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The forward- and the equity-premium puzzles:  
two symptoms of the same illness?

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# The forward- and the equity-premium puzzles: two symptoms of the same illness?

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# The Forward- and the Equity-Premium Puzzles: Two Symptoms of the Same Illness?

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## Abstract

Using information on US domestic financial data only, we build a stochastic discount factor—SDF— and check whether it accounts for foreign markets stylized facts that escape consumption based models. By interpreting our SDF as the projection of a pricing kernel from a fully specified model in the space of returns, our results indicate that a model that accounts for the behavior of domestic assets goes a long way toward accounting for the behavior of foreign assets prices. We address predictability issues associated with the forward premium puzzle by: *i*) using instruments that are known to forecast excess returns in the moments restrictions associated with Euler equations, and; *ii*) by pricing Lustig and Verdelhan (2007)'s foreign currency portfolios. Our results indicate that the relevant state variables that explain foreign-currency market asset prices are also the driving forces behind U.S. domestic assets behavior. *Keywords: Equity Premium Puzzle, Forward Premium Puzzle, Return-Based Pricing Kernel. J.E.L. codes: G12; G15*

## 1 Introduction

The Forward Premium Puzzle—henceforth, FPP—is how one calls the systematic departure from the intuitive proposition that, conditional on available information, the ex-

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pected return to speculation in the forward foreign exchange market should be zero.

One of the most acknowledged puzzles in international finance, the FPP was, in its infancy, investigated by Mark (1985) within the framework of the consumption capital asset pricing model—CCAPM. Using a non-linear GMM approach, Mark estimated extremely high values for the representative agents' risk aversion parameter, evidencing the canonical consumption model's inability to account for its over-identifying restrictions. Similar findings in equity markets by Hansen and Singleton (1982), Hansen and Singleton (1984) and Mehra and Prescott (1985), lead to the establishment of the so called Equity Premium Puzzle—henceforth, EPP.

It may, at first, seem surprising that such similar results were never properly linked, and we may only conjecture why the literature on the FPP and the EPP drifted apart after the Mark (1985)'s work.

Most probably, the existence of an early specificity for the FPP with no parallel in the case of the EPP—the predictability of returns based on interest rate differentials<sup>1</sup>—may have led many early researchers to believe that even if the CCAPM was capable of accounting for the equity premium it would not solve the FPP.

Indeed, representative of this latter point is the following quote from Engel (1996)'s very comprehensive survey: “[...] it is tempting to draw parallels between the empirical failure of models of the foreign exchange premium, and the closed-economy asset pricing models. [...] However, the forward discount puzzle is not so simple as the equity premium puzzle. International economists face not only the problem that a high degree of risk aversion is needed to account for the estimated values of  $rp_t^{re}$  [rational expectations risk premium]. There is also the question of why the forward discount is such a good predictor of  $s_{t-1} - f_t$  [the difference between the (log of) next period's spot exchange rate and the (log of) forward rate associated with the same period]. There is no evidence that the proposed solutions to the puzzles in domestic financial assets can shed light on this problem.”

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<sup>1</sup>We do not claim that returns on equity are not predictable. In fact, it is now established that dividend-price ratios, investment-capital ratio and other variables are capable of predicting returns. The point is that this empirical regularity was not seen, in the early days of research with the CCAPM, as a defining feature of the EPP. Nowadays an empirically successful model ought to take care of this (and many other) non-trivial aspects of asset behavior, as well.

The purpose of this paper is to offer evidence to the contrary; that the solution to the puzzle in domestic assets is likely to account for the analogous puzzle in foreign-asset pricing. Our take on the FPP is that it is not only an international-economics issue, but an asset-pricing problem with important repercussions to monetary economics.<sup>2</sup> Due to the broad range of the two puzzles (FPP and EPP), and to the likely existence of a common explanation, we believe that the efficient approach for the profession is to join efforts in solving them. Our results invite researchers to refrain from diverting resources on the search of foreign-market specific shocks and concentrate on improving the performance of domestic asset-pricing models.

Because a direct answer to the question of whether a consumption based model that accounted for the EPP would account for the FPP cannot be given without actually writing down such model, we devise a strategy to provide an indirect answer. What we do here is to build a pricing-kernel estimate using only U.S. domestic assets, and show that such pricing kernel generates a risk premium able to accommodate the FPP. This suggests that the relevant state variables that drive foreign-currency market puzzles are already driving the behavior of U.S. domestic assets.

The return based kernel is the unique projection of a stochastic discount factor—henceforth, SDF—on the space of returns, i.e., the SDF mimicking portfolio. One way to rationalize this SDF mimicking portfolio is to realize that it is the projection of a proper economic model (yet to be written) on the space of payoffs. Thus, the pricing properties of this projection are no worse than those of the proper model—a key insight of Hansen and Jagannathan (1991). An advantage of concentrating on the projection is that we can approximate it arbitrarily well in-sample using statistical methods and asset returns alone. Therefore, using such projection not only circumvents the non-existence of a proper consumption model but is also guaranteed not to under-perform in-sample such ideal model.

In order to estimate the SDF mimicking portfolio on this paper, we employ the unconditional linear multi-factor model, which is perhaps the dominant model in discrete-time empirical work in finance.

Our main tests are based on Euler equations. We exploit the theoretical lack of corre-

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<sup>2</sup>Indeed, the uncovered interest parity condition is central to many influential models in monetary economics at least since Mundell (1963). Understanding why this principle is rejected, is paramount to building better models in such an important field in economics.

lation between discounted risk premia and variables in the conditioning set, or between discounted returns and their respective theoretical means, employing both discounted scaled excess-returns and discounted scaled returns in testing. We test the hypothesis that pricing errors are statistically zero and also scaled pricing errors using standard t-statistic, Wald, and over-identifying restriction tests in a generalized method-of-moments framework.

Our main results are clear cut: return-based pricing kernels built using U.S. assets alone, which account for domestic stylized facts, seem to account for the behavior of foreign assets as well. Hence, our SDF prices correctly the expected return to speculation in forward foreign-exchange markets for the widest group possible of developed countries with a long enough span of future exchange-rate data (Canada, Germany, Japan, Switzerland and the United Kingdom). It is important to stress the out-of-sample character of this exercise, avoiding a critique of in-sample over-fitting often used in this literature; see, e.g., Cochrane (2001). We also explicitly consider the eight dynamic foreign currency portfolios of Lustig and Verdelhan (2007).

More specifically, in our empirical tests, we first examine the properties of our pricing kernel with regards to representative domestic assets—S&P500, 90 day Treasury Bill and six Fama and French (1993) benchmark portfolios. We fail to reject the null that average discounted returns are equal to unity and that excess returns are equal to zero. We, then, take this pricing kernel to try to price foreign assets. For all countries, we fail to reject the null that average discounted excess returns on currency speculation is zero and that average discounted returns are equal to one, using instruments known to having strong forecasting power for currency movements—one of the defining features of the FPP. We also show that our U.S. based kernel is successful in pricing all but one of Lustig and Verdelhan (2007) portfolios.

To try to convey the relevance of our findings note that, as of this moment, the question of what approach to adopt in trying to account for the FPP is still open. Recent work by Lustig and Verdelhan (2007), for example, tries to account for the forward premium behavior using a consumption based model. They build eight different foreign assets portfolios using information on differential interest rates and compute pricing errors implied by a representative agent's Euler Equation from a linear consumption model which they

calibrate by borrowing structural parameters from Yogo (2006). Burnside (2007), however, disputes their empirical procedure. He argues that one cannot reject the hypothesis that consumption risk explains none of the cross-sectional variation in the expected excess returns in their data set. The empirical evidence offered by Lustig and Verdelhan (2007) based on the low power of their statistical tests cannot support their main claim regarding the ability of consumption-based model to account for foreign asset pricing puzzles.

On the other side of the spectrum, Burnside et al. (2007) show how bid-ask spreads and price pressure may produce non-negligible effects in currency markets transactions. They argue that microstructure issues underlie the behavior of international forward premia.

Our exercise cannot tell whether a consumption model does the job, nor does it allow one to rule out the presence of significant effects of microstructure issues on foreign exchange market prices. It does, however, offer some support to the view that microstructure issues specific to foreign markets are not first order in generating the FPP, and that a single explanation (whether consumption based or not) should apply to domestic and foreign markets.<sup>3</sup>

The remainder of the paper is organized as follows. Section 2 gives an account of the literature that tries to explain the FPP and is related to our current effort. Section 3 discusses the techniques used to estimate the SDF and the pricing tests implemented in this paper. Section 4 presents the empirical results obtained in this paper. Concluding remarks are offered in Section 5.

## **2 Critical Appraisal of Current Debate on FPP**

### **2.1 The puzzles and the CCAPM**

Fama (1984) recalls that rational expectations alone does not restrict the behavior of forward rates, since it is always possible to include a risk-premium term that reconciles the time series behavior of the associated data. The relevant question is, thus, whether

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<sup>3</sup>Put differently, although market microstructure issues may be important, our findings suggest that had we, as a profession been able to produce a model with the pricing kernel we use here, we would not be talking about a Forward Premium Puzzle. The order of magnitude of the failure of our current models is well above what we would see, if we had such model.



a theoretically sound economic model can offer a definition of risk capable of correctly pricing the forward premium.<sup>4</sup>

The natural candidate for a theoretically sound model for pricing risk is the CCAPM of Lucas (1978) and Breeden (1979). Assuming that the economy has an infinitely lived representative consumer, whose preferences are representable by a von Neumann-Morgenstern utility function  $u(\cdot)$ , the first order conditions for his(ers) optimal portfolio choice yields

$$1 = \beta \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^i \right] \quad \forall i, \quad (1)$$

and, consequently,

$$0 = \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} (R_{t+1}^i - R_{t+1}^j) \right] \quad \forall i, j, \quad (2)$$

where  $\beta \in (0, 1)$  is the discount factor in the representative agent's utility function,  $R_{t+1}^i$  and  $R_{t+1}^j$  are, respectively, the real gross return on assets  $i$  and  $j$  at time  $t + 1$  and,  $C_t$  is aggregate consumption at time  $t$ .

Under the CCAPM,  $\beta u'(C_{t+1})/u'(C_t)$ , is a stochastic discount factor, i.e., a random variable that discount returns in such a way that their prices are simply the assets discounted expected payoffs. We shall use  $M_{t+1}$  to denote SDF's, be them associated with the CCAPM or not.

Let the standing representative agent be a U.S. investor who can freely trade domestic and foreign assets.<sup>5</sup> Next, define the covered,  $R^C$ , and the uncovered return,  $R^U$ , on foreign government bonds trade as

$$R_{t+1}^C = \frac{{}_tF_{t+1}(1 + i_{t+1}^*)P_t}{S_t P_{t+1}} \quad \text{and} \quad R_{t+1}^U = \frac{S_{t+1}(1 + i_{t+1}^*)P_t}{S_t P_{t+1}}, \quad (3)$$

where  ${}_tF_{t+1}$  and  $S_t$  are the forward and spot prices of foreign currency in units of domestic currency,  $P_t$  is the dollar price level and  $i_{t+1}^*$ , the nominal net return on a foreign asset in terms of the foreign investor's currency. Then, substitute  $R^C$  for  $R^i$  and  $R^U$  for  $R^j$  in (2) to

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<sup>4</sup>Frankel (1979), however, argues that most exchange rate risks are diversifiable, there being no grounds for agents to be rewarded for holding foreign assets.

<sup>5</sup>Here, we are implicitly assuming the absence of short-sale constraints or other frictions in the economy. Our assumption is in contrast with that of Burnside et al. (2007) for whom bid-ask spreads' impact on the profitability of currency speculation plays the main role in generating the FPP.

get

$$0 = \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t(1 + i_{t+1}^*)[{}_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right]. \quad (4)$$

Assuming  $u(C) = C^{1-\alpha} (1 - \alpha)^{-1}$ , Mark (1985) applied Hansen (1982)'s Generalized Method of Moments (GMM) to (4). He estimated a coefficient of relative risk aversion,  $\hat{\alpha}$ , above 40. He then tested the over-identifying restrictions to assess the validity of the model, rejecting them when the forward premium and its lags were used as instruments.<sup>6</sup>

It is well known that similar results obtain when one considers the excess return of equity over risk-free short term bonds. In this case,  $R_{t+1}^i = (1 + i_{t+1}^{SP})P_t/P_{t+1}$  and  $R_{t+1}^j = (1 + i_{t+1}^b)P_t/P_{t+1}$ , in (2), where  $i_{t+1}^{SP}$  is nominal return on S&P500 and  $i_{t+1}^b$  nominal return on the U.S. Treasury Bill. This is the EPP in a nutshell.

**Returns and Excess Returns** When we substitute  $R^C$  for  $R^i$  and  $R^U$  for  $R^j$ , in (1) we get

$$1 = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1}) {}_tF_{t+1}(1 + i_{t+1}^*)P_t}{u'(C_t) S_t P_{t+1}} \right] \quad \text{and} \quad 1 = \mathbb{E}_t \left[ \beta \frac{u'(C_{t+1}) S_{t+1}(1 + i_{t+1}^*)P_t}{u'(C_t) S_t P_{t+1}} \right]. \quad (5)$$

In the canonical model, e.g., Hansen and Singleton (1982), Hansen and Singleton (1983) and Hansen and Singleton (1984), the parameter of risk aversion is the inverse of the inter-temporal elasticity of substitution. Even if one is willing to accept high risk aversion, one must also be prepared to accept implausibly high and volatile interest rates.

