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HOW WE GOT HERE? A METHODOLOGY TO STUDY THE EVOLUTION OF ECONOMIC INDICATORS[*]

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Abstract

This paper proposes a methodology to analyze the evolution of the economic development of countries. Our approach is based upon the definition of temporal trajectories of countries in a common bidimensional space yielded by a High-Order Singular Value Decomposition (HOSVD). These trajectories are defined with respect to a pre-selected set of macroeconomic indicators and are appropriate for comparison purposes. To show the applicability of the proposed methodology we have used data from the World Bank concerning the economic and financial development of EU-27 over a 14-year span, that goes from 1995 to 2008. Based on this data we group the EU-27 state members according to their economic development, which is indicated by the position of their trajectories on the plane. We further perform individual analyses of the trajectories of Luxembourg, Germany and Portugal, aiming to both detect and interpret trends and changes in these economies. The results show that this methodology is of importance for economic studies, since it can help the design, monitoring and evaluation of specific economic policies, as well as provide an overview of the evolution of the studied economic phenomenon.

Keywords: European Union, HOSVD, International Comparisons, Temporal Trajectories
JEL codes: C33, O52

1 Introduction

Our world is dynamic and has been marked by constant changes in the economic field. The identification of events behind deep structural changes and the understanding of how an economic system evolves can provide us a good basis to characterize the general evolution of economies. It also contributes to acquire a better background knowledge about the studied phenomenon, fostering the adoption of pro-active attitudes and the prediction of future

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trends. On the other hand, when the evolution of a given economy is studied in comparison with other economies, it is possible to identify those showing similar development patterns and perform a global economic positioning. For this reason, studies focusing on the evolution are gaining importance in almost every fields of knowledge. Regarding comparative studies, several papers resort to comparisons among European countries. For instance, Mackenbach et al. (2008) compared the magnitude of inequalities in mortality and self-assessed health among European countries in order to identify opportunities for the reduction of inequalities in health. Casu and Molyneux (2003) evaluated the determinants of European bank efficiency and the impact of the creation of the Single Internal Market in the convergence of productivity across European banking markets, using regression models and DEA estimations. Christensen et al. (2006) performed a comparative study of life satisfaction in European Union, by contrasting Denmark with other 14 state members, aiming to find the causes behind the consistent high happiness rates of this country. Finally, Goncalves et al. (2010) carried out an analysis of the evolution of the Portuguese economic activity sectors, using the Statis methodology. Statis is an exploratory three-way method which finds the common structure underlying data and comprises three main steps. One of these steps also involves the definition and interpretation of individual temporal trajectories in a bidimensional common space. Nevertheless, none of the mentioned studies focused on the comparative analysis of the European economies, as is our intention.

The goals of this research are twofold: first, we aim to provide a simple and intuitive methodology for conducting comparative studies, in a time-oriented perspective; second, we intend to draw interesting facts from the evolution of some European economies, by performing an analysis of the economic course of EU state members, for a time period of almost fifteen years, using the proposed methodology. The analysis is performed by means of the definition and interpretation of temporal trajectories of countries in a low-dimensional space yielded by the two most representative components of HOSVD. The trajectories are then explored in order to *group* similar countries based on their position on the plane and identify countries with successful economies, in terms of the measured dimensions. After, we carry out individual analyses of Luxembourg, Germany and Portugal in order to detect critical events, contextualize and interpret them, and briefly characterize the general evolution of these economies during a given time horizon.

The paper proceeds as follows. In Section 2 we describe in detail the proposed methodology. We begin by introducing the preliminaries of tensor algebra, namely the main concepts, terms and notation. Then, we present the foundations of HOSVD and explain how this technique can be explored in order to define temporal trajectories of objects in low-dimensional spaces. Section 3 is devoted to the study of the evolution of the European Union economies. In this section we provide the description of data, detail the process of application of HOSVD and interpret the results for three EU state members: Luxembourg, Germany and Portugal. Finally, in Section 4 we provide a brief summary of the paper's main points, underline the importance of using our methodology to perform comparative longitudinal studies and recommend future research.

2 Methodology

In this section we introduce the adopted methodology to study the evolution patterns of the economic development of EU countries. The proposed analysis is mainly visual, since it is based on the comparison of temporal trajectories of the EU countries. These trajectories are defined in a low-dimensional and representative space, yielded by a HOSVD. HOSVD is a general decomposition method for multilinear algebra problems. Based on this method it is possible to extract the relevant information comprised in multi-way arrays and represent it through direct mapping in low-dimensional subspaces.

To better understand the proposed methodology, here we present some of the concepts, terms and notation from tensor algebra. After presenting the preliminaries, we describe the idea behind HOSVD, more specifically, behind Tucker3 model [1], and explain how this technique can be explored in order to define trajectories of objects over time.

2.1 Tensors

2.1.1 The Concept of Tensor

Traditional data analysis techniques, such as Regression, Principal Component Analysis (PCA), Clustering and Linear Discriminant Analysis (LDA), were devised to extract relevant knowledge from two-way (or two-order) data, usually represented in matrix-form. In fact, this two-order data representation constitutes the basis of numerous and interesting analysis. However, in most of cases it can only provide a static view of the world, for a specific point in time. Since many phenomena are inherently multidimensional, in several settings one should adopt data representation schemes able to model simultaneously all dimensions (including the time dimension). The economic field is very rich in this kind of phenomena and has available lots of data that evolves over time. For instance, the great majority, if not all, of the existing economic indicators (e.g. Gross Domestic Product, Employment, Exports/Imports) are collected periodically and made freely available through institutional websites (e.g. World Bank, International Monetary Fund, Eurostat), thus making it more frequent and easier the undertaking of temporal studies. Therefore, and since it is our purpose to perform a comparative study of the development of EU economies by focusing on their temporal trajectories,

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[1] The HOSVD method, also known as multilinear SVD, is the same as a Tucker3 model with orthogonality constraints on the component matrices.

it is important to consider the temporal dimension, along with all other dimensions. In such cases, high-order *tensors* (also known as *hypermatrices*, *multi-way models* or *multi-way arrays*) appear as more natural and appropriate data representations than matrices, since they are able to explicitly model a higher number of dimensions (e.g. objects, variables and time) without collapsing the data and, therefore, without losing information about its mutual dependencies.

One of the desirable features of modeling three-dimensional data as tensors and explore techniques especially devised to deal with these data structures, is the possibility of preserving all mutual dependencies established between the different dimensions. By taking into account all interactions among the modes of the tensor, one can define a common low-dimensional and representative space where we can display compact information from the original tensor.

2.1.2 Tensor Notation

Regarding notation, we follow the typical conventions and, in this paper, we use the standardized notation and terminology for multi-way analysis as proposed by Kiers (2000).

As previously mentioned, a tensor is a N-way data array, where N is the *order* of the tensor. The *order*, *ways* or *modes* of a tensor are synonyms and refer to the number of dimensions. Formally, we denote a *scalar* (tensor of order zero) using normal lowercase letters (e.g. n) and a *vector* (tensor of first order) using boldface lowercase letters (e.g. $\mathbf{v}$). *Matrices* (tensor of order two) are denoted by boldface capital letters (e.g. $\mathbf{X}$) and higher-order *tensors* are denoted by calligraphic letter $\mathcal{X}$. The element (i, j, k) of a three-order tensor $\mathcal{X}$ is denoted by x_{ijk}. The same logic applies to lower orders: v_i denotes the ith entry of vector $\mathbf{v}$ and x_{ij} denotes element (i, j) of matrix $\mathbf{X}$. Indexes typically range from 1 to their capital version: $i = 1, ..., I$, $j = 1, ..., J$ and $k = 1, ..., K$.

The n-rank of a tensor $\mathcal{X}$ is denoted by $R_n = rank_n\mathcal{X}$ $(n = 1, ..., N)$ and it should be interpreted as the column rank of $\mathbf{X}_{(n)}$, where the subscript indicates the mode. Typically, $R_n \leq I_n$ for all $n = 1, ..., N$ and a tensor $\mathcal{X}$ can be referred as a rank-$(R_1, R_2, ..., R_n)$ tensor.

