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# Dynamic Aspects of Productivity Spillovers, Terms of Trade and The "Home Market Effects"

**Ippei Fujiwara* and Naohisa Hirakata****

**Abstract**

In this paper, we first set up a model that incorporates firm dynamics into the Global Economy Model (henceforth, GEM) developed by the IMF Research Department. Then, we show how the economic variables respond to the shocks that shift the production frontier outwards, namely productivity gains in manufacturing, efficiency gains in creating new firms, and an increase in the labor force. Contrary to the model used in previous research on the same topic by Corsetti, Martin and Pesenti (2007, henceforth, CMP), our model contains rich and realistic dynamics embedded in the GEM such as a time-to-build constraint for firm dynamics, and nominal price and wage stickiness. We show that (1) the analytical results of CMP are dependent on the elasticity of substitution between domestic and foreign goods, (2) short-run responses could be different from those in CMP because of the existence of price and wage stickiness, and (3) persistence of shocks also alters the direction of responses via the wealth effect. These results suggest that it is of great importance for policy institutions to acknowledge the dynamic aspects of productivity spillovers by simulating a model with richer dynamics like the GEM.

**Keywords:** Endogenous variety; Home market effect; Productivity spillover
**JEL classification:** F12, E32, J41

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# 1 Introduction

Reflecting the growing interest in understanding the role of firm heterogeneity or endogenous variety in international trade as the seminal research by Melitz (2003) represents, macroeconomists also began considering the consequences of incorporating firm dynamics in dynamic stochastic general equilibrium models. As noted in Bilbiie, Ghironi and Melitz (2007), the standard assumption in a widely used monetary business cycle model, as summarized in Woodford (2003), that monopolistically competitive firms can maintain positive profits without worrying about new entry, should be considered to be unrealistic.[1] At the same time, empirical research in trade theory demonstrates strong procyclical behavior of new producer entry, which cannot be explained without firm dynamics. Therefore, it is quite reasonable to seek a model that has more realistic assumptions about firm dynamics and that can explain the observed role of extensive margin in the business cycle. Among several papers following this argument, Ghironi and Melitz (2005) build a microfounded model with endogenous determination of tradedness to give a theoretical background for the Harrod–Balassa–Samuelson story of real exchange rate determination, where the relative price of nontradable goods and therefore aggregate price will rise if the tradable sector in a country experiences a productivity growth. They can reproduce endogenously persistent deviations from purchasing power parity by simulating a model where all goods are potentially tradables but only an endogenously determined number of goods are traded because of trade barriers. Bilbiie, Ghironi and Melitz (2006a) examine a dynamic stochastic general equilibrium model with free entry and show that the performance of such models evaluated by the implied second moment properties are at least as good as the traditional model with a fixed number of varieties. Bilbiie, Ghironi and Melitz (2006b), on the other hand, search for the normative implication of endogenous variety. They recommend policy institutions to preserve the optimal amount of monopoly profits when firm entry is costly. Marginal cost pricing remains efficient only when the required sales subsidies are financed with the optimal split of lump-sum taxation between households and firms. Bilbiie, Ghironi and Melitz (2007), while summarizing the main conclusions from the above two papers, also show that inflation can act as a distortionary tax on firm profits and therefore can be harmful to the firm's incentive to create new goods. Mancini-Griffoli (2006) introduces an endogenous source of inertia in a dynamic general equilibrium model with firm entry. His model can produce persistent and hump-shaped responses of consumption, investment, output and new entry to a technology shock that are consistent with the observed data. Bergin and Corsetti (2006) analytically study the role of stabilization policy in a model with free entry. They show that monetary policy should have the additional role of controlling the optimal number of entrants. However, their conclusion is that the optimal monetary policy rule obtained in a dynamic new Keynesian model without entry remains optimal even under the model with endogenous

[1] Usually, it is assumed that firms continuously exist within the unit mass between 0 and 1.

variety. Corsetti, Martin and Pesenti (2007, henceforth CMP), whose motivation is similar to that of Bilbiie, Ghironi and Melitz (2006a) and ours, research the properties of the model with free entry from a positive perspective. Their analysis focuses on the responses of international relative prices to the shocks that expand the production frontier. They derive all the results analytically, and they conclude that a reduction in a fixed cost of entry, namely the efficiency gains in creating new firms or goods, improves the terms of trade of the country facing such a shock. This result is contrary to a common view in trade and growth theory that an increased supply of goods from an economy with high productivity growth should be absorbed by the rest of the world at falling prices, which results in a deterioration of the terms of trade. However, at the same time, this positive comovement of technological growth and the terms of trade is consistent with recent empirical evidence shown in Corsetti, Dedola and Leduc (2005, 2006), Debaere and Lee (2004), Hummels and Klenow (2005), and Kang (2004).

The aim of this paper is to evaluate the analytical results of CMP in a model with richer and more realistic dynamics like the Global Economy Model (henceforth, GEM) developed by the IMF Research Department. We first set up a model that incorporates firm dynamics into the GEM, which contains richer and more realistic dynamics such as a time-to-build constraint for firm dynamics, and nominal price and wage stickiness. Then, we show how the economic variables respond to the shocks that shift the production frontier outwards, namely productivity gains in manufacturing, efficiency gains in creating new firms, and increases in the labor force. The latter two are simulations that cannot be conducted using standard dynamic general equilibrium models with a fixed number of products. As for the efficiency gains, the reason is obvious because there is no firm entry. Increases in the labor force or working population have no effects on the per capita variables in standard models without the home market effect that stems from the trade cost and the endogenous variety.

According to CMP, the terms of trade deteriorate and the real exchange rate evaluated by the welfare-based price index, which is computed considering goods varieties, depreciates because of the productivity gains. The effects through the decrease in the marginal cost, namely the adjustments in the intensive margin, dominate the effects through the changes in the number of firms, namely the adjustments in the extensive margin. Therefore, the terms of trade, the intensive margin of the international relative price, worsen. The real exchange rate, the extensive margin of the international relative price, depreciates in accordance with the terms of trade. On the other hand, the terms of trade improve while the real exchange rate appreciates because of the efficiency gains and increase in the labor force. This is contrary to the case for the productivity gains. For the efficiency gains and increase in the labor force, there is no direct change in the real marginal cost, but the number of firms changes significantly reflecting the lower entry cost and the home market effect. Reflecting dominant effects via the extensive margin, the international relative price in the extensive margin and that in the intensive margin respond in opposite directions.

We, however, show that (1) the analytical results of CMP are dependent on the elasticity of substitution between domestic and foreign goods, (2) the short-run responses could be different from those in CMP because of the existence of price and wage stickiness, and (3) the persistence of shocks also alters the direction of responses via the wealth effect. These results suggest that it is of great importance for policy institutions to acknowledge the dynamic aspects of productivity spillovers by simulating a model with richer dynamics like the GEM.

The structure of this paper is as follows. In the next section, we introduce our model. Then, section three analyzes the international implication of shocks, which expand the production frontier outwards, on international relative prices. We examine three shocks, namely productivity gains in manufacturing, efficiency gains in creating new firms, and increases in the labor force. We show how responses could be different in a model with richer and more realistic dynamics than those obtained analytically in CMP. Finally, in section four, we summarize our findings.

# 2 The Model

The model used in this paper is based on the recent literature, which combines the 'New Open Economy Macroeconomics' initiated by Obstfeld and Rogoff (1995) and the 'New Trade Theory' advocated by Krugman (1980).[2] Because the heterogeneous technology level among firms is not considered in this paper, our model can be interpreted as a dynamic extension of Corsetti, Martin and Pesenti (2005) or a two-country extension of Bilbiie, Ghironi and Melitz (2006a) and Bergin and Corsetti (2006). Contrary to these previous studies, our model includes richer and more realistic dynamics, such as nominal price and wage rigidities. Because our model is based on the GEM, it has richer dynamics and can reproduce the regularities in the observed data.

The model is a two-country (economy) model, which consists of home (domestic) and foreign countries. There are three agents in each country. They are households, firms, and the monetary authority. Households maximize their welfare from consumption of final goods $C$ and leisure after differentiated labor supply $l$ to domestic firms. The number of households in the domestic country is $L$, while that in the foreign country is $L^*$number of households is an exogenous variable. They own domestic firms and therefore receive profits as a dividend.[3]

The goods market is monopolistically competitive. Each firm produces differentiated products and therefore chooses the optimal price that maximizes profit subject to a downward sloping demand curve. The number of domestic goods is $n$. Unlike other standard papers in new open economy macroeconomics or new Keynesian models, they are endogenously determined, which is the main

[2] For example, Ghironi and Melitz (2005), Bilbiie, Ghironi and Melitz (2006a, 2006b, 2007), Corsetti, Martin and Pesenti (2007), and Bergin and Corsetti (2006).

[3] We can easily extend our model to incorporate the capital stock by assuming that households rent capital to firms. For simplicity of analysis, however, we omit capital formation in this paper. The conclusions in this paper will not be affected by incorporation of capital.

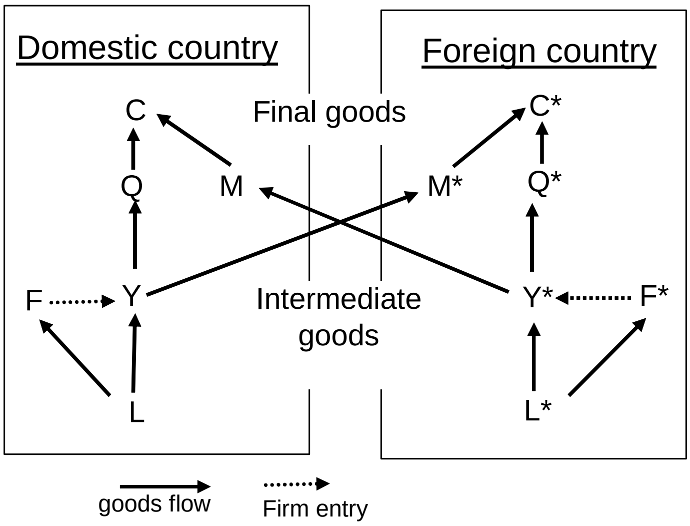


Figure 1: Model Concept

contribution of our model. When entering the market, each firm needs to incur fixed entry costs. Therefore, entry occurs when the net present value of the future profits exceeds entry cost. This eventually determines the macro production level as well as the number of goods, namely variety. On the exit side, a certain proportion $\delta$ of firms exit in each period.[4]

The production structure of this model can be summarized in the concept chart in Figure 1. Intermediate goods are produced using only labor. Then, they are used as intermediate inputs in final goods production[5] in either the domestic or the foreign country. All final goods produced this way are consumed.

Below, we first derive the structural equations from firms' and then households' optimization. In this model, $j \in [0, L_t]$ denotes the index of domestic households, and $j^* \in [0, L_t^*]$ denotes the index of foreign households, while $h \in [0, n_t]$ denotes the index of domestic firms and $f \in [0, n_t^*]$ denotes the index of foreign firms.

[4] Only relatively unproductive firms exit the market in a model that considers firms with heterogenous technology levels, such as Ghironi and Melitz (2005).

[5] As shown in Bilbiie, Ghironi and Melitz (2006a), regarding empirical problems associated with increasing returns to specialization and a CES production function, it may be better to model the household consuming a basket of goods defined over a continuum of goods. Neither specification, however, makes a difference in simulations conducted in this paper.