Accordingly, if one wants to identify the structural parameter  $\beta$ , in an econometric sense, one cannot resort to direct estimation of excess returns (e.g., 4), but rather to joint estimation of the two Euler equations for returns (e.g., 5), or to any linear rotation of them. It is, therefore, important to make a distinction between studies that test the over-identifying restrictions jointly implied by returns and those that test the ones implied by excess returns alone. For the latter no-rejection may be consistent with *any* value for  $\beta$ , including inadmissible ones.<sup>7</sup>

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<sup>6</sup>Similar results were reported later by Modjtahedi (1991). Using a different, larger data set, Hodrick (1989) reported estimated values of  $\hat{\alpha}$  above 60, but did not reject the over-identifying restrictions, while Engel (1996) reported some estimated  $\hat{\alpha}$ 's in excess of 100. A more recent attempt to use Euler equations to account for the FPP is Lustig and Verdelhan (2006), where risk aversion in excess of 100 is needed to price the forward premium on portfolios of foreign currency.

<sup>7</sup>This is for example the case of Lustig and Verdelhan (2007), as they point out in footnote 8, p. 95.

In our view, a successful consumption-based model must account for asset prices everywhere (domestically and abroad), as well as price returns, excess returns, and many new facts recently found in the extensive empirical research that has been in a great deal sparked by the theoretical developments of the late seventies—see, for example, Cochrane (2006).

## 2.2 Our strategy and main issues

Even though important progress has been made in building more successful consumption models, there is no consensus as of this moment on whether *any* model derived from the primitives of the economy accounts for either puzzle. The current state of the art, thus, precludes a direct answer to the question in the title of this paper. Hence, we take an indirect approach.

We extract a pricing kernel from U.S. return data alone and show that it prices both the domestic and the foreign-exchange returns and excess returns.

Following Harrison and Kreps (1979), Hansen and Richard (1987), and Hansen and Jagannathan (1991), we write the system of asset-pricing equations,

$$1 = \mathbb{E}_t \left[ M_{t+1} R_{t+1}^i \right], \quad \forall i = 1, 2, \dots, N. \quad (6)$$

Given free portfolio formation, the law of one price guarantees, through Riesz representation theorem, the existence of a SDF,  $M_{t+1}$  satisfying (6). Since (6) applies to all assets,

$$0 = \mathbb{E}_t \left[ M_{t+1} \left( R_{t+1}^i - R_{t+1}^j \right) \right], \quad \forall i, j. \quad (7)$$

We combine statistical methods with the Asset Pricing Equation (6) to devise pricing-kernel estimates as projections of SDF's on the space of returns, i.e., the SDF mimicking portfolio, which is unique even under incomplete markets. We denote the latter by  $M_{t+1}^*$ .

More specifically, our exercise consists in exploring a large cross-section of U.S. time-series stock returns to construct return-based pricing kernel estimates satisfying the Pricing Equation (6) for that group of assets. Then, we take these SDF estimates and use them to price assets not used in constructing them. Therefore, we perform a genuine out-of-sample forecasting exercise using SDF mimicking portfolio estimates, avoiding the in-sample over-fitting critique discussed in Cochrane (2001), for example.

We cannot overemphasize the importance of out-of-sample forecasting for our purposes. Our main point in this paper is to show that the forward- and the equity-premium puzzle are intertwined. Under the law of one price, a SDF that prices all assets exists, necessarily. Thus, an in-sample exercise would only provide evidence that the forward-premium puzzle is not simply a consequence of violations of the law of one price. We aim at showing more: a SDF can be constructed using only domestic assets, i.e., using the same source of information that guides research regarding the equity premium puzzle, and still price foreign assets. It is our view that this SDF is to capture the growth of the marginal utility of consumption in a model yet to be written.

In dispensing with consumption data, our paper parallels the works of Hansen and Hodrick (2006), Hodrick and Srivastava (1984), Cumby (1988), Huang (1989), and Lewis (1990), which implement latent variable models. They avoid the need for fully specifying a model for the pricing kernel by treating the return on a benchmark portfolio as a latent variable.<sup>8</sup> Also, Korajczyk and Viallet (1992) apply the arbitrage pricing theory—APT—to a large set of assets from many countries, and test whether including the factors as the prices of risk reduces the predictive power of the forward premium. They do not, however, perform out-of-sample exercises and do not try to relate the two puzzles.

Backus et al. (1995) ask whether a pricing kernel can be found that satisfies, at the same time, log-linearized versions of

$$0 = \mathbb{E}_t \left[ M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[{}_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right] \quad (8)$$

and

$$R_t^f = \frac{1}{\mathbb{E}_t(M_{t+1}^*)}, \quad (9)$$

where  $R_t^f$  is the risk-free rate of return. The nature of the question we answer is similar, albeit enlarging its scope by considering all the main domestic assets and focusing on much richer instrument sets.<sup>9</sup>

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<sup>8</sup>Their results met with partial success: all these papers reject the unbiasedness hypothesis but are in conflict with each other with regards to the rejection of restrictions imposed by the latent-variable model. However, contrary to what we do here, this line of research does not try to relate the EPP and the FPP.

<sup>9</sup>Anticipating our results, we should emphasize that we do not reject (9) for any of the instruments, as well, which means that our SDF satisfies both conditions presented by Backus et al. (1995).

**Microstructure and the FPP** Currency markets are thin and characterized by important transaction costs. Burnside et al. (2007) take this characteristic of markets seriously and show that bid-ask spreads are not negligible and that price pressures are important in understanding the persistence of apparently unexplored opportunities in those markets.

The fact that we are able to find a linear pricing functional suggests, however, that bid-ask spreads do not produce, at the frequency we examine, the magnitude of deviations from UIP that leads the empirical regularity to be considered a puzzle.

As for price pressures, the existence of a linear pricing functional does not rule out their presence. However, the fact that we construct our pricing kernel using domestic assets poses a challenge to any explanation that relies on microstructure issues *that are specific to foreign exchange markets*.

**Log-linear models** Most studies<sup>10</sup> report the FPP through the finding of  $\hat{\alpha}_1$  significantly smaller than zero when running the regression,

$$s_{t+1} - s_t = \alpha_0 + \alpha_1({}_t f_{t+1} - s_t) + u_{t+1}, \quad (10)$$

where  $s_t$  is the log of the exchange rate at time  $t$ ,  ${}_t f_{t+1}$  is the log of time  $t$  forward exchange rate contract and  $u_{t+1}$  is the regression error.<sup>11</sup>

Notwithstanding the possible effect of Jensen inequality terms, testing the uncovered interest rate parity (UIP) is equivalent to testing the null that  $\alpha_1 = 1$  and  $\alpha_0 = 0$ , along with the uncorrelatedness of residuals from the estimated regression. Not only is the null rejected in almost all studies, but the magnitude of the discrepancy is very large: according to Froot (1990), the average value of  $\hat{\alpha}_1$  is  $-0.88$  for over 75 published estimates across various exchange rates and time periods. A negative  $\alpha_1$  implies an expected domestic currency appreciation when domestic nominal interest rates exceed foreign interest rates, contrary to what is needed for the UIP to hold.

Getting to (10) from first principles, however, requires stringent assumptions on the underlying asset pricing model. As we shall see, some of these additional assumptions

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<sup>10</sup>See the comprehensive surveys by Hodrick (1987) and Engel (1996), and the references therein.

<sup>11</sup>In what follows, capital letters are used to represent variables in levels and small letters to represent the logs of these same variables.

may be behind the unexpected findings. In other words, by log-linearizing

$$1 = \mathbb{E}_t \left[ M_{t+1} R_{t+1}^i \right] \quad \forall i = 1, 2, \dots, N., \quad (11)$$

it is possible to justify regression (10), but not without unduly strong assumptions on the behavior of discounted returns.

Following Araujo and Issler (2008), consider a second-order Taylor expansion of the exponential function around  $x$ , with increment  $h$ ,

$$e^{x+h} = e^x + h e^x + \frac{h^2 e^{x+\lambda(h)h}}{2}, \quad \text{with } \lambda(h) : \mathbb{R} \rightarrow (0, 1). \quad (12)$$

Usually,  $\lambda(\cdot)$  depends on both  $x$  and  $h$ , but not for the exponential function. Indeed, dividing (12) by  $e^x$ , we get

$$e^h = 1 + h + \frac{h^2 e^{\lambda(h)h}}{2}, \quad (13)$$

which shows that  $\lambda(\cdot)$  depends only on  $h$ .<sup>12</sup> To connect (13) with the Pricing Equation (11), we assume  $M_{t+1} R_{t+1}^i > 0$  and let  $h = \ln(M_{t+1} R_{t+1}^i)$  to obtain<sup>13</sup>

$$M_{t+1} R_{t+1}^i = 1 + \ln(M_{t+1} R_{t+1}^i) + z_{i,t+1}, \quad (14)$$

where the higher-order term of the expansion is

$$z_{i,t+1} \equiv \frac{1}{2} \times \left[ \ln(M_{t+1} R_{t+1}^i) \right]^2 e^{\lambda(\ln(M_{t+1} R_{t+1}^i)) \ln(M_{t+1} R_{t+1}^i)}.$$

It is important to stress that (14) is not an approximation but an exact relationship. Also,  $z_{i,t+1} \geq 0$ .

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<sup>12</sup>The closed-form solution for  $\lambda(\cdot)$  is:

$$\lambda(h) = \begin{cases} \frac{1}{h} \times \ln \left[ \frac{2 \times (e^h - 1 - h)}{h^2} \right], & h \neq 0 \\ 1/3, & h = 0, \end{cases}$$

where  $\lambda(\cdot)$  maps from the real line into  $(0, 1)$ .

<sup>13</sup>This is *not* an innocuous assumption. By assuming no arbitrage (stronger than law of one price) we guarantee the existence of a positive  $M$ . Uniqueness of  $M$ , however, requires complete markets: a very strong assumption. Without uniqueness not all pricing kernels need to be positive.

Using past information to take the conditional expectation of both sides of (14), which we denote by  $\mathbb{E}_t(\cdot)$ , imposing the Pricing Equation, and rearranging terms, gives:

$$\mathbb{E}_t \left\{ M_{t+1} R_{t+1}^i \right\} = 1 + \mathbb{E}_t \left\{ \ln(M_{t+1} R_{t+1}^i) \right\} + \mathbb{E}_t(z_{i,t+1}), \text{ or,} \quad (15)$$

$$\mathbb{E}_t(z_{i,t+1}) = -\mathbb{E}_t \left\{ \ln(M_{t+1} R_{t+1}^i) \right\}. \quad (16)$$

Equation (16) shows that the behavior of the conditional expectation of the higher-order term depends only on that of  $\mathbb{E}_t \left\{ \ln(M_{t+1} R_{t+1}^i) \right\}$ . Therefore, in general, it depends on lagged values of  $\ln(M_{t+1} R_{t+1}^i)$  and on powers of these lagged values. This will turn out to a major problem when estimating (10). To see it, denote by  $\varepsilon_{i,t+1} = \ln(M_{t+1} R_{t+1}^i) - \mathbb{E}_t \left\{ \ln(M_{t+1} R_{t+1}^i) \right\}$  the innovation of  $\ln(M_{t+1} R_{t+1}^i)$ . Let  $\mathbf{R}_{t+1} \equiv (R_{t+1}^1, R_{t+1}^2, \dots, R_{t+1}^N)'$  and  $\boldsymbol{\varepsilon}_{t+1} \equiv (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \dots, \varepsilon_{N,t+1})'$  stack respectively the returns  $R_{t+1}^i$  and the forecast errors  $\varepsilon_{i,t+1}$ . From the definition of  $\boldsymbol{\varepsilon}_{t+1}$  we have:

$$\ln(M_{t+1} \mathbf{R}_{t+1}) = \mathbb{E}_t \left\{ \ln(M_{t+1} \mathbf{R}_{t+1}) \right\} + \boldsymbol{\varepsilon}_{t+1}. \quad (17)$$

Denoting  $\mathbf{r}_{t+1} = \ln(\mathbf{R}_{t+1})$ , with elements  $r_{t+1}^i$ , and  $m_{t+1} = \ln(M_{t+1})$  in (17), and using (16) we get

$$m_{t+1} = -r_{t+1}^i - \mathbb{E}_t(z_{i,t+1}) + \varepsilon_{i,t+1}, \quad \forall i. \quad (18)$$

Define the covered and uncovered return on trade in foreign assets as in (3). Then, using a forward version of (18) on both returns, and combining results, one gets

$$s_{t+1} - s_t = ({}_t f_{t+1} - s_t) - [\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})] + \varepsilon_{U,t+1} - \varepsilon_{C,t+1}, \quad (19)$$

where the index  $i$  in  $\mathbb{E}_t(z_{i,t+1})$  and  $\varepsilon_{i,t+1}$  in (18) is replaced by either  $C$  or  $U$ , in the case of covered and the uncovered return on trading foreign government bonds, respectively.

