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Working Papers

N° 2007-08

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April 2007

Documento de Investigación 2007-08

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Translog Cost Functions: An Application for Mexican Manufacturing*

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Abstract

We use translog cost functions to estimate own-price and substitution elasticities of input demands, economies of scale and average costs in Mexican manufacturing. Data from the Mexican Annual Industrial Survey is used for 1996, 2000 and 2003. We show that a model that allows for nonhomotheticity and nonunitary elasticities of substitution is appropriate to represent the production structure. Allen-Uzawa elasticities indicate the existence of substitution possibilities amongst inputs. The demand for electricity is essentially unitary elastic. All cross-price elasticities are less than one. Both scale economies and average costs diminish as the size of activity class increases. Economies of scale increased for any level of output. The differences in average costs between small and large activity classes were reduced and some disparities prevail in a number of manufacturing groups.
Keywords: Simultaneous equation models, Translog cost function, Manufacturing.
JEL Classification: C3, D24, L60

Resumen

Se utilizan funciones de costos translogarítmicas para estimar elasticidades precio y de sustitución de las demandas de insumos, economías de escala y costos medios en las manufactureras Mexicanas. Se analizan datos de la Encuesta Industrial Anual para 1996, 2000 y 2003. Se muestra que un modelo que no permite homoteticidad ni elasticidades unitarias en la función de costos es el más indicado para representar la estructura de producción. Elasticidades de Allen-Uzawa indican la existencia de posibilidades de sustitución entre los insumos. La demanda por electricidad es de elasticidad unitaria. Todas las elasticidades cruzadas son menores a uno. Tanto las economías de escala como los costos medios disminuyen conforme el tamaño de las clases de actividad aumenta. Las economías de escala aumentaron para cualquier nivel de producción. Las diferencias en costos medios entre pequeñas y grandes clases de actividad se han reducido y algunas disparidades se mantienen en ciertos grupos manufactureros.
Palabras Clave: Modelos con ecuaciones simultáneas, Función de costos translogarítmica, Manufacturas.

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*Daniela Flores Rico provided outstanding research assistance. We are very grateful to Daniel Chiquiar, Alejandro Díaz de León, Kenjiro Hori and Eduardo Martínez for revising earlier versions of the paper. We also thank seminar participants at Banco de México. Gerardo Leyva, Abigail Durán and Othoniel Soto were of great help answering our doubts about the data. Any errors and omissions are the sole responsibility of the authors.

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1 Introduction

The main purpose of this paper is to estimate transcendental logarithmic cost functions that include capital, labour, electricity and transport as factors of production for Mexico's manufacturing sector. The estimation of such functions will allow to study elasticities of input demands, economies of scale and average costs. Data from the Annual Industry Survey (AIS) by the Mexican Institute for Statistics, Geography and Informatics (INEGI is its acronym in Spanish) for the 1996-2003 period is used.

There are very few studies for Mexico that follow the overall approach presented here (i.e. use a translog cost function). For instance, Sterner (1989), studied energy use in Mexican Manufacturing using a translog cost function to estimate elasticities of substitution and factor demand during 1966-1981. The study was based on yearly censuses and 18 industries were selected. The restrictions of homotheticity and neutral technical change are tested and rejected most of the times, hence the unrestricted model was maintained. The estimated own-price elasticities were negative. Price elasticities were -0.4 for electricity, -0.6 for fuel, -0.5 for labour, -0.3 for materials and -0.2 for capital. In order to measure the extent to which factors of production can be substituted for one another, the Allen partial elasticities of substitution were calculated; it was found that, at the industry level, materials were the most common substitutes for energy, whilst for the total sector, labour was the main substitute for fuel. Moreover, only the cost shares in the system were estimated, that is, the total cost function was not included.[1]

Truett *et al.* (1994) introduced imports as a factor of production and used data from 1960 to 1988 to estimate an aggregate translog cost function; they found that: *i*) the price elasticities of demand for the inputs (capital, labour and imports) were nearly all negative and inelastic, and *ii*) that capital and imports and capital and labour were substitutes. Basically, they argue that, everything else constant, a reduction in the price of imports reduces the demand for both domestic labour and capital.

Truett and Truett (1998) analysed the existence of economies of scale in the Mexican nonelectrical machinery industry and calculated both direct and cross price elasticities of demand for its inputs (capital, labour and intermediate goods) using annual data from 1970 to 1992 to estimate a translog cost function. They rejected the restriction of homotheticity and homogeneity and considered the translog equation with the basic restrictions as the final model. Moreover, they found that the industry exhibits economies of scale, and that the direct demand elasticities for the inputs were negative and less than one; capital exhibited a higher price elasticity of demand than labour and intermediate goods. The input cross price elasticities indicated that these are substitutes. The coefficient of the neutral technological change variable was negative but not significative.

In a more recent paper, Truett and Truett (2005) estimated a translog cost function for both the Mexican automobile industry as a whole using data for the period 1970-1997. They found evidence of constant returns to scale in both industries and that capital, labour and foreign intermediate goods were all substitutes for one another, as are capital and domestic intermediate goods, but labour and domestic intermediate goods were complements.

Compared to the aforementioned studies, this paper uses data from the AIS and more specific variables, including electricity and transport, in the cost function. Also, it allows for a less restrictive production function as it presents various tests for homotheticity, homogeneity, constant returns to scale and unitary elasticities of substitution. In other words, an advantage of undertaking this approach is precisely that one

[1] This will be further explained in section 4.

can exploit duality theory without imposing any limitation in the underlying technology.[2] More precisely, and according to Shepard (1970), there is a unique correspondence between the production and cost functions. All of the information about the underlying technology is contained in both functions.

By following the approach of Christensen and Greene (1976), special emphasis is put on economies of scale and average costs (AC); in particular, these authors explain that the most convenient way to do so is to use cross-sectional data. Also, an important advantage of using this sort of data is that it allows to take into account some sources that may lead to changes in average costs, for example, economies of scale and technical change. An additional advantage of cross-sectional data is that it allows us to have a wider perspective on how economies of scale vary over time. Therefore, in this paper we consider three cross-section periods, the first for 1996, the second for 2000 and the third for 2003.

Lastly, similar to most micro-empirical studies, this document faces some limitations that could bias the results. For example, and as will be revised later: *i*) the time horizon is relatively short (eight years), which might not be enough to conclude anything related to technical change; *ii*) we do not have establishments, firms or companies as a unit of study, instead, each data point corresponds to an "activity class", that conglomerates a number of manufacturing establishments into it, implicitly this is assuming that every establishment in each activity class is somehow alike in terms of its manufacturing processes, technology, etc; *iii*) we cannot estimate translog cost functions for specific manufacturing subsectors/groups due to the few observations available, hence, we focus on the total manufacturing sector; *iv*) given the previous points, some of our findings should be taken with some reservation as we are comparing quite diverse manufacturing activities (i.e. production of sugar or coffee with production of automobiles or computers); and *v*) the construction of variables such as cost of capital, price of electricity and price of transport may suffer from the typical issues in measurement and construction of variables (i.e. simple models of depreciation, lack of data, use of proxies for variables, deflated variables with general price indexes, etc.).

The remainder of the paper is organised as follows. Section 2 discusses the data and variables used in the estimation. Section 3 offers a brief overview of the recent trends of factor prices and real unit costs for each factor in the Mexican manufacturing sector. Section 4 explains the model and the estimation strategy. Section 5 presents the main results and section 6 summarises.

2 Data and Variables

We use, mainly, the AIS from INEGI, which provides information on manufacturing regarding the following aspects: output, employment, investment and capital stocks, electricity consumption, and transport expenditure.[3] The AIS has been published since 1963. At first, it considered only 29 activity classes, but was extended in 1993 taking advantage of the Industrial Census (IC), considering as population all the manufacturing establishments existing at that time; thus, a new sampling was made for the AIS, which until 2003 included over 5,400 establishments grouped into 205 activity classes corresponding to the 9 subsectors of the Mexican Activity and Product Classification (CMAP is its acronym in Spanish).[4] The surveyed establishments produce nearly 85% of total manufacturing output and employ about 65% of the sector's labour

[2] The relationship between any constrained maximisation problem and its related "dual" constrained minimisation problem.

