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**THE IMPACTS OF STRUCTURAL CHANGES IN THE  
LABOR MARKET:  
A COMPARATIVE STATICS ANALYSIS USING  
HETEROGENEOUS-AGENT FRAMEWORK**

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**Abstract**

In this paper we aim at analyzing the impacts on welfare and wealth and consumption distribution across different labor market structural features. In particular, we pursue a steady-state analysis to assess the impacts of unit vacancy costs, unemployment replacement ratio or the job destruction rate, when they are changed in order to promote a given reduction in the unemployment rate.

We combine a labor market search and matching framework with unions, based on Mortensen and Pissarides (1994) with a heterogeneous-agent framework close to Imrohorglu (1989) in a closed economy model. Such approach enables the joint assessment of macroeconomic welfare and inequality together with implications derived from institutional changes in labor market. Moreover, the transition matrix between worker's states is endogenous, fully derived from labor market conditions.

Using feasible calibration to the Euro Area, we conclude that different institutional changes to promote unemployment reduction have non-neutral and differentiated effects on welfare and inequality. While changing unit vacancy costs and job destruction can be ranked, changes in the unemployment benefit replacement ration involve a trade-off between gains in welfare and in consumption/income distribution.

JEL Classification: E21, E24, E27, I30, J64.

Keywords: Labor market institutions, search and matching models, heterogeneous-agent models, welfare and inequality.

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# 1 Introduction

In this paper we aim at analyzing the impacts on welfare and on wealth and consumption/income distribution across different labor market structural features. In particular, we pursue a steady-state analysis to assess the impacts of unit vacancy costs, unemployment replacement ratio or the job destruction rate, when they are changed in order to promote a given reduction in the unemployment rate.

Heterogeneous agents are embedded in a labor market structure for which they do not have absolute control. In this context we use a labor market search and matching model, inspired in Mortensen and Pissarides (1994), combined with a heterogeneous-agent model where worker saving choices are defined as in Imrohoroglu (1989). We complement the labor market model with an additional source of frictions: the behavior of a Trade Union. The Trade Union bargains over wages on behalf of the labor force irrespectively of the idiosyncratic characteristics of each individual. Given the heterogeneity in the decision for asset holdings, the presence of the Union insures a unique equilibrium wage instead of multiple wages arising from individual bargaining. This emphasizes the influence of the institutions over the individual.

Deliberately choosing a parsimonious approach of both frameworks, there is no on-the-job search, no endogenous job destruction nor any job-search effort. As for the heterogeneous-agent framework, we consider that utility is driven only by consumption and that no borrowing is allowed. The timing of model-solving consists of two stages. First, firms with vacancies and unemployed workers interact in search of a match. In parallel, an Employers' Association and a Trade Union set the wage. Both mechanisms define labor market conditions. Second, given labor market conditions, optimal behavior of firms and workers determine, respectively, the demand and supply of capital.

There are only few works that combine these two frameworks. One is Krusell et al. (2010) who use a search and matching model of the labor market where savings work as an insurance against unemployment (following the works of Merz (1995); Andolfatto (1996); den Haan et al. (2000)). Their paper focuses, following the debate in the search and matching literature that began with Shimer (2005), on the inability of matching models to replicate the observed volatility on unemployment and vacancy rates. The model incorporates the worker savings behavior in the wage bargaining and takes the interest rate as exogenous. The framework closest to ours is that of Nakajima (2010). As in Krusell et al. (2010), this work also addresses the volatility problematic in the search and matching model by introducing self-insurance. As in our work, an income tax is introduced to finance unemployment insurance but it additionally includes leisure in the utility function of the worker; interest rate is also endogenous. Shao and Silos (2007) also use a framework very close to this one. Also with the aim at analyzing the volatility properties of unemployment, vacancies, wages and labor market tightness, interest rate and the transition matrix are exogenous and borrowing is allowed. In contrast, Bayer and Walde (2010) focus on the dynamic and equilibrium properties of the matching model with savings. They

use a continuous time model where the behavior of risk-averse individuals is analyzed in steady state equilibrium; they also introduce the dynamics of transition. Interest rate is exogenous and no comparative statics is performed.

In none of these works the equilibrium is found by allowing the interest rate to be endogenously determined through the interaction between the optimizing behaviors of both the firms and the workers. Moreover, the use of a transition probability matrix is, in our work, truly endogenously determined and derives from the labor market conditions. Finally, we additionally consider collective bargaining. We leave aside the volatility puzzle that dominates most of the related literature; instead, we intend to perform steady state comparative statics across different unemployment replacement ratios, vacancy unit costs and the job separation rates, in order to get insights of their implications on welfare and wealth and consumption/income inequality.

This work is organized as follows: section 2 describes the theoretical model; section 3 describes the computational approach to solve the model in steady state and the comparative statics results; section 4 concludes.

## 2 Model description

The model combines a search and matching framework based on Mortensen and Pissarides (1994) with an heterogeneous-agent framework closely following Imrohorglu (1989). It applies to a closed economy or to an open economy, large enough to drive international capital markets; by simplification, interest rate is endogenously determined only by pressures in the country's capital market.

The model considers a mixed system of insurance against unemployment: self-insurance, through private savings, and an unemployment benefit provided by the government. In the former, the decision is endogenous to the worker; in the latter, a fraction over the net wage endogenously set by the government and fully financed by taxes levied on the labor income of employed workers, such that the government's budget is always balanced.

No firm-worker bargaining is allowed. Real wages are settled in collective bargaining between a Employers' Association (representing firms) and a Trade Union (representing workers). Since savings as a self-insurance against unemployment derives from individual's choices, the Trade Union engages in collective bargaining where the fallback option includes only the unemployment benefit, equally spread across workers.

Perfect competition amongst firms imply that all firms are identical but also that a free-entry condition holds. All capital used by firms in productive activities are rented from workers and all savings are available for renting by firms. Thus, savings, capital and asset holdings refer, in substance, to the same; therefore, we will use these expressions interchangeably throughout the work.

The model period is one year and the interest rate and wage are referred in real terms henceforth.

## 2.1 Firms

There are a large number of firms under perfect competition, owned by all individuals in equal parts. Each firm holds one job, either occupied by a single worker or vacant. If occupied, the job-worker pair uses capital to produce a capital-equivalent good with homogeneous technology described as:

$$y \equiv y(k) = k^\alpha \quad (1)$$

where  $k$  and  $y$  represent the *per capita* stock of capital and output. Additionally, firms rent capital from workers at interest rate  $r$ .

The present value of an occupied job,  $J \equiv J(y, \tau, w, V)$ , corresponds to the expected output less rent costs, depreciation of capital,  $\delta$ , and the gross wage paid to the worker (upon which a fraction  $\tau$  is levied as income tax). In addition, there is an instantaneous probability  $\lambda$  of match separation and, consequently, of losing the value of the occupied vacancy. On the other hand, the vacant job,  $V \equiv V(q, J)$ , supports a fix cost,  $c$ , and has an instant probability,  $q$ , of becoming occupied. Thus, the present values of the occupied and vacant jobs satisfies:

$$rJ = y - (r + \delta)k - \frac{w}{1 - \tau} - \lambda J \quad (2)$$

$$rV = -c + qJ \quad (3)$$

Additionally, in equilibrium, free-entry implies that all profit opportunities for the vacant job are exhausted, that is,  $V = 0$ , then in (3) the free-entry condition is:

$$J = \frac{c}{q} \quad (4)$$

Given that  $q^{-1}$  is the average waiting time of the vacant job, the previous expression states that, in equilibrium, the expected value for the occupied vacancy must equal the cost of holding a vacant job.