Under  $\alpha_1 = 1$  and  $\alpha_0 = 0$  in (10), taking into account (19), allows concluding that:

$$u_{t+1} = -[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})] + \varepsilon_{U,t+1} - \varepsilon_{C,t+1}.$$

Hence, by construction, the error term  $u_{t+1}$  is serially correlated because it is a function of current and lagged values of observables.<sup>14</sup> However, in most empirical studies, lagged

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<sup>14</sup>Of course, one can get directly to (10) when  $\alpha_1 = 1$  and  $\alpha_0 = 0$  using (11) under log-Normality and Homoskedasticity of  $M_t R_{i,t}$ . One can also do it from (11) if  $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$  is constant. However,

observables are used as instruments to estimate (10) and test the null that  $\alpha_1 = 1$ , and  $\alpha_0 = 0$ . In that context, estimates of  $\alpha_1$  are biased and inconsistent and hypothesis tests are invalid. This may explain the finding that the average value of  $\hat{\alpha}_1$  is  $-0.88$  for over 75 published estimates across various exchange rates and time periods, and the fact that the null  $\alpha_1 = 1, \alpha_0 = 0$  is overwhelmingly rejected in these studies.

Although, many authors criticize the empirical use of the log-linear approximation of the Pricing Equation (11) leading to (10), as far as we know, this is the first time the criticism above is applied to the use of (10).

Aligned with these theoretical considerations we should mention a recent work by da Costa et al. (2008). They propose and test a bivariate GARCH-in-mean approach, using pricing kernels built along the lines of the ones used herein. According to their main results, although the omitted term is able to ‘explain’, in the sense of the null not being rejected in Euler Equation tests—this is particularly true in their in-sample tests—the inclusion of a risk premium term in the conventional regression changes neither the significance nor the magnitude of the forward premium forecasting power. Their results suggest that the lognormality assumption of conditional returns may be too off the mark for one to rely on the usual regression and its extensions.<sup>15</sup>

**Predictability and the FPP** Another defining characteristic of the FPP is the predictability of returns on currency speculation. Because  $\hat{\alpha}_1 < 0$  and significant, given that the auto-correlation of risk premium is very persistent, interest-rate differentials predict excess returns. Although predictability in equity markets has by now been extensively documented, it was not viewed as a defining feature of the EPP, back then.

We take predictability very seriously in our tests. Because we refrain from using log-  


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the conditions are very stringent in both cases: there is overwhelming evidence that returns are not log-Normal and homoskedastic, and to think that  $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$  is constant can only be justified as an algebraic simplification for expositional purposes.

Even under log-Normality, if returns are heteroskedastic,  $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$  will be replaced by the difference in conditional variances. Again, this is projection on lagged values of observables, and the same problems alluded above are present.

<sup>15</sup> Also, in articles similar to this one in scope, Gomes and Issler (2006) show that log linearizing the consumer Euler equation may be an explanation for the common finding of rule-of-thumb behavior in consumption decisions even when it is not present in the data.



linearizations, we incorporate predictability in time series and cross-section, respectively, by using forwards, price-dividend ratios, and other variables known to forecast returns as instruments, and by using differential interest rates to build portfolios of foreign assets, following the approach of Lustig and Verdelhan (2007).

### 3 Econometric Tests

Assume that we are able to approximate well enough a time series for the pricing kernel,  $M_{t+1}^*$ . First, we briefly discuss how to construct this time series for  $M_{t+1}^*$  using only asset-return information. Then, we show how to use this approximation to implement direct pricing tests for the forward and the equity-premium, in an Euler equation framework, as well as, to tackle some “puzzling aspects” in the U.S. stock and foreign currency markets.

#### 3.1 Return-Based Pricing Kernels

The basic idea behind estimating return-based pricing kernels with asymptotic techniques is that asset prices (or returns) convey information about the intertemporal marginal rate of substitution in consumption. If the Asset Pricing Equation holds, all returns must have a common factor that can be removed by subtracting any two returns. A common factor is the SDF mimicking portfolio  $M_{t+1}^*$ . Because every asset return contains “a piece” of  $M_{t+1}^*$ , if we combine a large enough number of returns, the average idiosyncratic component of returns will vanish in limit. Then, if we choose our weights properly, we may end up with the common component of returns, i.e., the SDF mimicking portfolio.

Although the existence of a strictly positive SDF can be proved under no arbitrage, uniqueness of the SDF is harder to obtain, since under incomplete markets there is, in general, a continuum of SDF’s pricing all traded securities. However, each  $M_{t+1}$  can be written as  $M_{t+1} = M_{t+1}^* + v_{t+1}$  for some  $v_{t+1}$  obeying  $\mathbb{E}_t [v_{t+1} R_{t+1}^i] = 0, \forall i$ . Since the economic environment we deal with must be that of incomplete markets, it only makes sense to devise econometric techniques to estimate the unique SDF mimicking portfolio— $M_{t+1}^*$ .

There are some techniques that could be employed to estimate a  $M_{t+1}^*$ : the consumption-

based and the preference-free ones. Here, we use principal-component and factor analyses, in a method that can be traced back to the work of Ross (1976), developed further by Chamberlain and Rothschild (1983), and Connor and Korajczyk (1986), Connor and Korajczyk (1993). A recent additional reference is Bai (2005). This method is asymptotic: either  $N \rightarrow \infty$  or  $N, T \rightarrow \infty$ , relying on weak law-of-large-numbers to provide consistent estimators of the SDF mimicking portfolio—the unique systematic portion of asset returns.

For sake of completeness, we present a more complete description of our method in Section A of the Appendix.

### 3.2 Main exercises

We employ an Euler-equation framework, something that was missing in the forward-premium literature after Mark (1985). Since the two puzzles are present in logs and in levels, by working directly with the Pricing Equation we avoid imposing stringent auxiliary restrictions in hypothesis testing, while keeping the possibility of testing the conditional moments through the use of lagged instruments along the lines of Hansen and Singleton (1982), Hansen and Singleton (1984) and Mark (1985). We hope to have convinced our readers that the log-linearization of the Euler equation is an unnecessary and dangerous detour.<sup>16</sup>

Euler equations (6) and (7) must hold for all assets and portfolios. If we had observations on  $M^*$ , then return data could be used to test directly whether they held. Of course,  $M^*$  is a latent variable, and the best we can do is to try and find a consistent estimator for  $M^*$ . Using return data, and a large enough sample, so that  $M^*$  and their estimators are “close enough,” we could still directly test the validity of these Euler equations. Note that not only are the estimators of  $M^*$  functions of return data but only return data are used to investigate whether the Asset Pricing Equation holds.

In this section we explain our pricing tests. Our main tests are out-of-sample exercises in which  $M^*$  is estimated with domestic assets alone and then used to price foreign assets.

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<sup>16</sup>In a recent study, da Costa et al. (2008) propose and test an Asset Pricing Theory-based regression, that has the conventional one as a particular case, suggesting that the lognormality assumption of conditional returns may be to blame.

Here, we verify if the relevant state variables that drive foreign-currency market puzzles are already driving the behavior of U.S. domestic assets. We do, however, conduct in-sample exercises, just as a homework.

The first exercise, discussed in the beginning of section 4.3 consists in using this SDF to price domestic asset returns: S&P500, 90-day T-bill and Fama-French portfolios. Its purpose is to investigate if the EPP or other anomalies in the behavior of domestic assets are present using our pricing-kernel estimate. Being assured that our pricing kernel does a good job in pricing in-sample the relevant domestic assets, we proceed to our main exercise. We test the pricing properties of the same kernel for foreign assets: now, an out of sample exercise.

Our procedure is quite similar in spirit to the one performed in da Costa et al. (2008), in the sense that finding a theoretical pricing kernel satisfying (6) for assets in the equity and foreign currency markets, we provide a linear functional that prices these assets. Such finding rules out microstructure explanations that generates violations of the law of one price, which underlie some market microstructure explanations.

In general, our pricing tests rely on two variants of the Pricing Equation:

$$1 = \mathbb{E}_t \left[ M_{t+1} R_{t+1}^i \right], \quad \forall i = 1, 2, \dots, N, \text{ or,} \quad (20)$$

$$0 = \mathbb{E}_t \left[ M_{t+1} \left( R_{t+1}^i - R_{t+1}^j \right) \right], \quad \forall i, j. \quad (21)$$

Consider  $z_t$  to be a vector of instrumental variables, which are all observed up to time  $t$ , therefore measurable with respect to  $\mathbb{E}_t(\cdot)$ . Employing *scaled returns* and *scaled excess-returns*—defined as  $R_{t+1}^i \times z_t$  and  $(R_{t+1}^i - R_{t+1}^j) \times z_t$ , respectively—we are able to test the conditional moment restrictions associated with (20) and (21) and consequently to derive the implications from the presence of information. This is particularly important for the FPP, since, when the CCAPM was employed or when returns were not discounted, the over-identifying restriction associated with having the own current forward premium as an instrument was usually rejected: a manifestation of its predictive power.

**Forward Premium Puzzle** For the FPP, multiply

$$0 = \mathbb{E}_t \left[ M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[{}_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right], \quad (22)$$

and

$$1 = \mathbb{E}_t \left[ M_{t+1}^* \frac{S_{t+1}(1+i_{t+1}^*)P_t}{S_t P_{t+1}} \right], \quad (23)$$

by  $z_t$  and apply the Law-of-Iterated Expectations to get

$$0 = \mathbb{E} \left\{ \left[ \begin{array}{c} M_{t+1}^* \frac{P_t(1+i_{t+1}^*)[{}^tF_{t+1}-S_{t+1}]}{S_t P_{t+1}} \\ M_{t+1}^* \frac{S_{t+1}(1+i_{t+1}^*)P_t}{S_t P_{t+1}} - 1 \end{array} \right] \otimes z_t \right\}. \quad (24)$$

The system of orthogonality restrictions (24) can be used to assess the pricing behavior of estimates of  $M^*$  with respect to the components of the forward premium for each currency or for any linear rotation of them. Equations in the system can be tested separately or jointly. In testing, we employ a generalized method-of-moments (GMM) perspective, using (24) as a natural moment restriction to be obeyed. Consider parameters  $\mu_1$  and  $\mu_2$  in:

$$0 = \mathbb{E} \left\{ \left[ \begin{array}{c} M_{t+1}^* \frac{P_t(1+i_{t+1}^*)[{}^tF_{t+1}-S_{t+1}]}{S_t P_{t+1}} - \mu_1 \\ M_{t+1}^* \frac{S_{t+1}(1+i_{t+1}^*)P_t}{S_t P_{t+1}} - (1 - \mu_2) \end{array} \right] \otimes z_t \right\}. \quad (25)$$

Parameters  $\mu_1$  and  $\mu_2$  should be interpreted as pricing errors in (22) and (23) respectively. We assume that there are enough elements in the vector  $z_t$  for  $\mu_1$  and  $\mu_2$  for the system to be over-identified. In order for (24) to hold, we must have  $\mu_1 = 0$  and  $\mu_2 = 0$ , which can be jointly tested using a Wald test. Alternatively, we could verify whether or not the over-identifying restrictions are not rejected using the  $T \times J$  test.

**Forward and Equity Premium Puzzles jointly** If our statistical proxies really represent a “true pricing kernel,” they should be able to price assets in both domestic and foreign markets jointly. Consequently, we include a joint test of the return-based pricing kernels, rather than estimating Euler equations separately for equity and foreign currency returns, whenever feasible.

A similar procedure can be implemented for both foreign currency and domestic markets, multiplying by  $z_t$  and applying the Law-of-Iterated Expectations, not only for (22)

and (23), but also for the following system of conditional moment restrictions

$$0 = \mathbb{E}_t \left[ M_{t+1}^* \frac{(i_{t+1}^{SP} - i_{t+1}^b) P_t}{P_{t+1}} \right] \quad (26)$$

and

$$1 = \mathbb{E}_t \left[ M_{t+1}^* \frac{(1 + i_{t+1}^{SP}) P_t}{P_{t+1}} \right], \quad (27)$$

where  $i_{t+1}^{SP}$  and  $i_{t+1}^b$  are respectively the returns on the S&P500 and on a U.S. government short-term bond. We are then able to perform a second econometric testing procedure given by

$$0 = \mathbb{E} \left\{ \begin{array}{c} \left[ \begin{array}{c} M_{t+1}^* \frac{P_t(1+i_{t+1}^*)}{S_t P_{t+1}} [{}_{t|}F_{t+1} - S_{t+1}] - \mu_1 \\ M_{t+1}^* \frac{S_{t+1}(1+i_{t+1}^*) P_t}{S_t P_{t+1}} - (1 - \mu_2) \\ M_{t+1}^* \frac{(i_{t+1}^{SP} - i_{t+1}^b) P_t}{P_{t+1}} - \mu_3 \\ M_{t+1}^* \frac{(1+i_{t+1}^{SP}) P_t}{P_{t+1}} - (1 - \mu_4) \end{array} \right] \otimes z_t \end{array} \right\} \quad (28)$$

to examine whether both EPP and FPP hold jointly. In this case, all pricing errors must be zero, i.e., we must have  $\mu_1 = 0, \mu_2 = 0, \mu_3 = 0$  and  $\mu_4 = 0$ , which could be tested using a Wald test. Alternatively, we could employ the over-identifying restriction  $T \times J$  test.