The notion of *tensor* encapsulates arrays with different number of orders: a vector of order I is a tensor in $\mathbb{R}^I$, a $I \times J$ matrix is a tensor in $\mathbb{R}^{I \times J}$, an $I \times J \times K$ three-order array is a tensor in $\mathbb{R}^{I \times J \times K}$ and a N-order array is a tensor in $\mathbb{R}^{I_1 \times I_2 \times ... \times I_N}$. For illustration purposes, in Figure 1 is depicted a three-order tensor $\mathcal{X} \in \mathbb{R}^{I \times J \times K}$. The entities along the vertical axis are indicated by the first index i and pertain to mode A. Those along the horizontal axis are indicated by the second index j and belong to mode B. Finally, entities along the depth axis are indicated by the third index k and pertain to mode C. The three sets of entities define the three ways, or three dimensions, of the three-order tensor. From now on, we will use the term *mode*, instead of *ways* or *dimensions*, to refer to a set of entities.

Sub-arrays are formed when a subset of indexes is fixed. For matrices, there are two types

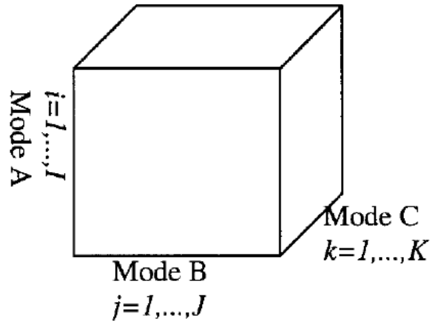


Figure 1: A three-order tensor (adapted from Kiers (2000)).

of indexes (or modes): indexes for the rows (i) and indexes for the columns (j). A colon is usually used to indicate all elements of a mode. Thus, the ith row of a matrix $\mathbf{X}$ is denoted by $x_{i,:}$, and the jth column of $\mathbf{X}$ is denoted by $x_{:,j}$ (Kolda and Bade, 2009).

The symbol $\circ$ denotes the **outer product** of vectors; for example, if $\mathbf{a} \in \mathbb{R}^I$, $\mathbf{b} \in \mathbb{R}^J$ and $\mathbf{c} \in \mathbb{R}^K$, then $\mathcal{X} = \mathbf{a} \circ \mathbf{b} \circ \mathbf{c}$ if and only if $x_{ijk} = a_i b_j c_k$ for all $1 \leq i \leq I$, $1 \leq j \leq J$, $1 \leq k \leq K$. The symbol $\otimes$ denotes the **Kronecker product** of vectors; for instance, $\mathbf{x} = \mathbf{a} \otimes \mathbf{b}$ means $x_l = a_i b_j$, with $l = j + (i-1)(J)$, for all $1 \leq i \leq I$, $1 \leq j \leq J$. The symbol $*$ denotes the **elementwise matrix product** (Dunlavy et al., 2006).

The **norm** of a tensor, which is the higher-order analogue of a matrix **Frobenius norm**, is the square root of the sum of the squares of all its elements. For a three-mode tensor $\mathcal{X} \in \mathbb{R}^{I \times J \times K}$, the norm is given by $\|\mathcal{X}\|^2 = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk}^2$.

2.1.3 Tensor Standard Operations

Albeit there are plenty of standard tensor operations, used for the purpose of multi-way analysis, here we will only introduce two of them. Once we do not intend to go into details in the explanation of the technique for decomposing tensors, we choose to present only the operations that are elementary and indispensable for understanding it (for further details see Kolda (2006)). The first operation, typically referred as *matricization*, is quite useful for computations, since it transforms the indexes of a tensor, so that it can be represented as a matrix (and vice-versa). The second operation, the *Kronecker product*, returns the tensor product between two matrices with respect to a standard choice of basis. This operation is useful for multi-way analysis since it mathematically expresses the relations between the original tensor and the corresponding decomposition in a simple form.

Definition 1 - Matricization, Matrix Unfolding or Matrix Flattening:
Matricization is the process of transforming a *tensor* into a *matrix*. The mode-d matricization of a N-order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ are vectors in $\mathbb{R}^{I_d}$ obtained by keeping index d

fixed and varying the other indexes. Therefore, the mode-d matricization $\mathbf{X}_{(d)}$ belongs to space $\mathbb{R}^{(\prod_{i \neq d} I_i) \times I_d}$ (Sun et al., 2006).

Definition 2 - KRONECKER PRODUCT OF MATRICES:
The *Kronecker product*, also known as the *direct product* or *tensor product*, is an operation on two matrices of arbitrary size. The *Kronecker product* of matrices $\mathbf{A} \in \mathbb{R}^{P \times Q}$ and $\mathbf{B} \in \mathbb{R}^{M \times N}$, is commonly denoted by the symbol $\otimes$, which applied as $\mathbf{A} \otimes \mathbf{B}$ yields the element-by-element multiplication of $\mathbf{B}$ with the elements from $\mathbf{A}$, returning a block matrix of size $(PM) \times (QN)$, as defined in Equation 1 (Brewer, 1978):

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \dots & a_{1q}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \dots & a_{2q}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1}\mathbf{B} & a_{p2}\mathbf{B} & \dots & a_{pq}\mathbf{B} \end{bmatrix} \tag{1}$$

2.2 Higher-Order Singular Value Decomposition

Three-way methods are multivariate data analysis tools that compress and visualize simultaneous variation of combinations of variables and objects (Smilde, 1992).

The most widely known three-way methods are the CANDECOMP/PARAFAC (CP) (Carroll and Chang, 1970; Harshman, 1970) and the Tucker3 decomposition (Tucker, 1963, 1966), which can be thought as higher-order generalizations of the Singular Value Decomposition (SVD). In this article we focus on Tucker decomposition since it is more flexible, easier to interpret and has less constraints (in fact, the CP model can be seen as a constrained variant of the three-way Tucker model, where the core tensor is superdiagonal).

There are also classic statistical methods, such as PDA (Panel Data Analysis) (a good survey is provided by (Urga, 1992)), focused on the study of dynamics and able to extract knowledge from temporal data. Panel data can be seen as a set of individuals (or objects) which features are repeatedly collected at two or more points in time. The analysis of this data aims to model the heterogeneity, or differences, between individuals, in order to capture its dynamics. This is usually done using regressions. However, this technique is supported by a bidimensional representation of data (or matrices), which does not preserve the original information and the interaction between all dimensions. Therefore, we envisage Tucker tensor decomposition as a better alternative of PDA in the study of evolution.

Tucker (1963) introduced the tensor decomposition, which inherits his name, in 1963. Refinements of this model were then performed by Levin (1963) and Tucker (1966). An in-depth study of Tucker3 decomposition was undertaken by De Lathauwer (2000), which coined its orthogonality-constrained version as HOSVD. We briefly introduce the foundations of this decomposition following closely the definitions provided by Skillicorn (2007), Kolda and Bade (2009), Tucker (1966) and Kroonenberg (1983).

The Tucker decomposition can be thought as a form of higher-order principal component analysis. The three-way version of this decomposition is usually called *Tucker3 model.* The term derive from the fact that the reduction of data is performed in all three modes of the tensor (in our case, countries, macroeconomic variables and time). The general Tucker3 model can be formulated as the factorization of the original three-order tensor $\mathcal{X}$, such that

$$\mathcal{X} \approx \mathcal{G} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} g_{pqr} \cdot a_p \circ b_q \circ c_r \tag{2}$$

The matricized form of this decomposition, for each one of the three modes, can be represented as follows:

$$\mathbf{X}_{(1)} \approx \mathbf{A}\mathbf{G}_{(1)}(\mathbf{C} \otimes \mathbf{B})^T \tag{3}$$

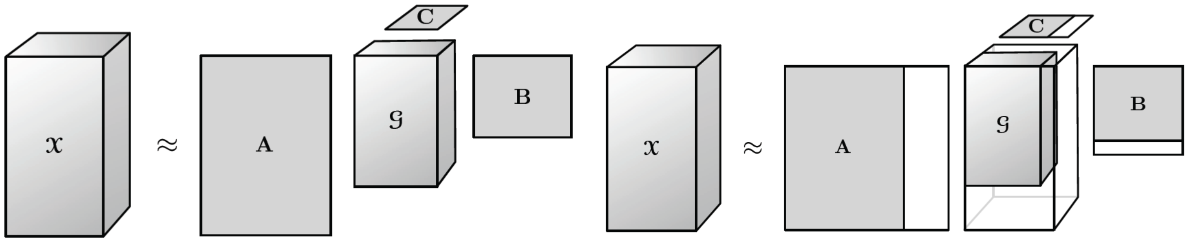
$$\mathbf{X}_{(2)} \approx \mathbf{B}\mathbf{G}_{(2)}(\mathbf{C} \otimes \mathbf{A})^T \tag{4}$$

$$\mathbf{X}_{(3)} \approx \mathbf{C}\mathbf{G}_{(3)}(\mathbf{B} \otimes \mathbf{A})^T \tag{5}$$