The current asset value from an occupied job, which underlies the formulation of (2), can be decomposed into the respective income-related components:

$$J_t = y_t - (r_t + \delta)k_t - T_t w_t \quad (5)$$

where the left-hand side of (5) refers to the period's asset value - or current profit. In the right-hand side, the first term represents output, the second term represents capital factor payments (rent and depreciation) and, finally, the last term represents the labor factor payment.

## Optimization problem

In this context, the firms choose the amount of capital to use when a job-worker pair is formed and, the respective labor demand. The problem of the firm is then one of maximizing the present value of a filled vacancy,  $J$ , by choosing the optimum amount of capital and labor, that is:

$$\max_{k,w} \{J\} \tag{6}$$

The solution to the optimization problem (6) is expressed in the following proposition:

**Proposition 1.** *At the optimum, the firm chooses the amount of capital and labor factor as to verify:*

$$w \equiv w(y, \tau, q) = (1 - \tau) \left[ y - (\delta + r)k - \frac{(r + \lambda)c}{q} \right] \tag{7}$$

$$k = \left( \frac{\alpha}{\delta + r} \right)^{\frac{1}{1-\alpha}} \tag{8}$$

Expression (8) states that the marginal product of capital must cover for the depreciation and the rent of capital. As for expression (7), the optimum wage for the firm, which follows from the optimum choice of capital, depends positively on output and on the probability of job matching; negatively on depreciation and rent costs of capital, on unit vacancy cost (which are direct costs to production) and on the probability of match separation (which acts as an indirect cost to production). In fact, any increase in direct or indirect costs of production have negative impacts on wages. The wage expression,  $w$ , is the inverse of the demand for labor function of the firm.

Since firms are homogeneous and own a single vacancy each, the individual demand for capital corresponds to the average aggregate demand for capital of the whole economy:

$$D_k = k \tag{9}$$

## 2.2 Labor market functioning

### Search and matching

As in Pissarides (2000) we consider an economy composed of  $L$  workers in the labor force. A worker can either be employed in a single job or unemployed, therefore, the labor market is decomposed as  $L = E + Un$ . Normalizing to the labor force,  $1 = e + u$  and thus  $u$  and  $e$  are the unemployment and employment rates. Similarly, there are  $N$  firms in the labor

market, either occupied by a single employee or vacant, that is,  $N = E + Vc$ . Normalizing to the labor force,  $\frac{N}{L} = e + v$  and thus  $v$  is the vacancy rate. The aggregate labor market conditions are measured by the relation between the rate of vacancies,  $v$ , and the rate of unemployment,  $u$ . For this purpose,  $\frac{v}{u} = \theta$  represents labor market tightness.

The process through which unemployed and vacancies meet is described by the matching function  $M = m(v, u)$ . As proposed by Blanchard and Diamond (1989) and Pissarides et al. (1986), the function  $m(v, u)$  is assumed to be homogeneous of degree one, concave, continuous and increasing in both arguments. In each period an amount  $M$  of matches are produced; of course, we impose that  $M = \min\{m(v, u), \text{vacancies}, \text{unemployed}\}$ ,  $m(0, u) = 0$  and  $m(v, 0) = 0$ . On the vacancy side, firms post job vacancies that are filled at rate  $q(\theta) \equiv \frac{m(v, u)}{v} = m\left(1, \frac{1}{\theta}\right)$ , *i.e.*, the vacancy matching rate. On the unemployment side, the unemployed worker finds a job at rate  $p(\theta) \equiv \frac{m(v, u)}{u} = m(\theta, 1)$ , *i.e.*, the unemployment matching rate. More, the dynamics of unemployment exiting and of vacancy matching follow *Poisson* processes, that is, the instantaneous endogenous probability of unemployment exiting and of any vacancy being occupied are, respectively,  $p(\theta)$  and  $q(\theta)$ . We also consider a *Poisson* process for job separation, with an instantaneous separation rate  $\lambda$ , exogenous. In particular, and following the literature, we assume the Cobb-Douglas type matching function:

$$m(v, u) = v^\eta u^{1-\eta}, \eta \in ]0, 1[ \quad (10)$$

which implies that

$$p \equiv p(\theta) = \theta^\eta \quad (11)$$

$$q \equiv q(\theta) = \theta^{\eta-1} \quad (12)$$

Notice that, the instantaneous probability of an unemployed worker finding a job,  $p$ , increases when labor market conditions improve, that is, for a given number of vacancies, a decrease in the number of the unemployed increases the probability of finding a job. Obviously, the reverse applies for the instantaneous probability of filling a vacant job,  $q$ .

At any given moment, a worker can be at one of two states ( $s$ ): employed ( $e$ ) or unemployed ( $d$ ),  $s = \{e, d\}$ . Following Imrohorglu (1989), labor market flows are determined by transition probabilities between states defined in the following matrix,  $\Pi$ :

$$\Pi \equiv \Pi(\theta) = \begin{bmatrix} p_{ee} & p_{de} \\ p_{ed} & p_{dd} \end{bmatrix} = \begin{bmatrix} 1 - \lambda & \lambda \\ p & 1 - p \end{bmatrix} \quad (13)$$

where  $p_{ji}$  represents the instantaneous probability that a worker is state  $i$  becomes a worker in state  $j$ .

### Centralized collective bargaining

In the labor market, workers are represented by a Trade Union and the firms by a Employers' Association. The sole purpose of these two institutions is to settle on a wage.

When negotiating, the Trade Union maximizes the rent of the employment state,  $W \equiv W(w, U)$ , over the fallback option of the unemployment state,  $U \equiv U(w, p, W)$ :

$$rW = w - \lambda[W - U] \quad (14)$$

$$rU = bw + p[W - U] \quad (15)$$

The employed worker receives a wage and faces an instantaneous probability  $\lambda$  of job separation and, therefore, of losing the surplus of being employed over being unemployed. If unemployed, the worker receives an unemployment benefit and faces an instantaneous probability  $p$  of job matching and, therefore, of gaining a surplus over being unemployed.

On the other side, given all firms are equal, the objective function of the Employers' Association is identical to that of the individual firm it represents, that is, maximize (2) over (3).

### Optimization problem

We assume that a successful match produces a nonnegative surplus for both the firm and the worker, that is, both agents are at least better off matched. Being so, let  $S = (W - U) + (J - V)$  be the total surplus of a successful match in the context of a collective bargaining. The problem at hand for the Employers' Association and the Trade Union is one of agreeing on the wage that maximizes the total surplus of a successful match. Formally, we will make use of the Nash Bargaining Solution (Nash (1950)):

$$\max_{(W-U), (J-V)} \{(W - U)^\mu J^{1-\mu}\} \quad (16)$$

$$s.t. : S = (W - U) + (J - V) \quad (17)$$

where  $\mu \in ]0, 1[$  is the bargaining strength of the Trade Union. The solution to the maximization problem yields:

$$\mu(J - V) = (1 - \mu)(W - U) \quad (18)$$

The explicit solution to the collective bargaining problem is expressed in the following proposition:

**Proposition 2.** *The wage that maximizes the total surplus of a successful match is:*



$$w \equiv \tilde{w}(y, \tau, k, \theta) = \frac{(1 - \tau) \mu}{\mu + (1 - \tau)(1 - \mu)(1 - b)} (y - (\delta + r)k + c\theta) \quad (19)$$

The wage agreed between the Employers' Association and the Trade Union decreases with depreciation and rent cost of capital and increases with output and labor market tightness. Higher output increases the firm resources to better reward labor; the inverse occurs when depreciation and capital rent costs increase. Expression (19) is the effective labor supply.

