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Abstract

Most New Keynesian models are derived under the assumption that inflation is equal to zero in the steady-state and yet most central banks around the world have inflation targets that are greater than such a number. In this paper we consider the open economy (welfare) implications of non-zero steady-state inflation rates both in the domestic and foreign economies. We show that higher inflation rates in the steady-state, both in the domestic and foreign economies, reduce welfare in the domestic economy. We also show that high domestic inflation rates in the steady-state have a more adverse effect on domestic welfare than high foreign inflation rates.

Keywords: Optimal Monetary Policy, Trend Inflation, Open Economy Macroeconomics.
JEL Classification:E32,E52.

Resumen

La mayoría de los modelos NeoKeynesianos se desarrollan bajo el supuesto que la inflación es igual a cero en el estado estacionario, aunque en la realidad la mayoría de los bancos centrales con esquemas de metas de inflación, tienen metas de dicha variables distintas a cero. En este documento, consideramos las implicaciones (de bienestar) de inflaciones domesticas y foráneas distintas a cero en el estado estacionario en un modelo de economía abierta. Demostramos que valores de inflación más altos, tanto en la economía domestica como en la foránea, reducen el bienestar en la economía domestica. También demostramos que valores altos de inflación domestica en el estado estacionario, tienen un efecto más adverso en el bienestar que valores altos de inflación foránea.

Palabras Clave: Política Monetaria Optima, Inflación, Macroeconomía de Economías Abiertas.

1 Introduction and motivation

New Keynesian models have become useful tools for the evaluation of monetary policy. For example Clarida et al (1999) and Walsh (2003) demonstrate how these models can be used to answer many important issues related to monetary policy such as the evaluation of monetary policy rules, commitment versus discretion, credibility, inflation bias, etc. However, this analysis assumes that inflation is equal to zero in the steady-state. Recently a group of authors have relaxed this assumption and have examined the implications of non-zero steady-state inflation[1] in these (closed) economy models (for example Ascari (2004), Kiley (2004), Ascari and Ropele (2006), and Blake and Fernandez-Corugedo (2006)). Their motivation stems from the fact that for most countries, the level of inflation targeted by the monetary policy authority is different from zero.[2] These authors show that deviations from zero inflation in the steady-state affect the dynamic properties of the model and an implication of this result is that it can be shown that higher levels of inflation in the steady-state may lead to increased inflation and output volatility. Indeed, Kiley (2004) motivates his paper by showing that for the G-7, higher levels of moderate inflation rates are associated with higher inflation volatility. Table 1 below extends Kiley's sample by including a number of OECD countries and also shows that higher inflation rates are associated with higher inflation volatility.

Almost at the same time, recent research has attempted to augment the simple closed economy New Keynesian models to capture the open economy aspect of many economies (for example McCallum and Nelson (2000), Corsetti and Pesenti (2001), Clarida, Gali and Gertler (2002), Gali and Monacelli (2005), etc to name but a few). These models, like their closed economy counterparts, have also become useful tools for the examination of monetary policy. However, none of these open economy models allow for positive inflation rates in the steady-state. Therefore a number of important policy questions related to non-zero steady-state inflation rates remain unanswered in these models: do higher levels of steady-state inflation (both domestic and foreign) result in more volatile output and inflation as closed economy models predict? Should domestic policy makers care equally about non-zero

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[1]Non-zero steady-state inflation is referred to trend inflation in this literature. In this paper we will use both terms.

[2]In this paper, as is common in most of the literature that examines trend inflation, we do not seek to answer the question of what is the optimal inflation rate the policy maker should target. We simply assume that the level of inflation can be different from zero in the steady state. A possible explanation for why inflation may be different from zero in the steady-state is that inflation can be measured with errors.

steady-state domestic and foreign inflation rates? Do low steady-state foreign inflation rates result in lower (domestic and CPI) inflation and output volatility?

In this paper we seek to answer these questions. We show that higher inflation rates in the steady-state, both in the domestic and foreign economies, reduce welfare (proxied by the variances of output and inflation) in the domestic economy. We also show that high domestic inflation rates in the steady-state have a more adverse effect on domestic welfare than high foreign inflation rates. This result suggests that, although importing low inflation rates from the rest of the world (due to globalization or better policy elsewhere, say) help to improve welfare in the domestic economy, it is paramount for the domestic economy to follow sensible monetary policy to achieve higher welfare. Indeed, these theoretical results appear to be broadly consistent with some of thestylised facts observed in Table 1. We can see clearly in Table 1 that for most countries there exist two "transition periods" from high to low inflation: the first period is around the early/mid 1980s (Belgium, Canada, Denmark, Finland, France, Germany, Holland, Ireland, Korea, Luxembourg, Switzerland and the US) whereas the second period is around the early 1990s (Australia, Iceland, Italy, New Zealand, Portugal, Spain, Sweden, the UK) [3] This may suggest is that the period of "low inflation", defined as anual inflation rates below 5 per cent appears to coincide for many countries that have strong trade links and perhaps suggests that importing low inflation from neighbouring countries helped reduce inflation in the domestic economy. At the same time, both periods also coincide with two periods thought to be important in the "fight against inflation": for example the Volcker disinflation in the US in the early 1980s and the implementation of inflation targeting in New Zealand (1991), the UK (1992), Sweden (1993) and Spain (1994)). Therefore, the periods of low inflation would appear to be both consistent with a "commitment " to fighting inflation whilst at the same time also being consistent for a number of countries with importing low inflation from abroad.

In this paper we present a very stylised Open Economy New Keynesian model, based on Clarida et al (2002) - CGG henceforth, where we assume that the level of inflation in the steady-state in the domestic and foreign economies can be different from zero. The model can be used to demonstrate that lower inflation in the domestic economy as well as in the rest of the world could theoretically account for the period of stability observed in the 1990s and 2000s.

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[3]The exceptions are Japan whose transition occurred in 1980, Greece whose transition occurred around 1997, and Mexico whose transition occurred around 2000.