[3] The construction of the variables followed OECD (2001).

[4] It is important to note that despite the number of establishments, the AIS sample is somehow biased towards relatively large establishments: more than a hundred employees, with a few exceptions.

force. We consider three years: 1996, 2000 and 2003.[5] The most recent AIS is for 2004. However, it suffered considerable changes that do not allow us to consider it in this study (for example, we cannot calculate neither the price of electricity nor remunerations per hour anymore). The variables considered are:

Value of Finished Products (Y).[6] It is the market value of the output of each activity class, using for its calculation an average wholesale price. This variable includes what is produced with the inputs used in a given year, regardless whether the products are sold or not. Therefore, the use of this variable considers the variation in the establishments' inventories. Its value is deflated with price indices specific to each activity class elaborated by INEGI.[7]

Cost of Capital. The cost of capital is proxied for by the gross rate of return paid to capital (R), which is equal to the net rate of return in Mexican manufacturing plus the activity class' depreciation rate. This net rate of return is calculated as the income attributable to the capital input divided by the total stock that generates such income. This is done with information for the manufacturing sector provided by the National Income and Product Accounts (NIPA).

First, the main component of capital income is the Gross Operational Surplus (GOS). However, it is important to mention that this concept includes other factor payments that are not specifically accounted for in the NIPA statistics, for example: the freelance labour income, informal sector and indirect taxes. Therefore, based on Mena (1997) and OECD (2001), it is necessary to extract from the GOS the payment made exclusively to capital. In particular, this is made according to

$$r_t K_t = GOS_t^m - OSLI_t^m - \frac{Y_t^m}{GDP_t} ISR_t - \delta K_t, \tag{1}$$

where r is the capital net rate of return, K is the capital stock, $OSLI$ are other sources of labour income, Y is the value of finished products, GDP is the Gross Domestic Product, ISR is the federal government tax revenue from corporate profits (direct tax), δ is the depreciation rate of the physical capital,[8] m stands for the manufacturing sector and t is a year. All series, except ISR, are available in 1993 constant pesos in the NIPA statistics. The ISR is deflated with the implicit GDP prices index taken from NIPA statistics.

Second, the capital stock that generates such income is calculated according to the perpetual inventories methodology as outlined in OECD (2001); for this calculation, the initial capital stock in the manufacturing sector is proxied for by $K_{88} = \frac{I_{88}}{c+\delta}$, where I_{88} is the gross fixed capital formation in 1988 and c is its average growth rate between 1988 and 2003 (see for example Hall and Jones, 1999).

Remunerations per Hour. Calculated as the total remunerations divided by the total hours worked by the occupied personnel.

Price of Electricity. It is calculated based on electricity expenditure and electricity consumption. It is the price paid per Kw/h consumed.[9]

[5] We do not consider the crisis period 1994-1995 as this could bias our results.

[6] The forthcoming results do not change dramatically if we use Value Added instead (i.e. Output minus Intermediate Materials).

[7] Y is deflated by specific price indices for each activity class; however, the majority of the establishments produce more than a single and homogeneous product, that is, one price index is considered for different establishments with different product ranges in the same activity class.

[8] The depreciation rate is obtained with data from the IC of 1998.

[9] The calculation of electricity price, based on the AIS, has the shortcoming that, for 2003, there are some observations with zeroes for consumption (thousands of Kw/h), making it impossible to calculate the price paid per Kw/h. This problem is solved as follows. For each activity class, the price per Kw/h in 2003 equals the 2002 price plus the annual average increase between 1996-2002. Around 29% of the 205 activity classes report a consumption equal to zero for 2003.

Price of Transport. The AIS does not provide information neither on the main type of transportation used (by air, rail, road or sea) nor on the destination (kilometers) of their products. Hence, given the lack of specific data, we use, as a proxy for this price, the expenditure on transportation of finished products divided by the value of finished products.[10]

3 Recent Developments

This section offers a general analysis of input prices and real unit costs for each factor considered. As mentioned, the present document studies 205 activity classes included in the AIS as a whole (i.e. total sector). However, for this particular section, we analyse in more detail specific subsectors, particularly Machinery & Equipment. To do so, we classify the 205 activity classes into 14 comprehensive groups, based on the North American Industry Classification System (NAICS).

The description and correspondence between both classifications are detailed in Table 1, which presents: *i*) the 9 CMAP subsectors, *ii*) the 14 NAICS groups, and *iii*) how the 9 CMAP subsectors have been reorganised into the 14 NAICS groups. For instance, subsector 3, Lumber & Wood, contains five activity classes (the numbers in parentheses in Table 1); these same five activity classes are reclassified into two NAICS groups, G3 Lumber & Wood and G14 Miscellaneous, with three activity classes going to G3 and the two remaining ones going to G14. Information on the main activities and products in each group can be found in the Appendix.

An obvious path to follow for this research would be to estimate translog cost functions for every CMAP subsector and/or NAICS group; however, as shown, these subsectors and/or groups have relatively few activity classes (i.e. G5 Petroleum & Coal, is composed by only two activity classes), which may lead to non-robust econometric estimates or simply to the impossibility of estimating the translog model. Hence, we do not consider any particular subsectors or group in Section 5. Nonetheless, here we do present information on input prices relevant for the 14 NAICS groups.

3.1 Input Prices and Unit Input Costs

In this section, an overview of: *i*) input prices, and *ii*) unit labour and total costs is presented.

Input Prices

The graphs in Figure 1 show the levels in 2003 (in 1996 pesos) and Table 2 presents the average annual rate of change in real terms during 1996-2003 of input prices for each group and for total manufacturing.

First, as expected, the cost of capital is very similar for all groups. Second, labour remunerations per hour worked show considerable differences amongst groups, for instance, the two groups with the highest remunerations per worked hour in 2003 were G5 Petroleum & Coal and G6 Chemicals, whilst the lowest remunerations were paid by G2 Textile, Apparel, Fur, Leather & Footwear and G3 Lumber & Wood, thus reflecting the prevalent heterogeneity in human capital and/or the existence of differences in the degree of possible rent extractions in each group (e.g. some benefits associated with the presence of unions). Third, there are considerable differences in the costs of electricity amongst groups, which could be explained by the fact that Comisión Federal de Electricidad (henceforth CFE, which is the largest Mexican state-owned

[10] Actually, the forthcoming results hold whether we consider three factors or four.

electricity supplier) follows tariff adjustment rules that differentiate amongst voltage capacities of industrial-use electricity (see CFE, 2006).[11] Finally, the price of transport also shows some disparity amongst groups, with G7 Non-metallic & Glass, G3 Lumber & Wood and G6 Chemicals paying the highest prices per unit of output, whilst the groups paying the lowest prices were G10 Electrical Equipment, Appliances & Components and G13 Computer & Electronic Products.

Table 1: Correspondence: Subsectors (CMAP) and Groups (NAICS)

Subsector		Group		
S1	(38)	G1	(38)	Food, Beverage & Tobacco
S2	(32)	G2	(32)	Textile, Apparel, Fur, Leather & Footwear
S3	(5)	G3	(3)	Lumber & Wood
		G14	(2)	Miscellaneous
S4	(9)	G4	(9)	Paper, Printing, Publishing & Reproduction
S5	(38)	G2	(1)	Textile, Apparel, Fur, Leather & Footwear
		G5	(2)	Petroleum & Coal
		G6	(32)	Chemicals
		G13	(1)	Computer & Electronic Products
		G14	(2)	Miscellaneous
S6	(16)	G7	(16)	Non-metallic & Glass
S7	(7)	G8	(7)	Primary & Fabricated Metal
S8	(57)	G8	(12)	Primary & Fabricated Metal
		G9	(11)	Machinery
		G10	(10)	Electrical Equipment, Appliances & Components
		G11	(7)	Automobiles
		G12	(4)	Other Transportation Equipment
		G13	(12)	Computer & Electronic Products
		G14	(1)	Miscellaneous
S9	(3)	G14	(3)	Miscellaneous

Number of Activity Classes in parentheses

With respect to relative changes in real terms of input prices between 1996 and 2003, it can be seen from Table 2 that the increase in the price of electricity shows the greatest disparity, whilst changes in labour remunerations per hour worked are relatively homogeneous amongst the manufacturing groups.