## 2.1 Firms

### 2.1.1 Final goods production

The final goods consumed by household $j$, namely $C_t(j)$, are produced by the following CES technology using a basket of home goods $Q_t(j)$ and a basket of foreign goods $M_t(j)$:

$$C_t(j) = \left[\nu^{\frac{1}{\varepsilon}} Q_t(j)^{1-\frac{1}{\varepsilon}} + (1-\nu)^{\frac{1}{\varepsilon}} M_t(j)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $\varepsilon$ denotes the elasticity of substitution between home and imported goods, and $\nu$ is the home bias parameter. By minimizing total expenditure defined as the sum of $P_{Q,t}Q_t(j)$ and $P_{M,t}M_t(j)$, where $P_{Q,t}$ is the aggregate price index for domestic goods and $P_{M,t}$ is that for foreign goods, subject to equation (1), we can obtain the demand for $Q_t(j)$ and $M_t(j)$:

$$Q_t(j) = \nu\left(\frac{P_{Q,t}}{P_t}\right)^{-\varepsilon} C_t(j), \tag{2}$$

and

$$M_t(j) = (1-\nu)\left(\frac{P_{M,t}}{P_t}\right)^{-\varepsilon} C_t(j), \tag{3}$$

and the utility-based consumer price index $P_t$ as a Lagrange multiplier on the constraint:

$$P_t = \left[\nu P_{Q,t}^{1-\varepsilon} + (1-\nu) P_{M,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Furthermore, baskets for home and foreign goods are also expressed as the CES aggregator of each good provided by different firms indexed by $h$ and $f$:

$$Q_t(j) \equiv A_{Q,t}\left[\int_0^{n_t} Q_t(h,j)^{1-\frac{1}{\theta}}\, dh\right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

and

$$M_t(j) \equiv A_{Q,t}^*\left[\int_0^{n_t^*} M_t(f,j)^{1-\frac{1}{\theta^*}}\, df\right]^{\frac{\theta^*}{\theta^*-1}}, \tag{5}$$

where $\theta(\theta^*) > 1$ denotes the elasticity of substitution among intermediate home (foreign) goods.[6] $A_{Q,t}$ and $A_{Q,t}^*$ determine the degree of taste for variety and take the forms:

$$A_{Q,t} \equiv (n_t)^{\gamma-\frac{\theta}{\theta-1}},$$

and

$$A_{Q,t}^* \equiv (n_t^*)^{\gamma^*-\frac{\theta^*}{\theta^*-1}},$$

[6] As examined in Tille (2001), we set different elasticities of substitution for intermediate goods between foreign and domestic goods. This is contrary to CMP and Corsetti and Pesenti (2001).

where $\gamma$ denotes the degree of taste for goods variety. As shown in Benassy (1996), $1-\gamma$ denotes the marginal utility (productivity) gain for increasing a given amount of consumption on a basket that includes one additional good variety. If $\gamma = \frac{\theta}{\theta-1}$ ($\gamma^* = \frac{\theta^*}{\theta^*-1}$), equations (4) and (5) collapse to the standard Dixit–Stiglitz aggregator.

Each household takes the prices of the home differentiated goods $p_t(h)$ as given and minimizes total expenditure expressed as $\int_0^{n_t} p_t(h)\, Q_t(h,j)\, dh$ subject to equation (4). The cost-minimizing price of one unit of the home goods basket, $P_{Q,t}$, obtained from this optimization problem is:

$$P_{Q,t} = \frac{1}{A_{Q,t}} \left[ \int_0^{n_t} p(h)^{1-\theta}\, dh \right]^{\frac{1}{1-\theta}}.$$

Similarly, that of the foreign goods basket, $P_{M,t}$ is defined as:

$$P_{M,t} = \frac{1}{A^*_{Q,t}} \left[ \int_0^{n^*_t} p(f)^{1-\theta^*}\, df \right]^{\frac{1}{1-\theta^*}}.$$

As is the case with final goods production, we can obtain the demand for each domestically produced goods:

$$\begin{aligned} Q_t(h,j) &= A_{Q,t}^{\theta-1} \left( \frac{p_t(h)}{P_{Q,t}} \right)^{-\theta} Q_t(j) \\ &= \nu A_{Q,t}^{\theta-1} \left( \frac{p_t(h)}{P_{Q,t}} \right)^{-\theta} \left( \frac{P_{Q,t}}{P_t} \right)^{-\varepsilon} C_t(j), \end{aligned}$$

as well as that for each foreign goods:

$$\begin{aligned} M_t(f,j) &= \left( A^*_{Q,t} \right)^{\theta^*-1} \left( \frac{p_t(f)}{P_{M,t}} \right)^{-\theta^*} M_t(j) \\ &= (1-\nu) \left( A^*_{Q,t} \right)^{\theta^*-1} \left( \frac{p_t(f)}{P_{M,t}} \right)^{-\theta^*} \left( \frac{P_{M,t}}{P_t} \right)^{-\varepsilon} C_t(j). \end{aligned}$$

### 2.1.2 Intermediate goods production

**Production technology** Intermediate goods are produced in a monopolistically competitive market. Production $Y_t$ requires one factor, labor $l_t$. Each domestic firm $h$ has a linear production function:

$$Y_t(h) = Z_t l_t(h), \tag{6}$$

where $Z_t$ is a labor productivity that is common to all domestic firms. Therefore, the nominal marginal cost is computed as:

$$MC_t(h) = \frac{W_t}{Z_t}, \tag{7}$$

where $W_t$ is the nominal wage index. In addition to this production cost, an entrant $e$ incurs sunk entry costs of $f_{E,t}$ units of labor prior to entry:

$$f_{E,t} = l_t\left(e\right).$$

**Price setting** Each incumbent firm $h$ sets two prices, $p_t\left(h\right)$ in the home market and $p_t^*\left(h\right)$ in the foreign market, so that the present discounted value of profit is maximized. Each firm $h$ takes into account the demand in the home market $Q_t\left(h,j\right)$ as well as the foreign market $M_t^*\left(h,j^*\right)$. We assume that there are sluggish price adjustment costs measured in terms of total profits, namely the Rotemberg-type (1982) adjustment cost. Thus, the maximization problem of each incumbent firm $h$ to set its prices is now expressed as follows:

$$\max_{p_\tau(h),p_\tau^*(h)} \mathrm{E}_t \sum_{\tau=t}^{\infty} \left(1-\delta\right)^{\tau-t} D_{t,\tau}\left(j\right)\Pi_\tau\left(h\right),$$

where $D_{t,i}\left(j\right)$ is the stochastic discount factor between $t$ and $i$,[7] and the profit $\Pi_t(h)$ is defined as:

$$\begin{aligned}\Pi_t\left(h\right) &\equiv \left[p_t\left(h\right) - MC_t(h)\right]\int_0^{L_t} Q_t\left(h,j\right)dj\left[1-\Gamma_{Q,t}(h)\right] \\ &+\left[\mathcal{E}_t p_t^*\left(h\right) - MC_t(h)\left(1+\tau\right)\right]\int_0^{L_t^*} M_t^*\left(h,j^*\right)dj^*\left[1-\Gamma_{M,t}^*(h)\right].\end{aligned} \tag{8}$$

and $\mathcal{E}_t$ is home currency per unit of foreign currency,[8] $\tau$ is the transportation cost of foreign goods, and $\Gamma_{Q,t}\left(h\right)$ and $\Gamma_{M,t}^*\left(h\right)$ are the Rotemberg-type adjustment costs defined as:

$$\Gamma_{Q,t}(h) \equiv \frac{\phi_Q}{2}\left[\frac{p_t\left(h\right)/p_{t-1}\left(h\right)}{P_{Q,t-1}/P_{Q,t-2}} - 1\right]^2,$$

$$\Gamma_{M,t}^*(h) \equiv \frac{\phi_M^*}{2}\left[\frac{p_t^*\left(h\right)/p_{t-1}^*\left(h\right)}{P_{M,t-1}^*/P_{M,t-2}^*} - 1\right]^2,$$

where $\phi_Q$ and $\phi_M^*$ define the size of the adjustment costs. By solving for the first-order condition with respect to $p_t(h)$, we can obtain a price-setting relation for domestically consumed goods:

$$\begin{aligned}0 &= \left[1-\Gamma_{Q,t}\left(n\right)\right]\left[p_t\left(h\right)\left(1-\theta\right)+\theta MC_t(h)\right] \\ &-\left[p_t\left(h\right)-MC_t(h)\right]\frac{\phi_Q p_t\left(h\right)/p_{t-1}\left(h\right)}{P_{Q,t-1}/P_{Q,t-2}}\left[\frac{p_t\left(h\right)/p_{t-1}\left(h\right)}{P_{Q,t-1}/P_{Q,t-2}}-1\right] \\ &+\mathrm{E}_t\left(1-\delta_D\right)D_{t,t+1}\left[p_{t+1}(h)-MC_{t+1}\right] \\ &\times\frac{\int_0^{L_{t+1}} Q_{t+1}\left(h,j\right)dj}{\int_0^{L_t} Q_t\left(h,j\right)dj}\frac{\phi_Q p_{t+1}\left(h\right)/p_t\left(h\right)}{P_{Q,t}/P_{Q,t-1}}\left[\frac{p_{t+1}\left(h\right)/p_t\left(h\right)}{P_{Q,t}/P_{Q,t-1}}-1\right].\end{aligned}$$

[7] The stochastic discount factor is determined as the households' Euler condition, which also takes the welfare-based CPI index into account.

[8] Producer currency pricing (PCP) is assumed in this paper.

Similarly, with respect to $p_t^*(h)$, the price-setting equation for exported goods is:

$$
\begin{aligned}
0 &= \left[1-\Gamma_{M,t}^{*}\left(h\right)\right]\left[\mathcal{E}_t p_t^*\left(h\right)\left(1-\theta\right)+\theta MC_t(h)\left(1+\tau_t\right)\right] \\
&-\left[\mathcal{E}_t p_t^*\left(h\right)-MC_t(h)\left(1+\tau_t\right)\right]\frac{\phi_M^* p_t^*\left(h\right)/p_{t-1}^*\left(h\right)}{P_{M,t-1}^*/P_{M,t-2}^*}\left[\frac{p_t^*\left(h\right)/p_{t-1}^*\left(h\right)}{P_{M,t-1}^*/P_{M,t-2}^*}-1\right] \\
&+\mathrm{E}_t\left(1-\delta_D\right)D_{t,t+1}\left[\mathcal{E}_{t+1}p_{t+1}^*\left(h\right)-MC_{t+1}(h)\left(1+\tau_t\right)\right] \\
&\times\frac{\int_0^{L_{t+1}^*}M_{t+1}^*\left(h,j^*\right)dj^*}{\int_0^{L_t^*}M_t^*\left(h,j^*\right)dj^*}\frac{\phi_M^* p_{t+1}^*\left(h\right)/p_t^*\left(h\right)}{P_{M,t}^*/P_{M,t-1}^*}\left[\frac{p_{t+1}^*\left(h\right)/p_t^*(h)}{P_{M,t}^*/P_{M,t-1}^*}-1\right].
\end{aligned}
$$

Under the flexible price equilibrium, where $\phi_Q$ and $\phi_M^*$ are zero, firms set prices that reflect the markup $\theta/\left(\theta-1\right)$ over marginal cost. Therefore, prices in the steady state become:

$$
p_t\left(h\right)=\frac{\theta}{\theta-1}MC_t(h), \tag{9}
$$

and

$$
\mathcal{E}_t p_t^*\left(h\right)=\frac{\theta^*}{\theta^*-1}MC_t(h)\left(1+\tau\right).
$$

With these equations, if $\theta=\theta^*$, we can derive the following relationship:

$$
\mathcal{E}_t p_t^*\left(h\right)=p_t\left(h\right)\left(1+\tau\right).
$$

This implies that the law of one price does not hold because of the transportation cost, even if prices are fully flexible and no difference exists in the price markup between the two countries.

**Free entry and value of firms** We model firms' entry/exit decisions following Ghironi and Melitz (2005). Merit of entry is dependent on the net present value of profit after entry, namely $\left\{\Pi_\tau\left(h\right)\right\}_{\tau=t+1}^{\infty}$ discounted by a stochastic discount factor because firms are eventually owned by households. At the same time, firm $h$ faces a shock such that it needs to exit the market with a constant positive probability $\delta$. Such an exiting probability also needs to be considered when discounting future profits. The expected profit $\varpi_t\left(h\right)$ of firm $h$ at $t$ is now expressed as follows:

$$
\varpi_t\left(h\right)=\mathrm{E}_t\sum_{\tau=t}^{\infty}\left(1-\delta\right)^{\tau-t}D_{t,\tau}\left(j\right)\Pi_\tau\left(h\right). \tag{10}
$$

Firms enter the market until the sunk cost[9] equals the expected profit. Hence, the free entry condition is obtained as:

$$
\varpi_t(h)=f_{E,t}W_t. \tag{11}
$$

[9] This cost can be considered an investment although there is no endogenous capital formation.