### 2.3 Workers

Following Imrohorglu (1989), we will use the transition probabilities between states of the worker (13) as a constraint to the maximization of the utility of the worker. From solving the worker optimization problem we can compute the average aggregate amount of savings, *i.e.*, the capital supply-side of the model.

A worker maximizes utility taking wage, labor market conditions and interest rate as given. Being so, let  $a_t, c_t \geq 0$  be, respectively, the amount of asset holdings and consumption level in period  $t$ ,  $\beta = (1 + r)^{-1}$  the discount factor and  $\Upsilon$  the following utility function that verifies the usual neoclassical properties:

$$\Upsilon_t \equiv \Upsilon(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} \quad (20)$$

where  $\sigma$  is a risk aversion parameter.

In each period, the worker must decide on the amount of assets to hold in the following period,  $a_{t+1}$ , subject to the following budget constraint:

$$c_t \equiv c_t(a_t, a_{t+1}, s_t, \pi_t) = \omega_t + \pi_t + (1 + r)a_t - a_{t+1} \quad (21)$$

where  $\omega_t = \{w_t, bw_t\}$  is the current income (for the employed or unemployed state) and  $\pi_t$  is the share of aggregate profits.<sup>1</sup>

#### Optimization problem

The worker must choose a plan  $\{a_{t+1}\}_{t=0}^{\infty}$  for a given  $(a_t, s_t)$  as to maximize the discounted utility,  $H_t \equiv H(\Upsilon_t, \Pi_t, H_{t+1})$ . Formally, we are faced with the following dynamic programming problem:

$$H_t = \max_{a_{t+1}} \left\{ \Upsilon_t + \beta \sum_s \Pi_t H_{t+1} \right\} \quad (22)$$

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<sup>1</sup>Firms are owned by workers. The amount of each period's aggregate profits are transferred as a lump sum to workers regardless of state. Since all firms are alike and only the occupied jobs yield positive returns, the share of the aggregate average profit each worker is entitled to is  $\frac{JE}{L} = Je = J(1 - u) \equiv \pi_t$ .

The solution to (22) yields the decision rule for next period's asset holdings, *i.e.*, the asset supply function that depends on the pair  $\{\text{current employment state, current amount of asset holdings}\}$ :

$$a_{t+1} = \Gamma(s_t, a_t) \quad (23)$$

Since we define a probabilistic framework for the transitions between worker states, more precisely, a discrete Markov process with a finite number of states with the ergodic property (13), then an invariant distribution can be determined. Being so, let

$$\rho \equiv \rho(s) \quad (24)$$

be the density function for the invariant distribution of asset holdings across workers depending on the worker state. Once determined, it is immediate to calculate the average aggregate equilibrium asset holding - the capital supply:

$$S_k \equiv a = \text{Avg}\{\rho\} \quad (25)$$

## 2.4 Steady state equilibrium

**Definition 1.** *Recursive stationary equilibrium.* The recursive stationary equilibrium of the economy is the set of functions  $S = \{w^*, \theta^*, u^*, \tau^*, D_k, S_k, r^*\}$  which are the solution to

1. *Optimization problem of the firm: the amount of capital used by the firm maximizes the value of the occupied vacancy (6), where  $k$  is the optimum per capita stock of capital and  $w$  is the inverse of the labor demand function;*
2. *Collective bargaining optimization problem: the wage agreed between the Employers' Association and the Trade Union maximizes the match surplus (16), where  $\tilde{w}$  is the labor supply function;*
3. *Labor market equilibrium:*

(a) *the equilibrium wage is such that labor demand equals labor supply:*

$$w^* = \tilde{w}(y^*, \tau^*, k^*, \theta^*) = w(y^*, \tau^*, \theta^*) \quad (26)$$

*implicit is the labor market tightness index:*

$$\theta^* = \{\theta : \tilde{w}(y, \tau, k, \theta) = w(y, \tau, \theta), \theta \in \mathbb{R}^+\} \quad (27)$$

- (b) *the unemployment rate is constant, i.e. the flows to and out from unemployment are equal:*

$$(1 - u) \lambda = up \quad (28)$$

*yielding the endogenous unemployment and vacant rates*

$$u^* \equiv u^*(\theta) = \frac{\lambda}{\lambda + p} \quad (29)$$

$$v^* \equiv v^*(\theta) = \frac{\lambda\theta}{\lambda + p} \quad (30)$$

4. *Government budget equilibrium: unemployment benefits are fully financed by income taxes levied over employees:*

$$\tau(1 - u)w^{gross} = ubw \quad (31)$$

*yielding the endogenous income tax rate:*

$$\tau^* \equiv \tau^*(\theta) = \frac{b\lambda}{b\lambda + p} \quad (32)$$

5. *Optimization problem of the worker: the amount of asset holdings, given wage, interest rate and labor market conditions, maximizes inter-temporal utility of the worker; and*
6. *Capital market equilibrium: the equilibrium interest rate of the economy is such that balances average aggregate capital demand (8) with average aggregate capital supply (25):*

$$r^* = \{r : D_k = S_k, r \in \mathbb{R}_0^+\} \quad (33)$$

### 3 Model implementation and output analysis

#### 3.1 Computational approach<sup>2</sup>

This model builds on two block linked by an interest rate, the demand and supply blocks. The demand block determines the asset demand level, which results from the maximizing behavior of the firm in terms of capital stock used, given wages (and correspondent  $\theta$  and  $u$ ). The labor costs are the result of wage negotiation between firms and the trade union. From the solution to the wage negotiation and the problem of the firm we get the optimal capital stock, wages and labor market conditions for different interest rates. The second block determines the aggregate savings level, which is supplied by households to firms. The solution to the worker utility maximization problem, given interest rate, wage and labor market conditions yields an optimal asset holdings level. The interest rate will make the

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<sup>2</sup>for an extensive description on the computational implementation see da Silva (2010)

link between the two blocks and will be such that, in equilibrium, the capital demanded by the firms equals the asset holdings by households and it will be found by trial and error.

In order to solve the model, one has to address each block at a time. First, for a given  $r$ , the demand block is solved by:

1. solving (1) to get the value of the demand for capital,  $D_k = k$ ;
2. solving (26) together with the previously obtained value for  $k$ , and the value for  $\theta$  is found;
3. solving (19) together with  $k$  and  $\theta$ : the equilibrium wage ( $w$ ) is found together with the unemployment level ( $u$ ) by using (28); and
4. using (28) into (13) to determine the transition probability matrix,  $\Pi$ .

Once the key values from the demand block are calculated, which are  $w$  and  $\Pi$ , we can solve the supply block, for the same value of  $r$ , by:

1. solving the dynamic programming problem (22) through numerical methods to obtain the decision rule (23),  $a_{t+1} = \Gamma(s_t, a_t)$ ;
2. solving (23) in order to determine the invariant distribution (24),  $\rho$ ; and
3. using (24) to find the supply of capital (25),  $S_k \equiv a = E\{\rho\}$ .