Country	High inflation period (Inflation above 5%)		Low inflation period (Inflation below 5%)	
	Average (period)	Std Deviation (period)	Average (period)	Std Deviation (period)
Canada	9.28 (1973−1983)	1.95 (1973−1983)	2.82 (1984−2005)	1.46 (1984−2005)
France	10.27 (1973−1985)	2.48 (1973−1985)	2.09 (1986−2005)	0.82 (1986−2005)
Germany	5.17 (1973−1982)	1.45 (1973−1982)	2.04 (1982−2005)	1.29 (1982−2005)
Italy	11.88 (1973−1992)	5.50 (1973−1992)	2.93 (1993−2005)	1.13 (1993−2005)
Japan	9.97 (1973−1980)	6.11 (1973−1980)	1.07 (1981−2005)	1.48 (1981−2005)
UK	10.16 (1973−1991)	5.75 (1973−1991)	2.63 (1992−2005)	1.56 (1992−2005)
US	8.75 (1973−1983)	2.69 (1973−1983)	3.11 (1984−2005)	0.99 (1984−2005)
Australia	9.66 (1973−1990)	2.95 (1973−1990)	2.49 (1991−2005)	1.33 (1991−2005)
Belgium	7.88 (1973−1984)	2.66 (1973−1984)	2.21 (1985−2005)	0.95 (1985−2005)
Denmark	10.11 (1973−1984)	2.38 (1973−1984)	2.64 (19854−2005)	1.07 (1985−2005)
Finland	10.77 (1973−1985)	3.81 (1973−1985)	2.47 (1986−2005)	1.90 (1986−2005)
Greece	16.44 (1973−1997)	5.41 (1973−1997)	2.44 (1998−2005)	2.64 (1998−2005)
Iceland	35.67 (1973−1991)	19.96 (1973−1991)	3.29 (1992−2005)	1.55 (1992−2005)
Ireland	12.74 (1973−1985)	5.10 (1973−1985)	2.97 (1986−2005)	1.17 (1986−2005)
Korea	16.82 (1973−1982)	8.31 (1973−1982)	4.48 (1984−2005)	2.19 (1984−2005)
Luxembourg	7.44 (1973−1984)	2.3 (1973−1984)	1.97 (1985−2005)	1.32 (1985−2005)
Mexico	36.05 (1973−2000)	34.21 (1973−2000)	4.56 (2001−2005)	0.88 (2001−2005)
Holland	7.05 (1973−1982)	2.09 (1973−1982)	2.12 (1983−2005)	1.09 (1983−2005)
New Zealand	12.05 (1973−1990)	4.09 (1973−1990)	1.99 (1991−2005)	0.95 (1991−2005)
Portugal	17.08 (1973−1994)	7.57 (1973−1994)	3.03 (1995−2005)	0.74 (1995−2005)
Spain	12.13 (1973−1992)	5.49 (1973−1992)	3.32 (1993−2005)	0.95 (1993−2005)
Sweden	8.66 (1973−1991)	2.54 (1973−1991)	1.52 (1992−2005)	1.32 (1992−2005)
Switzerland	5.06 (1972−1982)	2.97 (1972−1982)	1.99 (1991−2005)	1.60 (1991−2005)

Table 1: Inflation and the variance of inflation for a number of selected OECD countries

The paper is structured as follows: Section 2 presents the model. Section 3 examines the model's equilibrium, its steady-state and its dynamics. Section 4 considers monetary policy. Section 5 concludes. An appendix describes some of the algebraic equations in the text.

2 The model

As our model is a simplified version of the model of CGG (2002) we only present its main equations. When the model is log-linearised (in section 3) we show how trend inflation enters the model and how it affects its dynamics.

There are two countries, home and foreign that differ in size but are otherwise symmetric. The home country (H) has a mass of households ($1 - \gamma$) and the foreign country (F) has a mass γ. Both countries are assumed to have the same preferences, technology and market structure, though shocks may be imperfectly correlated. Each economy comprises households, firms and the policy maker. Each is now defined in turn.

2.1 Households

Households in both countries have access to a complete set of Arrow-Debreu securities which can be traded both domestically and internationally and seek to maximise the following utility function:

$$E_t \sum_{k=0}^{\infty} \beta^k \frac{C_{t+k}^{1-\rho}}{1-\rho} - \frac{N_{t+k}^{1+\sigma}}{1+\sigma} \tag{1}$$

where N are hours of labour and C is a composite consumption index defined by:

$$C_t = \frac{\left(C_t^h\right)^{1-\gamma} \left(C_t^f\right)^{\gamma}}{(1-\gamma)^{1-\gamma} (\gamma)^{\gamma}} = \frac{\left(C_t^h\right)^{1-\gamma} \left(C_t^f\right)^{\gamma}}{k} \tag{2}$$

where h denotes domestic goods and f are foreign goods. The associated price indices are:

$$P_t = \frac{\left(P_t^h\right)^{1-\gamma} \left(P_t^f\right)^{\gamma}}{k} = \frac{P_t^h S_t^{\gamma}}{k} \tag{3}$$

where $S_t \equiv \frac{P_t^f}{P_t^h}$ are the terms of trade.[4] The maximisation of (1) is subject to the following budget constraint:

$$P_t C_t + E_t D_{t+1} Q_{t,t+1} \leq D_t + W_t N_t - T_t + \Gamma_t \tag{4}$$

where D is the payoff of the portfolio held by the household, Q is a corresponding stochastic discount factor, W are nominal wages, T are lump sum taxes and Γ are firms' profits.

The first order necessary conditions for consumption allocation and intertemporal optimisation are:

$$C_t^h = (1 - \gamma) \left(\frac{P^h}{P_t} \right)^{-1} C_t \tag{5}$$

$$C_t^f = \gamma \left(\frac{P^f}{P_t} \right)^{-1} C_t \tag{6}$$

$$\beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} = E_t Q_{t,t+1} \left(\frac{P_{t+1}}{P_t} \right). \tag{7}$$

Defining $R_t^{-1} = E_t Q_{t,t+1}$ as the gross nominal yield on a one-period discount bond and taking expectations yields the Euler equation:

$$\beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{P_t}{P_{t+1}} \right) R_t = 1. \tag{8}$$

The first order condition for labour supply is:

$$C_t^\rho N_t^\sigma = \frac{W_t}{P_t}. \tag{9}$$

A symmetric set of first order conditions holds for citizens of the foreign country. The intertemporal efficiency condition can be written as

$$\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\rho} \left(\frac{P_t^*}{P_{t+1}^*} \right) \left(\frac{\xi_t}{\xi_{t+1}} \right) = Q_{t,t+1} \tag{10}$$

where ξ denotes the nominal exchange rate between the two countries. The Law of One Price is assumed to hold, $P_t = \xi_t P_t^*$, which together with (7) and (10) implies that

$$C_t = C_t^*. \tag{11}$$

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[4]Note that the assumption that the composite consumption index are Cobb-Douglas implies in this model that the trade balance is equal to zero if it begins in equilibrium (see Corsetti and Pesenti (2001), CGG (2002), Gali and Monacelli (2005), and below).