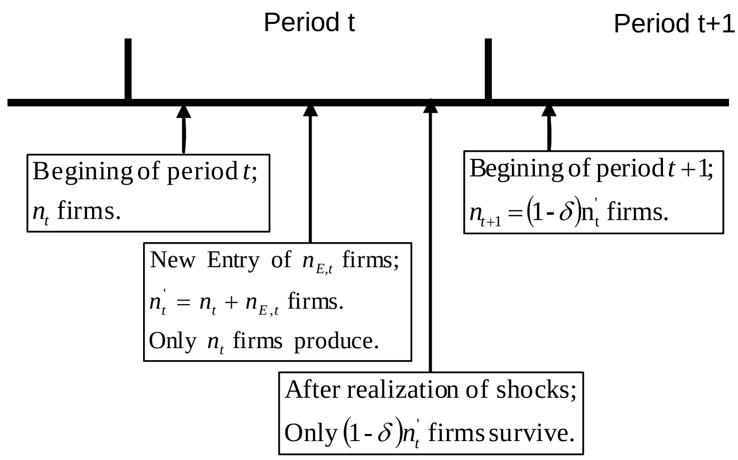


Figure 2: Timing of Entry and Exit

A firm that enters the market at $t$ can only start producing for profit at $t+1$ because of time-to-build constraints. At the beginning of $t$, there already exist $n_t$ firms. During period $t$, $n_{E,t}$ new firms enter. As a result, there are $n_t' \equiv n_t + n_{E,t}$ firms in period $t$. At the same time, production takes place with only $n_t$ firms. However, a shock then makes $\delta$ firms exit from the market at the end of each period. Therefore, at the end of period $t$, as well as at the beginning of period $t+1$, the number of firms is $(1-\delta)(n_t + n_{E,t})$. This also defines the number of firms that distribute profits to households, because profits are assumed to be distributed at the beginning of each period. $\delta n_{E,t}$ firms that enter at $t$ exit the market without any production. This can be seen in Figure 2.

Forward iteration of equation (10) and the absence of speculative bubbles yield the asset price solution:

$$\varpi_t(h) = (1-\delta) D_{t,t+1}(j) \left[\Pi_{t+1}(h) + \varpi_{t+1}(h)\right].$$

In the steady state,

$$\varpi(h) = \frac{(1-\delta)D(j)\Pi(h)}{1-(1-\delta)D(j)}.$$

From the free entry condition in equation (11), we can derive:

$$f_E W = \frac{(1-\delta)D(j)\Pi(h)}{1-(1-\delta)D(j)}. \tag{12}$$

By using equation (9), the corporate profit in equation (8) in the steady state becomes:

$$\Pi(h) = \frac{MC(h)}{\theta - 1} Y(h). \tag{13}$$

because

$$\mathcal{E}_t p_t^* (h) = p_t (h) (1 + \tau),$$

and the resource constraint of the product of firm $h$ is:

$$Y (h) = \int_0^{L_t} Q_t (h, j) \, dj + (1 + \tau) \int_0^{L_t^*} M_t^* (h, j^*) \, dj^*.$$

By plugging equation (13) into (12), we can obtain:

$$Y (h) = (\theta - 1) f_E Z \frac{1 - (1 - \delta) D (j)}{(1 - \delta) D (j)}.$$

In the steady state, the production level of each firm is determined by the degree of economies of scale $f_E Z$ and the product differentiation $\theta$ in addition to the discount rate $\frac{1-(1-\delta)D(j)}{(1-\delta)D(j)}$.

## 2.2 Household

Consumer $j$ receives utility from goods consumption $C (j)$ and disutility from labor supply $l (j)$. Consumer $j$ maximizes the lifetime expected utility as follows:

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ U_\tau \left[ C_\tau (j) \right] - V_\tau \left[ l_\tau (j) \right] \right\}, \tag{14}$$

where

$$U_t \left[ C_\tau (j) \right] = Z_{U,t} \frac{(1 - b_C)^\sigma \left[ C_t (j) - b_C C_{t-1} (j) \right]^{1-\sigma} - 1}{1 - \sigma}, \tag{15}$$

$$V_t \left[ l_\tau (j) \right] = Z_{V,t} \frac{(1 - b_l)^{-\zeta} \left[ l_t (j) - b_l l_{t-1} (j) \right]^{1+\zeta}}{1 + \zeta}. \tag{16}$$

$b_C$ and $b_l$ are habit formation parameters in consumption and labor supply respectively. $\sigma$ and $\varsigma$ determine the intertemporal elasticity of substitution in consumption and labor supply.

### 2.2.1 Budget constraint

Because we incorporate the dynamic entry behavior of firms, the number of firms is determined endogenously. Therefore, household income depends on the number of new-entry and incumbent firms. A firm's operating profit $\Pi_t (h)$ is paid to households as a dividend income through the purchase of the mutual fund. Households recognize that some shares are not carried into the next period. The dividend income, the value of selling and purchasing shares, also depends on the number of firms.

During period $t$, the representative home household buys $x_{t+1}$ shares in the mutual fund of $n_t' \equiv n_t + n_{E,t}$ home firms (those already operating at time $t$ and the new entrants). Only $n_{t+1} = (1 - \delta) n_t'$ firms produce and pay dividends at time $t+1$. As explained above, the dynamics of $n_t$ are already defined as follows:

$$n'_t \equiv n_t + n_{E,t}, \tag{17}$$
$$n_{t+1} = (1-\delta)\, n'_t,$$

and therefore:

$$n_{t+1} = (1-\delta)\,(n_t + n_{E,t})\,.$$

Reflecting these dynamics, the budget constraint of household $j$ now becomes:

$$\begin{aligned} & \mathcal{E}_t B_{F,t+1}(j) + B_{H,t+1}(j) + x_{t+1}(j) \int_0^{n'_t} \varpi_t(h)\, dh \\ \leq \; & (1+i^*)\left[1-\Gamma_{B,t}(j)\right] \mathcal{E}_t B_{F,t}(j) + (1+i_t)\, B_{H,t}(j) \\ & + W_t(j)\, l_t(j) \left[1-\Gamma_{W,t}(j)\right] \\ & + x_t(j) \int_0^{n_t} \left[\Pi_t(h) + \varpi_t(h)\right] dh \\ & - P_t C_t(j)\,. \end{aligned} \tag{18}$$

Several adjustment costs are assumed in this budget constraint. First, following the standard GEM, we assume that each household faces the following cost when trading bonds denoted in foreign currency:[10]

$$\Gamma_{B,t}(j) = \phi_{B1} \frac{\exp\left[\phi_{B2} \frac{\mathcal{E}_t B^*_{F,t}(j)}{P_t} - Z_{B0}\right] - 1}{\exp\left[\phi_{B2} \frac{\mathcal{E}_t B^*_{F,t}(j)}{P_t} - Z_{B0}\right] + 1} + Z_{B,t}. \tag{19}$$

Because household $j$ is the monopolistic supplier of the differentiated labor supply, it has wage-setting power. Similar to the price setting by firms, each household faces a Rotemberg-type nominal wage adjustment cost as below:

$$\Gamma_{W,t}(j) = \frac{\phi_W}{2} \left[\frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1\right]^2. \tag{20}$$

The consumer's optimization problem is to maximize equation (14) subject to equations (15) to (20) with respect to $C_t(j)$, $W_t(j)$, $B_{H,t+1}(j)$, $B_{F,t+1}(j)$, and $x_{t+1}$.

### 2.2.2 Euler equation: bonds

The Euler equation below is obtained by differentiating the objective with respect to home $B_{H,t+1}$ and foreign bond holding $B_{F,t+1}$.

$$1 = (1+i_t)\, E_t D_{t,t+1}(j) = (1+i^*_t)\left[1-\Gamma_{B,t+1}(j)\right] \mathrm{E}_t \left[D_{t,t+1}(j) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right],$$

where $D_{t,\tau}(j)$ is the stochastic discount factor:

$$D_{t,\tau}(j) \equiv \beta^{\tau-t} \frac{P_t U'\left[C_\tau(j)\right]}{P_\tau U'\left[C_t(j)\right]}.$$

[10] Without such a cost, we cannot obtain the unique rational expectation equilibrium. For details, see Schmitt-Grohe and Uribe (2003) and Bodenstein (2006).

### 2.2.3 Euler equation: shares

Concerning the optimal level of the mutual fund position, the first-order condition below is obtained:

$$\varpi_t(h) = (1-\delta)\,\mathrm{E}_t D_{t,t+1}(j)\left[\Pi_{t+1}(h) + \varpi_{t+1}(h)\right].$$

This equation equates returns, namely it produces a nonarbitrage condition from holding of $B_{H,t+1}$, $B_{F,t+1}$, and $x_{t+1}$.

### 2.2.4 Wage equation (labor supply)

Wage setting by households is expressed as:

$$\begin{aligned}\psi_t \frac{V_t'(j)}{U_t'(j)}\frac{P_t}{W_t(j)} &= (\psi_t - 1)\left[1-\Gamma_{W,t}(j)\right] + \left[W_t(j)\frac{\partial\Gamma_{W,t}(j)}{\partial W_t(j)}\right] \\ &\quad + \mathrm{E}_t\left[D_{t,t+1}(j)\frac{l_{t+1}(j)}{l_t(j)}W_{t+1}(j)\frac{\partial\Gamma_{W,t}(j)}{\partial W_t(j)}\right],\end{aligned}$$

where

$$V_t'(j) = Z_{V,t}\left[\frac{l_t(j) - b_l l_{t-1}(j)}{1-b_l}\right]^{\zeta}.$$

Each household $j$ supplies differentiated labor $l(h,j)$ at wage $W(j)$. We assume that firm $h$ has the CES aggregator of the differentiated labor $l(j)$:

$$l_t(h) = A_{l,t}\left[\int_0^{L_t} l_t(h,j)^{1-\frac{1}{\psi_t}}\,dj\right]^{\frac{\psi_t}{\psi_t-1}},$$

where

$$A_{l,t} \equiv L_t^{\gamma_l - \frac{\psi_t}{1-\psi_t}}.$$

Cost minimization implies that firm $h$'s demand for labor input $l(h,j)$ is a function of the relative wage:

$$l_t(h,j) = A_{l,t}^{\psi_t-1}\left[\frac{W_t(j)}{W_t}\right]^{-\psi_t} l_t(h),$$

and

$$l_t(e,j) = A_{l,t}^{\psi_t-1}\left[\frac{W_t(j)}{W_t}\right]^{-\psi_t} l_t(e)$$

where $W(j)$ is the nominal wage paid to home labor input $j$, and the wage index is defined as:

$$W_t = \frac{1}{A_{l,t}}\left[\int_0^{L_t} W_t(j)^{1-\psi_t}dj\right]^{\frac{1}{1-\psi_t}}.$$

### 2.2.5 Net foreign asset and exchange rate

We can define the holdings of net foreign assets as follows:

$$F_t(j) = (1 + i^*) \left[1 - \Gamma_{B,t}(j)\right] \mathcal{E}_t B_{F,t}(j).$$

Then, the nominal exchange rate is determined by the identity of the balance of payments:

$$\begin{aligned} \mathrm{E}_t D_{t,t+1} L_{t+1} F_{t+1}(j) &= L_t F_t(j) + (1 + i^*_{t-1}) \Gamma_{B,t-1}(j) \mathcal{E}_t L_t B_{F,t}(j) \\ &\quad + \mathcal{E}_t P^*_{M,t} L^*_t M^*_t(j^*) - P_{M,t} L_t M_t(j). \end{aligned}$$

## 2.3 Monetary Authority

Because there is no tax collection, only the central bank conducts stabilization policy. The central bank facilitates households' consumption smoothing and at the same time works as a nominal anchor to obtain a unique rational expectations equilibrium. Following Juillard, Karam, Laxton and Pesenti (2005), we employ a simple monetary policy instrument rule:[11]

$$\begin{aligned} (1 + i_t)^4 - 1 &= \omega_i \left[(1 + i_{t-1})^4 - 1\right] + (1 - \omega_i) \left[\left(1 + \frac{\pi_{t+1}}{\beta}\right)^4 - 1\right] \\ &\quad + \omega_1 E_t (\pi_t - \overline{\pi}), \end{aligned}$$

where $\overline{\pi}$ is the target level of inflation, and

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

## 2.4 Market Clearing

### 2.4.1 Industry equilibrium

To close the model, we need to clear goods markets globally. This requires total revenue to equal world expenditure:

$$\begin{aligned} &\int_0^{n_t} \left[ p_t(h) \int_0^{L_t} Q_t(h, j)\, dj + \mathcal{E}_t p^*_t(h) \int_0^{L^*_t} M^*_t(h, j^*)\, dj^* \right] dh \\ = \; & \mu_t \int_0^{L_t} E_t(j)\, dj + (1 - \mu^*_t) \mathcal{E}_t \int_0^{L^*_t} E^*_t(j^*)\, dj^*. \end{aligned}$$

The left-hand side is the total industry revenue in the home and foreign markets, while the right hand side is the industry expenditure in each country. $E_t(j)$ denotes the total spending by consumer $j$, which is defined as:

$$E_t(j) = P_t C_t(j). \tag{21}$$

[11] A monetary policy rule can change the reaction of the markup and therefore the persistence and the size of the impulse responses. Therefore, as a benchmark, we show the responses from flexible price models, where monetary policy completely stabilizes the markup, in the next section.