The Tucker decomposition in Equation 2 can also be written elementwise as:

$$\hat{x}_{ijk} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip} b_{jq} c_{kr} g_{pqr} \tag{6}$$

where $i = 1, ..., I$, $j = 1, ..., J$ and $k = 1, ..., K$. Here, the coefficients a_{ip}, b_{jq} and c_{kr} represent the entries of orthonormal matrices, also referred to as *component* or *factor* matrices $\mathbf{A} \in \mathbb{R}^{I \times P}$, $\mathbf{B} \in \mathbb{R}^{J \times Q}$ and $\mathbf{C} \in \mathbb{R}^{K \times R}$. These matrices can be thought as the principal components in each mode. In turn, the coefficient g_{pqr} represents the entry of the so-called *core tensor* $\mathcal{G} \in \mathbb{R}^{P \times Q \times R}$. The number of entities (i.e. number of rows) in each mode are represented by letters I, J and K. The number of components, or levels (i.e. number of columns of the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$) in the first, second and third mode of the tensor are represented by letters P, Q and R, respectively. We can see the *core tensor* $\mathcal{G}$ as a compressed version of the original tensor $\mathcal{X}$ if P, Q and R are smaller than I, J and K. Tucker suggested interpreting the *core tensor* as describing the latent structure in data, since it has information about the level of interaction between the different components, and the *component matrices* as mixing this structure to give the observed data (Tucker, 1966). The *core tensor* can also be interpreted as a generalization of the eigenvalues, or



(a) (b)

Figure 2: (a) Basic Tucker decomposition and (b) truncated Tucker decomposition of a three-way array (Kolda and Bade, 2009).

of the singular values of SVD, and it constitutes a further partitioning of the "explained" variation as is indicated by the eigenvalues of the standard PCA. The square of each entry of $\mathcal{G}$ is proportional to the amount of variance that the entry explains and its value indicates how the various components relate to each other. Matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are assumed to be columnwise orthogonal. The orthogonality is desirable since it facilitates the analysis and hastens the computation of the decomposition.

The basic idea of the tensor decomposition proposed by Tucker (1966) is to find those components that best capture the variation in each mode n. Or, in other words, the goal of Tucker's method is to find a set of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, and a small tensor $\mathcal{G}$ that, in general, have less dimensionality than the original tensor, but are able to reconstruct the most important information contained in data. When $R_n \leq rank_n(\mathcal{X})$, for one or more modes, the decomposition is called truncated. The truncated decomposition is able to describe data in a more condensed form than the original data array, as depicted in Figure 2-(b). However, the found solution may have the same dimensionality as the initial data, as illustrated in Figure 2-(a).

The problem of decomposing a tensor, which can be translated into finding the best estimation of the model presented in Equation 2, can be reduced to a straightforward optimization problem (see Equation 7). Assuming the tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$, the goal is to minimize the difference between the original tensor and the estimated model, so the decomposed tensor can describe original data as accurately as possible. The accuracy of the estimated model is measured in terms of the "explained" variation.

$$\min_{\mathcal{G},\mathbf{A}^{(1)},...,\mathbf{A}^{(N)}} \|\mathcal{X} - |\mathcal{G}; \mathbf{A}^{(1)}, \mathbf{A}^{(2)}, ..., \mathbf{A}^{(N)}|\|| \tag{7}$$

subject to $\mathcal{G} \in \mathbb{R}^{R_1 \times R_2 \times \ldots \times R_N}$,

$\mathbf{A}^{(n)} \in \mathbb{R}^{I_n \times R_n}$ and columnwise orthogonal for $n = 1, ..., N$

Usually, this optimization problem is solved using an **Alternating Least Squares** (ALS) approach. More detailed information about the ALS algorithm can be found in Kroonenberg (1983).

Today, the class of multi-way methods proposed by Tucker (1966), namely, the Tucker3 model in its orthogonality-constrained version, is better known as HOSVD (De Lathauwer, 2000).

2.3 Visualization of Evolving Data

After compressing the temporal data, by extracting its fundamental properties using HOSVD, we explore this information in order to visualize the evolution of each object (in our case, the EU countries correspond to the objects) in a natural, intuitive and compact way. To do so, we resort to the concept of trajectory, as will be explained below.

2.3.1 Definition and Interpretation of Trajectories

A trajectory can be defined as a set of time-ordered states of an object in a dynamical system. Typically, these trajectories are defined in low-dimensional representative subspaces and are graphically represented by a line that connects the coordinates of an object for different time points. It is common to resort to 2D, instead of 3D subspaces, since they are simpler to analyze and, at the same time, allow for an effective data analysis. Thus, we use two-dimensional projections and encode the third dimension as a trajectory over the plane. In such way, we are able to map a given country's trajectory along time, by simply using two-dimensional projections, thus producing a compact, clear and informative representation of data evolution.

The appealing feature of trajectories is that they render temporal visualization more appealing to human eye, promoting an efficient dissemination of temporal results. Besides, they help achieve a faster insight into the evolution of a country's performance, allowing for an intuitive detection of structural changes that may occur. When all the trajectories of a group of countries are represented in the same plot, the trajectory is also able to show the relative position of each country compared to all other countries, thus allowing the undertaking of comparative studies.

In order to define the trajectories of each country we decompose the original three-order tensor by estimating a HOSVD, or Tucker3 model with orthogonality constraints, as introduced before. By decomposing the original tensor we obtain a more compact, yet accurate, representation of the structural properties of data. Thereafter, we consider the two-dimensional subspace spanned by the two most representative components of matrix $\mathbf{B}$, and define the x and y coordinates for each time point k $(k = 1, ..., K)$ of the trajectory. We obtain these coordinates for each country i $(i = 1, ..., I)$, by computing the **dot product** between $x_{i,:,k}$ (horizontal fibers of $\mathcal{X}$) and each column of component matrix $\mathbf{B}$ (the first and the second components are assigned to the x-axis and y-axis, respectively). This vector operation returns the coordinates of the time points in the projection axis, for each considered European country. The last step of this analysis is the interpretation of the results. To do so, we consider that a country's trajectory can be characterized by a **direction** (upwards, downwards, leftwards, rightwards, or combinations of them), that can be more regular or more irregular; and by an **amplitude**, which can be higher, thus covering a larger space area, or lower, by keeping its position in the plane almost unchanged over time. Also, both the **shape** and the **position** of the trajectory can be used to identify countries with similar economic development. We will take these features into consideration when analyzing the trajectories in the case study we will present in the next section.

The proposed methodology can be of importance for economic studies, more specifically, for international longitudinal comparisons, since it is able to give valuable insights into the driving forces in an economy and thus be the basis for the design, monitoring and evaluation of specific national economic policies. In the context of the case study that will be presented, the definition of trajectories is not only useful for understanding the social and economic course of a given country, but it can also work as a complementary tool for the monitoring and evaluation of the success of certain EU policies.

3 Comparative Study of the Development of EU Economies

In this section, we perform a comparative analysis of the evolution of the actual EU state members over a time horizon of 14 years, using as a basis publicly available data from the World Bank. As previously mentioned, this temporal analysis is carried out by means of the definition of trajectories of countries in the space spanned by the two most representative components of the variables mode, returned by a three-order SVD. The main goals of this study are twofold: characterization of the evolution of the economic development of representative EU state members based on their position on the bidimensional space; and detection of trends and significant changes in the evolution of each economy through the analysis of the layout of the trajectories, and further interpretation.

3.1 Description of Economic Data

To conduct this investigation we extracted publicly available data from the World Bank website[2] concerning the economic and financial development of a set of European countries,

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[2] http://www.worldbank.org

for a time horizon of 14 years that goes from 1995 to 2008 [3]. The source of data was mainly the World Development Indicators database (WDI), which is the primary World Bank database for development data from officially-recognized international sources.