2.2 Firms

There are two types of firms in each economy. There is a continuum of intermediate goods firms, each producing a differentiated material input. Final good producers then combine these inputs into output, which they sell to households. The production function for each of these final good producers is:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{12}$$

where Y denotes aggregate output and $Y(i)$ denotes the input produced by an intermediate goods firm, i. Both variables are expressed in per capita terms and are thus normalised by population size $(1 - \gamma)$. Profit maximisation yields the following demand equations for each of the intermediate inputs:

$$Y_t(i) = \left(\frac{P_t^h(i)}{P_t^h} \right)^{-\varepsilon} Y_t \tag{13}$$

as well as the domestic price index:

$$P_t^h = \left(\int_0^1 P_t^h(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \tag{14}$$

It is assumed that the number of final goods firms within each country equals the number of households.

Intermediate goods producers are monopolistic competitors that produce differentiated products and set nominal prices on a staggered basis. These firms access the following technology:

$$Y_t(i) = A_t N_t(i) \tag{15}$$

where A is a technology shock component which is common to all firms. The real marginal cost (expressed in terms of domestic prices) is:

$$MC_t = \frac{W_t}{P_t^h A_t} = \frac{W_t}{P_t S_t^{-\gamma} A_t k}. \tag{16}$$

Firms face nominal price rigidities a la Calvo (1983). They face a constant probability, θ, of not being able to change prices each period. The optimal pricing decisions for firms of

this type is:

$$\frac{\overline{P}_t^h(j)}{P_t^h} = X_t^h = \frac{\varepsilon}{\varepsilon - 1}\frac{\Psi_t}{\Omega_t} \tag{17}$$

$$\Psi_t = E_t \sum_{k=0}^{\infty} (\theta\beta)^k \Delta_{t,t+k} Y_{t+k} MC_{t+k} \left(\frac{P_t^h}{P_{t+j}^h}\right)^{-\varepsilon} \tag{18}$$

$$\Omega_t = E_t \sum_{k=0}^{\infty} (\theta\beta)^k \Delta_{t,t+k} Y_{t+k} \left(\frac{P_t^h}{P_{t+j}^h}\right)^{1-\varepsilon} \tag{19}$$

where $\overline{P}_t^h(j)$ denotes the optimal price set by one of the optimising firms, X^h is the dispersion in domestic prices and Δ is a discount factor. One can think of Ψ_t and Ω_t as representing present and future marginal revenues and marginal costs respectively. With Calvo prices the evolution of domestic prices follows:

$$\left(P_t^h\right)^{1-\varepsilon} = \theta \left(P_{t-1}^h\right)^{1-\varepsilon} + (1-\theta)\left(\overline{P}_t^h\right)^{1-\varepsilon}. \tag{20}$$

What remains is to define the preferences for the domestic (and foreign) policy maker. We do this in section 4 where we discuss monetary policy in more detail.

3 Equilibrium, the steady-state and the dynamics of the model

Goods market clearing in the domestic and foreign countries are given by:

$$(1-\gamma)Y_t = (1-\gamma)C_t^h + \gamma C_t^{h,*} \tag{21}$$

$$\gamma Y_t^* = (1-\gamma)C_t^f + \gamma C_t^{f,*} \tag{22}$$

where $C_t^{h,*}$ denotes consumption by foreign consumers of the home good, whilst $C_t^{f,*}$ denotes foreign consumption of the foreign good. Combining (5), (6), their equivalent expressions for the foreign economy, and the Law of One Price we have:

$$\frac{\xi_t P_t^*}{P_t} = 1$$

so that the CPI based real exchange rate is unity. The trade balance is always in equilibrium:

$$P_t^h Y_t = P_t C_t \tag{23}$$

$$P_t^{f,*} Y_t^* = P_t^* C_t^*. \tag{24}$$

Combining (3) with (23) yields:

$$Y_t = \frac{\left(\frac{P_t^f}{P_t^h}\right)^\gamma C_t}{(1-\gamma)^{(1-\gamma)}\gamma^\gamma} = \frac{S_t^\gamma C_t}{(1-\gamma)^{(1-\gamma)}\gamma^\gamma} \tag{25}$$

where

$$\frac{P_t^f}{P_t^h} = S_t = \frac{Y_t}{Y_t^*}. \tag{26}$$

Equations (25), (26) and (8) determine aggregate demand. Note that we can re-write the Euler equation to resemble an IS curve:

$$\beta E_t \left(\frac{Y_{t+1}S_{t+1}^{-\gamma}}{Y_t S_t^{-\gamma}}\right)^{-\rho} \left(\frac{P_t}{P_{t+1}}\right) R_t = 1. \tag{27}$$

On the supply side, the aggregated production function is:

$$N_t = \frac{Y_t}{A_t}$$

which together with (16) plus (9) yield:

$$MC_t = (Y_t)^{\rho+\sigma+\gamma(1-\rho)} (Y_t^*)^{\gamma(\rho-1)} A_t^{-(1+\sigma)} k^{\rho-1}. \tag{28}$$

Note that with logarithmic preferences in consumption, marginal costs will not depend on foreign output:

$$MC_t = (Y_t)^{1+\sigma} A_t^{-(1+\sigma)}. \tag{29}$$

To close the model we need to specify monetary policy. We take up this issue in section 4.

3.1 The steady-state

The steady-state of the model is very similar to the closed economy steady-state (see Ascari (2004) and Blake and Fernandez-Corugedo (2006) for the expressions for the closed economy steady-state). In the open economy case, the appropriate expressions for the steady-state are (see Appendix A for their derivation) are given in Table 2.