$\mu_t$ is home consumer $j$'s expenditure share for home country goods, and $\mu_t^*$ is foreign consumer $j^*$'s expenditure share for foreign country goods, which are defined using equations (2) and (3) as:

$$\mu_t = \frac{P_{Q,t} Q_t(j)}{E_t(j)} = \frac{P_{Q,t} Q_t(j)}{P_t A_t(j)} = \nu \left( \frac{P_{Q,t}}{P_t} \right)^{1-\epsilon_{QM}},$$

and

$$\mu_t^* = \frac{P_{Q,t}^* Q_t^*(j)}{E_t^*(j^*)} = \frac{P_{Q,t}^* Q_t^*(j)}{P_t^* A_t^*(j^*)} = \nu^* \left( \frac{P_{Q,t}^*}{P_t^*} \right)^{1-\epsilon_{QM}^*}.$$

Furthermore, this relationship can be transformed to derive the number of firms in each industry:

$$n_t = \frac{\mu_Q \int_0^{L_t} E_t(j) dj + (1-\mu_Q^*) L_t^* \mathcal{E}_t E_t^*(j^*)}{p_t(h) L_t Q_t(h,j) + \mathcal{E}_t p_t^*(h) L_t^* M_t^*(h,j^*)}.$$

### 2.4.2 General equilibrium

In a general equilibrium, spending for goods in each period must be equal to the sum of the factor income of labor, the net income from bonds and investment income where costs of investing in new firms are excluded. Thus,

$$\begin{aligned} \int_0^{L_t} E_t(j)\, dj \;=\; & \int_0^{L_t} (1+i_t^*) \left[1 - \Gamma_{B,t}(j)\right] \mathcal{E}_t B_{F,t}(j)\, dj + \int_0^{L_t} (1+i_t) B_{H,t}(j)\, dj \\ & - \mathcal{E}_t \int_0^{L_t} B_{F,t+1}(j)\, dj - \int_0^{L_t} B_{H,t+1}(j)\, dj + \int_0^{L_t} W_t(j)\, l_t(j) \left[1 - \Gamma_{W,t}(j)\right] dj \\ & + \int_0^{n_t} \Pi_t(h)\, dh - \int_0^{n_{E,t}} \varpi_t(h)\, dh. \end{aligned} \tag{22}$$

In addition, the following resource constraint, namely the goods market clearing condition, needs to be satisfied:

$$Y_t(h) = \int_0^{L_t} Q_t(h,j) dj + (1+\tau) \int_0^{L_t^*} M_t^*(h,j^*) dj.$$

The factor market clearing conditions can be written as:

$$l_t(j) = \int_0^{n_t} l_t(h,j) dh + \int_0^{n_{E,t}} l_t(e,j)\, de,$$

These equations imply that labor and capital market clearing conditions are required so that the total supply of the factor should be equal to the factor demand for production and entry. Market clearing in the bond market requires:

$$\int_0^{L_t} B_{H,t}(j) dj = 0,$$

$$\int_0^{L_t} B_{F,t}(j) dj + \int_0^{L_t^*} B_{F,t}^*(j^*) dj^* = 0.$$

All the equations in the model are shown in the appendix.

## 2.5 Calibration

In this subsection, we show the calibration of major parameters. Basically, parameters are set following the previous research using the GEM. Table 1 shows the values of major parameters. The parameters that determine the nominal

| Parameter | Value | Description and Definitions |
|---|---|---|
| $\varepsilon$ | 1.5 | Elasticity of substitution between domestic and foreign goods |
| $\zeta$ | 3 | Inverse of Frisch elasticity |
| $\psi$ | 6 | $\psi/(\psi-1)$ is wage markup |
| $\theta$ | 6 | $\theta/(\theta-1)$ is price markup |
| $\phi_Q$ | 400 | Adjustment cost for price setting in domestic market |
| $\phi_M$ | 400 | Adjustment cost for price setting in foreign market |
| $\phi_W$ | 400 | Adjustment cost for nominal wage setting |
| $\gamma$ | 1.2 | Degree of taste for variety in goods |
| $\gamma_l$ | 1 | Degree of taste for variety in labor |
| $\beta$ | $1.03^{-0.25}$ | Subjective discount factor |
| $\delta$ | 0.025 | Firm exit shock |
| $\nu$ | 0.5 | Home bias parameter |
| $\phi_{B1}$ | 0.05 | Transaction−cost parameter in the bond market |
| $\phi_{B2}$ | 0.1 | Transaction−cost parameter in the bond market |
| $b_c$ | 0.83 | Habit persistence parameter in consumption |
| $b_l$ | 0 | Habit persistence parameter in labor supply |
| $1/\sigma$ | 0.8 | Intertemporal elasticity of substitution |

Table 1: Calibration

rigidity ($\phi_Q, \phi_M, \phi_W$), elasticity of substitution in production of intermediate goods and raw materials ($\xi, \xi^O$) and transaction cost parameters in the bond market ($\phi_{B1}, \phi_{B2}$) are set following Laxton and Pesenti (2003). The elasticity of substitution between domestic and imported goods ($\varepsilon$) is set to 1.5 according to Smets and Wouters (2002) and Chari, Kehoe and McGrattan (2002). We set the inverse of the Frisch elasticity($\zeta$), the habit persistence parameter in consumption ($b_c$) and the intertemporal elasticity of substitution ($1/\sigma$) following Julliard, Karam, Laxton and Pesenti (2005). We assume that there is no home bias in the calibration of $\nu$,. In the simulations below, symmetry between the home and foreign country is assumed. Furthermore, shocks are assumed to be very persistent. The AR (1) parameter for each shock is set to 0.9.

# 3 International Implications of Expanding Production Frontier

In this section, we check whether the directions of responses analytically derived by CMP remain valid in a model with richer and more realistic dynamics like ours. We examine three shocks, namely (1) productivity gains in manufacturing, (2) efficiency gains in creating new firms, and (3) increases in the labor force. The latter two are simulations that cannot be conducted using standard dynamic general equilibrium models with a fixed number of products. As for the efficiency gains, the reason is obvious because there is no firm entry. Increases in the labor force or working population have no effect on the per capita variables in standard models without the home market effect that stems from trade cost and endogenous variety. In each shock simulation, we first introduce the intuitive explanation of responses described in CMP. Then, we show and explain the impulse responses in our model.

There exist three major differences between simulations in this paper and those examined in CMP. First, as has been mentioned, the model is different. Our model contains much richer and more realistic dynamics such as the time-to-build constraint in the firm dynamics, and nominal price and wage rigidities. Second, the elasticity of substitution between domestic and foreign goods is set equal to that between domestic or foreign goods in CMP. This is somewhat contrary to the standard calibration in the literature of new open economy macroeconomics. Moreover, the functional form of the final goods aggregator employed in CMP is different from the one in our model. While the technology term stemming from the taste for variety is excluded from the aggregator in CMP, it is included in aggregated domestic and foreign goods in our baseline model. Finally, the model considered in CMP does not have any intrinsic dynamics. Hence, the direction of responses obtained in CMP is considered to be that of the steady-state (long-run) responses rather than the (short-run) dynamic responses, which reflect richer dynamics.

Regarding the second point above, to be able to compare the results from our model with those from CMP, we demonstrate the direction of responses in a modified model, where the same functional form for the consumption aggregator as in CMP is employed. Instead of using equations (1), (4), and (5), we use the same aggregating function for final goods as employed in CMP:

$$C_t(j) = A_t \left[ \int_0^{n_t} C(h,j)^{1-\frac{1}{\theta}} dh + \int_0^{n_t^*} C(f,j)^{1-\frac{1}{\theta}} dh \right]^{\frac{\theta}{1-\theta}},$$

where

$$A_t = (n_t + n_t^*)^{\gamma - \frac{\theta}{\theta-1}}.$$

We name this model the "elastic model" because the elasticity of substitution between home and foreign goods is set to be higher here than in our baseline model.[12]

Regarding the first point, we will show the simulation results of the flexible price counterpart[13] of both the baseline and the elastic model.[14] Although there

[12] We also conduct a sensitivity analysis for the different values of the intertemporal elasticity of substitution, whose importance in international spillovers is emphasized in CMP. Indeed, the intertemporal elasticity of substitution alters the directions of responses in both the short run and the long run, but it does not produce any puzzling responses compared with CMP within our range of the examined values for the intertemporal elasticity of substitution.

[13] To understand the role of the time-to-build constraint for firm dynamics is left for our future research. Instead of using time-to-build, we now consider incorporating the simple firm dynamics employed in Bergin and Corsetti (2006), where all firms exist only for a period.

[14] We set the Rotemberg-type adjustment cost for nominal price and wage rigidities to zero

is no nominal rigidity in this model, the welfare-based price level still fluctuates as the number of firms existing in the economy changes. Regarding the third point, the short-run as well as the long-run—namely the steady-state—effects of these shocks will also be demonstrated in each model simulation.

The variables of interest are, as in CMP, the number of firms, the terms of trade, the real exchange rate and the welfare-based CPI index, because we focus our attention on the international spillovers of production-enhancing technology via international relative prices. The terms of trade are on a firm basis. Therefore, it is the international relative price in the intensive margin. On the other hand, the real exchange rate is denominated by the welfare-based CPI index, which considers the taste for variety. Hence, this is considered to be the international relative price in the extensive margin. Because the terms of trade in this paper are defined as the ratio of import prices over export prices, the positive reaction means worsening of terms of trade. On the other hand, the real exchange rate is defined in a standard manner where it is appreciating when it becomes smaller. In this paper, the numeraire is assumed to be the welfare-based CPI index while it is the nominal wage in CMP. Hence, the responses of the welfare-based price index below are inverse to those of the real wage denominated by the welfare-based CPI index.

## 3.1 Productivity Gains in Manufacturing

According to CMP, from a microeconomic perspective, an increase in technology results in lower marginal costs. Therefore, each firm intends to lower its price so as to expand its sales share in the market. On the other hand, from a macroeconomic perspective, this intensified competition among firms lowers the profits of each firm because of the reduced price. If the wealth effect stemming from productivity gains in manufacturing on leisure is strong, increases in consumption demand become mild. As a result, profits are not sufficient to cover the entry cost, and some firms exit from the market. If such wealth effects are not very strong, several new firms consider it profitable to enter the market because the expected profit stream is large enough to cover the entry cost. As for the effects on the terms of trade, reflecting the lower price charged by the domestic firms, the terms of trade worsen in the domestic country reflecting the productivity gains. The domestic price level becomes lower because of the reduced marginal cost as well as the increased number of varieties, causing the welfare-based price level to decrease. Reflecting these developments in prices, the real exchange rate depreciates. According to CMP, directions of responses of the foreign variables, namely the number of firms and the welfare-based price level, are ambiguous.