**Lustig and Verdelhan (2007) portfolios** A more direct approach for dealing with predictability in foreign government bond markets is to use dynamic portfolios built using variables with good forecasting properties regarding the returns of these bonds. We follow Lustig and Verdelhan (2007), in building eight different zero-cost foreign-currency portfolios, by selling 90 day T-bills and using the proceeds to buy government bonds of various countries. Portfolios are ranked according to interest rate differential with respect to the T-bill—that is, excess returns here in units of foreign currency over units of U.S. currency, hence measurable with respect to  $t$ .

More specifically, at the end of each period  $t$ , we allocate countries to eight portfolios on the basis of the nominal interest rate differential,  $i_t^* - i_t^b$ , observed at the end of period  $t$ . In this paper, each of these foreign currency portfolios is rebalanced every quarter.

We rank these portfolios from low to high interest rate differential, portfolio 1 being the portfolio with the lowest interest rate currencies and portfolio 8 being the one with the highest interest rate currencies. Finally, for  $j = 1, \dots, 8$ , we compute  $R_{t+1}^{j,e}$  as the U.S.\$ real excess return on foreign currency portfolio  $j$  over T-bill.

Applying the conditioning information procedure, we have the following econometric test to price these dynamic portfolios:

$$0 = \mathbb{E} \left\{ \begin{bmatrix} M_{t+1}^* R_{t+1}^{1,e} - \mu_1 \\ M_{t+1}^* R_{t+1}^{2,e} - \mu_2 \\ \cdot \\ \cdot \\ M_{t+1}^* R_{t+1}^{8,e} - \mu_8 \end{bmatrix} \otimes z_t \right\}. \quad (29)$$

Once again, we could use a Wald test to verify whether  $\mu_j = 0$ , for  $j = 1, \dots, 8$ , and check the appropriateness of the over-identifying restrictions using Hansen's  $T \times J$  test.

### 3.3 The U.S. domestic financial market: an in-sample exercise

**Equity Premium Puzzle** To implement an in-sample test for the domestic market, we follow the standard procedure above, but considering only the conditional moment restrictions (26) and (27), obtaining:

$$0 = \mathbb{E} \left\{ \begin{bmatrix} M_{t+1}^* \frac{(i_{t+1}^{SP} - i_{t+1}^b) P_t}{P_{t+1}} - \mu_1 \\ M_{t+1}^* \frac{(1 + i_{t+1}^{SP}) P_t}{P_{t+1}} - (1 - \mu_2) \end{bmatrix} \otimes z_t \right\} \quad (30)$$

to test whether or not the EPP holds: we test whether pricing errors are zero using a Wald test, i.e.,  $\mu_1 = 0$  and  $\mu_2 = 0$ , and check the appropriateness of the over-identifying restrictions using Hansen's  $T \times J$  test.

**Predictability (in the cross-section) for the domestic financial market** Beyond the high equity Sharpe ratio or the reported power of the dividend-price ratio to forecast stock-market returns, the pattern of cross-sectional returns of assets exhibit some "puzzling

aspects” as the “size” and the “value” effects—e.g. Fama and French (1996) and Cochrane (2006)—, i.e., the fact that small stocks and stocks with low market values—relative to book values—tend to have higher average returns than other stocks. We also perform in-sample tests to check whether these pricing anomalies occur in our data set by pricing the six Fama and French (1993) benchmark portfolios, dynamic portfolios extracted from the Fama-French library.

These Fama-French portfolios sort stocks according to size and book-to-market, because both size and book-to-market predict returns. At the end of each quarter  $t$ , NYSE, AMEX and Nasdaq stocks are allocated to two groups (small,  $S$ , or big,  $B$ ) based on whether their market equity ( $ME$ ) is below or above the median  $ME$  for NYSE stocks. Once more, these stocks are allocated in an independent sort to three book-to-market equity ( $BE/ME$ ) groups (low,  $L$ , medium,  $M$ , or high,  $H$ ) based on the breakpoints for the bottom 30 percent, middle 40 percent and top 30 percent of the values of  $BE/ME$  for NYSE stocks. Then, six size- $BE/ME$  portfolios (labelled  $B/H, B/L, B/M, S/H, S/L$  and  $S/M$ ) are defined as the intersections of the two  $ME$  and the three  $BE/ME$  groups.

According to Fama and French (1993), one can obtain a time series for the three factors based on these six portfolios. The  $HML$  (High Minus Low) factor is the average return on the two value portfolios minus the average return on the two growth ones. The  $SMB$  (Small Minus Big) factor is the average return on the three small portfolios minus the average return on the three big ones. The third factor  $RmRf$  is the excess return on the market.

We apply the same conditioning information procedure to get the following econo-

metric test to price these dynamic portfolios:

$$0 = \mathbb{E} \left\{ \begin{array}{c} \left[ \begin{array}{c} M_{t+1}^* R_{t+1}^{B/H} - \mu_1 \\ M_{t+1}^* R_{t+1}^{B/L} - \mu_2 \\ M_{t+1}^* R_{t+1}^{B/M} - \mu_3 \\ M_{t+1}^* R_{t+1}^{S/H} - \mu_4 \\ M_{t+1}^* R_{t+1}^{S/L} - \mu_5 \\ M_{t+1}^* R_{t+1}^{S/M} - \mu_6 \end{array} \right] \otimes z_t \end{array} \right\} \quad (31)$$

Again, we use a Wald test to check whether  $\mu_j = 0$ , for  $j = 1, 2, \dots, 6$ , and verify the appropriateness of the over-identifying restrictions using Hansen's  $T \times J$  test.

### 3.4 GMM estimation setup

As argued above, since our tests are based on Euler-equation restrictions it is natural to estimate the pricing errors  $\mu_i$  by GMM. As in any Finance study employing GMM estimation, one must choose first how to weight the different moments used for identification of the parameters of interest ( $\mu_i$  in our setup). Hansen (1982) shows how to get asymptotic efficient GMM estimates by choosing as weights the inverse of the asymptotic variance-covariance matrix of sample moments. There is a direct analogy between this choice and the choice of OLS vs. GLS estimation in the context of linear regression models: OLS weights all errors equally – the identity matrix is used as weight – while GLS weights different errors based upon the inverse of the variance-covariance matrix of the errors. In the latter choice, there is a natural trade-off between attaining full efficiency and correctly specifying the structure of the variance-covariance matrix. As is well known, both OLS and GLS are consistent under correct specification of the variance-covariance matrix. However, in trying to achieve full-efficiency, one can render GLS estimates inconsistent if the structure chosen is incorrectly specified. Thus, OLS is a robust estimate in the sense



that it does not rely on a correct choice for the weighting matrix. For that reason, most applied econometric studies use OLS in estimation and properly estimate its standard errors using the methods advanced by White (1980) and later generalized by Newey and West (1987). Here, because we want to rely on robust estimates, we will use the identity matrix in weighting orthogonality conditions and will correct the estimate of their asymptotic variance-covariance matrix for the presence of serial correlation and heteroskedasticity of unknown form using the techniques in Newey and West.

There is an additional reason why it may be hard to achieve an optimal weighting scheme in GMM estimation in this paper, favoring the use of the identity matrix in weighting moment restrictions. Suppose that we use  $N$  pricing equations with  $k$  instruments to form the  $N \times k$  orthogonality conditions used in GMM estimation. The variance-covariance matrix of sample moments associated with these orthogonality conditions is of order  $(N \times k) \times (N \times k)$  and has  $(N \times k) \times (N \times k + 1) / 2$  parameters. These must be estimated using  $T$  time-series observations. If  $N \times k$  is large vis-a-vis to  $T$ , it may be infeasible to estimate the variance-covariance matrix of sample moments. Even if estimation is feasible, the estimate of this variance-covariance may be far away from its asymptotic probability limit, which is a problem for asymptotic tests. A simple example suffices here. In this paper,  $T = 112$ , while  $k = 2, 3$ , or  $4$ , and  $N = 2, 4$ , or  $8$ . It is obvious that, in the more extreme cases, the use of asymptotic tests is jeopardized.<sup>17</sup>

If the identity matrix is used in weighting moments, it makes little sense to use Hansen's over-identifying restrictions  $T \times J$  test to verify whether or not pricing errors are zero, since possible covariance terms between different scaled pricing errors are disregarded by using a diagonal weighting matrix. This fact, coupled with the well known problem of the lack of power of the  $T \times J$  test, suggest the use of the Wald test to check whether or not pricing errors are jointly statistically zero<sup>18</sup>. In this paper, this is our final choice of testing method for verifying if the implied moment restrictions in each case hold or not.

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<sup>17</sup>We avoid joint tests of the "puzzling aspects" related to predictability in cross-section for both foreign currency and equity markets. This exercise would leave us with 14 sample moment conditions, for which our 112 time-series observations cannot yield an invertible variance-covariance matrix of sample moments.

<sup>18</sup>Notice that the Wald test uses a non-diagonal matrix in weighting pricing errors, even if the identity matrix is used in weighting moments.

### 3.5 Instruments

The question of which variables are good predictors for returns is still open. But the choice of a representative set of forecasting instruments highlights the relevance of the conditional tests we choose. Here, we use mainly specific financial variables as instruments, carefully choosing them based on their forecasting potential. For the EPP, we employ the dividend-price ratio and the investment-capital ratio, following Campbell and Shiller (1988), Fama and French (1988) and Cochrane (1991), who show evidence that these variables are good predictors of stock-market returns. Regarding the FPP, we use the current value of the respective forward premium,  $({}_tF_{t+1} - S_t)/S_t$ , since its forecasting ability is a defining feature of this puzzle and this series is measurable with respect to the information set used by the representative consumer.

Regarding predictability in the cross-sectional dimension for domestic and foreign currency-markets, we follow the original papers using as instruments, respectively, the Fama-French factors,  $RMRF_t$ ,  $SMB_t$  and  $HML_t$  and  $\Delta\tilde{R}_t$ —the cross-sectional average interest rate difference on portfolios 1 through 8, normalized to be positive. All these series are measurable with respect to the information set used by the representative consumer.

In addition to these variables, as a robustness check, we also used as instruments lagged values of returns for the assets being tested. Taking into account the fact that expected returns and business cycles are correlated, e.g., Fama and French (1989), we also use as instruments macroeconomic variables with documented forecasting ability regarding financial returns, such as real consumption and GDP instantaneous growth rates and the consumption-GDP ratio.

## 4 Empirical Results

### 4.1 Data and Summary Statistics

In principle, whenever econometric or statistical tests are performed, it is preferable to employ a large data set either in the time-series ( $T$ ) or in the cross-sectional dimension ( $N$ ). In choosing return data, we had to deal with the trade-off between  $N$  and  $T$ . In order to get a larger  $N$ , one must accept a reduction in  $T$ : disaggregated returns are only available for smaller time spans than aggregated returns.

In terms of sample size, our main limitation for the time-series span used here is regarding FPP tests. Foreign-exchange only floated post-Bretton Woods Agreements: the Chicago Mercantile Exchange, the pioneer of the financial-futures market, only launched currency futures in 1972. In addition to that, only futures data for a few developed countries are available since then. In order to have a common sample for the largest set of countries possible, we considered here U.S. foreign-exchange data for Canada, Germany, Japan, Switzerland and the U.K., covering the period from 1977:1 to 2004:4, on a quarterly frequency, comprising 112 observations.

Returns on covered and uncovered trading with foreign government bonds were transformed into U.S.\$ real returns using the consumer price index in the U.S. The forward-rate series were extracted from the Chicago Mercantile of Exchange database, while the spot-rate series were extracted from the Bank of England and the IFS databases. To study the EPP we used the U.S.\$ real returns on the S&P500 and on 90-day T-bill. The latter were extracted in nominal terms from the CRSP and the IFS database, respectively. Real returns were obtained using the U.S. CPI as deflator.

In order to account for other domestic and international markets stylized facts that escape consumption based models, we also used U.S.\$ real returns on the Fama and French (1993) benchmark portfolios formed on size and book-to-market, extracted from the French data library, as well as Lustig and Verdelhan (2007) eight foreign currency portfolios, here constructed using quarterly series of spot exchange rates and short-term foreign government bonds available in IFS database.<sup>19</sup>

Following Lustig and Verdelhan (2007)'s procedure, we use as the foreign interest rate, the interest rate on a 3-month government security (e.g. a U.S. T-bill) or an equivalent instrument, taking into account that as data became available, new countries are added (or subtracted) to these portfolios.

A second ingredient for testing these two puzzles is to estimate return-based pricing kernels.

The first database used to estimate SDF's is comprised of U.S.\$ real returns on all U.S. stocks traded along the period in question, in a totality of 464 stocks, according to CRSP database. This is used to construct what we labelled the domestic SDF. It is com-

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<sup>19</sup>The entire database of foreign government bonds is in the section B in the Appendix.

pletely U.S. based and available at a very disaggregated level. Domestic SDF's are later used to test the Pricing Equation for foreign assets—a direct response to Cochrane (2001)' critique—making our pricing tests out-of-sample in the cross-sectional dimension (assets). The second database used here to estimate SDF's is comprised of US\$ real returns on one hundred size-*BE/ME* portfolios.