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Home	Foreign
$N = \frac{Y}{A}$	$N^* = \frac{Y^*}{A^*}$
$N = \left[\frac{W}{P} C^{-\rho}\right]^{1/\sigma}$	$N^* = \left[\frac{W^*}{P^*}(C^*)^{-\rho}\right]^{1/\sigma}$
$G_C = (R^r \beta k)^{1/\rho}$	$G_C^* = ((R^r)^* \beta)^{1/\rho}$
$Y = \frac{S^\gamma C}{k}$	$Y^* = \frac{(S^*)^{\gamma-1}C^*}{k}$
$X^h = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1-\theta\beta(\Pi^h)^{\varepsilon-1}}{1-\theta\beta(\Pi^h)^\varepsilon}\right) MC$	$X^{f,*} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{1-\theta\beta(\Pi^{f,*})^{\varepsilon-1}}{1-\theta\beta(\Pi^{f,*})^\varepsilon}\right) MC^*$
$X^h = \left(\frac{1-\theta(\Pi^h)^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}$	$X^{f,*} = \left(\frac{1-\theta(\Pi^{f,*})^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}$
$MC = \frac{(Y)^{\rho+\sigma+\gamma(1-\rho)}(Y^*)^{\gamma(\rho-1)}A^{-(1+\sigma)}}{k^{1-\rho}}$	$MC^* = \frac{(Y^*)^{(1-\gamma)(1-\rho)+\sigma+\rho}(Y)^{(1-\gamma)(\rho-1)}(A^*)^{-(1+\sigma)}}{k^{1-\rho}}$
$Y = A\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(X^h\right)^{1+\varepsilon\sigma_n}\left(\frac{1-\theta\beta(\Pi^h)^\varepsilon}{1-\theta\beta(\Pi^h)^{\varepsilon-1}}\right)\right]^{\frac{1}{1+\sigma_n}}$	$Y^* = A^*\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(X^{f,*}\right)^{1+\varepsilon\sigma_n}\left(\frac{1-\theta\beta(\Pi^{f,*})^\varepsilon}{1-\theta\beta(\Pi^{f,*})^{\varepsilon-1}}\right)\right]^{\frac{1}{1+\sigma_n}}$
$S = \frac{Y}{Y^*}$	

Table 2: Steady-state expressions

In table 2, Π^h denotes the inflation rate of the domestic economy in the steady-state, $\Pi^{f,*}$ the steady-state inflation rate in the rest of the world and R^r denotes the real interest rate.[5] Note that only *domestic* inflation rates affect *domestic* variables in the steady-state and, similarly, only foreign inflation rates affect foreign variables, that is, trend inflation in one country does not affect the variables of the other country. The reason for this result is due to the terms of trade, which compensate the effect of foreign output reductions brought about by higher foreign trend inflation on domestic output (see Appendix A). Therefore, the expressions for domestic output are the same as those for the closed economy model of Ascari (2004) and Blake and Fernandez-Corugedo (2006).

3.2 Dynamics

We now present the model in log-linear form. In the remainder of the paper, lower case letters will denote log deviations of a variable from its deterministic steady-state, that is $x_t = X_t - X$. Linearisation of (27) yields the IS curve:

$$y_t \simeq E_t y_{t+1} - \frac{1}{\rho} E_t \left(r_t - \pi_{t+1}\right) - \gamma E_t \Delta s_{t+1}. \tag{30}$$

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[5]In order to generate non-zero steady-state inflation rates in the domestic and foreign economies, the policy makers in both countries must inject money at a rate of growth that is consistent with the inflation rate in the steady-state (see for example, Ascari and Ropele (2006) and Blake and Fernandez-Corugedo (2006)).

Using the definition of CPI inflation $\pi_t \simeq (1 - \gamma) \pi_t^h + \gamma \pi_t^f \simeq \pi_t^h + \gamma \Delta s_t$ implies that

$$y_t \simeq E_t y_{t+1} - \frac{1}{\rho} E_t \left(r_t - \pi_{t+1}^h \right) - \gamma \left(\frac{\rho - 1}{\rho} \right) E_t \Delta s_{t+1}. \tag{31}$$

The terms of trade enter the IS curve through two channels. First, the resource constraint states that domestic output is equal to consumption plus terms of trade, and second, in the Euler equation, what matters to consumers is CPI and not domestic inflation. Since there is a relationship between CPI inflation, domestic inflation and the terms of trade, these enter through this second channel. These two channels cancel when households have log-preferences for utility. Note that, as in closed economy models, steady-state inflation does not enter the IS curve. There is an equivalent IS expression for the foreign country (see (42) below).

The New Keynesian Phillips Curve for the domestic economy in the face of trend inflation is given by:

$$\pi_t^h \simeq \underbrace{\lambda \left(\Pi^h \right) \left(\Pi^h - 1 \right) \left(\rho - 1 \right)}_{\text{``output''}} y_t + \underbrace{\lambda \left( \Pi^h \right) \left( \Pi^h - 1 \right) \gamma \rho}_{\text{``terms of trade''}} s_t + \underbrace{\lambda \left(\Pi^h \right) \left(1 - \theta \beta \left(\Pi^h \right)^\varepsilon \right)}_{\text{``marginal cost''}} mc_t$$

$$+ \beta \Pi^h E_t \pi_{t+1}^h + \underbrace{\lambda \left(\Pi^h \right) \left(\Pi^h - 1 \right)}_{\text{``contract inflation''}} \Omega_t + \upsilon_t \tag{32}$$

$$\Omega_t \simeq \left(1 - \theta \beta \left(\Pi^h \right)^{\varepsilon - 1} \right) \gamma \rho s_t + \left(1 - \theta \beta \left(\Pi^h \right)^{\varepsilon - 1} \right) \left(1 - \rho \right) y_t \tag{33}$$

$$+ \left(\varepsilon - 1 \right) \theta \beta \left(\Pi^h \right)^{\varepsilon - 1} E_t \pi_{t+1}^h + \theta \beta \left(\Pi^h \right)^{\varepsilon - 1} E_t \Omega_{t+1} \tag{34}$$

$$\upsilon_t = \phi \upsilon_{t-1} + \upsilon_t \tag{35}$$

where[6]

$$\lambda \left(\Pi^h \right) = \frac{1 - \theta \left(\Pi^h \right)^{\varepsilon - 1}}{\theta \left(\Pi^h \right)^{\varepsilon - 1}}. \tag{36}$$