### 3.1.1 Baseline model

Figure 3 below shows the dynamic responses for the productivity gains in manufacturing. The directions of responses are consistent with those obtained

and the coefficient on the inflation dynamics in a monetary policy rule to be very large.

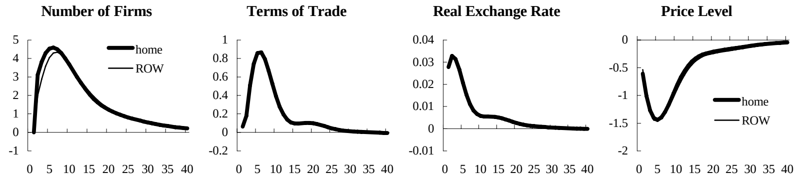


Figure 3: Baseline

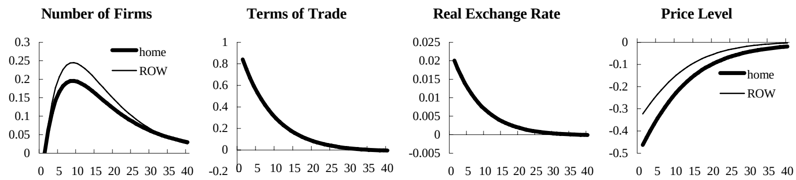


Figure 4: Baseline (Flex Price)

in CMP. In our model simulation, the increased technology in manufacturing causes profits to increase high enough for new firms to start business. As explained in CMP, the terms of trade deteriorate because the lowered marginal cost reduces the price of each domestic good. The welfare-based price level in the domestic country is also decreased because of the increased number of firms and the reduced marginal cost. The difference between the domestic and foreign welfare-based price levels is very small, but both the increase in the number of firms and the degree of reduction in the marginal cost is larger in the domestic than in the foreign country. The fact that the domestic price level becomes smaller than the foreign counterpart contributes to the deprecation in the real exchange rate. However, the depreciation intrinsically stems from the changes in the international relative price of the intensive margin, namely the terms of trade, because the effects via the lowered marginal cost dominates that via changes in the number of firms. Responses in a flexible price model are demonstrated in Figure 4. Because of the time-to-build constraint in the firm dynamics, the number of firms increases only gradually as observed in the baseline model. The directions of responses are consistent with those predicted in CMP. It is now clear that because of the lowered marginal cost, the welfare-based CPI index in the domestic countries becomes smaller than that in the foreign country. As a result of these price developments, the real exchange rate significantly depreciates for the domestic country.

We also checked the long-run directions of the responses for the productivity gain, namely the changes in the steady state after the increase in technology, which are shown in Table 2 below. They turn out to be the same as those of

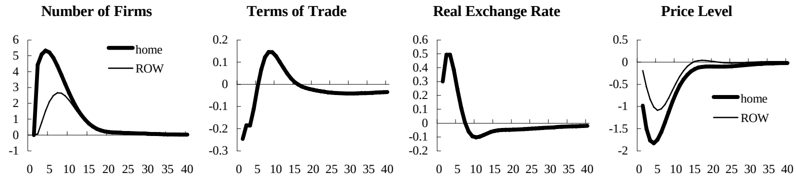


Figure 5: Elastic

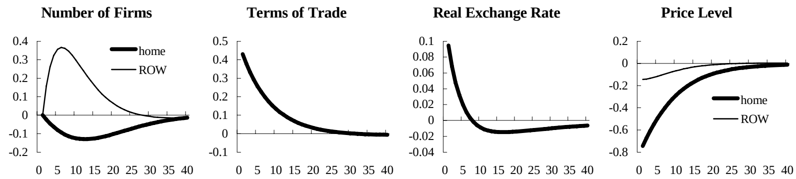


Figure 6: Elastic (Flex Price)

the dynamic responses.[15]

### 3.1.2 Elastic model

Figure 5 demonstrates the responses in a model where the same functional form of the consumption aggregator used in CMP is employed. Because the elasticity of substitution between foreign and domestic products is high in this model, the necessity of purchasing foreign goods becomes smaller. Furthermore, nominal rigidities prevent domestic firms from reducing their profits too much through competition because prices set by the individual firms do not fall significantly and immediately. Consequently, the increase in the number of domestic firms is much greater than in the former two cases. Contrary to the CMP prediction, the terms of trade improve albeit temporarily. As you can clearly see from Figure 6, this is because of the existence of nominal stickiness in our model. Because the price and nominal wage are adjusted only sluggishly in the former exercise in Figure 5, the initial decrease in the prices set by individual firms becomes mild. This is the main reason for the initial improvement in the terms of trade in Figure 5. Although the responses in Figure 6 are consistent with the prediction in CMP, an interesting finding here is that the number of domestic varieties now decreases while it increases in Figure 5. Without nominal rigidities, prices set by the individual firms become much lower than those by foreign firms. In other words, the intensified competition among firms lowers the profits in each domestic firm more significantly without nominal rigidities. As a result, the

[15] Because nominal rigidities have no influence on the steady state, the long-run responses in this flexible price model are the same as the baseline model. Therefore, we will not mention the long-run responses in the flexible price model henceforth.

number of firms decreases in the domestic country. The fact that the welfare-based price level in the domestic country is still smaller than that in the foreign country even in the presence of such firm dynamics seems somewhat puzzling. Yet, this just reflects the fact that the effects through changes in the intensive margin, namely the decrease in the real marginal cost, dominate the effects through the extensive margin.

The long-run directions of the responses are the same as those of the dynamic responses, except for the number of firms in the domestic country, as shown in Table 2. This suggests that the persistence of the shock is also a key factor in determining the directions of the responses because it can change the size of the wealth effects.

### 3.1.3 Summary

Table 2 summarizes the directions of responses in both CMP and our model. We can conclude that concerning the productivity gain, the results obtained in

| | CMP | baseline (low elasticity) | | | high elasticity | | |
|---|---|---|---|---|---|---|---|
| | | short-run | | long-run | short-run | | long-run |
| | | sticky | flex. | | sticky | flex | |
| n | **?** | + | + | + | + | - | + |
| n* | **?** | + | + | + | + | + | + |
| TOT | + | + | + | + | - **to** + | + | + |
| RER | + | + | + | + | + | + | + |
| P | - | - | - | - | - | - | - |
| P* | **?** | - | - | - | - | - | - |

Table 2: Summary Table for Productivity Gains

CMP hold in a model with much richer dynamics than the GEM. There are several intriguing points. First, both the nominal rigidities and the elasticity of substitution between domestic and foreign goods are crucial in determining the directions of short-run responses. Furthermore, even though the shock is assumed to be very persistent, the directions of initial responses to a temporary shock can be different from those to a permanent shock. This suggest that for a proper conduct of monetary policy, we need to check simulations not only from the theoretical model but also from a model with richer dynamics like the GEM.

## 3.2 Efficiency Gains in Creating New Firms (Goods)

Now, we check the results obtained in CMP for the efficiency gain associated with creating new firms. CMP conclude that the lower fixed cost to entry naturally increases the number of varieties in the domestic economy. As there are no direct changes[16] in the real marginal cost, prices set by individual firms do not change at all. Under such circumstances, the terms of trade correspond one-to-one with nominal exchange rates. As nominal exchange rates appreciate reflecting an increase in the relative demand for home labor, the home terms of

[16] There exist indirect effects to alter the real marginal cost because of the assumption of sticky price.

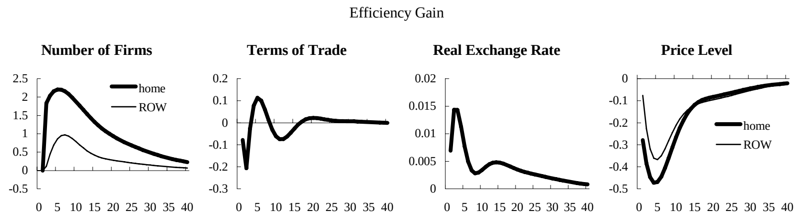


Figure 7: Baseline

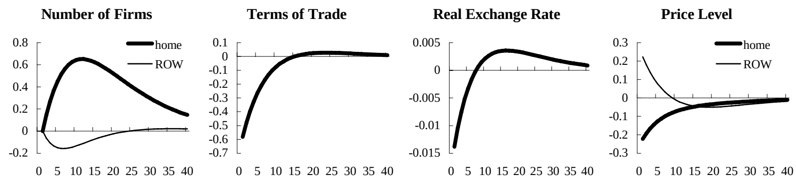


Figure 8: Baseline (Flex Price)

trade improve. This is because the prices set by the individual firms are solely determined by the nominal wage as shown in equations (7) and (9) because labor is the only production factor in CMP and our model. However, as the number of firms increases more in the domestic country, the welfare-based price index becomes smaller in the domestic country than in the rest of the world as the array of home products increases more. Reflecting these developments in prices, the real exchange rate depreciates for the domestic country.

### 3.2.1 Baseline model

Figure 7 shows the responses in the baseline model. All responses are consistent with the prediction by CMP. The number of firms increases, the terms of trade improve, but the real exchange rate depreciates because the welfare-based CPI index becomes smaller in the domestic country than in the foreign country. The direction of the response of the terms of trade is not significant. It tends to fluctuate rather than improving as predicted by CMP. This reflects the fluctuations in the markup because of nominal rigidities.[17] As shown in Figure 8, the response of the terms of trade is exactly what is expected by CMP. The fact that the real exchange rate initially appreciates is somewhat puzzling. Yet, as the welfare-based price level is reduced more significantly in the domestic than in the foreign country, the appreciation in the nominal exchange rate is very strong in this economy. Because the terms of trade correspond one-to-one with

[17] As will be shown below, the elasticity of substitution between home and foreign products also affects the direction of the terms of trade.

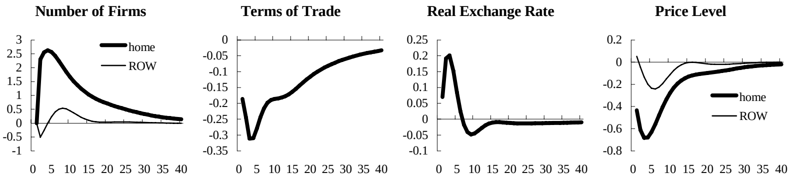


Figure 9: Elastic

the nominal exchange rate, the nominal exchange rate is appreciating thanks to the strong demand (for labor) in the domestic country. A reduction in the entry cost induces investment for creating new firms, but, because of the existence of the time-to-build constraint for firm creation, an immediate increase in the number of firms is prevented.[18] As a result, the channel to increase the demand for investing in new firms becomes more significant soon after the shock hits the economy. According to this channel, a reduction in the entry fixed cost, as if it were a standard demand shock, raises the price set by each domestic firm in the early stage of responses.[19]

As for the long-run responses, Table 3 below demonstrates that they are consistent with the prediction in CMP. However, responses of some variables in the long run differ from those in the short run because the wealth effects becomes more prevalent with the permanent shock.

### 3.2.2 Elastic model

The responses in a model with a high elasticity of substitution between foreign and domestic products are very much in accordance with those obtained in CMP as demonstrated in Figure 9. The home households do not have to rely on the supply of the foreign products in this case. Therefore, the number of products increases in the domestic country as stressed in CMP, while that in the foreign country decreases. Such firm dynamics with the existence of the transportation cost improves the home demand condition and tightens the home labor market. Therefore individual goods prices become higher in the domestic than in the foreign country. This home market effect results in the secure improvement in the terms of trade.

Responses in the flexible price counterpart to this model are shown in Figure 10. Because there are no fluctuations in the markup, entering the domestic market becomes more profitable reflecting the clearly improved terms of trade. For the same reason mentioned for responses in Figure 8, the real exchange rate initially appreciates.