Hence, this implies that the change in the price of electricity relative to the change in the price of other inputs has been higher in some groups than in others. For instance, and related to the previous paragraph,

[11] For example, high voltage tariffs are adjusted in 59% according to inflation in three sub-indexes of the PPI (namely Machinery and Equipment, Basic Metal, and Other Manufacturing) and in 41% according to fuels inflation (including fuel oil, natural gas, diesel, and coal), while the tariff for low voltage electricity raises in 80% and in 20% according to the inflation in these two groups of price indexes, respectively. Therefore, an establishment whose productive machinery and equipment is more likely to use high voltage electricity (like an establishment in the G11 Automobiles group, for instance) might face both higher electricity prices and annual increases than an establishment that is more likely to consume low voltage electricity (like one in the G1 Food, Beverage & Tobacco group).

in one extreme is G11 Automobiles, with an average increase in the price of electricity of 12.4% per year, and an increase of only 1.5% per year in labour remunerations; in the other extreme is G1 Food, Beverage & Tobacco, with an average decrease of 13% per year in the price of electricity and an average increase in labour remunerations of 2.8% per year. The change in the price of transport has been also somewhat heterogeneous, ranging between -11.7% in G13 Computer & Electronic Products and 8.1% in G12 Other Transportation Equipment.

Figure 1: Levels of Input Prices, 2003

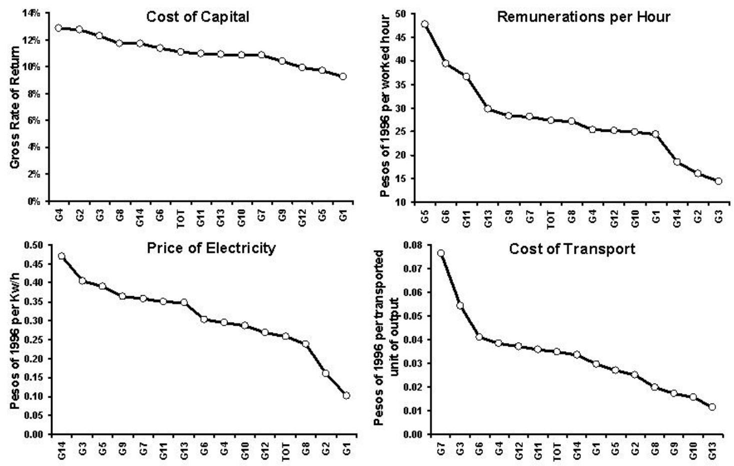


Table 2: Average Annual Change of Input Prices (Percent)

		Remunerations per Hour	Price of Electricity	Price of Transport
G11	Automobiles	1.52	12.39	1.09
G14	Miscellaneous	3.67	7.66	3.11
G7	Non-metallic & Glass	2.42	3.72	2.39
G5	Petroleum & Coal	3.70	2.69	4.46
G4	Paper, Printing, Publishing & Reproduction	1.50	2.47	-1.76
G3	Lumber & Wood	2.00	1.79	-2.40
G10	Electrical Equipment, Appliances & Components	2.05	0.34	-2.80
G8	Primary & Fabricated Metal	0.48	0.13	-6.35
G6	Chemicals	2.30	-0.05	-1.50
G9	Machinery	1.04	-0.50	-0.83
G13	Computer & Electronic Products	0.69	-0.64	-11.69
G12	Other Transportation Equipment	4.04	-4.47	8.13
G2	Textile, Apparel, Fur, Leather & Footwear	1.73	-11.75	3.20
G1	Food, Beverage & Tobacco	2.78	-12.99	1.26
G15	Total Manufacturing	2.12	-0.26	0.02

Groups ranked with respect to average annual change of price of electricity

Unit Input Costs

In this subsection we present the real labour and total costs per unit of product. Unit labour costs are calculated as the expenditure made in labour input divided by the value of finished products, both amounts expressed in constant prices of 1993.[12] In a similar fashion, a measure of average costs is calculated, where the numerator is equal to the sum of the expenditure on the four inputs herein considered. Indices for unit labour costs and for average costs are calculated for each group and for the whole sector, normalizing to 100 the cost in 1996. The value of the indices in 2003, as well as their average annual percent change during 1996-2003, are shown in Table 3.[13]

First, regarding the unit labour costs, G14 Miscellaneous and G2 Textile, Apparel, Fur, Leather & Footwear have shown minor decreases, whilst G13 Computer & Electronic Products is the group that clearly has had the greatest decrease in its labour costs. Second, with respect to average real costs, it is found that the groups with the major decreases are G13 Computer & Electronic Products and G10 Electrical Equipment, Appliances and Components, whilst those showing the minor decreases are G14 Miscellaneous and G2 Textile, Apparel, Fur, Leather & Footwear.

Table 3: Costs per Unit of Product (1996=100)

		Labour		Total	
		2003	Avg. Annual Change	2003	Avg. Annual Change
G13	Computer & Electronic Products	33.5	-14.5	36.9	-13.3
G10	Electrical Equipment, Appl. & Comp.	66.3	-5.7	64.7	-6.0
G5	Petroleum & Coal	73.4	-4.3	69.7	-5.0
G9	Machinery	74.8	-4.1	73.9	-4.2
G1	Food, Beverage & Tobacco	74.9	-4.0	73.5	-4.3
G8	Primary & Fabricated Metal	75.3	-4.0	73.8	-4.3
G11	Automobiles	77.5	-3.6	85.2	-2.3
G3	Lumber & Wood	78.9	-3.3	66.8	-5.6
G4	Paper, Printing, Publishing & Rep.	83.0	-2.6	71.6	-4.7
G7	Non-metallic & Glass	86.6	-2.0	78.2	-3.5
G12	Other Transportation Equipment	91.2	-1.3	77.5	-3.6
G6	Chemicals	92.6	-1.1	89.5	-1.6
G14	Miscellaneous	94.7	-0.8	90.5	-1.4
G2	Textile, Apparel, Leather & Footwear	104.0	0.6	92.9	-1.0
G15	Total Manufacturing	79.5	-3.2	77.0	-3.7

Groups ranked with respect to the average annual change of unit labour costs

[12] Specific price indices for each input and for the product are used. Moreover, when instead of Y we use value added, the ranking does not change drastically.

[13] The calculation of these costs is based on the measure of unit labour costs of Mexican manufacturing at constant prices elaborated by INEGI, who defines this indicator as the cost in real terms of the labour required to generate one unit of product.

4 The Translog Cost Function

4.1 The Model

The cost function has as its arguments the level of output and input prices. In particular, the translog cost function could be considered as a second-order Taylor's series approximation in logarithms to an arbitrary cost function (see Christensen *et al.*, 1973). The more general specification of the translog cost function imposes no prior restriction on the production structure, that is, it does not impose, ex ante, neutrality, homotheticity, homogeneity, constant returns to scale, or unitary elasticities of substitution; in fact, it allows to test these alternative production configurations.