We conduct this study at the country level of analysis, since our focus is directed towards the current 27 state members of the European Union (EU), namely: Austria (AUT), Belgium (BEL), Bulgaria (BGR), Cyprus (CYP), Czech Republic (CZE), Denmark (DNK), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Hungary (HUN), Ireland (IRL), Italy (ITA), Latvia (LVA), Lithuania (LTA), Luxembourg (LUX), Malta (MLT), Netherlands (NLD), Poland (POL), Portugal (PRT), Romania (ROM), Slovak Republic (SVK), Slovenia (SVN), Spain (ESP), Sweden (SWE) and United Kingdom (GBR). The terms in brackets are the abbreviations of the corresponding countries, according to the *The Roots Web Surname List*, which uses 3 letters standardized abbreviations to designate countries and other regional locations. Since the reported temporal analysis covers the period $[1995, 2008]$ is important to recall that Austria, Finland and Sweden joined the European Union in year 1995, followed by ten other European countries (Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia), who joined the Union in 2004. Later, in 2007, the entry of Romania and Bulgaria to EU, increased the number of members states to the actual 27.

In order to measure the economic development of the mentioned economies, we characterize each country based on a set of 9 macroeconomic indicators. We tried to select those indicators that better represent the economic development of a country, mainly based on the information provided by Frumkin (2000). These indicators can be briefly described as follows [4]:

- **CO2 emissions:** carbon dioxide emissions are those stemming from the burning of fossil fuels and the manufacture of cement. They include carbon dioxide produced during consumption of solid, liquid, and gas fuels and gas flaring, and are expressed as metric tons per capita.

- **GDP growth:** annual percentage growth rate of Gross Domestic Product (GDP) at market prices based on constant local currency. Aggregates are based on constant 2000 U.S. dollars.

- **GDP per capita:** GDP per capita is gross domestic product divided by midyear population. GDP is the main economic growth indicator and is frequently referred as a proxy for a country's income. Based on the information provided by the World Bank, GDP is computed as the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of

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[3] The time horizon ends in 2008 due to the current unavailability of some macroeconomic indicators for the following years.

[4] Most of the adopted descriptions are the ones provided by the World Bank.

the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. GDP is, therefore, an aggregated measure of the total economic production of a country, measuring the overall national output. In turn, GDP per capita is widely used for the comparison of living standards, or to monitor the process of convergence across the European Union. This indicator is expressed in current U.S. dollars.

- **Foreign Direct Investment, net inflows:** Foreign Direct Investment (FDI) is a category of international investment, and can be described as the net inflows of investment to acquire a lasting management interest (10 percent or more of voting stock) in an enterprise operating in an economy other than that of the investor. It is the sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments. This indicator is expressed as a share of GDP.
- **Gross Savings:** computed as gross national income less total consumption, plus net transfers. It can also be defined as income not spent or deferred consumption of a country. This indicator is expressed in current U.S. dollars.
- **Gross Savings (% of GDP):** this indicator distinguishes from the previous one, since the Gross Savings are expressed as a share of the GDP of the corresponding country.
- **Employment to population ratio:** proportion of a country's population that is employed. Ages 15 and older are generally considered the working-age population. It is an indicator of the available labor force of a given country.
- **Balance of Trade:** the trade balance of a country, also known as *net exports*, is given by the difference between the monetary value of a country's exports and imports of goods and services. There is a trade surplus when the exports exceed the imports and, analogously, there is a trade deficit if the imports of a country are higher than its exports. Since the Balance of Trade is a component of GDP, ceteris paribus, if there is a trade surplus the GDP increases and if there is a trade deficit the GDP decreases. This indicator is expressed in current U.S. dollars.
- **Health expenditure per capita:** total health expenditure is the sum of public and private health expenditures as a ratio of total population. It covers the provision of health services (preventive and curative), family planning activities, nutrition activities, and emergency aid designated for health but does not include provision of water and sanitation. This indicator is expressed in current U.S. dollars.

Since these indicators are expressed in different units of measurement, we standardized them, via z-scores transformation, in order to nullify the effect of different scales and magnitudes in the computation of tensor decomposition.

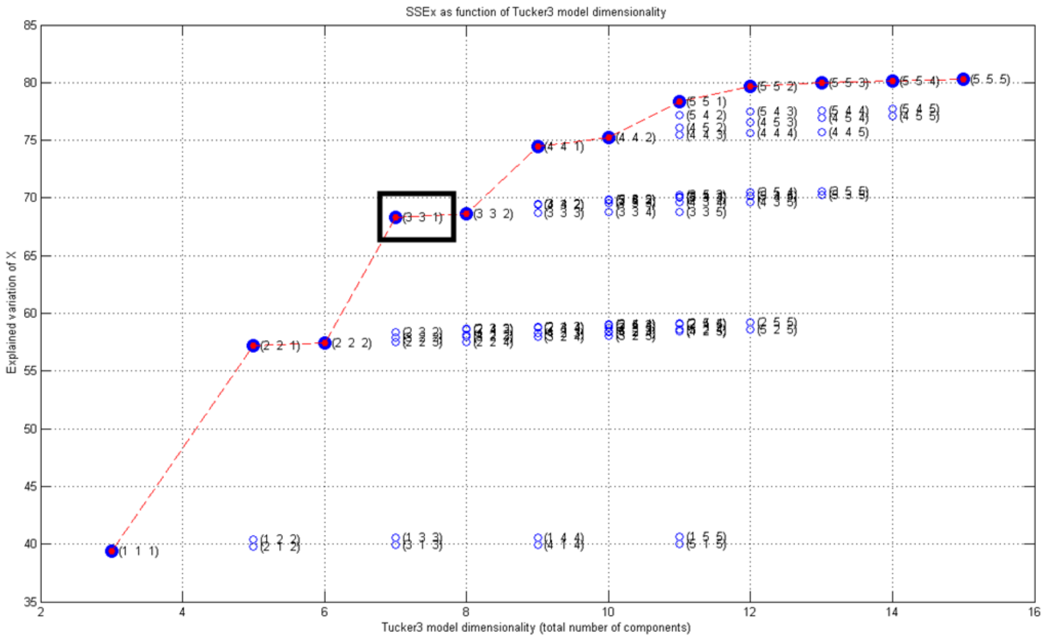

Figure 3: Scree plot for the World Bank data showing the percentage of variation explained by each possible order of the Tucker3 model.

3.2 Economic Data Representation and Decomposition

After conducting the process of extracting and standardizing data, we organized the initial matrices, comprising the yearly economic information for the selected European countries, as a three-order tensor $\mathcal{X} \in \mathbb{R}^{27\times 9\times 14}$, where the first mode corresponds to the 27 EU countries, the second mode represents the 9 macroeconomic indicators, and the third mode corresponds to the 14 analyzed years. Then, we proceeded to the decomposition of $\mathcal{X}$ into a small core tensor $\mathcal{G}$ and a set of component matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, each one summarizing the dimensionality of each original mode into a few representative components. To do so, we estimated an orthogonality-constrained Tucker3 model of order $(3 \times 3 \times 1)$, which explains 68.28% of the total data variation. This order is a parameter of the model and refers to the number of components (or factors) retained in each mode ($P = 3$, $Q = 3$ and $R = 1$). Its choice was guided by the analysis of a scree plot that indicates the potential ability of a Tucker3 model to explain the original data, for each possible combination of number of components. This plot is depicted in Figure 3. The *core tensor* $\mathcal{G}$ contains the weights of all possible triads (combination of components, for the three modes) and these weights reflect the importance of the interaction between components, thus revealing the underlying variation pattern. The results tell us that the interaction of components that explains the higher portion of the sum of squares and, therefore, is the most important for understanding the data structure, is the interaction $(1, 1, 1)$ (explains 57.65% of the initial 68.28% variation). The second most relevant interaction is $(2, 2, 1)$, which explains 26.15% of the total variation