[18] In particular, in the period when the shock hits the economy, the number of firms cannot increase.

[19] This implies that the time-to-build constraint may also affect the dynamic responses for expanding the production frontier.

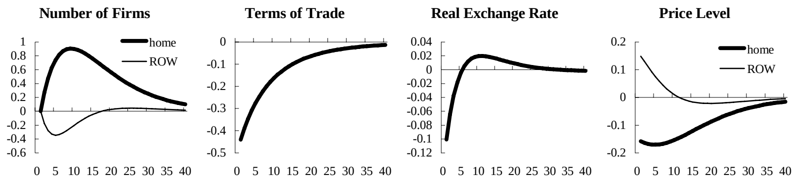


Figure 10: Elastic (Flex Price)

Again, the long-run responses are consistent with the prediction in CMP, but some differences between short-run and long-run responses are still observed as shown in Table 3.

#### 3.2.3 Summary

Directions of both short-run and long-run responses are summarized in Table 3. Again, as for the case with the baseline model, except for several short-run

| | CMP | baseline (low elasticity) | | | high elasticity | | |
|---|---|---|---|---|---|---|---|
| | | short-run | | long-run | short-run | | long-run |
| | | sticky | flex. | | sticky | flex | |
| n | + | + | + | + | + | + | + |
| n* | **?** | + | - | + | **- to +** | - | + |
| TOT | - | **- to +** | - | - | - | - | - |
| RER | + | + | + | + | + | **- to +** | + |
| P | - | - | - | - | - | - | - |
| P* | **?** | - | - | - | - | + | - |

Table 3: Summary Table for Efficiency Gains

responses, the prediction by CMP still holds in the case with efficiency gains from creating new firms. The differences in directions between short-run and long-run responses again suggest the importance of the degree of the wealth effects, and the dynamics embedded in our model such as the time-to-build constraint and the nominal rigidities.

### 3.3 Increase in Labor Force

CMP concludes that the responses for an increase in the labor force should be similar to those for the efficiency gain from firm entry. However, if the elasticity of substitution between home and foreign goods is low, we can obtain different result from those obtained in the previous subsection. We can see a clear difference in impulse responses between a model with richer dynamics and the theoretical model considered in CMP.

#### 3.3.1 Baseline model

Figure 11 demonstrates the responses for an exogenous increase in the labor force. Contrary to the prediction by CMP, the terms of trade deteriorate in the

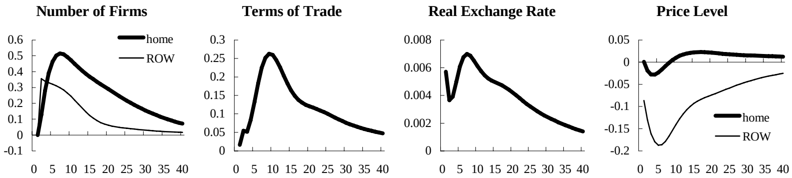


Figure 11: Baseline

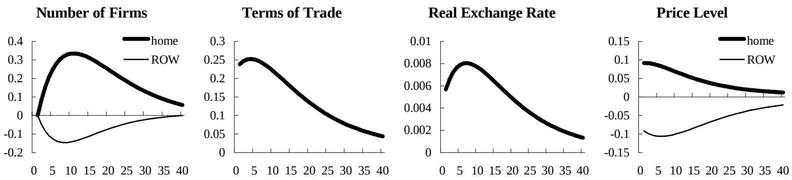


Figure 12: Baseline (Flex Price)

baseline model. CMP notes that "a larger home market appreciates the price of home labor relative to its foreign counterparts, thus improving the home terms of trade .... Therefore, the 'home market effect' here takes the form of a terms of trade appreciation." As the elasticity of substitution between home and foreign goods is low in our baseline model compared with CMP, an increased number of households need to purchase lots of foreign goods, which need to be produced in the foreign country with an unchanged working population. As a result, in our model, a larger home market appreciates the price of foreign labor relative to its home counterparts. Therefore, the terms of trade in the domestic country weaken.

These developments will be understood more intuitively by looking at the responses in a baseline model under the flexible price assumption shown in Figure 12. Similarly to the case with the reduction in the entry cost examined above, an exogenous increase in the working population does not have any direct effects on the real marginal cost. Therefore, the terms of trade move in line with nominal exchange rates. This is the main reason for the real exchange rate depreciation. In this paper, the response of the welfare-based price level is defined as the inverse of the real wage so that the measure of the welfare-based price index is the same as the one defined in CMP, which assumes the nominal wage in the home country to be a numeraire. The welfare-based price index implies a higher real wage in the foreign than in the domestic country. This is also the reason for the nominal exchange depreciation that results in the worsening of the terms of trade and the real exchange rate depreciation.

As for the long-run responses shown in Table 4, the terms of trade deteriorate and the welfare-based CPI index in the home country increases for a

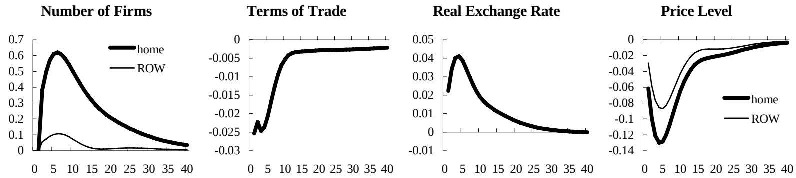


Figure 13: Elastic

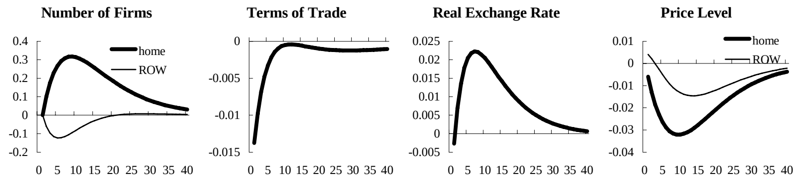


Figure 14: Elastic (Flex Price)

permanent increase in the labor force as observed in the short-run responses above. Therefore, shock persistence is not the reason for this difference from CMP's results. We seek the reason for this divergence below.

### 3.3.2 Elastic model

Responses in the elastic model are shown in Figure 13. Contrary to the simulation with the baseline model, the terms of trade appreciate reflecting the home market effects. The welfare-based price index decreases in the domestic country. By using the same consumption aggregator employed in CMP, the elasticity of substitution between home and foreign goods becomes larger in this model than in the baseline model. Now, the domestic market expansion does not affect the foreign economy very significantly because the domestic households can increase their welfare by purchasing more domestic goods. Hence, the price of each domestic good becomes higher than its foreign counterpart. As a result, the terms of trade in the domestic country improve thanks to this home market effect. Furthermore, to be consistent with the prediction by CMP, the larger domestic market reduces the domestic welfare-based price index thanks to the increase in the number of domestic firms.

The responses in the flexible price counterpart model in Figure 14 show similar results. We can see that fluctuations in the real marginal cost because of nominal rigidities change the responses of international relative prices and therefore the responses of the number of firms in the foreign country.

The long-run responses are also consistent with the theoretical prediction made by CMP as demonstrated in Table 4. They are only slightly different from the short-run responses.

### 3.3.3 Summary

The simulation results for the larger domestic market are summarized in Table 4. The main conclusion from this subsection is that by setting the elasticity of

| | CMP | baseline (low elasticity) | | | high elasticity | | |
|---|---|---|---|---|---|---|---|
| | | short-run | | long-run | short-run | | long-run |
| | | sticky | flex. | | sticky | flex | |
| n | + | + | + | + | + | + | + |
| n* | **?** | + | - | + | + | - | + |
| TOT | - | + | + | + | - | - | - |
| RER | + | + | + | + | + | + | + |
| P | - | **- to** + | + | + | - | - | - |
| P* | **?** | - | + | - | - | - | - |

Table 4: Summary Table for Increased Labor Force

substitution between home and foreign goods to be equal to that in CMP, we can obtain the same direction of responses. Therefore, we can conclude that this elasticity is very important in determining the responses of international relative prices, namely the degree of international spillovers, in a model with endogenous variety.

# 4 Conclusion

In this paper, we have shown how economic variables respond to shocks that shift the production frontier outwards, namely productivity gains in manufacturing, efficiency gains in creating new firms, and increases in the labor force, in a two-country model. For this purpose, contrary to the theoretical model used in CMP, we set up a model that contains richer and more realistic dynamics embedded in the GEM such as nominal price and wage stickiness. Our main conclusions are: (1) the elasticity of substitution between domestic and foreign goods is crucial in determining the direction of responses for a shock that expands the production frontier; (2) nominal rigidities also alter the short-run responses by changing the responses of the markup, and (3) persistence in shocks also matters for the determination of the direction of responses because it changes the size of the wealth effect. The elasticity of substitution between home and foreign goods is important in determining the size of international spillovers, and the nominal rigidities have significant effects on the price setting and therefore the markup determination, while the magnitude of the wealth effects is dependent on the persistence of the shocks. As a result, these factors are naturally considered to be very important determinants of the impulse responses. These results suggest that for a proper conduct of monetary policy, we need to check simulations not only from a theoretical model but also from a model with richer and more realistic dynamics like the GEM.

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Appendix

# Model Equations

## Final Goods Production

$$C_t(j) \qquad C_t(j) = \left[\nu_Q^{\frac{1}{\varepsilon}} Q_t(j)^{1-\frac{1}{\varepsilon}} + (1-\nu_Q)^{\frac{1}{\varepsilon}} M_t(j)^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_t^*(j^*) \qquad C_t^*(j^*) = \left[(\nu_Q^*)^{\frac{1}{\varepsilon^*}} (Q_t^*(j^*))^{1-\frac{1}{\varepsilon^*}} + (1-\nu_Q^*)^{\frac{1}{\varepsilon^*}} (M_t^*(j^*))^{1-\frac{1}{\varepsilon^*}}\right]^{\frac{\varepsilon^*}{\varepsilon^*-1}}$$

## Demand for Intermediate Goods

Household's demand for aggregated intermediate goods

$$Q_t(j) \qquad Q_t(j) = \nu_Q \left(\frac{P_{Q,t}}{P_t}\right)^{-\varepsilon} C_t(j)$$

$$Q_t^*(j^*) \qquad Q_t^*(j^*) = \nu_Q^* \left(\frac{P_{Q,t}^*}{P_t^*}\right)^{-\varepsilon^*} C_t^*(j^*)$$

$$M_t(j) \qquad M_t(j) = (1-\nu_Q) \left(\frac{P_{M,t}}{P_t}\right)^{-\varepsilon} C_t(j)$$

$$M_t^*(j^*) \qquad M_t^*(j^*) = (1-\nu_Q^*) \left(\frac{P_{M,t}^*}{P_t^*}\right)^{-\varepsilon^*} C_t^*(j^*)$$

Household's demand for aggregated intermediate goods produced by each firm

$$Q_t(h,j) \qquad Q_t(h,j) = A_{Q,t}^{\theta-1} \left(\frac{p_t(h)/P_t}{P_{Q,t}/P_t}\right)^{-\theta} Q_t(j)$$

$$Q_t^*(f,j^*) \qquad Q_t^*(f,j^*) = (A_{Q,t}^*)^{\theta^*-1} \left(\frac{p_t^*(f)/P_t^*}{P_{Q,t}^*/P_t^*}\right)^{-\theta^*} Q_t^*(j^*)$$

$$M_t(f,j) \qquad M_t(f,j) = \left(A_{Q,t}^*\right)^{\theta^*-1} \left(\frac{p_t(f)/P_t}{P_{M,t}/P_t}\right)^{-\theta^*} M_t(j)$$

$$M_t^*(h,j^*) \qquad M_t^*(h,j^*) = (A_{Q,t})^{\theta-1} \left(\frac{p_t^*(h)/P_t^*}{P_{M,t}^*/P_t^*}\right)^{-\theta} M_t^*(j^*)$$

$$A_{Q,t} \qquad A_{Q,t} \equiv n_t^{\gamma-\frac{\theta}{\theta-1}}$$

$$A_{Q,t}^* \qquad A_{Q,t}^* \equiv (n_t^*)^{\gamma_Q^*-\frac{\theta^*}{\theta^*-1}}$$