At this point, we have determined the value for the demand and supply of capital for a given  $r$ . The interest rate equilibrium value is achieved by guessing across different values until convergence, that is, until  $|D_k(r) - S_k(r)| \leq \epsilon$  verifies, with  $\epsilon$  being an arbitrarily small positive value.

## Welfare and inequality

As we aim at assess the impacts of changing structural features of the labor market, we compute the steady state value function (22) to measure welfare. Moreover, and to assess potential inequality impacts of the measures, we complement welfare analysis with consumption/income dispersion, using the *Gini* coefficient.<sup>3</sup>

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<sup>3</sup>The technique used for solving the model requires a discrete asset space, therefore, the calculation of *Gini coefficient* will be:

$$Gini = 1 - \sum_{i=0}^n (\tilde{\rho}_i - \tilde{\rho}_{i-1}) (\tilde{a}_i + \tilde{a}_{i-1})$$

where  $\tilde{\rho}$  and  $\tilde{a}$  corresponds to the cumulative value for the population and asset holdings at the  $i^{th}$  quantile, respectively. This index of inequality measures the distance of the variable cumulative distribution from absolute equality dispersion. It ranges from  $Gini = 0$  (absolute equality) to  $Gini = 1$  (absolute inequality).

### 3.2 Calibration

When available we use Euro area data, as collected in Tables 5.3-5.2 in appendix and in Figure 3.1, which also serve as reference to model's output. We define  $EA(12)$  as the Euro area first 12 member countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.

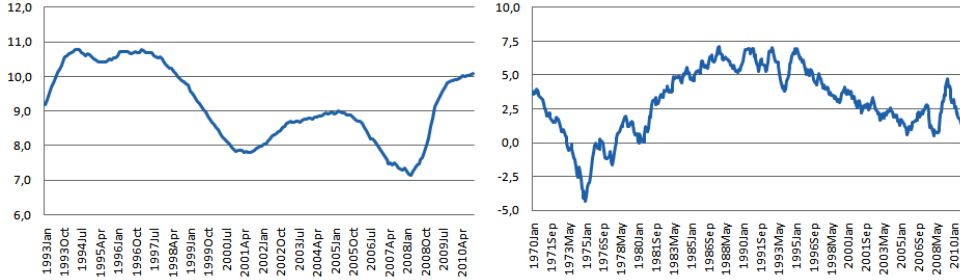


Figure 3.1: Unemployment rate and long term real interest rate (monthly)

For parameter calibration purposes, data show that the average job duration is 11.15 years, hence,  $\lambda$  is calibrated as 0.082.<sup>4</sup> Unemployment rate registers an average of 9.17%, while annualized real interest rate<sup>5</sup> has an average of 3.06%. Given that data on the unit vacancy costs,  $c$ , for the  $EA(12)$  countries are, to our knowledge, nonexistent, we chose, residually, the value for  $c$  as well as the maximum asset level a worker can hold,  $a_{max}$ , in order to obtain steady state equilibrium interest and unemployment rates as close as possible to the data. As for the minimum asset level a worker can hold,  $a_{min}$ , the no-borrowing model imposition implies that this parameter is set to zero.<sup>6</sup>

Data for the calibration of the production function and matching function are also incomplete, that is, we could not find data for a significant number of countries and the series were also small preventing any relevant estimation of the parameters. Given this we have set these parameters according to the related literature. Of course, in connection with the production technology, we have also calibrated the depreciation rate of capital accordingly. For this purpose, we chose the calibration used in Ljungqvist and Sargent (2007) of the related parameters. As for the utility function, we used the calibration of  $\sigma$  as in Imrohorglu (1989), that is, 6.2. Finally, for the unemployment replacement ratio,  $b$ , we rely on Campolmi and Faia (2010), this rate varies between 0.39 for Spain and 0.89

<sup>4</sup>Given the assumption of a Poisson process for job separation rate, if a job is expected to end once every 11.15 years, the probability that it will end within a single year is of  $\lambda = \frac{1}{11.15} e^{-\frac{1}{11.15}} = 0.08199266$ .

<sup>5</sup>Source: ECB. The long term real interest rate is computed as the difference between the 10 year government bonds interest rate and the HCPI changes relative to the homologous period.

<sup>6</sup>As in Imrohorglu (1989), we use a discrete domain for asset holdings possibilities, *ie*, we divide it into equally spaced intervals - or points; larger number of points (smaller intervals) imply more accuracy. This particular technique implies a trade-off between the solution's accuracy and the computational effort. Being so, we chose the number of points in the asset holdings domain as to insure the robustness of the results, that is, we determined the minimum amount of points so that no significant changes in the outcome of the model are registered when we reduce the size of the intervals.

for Ireland. We chose the intermediate value of 0.65. Also, taken the data (see Table 5.2 in appendix) the compensation of employees averages at 48.75% of GDP.

Table 3.1 summarizes the parameter calibration of the model.

	PARAMETER	VALUE	SOURCE
$\alpha$	Production function index	0.3333	Ljungqvist and Sargent (2007)
$\delta$	Depreciation rate of capital	0.0847	Ljungqvist and Sargent (2007)
$\lambda$	Job separation rate	0.0820	Data on <i>EA(12)</i>
$\mu$	Worker bargaining power	0.5000	Ljungqvist and Sargent (2007)
$\eta$	Matching function index	0.5000	Ljungqvist and Sargent (2007)
$\sigma$	Utility function index	6.2000	Imrohoroglu (1989)
$c$	Unit vacancy cost	0.4636	Residual
$b$	Wage replacement ratio	0.6500	Campolmi and Faia (2010)
$a_{max}$	Maximum level of asset holdings	4,7531	Residual
$a_{min}$	Minimum level of asset holdings	0.0000	No-borrowing constraint

Table 3.1: Parameter calibration

### 3.3 Numerical simulations

#### Steady state results

Figure 3.2 shows the stationary distribution of assets depicting an accumulation point in

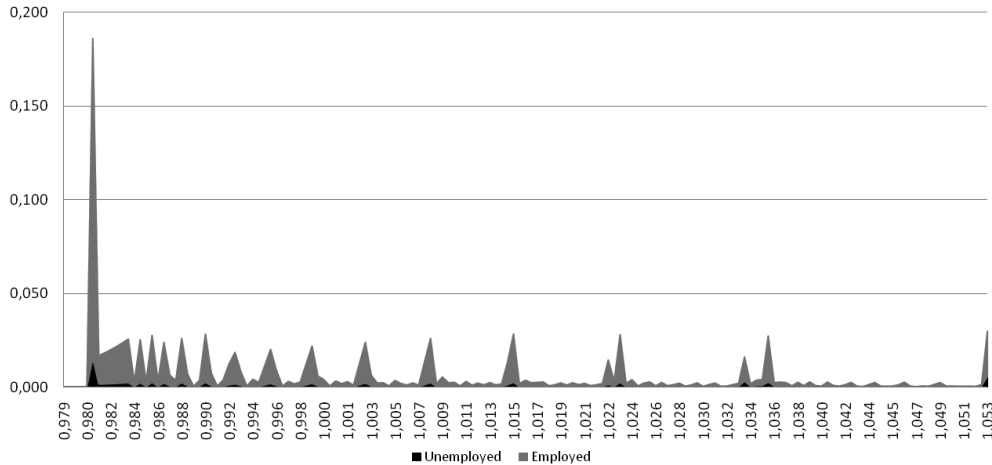


Figure 3.2: Employed and unemployed asset holdings distribution (% of the population in the  $y$  axis and the asset holdings level, normalized to  $k$ , in the  $x$  axis).

the lower end of the asset domain; this is an idiosyncratic result of this type of modeling [see, for example, Imrohoroglu (1989) (Figure 1, page 1376) or Nakajima (2010) (Figure 1 (e), page 23) for a similar result in a discrete context or Krusell et al. (2010) (Figure 2, page 19) for the continuous time case]. The small dimension of the transition matrix  $\Pi$  and the choice of utility function, depending on consumption alone, are the reasons, we believe, for

this behavior. Another consistent feature with similar works is that the employed worker holds much more assets than the unemployed (Imrohoroglu (1989); Nakajima (2010) and Krusell et al. (2010)). This is an expected result since, given the homogeneity of the household behavior, the lower income for the unemployed workers implies a smaller ability to accumulate wealth.