In the first case, we extract from these 464 stocks factors which are useful to derive a linear model to the SDF, while in the second one, the factors are computed from Fama-French portfolios, also extracted from the French data library.

The two data sets described above were employed to estimate the multi-factor versions of the SDF's: the first one characterizing essentially the stock market, labelled  $\widetilde{M}_t^{*ST}$ , and the second one capturing the size-book effects, labelled  $\overline{M}_t^{*FF}$ .

All macroeconomic variables used in econometric tests were extracted from FED's FRED database. We also employed additional forecasting financial variables that are specific to each test performed, and are listed in the appropriate tables of results.

Table 1 presents a summary statistic for real returns and excess returns on the assets to be priced over the period 1977:1 to 2004:4. The average real return on the covered trading of foreign government bonds ranges from 1.77% to 2.60% a year, except for the Swiss bonds with 7.88%, while that of uncovered trading ranges from 2.72% to 5.14%. The real return on the S&P500 is 9.23% at an annual rate, while that of the 90-day T-bill is 1.78%, with a resulting excess return (equity premium) of 7.35%. As expected, real stock returns are much more volatile than the U.S. Treasury Bill return—annualized standard deviations of 16.64% and 1.61% respectively. Some of the foreign-currency portfolios in the Lustig and Verdelhan (2007) database have a negative real excess return but, in general, their volatilities seem to be modest, corroborating these same statistics for original annually re-balanced portfolios, as one can observe in Figure 2. In contrast, means and standard deviations of the Fama and French portfolios are quite high, reaching more than 17% and 29% respectively.

We computed the Sharpe ratio for the U.S. stock market to be 0.44, while the Sharpe ratio of the uncovered trading of foreign bonds ranges from  $-0.20$  to  $0.38$ , depending on the country being considered. According to Shiller (1982), Hansen and Jagannathan (1991), and Cochrane and Hansen (1992), an extremely volatile SDF is required to match

the high equity Sharpe ratio of the U.S. Hence, the smoothness of aggregate consumption growth is the main reason behind the EPP. Since the higher the Sharpe ratio, the tighter the lower bound on the volatility of the pricing kernel, a natural question that arises is the following: may we regard this fact as evidence that a kernel that prices correctly the equity premium would also price correctly the forward premium? In this sense, our results can be useful to answer this question in an indirect way.

## 4.2 SDF Estimates

Before presenting the results for SDF estimates, we tested all returns used either in estimating SDF's or in the testing procedures for the presence of a unit root in the autoregressive polynomial.<sup>20</sup> Because returns are prone to displaying heteroskedasticity, we used the Phillips and Perron (1988) unit-root test including an intercept in the test regression. For almost all cases we rejected the null of a unit root at the 5% level and rejected the null for all cases at the 10% level. We take this as evidence that all returns and the pricing kernels are well behaved.<sup>21</sup>

The first step in constructing  $\widetilde{M}_t^{*ST}$ , from the real returns on all traded U.S. stocks, is to choose the number of factors. Following Lehmann and Modest (1988), Connor and Korajczyk (1988), Tsay (2001) and most of the empirical literature, we took a pragmatic, informal, but useful view, examining the time plot of the eigenvalues ordered from the largest to the smallest. We are able to determine appropriately that five factors are needed to account for about a representative portion of the variation of our 464 stock returns, a choice (five factors) that is close to the one in Connor and Korajczyk (1993), who examined returns from stocks listed on the New York Stock Exchange and the American Stock Exchange. For the second SDF, with only three factors, more than 99% of the variation of the one hundred Fama-French portfolios is explained.

The estimates of  $M_t^*$ — $\widetilde{M}_t^{*ST}$  and  $\overline{M}_t^{*FF}$ —are plotted in Figure 1, which also include their summary statistics. Their means are slightly below unity. The pricing kernel extracted from the Fama-French portfolios is more volatile than the first one and the correlation coefficients between them is 0.357.

<sup>20</sup>See Hansen (1982) and Ogaki (1992) for more details.

<sup>21</sup>Unit-root test results for all series used in the paper are available upon request.

### 4.3 Pricing-Test Results

Tables 2 and 3 present results on the behavior of our return-based pricing kernels,  $M_t^{*ST}$  and  $\overline{M}_t^{*FF}$ —formed using only domestic U.S. returns—in pricing returns related to the EPP and other domestic returns discussed in the literature. These are in-sample tests. Table 2 tests the system of orthogonality restrictions (30), accounting for the equity premium,  $R_{t+1}^{SP} - R_{t+1}^b$ , and for the return on S&P500,  $R_{t+1}^{SP}$ . We use a constant as instrument as well as  $(D/P)_t$  and  $(I/C)_t$ . Table 3 presents pricing tests on the Fama-French benchmark portfolios using as instruments a constant and the Fama-French factors.

These in-sample tests show no evidence that pricing errors are relevant: at the 5% significance level, pricing errors are all statistically zero (individually or jointly) using the Wald test. Notice that pricing errors and the respective standard errors are multiplied by 100.

Tables 4, 5 and 6 present our main results, i.e., the results of out-of-sample pricing tests, where  $\widehat{M}_t^{*ST}$  and  $\overline{M}_t^{*FF}$  are respectively used to price foreign assets related to the FPP and other foreign returns on dynamic portfolios discussed in the literature.

Table 4 presents the results of tests for the system of orthogonality restrictions (25). There are two Euler equations: one with the excess return on uncovered over covered trade with foreign government bonds ( $R_{t+1}^U - R_{t+1}^C$ ), and one with the uncovered return,  $R_{t+1}^U$ . The instrument set includes a constant and the variable that generates the FPP in regression (10),  $({}_tF_{t+1}^j - S_t^j)/S_t^j$ , making the result of these tests very informative with regards to the FPP. At the 5% significance level, all individual pricing errors are statistically zero for all foreign currencies studied here. Wald-test results show no sign of joint significance for currency-pricing errors. Moreover, the size of the pricing errors is relatively modest, lower than unity with a mean value of 0,5%, when trying to price returns and excess returns simultaneously.

In Table 5 we present joint tests of the FPP and of the EPP for Canada, Germany, Japan, Switzerland and the U.K. These pricing restrictions stack the Euler equations that characterize the EPP tested in Table 2 and those tested in Table 4, i.e., the system (28). The instrument set is the union of the sets previously considered: a constant,  $({}_tF_{t+1}^j - S_t^j)/S_t^j$ ,  $(D/P)_t$  and  $(I/C)_t$ . In this strongest test of the pricing kernels results confirm that return-based kernels generate no puzzles whatsoever in pricing returns and excess

returns for the U.K., Germany, and Switzerland. Moreover, for all countries, all individual tests for pricing errors are not significant at usual significance levels, whereas joint Wald tests reject that all pricing errors are zero for Canada and Japan at these same levels. A way to reconcile this last set of results is to imagine that pricing errors are indeed correlated, perhaps strongly. If this is case, it is possible for individual tests not to reject the null while joint tests produce the opposite result.<sup>22</sup>

Table 6 presents pricing tests for the dynamic Lustig and Verdelhan (2007) foreign currency portfolios, when  $\overline{M}_t^{*FF}$  is used in testing (29). The pricing error of portfolio 1 is significantly different from zero at the 5% level in out-of-sample tests. Portfolio 1 is the portfolio built with the lowest interest rate currencies relative to the U.S. T-bill. Due to rejection of Portfolio 1 in these pricing tests, joint Wald-tests also reject the group of portfolios 1 - 8. Further investigation on pricing errors confirms that mispricing comes indeed from Portfolio 1 alone. Once we remove it from the Lustig and Verdelhan set of portfolios, there is no evidence of mispricing at the 5% level.

The results in Tables 4, 5 and 6 show strong evidence that  $\widetilde{M}_t^{*ST}$  and  $\overline{M}_t^{*FF}$  are able to account for most of the puzzling aspects in asset pricing, as the EPP and the FPP. Also, there is strong evidence that they price correctly all the dynamic Fama-French portfolios and all the Lustig and Verdelhan portfolios—with the exception of Portfolio1.

Here, we should stress that  $\widetilde{M}_t^{*ST}$  and  $\overline{M}_t^{*FF}$  were constructed using domestic asset returns alone. Therefore, these exercises are out-of-sample. Moreover, our results do not come from increased variance. Indeed, when using the multi-factor SDF estimate, the discounted return, i.e., scaled by  $M^*$ , displayed lower variance than the undiscounted return. The high p-values found in this paper come from lower pricing errors, not higher variance.<sup>23</sup>

Table 7 displays the performance of  $\widetilde{M}_t^{*ST}$  in predicting returns and excess returns on

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<sup>22</sup>Hansen (1982)'s  $J$ -test does not reject the null because it evaluates the quadratic form at the optimal level, using the identity matrix as a weighting factor. This, of course, does not take into account the cross-correlation of the pricing errors.

<sup>23</sup>To be more specific, when the multi-factor pricing kernel is used the only exceptions are real returns on uncovered trading with Japanese and Canadian bonds. For returns and excess returns on all other foreign bonds, return on S&P500, market excess return, Fama-French and Lustig-Verdelhan portfolios, the volatility is lower for SDF-discounted returns.

uncovered and covered trading of British, Canadian, German, Japanese and Swiss bonds. Table 8 presents the same result for  $\overline{M}_t^{*FF}$  in predicting Lustig and Verdelhan (2007)'s foreign currency portfolios.

In Figures 3 and 4 we show predicted versus realized average return (to be precise, net return) and excess return for each pricing kernel. Pricing errors are measured by the distance to the 45-degree line (dotted line). The pricing kernels do a reasonable job in predicting excess returns on trading with foreign government bonds and on dynamic foreign currency portfolios. The main exceptions are the excess return on Swiss bonds and the two extreme portfolios of Lustig and Verdelhan, for which pricing errors are large.

As expected, in-sample forecasts are in general better than out of sample forecasts. Evidence toward this fact is found in Table 7, where we display the forecasting gains of using  $\widetilde{M}_t^*$ : a global pricing kernel constructed by da Costa et al. (2008) from aggregate returns on stocks and government bonds for G7 countries and on Lustig and Verdelhan (2007)'s eight foreign dynamic currency portfolios. The gain in using such pricing kernel *vis a vis*  $\widetilde{M}_t^{*ST}$  is about 20%. Based on results reported in Table 8, the out-of-sample forecast are quite reasonable, more so if we consider the high values for the excess returns on portfolios in question.

Finally, according to Hansen and Jagannathan (1997), "[...] pricing errors may occur either because the model is viewed formally as an approximation, [...] or because the empirical counterpart to the theoretical stochastic discount factor is error ridden". In this sense, we are able to show that  $\widetilde{M}_t^{*ST}$ , a SDF constructed to account only for domestic puzzles, not only does a good job in accounting for foreign market results, based on  $\chi^2$  statistics, but, in fact, performs better than  $\widetilde{M}_t^*$ , according to the pricing error measure developed by these authors. The measures are 0.41 and 0.36, respectively.

**Discussion** Notwithstanding the rejection for one of the Lustig and Verdelhan (2007)'s portfolios in pricing tests, and two joint rejections on Wald tests, the favorable out-of-sample tests results obtained in Tables 4 through 6 are in sharp contrast with those formerly obtained in log-linear tests of the FPP and the results obtained using consumption-based kernels. As argued above, this may suggest the inappropriateness of log linearizing the Euler equation and/or using current consumption-based pricing kernels.



As in any empirical exercise, it is important to verify whether results are robust to changes in the environment used in testing. Here, we changed the conditioning set used through Tables 2 to 6 as well as the assets directly being tested in pricing. In both instances the basic results remain unchanged and are available upon request.

Finally, a crucial element in our analysis is showing that the returns of investments in foreign assets are accounted for by an adequate pricing kernel. This is indeed what we found when the ratio  ${}_tF_{t+1}/S_t$  was used as an instrument and the theoretical restrictions tested were not rejected. This suggests that  ${}_tF_{t+1}/S_t$  has no predictive power for

$$M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)}{P_{t+1}} \frac{{}_tF_{t+1} - S_{t+1}}{S_t},$$

despite having predictive power for

$$\frac{P_t(1 + i_{t+1}^*)}{P_{t+1}} \frac{{}_tF_{t+1} - S_{t+1}}{S_t}.$$

Hence, although the excess returns on uncovered over covered trading with foreign bonds are predictable, “risk adjusted” excess returns are not. Another piece of evidence on this regard is the relative success in pricing Lustig and Verdelhan’s portfolios. We have shown that discounted excess return of Lustig and Verdelhan (2007)’s dynamic currency portfolios are zero, with the exception of one portfolio: the one with the lowest returns.

## 5 Conclusion

Previous research has cast doubt on whether a single asset pricing model was capable of correctly pricing the equity and the forward premium, which lead to the emergence of two separate literatures. We challenge this position and propose a fresh look into the relationship between the Equity and the Forward Premium Puzzles.

We do so by extracting SDF estimates using U.S. return data and showing them to be able to properly price returns and excess returns of assets that comprise the equity premium and the forward premium puzzles. We, thus, establish the common nature of the FPP and the EPP. By using consistent estimates of the SDF mimicking portfolio, we have shown that the projection of a proper CCAPM model—yet to be written—on the space of returns is able to properly price assets that comprise the EPP and the FPP. We have

not found this proper model, yet. Nevertheless, we rely on the fact its pricing properties cannot be better than those of its projection on the space of returns.