## Relative price of intermediate goods

Aggregate price

$$\frac{P_{Q,t}}{P_t} \qquad \frac{P_{Q,t}}{P_t} = \frac{1}{A_{Q,t}} n_t^{\frac{1}{1-\theta}} \frac{p_t(h)}{P_t}$$

$$\frac{P_{Q,t}^*}{P_t^*} \qquad \frac{P_{Q,t}^*}{P_t^*} = \frac{1}{A_{Q,t}^*} (n_t^*)^{\frac{1}{1-\theta^*}} \frac{p_t^*(f)}{P_t^*}$$

$$\frac{P_{M,t}}{P_t} \qquad \frac{P_{M,t}}{P_t} = \frac{1}{A_{Q,t}^*} (n_t^*)^{\frac{1}{1-\theta^*}} \frac{p_t(f)}{P_t}$$

$$\frac{P_{M,t}^*}{P_t^*} \qquad \frac{P_{M,t}^*}{P_t^*} = \frac{1}{A_{Q,t}} n_t^{\frac{1}{1-\theta}} \frac{p_t^*(h)}{P_t^*}$$

## Price Setting

$$
\begin{array}{ll}
\frac{p_t(h)}{P_t} & 0 = (1-\Gamma_{Q,t}(h))\left[(1-\theta)\frac{p_t(h)}{P_t} + \theta\frac{MC_t(h)}{P_t}\right] - \left[\frac{p_t(h)}{P_t} - \frac{MC_t(h)}{P_t}\right] p_t(h)\Gamma'_{Q,t}(h) \\
 & -E_t\left[(1-\delta_D)D_{t,t+1}(j)\pi_{t+1}\frac{n_{H,t+1}Q_{t+1}(h,j)}{n_{H,t}Q_t(h,j)}\left(\frac{p_{t+1}(h)}{P_{t+1}} - \frac{MC_{t+1}(h)}{P_{t+1}}\right)\Gamma'_{Q,t+1}(h)\right] \\
\frac{p^*_t(h)}{P^*_t} & 0 = (1-\Gamma^*_{M,t}(h))\left[(1-\theta^*)\frac{p^*_t(h)}{P^*_t}\frac{\mathcal{E}_t P^*_t}{P_t} + \theta^*\frac{MC_t(h)}{P_t}(1+\tau_t)\right] \\
 & -\left[\frac{p^*_t(h)}{P^*_t}(1-\theta^*)\frac{\mathcal{E}_t P^*_t}{P_t} - \theta^*\frac{MC_t(h)}{P_t}(1+\tau_t)\right] p^*_t(h)\Gamma^{*\prime}_{M,t}(h) \\
 & -E_t\left[(1-\delta_D)D_{t,t+1}(j)\pi_{t+1}\frac{n^*_{H,t+1}M^*_{t+1}(h,j^*)}{n^*_{H,t}M^*_t(h,j^*)}\right. \\
 & \left.\times\left[\frac{p^*_{t+1}(h)}{P^*_{t+1}}\frac{\mathcal{E}_{t+1}P^*_{t+1}}{P_{t+1}} - \frac{MC_{t+1}(h)}{P_{t+1}}(1+\tau_t)\right] p^*_{t+1}(h)\Gamma^{*\prime}_{M,t+1}(h)\right] \\
\frac{p_t(f)}{P_t} & 0 = (1-\Gamma_{M,t}(f))\left[\frac{p_t(f)}{P_t}(1-\theta)\frac{P_t}{\mathcal{E}_t P^*_t} + \theta\frac{MC^*_t(f)}{P^*_t}(1+\tau^*_t)\right] \\
 & -\left[\frac{p_t(f)}{P_t}\frac{P_t}{\mathcal{E}_t P^*_t} - \frac{MC^*_t(f)}{P^*_t}(1+\tau^*_t)\right] p_t(f)\Gamma'_{M,t}(f) \\
 & -E_t\left[(1-\delta^*_D)D_{t,t+1}\frac{n_{H,t+1}M_{t+1}(f,j)}{n_{H,t}M_t(f,j)}\left[\frac{p_{t+1}(f)}{P_{t+1}}\frac{P_{t+1}}{\mathcal{E}_{t+1}P^*_{t+1}} - \frac{MC^*_{t+1}(f)}{P^*_{t+1}}(1+\tau^*_t)\right]\right. \\
 & \left.\times p_{t+1}(f)\Gamma'_{M,t+1}(f)\right] \\
\frac{p^*_t(f)}{P^*_t} & 0 = (1-\Gamma^*_{Q,t}(f))\left[(1-\theta^*)\frac{p^*_t(f)}{P^*_t} + \theta^*\frac{MC^*_t(f)}{P^*_t}\right] - \left[\frac{p^*_t(f)}{P^*_t} - \frac{MC^*_t(f)}{P^*_t}\right] p^*_t(f)\Gamma^{*\prime}_{Q,t}(f) \\
 & -E_t\left[(1-\delta^*_D)D^*_{t,t+1}\frac{n^*_{H,t+1}Q^*_{t+1}(f,j^*)}{n^*_{H,t}Q^*_t(f,j^*)}\left(\frac{p^*_{t+1}(f)}{P^*_{t+1}} - \frac{MC^*_{t+1}(f)}{P^*_{t+1}}\right) p^*_{t+1}(f)\Gamma^{*\prime}_{Q,t+1}(f)\right]
\end{array}
$$

## Price adjustment costs for differentiated intermediate goods

$$
\begin{array}{ll}
\Gamma_{Q,t}(h) & \Gamma_{Q,t}(h) \equiv \frac{\phi_Q}{2}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}} - 1\right)^2 \\
\Gamma'_{Q,t}(h) & \left[p_t(h)\Gamma'_{Q,t}(h)\right] = \phi_Q\frac{\pi_t(h)}{\pi_{Q,t-1}}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}} - 1\right) \\
\Gamma'_{Q,t+1}(h) & \left[D_{t,t+1}p_{t+1}(h)\Gamma'_{Q,t+1}(h)\right] = -D_{t,t+1}\phi_Q\frac{\pi_t(h)^2}{\pi_{Q,t-1}}\left(\frac{\pi_t(h)}{\pi_{Q,t-1}} - 1\right) \\
\Gamma^*_{Q,t}(f) & \Gamma^*_{Q,t}(f) \equiv \frac{\phi^*_Q}{2}\left(\frac{\pi^*_t(f)}{\pi^*_{Q,t-1}} - 1\right)^2 \\
\Gamma^{*\prime}_{Q,t}(f) & \left[P^*_{Q,t}\Gamma^{*\prime}_{Q,t}(f)\right] = \phi^*_Q\frac{\pi^*_t(f)}{\pi^*_{Q,t-1}}\left(\frac{\pi^*_t(f)}{\pi^*_{Q,t-1}} - 1\right) \\
\Gamma^{*\prime}_{Q,t+1}(f) & \left[D^*_{t,t+1}P^*_{Q,t+1}\Gamma^{*\prime}_{Q,t+1}(f)\right] = -D^*_{t,t+1}\phi^*_Q\frac{\pi^*_t(h)^2}{\pi^*_{Q,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{Q,t-1}} - 1\right) \\
\Gamma_{M,t}(f) & \Gamma_{M,t}(f) \equiv \frac{\phi_M}{2}\left(\frac{\pi_t(f)}{\pi_{M,t-1}} - 1\right)^2 \\
\Gamma'_{M,t}(f) & \left[P_{M,t}\Gamma'_{M,t}(f)\right] = \phi_M\frac{\pi_t(f)}{\pi_{M,t-1}}\left(\frac{\pi_t(f)}{\pi_{M,t-1}} - 1\right) \\
\Gamma'_{M,t+1}(f) & \left[D_{t,t+1}P_{M,t+1}\Gamma'_{M,t+1}(f)\right] = -D_{t,t+1}\phi_M\frac{\pi_t(h)^2}{\pi_{M,t-1}}\left(\frac{\pi_t(h)}{\pi_{M,t-1}} - 1\right) \\
\Gamma^*_{M,t}(h) & \Gamma^*_{M,t}(h) \equiv \frac{\phi^*_M}{2}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}} - 1\right)^2 \\
\Gamma^{*\prime}_{M,t}(h) & \left[p^*_t(h)\Gamma'_{M,t}(h)\right] = \phi^*_M\frac{\pi^*_t(h)}{\pi^*_{M,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}} - 1\right) \\
\Gamma^{*\prime}_{M,t+1}(h) & \left[D^*_{t,t+1}P^*_{M,t+1}\Gamma^{*\prime}_{M,t+1}(h)\right] = \\
 & -D^*_{t,t+1}\phi^*_M\frac{\pi^*_t(h)^2}{\pi^*_{M,t-1}}\left(\frac{\pi^*_t(h)}{\pi^*_{M,t-1}} - 1\right)
\end{array}
$$

## Marginal costs

Intermediate goods producers

$$
\begin{array}{ll}
\frac{W_t}{P_t} & \frac{MC_t(h)}{P_t} = \frac{1}{Z_{Q,t}} \left(\frac{W_t}{P_t}\right) \\
\frac{W_t^*}{P_t^*} & \frac{MC_t^*(f)}{P_t^*} = \frac{1}{Z_{Q,t}^*} \left(\frac{W_t^*}{P_t^*}\right)
\end{array}
$$

# Labor market

Taste for the variety of labor input

$$
\begin{array}{ll}
A_{l,t} & A_{l,t} = L_t^{\gamma_l - \frac{\psi}{\psi-1}} \\
A_{l,t}^* & A_{l,t}^* = (L_t^*)^{\gamma_l^* - \frac{\psi^*}{\psi^*-1}}
\end{array}
$$

Aggregate wage

$$
\begin{array}{ll}
\frac{W_t(j)}{P_t} & \frac{W_t}{P_t} = \frac{1}{A_{l,t}} L_t^{\frac{1}{1-\psi}} \frac{W_t(j)}{P_t} \\
\frac{W_t^*(j^*)}{P_t^*} & \frac{W_t*}{P_t^*} = \frac{1}{A_{l,t}^*} (L_t^*)^{\frac{1}{1-\psi^*}} \frac{W_t^*(j^*)}{P_t^*}
\end{array}
$$

Wage for each household

$$
\begin{array}{ll}
V_t'(j) & \psi \frac{V_t'(j)}{U_t'(j)} \frac{P_t}{W_t(j)} = (\psi - 1)\left(1 - \Gamma_{W,t}(j)\right) + W_t(j) \Gamma'_{W,t}(j) \\
 & + E_t \left[ D_{t,t+1} \frac{l_{t+1}(j)}{l_t(j)} W_{t+1}(j) \Gamma'_{W,t+1}(j) \right] \\
V_t^{*\prime}(j^*) & \psi^* \frac{V_t^{*\prime}(j^*)}{U_t^{*\prime}(j^*)} \frac{P_t^*}{W_t^*(j^*)} = (\psi^* - 1)\left(1 - \Gamma_{W,t}^*(j^*)\right) + W_t^*(j^*) \Gamma^{*\prime}_{W,t}(j) \\
 & + E_t \left[ D_{t,t+1}^* \frac{l_{t+1}^*(j^*)}{l_t^*(j^*)} W_{t+1}^*(j^*) \Gamma^{*\prime}_{W,t+1}(j) \right]
\end{array}
$$

# Utility functions

$$
\begin{array}{ll}
U_t'(j) & U_t'(j) = Z_{U,t} \left(\frac{C_t(j) - b_C C_{t-1}(j)}{1 - b_C}\right)^{-\sigma} \\
U_t^{*\prime}(j) & U_t^{*\prime}(j^*) = Z_{U,t}^* \left(\frac{C_t^*(j^*) - b_C^* C_{t-1}^*(j^*)}{1 - b_C^*}\right)^{-\sigma^*} \\
l_t(j) & V_t'(j) = Z_{L,t} \left(\frac{l_t(j) - b_l l_{t-1}(j)}{1 - b_l}\right)^{\zeta} \\
l_t^*(j^*) & V_t^{*\prime}(j^*) = Z_{V,t}^* \left(\frac{l_t^*(j^*) - b_l^* l_{t-1}^*(j^*)}{1 - b_l^*}\right)^{\zeta^*}
\end{array}
$$