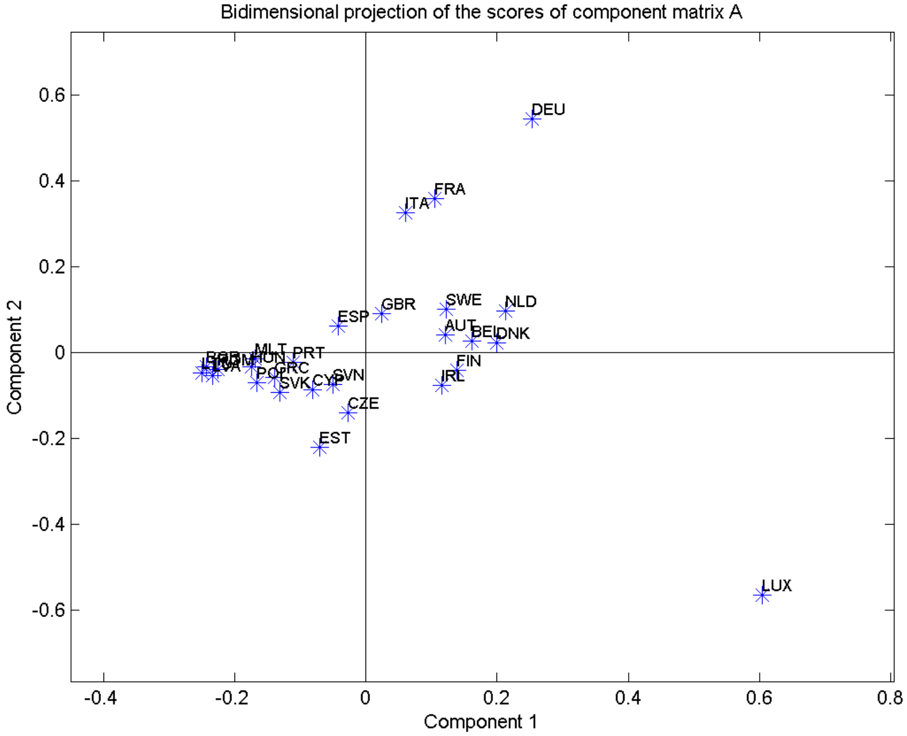

Figure 4: Projection of the coefficients of matrix **A** in the bidimensional space defined by the two most representative components of matrix **A**. Recall that this matrix is associated to the row-entities (i.e. EU state members) of the original tensor $\mathcal{X}$.

explained by the estimated model. In turn, the entries of the *component matrices* **A**, **B** and **C**, represent the weights (also referred to as *scores* or *coefficients*) of the corresponding entities (countries, macroeconomic indicators and years, respectively) in a given level of a given mode. Note that these *component matrices* have as many columns, or levels, as the number of components defined in the order of the estimated Tucker3 model.

3.3 Interpretation of the Axes of the Bidimensional Space

Before presenting the trajectories, we first need to interpret the meaning of each component of mode B that will define the plane where we represent the trajectories. To help this interpretation, we project the coefficients of each indicator in the space spanned by the two most representative components of the mentioned mode, as shown in Figure 5. We focus on indicators having extreme scores, since those are the ones with higher contribution to the formation of the axis. We perform the same analysis with component matrix **A**, in order to find the countries associated to mode B's components.

Based on the analysis of the scores of the first component of matrix **B**, denoted by $\mathbf{B}_{:,\mathbf{1}}$, we observe that almost every entity was assigned a positive score, being the higher scores the ones assigned to the following indicators:

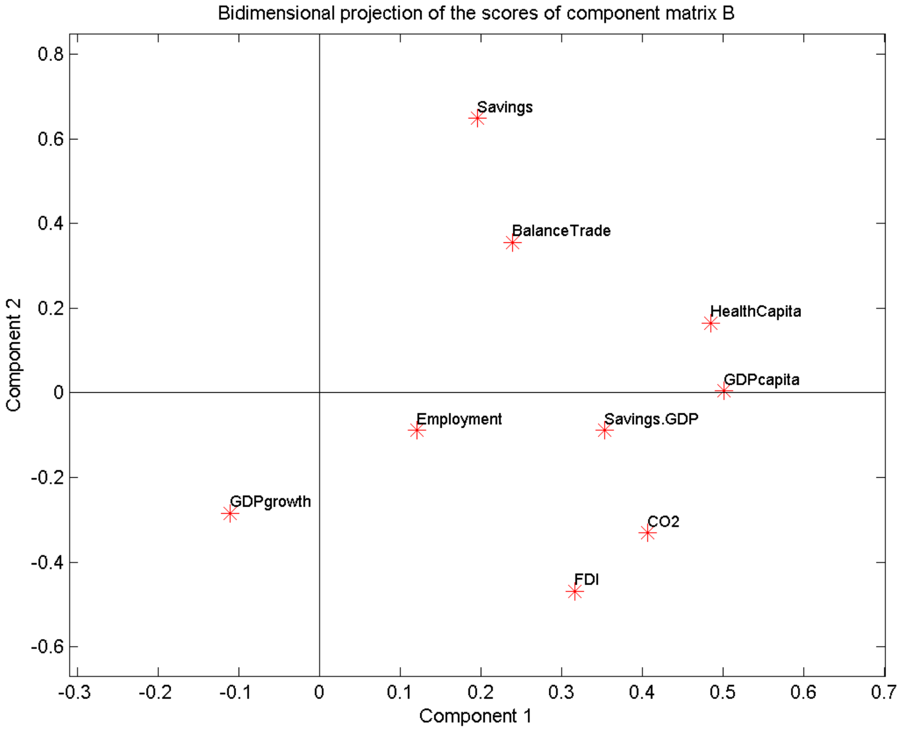

Figure 5: Projection of the coefficients of matrix **B** in the bidimensional space defined by the two most representative components of matrix **B**. Recall that this matrix is associated to the column-entities (i.e. macroeconomic indicators) of the original tensor $\mathcal{X}$.

- GDP per capita ($score = 0.501$)
- Health expenditure per capita ($score = 0.4855$)

On the negative side of the axis there is only one macroeconomic indicator (the GDP growth) but with a paltry contribution ($score = -0.1101$).

The strong association between GDP and health expenditure, evidenced by the analysis of mode B, agrees with findings reported in the literature. Joseph Newhouse was the first one to draw attention to this issue by studying the relationship between a country's medical-care expenditures with its income (Newhouse, 1977). His results confirmed the importance of GDP per capita as a determinant of per capita health spending. Several research studies followed on from this work and most of them provide broad support for the original conclusions, confirming that, in fact, there is a strong positive relationship between health expenditure per capita and GDP per capita. Since both the income, measured by GDP per capita, and health, measured by health expenditure per capita, are dimensions of the composite *Human Development Index*[5], we define this axis as the *social and economic development axis*, where the social development is measured by the investment on health and the economic development is measured by the overall national output. In order to understand which side

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[5] The Human Development Index (HDI) is a comprehensive measure of overall well being of the individuals of a country.

of the axis is associated to high social and economic development and which side is associated to low development, we analyzed the scores of the countries positioned in each side of the first component, as depicted in Figure 4. To perform the comparison, we chose countries that would represent extreme situations, namely, Luxembourg (positive side) and Lithuania (negative side). Then, we compared their coefficients for the previous indicators. Based on this comparison, we conclude that negative scores are associated to low social and economic development and, in contrast, positive scores are assigned to countries with commendable social and economic level.

Regarding the second component of the same matrix, denoted by $\mathbf{B}_{:,\mathbf{2}}$, the same kind of analysis highlighted the contributions of Gross Savings (GS), on the positive side of the y-axis ($score = 0.6485$), and of Foreign Direct Investment inflow (FDI), on the negative side of the axis (-0.4689). The association between these two indicators is not obvious, since FDI measures the foreign ownership of productive assets, while GS can be understood as income not spent by a country. Therefore, we simply define this component as being the *Savings VS Foreign Direct Investment axis.* The EU state member with highest association to GS or, in other words, to the positive side of the second component, is Germany ($score = 0.5444$). This country is followed by France ($score = 0.3584$) and by Italy ($score = 0.3255$). In turn, we only find one country significantly related to the negative side of the y-axis, which is Luxembourg ($score = -0.5652$). Therefore, we conclude that countries having high positive coefficients in this axis have significant savings rates, whereas countries showing high negative scores benefit from high rates of FDI inflow. Those countries positioned close to the origin do not have either high savings neither high FDI inflow. Also, we observe that these scores are independent from the opposite indicator, i.e. countries with high savings do not have, necessarily, low rates of FDI and vice versa.