	VARIABLE	DATA	MODEL	
$r$	Interest rate	3.058	3.058	%
$u$	Unemployment rate	9.173	9.173	%
$\theta$	Labor market tightness	[3.130 , 36.760]	65.907	
$v$	Vacancy rate	[0.210 , 1.890]	6.046	%
$\frac{1}{p}$	Average unemployment duration	1.396	1.232	periods
	Compensation of employees	48.750	64.103	% GDP
	Overall <i>Gini</i> coefficient	[26 , 38]	43.874	%

Table 3.2: *EA(12)* data vs base case model output

Table 3.2 summarizes the main results of the model in steady state equilibrium compared with the average data collected for the *EA(12)* countries.

The model is unable to match the vacancy rate and, consequently, the equilibrium labor market tightness. It is worth mentioning that, on the firm's side, the optimal behavior takes into account the vacancy unit cost for which, lacking micro data, we have calibrated residually. But this calibration strategy may not be the cause of such a high model vacancy rate result since the value used is within the range used in related literature (see Pissarides (2009); Krusell et al. (2010) and Bartelsman et al. (2010)). It is our conviction that, once again, the dimension of the transition matrix  $\Pi$  is the cause of such a high value for the vacancy rate and, consequently, of the high labor market tightness outcome. Therefore, increasing the range of states and, perhaps, endogenizing the in-here exogenous  $\lambda$  could improve the prediction on the vacancy rate.

The share of the labor factor in GDP is higher than observed in 15.35pp. This is due, as discussed in the previous paragraph, to the abnormal result in the vacancy rate, that is, we believe the causes of the unbalance between labor and capital weights in output have the same origins as those from the vacancy rate. See that, one key expression of the model is the vacancy creation condition (4), which is crucial, not only to the labor market conditions outcome, but also to output decomposition (5).

The average unemployment duration is lower than data shows: while the model predicts 14.8 months unemployment duration, data shows that it takes 2 months longer for the unemployed to find a job.

The *Gini* coefficient for consumption/income is 5.85pp, larger than the high end of the data interval.

## Comparative Statics

The aim of this work is to assess how changes in some institutional labor market variables

		$b$	$\lambda$	$c$	
$\Delta$	Required change	-6.5544	-5.4473	-10.3069	%
$\frac{\Delta}{\Delta\%u}$	Required change to percentage change in $u$ ratio	1.2025	0.9994	1.8909	
$r$	Interest rate	0.0714	-0.0089	-0.0263	pp
$k \equiv a$	Average per capita asset holdings	-0.8714	0.1089	0.3241	%
$a^e$	Assets of the employed	-0.3056	0.6631	0.9009	%
$a^u$	Assets of the unemployed	-6.4454	-5.3506	-5.3582	%
$a^u/a$	Unemployed assets to average assets ratio	-0.0562	-0.0545	-0.0566	pp
$a^u/a^e$	Unemployed assets to employed assets ratio	-0.6252	-0.6064	-0.6297	pp
$y$	Output	-0.2913	0.0363	0.1079	%
$c$	Overall Consumption	-3.5955	0.8488	0.9318	%
$c^e$	Consumption of the employed	0.3231	0.4595	0.5308	%
$w$	Employed current income	0.1647	0.5480	0.6759	%
$ra^e$	Employed savings income	2.0227	0.3717	0.0332	%
$w/c^e$	Current income on consumption	-0.1840	0.0174	0.0635	pp
$c^u$	Consumption of the unemployed	-5.0472	0.4187	0.4216	%
$bw$	Unemployed current income	-6.4004	0.5480	0.6759	%
$ra^u$	Unemployed savings income	-4.2605	-5.6246	-6.1721	%
$bw/c^e$	Current income on consumption	-0.0433	0.1164	0.1290	pp
$\Upsilon$	Overall welfare	-0.3577	0.1643	0.1732	%
$\Upsilon^e$	Welfare of the employed	0.0239	0.0317	0.0356	%
$\Upsilon^u$	Welfare of the unemployed	-0.3536	0.0829	0.0831	%
$Gini a$	Overall <i>Gini</i> for assets	0.1304	0.0934	0.0933	pp
$Gini a^e$	Employed <i>Gini</i> for assets	0.1113	0.0970	0.0628	pp
$Gini a^u$	Unemployed <i>Gini</i> for assets	0.1403	0.0590	0.1055	pp
$Gini C$	Overall <i>Gini</i> for consumption	-1.0718	-0.1844	-0.1499	pp
$Gini C^e$	Employed <i>Gini</i> for consumption	0.0149	0.0118	0.0069	pp
$Gini C^u$	Unemployed <i>Gini</i> for consumption	0.0323	0.0109	0.0181	pp
$\theta$	Labor market tightness	0.1310	0.0111	0.1310	
$v$	Vacancy Rate	0.4191	-0.2660	0.4191	pp
$p_{uu}$	Probability of remaining unemployed	-5.1528	-0.4499	-5.1528	pp
$\frac{1}{q}$	Vacancy spell	6.3471	0.5541	6.3471	%
$\frac{1}{p}$	Unemployment spell	-5.9683	-0.5511	-5.9683	%
$\tau$	Income tax rate	-0.7066	-0.3463	-0.3463	pp
$J$	Average firm profit	7.0218	-3.5144	-4.8368	%
$\pi$	Average <i>per capita</i> profit	7.6110	-2.9833	-4.3129	%

Table 3.3: Impacts of changes in labor market institutional variables in order to promote a 0.5pp reduction in equilibrium unemployment rate



affect the equilibrium unemployment rate and analyze their implications on welfare and wealth and consumption/income inequality. In order to make analysis comparable across institutional changes, we have computed for the required change in the unit vacancy cost,  $c$ , in the unemployment replacement ratio,  $b$ , and on the job destruction rate,  $\lambda$ , in order to accomplish the same reduction (0.5pp) in the unemployment rate; given the baseline case solution, this comprises to a 5,47% reduction in the unemployment rate. The focus on these specific institutional variables derives from the fact that: (i) the unemployment replacement ratio is a direct labor market institutional variable as it captures the amount of income that an unemployed is entitled to as insurance for becoming unemployed; (ii) regarding the job separation rate, it captures, though indirectly, job destruction regulation (job protection legislation, firing costs or age retirement restrictions, for instance, are strong constraints on the job separation decision); (iii) although the unit vacancy cost is not a labor market institutional variable, vacancy costs are crucial in the job creation decision of the firm and, hence, they are meaningful as they impact strongly on unemployment and labor market functioning. This variable can be an instrument of labor market intervention, though, if, for example, upon it incides an employment subsidy to firms or some other form of intervention that reduces the firm's cost on job creation.