The translog cost function can be written as

$$\ln C = \alpha_0 + \sum_{i=1} \alpha_i \ln P_i + \frac{1}{2} \sum_{i=1} \sum_{j=1} \gamma_{ij} \ln P_i \ln P_j + \alpha_y \ln Y + \frac{1}{2} \gamma_{yy} (\ln Y)^2 + \sum_{i=1} \gamma_{iy} \ln P_i \ln Y, \qquad (2)$$

where $i, j = 1, ..., N$ index the N different inputs considered and $\gamma_{ij} = \gamma_{ji}$, C is total cost, Y is output and the P_i's are the prices of the factor inputs. For a cost function to be well behaved it must be homogeneous of degree one in prices, implying that, for a fixed level of output, total cost must increase proportionally when all prices increase proportionally. Thus, the following restrictions on equation (2) apply

$$\sum_{i=1} \alpha_i = 1, \qquad (3)$$

$$\sum_{i=1} \gamma_{iy} = 0, \qquad (4)$$

$$\sum_{i=1} \gamma_{ij} = \sum_{j=1} \gamma_{ij} = \sum_{i=1} \sum_{j=1} \gamma_{ij} = 0. \qquad (5)$$

As mentioned, a number of additional parameter restrictions can be imposed on the translog cost function, which implicitly represent the underlying technology. Homotheticity means that the cost function can be written as a separable function in output and factor prices.[14] For homotheticity, it is necessary and sufficient that

$$\gamma_{iy} = 0, \ \forall_i. \qquad (6)$$

The cost function is homogeneous in output if the elasticity of cost with respect to output is constant, this occurs with the following restrictions

$$\gamma_{iy} = 0, \quad \gamma_{yy} = 0, \qquad (7)$$

in this case the degree of homogeneity equals $\frac{1}{\alpha_y}$. There are constant returns to scale (CRS) of the dual production function when, in addition to equation (7)

$$\alpha_y = 1. \qquad (8)$$

[14]With nonhomothetic cost functions their ratios of cost-minimising inputs demands are allowed to depend on the level of output, by contrast, with homothetic functions relative input demands are independent of the level of output.

Ultimately, the translog function becomes a constant returns to scale Cobb-Douglas function if, besides to the previous restrictions, each of the

$$\gamma_{ij} = 0,\ \forall_i. \tag{9}$$

Actually, the elasticities of substitution can all be restricted to unity by the elimination of the second-order terms in the prices from the translog cost function and can be applied to the translog, homothetic, homogeneous and/or the CRS models.

Direct estimation of equation (2) can be carried out. However, gains in efficiency can be obtained if the optimal cost-minimising input demand equations, cost-share equations, are estimated jointly with equation (2). More specifically, with a set of cost-share equations directly related with the translog cost function as implied by duality theory. Following Shepard's Lemma, the derived demand for an input is obtained by partially differentiating the cost function with respect to input prices $\frac{\partial C(\cdot)}{\partial P_i} = Z_i$. Thus, partially differentiating equation (2) and using Shepard's Lemma, such cost-share equations can be obtained

$$\frac{\partial \ln C}{\partial \ln P_i} = \frac{P_i}{C}\frac{\partial C}{\partial P_i} = \frac{P_i X_i}{C} = \alpha_i + \sum_{j=1} \gamma_{ij} \ln P_j + \gamma_{iy} \ln Y, \tag{10}$$

where $\sum_{i=1} P_i X_i = C$. If $S_i \equiv \frac{P_i X_i}{C}$, then $\sum_{i=1} S_i = 1$.

The necessary restrictions given by equations (3), (4), and (5) are imposed to the constraint $\sum_{i=1} S_i = 1$ as well, which also implies that only $N-1$ of the share equations in (10) are linearly independent.

Once the coefficients are estimated, one can construct Allen partial elasticities of substitution between two factors i and j (Uzawa, 1962). These elasticities are crucial to describe the pattern and degree of substitutability and complementarity amongst the factors of production. Basically, they measure the percentage change in factor proportions due to a one-percent change in their relative prices. For the translog this implies

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \quad \text{for } i \neq j. \tag{11}$$

In addition, one can compute own - and cross-price elasticities of factor demand (*ceteris paribus*, how the demand for input i responds with respect to changes in its own price or to changes in the price of input j) as $\eta_{ij} = S_i \sigma_{ij}$, which can be calculated as

$$\begin{aligned} \eta_{ii} &= \frac{\gamma_{ii}}{S_i} + S_i - 1, \\ \eta_{ij} &= \frac{\gamma_{ij}}{S_i} + S_j \quad \text{for } i \neq j. \end{aligned} \tag{12}$$

Based on Hanoch (1975), economies of scale must be evaluated along the expansion path, that is, where factor prices are constant and costs are minimised at every level of output; whereas returns to scale are usually defined along an arbitrary input-mix ray. In fact, if the production function is homothetic, both returns to scale and economies of scale will be the same.

Economies of scale are defined in terms of the relative increase in output resulting from a proportional increase in all inputs. This is expressed as one minus the elasticity of total cost with respect to output

$$\Psi = 1 - \frac{\partial \ln C}{\partial \ln Y}, \tag{13}$$

Ψ is positive for scale economies and negative for diseconomies of scale. Specifically, Ψ is calculated, for the unconstrained specification as $1-(\alpha_y+\gamma_{yy}\ln Y+\sum_i \gamma_{iy}\ln P_i)$, for the homotheticity case as $1-(\alpha_y+\gamma_{yy}\ln Y)$, and for the homogeneity model as $1-\alpha_y$.

4.2 Estimation Strategy

Joint estimation of the cost function and the cost share equations by Full Information Maximum Likelihood is used. As shown by Christensen and Greene (1976), the inclusion of the cost share equations means the addition of degrees of freedom without the addition of any unrestricted regression coefficients, resulting in more efficient parameter estimates. Only $N-1$ share equations are included in the system to avoid singularity problems. For more on this matter, see Barten (1969) and Kmenta and Gilbert (1968).

In this study, the models are estimated after imposing the translog symmetry condition and the constraints for linear homogeneity in factor prices.

Particularly, the considered models are: *i*) an unconstrained translog cost function (A), *ii*) a homothetic cost structure (B), *iii*) a cost structure with homogeneity imposed (C), *iv*) a constant returns to scale structure (D), and *v*) models E, F, G and H that correspond to models A, B, C and D, respectively, with unitary elasticities of substitution imposed in each case. In sum, eight different models are estimated for three different periods (1996, 2000, 2003).

The acceptance of any of the previous structures is determined by likelihood ratios tests, since maximum likelihood estimates of the parameters are obtained. In particular

$$\text{LR test statistic: } -2\left(\Omega_R-\Omega_U\right), \tag{14}$$

where Ω_R and Ω_U are the log likelihood values under the restricted and unconstrained versions, respectively. Equation (14) is distributed asymptotically as a chi-square random variable, with degrees of freedom equal to the number of independent restrictions being imposed.

5 Empirical Results

As argued, it is desirable to start analysing the structure of production of any given economic industry with the least restrictive model possible, in our case a model that allows nonhomotheticity and nonunitary elasticities of substitution. In section 5.1, the results for the total Mexican manufacturing cost function estimation are presented and in section 5.2 scale economies and average costs are assessed.

5.1 Cost Functions for Mexican Manufacturing

This section presents the results of the cost function estimation of the eight models A–H for three years: 1996, 2000 and 2003. Capital, labour, electricity and transport are, respectively, denoted by the K, L, E and T subscripts.

The restrictions of linear homogeneity of degree one in input prices were imposed in all models to ensure that the estimated cost functions represent well-behaved production structures.

For the particular case of 2003, Table 4 and Table 5 present the complete set of parameter estimates with the different models, the former with nonunitary elasticities of substitution (models A-D) and the latter with unitary elasticities of substitution imposed (models E-H). Table 6 presents the estimated parameters corresponding to model A for the 1996 and 2000 data sets.

The estimates point to statistically significant nonhomotheticity involving capital (γ_{KY}) and transport (γ_{TY}) for 2003, transport (γ_{TY}) and labour (γ_{LY}) for 2000, and transport (γ_{TY}) for 1996. Moreover, the substitution parameters γ_{KT} and γ_{LT} are statistically significant for the three years as well. This sheds initial light that neither the homotheticity nor the unitary elasticities of substitution hypotheses are consistent with the data.

As explained, we do need to test for the validity of the restrictions imposed to models B-H; hence, Table 7 shows the likelihood ratio statistics for the three data sets. We can comfortably reject all the hypotheses on parameter restrictions for the three samples at the 5% significance level. Therefore, model A, which allows for nonhomotheticity and nonunitary elasticities of substitution, is the one that better represents the production structure of Mexican manufacturing.