To better understand the relationship between FDI and GS, and similarly to what we did for the first component, we looked for previous studies that pursued this question. According to Salahuddin (2010), FDI inflow serves as a strong mechanism for the promotion and spread of business opportunities throughout the developing and industrialized economies. Also, it is believed that FDI inflow stimulates the economic growth (Chung, 1995), for instance, through knowledge transfers, and can act as a catalyst for sustainable development. Salahuddin (2010) investigated both the long-run and the short-run relationship between FDI and Gross Domestic Savings (GDS), in Bangladesh, for the time period that goes from 1985 to 2007. Based on their study, they found out that there is a bidirectional causal relationship between FDI and GDS, for Bangladesh, and as a result they concluded that these indicators are complements.

Aiming to verify the validity of such conclusions for the analyzed EU state members we computed the correlation between GS and FDI, using our data, and we found that there is a weak negative correlation between these macroeconomic indicators (corr(GS,FDI)$= -0.113$). Therefore, in our case, we cannot generalize the conclusions of the mentioned study and, thus, claim that these two indicators are complementary. This corroborates the results from the Tucker3 model, once these two macroeconomic indicators were assigned opposite signs in the second component of mode B.

In short, we can conclude that the best positions in the component's space are both the first and the fourth quadrants. These quadrants are closely associated to the most developed EU economies, such as Germany, Luxembourg, France, Italy and the Netherlands, just to name a few. In contrast, the worst positions are the second and the third quadrants, which are associated to countries with low economic performances. Examples of such countries are Lithuania, Bulgaria, Latvia and Romania. Therefore, countries whose trajectories take the **direction** of the first quadrant are improving their social and economic development, as measured by GDP and health expenditure per capita, and increasing their gross savings. Those showing a trajectory towards the fourth quadrant are also improving their social and economic development, but also receiving more investment from abroad. Otherwise, if moving in the direction of the second/third quadrant, they are worsening their economic position. Moreover, if this direction is taken towards the origin of the y-axis, both GS and FDI inflow are deteriorating. Regarding the **amplitude**, if most of the time points of a given country's trajectory has the same coordinates, then one can assume that its economic situation is stable. An analogous reasoning holds for the opposite scenario.

After decomposing the tensor and assigning a meaning to the components, we define the trajectories of each country following the procedure described in the Methodology section.

3.4 Analysis of Temporal Trajectories

From the joint analysis of the trajectories of all studied countries, projected in the space spanned by the two most representative components of matrix $\mathbf{B}$ and depicted in Figure 6, we can observe that there are some trajectories that are closer to each other than to other trajectories. The relative proximity of a given pair of trajectories in this space can be indicative of similarity between the corresponding countries, with respect to the meaning of the components. For instance, Germany, France and Italy show similar positions on the plane, since they are all located on the middle top of the first quadrant. From these three countries, two of them, namely France and Italy, are more similar to each other than to Germany. In fact, if we also take into consideration the visual aspect of the trajectories of these two countries we observe that even the layout of their trajectories are quite similar. Bearing this in mind, and due to space constraints, we chose to analyze the evolution of,

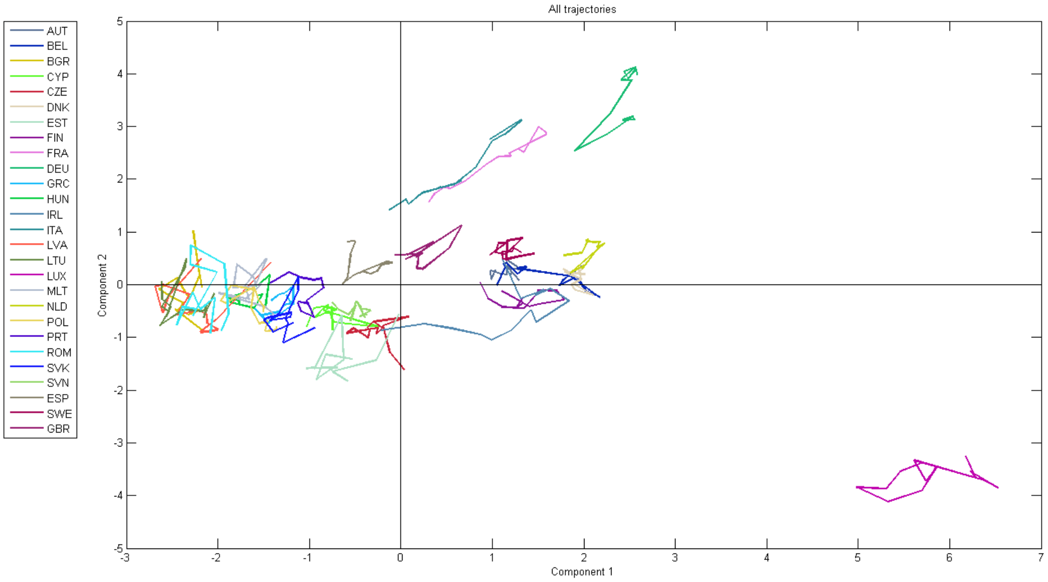

Figure 6: Trajectories of all countries in the space spanned by the two most representative components of matrix **B**.

Table 1: Standardized values of *GDP per capita*, *Health expenditure per capita*, *FDI* and *Gross Savings* of Luxembourg for the time period that goes from 1995 to 2008.

Year	GDP per capita	Health Expenditure per capita	FDI	Gross Savings
1995	2.6793	1.4615	4.8711	-0.5372
1996	2.6109	1.4218	4.9354	-0.5499
1997	2.5035	1.3462	4.9177	-0.5673
1998	2.5255	1.3875	4.9457	-0.5736
1999	2.7215	1.6158	4.9758	-0.5751
2000	2.7754	2.37	4.8958	-0.5904
2001	2.7494	2.2347	4.9909	-0.5989
2002	2.7997	2.5234	4.9956	-0.6085
2003	2.9207	2.3027	4.9872	-0.6185
2004	2.9507	2.4893	4.9842	-0.5882
2005	3.1054	2.5474	4.9624	-0.5901
2006	3.2761	2.6416	4.9665	-0.5970
2007	3.3974	2.5197	4.9444	-0.5815
2008	3.5309	2.3755	4.7997	-0.5813

not all, but only a small set of selected countries whose trajectories deviate more from the norm (e.g. Luxembourg and Germany) or are representative of a given *group* [6]. Thus, in the following subsections we describe and interpret the trajectories of the following countries:

1. Luxembourg

2. Germany

3. Portugal

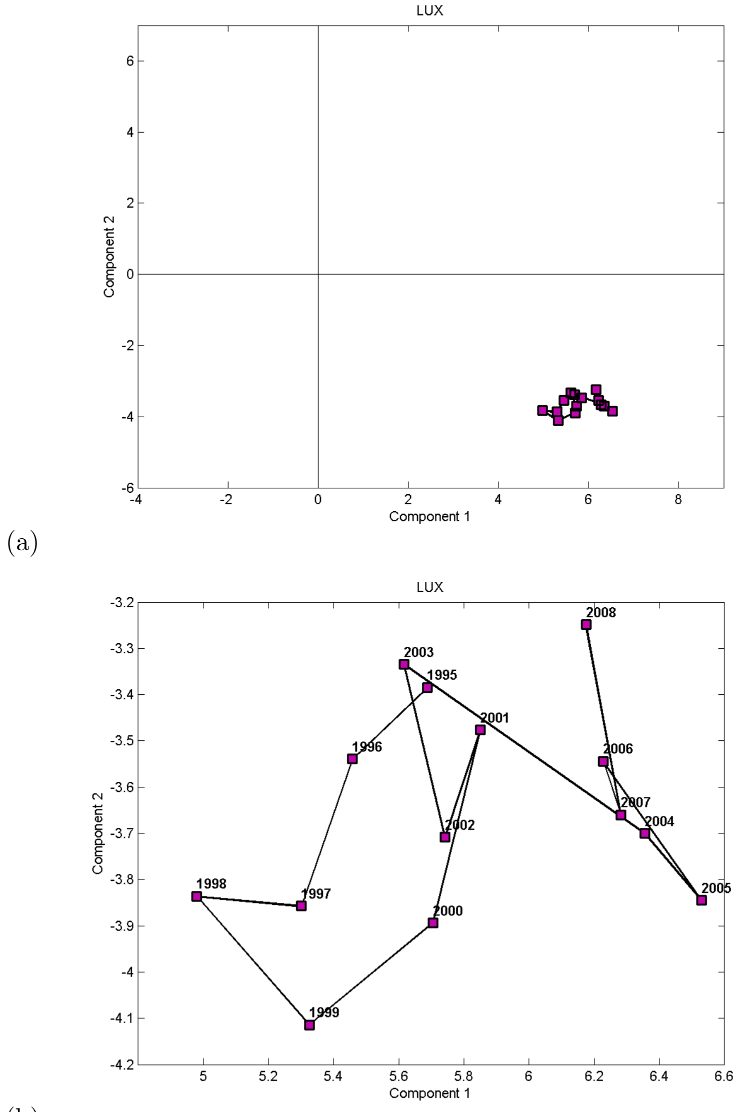

Figure 7: Trajectory of Luxembourg. The figure at the bottom is a closer look of the trajectory.