There is an equivalent expression for the foreign economy (see below). Ω_t is a term that discounts marginal revenues which was termed "contract inflation" by Currie and Levine (1993). For further reference, note that the domestic output gap enters the NKPC through three channels, these have been marked "output", "terms of trade" and "marginal cost"; foreign output gaps will enter through those last two channels. The first two channels are not present in models where inflation is equal to zero in the steady-state (note that the

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[6]The derivation of this equation is very similar to the derivation for the closed economy (see Ascari and Ropele (2006) and Blake and Fernandez-Corugedo (2006) for more).

coefficients are zero when $\Pi^h = 1$). To understand how these terms come about consider the optimal pricing decision of firms re-expressed as:[7]

$$X_t^h = \frac{\varepsilon}{\varepsilon - 1} \frac{\Psi_t}{\Omega_t} = \frac{\left[U_c(t) Y_t MC_t + \theta \beta E_t \left(\Pi_{t+1}^h \right)^\varepsilon \Psi_{t+1} \right]}{U_c(t) Y_t + \theta \beta E_t \left(\Pi_{t+1}^h \right)^{\varepsilon-1} \Omega_{t+1}} \tag{37}$$

where $U_c(t)$ represents the marginal utility of consumption at time t. The key terms are the discount factors for Ψ_t and Ω_t ($\theta \beta E_t \left(\Pi_{t+1}^h \right)^\varepsilon$ and $\theta \beta E_t \left(\Pi_{t+1}^h \right)^{\varepsilon-1}$ respectively). These discount factors "increase" as trend inflation increases: as inflation is higher, future marginal revenues and costs are eroded more rapidly and so are profits (note that marginal revenues are eroded more rapidly than marginal costs since $\varepsilon > 1$). Firms, must therefore be "more forward-looking" in the sense that they will discount future streams of revenues and costs higher.

By substituting the expressions for the domestic and foreign marginal costs plus the terms of trade ($s_t \simeq y_t - y_t^*$) we reduce the model to

$$y_t \simeq E_t y_{t+1} - \frac{1}{\rho_0} E_t \left(r_t - \pi_{t+1}^h \right) + \frac{\kappa_0}{\rho_0} E_t y_{t+1}^* - \frac{\kappa_0}{\rho_0} y_t^* \tag{38}$$

$$\pi_t^h \simeq C_1 y_t + C_2 y_t^* - C_3 a_t + C_4 \Omega_t + C_5 E_t \pi_{t+1}^h + \upsilon_t \tag{39}$$

$$\Omega_t \simeq C_6 y_t - C_7 y_t^* + C_8 E_t \pi_{t+1}^h + C_9 E_t \Omega_{t+1} \tag{40}$$

$$\upsilon_t = \phi \upsilon_{t-1} + \upsilon_t \tag{41}$$

$$y_t^* \simeq E_t y_{t+1}^* - \frac{1}{\rho_0^*} E_t \left(r_t^* - \pi_{t+1}^{f,*} \right) + \frac{\kappa_0^*}{\rho_0^*} E_t y_{t+1} - \frac{\kappa_0^*}{\rho_0^*} y_t \tag{42}$$

$$\pi_t^{f,*} \simeq C_1^* y_t^* + C_2^* y_t - C_3^* a_t^* + C_4^* \Omega_t^* + C_5^* E_t \pi_{t+1}^{f,*} + \upsilon_t^* \tag{43}$$

$$\Omega_t^* \simeq C_6^* y_t^* - C_7^* y_t + C_8^* E_t \pi_{t+1}^{f,*} + C_9^* E_t \Omega_{t+1}^* \tag{44}$$

$$\upsilon_t^* = \phi^* \upsilon_{t-1}^* + \upsilon_t^* \tag{45}$$

where

---

[7]The discussion here closely follows Ascari and Ropele (2006).

$$
\begin{aligned}
C_1 &= \lambda\left(\Pi^h\right)\left[\mu\left(\Pi^h\right)\kappa - \left(\Pi^h - 1\right)\kappa_1\right], & C_1^* &= \lambda\left(\Pi^{f,*}\right)\left[\mu\left(\Pi^{f,*}\right)\kappa^* - \left(\Pi^{f,*} - 1\right)\kappa_1^*\right], \\
C_2 &= \lambda\left(\Pi^h\right)\left[\mu\left(\Pi^h\right)\kappa_0 + \left(\Pi^h - 1\right)\rho\gamma\right], & C_2^* &= \lambda\left(\Pi^{f,*}\right)\left[\mu\left(\Pi^{f,*}\right)\kappa_0^* + \left(\Pi^{f,*} - 1\right)\rho\left(1 - \gamma\right)\right], \\
C_3 &= \lambda\left(\Pi^h\right)\mu\left(\Pi^h\right)\left(1 + \sigma\right), & C_3^* &= \lambda\left(\Pi^{f,*}\right)\mu\left(\Pi^{f,*}\right)\left(1 + \sigma\right), \\
C_4 &= \lambda\left(\Pi^h\right)\left(\Pi^h - 1\right), & C_4^* &= \lambda\left(\Pi^{f,*}\right)\left(\Pi^{f,*} - 1\right), \\
C_5 &= \beta\Pi^h, & C_5^* &= \beta\Pi^{f,*}, \\
C_6 &= \left(1 - \tau\left(\Pi^h\right)/\Pi^h\right)\kappa_1, & C_6^* &= \left(1 - \tau\left(\Pi^{f,*}\right)/\Pi^{f,*}\right)\kappa_1^*, \\
C_7 &= \left(1 - \tau\left(\Pi^h\right)/\Pi^h\right)\gamma\rho, & C_7^* &= \left(1 - \tau\left(\Pi^{f,*}\right)/\Pi^{f,*}\right)\left(1 - \gamma\right)\rho, \\
C_8 &= \left(\varepsilon - 1\right)\tau\left(\Pi^h\right)/\Pi^h, & C_8^* &= \left(\varepsilon - 1\right)\tau\left(\Pi^{f,*}\right)/\Pi^{f,*}, \\
C_9 &= \tau\left(\Pi^h\right)/\Pi^h & C_9^* &= \tau\left(\Pi^{f,*}\right)/\Pi^{f,*}
\end{aligned}
$$