Table 4: Cost Funtion Estimation for All Models with Nonunitary Elasticities of Substitution, 2003

	Translog	Homotheticity	Homogeneity	CRS		Translog	Homotheticity	Homogeneity	CRS
	A	B	C	D		A	B	C	D
α_0	5.213**	3.452**	1.226**	-0.742*	γ_{KK}	0.036*	0.047**	0.044**	0.037*
	(1.114)	(1.203)	(0.422)	(0.325)		(0.016)	(0.016)	(0.015)	(0.015)
α_Y	0.324*	0.498**	0.841**	1.000	γ_{LL}	0.034	0.026	0.026	0.039*
	(0.135)	(0.155)	(0.020)	-		(0.022)	(0.021)	(0.019)	(0.019)
α_K	0.466**	0.337**	0.324**	0.316**	γ_{EE}	0.001	-0.003	-0.003	-0.003
	(0.082)	(0.098)	(0.089)	(0.091)		(0.006)	(0.006)	(0.006)	(0.006)
α_L	0.542**	0.389**	0.389**	0.314**	γ_{TT}	0.061**	0.058**	0.059**	0.061**
	(0.137)	(0.124)	(0.113)	(0.112)		(0.002)	(0.002)	(0.002)	(0.002)
α_E	-0.097	-0.035	-0.027	0.006	γ_{KL}	-0.009	-0.016	-0.015	-0.013
	(0.074)	(0.057)	(0.057)	(0.059)		(0.016)	(0.016)	(0.014)	(0.015)
α_T	0.089*	0.310**	0.314**	0.363**	γ_{KE}	-0.007	-0.006	-0.005	-0.003
	(0.040)	(0.029)	(0.031)	(0.029)		(0.007)	(0.006)	(0.006)	(0.007)
γ_{YY}	0.034**	0.026*			γ_{KT}	-0.020**	-0.024**	-0.024**	-0.020**
	(0.009)	(0.011)				(0.005)	(0.005)	(0.005)	(0.005)
γ_{KY}	-0.012**				γ_{LE}	0.011	0.017	0.016	0.011
	(0.005)					(0.011)	(0.010)	(0.010)	(0.010)
γ_{LY}	-0.015				γ_{LT}	-0.036**	-0.026**	-0.027**	-0.036**
	(0.008)					(0.005)	(0.005)	(0.005)	(0.005)
γ_{EY}	0.007				γ_{ET}	-0.005	-0.008**	-0.008**	-0.004
	(0.004)					(0.003)	(0.002)	(0.002)	(0.003)
γ_{TY}	0.020**								
	(0.002)								
Rests.	None	3	4	5		None	3	4	5

Standard errors in parenthesis. * or ** indicate statistical significance at 5 or 1 percent, respectively.

Table 5: Cost Funtion Estimation for Models with Unitary Elasticities of Substitution, 2003

	Translog	Homotheticity	Homogeneity	CRS		Translog	Homotheticity	Homogeneity	CRS
	E	F	G	H		E	F	G	H
α_0	5.086**	4.649**	2.079**	-1.730**	γ_{YY}	0.034**	0.026*		
	(1.163)	(1.213)	(0.292)	(0.103)		(0.009)	(0.011)		
α_Y	0.246	0.322	0.721**	1.000	γ_{KY}	-0.012**			
	(0.168)	(0.179)	(0.021)	-		(0.005)			
α_K	0.426**	0.244**	0.244**	0.244**	γ_{LY}	-0.015			
	(0.050)	(0.009)	(0.009)	(0.009)		(0.008)			
α_L	0.707**	0.597**	0.597**	0.595**	γ_{EY}	0.007			
	(0.086)	(0.012)	(0.012)	(0.012)		(0.004)			
α_E	-0.051	0.068**	0.068**	0.070**	γ_{TY}	0.020**			
	(0.050)	(0.007)	(0.007)	(0.007)		(0.002)			
α_T	-0.081*	0.091**	0.091**	0.091**					
	(0.038)	(0.006)	(0.006)	(0.006)					
Rests.	6	9	10	11		6	9	10	11

Standard errors in parenthesis. * or ** indicate statistical significance at 5 or 1 percent, respectively

Table 6: Cost Funtion Estimation for Model A, 2000 and 1996

	2000	1996		2000	1996		2000	1996
α_0	3.460*	4.456**	γ_{KY}	-0.004	-0.007	γ_{TT}	0.062**	0.051**
	(1.355)	(1.491)		(0.005)	(0.007)		(0.002)	(0.002)
α_Y	0.528**	0.400	γ_{LY}	-0.021**	-0.014	γ_{KL}	-0.001	-0.003
	(0.180)	(0.215)		(0.008)	(0.009)		(0.017)	(0.021)
α_K	0.320**	0.440**	γ_{EY}	0.005	0.006	γ_{KE}	0.001	-0.003
	(0.087)	(0.091)		(0.004)	(0.004)		(0.008)	(0.010)
α_L	0.638**	0.529**	γ_{TY}	0.020**	0.016**	γ_{KT}	-0.017**	-0.021**
	(0.126)	(0.117)		(0.002)	(0.002)		(0.005)	(0.005)
α_E	-0.072	-0.068	γ_{KK}	0.018	0.028	γ_{LE}	0.011	0.008
	(0.055)	(0.037)		(0.018)	(0.022)		(0.010)	(0.010)
α_T	0.113**	0.098*	γ_{LL}	0.029	0.024	γ_{LT}	-0.039**	-0.029**
	(0.037)	(0.036)		(0.022)	(0.025)		(0.005)	(0.005)
γ_{YY}	0.023	0.032*	γ_{EE}	-0.007	-0.005	γ_{ET}	-0.006	0.000
	(0.013)	(0.016)		(0.007)	(0.007)		(0.003)	(0.004)

Standard errors in parenthesis. * or ** indicate statistical significance at 5 or 1 percent, respectively.

Table 7: Test Statistics for Restrictions on Model A

	Nonunitary Elasticities of Substitution			Unitary Elasticities of Substitution			
	Homotheticity	Homogeneity	CRS	Translog	Homotheticity	Homogeneity	CRS
# of Restrictions	3	4	5	6	9	10	11
Critical χ^2 (5%)	7.81	9.49	11.07	12.59	16.92	18.31	19.68
χ^2 for 1996	55.66	57.96	124.88	244.27	266.54	270.74	368.04
χ^2 for 2000	83.12	84.07	145.11	292.82	315.34	316.69	434.77
χ^2 for 2003	112.70	119.45	184.23	316.05	362.46	367.41	494.04

As argued, a measure of the ease or difficulty with which one input can substitute for one another is given by the Allen-Uzawa partial elasticities of substitution, which measure the percentage change in factor proportions due to a one percent change in their relative prices. The substitution possibilities prevailing in Mexican manufacturing are presented in Table 8 given by the estimated Allen-Uzawa partial elasticities of substitution.

All partial elasticities indicate the existence of substitution possibilities amongst the other inputs for the three years.[15] Particularly, considering the statistically significant elasticities of Table 8, there exists important substitutability, for the three years, between capital and labour (σ_{KL}), and labour and electricity (σ_{LE}). Also, for 1996, there is significant substitutability between electricity and transport (σ_{ET}), and for 2000 between capital and electricity (σ_{KE}).

Table 8: Allen-Uzawa's Elasticities of Substitution

	1996	2000	2003
σ_{KL}	0.985**	0.992**	0.938**
	(0.115)	(0.0105)	(0.114)
σ_{KE}	0.827	1.053*	0.575
	(0.558)	(0.486)	(0.608)
σ_{KT}	0.180	0.269	0.052
	(0.730)	(0.579)	(0.778)
σ_{LE}	1.298**	1.329**	1.260**
	(0.417)	(0.373)	(0.333)
σ_{LT}	0.222	0.230	0.314
	(0.617)	(0.517)	(0.466)
σ_{ET}	1.016**	-0.107	0.174
	(0.035)	(1.281)	(1.017)

Standard errors in parenthesis

* or ** indicate statistical significance at 5 or 1 percent, respectively

[15] Complementarities seem to arise between electricity and transport (σ_{ET}) only for the year 2000 but this is not statistically significant.

