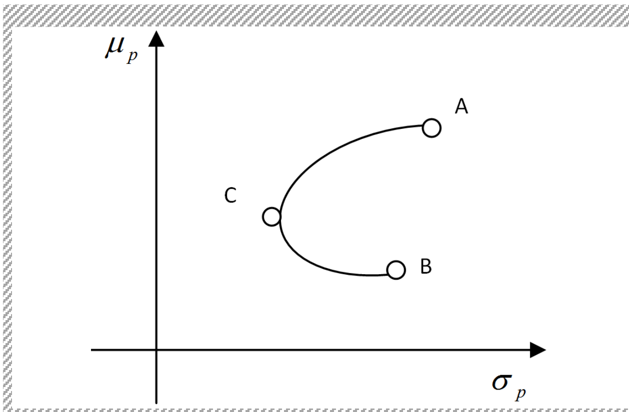

Figure 3. Relationship risk–profitability for $-1 < \rho < 1$

Comments for the case $\rho = -1$:

- Only the segment (CB) represents the frontier of efficiency: a cautious investor cannot accept the segment (AC), because any portfolio on the segment AC is strictly dominated by one on the segment CA', which has the same risk, but a better profitability.
- It is clear the gain obtained by the diversification of the portfolio:

  - We start from A with $X = 1$
  - When „X" decreases, we move towards C, realising the following: the growth of the average of the profitability and the diminution of the risk; on the segment AC, the marginal risk of the asset A_2 is negative; therefore, the growth of the weight of the asset 2 within a portfolio leads to a diminution of the risk.
  - The point „C" represents a portfolio without risk, composed by two risky assets.

On the segment (CB) is displayed yet another risk for any growth of the average: therefore, the marginal risk of the asset 2 on the segment (CB) becomes positive.

C) Case $-1 < \rho < 1$:

In this case, σ_p^2 is not a perfect square: the graph of the function implicitly expressed by the functions (1), (2) is presented below. *Figure 3* contains the parabola (ACB) which is exactly the frontier of the efficiency of the portfolio.

Comments
1) The minimal risk achieved in the point (C) is not zero.
2) Only the segment (CA) is a frontier of rentability: a prudent investor cannot accept the segment (BC) because any portfolio on the segment (BC) is strictly dominated by a portfolio on the segment (CA) which has the same risk but a better rentability.

4. Application of Optimization of a Portfolio of Stocks Listed on BSE

4.1. Financial ratios used in the actions evaluation

Evaluation of the stocks will be performed, as usual, using two specific methods of financial analysis: fundamental analysis and technical analysis.

Fundamental analysis attempts to determine a value closer to the reality of stocks based on information on the company's financial situation, the area in which they operate, investments, property etc. The purpose of this analysis is to select those stocks which, at the time, the market price is lower than the value of the outcome of the analysis, thus creating the premises for future market to recognize the value and price to raise. This means that fundamental analysis attempts to predict the direction of share price development of medium and long term from past and present achievements of the company and to estimate its future. Such fundamental analysis relies on a direct cause-effect relationship between the economic value of a stock and its market price developments.

Technical analysis studies the evolution of the trading price, it assumes that all relevant information to the market is already included in price, except for natural disasters such shock events, investor psychology etc.

Depending on access to information, the time for analysis and investment strategy chosen, each investor chooses the type of analysis that fits better. Thus speculators go long on technical analysis; long-term investors go on fundamental analysis. Ideally the two should be used together to confirm the purchase or sale signals that they offer. We will present some of the most important financial indicators that we will use in our study.