and where $\mu\left(\Pi^j\right) = \left(1 - \tau\left(\Pi^j\right)\right)$, $\tau\left(\Pi^j\right) = \theta\beta\left(\Pi^j\right)^\varepsilon$, $j = h; f, *$, $\rho_0 = \left(\rho - \kappa_0\right), \kappa = \rho + \sigma - \gamma\left(\rho - 1\right) = \rho + \sigma - \kappa_0$, $\kappa_0 = \gamma\left(\rho - 1\right), \kappa_1 = \gamma\rho - \left(\rho - 1\right)$, $\rho_0^* = \left(\rho - \kappa_0^*\right)$, $\kappa^* = \left(\sigma + \rho - \kappa_0^*\right), \kappa_0^* = \left(1 - \gamma\right)\left(\rho - 1\right), \kappa_1^* = \left(1 - \gamma\right)\rho - \left(\rho - 1\right), \lambda\left(\Pi^{f,*}\right) = \frac{1 - \theta\left(\Pi^{f,*}\right)^{\varepsilon - 1}}{\theta\left(\Pi^{f,*}\right)^{\varepsilon - 1}}$. As (38)-(44) demonstrate, both domestic and foreign output matter in the model in both the IS and NKPCs. Equation (39) is a generalisation of CGG (2002) for if $\Pi^h = 1$ (39) reduces to:

$$
\pi_t^h \simeq \delta\kappa y_t + \delta\kappa_0 y_t^* - \delta\left(1 + \sigma\right)a_t + \beta E_t\pi_{t+1}^h + \upsilon_t
$$

where $\delta = \frac{\left(1 - \theta\right)\left(1 - \theta\beta\right)}{\theta}$.

The importance of trend inflation for the dynamics of the NKPC is shown in Figure 1 which plots the key term $\lambda\left(\Pi^j\right)$, $j = h; f, *$ that enters in all variables dated at t in the NKPC. We see that $\lambda\left(\Pi^j\right)$ is a decreasing function of Π^j and that the elasticity of substitution between goods is also an important parameter (recall that these two parameters act as a kind of discount factor for the future streams for marginal revenues and costs of firms): the rate of decrease in $\lambda\left(\Pi^j\right)$ as trend inflation increases, is greater the higher the elasticity of substitution. And since $\lambda\left(\Pi^j\right)$ only affects variables dated at t, then the coefficients on those variables in the Phillips Curve decrease as trend inflation increases (this represents the discounting effect previously discussed). This is a key insight and will drive the results in the next section. This is because the "controllability" of our model depends on the coefficients of the variables dated at time t and in particular on the coefficient of output in the Phillips Curve.

There is another interesting implication of trend inflation. In CGG (2002), where there is no trend inflation, foreign output enters through the marginal cost channel. However, when preferences are logarithmic, foreign output vanishes from the model (see equations (29) and (42)). In our model, this is not the case because whilst in the IS curve foreign output vanishes, in the NKPC it does not (it vanishes in the marginal cost term but not in

β	θ	ρ	σ	γ	ε	ϕ, ϕ^*	σ_u, σ_{u^*}	$cov(u, u^*)$
0.99	0.75	2	2.5	0.25	6	0.5	0.1	0.001

Table 3: Parameter values

other terms). To see this, note that when $\rho = 1$ we have:

$$C_2 = \gamma \lambda \left(\Pi^h \right) \left(\Pi^h - 1 \right) \quad C_2^* = (1 - \gamma) \lambda \left(\Pi^{f,*} \right) \left(\Pi^{f,*} - 1 \right).$$

Therefore, unlike CGG (2002), where $\Pi^h = \Pi^{f,*} = 1$, foreign output enters the domestic Phillips Curve, and domestic output enters the foreign Phillips Curve. This is due to the discounting factors discussed previously.

3.3 Model Calibration

Before we examine monetary policy we present the calibration of the model. The parameter values used are given in table 3 and are standard in the literature (see eg McCallum and Nelson (2000), Pappa (2004), Gali and Monacelli (2005), etc). We assume for simplicity that the persistence of domestic and foreign cost push shocks is the same. As we are interested in monetary policy, we abstract from technology shocks and assume that $C_3 = C_3^* = 0$.

4 Discretionary Monetary Policy

We now examine how trend inflation affects monetary policy. We closely follow CGG (1999, 2002) and Ascari and Ropele (2006). To compare results with CGG (2002) we consider monetary policy under discretion, paying attention to the impact that domestic and foreign trend inflation rates have on the variances of inflation and the output gap (which can be thought of as proxies for welfare). Because most central banks around the world do not target domestic inflation but CPI inflation, we will also consider discretionary monetary policy under CPI targeting.

4.1 Domestic inflation targeting

Although inflation targeting central banks usually target CPI inflation, we first consider domestic inflation targeting to compare results with CGG (2002). In this case the policy maker targets domestic inflation, π_t^h. Moreover, as considered by CGG (2002), we assume

that *the domestic economy takes foreign variables as given*. However, because foreign output affects domestic inflation, it is necessary to specify foreign monetary policy. We assume for simplicity that both domestic and foreign policy makers take each other's policies as given and in particular that foreign monetary policy is undertaken under discretion and that the foreign policy maker ignores events in the domestic economy (it takes foreign variables as given). The solution for the foreign country is:

$$y_t^* = -\frac{(C_1^* + C_4^* C_6^*)}{\chi^f} \pi_t^{f,*} \tag{46}$$

where χ^f denotes the weight given by the policy maker to output stabilization. Since the foreign policy maker is assumed to ignore the domestic cost push shocks, the solution of the optimisation problem for output, inflation and the interest rate expressed in terms of foreign shocks is given by $y_t^* = \Xi_y^d v_t^*$, $\pi_t^{f,*} = \Xi_\pi^d v_t^*$ and $r_t^* = \Xi_r^d v_t^*$ where Ξ_y^d, Ξ_π^d and Ξ_r^d are expressions given in Appendix B.[8][9]

We finally assume that the loss function for the domestic policy maker is given by[10]

$$L_t^h = \frac{1}{2} \left[\chi \left(y_t\right)^2 + \left(\pi_t^h\right)^2 \right]. \tag{47}$$

The domestic policy maker optimises (47) subject to the IS and Phillips Curves for the domestic and foreign economies. The parameter χ determines how much weight policy makers give to output. If $\chi = 0$ the policy maker is assumed to follow strict inflation targeting.