By working with a U.S. based version of SDF estimates, we were able to show that the factors contained in domestic (U.S.) market returns yield neither evidence of the EPP nor evidence of the FPP for most cases.

Our starting point is the Asset Pricing Equation, coupled with the use of consistent estimators of the SDF mimicking portfolio. We first show that, our return-based kernels constructed using domestic (U.S.) returns alone price correctly the equity premium, the 90-day T-bill and the Fama-French portfolios. When discounted by our pricing kernels, excess returns are shown to be orthogonal to past information that is usually known to forecast undiscounted excess returns. Based on these results, we go one step further and ask whether the EPP and the FPP are but two symptoms of the same illness—the inability of standard (and augmented) consumption-based pricing kernels to price asset returns or excess-returns. In our tests, we found that the *ex-ante* forward premium is not a predictor of discounted excess returns, despite their being so for undiscounted excess returns.

We believe to have offered evidence that the answer to the question posed in the title of this paper is in the affirmative. A different and interesting issue is whether the forward should premium be regarded as a reward for risk taking? If we take the covariance with  $M_{t+1}^*$  as the relevant measure of risk, then our answer is yes. This position is not without controversy, however. Citing (Engel (1996), p. 162): “If the [CAPM] model were found to provide a good description of excess returns in foreign exchange markets, there would be some ambiguity about whether these predicted excess returns actually represent premiums.” We do however side with the position implicit in Brandt et al. (2006) for whom the behavior of the SDF is equal to that of the intertemporal marginal rate of substitution for a model of preferences and/or market structure yet to be written. In this sense, we have provided grounds to believe that the striking similarity in the results found in trying to apply the CCAPM for the two markets is not accidental.

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## A Return-Based Pricing Kernels and the SDF Mimicking Portfolio

Factor models summarize the systematic variation of the  $N$  elements of the vector  $\mathbf{R}_t = (R_{1,t}, R_{2,t}, \dots, R_{N,t})'$  using a reduced number of  $K$  factors,  $K < N$ . Consider a  $K$ -factor model in  $R_{i,t}$ :

$$R_{i,t} = a_i + \sum_{k=1}^K \beta_{i,k} f_{k,t} + \eta_{it}, \quad (32)$$

where  $f_{k,t}$  are zero-mean pervasive factors and, as is usual in factor analysis and

$$\text{plim}_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \eta_{i,t} = 0.$$

Denote by  $\Sigma_r = \mathbb{E}(\mathbf{R}_t \mathbf{R}_t') - \mathbb{E}(\mathbf{R}_t) \mathbb{E}(\mathbf{R}_t')$  the variance-covariance matrix of returns. The first principal component of the elements of  $\mathbf{R}_t$  is a linear combination  $\theta' \mathbf{R}_t$  with maximal variance subject to the normalization that  $\theta$  has unit norm, i.e.,  $\theta' \theta = 1$ . Subsequent principal components are identified if they are all orthogonal to the previous ones and are subject to the same normalization. The first  $K$  principal components of  $\mathbf{R}_t$  are consistent estimates of the  $f_{k,t}$ 's. Factor loadings can be estimated consistently by simple OLS regressions of the form (32).

That is, principal components are simply linear combinations of returns. They are constructed to be orthogonal to each other, to be normalized to have a unit length and to deal with the problem of redundant returns, which is very common when a large number of assets is considered. They are ordered so that the first principal component explains the largest portion of the sample variance-covariance matrix of returns, the second one explains the next largest portion, and so on.

To understand why we need the unit-norm condition, recall that the variance of the first principal component  $\theta' \mathbf{R}_t$  is  $\theta' \Sigma_r \theta$ . As discussed in Dhrymes (1974), because we want to maximize this variance, the problem has no unique solution—we can make the variance as large as we want by multiplying  $\theta$  by a constant  $\kappa > 1$ . Indeed, we are facing a *scale* problem, which is solved by imposing unit norm, i.e.,  $\theta' \theta = 1$ .

When a multi-factor approach is used, the first step is to specify the number of factors by means of a statistical or theoretical method and then to consider the estimation



of the model with known factors. Here, we are not particularly concerned about identifying the factors themselves. Rather, we are interested in constructing an estimate of the unique SDF mimicking portfolio  $M_t^*$ . For that, we shall rely on a purely statistical model, combined with the Pricing Equation.

It is straightforward to connect principal-component and factor analyses with the Pricing Equation, delivering a consistent estimator for  $M_t^*$ . Given estimates of  $a_i$ ,  $\beta_{i,k}$ , and  $f_{k,t}$  in (32), one can write their respective expected-beta return expression:

$$\mathbb{E}(R_i) = \gamma + \sum_{k=1}^K \beta_{i,k} \lambda_k, i = 1, 2, \dots, N$$

where  $\lambda_k$  is interpreted as the price of the  $k$ -th risk factor. The fact that the zero-mean factors  $f \equiv \tilde{f} - \mathbb{E}(\tilde{f})$  are such that  $\tilde{f}$  are returns with unitary price allows us to measure the  $\lambda$  coefficients directly by

$$\lambda = \mathbb{E}(\tilde{f}) -$$

and consequently to estimate only  $\gamma$  via a cross-sectional regression<sup>24</sup>. Given these coefficients, one can easily get an estimate of  $M_t^*$ ,

$$\tilde{M}_t^* \equiv a + \sum_{k=1}^K b_k f_{k,t}$$

where  $(a, b)$  is related to  $(\lambda, \gamma)$  through

$$a \equiv \frac{1}{\gamma} \text{ and } b \equiv -\gamma [\text{cov}(ff')]^{-1} \lambda,$$

It is easy then to see the equivalence between the beta pricing model and the linear model for the SDF. More, it is immediate that

$$\mathbb{E}(\tilde{M}_t^* R_{i,t}) = 1, \quad i = 1, 2, \dots, N.$$

The number of factors used in the empirical analysis,  $K$ , is expected to be rather small. We followed Lehmann and Modest (1988) and Connor and Korajczyk (1988), taking the pragmatic view whereby increasing  $K$  until the estimate of  $M_t^*$  changed very little due to the last increment in the number of factors.

It is important to stress that we need to impose a scale whenever a factor-model is used. Here, it is implicitly imposed that  $\text{VAR}(\tilde{M}_t^*) = 1$ ; see Cochrane (2001).

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<sup>24</sup>Note that we are not assuming the existence of a risk free rate.

## **B Foreign currency portfolios - database**

Our panel includes 80 countries. We include each of the following countries for the dates noted in parenthesis: Albania (1998:2 - 2004:4), Algeria (1998:3 - 2004:4), Antigua and Barbuda (1998:2 - 2004:4), Armenia (1998:2 - 2004:4), Australia (1977:1 - 2000:3), Austria (1977:1 - 2004:4), Bahamas (1977:1 - 2004:4), Belgium (1977:1 - 2004:4), Belize (1979:1 - 2004:4), Bulgaria (1998:2 - 2004:4), Bahrain (1998:2 - 2004:4), Brazil (1995:2 - 2004:4), Barbados (1977:1 - 2004:4), Canada (1977:1 - 2004:4), Cyprus (1977:1 - 2004:4), Czech Republic (1998:2 - 2004:4), Denmark (1977:1 - 2004:4), Dominica (1998:2 - 2004:4), Egypt (1998:2 - 2004:4), Ethiopia (1998:2 - 2004:4), Fiji (1977:1 - 2004:4), France (1977:1 - 2004:4), Greece (1998:2 - 2004:4), Hong-Kong (1998:2 - 2004:4), Germany (1977:1 - 2004:4), Granada (1977:1 - 2004:4), Guyana (1977:1 - 2004:4), Honduras (1998:2 - 2004:4), Hungary (1998:2 - 2004:4), India (1977:1 - 2004:4), Iceland (1998:2 - 2004:4), Israel (1998:2 - 2004:4), Italy (1977:1 - 2004:4), Jamaica (1977:1 - 2004:4), Japan (1977:1 - 2004:4), Kenya (1977:1 - 2004:4), Kuwait (1998:2 - 2004:4), Kazakhstan 1998:2 - 2004:4), Kyrgyz Republic (1998:2-2004:4), Lao People's Dom. Republic (1998:2 - 2004:4), Lebanon (1998:2 - 2004:4), Lesotho (1998:2 - 2004:4), Lithuania (1998:2 - 2004:4), Latvia (1998:2 - 2004:4), Malawi (1998:2 - 2004:4), Malaysia (1977:1 - 2004:4), Malta (1998:2 - 2004:4), Mexico (1998:2 - 2004:4), Moldova (1998:2 - 2004:4), Namibia (1977:1 - 2004:4), Netherlands (1977:1 - 2004:4), Netherlands Antiles (1998:2 - 2004:4), Nigeria (1998:2 - 2004:4), Norway (1977:1 - 2004:4), Nepal (1998:2 - 2004:4), New-Zealand (1977:1 - 2004:4), Papua New Guinea (1977:1-2004:4), Philippines (1977:1 - 2004:4), Poland (1998:2 - 2004:4), Portugal (1977:1 - 2004:4), Romania (1998:2 - 2004:4), Russian Federation (1994-2002), Seychelles (1998:2 - 2004:4), Sierra Leona (1977:1-2004:4), Singapore (1977:1 - 2004:4), Slovenia (1998-2002), Solomon Island (1998:2 - 2003:2), Spain (1977:1 - 2004:4), Sri Lanka (1998:2 - 2004:4), St. Kitts and Nevis (1998:2 - 2004:4), St. Lucia (1998:2 - 2004:4), St. Vincent and Grens (1998:2 - 2004:4), Sweden (1977:1 - 2004:4), Swaziland (1992:1 - 2004:4), Switzerland (1977:1 - 2004:4), Tanzania (1998:2-2004:4), Trinidad and Tobago (1977:1 - 2004:4), Turkey (1985:1 - 1993:4), United Kingdom (1977:1 - 2004:4), United States (1977:1 - 2004:4), Venezuela (1994:2 - 2004:4), Vietnam (1998:2-2004:4), South Africa (1977:1 - 2004:4), Zambia (1977:1 - 2004:4), Zimbabwe (1977:1 - 2004:4). The exchange and T-bill rates were downloaded from IFS/ IMF. The maturity of the T-bill rates is 3 months.

Table 1

Summary statistics for the quarterly real returns and excess returns (% per year). From 1977:1 to 2004:4, 112 observations.

International data: Foreign government bonds					
US\$ real net return on the covered trading of foreign government bond		US\$ real net return on the uncovered trading of foreign government bond		US\$ real excess return on the uncovered over the covered trading of foreign government bond	
Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
British bonds	1.770	2.214	1.058	3.322	10.838
Canadian bonds	2.287	2.213	5.622	0.428	5.088
German bonds	1.972	2.692	11.941	1.128	12.349
Japanese bonds	2.597	1.801	13.300	1.451	13.201
Swiss bonds	7.880	15.904	16.895	-4.134	20.465

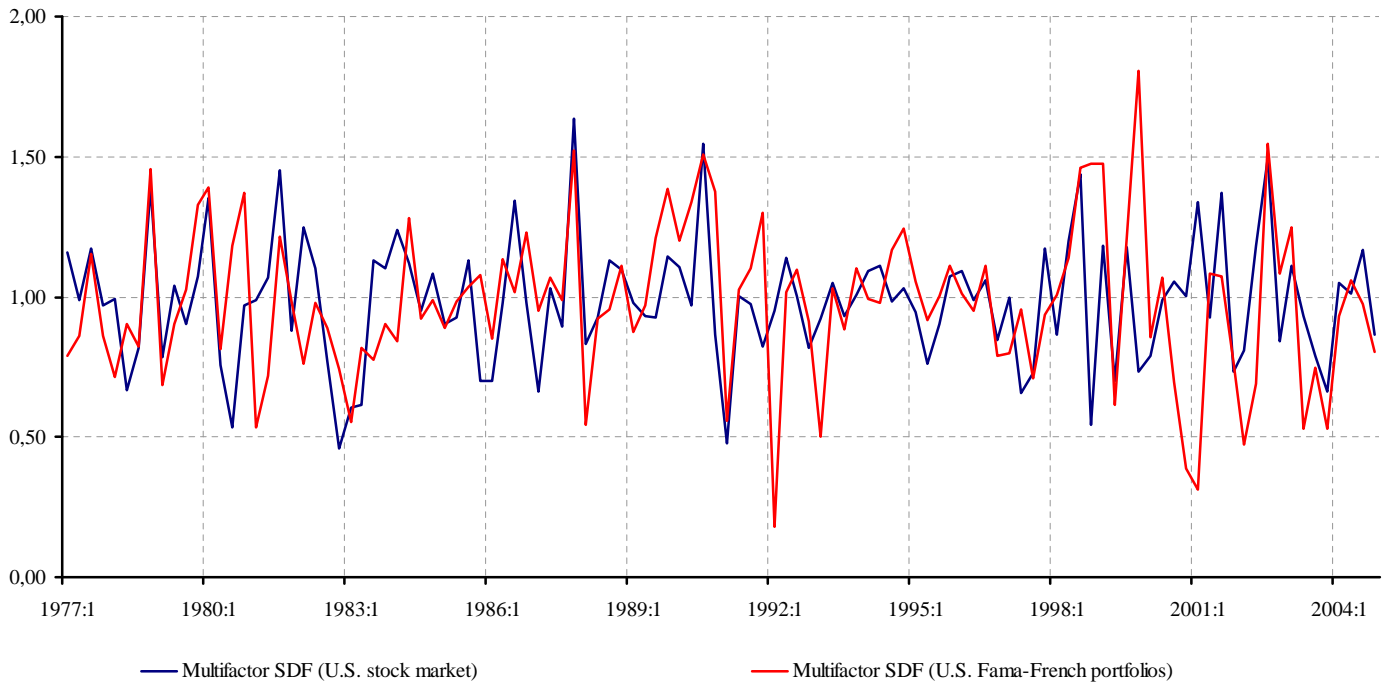
International data: Foreign currency portfolios, following Lustig and Verdelhan (2007)					
US\$ real excess return on foreign currency portfolios			US\$ real excess return on foreign currency portfolios		
Mean	Standard deviation		Mean	Standard deviation	
Foreign currency #1	-3.394	4.177	Foreign currency #5	-1.547	9.749
Foreign currency #2	-0.611	6.946	Foreign currency #6	-1.973	9.015
Foreign currency #3	0.301	8.192	Foreign currency #7	1.185	9.226
Foreign currency #4	1.261	8.145	Foreign currency #8	4.134	12.267