- *PER indicator (net income per share)* is calculated by dividing the current market price to the value of net profit per share for the past four consecutive quarters,

- *The P / BV (book value of shares)* is calculated by dividing the current trading price to book value per share determined according to the latest financial reporting, accounting value of a share is calculated by dividing the total equity value of the company to the total of it shares issued and outstanding; equity value is determined by deducting total liabilities from total assets owned company and is "shareholder wealth", which is what remains to be recovered if the assets and liabilities would be paid.
- *The ratio of value traded in last 52 weeks and market capitalization*, the report shows the liquidity action
- *Evolution of price*: to observe the price level at a given time we take into account the maximum price and minimum price achieved in the last 6 months
- *Divy index* measures the performance of dividend and is calculated as the ratio between the amount of the dividend and book value or market value of the action. Divy index assesses the efficiency of investment in an asset.

We used information on a total of 60 stocks representing stocks of Class I and II, traded on the Bucharest Stock Exchange on 11.02.2011. Then we considered only the actions for which it is possible to calculate most the indicators mentioned resulting in 40 stocks. The aim of our study is to find similarities and differences between the current analysis and build portfolio that is representative for BSE. Since the Bucharest Stock Exchange is not mature enough, we cannot afford to use a single financial index, such as, for example, the closing price. So we take into account several characteristics for each asset, we use data analysis techniques in order to process this vast amount of information. We consider the values of the five financial indices for each asset. *Table 1* lists, for each of the 40 stocks analyzed, the values of the five features; we used the data available on the Bucharest Stock Exchange on 11.02.2011.

4.2 Principal components analysis

We apply data analysis techniques to discover the similarities and differences between the stocks of the Bucharest Stock Exchange, using the package StatistiXL 1.8. *Figure 4* contains the tree resulted from PCA. Dendrogram usually begins with all assets as separate groups and shows a combination of mergers to a single root. Stocks belonging to the same cluster are similar in terms of features taken into account. In order to build a diversified portfolio, we first choose the number of clusters (for our study, we chose 2), which will be taken into account. We will then choose a stock from each group.

Comments: We observe the 2 classes in which the stocks were grouped. Those classes are presented further.

4.3 Risk estimation

We used the closing price values daily for each share, corresponding to a time horizon of 50 days to measure VaR for each stock. We used the data available on the Bucharest Stock Exchange from January 21 2011 - February 21 2011. The tables from Annex contain values of VaR for each stock and three levels of probability value.

Table 1

The value of the 5 features

No	Symbol	PER	P/BV	DIVY	Min/ P	Max/P
1	FP	10,91	0,69	13,33	1	1,05
2	SIF5	12,78	1,23	11,64	0,76	1,31
3	SIF2	7,1	1,38	4,95	0,69	1,13
4	BRD	16,06	1,77	2,05	0,8	1,07
5	TLV	22,87	0,87	2,62	0,84	1,14
6	BVB	31,64	3,54	2,46	0,76	1,04
7	BCM	18,64	0,37	7,78	0,85	1,53
8	SIF1	12,69	1,11	4,95	0,89	1,48
9	SIF3	23,38	0,87	5,71	0,87	1,47
10	SIF4	18,44	0,38	6,32	0,99	1,32
11	DAFR	18,26	0,77	0,00	0,77	1,06
12	TEL	-	0,63	2,48	0,71	1,06
13	COMI	12,65	0,66	0,00	1	1,05
14	TGN	8,24	1,36	4,73	0,73	1,03
15	BRK	14,96	0,91	0,00	0,89	1,23
16	SNP	9,96	0,47	0,00	0,83	1,02
17	ATB	15,29	1,09	2,87	0,8	1,05
18	AZO	2,77	0,65	0,00	0,83	1,15
19	BIO	12,11	1,77	0,00	0,71	1
20	PREH	51,8	0,49	4,37	0,85	1,23
21	ALR	19,22	1,61	5,79	0,8	1,06
22	SNO	27,76	0,52	9,41	0,98	1,44
23	SCD	8,15	1,48	0,00	0,68	1,05
24	VESY	-	0,39	4,5	0,83	1,4
25	OIL	37,83	0,79	0,29	0,86	1,26
26	AMO	6,3	0,19	0,00	1	1,7
27	COTR	49,9	0,21	0,00	0,81	1,4
28	RMAH	7,63	0,91	1,36	0,82	1,06
29	SPCU	1395	0,65	2,93	0,93	1,25
30	CEON	22,71	0,37	0,00	1	1,44
31	SIF2	7,1	1,38	4,95	0,7	1,06
32	PTR	6,96	0,81	4,71	0,97	1,42
33	RPH	22,44	5,39	0,00	0,9	1,22
34	ALT	13,66	0,32	0,00	0,87	1,13
35	MPN	-	0,85	4,31	0,86	1,27
36	ELJ	2,37	0,33	0,00	0,58	1,12
37	CMP	15,82	0,38	0,00	0,77	1,02
38	ROCE	-	0,45	0,65	0,91	1,54
39	ALU	25,6	0,68	5,71	1	1,78
40	CGC	102,1	0,19	0,00	1	1,66