Regarding own- and cross-price elasticities for input demands, these are presented in Table 9, where each element is the elasticity of demand for the input in the row after a price change of the input in the column, for the three studied years. All the own-price elasticities of factor demand, along the main diagonal, are consistent with microeconomic theory and have the correct negative sign. In general terms, we could state that the demand for electricity is, basically, unitary elastic, whilst the other inputs are inelastic.[16]

The cross-price elasticities, off-diagonal terms, contain the same information as the elasticities of substitution in Table 8, but cross-section elasticities are not symmetric since they depend on the input shares. It can be observed, for the three sample periods, that all cross-price elasticities are less than one.

Table 9: Own- and Cross-Price Elasticities for Input Demands

1996	Capital	Labour	Electricity	Transport
Capital	-0.564**	0.509**	0.042	0.013
	(0.098)	(0.138)	(0.040)	(0.054)
Labour	0.354**	-0.436**	0.066	0.016
	(0.105)	(0.125)	(0.496)	(0.047)
Electricity	0.297	0.671*	-1.042**	0.074
	(0.217)	(0.271)	(0.162)	(0.065)
Transport	0.065	0.115**	0.052	-0.231
	(0.263)	(0.050)	(0.035)	(0.543)
2000	Capital	Labour	Electricity	Transport
Capital	-0.665**	0.579**	0.062	0.023
	(0.102)	(0.151)	(0.061)	(0.053)
Labour	0.268*	-0.367**	0.079	0.020
	(0.106)	(0.133)	(0.071)	(0.050)
Electricity	0.285*	0.775**	-1.051**	-0.009
	(0.170)	(0.286)	(0.187)	(0.111)
Transport	0.073	0.134**	-0.006	-0.200
	(0.159)	(0.047)	(0.076)	(0.458)
2003	Capital	Labour	Electricity	Transport
Capital	-0.611**	0.568**	0.039	0.004
	(0.076)	(0.149)	(0.054)	(0.067)
Labour	0.225*	-0.338*	0.085	0.027
	(0.096)	(0.133)	(0.080)	(0.045)
Electricity	0.138	0.763**	-0.916**	0.015
	(0.157)	(0.268)	(0.100)	(0.090)
Transport	0.012	0.190**	0.012	-0.214
	(0.187)	(0.045)	(0.069)	(0.459)

Standard errors in parenthesis

* or ** indicate statistical significance at 5 or 1 percent, respectively

[16] All statistically significant, but transport, at 5% level of significance.

Two are the highest elasticities, both involving labour: $\eta_{KL} > 0.5$ and $\eta_{EL} > 0.67$. The cross-price elasticity between labour and capital, η_{LK}, is 0.35 for 1996 and 0.23 for 2003.[17] In 2000 there is a statistically significant elasticity between electricity and capital ($\eta_{EK} = 0.285$) and in 2003 it decreases to 0.138 but is not significant. The impact that price changes of both electricity and transport have on the other inputs is negligible and is not statistically significant. Such cross-price elasticities are important in order to affect the quantity of a specific input used as these elasticities allow to know in what direction prices should be changed.

5.2 Economies of Scale and Average Costs

By closely following Christensen and Greene (1976), this section studies economies of scale for the three years considered. An estimate of scale economies can be derived for each activity class by evaluating the formulas stated at the end of section 4.1 at the observed level of output and factor prices.

As done by Christensen and Greene (1976), the sample is partitioned into five 'clusters' of activity classes according to output (Cluster 1 is the smallest and Cluster 5 the highest one). In Tables 10 and 11, we present estimates of scale economies for the activity class with the median output in each cluster for the six models that allow such analysis.[18] Before commenting some main results, it is worth mentioning that the estimates in all models were statistically significant.

Table 10: Estimated Scale Economies for Models with Non-Unitary Elasticities of Substitution

	Translog (A)			Homotheticity (B)			Homogeneity (C)		
Size Cluster	1996	2000	2003	1996	2000	2003	1996	2000	2003
Cluster 1	0.293**	0.313**	0.372**	0.182**	0.179**	0.196**	0.157**	0.157**	0.159**
	(0.031)	(0.033)	(0.028)	(0.030)	(0.028)	(0.030)	(0.020)	(0.019)	(0.020)
Cluster 2	0.280**	0.345**	0.321**	0.159**	0.161**	0.160**	0.157**	0.157**	0.159**
	(0.025)	(0.029)	(0.022)	(0.021)	(0.018)	(0.022)	(0.020)	(0.019)	(0.020)
Cluster 3	0.275**	0.284**	0.318**	0.146**	0.152**	0.146**	0.157**	0.157**	0.159**
	(0.029)	(0.025)	(0.024)	(0.023)	(0.018)	(0.021)	(0.020)	(0.019)	(0.020)
Cluster 4	0.242**	0.296**	0.271**	0.132**	0.143**	0.124**	0.157**	0.157**	0.159**
	(0.032)	(0.029)	(0.025)	(0.029)	(0.022)	(0.023)	(0.020)	(0.019)	(0.020)
Cluster 5	0.181**	0.254**	0.271**	0.113*	0.129**	0.096**	0.157**	0.157**	0.159**
	(0.044)	(0.036)	(0.034)	(0.042)	(0.031)	(0.030)	(0.020)	(0.019)	(0.020)

Standard errors in parenthesis. * or ** indicate statistical significance at 5 or 1 percent, respectively

As known, the estimates of scale economies for the homogeneous models (C & G), are constant at all levels of output. What is interesting to note is that, from 1996 to 2003, economies of scale increased.[19]

With respect to the other models that allow scale economies to change with output (A, B, E, F), we observe that, for the three years analysed, scale economies diminish as the size of activity class increases (except for Model E in 2003). In addition, the estimates are somehow similar between models A, E and F,

[17] The elasticities η_{KL}, η_{EL} and η_{LK} are statistically significant.
[18] The results do not change considerably if we use the highest output or the mean output.
[19] Model C: from 0.157 to 0.159; Model G: from 0.22 to 0.28.

with Model B presenting smaller estimates. In fact, there is no considerable difference between the estimates amongst the five clusters in each model. For instance, based on the 2003 results, the estimate for Cluster 1 in Model A, which is the one that was accepted as a final model in the previous section, is 0.37 whilst for Cluster 5 is 0.27. It is also observed that for Model A, economies of scale increased between 1996 and 2003 for the five clusters.

Table 11: Estimated Scale Economies for Models with Unitary Elasticities of Substitution

	Translog (E)			Homotheticity (F)			Homogeneity (G)		
Size Clusters	1996	2000	2003	1996	2000	2003	1996	2000	2003
Cluster 1	0.292**	0.315**	0.271**	0.262**	0.288**	0.320**	0.222**	0.259**	0.279**
	(0.046)	(0.042)	(0.029)	(0.048)	(0.043)	(0.031)	(0.022)	(0.021)	(0.021)
Cluster 2	0.269**	0.314**	0.297**	0.228**	0.264**	0.277**	0.222**	0.259**	0.279**
	(0.024)	(0.024)	(0.020)	(0.026)	(0.023)	(0.021)	(0.022)	(0.021)	(0.021)
Cluster 3	0.249**	0.272**	0.292**	0.208**	0.252**	0.261**	0.222**	0.259**	0.279**
	(0.024)	(0.021)	(0.023)	(0.024)	(0.021)	(0.021)	(0.022)	(0.021)	(0.021)
Cluster 4	0.221**	0.271**	0.261**	0.188**	0.238**	0.235**	0.222**	0.259**	0.279**
	(0.031)	(0.028)	(0.027)	(0.032)	(0.027)	(0.025)	(0.022)	(0.021)	(0.021)
Cluster 5	0.161**	0.232**	0.286**	0.158**	0.219**	0.202**	0.222**	0.259**	0.279**
	(0.054)	(0.045)	(0.037)	(0.054)	(0.044)	(0.035)	(0.022)	(0.021)	(0.021)