US data: Stock market index and 90-day T-bill					
US\$ real net return and excess return			US\$ return on Fama-French portfolios		
Mean	Standard deviation		Mean	Standard deviation	
S&P500	9.225	16.644	Big Growth	9.050	19.387
T-bill	1.777	1.606	Big Neutral	9.926	16.112
S&P500 - T-bill	7.351	16.273	Big Value	10.123	16.655
			Small Growth	10.760	28.676
			Small Neutral	14.971	21.379
			Small Value	17.286	23.341

**Figure 1**  
**Multifactor pricing kernels**

Multifactor pricing kernels constructed from the real returns on 464 stocks traded on NYSE ( $\widetilde{M}_t^{*ST}$ ) and returns on one hundred size-*BE/ME* Fama-French portfolios ( $\overline{M}_t^{*FF}$ ). From 1977:1 to 2004:4, 112 observations.

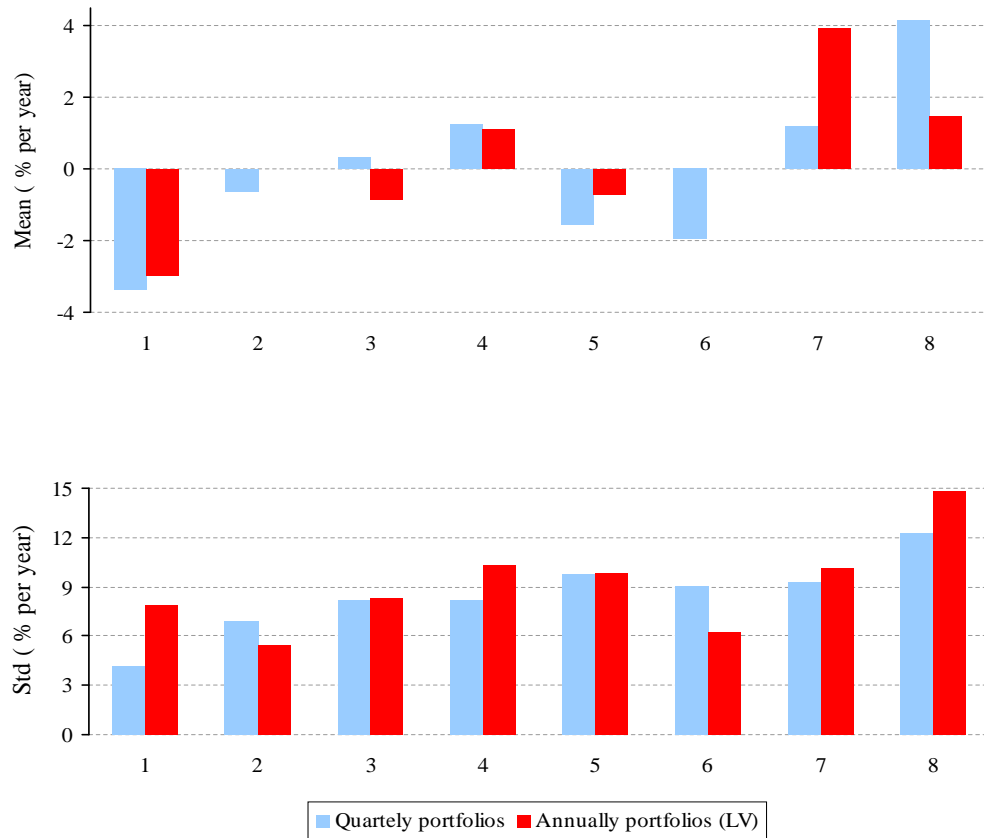


Quarterly summary statistics				
	Mean	Standar deviation	Min.	Max.
$\widetilde{M}_t^{*ST}$	0.984	0.222	0.458	1.635
$\overline{M}_t^{*FF}$	0.980	0.279	0.178	1.806

**Figure 2**  
**Foreign currency portfolios**

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This figure presents mean and standard deviation, of US\$ real excess return on 8 dynamic foreign currency portfolios. In this paper, we construct quarterly re-balanced portfolios and the data covers the period from 1977:1 to 2004:4. We also plot these statistics of the original Lustig and Verdelhan (2007) annually re-balanced portfolios for the period from 1971 to 2002.



**Table 2**  
**EPP test using  $\widetilde{M}_t^{*ST}$  (in-sample exercise for S&P500 and American 90-day T-bill).** <sup>a,b,c</sup>

$$\text{System: } \mathbb{E}_t \left\{ \begin{array}{l} \widetilde{M}_t^{*ST} \cdot (R_{t+1}^{SP} - R_{t+1}^b) - \mu_1 \\ \widetilde{M}_t^{*ST} \cdot R_{t+1}^{SP} - (1 - \mu_2) \end{array} \right\} = \mathbf{0} \quad \begin{array}{l} \text{Instrument set: } (D/P)_t, (I/C)_t \\ \text{and a constant} \end{array}$$

Wald test			Pricing errors			Overidentifying restrictions		
H <sub>0</sub> : $\mu_i = 0, i = 1, 2$			$\widehat{\mu}_1$	0.358	(0.789)	[0.650]	J-statist.	0.0001
$\chi^2$	0.549	[0.760]	$\widehat{\mu}_2$	0.968	(1.393)	[0.488]	p-value	[0.999]

\* Indicates the rejection of the null hypothesis of insignificant pricing errors at 5% level.

\*\* Indicates the rejection of the  $T \times J$  statistic at 5% significance level. <sup>a</sup> Hansen's (1982)

Generalized Method of Moments technique used to test Euler equations and to estimate

the model parameters over the period from 1977:1 to 2004:4,112 observations. <sup>b</sup> Respective

standard errors are reported in parenthesis while p-values in the box brackets. <sup>c</sup> The pricing

errors and the standard errors are reported multiplied by 100.

**Table 3**  
**American stock market anomaly test using  $\overline{M_t^{*FF}}$  (in-sample exercise for**  
**Fama-French factors, zero-cost portfolios formed on value, size and**  
**book-to-market). <sup>a,b,c</sup>**

$$\text{System: } \mathbb{E}_t \left\{ \begin{array}{l} \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{B/H} - (1 - \mu_1) \\ \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{B/L} - (1 - \mu_2) \\ \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{B/M} - (1 - \mu_3) \\ \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{S/H} - (1 - \mu_4) \\ \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{S/L} - (1 - \mu_5) \\ \widetilde{M_{t+1}^{*US}} \cdot R_{t+1}^{S/M} - (1 - \mu_6) \end{array} \right\} = \mathbf{0}$$

Instrument set:  $HML, SMB_t, RmRf_t$   
and a constant

Wald test	Pricing errors				Overidentifying restrictions	
$H_0: \mu_i = 0, i = 1, \dots, 3$	$\widehat{\mu}_1$	0.362	(2.915)	[0.901]	J-statist.	0.0097
$\chi^2$ 10.428 [0.108]	$\widehat{\mu}_2$	-0.467	(3.355)	[0.889]	p-value	[1.000]
	$\widehat{\mu}_3$	0.121	(2.996)	[0.968]		
	$\widehat{\mu}_4$	-0.302	(2.511)	[0.904]		
	$\widehat{\mu}_5$	-0.185	(3.239)	[0.954]		
	$\widehat{\mu}_6$	-0.274	(2.683)	[0.919]		

\* Indicates the rejection of the null hypothesis of insignificant pricing errors at 5% level. \*\* Indicates the rejection of the  $T \times J$  statistic at 5% significance level. <sup>a</sup> Hansen's (1982) Generalized Method of Moments technique used to test Euler equations and to estimate the model parameters over the period from 1977:1 to 2004:4, 112 observations. <sup>b</sup> Respective standard errors are reported in parenthesis while p-values in the box brackets. <sup>c</sup> The pricing errors and the standard errors are reported multiplied by 100.

Table 4

**FWP test using  $\widehat{M}_{t+1}^{*ST}$  (out-of-sample exercise for each foreign currency).** <sup>a,b,c,d</sup>

$$\text{System: } \mathbb{E}_t \left\{ \begin{array}{l} \widehat{M}_{t+1}^{*ST} \cdot (R_{t+1}^{U,j} - R_{t+1}^{C,j}) - \mu_1 \\ \widehat{M}_{t+1}^{*ST} \cdot R_{t+1}^{U,j} - (1 - \mu_2) \end{array} \right\} = 0$$

Instrument set:  $({}_tF_{t+1}^j - S_t^j)/S_t^j$  and a constant

Results for British government bonds:

Wald test	Pricing errors	Over. restrictions
$H_0: \mu_i = 0, i = 1, 2$	$\widehat{\mu}_1$ 0.877 (0.609) [0.151]	J-statist. 0.0000
$\chi^2$ 2.485 [0.289]	$\widehat{\mu}_2$ 0.306 (1.892) [0.872]	p-value [1.000]

Results for German government bonds:

Wald test	Pricing errors	Over. restrictions
$H_0: \mu_i = 0, i = 1, 2$	$\widehat{\mu}_1$ 0.450 (0.722) [0.534]	J-statist. 0.0000
$\chi^2$ 0.839 [0.658]	$\widehat{\mu}_2$ 0.705 (1.961) [0.720]	p-value [1.000]

Results for Swiss government bonds:

Wald test	Pricing errors	Over. restrictions
$H_0: \mu_i = 0, i = 1, 2$	$\widehat{\mu}_1$ -0.860 (1.222) [0.482]	J-statist. 0.0015
$\chi^2$ 0.505 [0.777]	$\widehat{\mu}_2$ 0.542 (2.046) [0.791]	p-value [0.919]

Results for Canadian government bonds:

Wald test	Pricing errors	Over. restrictions
$H_0: \mu_i = 0, i = 1, 2$	$\widehat{\mu}_1$ 0.000 (0.280) [0.981]	J-statist. 0.0000
$\chi^2$ 0.361 [0.835]	$\widehat{\mu}_2$ 1.067 (1.799) [0.554]	p-value [1.000]

Results for Japanese government bonds:

Wald test	Pricing errors	Over. restrictions
$H_0: \mu_i = 0, i = 1, 2$	$\widehat{\mu}_1$ 0.200 (0.726) [0.784]	J-statist. 0.0000
$\chi^2$ 0.341 [0.843]	$\widehat{\mu}_2$ 0.793 (1.807) [0.661]	p-value [1.000]

\* Indicates the rejection of the null hypothesis of insignificant pricing errors at 5% level. \*\* Indicates the rejection of the  $T \times J$  statistic at 5% significance level. <sup>a</sup> Hansen's (1982) Generalized Method of Moments technique used to test Euler equations and to estimate the model parameters over the period from 1977:1 to 2004:4, 112 observations. <sup>b</sup> We use the superscript  $j$  for returns, forward and spot rates in order to associate these variables to the country  $j$ . <sup>c</sup> Respective standard errors are reported in the parenthesis while p-values in the box brackets. <sup>d</sup> The pricing errors and the standard errors are reported multiplied by 100.