Source: *Bursa de Valori Bucureşti http://www.bvb.ro/*

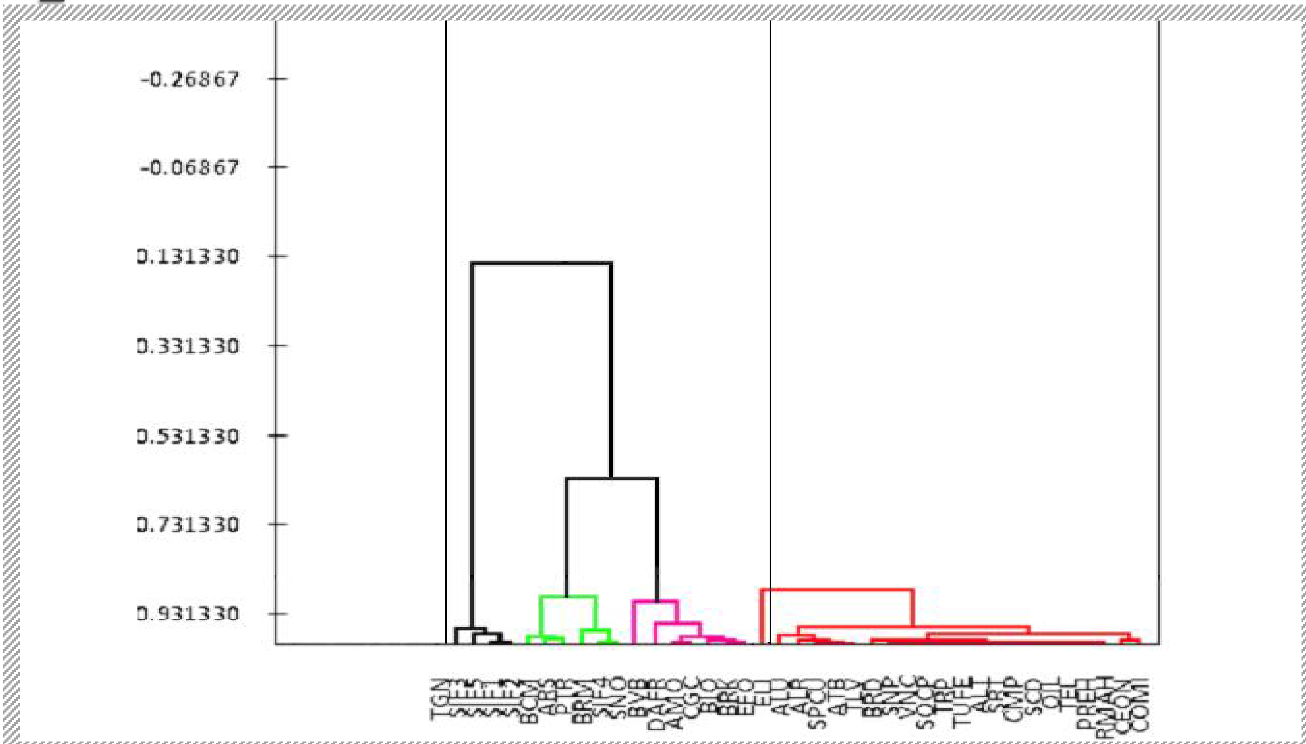

Figure 4. Group of stocks

Source: *generated using Package of Programs Statisti XL*

4.4 Construction of an optimal portfolio made of 2 stocks

We start from the classes we formed above and we choose from each of them the stock which has minimal VaR for the probability 0.99; we get the two stocks which form the portfolio to which we can construct the frontier of rentability: TLV, BIO

4.5 The efficient frontier of rentability for the portfolio made of 2 stocks

We selected the closing prices for the period 9.11-10.12.2010 for the 2 selected stocks: TLV and BIO.

We obtained the values:

TLV : 1,27; 1.29; 1,26; 1,25; 1,3; 1,3; 1,3; 1,23; 1,23; 1,23; 1,22; 1,23; 1,24; 1,24; 1,25 ; 1,27; 1,24; 1,24; 1,26; 1,26;1,27

BIO: 0,185; 0,187; 0,187; 0,187; 0,188; 0,19; 0,189; 0,189; 0,188; 0,188; 0,188; 0,188; 0,189; 0,193; 0,2; 0,196; 0,196; 0,201; 0,203; 0,202; 0,203

We will transform the prices in annual rentability by the formula (final price-initial price)x360/ initial price. We get the following repartition for the rentabilities:

$$X_{TLV}:\begin{pmatrix} -28{,}8 & -18 & -10{,}8 & -8{,}3 & 0 & 5{,}8 & 14{,}4 & 28{,}8 \\ \frac{1}{20} & \frac{2}{20} & \frac{2}{20} & \frac{3}{20} & \frac{4}{20} & \frac{2}{20} & \frac{4}{20} & \frac{2}{20} \end{pmatrix}$$

$$X_{BIO}:\begin{pmatrix} -7{,}2 & -1{,}8 & 0 & 1{,}8 & 3{,}6 & 7{,}2 & 10{,}8 & 14{,}4 \\ \frac{1}{20} & \frac{3}{20} & \frac{7}{20} & \frac{4}{20} & \frac{2}{20} & \frac{1}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix}$$

- We apply the formulae specific to statistics and we get:

$\mu_1 = 3{,}1$ and deviation $\sigma_1 = 16$

$\mu_2 = 1{,}7$ and deviation $\sigma_2 = 4{,}7$

$\rho_{12} = 0{,}12$

- Applying formula 1 we get

$$\mu_p = X \cdot 3{,}1 + (1-X)\cdot 1{,}7 = 1{,}7 + 1{,}4X \quad (7)$$

- Applying formula 2 we get

$$\sigma_p^2 = X^2 \cdot 256 + (1-X)^2 \cdot 22 + 2X\cdot(1-X)\cdot 0{,}12 \cdot 16 \cdot 4{,}7$$

which means

$$\sigma_p^2 = 260X^2 - 26X + 22 \quad (8)$$

To minimize the risk, we need x = - b/2a, thus x = 0,05, which means that, for the risk to be minimal, we must have : 5% stocks TLV and 95% stocks BIO. From relations 7 and 8 we get the frontier of efficiency as a solution of the equation $\sigma_p^2 = 127{,}4\mu_p^{\ 2} - 455\mu_p + 427{,}6$

Graphically (*Figure 5*), we get the below representation with the remark that the efficient frontier is just the curve AC.