Here we consider a problem where the policy maker reoptimises every period and thus takes expectations as given. As in CGG, (1999, 2002) the Central Bank chooses y_t and π_t^h to minimise (47) subject to the NKPC re-defined as

$$\pi_t^h = Y\left(\Pi^h\right) y_t + \beta \Pi^h E_t \pi_{t+1}^h + v_t + V\left(\Pi^h, \Pi^{f,*}\right) v_t^* + f_t + F_t \tag{48}$$

---

[8]One can think of this set-up as one where the foreign policy maker is the leader and the domestic economy the follower (see Blake and Kirsanova (2006)). To assume otherwise will imply a complicated yet interesting policy problem which is beyond the aim of this paper.

[9]Note that this solution is equivalent to the solution for the closed economy problem in the face of trend inflation.

[10]This loss function, which is not microfounded, is chosen to compare results with CGG (1999) and Ascari and Ropele (2006). Note further that since both terms entering the loss function enter as logarithmic deviations from their deterministic steady-state, we are implicitly assuming that the policy maker has an inflation target that is equal to the level of inflation in the steady-state.

where f denotes all other variables dated t, F denote all other variables dated $t+1$ and where

$$Y\left(\Pi^h\right) = \left(C_1 + C_4 C_6\right), \tag{49}$$

$$V\left(\Pi^h, \Pi^{f,*}\right) = -\left(C_4 C_7 - C_2\right) \Xi_y^d. \tag{50}$$

Whilst (49) depends on domestic trend inflation, (50) is a function of domestic and foreign trend inflation rates (where the foreign trend rates appear through the term Ξ_y^d).[11]

Anticipating some of the discussion that follows, Figure 2 plots the values of $Y\left(\Pi^h\right)$ and $V\left(\Pi^h, \Pi^{f,*}\right)$ as functions of both domestic and foreign trend inflation rates. We also perform sensitivity analysis on $V\left(\Pi^h, \Pi^{f,*}\right)$ to show how changes in χ^f affect this multiplier. Figure 2 shows that the values of both $Y\left(\Pi^h\right)$ and $V\left(\Pi^h, \Pi^{f,*}\right)$ decrease in absolute value for higher values of domestic trend inflation and that foreign trend inflation also decreases the absolute value of $V\left(\Pi^h, \Pi^{f,*}\right)$ (for moderate levels of χ^f). When $C_2 = C_2^* = C_7 = C_7^* = 0$, as was assumed by CGG (2002), foreign inflation and foreign shocks do not matter.

The problem for the policy maker is to minimise (47) subject to (48). The solution is:

$$y_t = -\frac{Y\left(\Pi^h\right)}{\chi} \pi_t^h. \tag{51}$$

Thus, as in closed (CGG (1999) and Ascari and Ropele (2006)) and open economy (CGG (2002)) models, the solution is to "lean against the wind" such that as inflation increases, the policy maker reduces the domestic output gap. As we saw in Figure 2, $Y\left(\Pi^h\right)$ decreases as domestic trend inflation increases. As in the closed economy model with trend inflation, see Ascari and Ropele (2006), the degree of "aggressiveness" with which the output gap responds to inflation along the optimal path decreases with higher domestic trend inflation. This implies that the policy maker will care less about inflation and more about output. This is because (as Ascari and Ropele (2006) argue) with higher trend inflation, the gain in reduced inflation per unit of output loss decreases. Thus, with higher trend inflation, the more domestic and foreign cost push shocks are passed onto inflation and less to output. To better understand this point, we derive the analytical solutions for output and inflation in

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[11]In the closed economy where $\gamma = 0$, we have the expressions obtained by Ascari and Ropele (2006):

$$Y\left(\Pi^h\right) = \frac{\left[\beta\Pi^h - \tau\left(\Pi^h\right)\right]\left[\mu\left(\Pi^h\right)\left(\rho + \sigma\right) + \frac{\left(\Pi^h - 1\right)\left(\rho - 1\right)\tau\left(\Pi^h\right)}{\Pi^h}\right]}{\tau\left(\Pi^h\right)}, V\left(\Pi^h, \Pi^*\right) = 0.$$

terms of the fundamental shocks hitting the economy, v_t and v_t^*. This will also allow us to derive the variances of output and inflation. Substitute (51) into (48) to yield:

$$y_t = y_v v_t + y_{v^*} v_t^* \tag{52}$$

$$\pi_t^h = \pi_v^h v_t + \pi_{v^*}^h v_t^* \tag{53}$$

where

$$y_v = \left(\frac{\chi(C_5\phi - 1) - C_1 Y(\Pi^h)}{Y(\Pi^h)} + \frac{C_4(C_6 - \phi C_8)}{1 - \phi C_9} \right)^{-1} \qquad y_{v^*} = \Xi_y^d \left(C_2 - \frac{C_4 C_7}{1 - \phi^* C_9} \right) y_v$$

$$\pi_v^h = -\frac{\chi y_v}{Y(\Pi^h)} \qquad\qquad\qquad\qquad\qquad \pi_{v^*}^h = -\frac{\chi y_{v^*}}{Y(\Pi^h)}.$$

Equations (52) and (53) imply that the policy maker responds to both domestic and foreign cost-push shocks. Both $y_v < 0$ and $\pi_v^h > 0$ are expressions similar to those one would obtain in a closed economy model. Note that these two coefficients do not depend on foreign trend inflation but depend on open economy parameters, as C_1 and C_6 show. Both y_{v^*} and $\pi_{v^*}^h$ depend on both foreign trend inflation (through Ξ_y^d) and open economy parameters, as C_1, C_2, C_6 and C_7 show. Figure 3 shows how these coefficients are affected by domestic and foreign trend inflation as well as by χ^f. The top four figures show the response of these coefficients to changes in domestic trend inflation and the bottom four the response of these coefficients to foreign trend inflation. As in the closed economy case, $\frac{\partial y_v}{\partial \Pi^h} < 0$ and $\frac{\partial \pi_v^h}{\partial \Pi^h} > 0$. The reason for this result was given above: as trend inflation increases, the reduction in the output gap needed to reduce inflation needs to increase; monetary policy is less effective in reducing inflation as trend inflation increases. Thus, the optimal response for monetary policy is to be "increasingly cautious and passive" since "low values of inflation variability can be obtained only at the expense of great output variability" (Ascari and Ropele, page 16).