Table 5

FPP test using  $\widehat{M}_t^{*ST}$  (joint out-of-sample exercise, including S&P500 and American 90-day T-bill, for each foreign currency) <sup>a,b,c,d</sup>

$$\text{System: } \mathbb{E}_t \left\{ \begin{array}{l} \widehat{M}_t^{*ST} \cdot (R_{t+1}^{Uj} - R_{t+1}^{Cj}) - \mu_1 \\ \widehat{M}_t^{*ST} \cdot R_{t+1}^{Uj} - (1 - \mu_2) \\ \widehat{M}_t^{*ST} \cdot (R_{t+1}^{SP} - R_{t+1}^b) - \mu_3 \\ \widehat{M}_t^{*ST} \cdot R_{t+1}^{SP} - (1 - \mu_4) \end{array} \right\} = 0$$

Instrument set:  $({}_tF_{t+1}^j - S_t^j)/S_t^j, (D/P)_t, (I/C)_t$  and a constant

Results for British government bonds:

Wald test	Pricing errors	Over. restrictions
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 4$	$\widehat{\mu}_1$ 0.819 (0.611) [0.181]	J-statist. 0.0007
$\chi^2$ 3.254 [0.516]	$\widehat{\mu}_2$ 0.484 (1.900) [0.799]	p-value [0.999]
	$\widehat{\mu}_3$ 0.358 (0.789) [0.650]	
	$\widehat{\mu}_4$ 0.968 (1.393) [0.488]	

Results for German government bonds:

Wald test	Pricing errors	Over. restrictions
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 4$	$\widehat{\mu}_1$ 0.457 (0.728) [0.531]	J-statist. 0.0008
$\chi^2$ 1.649 [0.800]	$\widehat{\mu}_2$ 0.843 (1.973) [0.669]	p-value [0.999]
	$\widehat{\mu}_3$ 0.358 (0.789) [0.650]	
	$\widehat{\mu}_4$ 0.968 (1.393) [0.488]	

Results for Swiss government bonds:

Wald test	Pricing errors	Over. restrictions
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 4$	$\widehat{\mu}_1$ -0.817 (1.230) [0.507]	J-statist. 0.0023
$\chi^2$ 3.136 [0.535]	$\widehat{\mu}_2$ 0.635 (2.061) [0.758]	p-value [0.999]
	$\widehat{\mu}_3$ 0.358 (0.789) [0.650]	
	$\widehat{\mu}_4$ 0.968 (1.393) [0.488]	

Results for Canadian government bonds:

Wald test	Pricing errors	Over. restrictions
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 4$	$\widehat{\mu}_1$ 0.000 (0.283) [0.981]	J-statist. 0.0012
$\chi^2$ 28.464* [0.000]	$\widehat{\mu}_2$ 1.146 (1.814) [0.528]	p-value [0.271]
	$\widehat{\mu}_3$ 0.358 (0.789) [0.650]	
	$\widehat{\mu}_4$ 0.968 (1.393) [0.488]	

Results for Japanese government bonds:

Wald test	Pricing errors	Over. restrictions
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 4$	$\widehat{\mu}_1$ 0.147 (0.726) [0.840]	J-statist. 0.0005
$\chi^2$ 21.155* [0.000]	$\widehat{\mu}_2$ 0.984 (1.810) [0.588]	p-value [0.999]
	$\widehat{\mu}_3$ 0.358 (0.789) [0.650]	
	$\widehat{\mu}_4$ 0.968 (1.393) [0.488]	

\* Indicates the rejection of the null hypothesis of insignificant pricing errors at 5% level. \*\* Indicates the rejection of the  $T \times J$  statistic at 5% significance level. <sup>a</sup> Hansen's (1982)

Generalized Method of Moments technique used to test Euler equations and to estimate the model parameters over the period from 1977:1 to 2004:4, 112 observations. <sup>b</sup> We use

the superscript  $j$  for returns, forward and spot rates in order to associate these variables to the country  $j$ . <sup>c</sup> Respective standard errors are reported in the parenthesis while

p-values in the box brackets. <sup>d</sup> The pricing errors and the standard errors are reported multiplied by 100.

Table 6

Foreign government bond market anomalies test using  $\overline{M_t^{*FF}}$  (out-of-sample exercise for Lustig-Verdelhan foreign currency portfolios) <sup>a,b,c</sup>

$$\text{System: } \mathbb{E}_t \left\{ \begin{array}{l} \overline{M_t^{*FF}} \cdot R_{t+1}^{1,e} - \mu_1 \\ \overline{M_t^{*FF}} \cdot R_{t+1}^{2,e} - \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \overline{M_t^{*FF}} \cdot R_{t+1}^{8,e} - \mu_8 \end{array} \right\} = \mathbf{0} \quad \text{Instrument set: } \Delta \widetilde{R}_t \text{ and a constant}$$

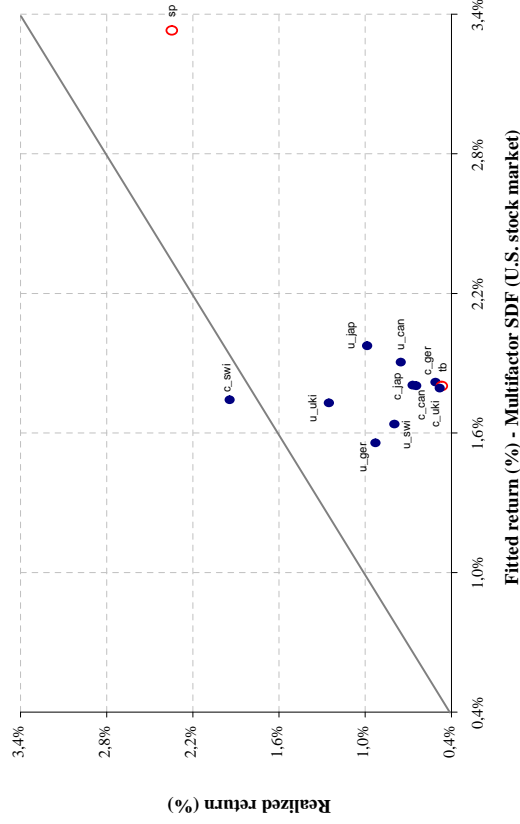
Wald test			Pricing errors				Overidentifying restrictions	
H <sub>0</sub> : $\mu_i = 0, i = 1, \dots, 8$			$\widehat{\mu}_1$	-0.789*	(0.259)	[0.001]	J-statist.	0.0000
$\chi^2$	32.918*	[0.000]	$\widehat{\mu}_2$	0.030	(0.428)	[0.945]	p-value	[1.000]
			$\widehat{\mu}_3$	0.294	(0.453)	[0.516]		
			$\widehat{\mu}_4$	0.493	(0.423)	[0.244]		
			$\widehat{\mu}_5$	-0.038	(0.430)	[0.930]		
			$\widehat{\mu}_6$	0.361	(0.512)	[0.481]		
			$\widehat{\mu}_7$	0.445	(0.592)	[0.453]		
			$\widehat{\mu}_8$	1.099	(0.649)	[0.091]		

\* Indicates the rejection of the null hypothesis of insignificant pricing errors at 5% level. \*\* Indicates the rejection of the  $T \times J$  statistic at 5% significance level. <sup>a</sup> Hansen's (1982) Generalized Method of Moments technique used to test Euler equations and to estimate the model parameters over the period from 1977:1 to 2004:4,112 observations. <sup>b</sup> Respective standard errors are reported in parenthesis while p-values in the box brackets. <sup>c</sup> The pricing errors and the standard errors are reported multiplied by 100.

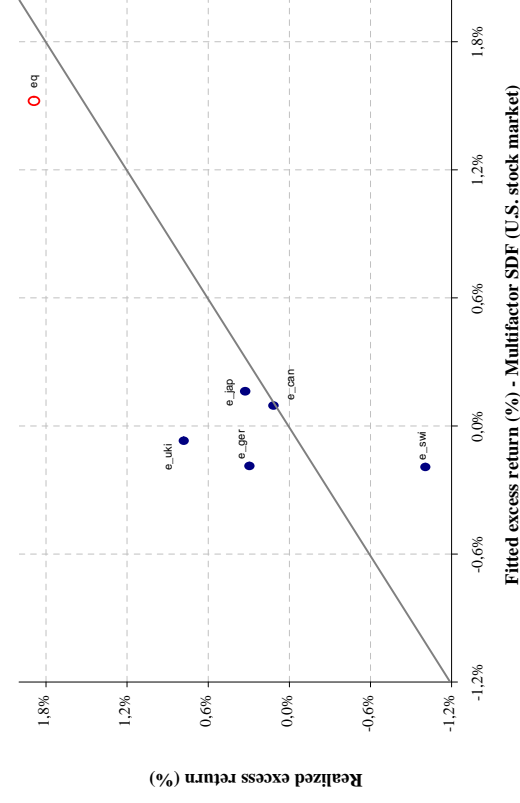
**Figure 3**

**Prediction exercise using  $\widehat{M}_t^{*ST}$**

This figure plots the average vs. the predicted returns on the assets in foreign currency and American stock markets. More specifically, in Figure 3 (a) we have an out-of sample exercise with a cross-section comprised of real returns on covered and uncovered trading with British, Canadian, German, Japanese and Swiss government bonds (filled dots) and also an in-sample exercise with returns on 90-day T-bill and S&P500 (empty dots). Figure 3 (b) is similar to the first one, but with excess returns on the respective assets. From 1977:1 to 2004:4, 112 observations.



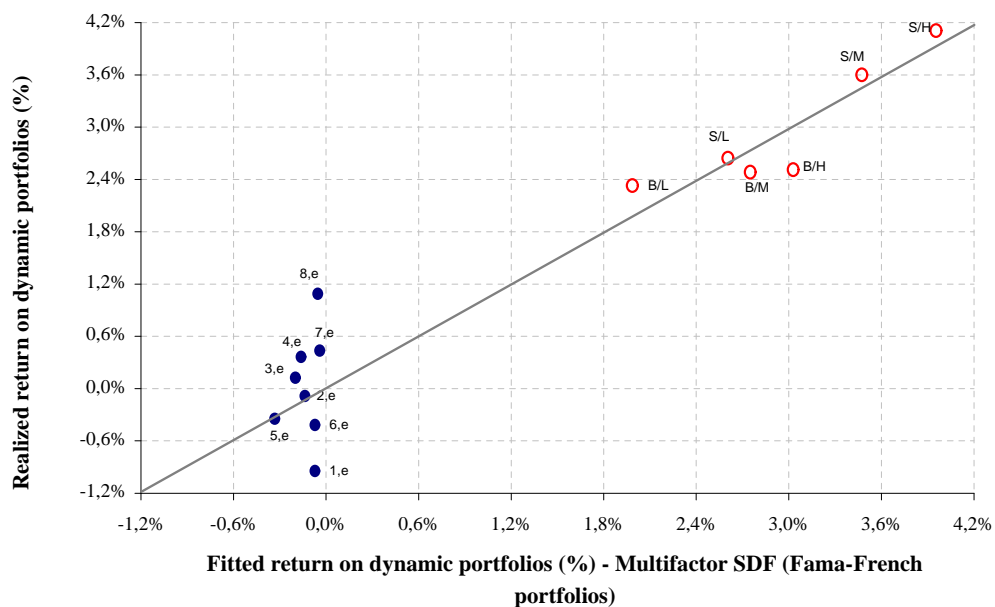
(a)



(b)

**Figure 4**  
**Prediction exercise using  $\overline{M_t^{*FF}}$ .**

This figure plots the average vs. the predicted returns on specific American and foreign dynamic portfolios. More specifically, we have an out-of sample exercise with a cross-section comprised of real excess returns on Lustig-Verdelhan portfolios (filled dots) and also an in-sample exercise with returns on Fama-French benchmark portfolios (empty dots). From 1977:1 to 2004:4, 112 observations.



**Table 7**  
**Multifactor pricing kernel (  $\widetilde{M}_t^{*ST}$  ) out-of-sample and in-sample prediction performance.<sup>a</sup>**

Pricing kernel	Foreign government bonds		U.S. stock market	
	Prediction performance measures		Prediction performance measures	
	Root mean-square error (RMSE)	Mean absolute error (MAE)	Root mean-square error (RMSE)	Mean absolute error (MAE)
$\widetilde{M}_t^{*ST}$	0.881%	0.774%	0.990%	0.900%
$\widetilde{M}_t^*$	0.740%	0.482%	---	---

<sup>a</sup> Prediction performance statistics display how the fitted return and excess return predictions compared to the realized observations. These statistics include average returns on uncovered and covered trading with British, Canadian, German, Japanese and Swiss foreign government bonds and the respective average excess returns vs. predictions of the pricing kernels extracted to account the EPP and the FPP,  $\widetilde{M}_t^{*ST}$ . We also include  $\widetilde{M}_t^*$  in-sample prediction as a benchmark. For the U.S. stock market, the statistics include returns on S&P500, T-Bill and the respective excess return. From 1977:1 to 2004:4, 112 observations.

**Table 8**  
**Multifactor pricing kernel (  $\overline{M}_t^{*FF}$  ) out-of-sample and in-sample prediction**

Pricing kernel	Foreign currency portfolios		Fama-French benchmak portfolios	
	Prediction performance measures		Prediction performance measures	
	Root mean-square error (RMSE)	Mean absolute error (MAE)	Root mean-square error (RMSE)	Mean absolute error (MAE)
$\overline{M}_t^{*FF}$	0.586%	0.464%	0.290%	0.233%

<sup>a</sup> Prediction performance statistics display how the fitted return and excess return predictions compared to the realized observations. These statistics include average excess returns on eight dynamic Lustig-Verdelhan portfolios vs. predictions of the pricing kernel extracted to account the size and book-to-market effects on American stock market,  $\overline{M}_t^{*FF}$ . In the in-sample exercise, the statistics include average return on six benchmak Fama-French portfolios formed on size and book-to-market effects. From 1977:1 to 2004:4, 112 observations.