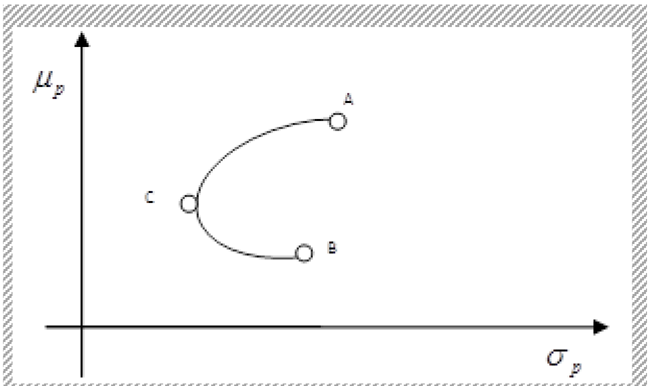

Figure 5. Efficient frontier

Conclusions

Many mathematical methods for portfolio optimization problems use a particular initial composition of the portfolio without specifying how assets have been chosen. The approach proposed by us has an important advantage, as guaranteed by the diversity of the portfolio and has the ability to improve portfolio performance because it starts with an initial portfolio comprising a wide range of low-risk assets. The proposed methodology has been highlighted by a case of study for the stocks listed on BSE. The most performing stocks, those that are used to construct the initial portfolio, are TLV and BIO. We computed the composition of the portfolio which guaranties the minimisation of the risk (5% stocks TLV and 95% stocks BIO) and we found the equation of the frontier of efficiency ($\sigma_p^2 = 127{,}4\mu_p^{\ 2} - 455\mu_p + 427{,}6$) which allows us to find for a desired profitability a certain level of the assumed risk.

Acknowledgement

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Annex

VaR for each stock

Class 1	0.9	0.95	0.99
ARS	0.0721	0.0891	0.1325
AMO	0.0721	0.089	0.135
BCM	0.031	0.082	0.2134
BIO	0.0235	0.0372	0.0434
BRK	0.0223	0.0424	0.0486
BVB	0.0270	0.0299	0.0676
COTR	0.0496	0.0702	0.1054
CGC	0.0512	0.0747	0.1289
DAFR	0.0328	0.0400	0.0468
ELJ	0.0531	0.0742	0.0969
EFO	0.0531	0.0742	0.0969
FP	0.0234	0.0344	0.0729
MPN	0.0615	0.0815	0.1045
PTR	0.0348	0.0448	0.0566
ROCE	0.0415	0.0705	0.1025
RPH	0.0428	0.0612	0.136
SIF1	0.0276	0.0317	0.0506
SIF2	0.0225	0.027	0.0329
SIF3	0.0255	0.0305	0.0543
SIF4	0.0237	0.028	0.0588
SIF5	0.0242	0.026	0.0495
SNO	0.0218	0.0623	0.1042
TGN	0.0167	0.0221	0.0527
VESY	0.0543	0.0I58	0.1057

Class 2	0.90	0.95	0.99
ALT	0.029	0.0371	0.0653
ALU	0.0236	0.0317	0.0529
ALR	0.0415	0.0705	0.1025
AZO	0.0265	0.0405	0.0669
ALR	0.0139	0.0171	0.0519
ATB	0.0221	0.0293	0.033
BCM	0.0460	0.0562	0.0815
BRD	0.0257	0.0301	0.0275
BRM	0.0572	0.0650	0.1074
CEON	0.0287	0.0455	0.0995
CMP	0.0228	0.0422	0.0701
COMI	0.0282	0.0313	0.042
OIL	0.0497	0.0723	0.1043
PREH	0.0568	0.0807	0.1476
RMAH	0.0465	0.055	0.1003
SCD	0.0248	0.0471	0.075
SOCP	0.0317	0.0485	0.0584
SRT	0.0534	0.0687	0.0748
SNP	0,0201	0.0248	0.0325
SPCU	0.0418	0.0585	0.2624
TEL	0.0247	0.0347	0.0492
TLV	0.0134	0.0204	0.0259
TUFE	0.0333	0.0388	0.0558
TRP	0.0421	0.0833	0.1034
VNC	0.0348	0.0383	0.0584

Source: *generated with Package of Programmes Statisti XL*