Accordingly, Table 3.3 presents the required changes in the labor market institutional features together with the corresponding steady state impacts on the most relevant endogenous variables and also on average welfare and wealth and consumption/income distribution by worker status.

We start by stating that the desired reduction on the unemployment rate can be achieved by reasonable changes in all the exogenous variables. It is by itself a positive outcome since it excludes extreme responses from these variables. For that matter, the model predicts that in order to decrease the unemployment rate in 0.5pp the replacement ratio needs to be reduced in 6.55%. Alternatively, the same result is achieved by reducing 5.45% in the job separation rate or by reducing the unit vacancy cost in 10.31%, *ceteris paribus*. While the effects are one-to-one of  $\lambda$  in  $u$ , it requires 1.2% and 1.9% changes in  $b$  and  $c$  to achieve a decrease 1% change in the unemployment rate.

Average *per capita* capital,  $k$ , falls with  $b$  and increases with  $\lambda$  and  $c$  which, by (1), pushes the average per capita output,  $y$ , in the same direction accompanied by opposite changes in the equilibrium interest rate. The greatest positive impact is obtained when we reduce the vacancy unit cost. For the same desired reduction in the unemployment rate, reducing a direct production cost results in an approximately 3 times greater increase in output than reducing the job separation rate. Moreover, the reduction in the unemployment replacement ratio achieves the desired reduction in unemployment but yields negative results on output. In all cases, the unemployed worker owns less assets,  $a^u$  falls, and, moreover, he owns relatively less than the average employed worker (both  $a^u/a$  and  $a^u/a^e$  ratios decreases).

Average overall welfare decreases only when  $b$  is used as a institutional tool towards unemployment reduction. This results from the large decline in the consumption level of the

unemployed which is not sufficiently compensated by the rise in the employed consumption; hence, average welfare moves directly with  $b$ . More precisely, the sharp reduction on the unemployed consumption is the result of both a fall in current labor-related and savings income but insufficient increase in the firms profits share: the increase in wage is overcome by the fall in  $b$  and the increase in the interest rate is overcome by a fall in asset holdings, resulting in a double downward effect not compensated by the great increase in  $\pi$  for the consumption of the unemployed. In contrast, in the case of  $\lambda$  and  $c$ , the fall in savings income and firms profits share is more than compensated by the rise in current labor-related income. As regards consumption of the employed worker, it rises due to the increase in both current and savings income. Even when the increase in asset holdings is not enough to overcome the decrease in interest rate, resulting in a marginal reduction on the savings income (as in case of a downward shock in  $c$ ), the increase in current labor-related income is enough to raise the consumption of the employed. Once again, a reduction in  $c$  dominates, yielding the best results in overall (and group-specific) consumption and, consequently, in overall (and group-related welfare).

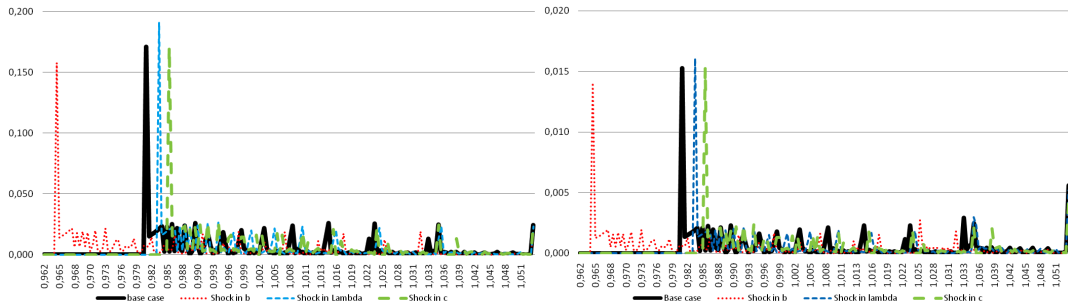


Figure 3.3: Impacts on asset holdings distribution of the employed and unemployed due to changes in labor market institutional variables in order to promote a  $0.5pp$  reduction in equilibrium unemployment rate. For the employed in the left side and for the unemployed in the right side. (% of the population in the  $y$  axis and the asset holdings level, normalized in order of the baseline case average asset holdings for each state, in the  $x$  axis)

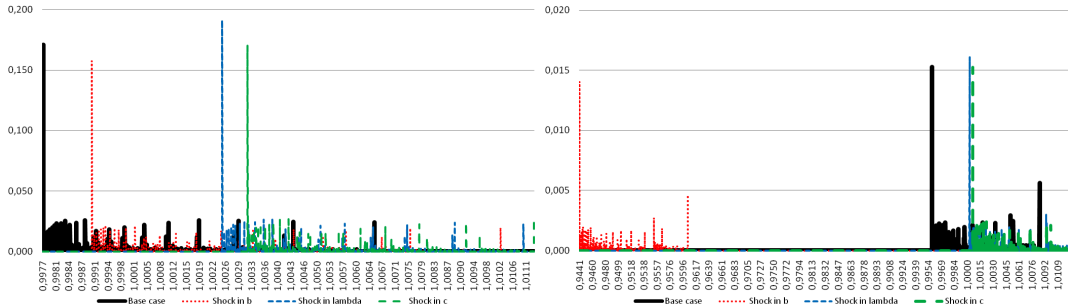


Figure 3.4: Impacts on the consumption distribution of the employed and unemployed due to changes in labor market institutional variables in order to promote a  $0.5pp$  reduction in equilibrium unemployment rate.

Regarding asset holdings, all inequality indicators worsen, in particular for  $b$ . Notice that, in the case of  $b$  for both the employed and unemployed, this is explained by a decrease in savings, that is, workers savings choices are spread across a much larger domains. While for the case of  $\lambda$  and  $c$  we observe that the savings interval narrows and the inequality increase derives from the fact that workers deviate for the upper and lower boundaries of the domain.

As for inequality in consumption, all institutional variables reduce asymmetries between groups but increases for all within group inequality. Given the increase in employed individuals, who benefit from larger consumption levels, a positive effect in overall consumption inequality is possible in spite of the negative effects in groups-specific inequality. This is the result of a smaller distance between the distributions of consumption of the employed and unemployed groups and, at the same time, a wider distribution of individuals throughout a larger consumption domain within groups, hence, increasing group-specific inequality. In the case of the reduction in  $b$ , the effects are ambiguous because this policy registers the largest reduction in overall inequality and, at the same time, the larger increases in groups-specific inequality.

As for labor market tightness,  $\theta$  improves in all cases, although with a clear smaller impact in the case of  $\lambda$  due to the opposite effects of a decrease in both unemployment and vacancy rates (in the case of  $b$  and  $c$ , a decrease in  $u$  is leveraged by an increase in  $v$ ). Labor market conditions are determined by the demand and supply for labor which, from (7), establish a positive relation between  $\lambda$  demand for labor, that is, as jobs last longer, the average job turnover falls, implying that less job vacancies are available, this effect dims the positive one resulting from a fall of unemployment in  $\theta$ . Consequently, the reduction in the expected duration of unemployment is less sharp and the expected time for a job vacancy to be filled increases by less in this case when compared with the alternative structural changes ( $b$  and  $c$ ).