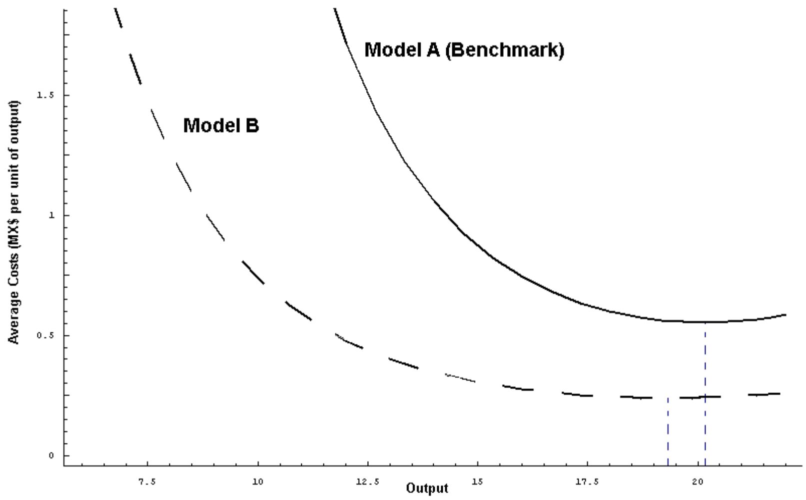
Standard errors in parenthesis. * or ** indicate statistical significance at 5 or 1 percent, respectively

For expositional purposes only, we derive the estimated average cost curves for the manufacturing sector based on the best two estimated models: A & B. The average cost curve is obtained by evaluating the average cost function for a range of outputs whilst holding the factor prices fixed at the sample means. Figure 2 shows the curves for these two models contained in Tables 10 for 2003 with the size distribution of activity classes beneath the horizontal axis together with their output share. It can be seen that there is an initial indication that no activity class was close to the minimum average cost point (indicated by the dotted vertical lines).

In Figure 3, we plot the average cost curves for 1996, 2000 and 2003, for model A, with the size distribution of activity classes beneath the horizontal axis for every year with their output share. It is observed that the 1996 average cost curve is flatter than the other two curves, a shape consistent with our previous finding of lower scale economies in 1996. Also, as shown by the box below the cost curves, activity classes are located to the left of the minima cost points (those for 1996 and 2000 are not shown in Figure 3). It should be mentioned that being away from the minimum point on the average cost curve does not necessarily mean that activity classes are operating sub-optimally: at the minimum point we know that Average Cost=Marginal Cost and so activity classes would be making zero excess profits; at output levels to the left of this point, then activity classes would be making excess profits.[20] Hence, this may be more of an issue about competition (market structure) in the output goods market.

[20] When the price is driven down, an activity class would increase its output to where average cost of production is lower, until it reaches the minimum-average cost point.

Figure 2: Average Cost Curves Models A & B, 2003

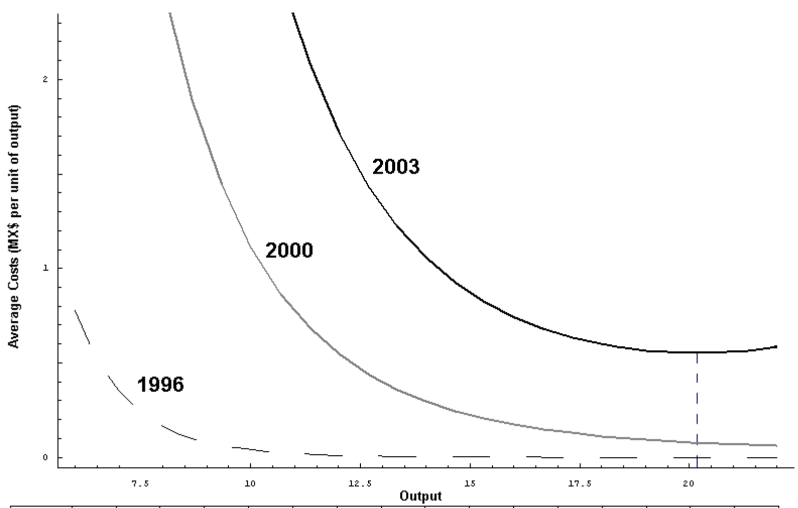


2003	1	1	1	1	7	17	27	66	42	32	8	2				
	(0.0002)	(0.0003)	(0.001)	(0.002)	(0.05)	(0.38)	(1.49)	(10.09)	(17.18)	(33.41)	(19.19)	(18.21)				

Size Distribution of Activity Classes

Note: Numbers in parentheses indicate output share.

Figure 3: Average Cost Curves for Model A: 1996, 2000 & 2003



1996		1		2	4	17	36	69	46	23	6	1				
		(0.0004)		(0.01)	(0.05)	(0.52)	(2.72)	(13.56)	(24.47)	(29.29)	(17.72)	(11.67)				
2000		1		1	6	16	28	59	51	34	7	1	1			
		(0.0004)		(0.003)	(0.05)	(0.35)	(1.71)	(8.66)	(18.96)	(33.65)	(17.05)	(5.13)	(14.44)			
2003	1	1	1	1	7	17	27	66	42	32	8	2				
	(0.0002)	(0.0003)	(0.001)	(0.002)	(0.05)	(0.38)	(1.49)	(10.09)	(17.18)	(33.41)	(19.19)	(18.21)				

Size Distribution of Activity Classes

Note: The 1996 and 2000 average cost curves reach their minima to the right of the 2003 curve minimum.
Numbers in parentheses indicate output share.

In addition, it can be seen that the minimum point of the cost curve in 2003 is located upwards and to the left (northwest) of the other two minima points (not shown in Figure 3), implying that less output is required to achieve the lowest cost region; moreover, this may provide some evidence that the average cost curve is shifting up because both fixed costs and variable costs are rising.

As mentioned in Section 1, one should be cautious about concluding on whether or not there might be an authentic negative technical change or a genuine increase in costs present in Figure 3 as diverse factors (i.e. measurement issues, data aggregation, lack of data and use of proxies, variable construction, possible changes in the composition/structure of the industry, etc.) could be exerting some biases in the results in the short time span we have.[21] Lastly, as argued, we are considering as a benchmark the total manufacturing sector, which might not be the most precise way to determine whether or not there is technical change when we are analysing rather different units of study (for example, meat processing vs dental equipment and instruments manufacturing); ideally, the analysis outlined here should be applied to specific manufacturing groups using data at the establishment level. Thus, it would be somehow odd to conclude that activity classes have become technically less efficient over the years.

To statistically formalise the existence or not of scale economies, we follow Christensen and Greene (1976) and present Table 12, which displays the number of activity classes and the share of total output in 1996, 2000 and 2003, located in each of three regions of the cost curve for the whole sector depicted in Figure 3: *i*) statistically significant scale economies; *ii*) no significant scale economies or diseconomies; and *iii*) statistically significant scale diseconomies.[22] Once again, our benchmark (the total manufacturing sector) may not be the best one as the diversity in manufacturing processes and products inherent in our data set is quite important.

It can be observed that, in 1996, there were eleven activity classes that represented 37.0% of total output showing no statistically significant scale economies or diseconomies. This number decreased to nil activity classes in 2000 and to two activity classes that represented 18.0% of total output in 2003.[23] Furthermore, no activity class showed statistically significant scale diseconomies in any of the three samples studied.

Table 12: Ranges of Significant Scale Economies

	1996	2000	2003
Significant scale economies			
Number of activity classes	194	205	203
Share of total output	0.63	1.000	0.82
No significant economies or diseconomies			
Number of activity classes	11	0	2
Share of total output	0.37	0	0.18
Significant scale diseconomies			
Number of activity classes	0	0	0
Share of total output	0	0	0

To conclude this section and to provide, perhaps, a more valid cost comparison between activity classes in each manufacturing group, we present Table 13, where entries in the table show the average cost for representative activity classes of a particular size for each group as a percentage of the minimum point on the average cost curve for the whole manufacturing sector in 2003.