3.4.1 Trajectory of Luxembourg

The trajectory of Luxembourg, which is the most peripheral trajectory on the plane, is quite unstable and irregular, with some ups and downs and frequent changes in the direction, showing no trend in the movement, as can be seen from Figure 7. The position of the trajectory, at the bottom of the fourth quadrant, indicates that Luxembourg is one of the most developed economies in EU, both in social and economic terms, and receives a considerable amount of FDI, which is almost 5 times higher than the overall mean. In fact, Luxembourg occupies, consistently, the best positions in the ranking of highest GDP per capita in the World and is one of the main actors in EU in terms of FDI, being the major recipient of FDI inflows from both the remaining EU state members and the rest of the world. The ability of Luxembourg to effectively attract foreign investment is closely related to the favorable climate created through the adopted government policies and incentives to encourage external investment (e.g. deferred corporate tax payment schedules, capital investment subsidies and financing of plant equipment). As a consequence, Luxembourg has achieved a commendable role in EU FDI, which is mainly explained by the importance of its financial intermediation activity and the country's expertise in the Banking sector.

By comparing the first time point (1995) with the last analyzed year (2008), we observe that the trajectory of this country slightly moves to its northeast direction. This suggests an improvement of the social and economic development and a small decrease of the FDI inflow. This visual analysis is corroborated by the observation of the standardized values of the representative macroeconomic indicators of each component, presented in Table 1. Nevertheless, though there are some small ups and downs in the FDI inflow over the analyzed time period, the standardized values do not vary significantly, keeping almost unchanged. The same observation holds for the remaining indicators. Still based on the analysis of the table, we verify that this country is characterized by auspicious and consistent values of GDP per capita, health expenditure per capita and FDI, which are always higher than the corresponding mean of EU-27. The same does not hold for gross savings, which is always below the overall mean, as can be ascertained from the position of the trajectory of this country at the bottom side of the fourth quadrant. The direction of the trajectory, though irregular, signalizes a consistent improvement of Luxembourg's economic development, measured by the interplay between GDP and health expenditure per capita, since it moves towards the right side of the plane.

Table 2: Standardized values of *GDP per capita*, *Health expenditure per capita*, *FDI* and *Gross Savings* of Germany for the time period that goes from 1995 to 2008.

Year	GDP per capita	Health Expenditure per capita	FDI	Gross Savings
1995	1.1371	1.7702	-0.3844	3.7669
1996	1.0372	1.7102	-0.3620	3.5791
1997	0.9372	1.5983	-0.3956	3.3649
1998	0.8926	1.5544	-0.3717	3.3040
1999	0.7989	1.4515	-0.2445	3.2334
2000	0.6837	1.2293	-0.1913	3.1569
2001	0.6687	1.2045	-0.2507	3.0533
2002	0.6059	1.0771	-0.2130	3.0593
2003	0.5610	1.0182	-0.2439	3.0966
2004	0.5154	0.8762	-0.2765	3.2966
2005	0.4446	0.8445	-0.2633	3.3249
2006	0.3910	0.7816	-0.2793	3.4601
2007	0.3591	0.7625	-0.2990	3.5160
2008	0.3668	0.7781	-0.3506	3.6323

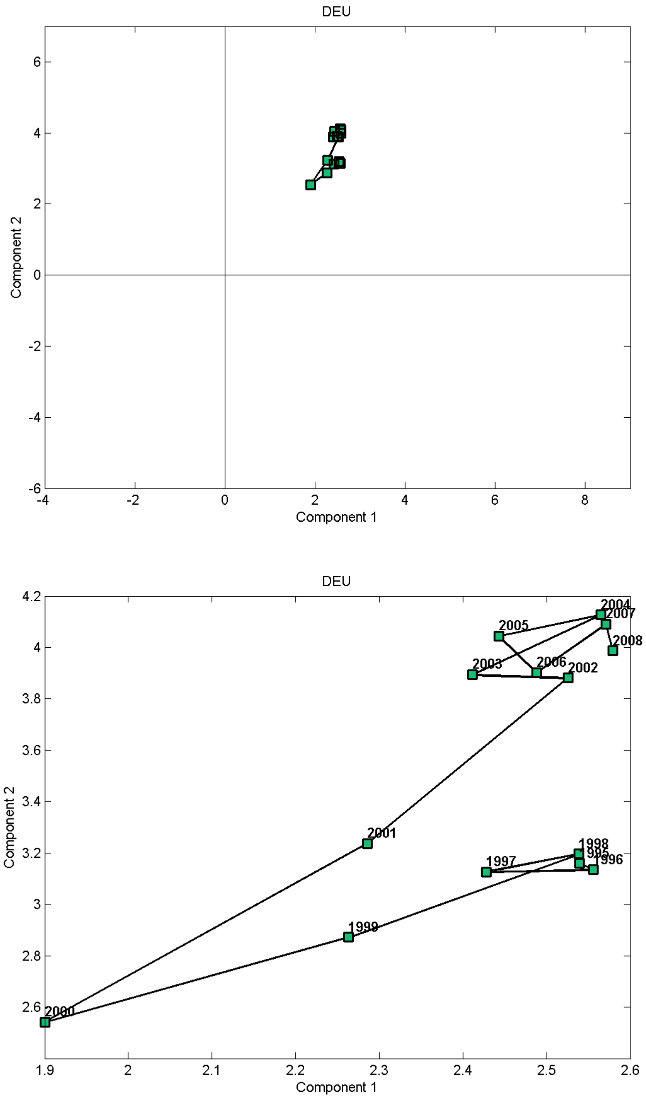

Figure 8: Trajectory of Germany. The figure at the bottom is a closer look of the trajectory.

3.4.2 Trajectory of Germany

Regarding the first quadrant of the bidimensional space, associated to countries with significant social and economic development and relatively high GS, we selected Germany as the representative country of this location. The trajectory of Germany, depicted in Figure 8, has the shape of a bird's beak, meaning that there are some stable periods (those in the base of the bird's beak), with minor movements on the space, intercalated with sharp leftwards (from 1998 to 2000) and rightwards movements (from 2000 to 2002). Based on Table 2, we verify that both the standardized values of GDP and health expenditure per capita show a decreasing trend, though they are always positive and, therefore, higher than the EU-27 mean, for the time period under analysis. The favorable situation suggested by the analysis of these macroeconomic indicators agrees with the intuition, since Germany is known for being the largest and more competitive European economy and one of the most auspicious economies in the World. The position of the Germany's trajectory in the rightmost side of the first quadrant clearly reflects this situation. In contrast with Luxembourg, Germany has a history of relatively low FDI inflows, when compared to the remaining member states of the European Union, since the standardized values are always below the overall mean. Nevertheless, we observe small improvements of the FDI inflows during the time period $[1998, 2002]$. These oscillations may be the cause behind the "bird's beak" behavior of the trajectory in the same interval of time. In fact, from 1998 until 2000, the trajectory of Germany is described by a left-downwards movement, motivated by a decrease in GS combined with an increase in FDI. This trend prevails only for the mentioned period, once the year 2000 acts as a tipping point in the behavior of the trajectory, which thereafter takes the opposite direction, by moving into the upper-right side of the space. Based on the interpretation of the axes, this movement signalizes an improvement of the general social and economic development of Germany, a growth of the country's savings and a deterioration of the FDI inflows. This opposite trend persists until 2002, which marks a period of more stability. However, it is important to stress out the decline in the standard of living, from 2002 to 2003, flagged by the leftwards movement of the trajectory. To better understand the meaning of such change, we complemented the visual observation with the analysis of the GDP growth and some Internet news. The interpretation of these sources of information led us to conclude that this change was closely related to the stagnation of the German economy during $[2002, 2003]$. This stagnation was confirmed by the analysis of the GDP growth rates, which was 0, in 2002, and -0.22%, in 2003. The main factor that influenced this stagnation was mainly the rose of the unemployment impelled by a high rate of job losses in the manufacturing during the mentioned period. From 2003 to 2004, the economy recovers and achieves a GDP growth

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[6] By *group* we mean a set of countries that are located near to each other in the defined bidimensional space, and are relatively far apart from other sets of countries.