Reflecting the improvement in labor market conditions, the wage negotiated between Employers' Association and the Trade Union rises for all strategies. The smallest increase is registered when  $b$  is reduced. Given that the opportunity cost of being unemployed ( $bw$ ) decreases with  $b$ , *ceteris paribus*, the Trade Union obtains a smaller wage improvement when negotiating with the Employers' Association. Apparently, the use of  $c$  and  $\lambda$  also results in higher wages, although for different reasons: on the one hand, low  $c$  corresponds to low expected job vacancy costs (4) and, therefore, firms are more willing to open new vacancies, increasing labor demand. This effect is expressed in (7) where the optimum wage reflects the reduction on vacancy costs to compensate for higher wages resulting from a raise in competition between workers. On the other hand, a low  $\lambda$  corresponds to a low uncertainty of firms on the asset value of the job; therefore, firms are willing to pay more to the worker in order to keep the vacancy filled (see also (7)); at the same time a low  $\lambda$  increases workers fallback option and, consequently, their bargaining power.

As expected, the equilibrium income tax rate decreases in all cases. A reduction in the number of unemployed requires fewer resources to pay for unemployment assistance and,

at the same time, there are more employed workers to tax. An additional effect works in the case of an intervention using  $b$  because, obviously, the replacement ratio is reduced, hence implying an even lower tax rate.

## 4 Conclusions

In order to assess the welfare and redistributive impacts of some institutional changes in the labor market, we use a labor market search and matching model with aggregate savings within a heterogeneous-agent framework. We have deliberately elected a simple formulation from the search and matching literature as well as from the heterogeneous-agent framework. One novelty is the structure of the model which relies in combining labor market equilibrium with capital market equilibrium. This specification allows interest rate to be determined endogenously. Second, the particular structure chosen for the labor market interactions, determines an equilibrium that is based on the behaviors of workers and firms but strongly constrained by labor market institutions, in particular, the use of a Trade Union on the wage bargaining process. Third, the model enables to assess, besides impacts on structural macroeconomic variables, welfare and wealth and consumption/income distribution impacts from structural changes in labor market functioning. Fourth, the transition matrix is endogenous; it is not produced according to exogenous shocks and derives from the equilibrium outcome.

We conclude that, for a reasonable parameter calibration - mostly based on the *EA(12)* economy, the model is accurate in predicting the equilibrium unemployment rate and interest rate. Some deviations are registered in what regards the unemployment duration, *Gini* coefficient for overall consumption/income and output decomposition. Although, vacancy rate and, consequently, labor market tightness, are higher than data shows. The results from the comparative statics analysis show that, to obtain a given reduction in the unemployment rate the reduction in the vacancy unit costs, induces both larger welfare improvement and a smaller inequality reduction when compared with a strategy relying on the reduction of the job separation rate. On the contrary, a reduction in the replacement ratio implies a trade-off between welfare and consumption/income distribution. The fact that a lower replacement ratio implies a worse welfare may be justified because the model ignores the effects that the replacement ratio has on the search intensity of the unemployed. It is commonly argued that there is a significant impact of unemployment insurance on the worker search intensity (see, for instance, Pissarides (2000)) and, therefore, on the probability of finding a job and thus, on welfare. Therefore, the immediate research agenda for this future work is the inclusion of search effort by the unemployed worker by allowing the probability of finding a job be a function of the replacement ratio level and/or by affecting unemployment duration. Furthermore, the worker utility function in the model should encompass a trade-off between consumption and leisure, namely to capture labor supply-side effects. Another caveat is the small dimension of the transition matrix; the inclusion of extended characteristics of the worker, such as education level, gender or tenure, would

certainly produce higher dispersion across households.

Finally, recall that this is a static analysis and, therefore, any results are computed only by comparing the initial system steady state with the end system steady state. No transition dynamics is accounted for, thus, we cannot determine if, in fact, the best steady state results still correspond to the most beneficial when including the transition path.

## References

- Andolfatto, D.: 1996, 'Business Cycles and Labor-Market Search'. *American Economic Review* **86**(1), 112–32.
- Bartelsman, E., P. Gautier, and J. De Wind: 2010, 'Employment Protection, Technology Choice, and Worker Allocation'. IZA Discussion Papers 4895, Institute for the Study of Labor (IZA).
- Bayer, C. and K. Walde: 2010, 'Matching and Saving in Continuous Time: Theory'. Working Papers 1004, Gutenberg School of Management and Economics, Johannes Gutenberg-Universität Mainz.
- Blanchard, O. J. and P. Diamond: 1989, 'The Beveridge Curve'. *Brookings Papers on Economic Activity* **20**(1989-1), 1–76.
- Campolmi, A. and E. Faia: 2010, 'Labor Market Institutions and Inflation Volatility in the Euro Area'. *Journal of Economic Dynamics and Control*.
- da Silva, C. A.: 2010, 'Computational implementation of A Matching Model with Aggregate Savings: A Comparative Statics Approach'. *MIMEO; Universidade do Porto, Faculdade de Economia*.
- den Haan, W. J., G. Ramey, and J. Watson: 2000, 'Job Destruction and Propagation of Shocks'. *American Economic Review* **90**(3), 482–498.
- Imrohorglu, A.: 1989, 'Cost of Business Cycles with Indivisibilities and Liquidity Constraints'. *The Journal of Political Economy* **97**(6), 1364–1383.
- Krusell, P., T. Mukoyama, and A. Sahin: 2010, 'Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations'. *Review of Economic Studies* **77**(4), 1477–1507.
- Ljungqvist, L. and T. J. Sargent: 2007, 'Understanding European unemployment with matching and search-island models'. *Journal of Monetary Economics* **54**(8), 2139–2179.
- Merz, M.: 1995, 'Search in the labor market and the real business cycle'. *Journal of Monetary Economics* **36**(2), 269–300.
- Mortensen, D. T. and C. A. Pissarides: 1994, 'Job Creation and Job Destruction in the Theory of Unemployment'. *Review of Economic Studies* **61**(3), 397–415.

- Nakajima, M.: 2010, ‘Business cycles in the equilibrium model of labor market search and self-insurance’. Working Papers 10-24, Federal Reserve Bank of Philadelphia.
- Nash, J. F.: 1950, ‘The Bargaining Problem’. *Econometrica* **18**(2), 155–162.
- Pissarides, C.: 2000, *Equilibrium unemployment theory*. MIT Press.
- Pissarides, C., R. Layard, and M. Hellwig: 1986, ‘Unemployment and Vacancies in Britain’. *Economic Policy* **1**(3), 500–559.
- Pissarides, C. A.: 2009, ‘The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?’. *Econometrica* **77**(5), 1339–1369.
- Shao, E. and P. Silos: 2007, ‘Uninsurable individual risk and the cyclical behavior of unemployment and vacancies’. Working Paper 2007-05, Federal Reserve Bank of Atlanta.
- Shimer, R.: 2005, ‘The Cyclical Behavior of Equilibrium Unemployment and Vacancies’. *American Economic Review* **95**(1), 25–49.