In particular, the representative activity classes are classified into three different levels/categories of output (proxy for size) in the 14 manufacturing groups. The three levels of output considered within each

[21] Christensen and Greene (1976) had data points for 1955 and 1970.

[22] Based on Christensen and Greene (1976), the confidence level used to determine statistically significant scale economies is set at the 95% level; any point on the average cost curve with a corresponding Ψ which is less than 1.96 times its standard error is considered to be in the "flat" region (with no significant scale economies or diseconomies).

[23] These two activity classes were: *i*) production, assembly and repair of computers, and *ii*) production and assembly of automobiles and trucks.

of the 14 manufacturing groups are as follows: *i*) Small category– output average of the activity classes that jointly represent an output share, in its respective manufacturing group, close to 15%;[24] *ii*) Medium category– the activity class in each group with the median level of output; and, *iii*) Large category– the activity class with the highest level of output in each group.[25]

Table 13: Average Cost Estimates for Manufacturing Groups in 2003
(As a Percentage of the Manufacturing Sector's Minimum Average Cost)

		Activity Class Size			Diff. (p.p.) in 2003	Diff. (p.p.) in 1996
	Group	Small	Medium	Large	Small−Large	Small−Large
G1	Food, Beverage & Tobacco	170.0	149.8	122.6	47.5	139.1
G2	Textile, Apparel, Leather	205.5	186.2	141.3	64.3	184.2
G3	Lumber & Wood	201.9	191.8	172.6	29.3	52.9
G4	Paper, Printing, Publishing & Rep.	173.5	164.8	130.0	43.5	121.5
G5	Petroleum & Coal	173.4	N/A	151.6	21.8	38.4
G6	Chemicals	173.0	156.3	124.3	48.7	150.3
G7	Non-metallic & Glass	189.3	173.3	129.3	60.1	167.3
G8	Primary & Fabricated Metal	172.5	155.2	122.0	50.5	153.2
G9	Machinery	189.8	180.6	141.2	48.7	107.0
G10	Electrical Equip., Appl. & Comp.	166.2	153.5	129.5	36.7	96.7
G11	Automobiles	150.0	151.9	109.1	40.9	127.1
G12	Other Transportation Equip.	239.7	213.5	183.1	56.6	114.7
G13	Computer & Electronic Products	179.7	183.4	112.8	66.9	197.0
G14	Miscellaneous	193.1	174.5	165.6	27.6	92.4
	Difference (p.p.) in 2003	89.7	63.7	74.0		
	Difference (p.p.) in 1996	242.7	165.0	255.1		

Consistent with our previous results the average cost decreases as activity class size increases in all manufacturing groups. In addition, the lowest percentages are found in the Large-category for G11 Automobiles and G13 Computer & Electronic Products, whilst the highest percentages correspond to G12 Other Transportation Equipment in both the Small and Medium categories and to G2 Textiles in the Small Category. This could suggest the influence or the existence of important fixed costs for particular groups.

Of more interest is to look at the cost differences: *i*) between size categories within groups and *ii*) between manufacturing groups within size categories. From the last rows of Table 13, which shows for each of the three size categories, the difference between the largest percentage and the lowest percentage across the 14 manufacturing groups, for 2003 and 1996, the lowest spread is encountered in the Medium-category. The last columns of Table 13, show the difference between activity class size (Small − Large) in each manufacturing group, for 2003 and 1996, respectively.[26] Moreover, it is observed that all size categories and all groups experienced reductions in their respective spreads; in other words, it seems that there was less dispersion in

[24] If we consider the smallest activity class instead, the intuition of the results does not change at all.
[25] In fact, with the exception of G1, the largest activity class in each group represents more than 15% of its group's output.
[26] Actually, quite similar results –ranking– are encountered for the initial year: 1996.

the scale of production in 2003 than in 1996.[27] This is in some way expected given that, as discussed, the minimum average cost for the total sector has moved upwards and to the left, and compared to 1996, more activity classes were closer to the minimum cost region in 2003.

6 Summary

This paper estimated transcendental logarithmic cost functions for Mexico's manufacturing sector, the factors of production included were capital, labour, electricity and transport. Data from the AIS for the 1996-2003 period was used.

By taking this route, a less restrictive production function was allowed as various tests for homotheticity, homogeneity, constant returns to scale and unitary elasticities of substitution were considered. Moreover, following Christensen and Greene (1976), particular focus was put on estimating economies of scale and average costs.

It was noted that labour remunerations per hour worked and the price of electricity present strong differences amongst the manufacturing groups. Regarding relative changes in real terms of input prices between 1996 and 2003, the increases in the prices of electricity and transport presented the greatest disparity, whilst changes in labour remunerations per hour worked were somehow homogeneous amongst the manufacturing groups.

In relation to the translog cost estimation, it was found that, after imposing several conditions into the basic equation, the best model for Mexican manufacturing would be one characterised by nonhomotheticity and nonunitary elasticities of substitution.

The calculated partial elasticities revealed that there are alternatives of substitution between the considered inputs. All the own-price elasticities of factor demand presented the correct negative sign; in particular, the demand for electricity is unitary elastic, followed by capital, labour and transport (inelastic). The obtained cross-price elasticities were less than one.

With respect to the estimation of scale economies and average costs in this sector, two interesting results are found. First, for the models in which scale economies are allowed to vary with output, it is observed that, in general, scale economies decrease with the level of output, implying gains in efficiency for activity classes with higher output. Second, there is a generalised increase in the estimated scale economies over time for any given level of output.

Consistent with the findings on scale economies, average costs decrease as activity class size increases. At the specific manufacturing group level, it is observed that in general, G2 Textiles, G3 Lumber & Wood and G12 Other Transportation Equipment show the higher average costs, whilst G11 Automobiles is the group with the lowest average costs. Although the differences in average costs between small and large activity classes have decreased in all the manufacturing groups in 2003, with respect to 1996, some important differentials remain in some groups. Still, there is indication of convergence and/or homogeneity between groups in the manufacturing industry.

As argued in the paper, there might be some biases in the results encountered due to different issues related to the time span, data availability, data aggregation, etc.

[27] In consequence, from 1996 to 2003, the average costs have decreased in every entry of Table 13.

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Appendix

Groups Composition

Each manufacturing group is composed mainly by the following activities and products:

G1. Food, Beverage & Tobacco: Meat processing, dairy, cereals, bakery, tortilla, sugar industry, sweets, coffee, alcohol, beverages, tobacco.

G2. Textile, Apparel, Fur, Leather & Footwear: Fibers, fabrics, clothes, leather goods, fur, shoes.

G3. Lumber & Wood: Wood processing, construction supplies and containers.

G4. Paper, Printing, Publishing & Reproduction: Manufacturing of paper products, prints, newspapers, books, magazines.

G5. Petroleum & Coal: Coke, mineral oils and additives.

G6. Chemicals: Basic oil chemistry, fertilizers, insecticides, resins, paints, pharmaceutical, perfumes, tires, rubber, tubing, plastic house supplies.

G7. Non-metallic & Glass: Construction materials, glass, cement, concrete, ceramic.

G8. Primary & Fabricated Metal: Iron, steel, alloys, aluminium, heaters.

G9. Machinery: Tractors, machinery, agricultural supplies, pumps, filters.

G10. Electrical Equipment, Appliances & Components: stoves, ovens, refrigerators, washing machines, heaters, boilers, batteries, bulbs, automotive electric components.

G11. Automobiles: Production, assembly and repair of automobiles, trucks, engines, motors, transmissions, suspensions, brakes.

G12. Other Transportation Equipment: Production, assembly and repair of navigation ships and boats, railroad equipment, motorcycles, bicycles and parts.

G13. Computer & Electronic Products: Computers, radios, TV sets, photography, medical equipment, measurement equipment, lenses, typewriters, cassettes, discs.

G14. Miscellaneous: Jewelry, toys, office supplies, mattresses, furniture.