Table 3: Standardized values of *GDP per capita*, *Health expenditure per capita*, *FDI* and *Gross Savings* of Portugal for the time period that goes from 1995 to 2008.

Year	GDP per capita	Health Expenditure per capita	FDI	Gross Savings
1995	−0.3731	−0.3682	−0.3731	−0.3582
1996	−0.3641	−0.3451	−0.2958	−0.3901
1997	−0.3706	−0.3310	−0.2756	−0.4041
1998	−0.3592	−0.3352	−0.3090	−0.3935
1999	−0.3426	−0.3022	−0.2601	−0.4005
2000	−0.3620	−0.2594	−0.2551	−0.4340
2001	−0.3639	−0.2880	−0.2055	−0.4399
2002	−0.3774	−0.3108	−0.2263	−0.4319
2003	−0.3923	−0.2877	−0.1889	−0.4395
2004	−0.4121	−0.2809	−0.2483	−0.4551
2005	−0.4408	−0.2862	−0.2554	−0.4898
2006	−0.4677	−0.3339	−0.2180	−0.5059
2007	−0.4969	−0.3	−0.3135	−0.5042
2008	−0.5158	−0.3362	−0.3171	−0.5159

rate of approximately 1.21%. After the "bird's beak" segment of the trajectory, we can observe that the Germany reached a higher level in the second component, meaning that the overall GS of this country consistently improved to a better situation. Comparing with the remaining 26 EU state members, we also observe that Germany is the country with higher national savings, since it is located in the top of the first quadrant of the bidimensional space. This is a reflection of the *save more, borrow less* German mentality and culture.

3.4.3 Trajectory of Portugal

The trajectory of Portugal, which is the country we selected as a representative of both the second and the third quadrants, has small amplitude and is quite stable when compared to other EU state members, as can be ascertained from Figure 9. Typically, countries located in these quadrants are the poorest economies of the European Union, with low social and economic development, low FDI inflows and undesirable national savings rate. This less desirable economic position is corroborated for Portugal by the analysis of Table 3, where we verify that the relevant macroeconomic indicators are permanently below the EU-27 mean, assuming negative standardized values. Therefore, we can deduce that in the last 14 years, Portugal has never stood out from the majority of the European countries, having a steady economic situation with paltry economic growth rates, especially after 2002. Through the analysis of Portugal's trajectory, we verify that, after year 2002, the trajectory starts to slightly move leftwards, assuming positions closer to the origin of the second component. This movement persists until 2008, meaning that in last years Portugal has experienced a progressive worsening of the general economic situation. Nevertheless, in the first analyzed years, namely, during the time period that goes from 1995 to 2001, Portugal benefited from a better economic environment, with its macroeconomic indicators assuming values closer to the EU-27 mean. The deterioration of the economic position, that approximately begun in 2002, may be related to the entrance in the monetary union and consequent join of the single currency, which took place exactly in this year. This monetary integration of Portugal

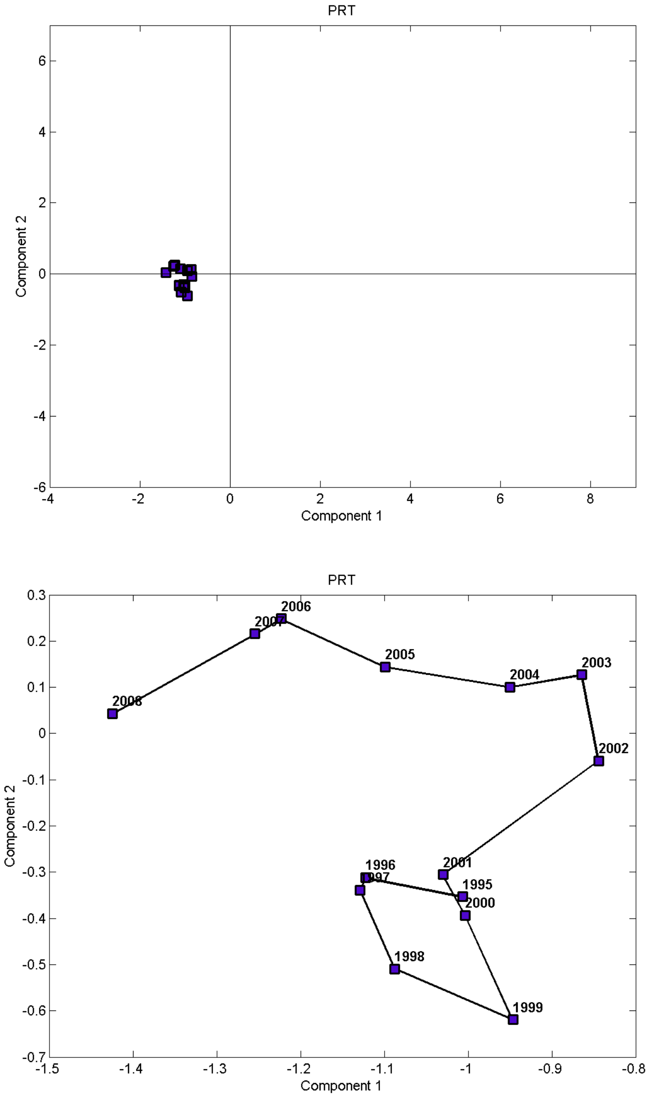

Figure 9: Trajectory of Portugal. The figure at the bottom is a closer look of the trajectory.

with several other EU countries, had impact in the autonomy of the country in what regards the control of both the exchange and monetary policies, which were transferred from the Bank of Portugal to the European Central Bank. These policies were one of the means Portugal had to attain their economic goals. The lack of such policies, compounded by the existence of structural problems, hindered the economic growth and the competitiveness of the Portuguese economy and led the country to a serious economic recession, that prevailed until today.

4 Concluding Remarks

Most of the widely used data analysis techniques, such as Principal Component Analysis, Clustering and Regression, were devised to analyze data in a specific moment in time, thus providing a static view of the world. Whilst aware that our world is dynamic, and not static, in this paper we study and present a methodology for understanding the evolution of multivariate economic data over time. To illustrate its applicability we undertake a comparative analysis of the evolution of the economic and financial development of EU-27 over a 14-year span, that goes from 1995 to 2008. The methodology is based upon the definition of temporal trajectories, at the country-level, which are projected in a representative bidimensional space spanned by the components yielded by a High-Order Singular Value Decomposition (HOSVD). The information extracted from the analysis of the relative position of the trajectories of the EU state members, in terms of macroeconomic development, provides an overview of the overall economic situation over a given time horizon, by means of the identification of important turning points and detection of trends (e.g. if the trajectories of the majority of EU state members are moving towards the space quadrant associated to high levels of economic development, then we can deduce that there is a trend of economic growth in the European Union). In turn, the study of the individual trajectories of a chosen set of countries (Luxembourg, Germany and Portugal) allowed us to draw some conclusions regarding the stability of their economic development, identify years of transition, flag important political and economic changes and detect trends of growth. Nevertheless, and albeit the simplicity and compactness of the information provided by the proposed methodology, we identified some limitations. First, the shape and the interpretation of the temporal trajectories is highly dependent on the selected macroeconomic indicators. Second, the interpretation of the axes of the bidimensional space can be quite demanding. Therefore, it is advisable a careful selection of the variables before applying the methodology, in order to generate a low-dimensional space that really measures what we intend to study.

In order to improve the proposed methodology, we plan to conduct more research aiming to introduce a forecasting step that explores information regarding past events, given by the properties of trajectories, as a way to define possible future scenarios (e.g. optimistic, pessimistic and realistic scenario). Therefore, the overall motivation of this research is to understand the past or, in other words, the course of events that led to the present situation, in order to predict the future.

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