## 5 Appendix

### 5.1 Proof for Proposition 1

PROPOSITION 1: *At the optimum, the firm chooses the amount of capital and labor factor as to verify:*

$$w \equiv w(y, \tau, q) = (1 - \tau) \left[ y - (\delta + r)k - \frac{(r + \lambda)c}{q} \right]$$

$$k = \left( \frac{\alpha}{\delta + r} \right)^{\frac{1}{1-\alpha}}$$

PROOF:

following Pissarides (2000) chap.1, the firm aims at maximizing the asset value of the occupied vacancy by choosing the optimum amount of capital and labor, therefore, the problem of the firm is:

$$\max_{k, w} \{J\} \tag{34}$$

in steady state  $V = 0$  and rearranging 2 as  $J = \frac{1}{r+\lambda} (y - (r + \delta)k) - \frac{w_t}{(r+\lambda)(1-\tau)}$ , the first order condition is:

$$\frac{dJ(k)}{dk} = 0 \Leftrightarrow f'_k = r + \delta \tag{35}$$

Our choice of production function implies that  $f'_k = \alpha k^{\alpha-1}$ , which from the optimizing choice of capital by the firm (8), the *per capital* equilibrium stock of capital is:

$$k = \left( \frac{\alpha}{\delta + r} \right)^{\frac{1}{1-\alpha}} \quad (36)$$

In equilibrium all profit opportunities are exhausted, therefore, the vacant job cannot yield any positive value implying that for (2), with use of (4) and (8), the inverse of the demand for labor function is:

$$w = (1 - \tau) \left[ y - (\delta + r) k - \frac{(r + \lambda) c}{q} \right] \quad (37)$$

◇

## 5.2 Proof for Proposition 2

PROPOSITION 2: *The wage that maximizes the total surplus of a successful match is*

$$w \equiv \tilde{w}(y, \tau, k, \theta) = \frac{(1 - \tau) \mu}{\mu + (1 - \tau)(1 - \mu)(1 - b)} (y - (\delta + r) k + c\theta)$$

PROOF:

Again, following Pissarides (2000) chap.1, in equilibrium, all profit opportunities for the vacant vacancy are exhausted, therefore  $V = 0$ . Rearranging (2) and (14) as  $J = \frac{1}{r+\lambda} (y - (\delta + r) k) - \frac{w}{(r+\lambda)(1-\tau)}$  and  $W = \frac{w+\lambda U}{r+\lambda}$ , respectively, we can substitute in (18):

$$\begin{aligned} \mu J &= (1 - \mu) (W - U) \\ \Leftrightarrow w &= \frac{(1 - \tau)}{((1 - \mu)(1 - \tau) + \mu)} [(1 - \mu) rU + \mu (y - (\delta + r) k)] \end{aligned} \quad (38)$$

On the other hand, we rearrange (15) as  $W - U = \frac{rU - bw}{p}$  and  $\frac{vm(u,v)}{um(u,v)} = \frac{p}{q} = \theta$ . Together with (4) we replace in (18) to achieve at:

$$\begin{aligned} \mu J = (1 - \mu) (W - U) &\Leftrightarrow \mu \frac{c}{q} = (1 - \mu) \frac{rU - bw}{p} \\ &\Leftrightarrow rU = \frac{\mu}{1 - \mu} c\theta + bw \end{aligned} \quad (39)$$

Finally, using (39) in (38) and making use of (8) yields the desired expression for the wage as a function of market conditions alone:

$$\begin{aligned} w &= \frac{(1 - \tau)}{((1 - \mu)(1 - \tau) + \mu)} [(1 - \mu) rU + \mu (y - (\delta + r) k)] \\ &= \frac{(1 - \tau) \mu}{\mu + (1 - \tau)(1 - \mu)(1 - b)} (y - (\delta + r) k + c\theta) \end{aligned} \quad (40)$$

△

### 5.3 Tables

YEAR	1986	1987	1988	1989	1990	1991	1992	1993
months	17.54	20.09	20.84	19.98	17.85	16.30	15.42	15.27
YEAR	1994	1995	1996	1997	1998	1999	2000	2001
months	16.75	18.23	18.09	18.28	18.32	17.67	17.42	15.98
YEAR	2002	2003	2004	2005	2006	2007	2008	2009
months	15.61	16.08	15.75	15.68	15.66	14.83	12.39	11.92
Average	<i>Source: OECD</i>							
16.75								

Table 5.1: Average unemployment duration in European countries

YEAR	1995	1996	1997	1998	1999	2000
% GDP	50.1	49.8	49.3	48.8	49.1	49.0
YEAR	2001	2002	2003	2004	2005	2006
% GDP	49.0	49.0	48.9	48.3	48.0	47.6
YEAR	2007	2008	2009	2010 <sup>f</sup>	2011 <sup>f</sup>	2012 <sup>f</sup>
% GDP	47.3	48.1	49.5	48.9	48.5	48.3
Average	<i>f: Forecast. Source: Eurostat</i>					
48.75						

Table 5.2: Average compensation of employees on *EA(12)* countries (as a percentage of GDP)



COUNTRY	GINI <sup>1</sup>	JOB TENURE <sup>3</sup> (years)	VACANCY RATE <sup>4</sup> (%)	UNEMPLOYMENT RATE <sup>5</sup> (%)	$\theta$ $v/u$
Austria	27.00	10.86	0.81	4.26	19.08
Belgium	27.00	11.73	1.10	8.49	12.93
Finland	27.00	10.46	0.76	9.47	8.05
France	28.00	11.31	0.84	9.54	8.79
Germany	30.00	10.63	0.90	8.43	10.68
Greece	32.00	13.44	-	-	-
Ireland	33.00	10.00	-	-	-
Italy	35.00	12.17	-	-	-
Luxembourg	26.00	11.01	0.48	3.03	15.91
Netherlands	27.00	10.05	1.89	5.15	36.76
Portugal	38.00	12.39	0.21	6.53	3.21
Spain	32.00	9.76	0.43	13.70	3.13
Average		11.15			13.17

<sup>1</sup> OCDE data on income distribution for the mid 2000's

<sup>3</sup> OCDE yearly data on average job tenure for the 1992-2009 period

<sup>4</sup> OCDE yearly data on vacancy rate for the 1981-2009 period

<sup>5</sup> OCDE yearly data on unemployment rate for the 1986-2009 period

Table 5.3: Selected data on the *EA(12)* countries

VARIABLE		VALUE		CONSUMPTION		VALUE
$r$	Interest rate	3.058	%	$c$	Overall	1.917
$k \equiv a$	Capital	4.514		$c^e$	Employed	1.132
$a^e$	Capital of the employed	4.098		$c^u$	Unemployed	0.785
$a^u$	Capital of the unemployed	0.416		UTILITY		Value
$a^u/a$	-	9.216	%	$\Upsilon$	Overall	0.038
$a^u/a^e$	-	10.150	%	$\Upsilon^e$	Employed	0.083
$w$	Wage	0.994		$\Upsilon^u$	Unemployed	-0.045
$\theta$	Labor market tightness	0.659		GINI FOR ASSETS		Value
$v$	Vacancy rate	6.046	%	Overall		0.383 %
$u$	Unemployment rate	9.173	%	Employed		0.412 %
$p_{uu}$	Prob. remain unemployed	18.817	%	Unemployed		0.291 %
$\frac{1}{q}$	Vacant vacancy spell	0.812		GINI FOR CONSUMPTION		Value
$\frac{1}{p}$	Unemployment spell	1.232		Overall		43.874 %
$\tau$	Income tax rate	6.160	%	Employed		0.053 %
JOB DECOMPOSITION		VALUE	%	Unemployed		0.052 %
$y$	Output	1.653				
$\frac{w}{(1-\tau)}$	Wage	1.059	(64.10%)			
$rk$	Capital rent	0.138	(8.35%)			
$\delta k$	Capital depreciation	0.413	(24.98%)			
$J$	Job asset value	0.042	(2.56%)			

Table 5.4: Baseline case model output