# Wage adjustment costs

| | |
|---|---|
| $\Gamma_{W,t}(j)$ | $\Gamma_{W,t}(j) = \frac{\phi_W}{2}\left(\frac{\pi_{W,t}(j)}{\pi_{W,t-1}} - 1\right)^2$ |
| $\Gamma'_{W,t}(j)$ | $\left[W_t(j)\Gamma'_{W,t}(j)\right] = \phi_W \frac{\pi_{W,t}(j)}{\pi_{W,t-1}}\left(\frac{\pi_{W,t}(j)}{\pi_{W,t-1}} - 1\right)$ |
| $\Gamma'_{W,t+1}(j)$ | $\left[D_{t,t+1}W_{t+1}(j)\Gamma'_{W,t+1}(j)\right] =$ $-D_{t,t+1}\phi_W \frac{\pi_{W,t+1}(j)^2}{\pi_{W,t}}\left(\frac{\pi_{W,t+1}(j)}{\pi_{W,t}} - 1\right)$ |
| $\Gamma^*_{W,t}(j^*)$ | $\Gamma^*_{W,t}(j^*) = \frac{\phi^*_W}{2}\left(\frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}} - 1\right)^2$ |
| $\Gamma^{*\prime}_{W,t}(j^*)$ | $\left[W^*_t(j)\Gamma^{*\prime}_{W,t}(j)\right] = \phi^*_W \frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}}\left(\frac{\pi^*_{W,t}(j)}{\pi^*_{W,t-1}} - 1\right)$ |
| $\Gamma^{*\prime}_{W,t+1}(j^*)$ | $\left[D^*_{t,t+1}W^*_{t+1}(j)\Gamma^{*\prime}_{W,t+1}(j)\right] =$ $-D^*_{t,t+1}\left[\phi^*_W \frac{\pi^*_{W,t+1}(j)^2}{\pi^*_{W,t}}\left(\frac{\pi^*_{W,t+1}(j)}{\pi^*_{W,t}} - 1\right)\right]$ |

## Demand for labor input by an intermediate goods producer

Incumbents

| | |
|---|---|
| $l_t(h,j)$ | $l_t(h,j) = A_{l,t}^{\psi-1}\left(\frac{W_t(j)/P_t}{W_t/P_t}\right)^{-\psi} l_t(h)$ |
| $l_t(h)$ | $l_t(h) = \frac{Y_t(h)}{Z_t}$ |
| $l^*_t(f,j^*)$ | $l^*_t(f,j^*) = (A^*_{l,t})^{\psi^*-1}\left(\frac{W^*_t(j^*)/P^*_t}{W^*_t/P^*_t}\right)^{-\psi^*} l^*_t(f)$ |
| $l^*_t(f)$ | $l^*_t(f) = \frac{Y^*_t(f)}{Z^*_t}$ |

Entrants

| | |
|---|---|
| $l_t(e,j)$ | $l_t(e,j) = A_{l,t}^{\psi-1}\left(\frac{W_t(j)/P_t}{W_t/P_t}\right)^{-\psi} l_t(e)$ |
| $l_t(e)$ | $l_t(e) = f_{E,t}$ |
| $l^*_t(e^*,j^*)$ | $l^*_t(e^*,j^*) = (A^*_{l,t})^{\psi^*-1}\left(\frac{W^*_t(j^*)/P^*_t}{W^*_t/P^*_t}\right)^{-\psi^*} l^*_t(e^*)$ |
| $l^*_t(e^*)$ | $l^*_t(e^*) = f^*_{E,t}$ |

## Profits and share prices

Euler equations of shares

| | |
|---|---|
| $\frac{\varpi_t(h)}{P_t}$ | $\frac{\varpi_t(h)}{P_t} = (1-\delta_D)D_{t,t+1}(j)\pi_{t+1}\left[\frac{\Pi_{t+1}(h)}{P_{t+1}} + \frac{\varpi_{t+1}(h)}{P_{t+1}}\right]$ |
| $\frac{\varpi^*_t(f)}{P^*_t}$ | $\frac{\varpi^*_t(f)}{P^*_t} = (1-\delta^*_D)D^*_{t,t+1}(j^*)\pi^*_{t+1}\left[\frac{\Pi^*_{t+1}(f)}{P^*_{t+1}} + \frac{\varpi^*_{t+1}(f)}{P^*_{t+1}}\right]$ |

Free entry conditions

| | |
|---|---|
| $\frac{MC_t(h)}{P_t}$ | $\frac{\varpi_t(h)}{P_t} = \phi_t(e)\frac{W_t}{P_t}$ |
| $\frac{MC^*_t(f)}{P^*_t}$ | $\frac{\varpi^*_t(h)}{P^*_t} = \phi^*_t(e^*)\frac{W^*_t}{P^*_t}$ |

Profits

$\frac{\Pi_t(h)}{P_t}$ $\quad \frac{\Pi_t(h)}{P_t} = \left(\frac{p_t(h)}{P_t} - \frac{MC_t(h)}{P_t}\right) n_t Q_t(h,j)][1-\Gamma_{Q,t}(h)]$
$\quad + \left[\frac{p_t^*(h)}{P_t^*}\frac{\mathcal{E}_t P_t^*}{P_t} - \frac{MC_t(h)}{P_t}(1+\tau_t)\right] n_t^* M_t^*(h,j^*)[1-\Gamma_{M,t}^*(h)]$
$\frac{\Pi_t^*(f)}{P_t^*}$ $\quad \frac{\Pi_t^*(f)}{P_t^*} = \left(\frac{p_t^*(f)}{P_t^*} - \frac{MC_t^*(f)}{P_t^*}\right) n_t^* Q_t(f,j)][1-\Gamma_{Q,t}^*(f)]$
$\quad + \left[\frac{p_t(f)}{P_t}\frac{P_t}{\mathcal{E}_t P_t^*} - \frac{MC_t^*(f)}{P_t}(1+\tau_t^*)\right] n_t M_t(f,j)[1-\Gamma_{M,t}(f)]$

# Dynamics of the number of firms

$n_t$ $\quad n_t = (1-\delta)(n_{t-1} + n_{E,t-1})$
$n_t^*$ $\quad n_t^* = (1-\delta^*)(n_{t-1}^* + n_{E,t-1}^*)$

# Exchange rates

$\frac{\mathcal{E}_t P_t^*}{P_t}$ $\quad E_t D_{t,t+1} L_{t+1}\frac{F_{t+1}(j)}{P_{t+1}}\pi_{t+1} = L_t\frac{F_t(j)}{P_t} + (1+i_{t-1}^*)\Gamma_{B,t-1}(j)\frac{\mathcal{E}_t P_t^*}{P_t} L_t \frac{B_{F,t}(j)}{P_{t-1}}\frac{1}{\pi_t}$
$+\frac{\mathcal{E}_t P_t^*}{P_t}\frac{P_{M,t}^*}{P_t^*} L_t^* M_t^*(j^*) - \frac{P_{M,t}}{P_t} L_t M_t(j)$

# Financial assets

$\frac{F_t(j)}{P_t}$ $\quad \frac{F_t(j)}{P_t} = (1+i_t^*)(1-\Gamma_{B,t}(j))\frac{1}{\pi_t^*}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t}(j)}{P_{t-1}^*}$

# Monetary Authority

$i_t$ $\quad (1+i_t)^4 - 1 = \omega_1\left[(1+i_{t-1})^4 - 1\right] + (1-\omega_1)\left[(1+\pi_{t+1}/\beta)^4 - 1\right]$
$\quad +\omega_2 E_t(\pi_t - \pi_{tar})$
$i_t^*$ $\quad (1+i_t^*)^4 - 1 = \omega_1^*\left[(1+i_{t-1}^*)^4 - 1\right] + (1-\omega_1^*)\left[(1+\pi_{t+1}^*/\beta^*)^4 - 1\right]$
$\quad +\omega_2^* E_t(\pi_t^* - \overline{\pi}^*)$

# Bond market

$D_{t,t+1}$ $\quad 1 = (1+i_t)E_t D_{t,t+1}$
$D_{t,t+1}^*$ $\quad 1 = (1+i_t^*)E_t D_{t,t+1}^*$
$\Gamma_{B,t}(j)$ $\quad 1 = (1+i_t^*)(1-\Gamma_{B,t+1}(j))E_t\left(D_{t,t+1}\frac{\mathcal{E}_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_t}{\mathcal{E}_t P_t^*}\frac{\pi_{t+1}}{\pi_{t+1}^*}\right)$
$\frac{B_{F,t+1}(j)}{P_t^*}$ $\quad \Gamma_{B,t}(j) = \phi_{B1}\frac{\exp\left(\phi_{B2}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t+1}(j)}{P_t^*} - Z_{B0,t}\right)-1}{\exp\left(\phi_{B2}\frac{\mathcal{E}_t P_t^*}{P_t}\frac{B_{F,t+1}(j)}{P_t^*} - Z_{B0,t}\right)+1} + Z_{B,t}$
$\frac{B_{F,t+1}^*(j)}{P_t}$ $\quad \frac{B_{F,t+1}^*(j^*)}{P_t} = -\frac{L_t}{L_t^*}\frac{B_{F,t+1}(j)}{P_t^*}$

# Market clearing

## Labor market

$n_{E,t}$ $\quad l_t(j) = n_t l_t(h,j) + n_{E,t} l_t(e,j)$
$n_{E,t}^*$ $\quad l_t^*(j^*) = n_t^* l_t^*(f,j^*) + n_{E,t}^* l(e^* j^*)$

Intermediate goods market

$Y_t(h) \quad Y_t(h) = L_t Q_t(h,j) + (1+\tau_t) L_t^* M_t^*(h,j^*)$
$Y_t^*(f) \quad Y_t^*(f) = L_t^* Q_t(f,j^*) + (1+\tau_t^*) L_t^* M_t(f,j)$

# Inflation rates

## Final goods

$\pi_t \quad D_{t,t+1} \equiv \beta \frac{U'_{t+1}}{\pi_{t+1} U'_t}$
$\pi_t^* \quad D_{t,t+1}^* \equiv \beta \frac{U^{*\prime}_{t+1}}{\pi_{t+1}^* U_t^{*\prime}}$

## Aggregate intermediate goods

$\pi_{Q,t} \quad \pi_{Q,t} = \frac{P_{Q,t}/P_t}{P_{Q,t-1}/P_{t-1}} \pi_t$
$\pi_{Q,t}^* \quad \pi_{Q,t}^* = \frac{P_{Q,t}^*/P_t^*}{P_{Q,t-1}^*/P_{t-1}^*} \pi_t^*$
$\pi_{M,t} \quad \pi_{M,t} = \frac{P_{M,t}/P_t}{P_{M,t-1}/P_{t-1}} \pi_t$
$\pi_{M,t}^* \quad \pi_{M,t}^* = \frac{P_{M,t}^*/P_t^*}{P_{M,t-1}^*/P_{t-1}^*} \pi_t^*$

## An intermediate good by each firm

$\pi_t(h) \quad \pi_t(h) = \frac{p_t(h)/P_t}{p_{t-1}(h)/P_{t-1}} \pi_t$
$\pi_t^*(f) \quad \pi_t^*(f) = \frac{p_t^*(f)/P_t^*}{p_{t-1}^*(f)/P_{t-1}^*} \pi_t^*$
$\pi_t(f) \quad \pi_t(f) = \frac{p_t(f)/P_t}{P_{t-1}(f)/P_{t-1}} \pi_t$
$\pi_t^*(h) \quad \pi_t^*(h) = \frac{p_t^*(h)/P_t^*}{p_{t-1}^*(h)/P_{t-1}^*} \pi_t^*$