Similar arguments can be used to explain the response of output and inflation to foreign cost-push shocks given by y_{v^*} and $\pi_{v^*}^h$. A foreign cost push shock decreases foreign output but increases foreign inflation. The increase in foreign inflation, through the terms of trade, leads to an increase in domestic output as domestic and foreign consumers substitute foreign goods for domestic ones. Thus two channels then affect domestic inflation. In the first channel, the fall in foreign output exerts (a direct) downward pressure on domestic inflation (since $C_2 > 0$). In the second channel domestic output increases because, one the on hand, foreign output falls (this is true when $\sigma > 1$, see CGG (2002)) and, on the other, foreign prices

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increase, thereby resulting in upward pressure on domestic inflation (since $C_1 > 0$). The first channel appears to dominate (as is the case in CGG (2002)). Note further that as domestic trend inflation increases, the absolute values of y_{v*} and π_{v*}^h fall (ie $\left|\frac{\partial y_{v*}}{\partial \Pi^h}\right| < 0$, $\left|\frac{\partial \pi_{v*}^h}{\partial \Pi^h}\right| < 0$). This is because, as before, the controllability of the foreign variables falls as domestic trend inflation increases (C_1 and C_2 fall). Higher values of foreign trend inflation have a similar impact, that is $\left|\frac{\partial y_{v*}}{\partial \Pi^{f,*}}\right| < 0$, $\left|\frac{\partial \pi_{v*}^h}{\partial \Pi^{f,*}}\right| < 0$. This is because as foreign trend inflation increases, it is optimal for foreign policy makers to have output respond less to foreign shocks. As foreign output responds less to these shocks, the impact of foreign cost-push shocks on domestic inflation falls.

Figure 4 plots the variances of output and domestic inflation for different values of domestic and foreign trend inflation as well as for different values of χ and χ^f. These variances were calculated using (52) and (53). Figure 4 has two columns: the first column evaluates the impact of domestic trend inflation and the policy preference parameters χ and χ^f on the variances of output and domestic inflation. The second column evaluates the impact of foreign trend inflation and the policy preference parameters χ and χ^f on the variances of output and domestic inflation. In all diagrams, the value of χ is gradually increased from 0 to 0.5, leading to the familiar inflation and output volatility trade-off (known as Taylor frontiers):[12] points on the north-west of each plotted line represent lower values of χ relative to points in the south-east. Each column has three diagrams: as we move from the top to the bottom one we increase the value of χ^f from 0.05 to 0.25 to 0.5; that is, we make the foreign policy maker care more about output gap stabilisation.

The most striking result emanating from Figure 4 is that as domestic trend inflation increases, the frontiers get-worse in the sense that they move to the north east and the volatility of both output and inflation increases. The impact of foreign trend inflation is not noticeable (the frontiers also move in a north-easterly direction that is not visible in the diagram) nor are changes to the preferences of the foreign policy maker. The intuition for these results is the same as that one for the closed economy: as trend inflation increases, for a given level of inflation volatility, higher output volatility is needed to affect inflation. Thus, the message emanating from Figure 4 is clear: when the domestic policy maker cares about domestic inflation and output, it is best to have low steady-state domestic inflation rates; neither foreign trend inflation nor the different preferences of the foreign policy maker seem to make much impact. These conclusions are robust to changes in some of the model's

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[12]See eg CGG (1999), page 1673.

parameters: increasing the degree of openness, γ, to 0.5, increasing the inverse of the elasticities of substitution for consumption and labour supply, ρ and σ, to 4, or changing the elasticity of demand, ε, to 10 do not change the qualitative nature of the results (see figures 5 and 6).[13]

Implications for CPI inflation We now examine the implications of domestic inflation targeting for the variance of CPI inflation. We do this to be able to compare results with the case where the central bank targets CPI inflation (discussed below). To obtain the solution for CPI inflation in terms of the fundamental shocks, first note that since CPI is defined as $\pi_t = (1-\gamma)\pi_t^h + \gamma\left(\pi_t^{f,*} + \Delta e_t\right)$ and since we have a solution for π_t^h and $\pi_t^{f,*}$, all we need is a solution for e_t. In Appendix B we show that the solution for e_t is given by $e_t = E_v v_t + E_{v^*} v_t^*$, where E_v and E_{v^*} depend on $y_v, y_{v^*}, \pi_v^h, \pi_{v^*}^h, \Xi_\pi^d$ and Ξ_y^d. Figure 7 plots E_v and E_{v^*}. We see that, consistent with CGG (2002), the response of the exchange rate to a domestic cost push shock is to appreciate it, whereas the response to a foreign cost push shocks is to depreciate it.

CPI inflation, expressed in terms of the fundamental shocks v_t and v_t^* is given by:

$$
\begin{aligned}
\pi_t &\simeq (1-\gamma)\left(\pi_v^h v_t + \pi_{v^*}^h v_t^*\right) + \gamma\left(\Xi_\pi^d v_t^* + E_v \Delta v_t + E_{v^*} \Delta v_t^*\right) \\
&\simeq \underbrace{\left[(1-\gamma)\pi_v^h + \gamma E_v\right]}_{\pi_v^{CPI}} v_t + \underbrace{\left[(1-\gamma)\pi_{v^*}^h + \gamma\Xi_\pi^d + \gamma E_{v^*}\right]}_{\pi_{v^*}^{CPI}} v_t^* - \gamma E_v v_{t-1} - \gamma E_{v^*} v_{t-1}^* \quad (54)
\end{aligned}
$$

Figure 7 also plots the values of π_v^{CPI} and $\pi_{v^*}^{CPI}$. In this case, both domestic and foreign cost-push shocks increase CPI inflation as one would expect. Moreover, as was the case for domestic inflation, increases in domestic trend inflation, increase π_v^{CPI}. Increases in foreign trend inflation in turn increase $\pi_{v^*}^{CPI}$.

The variance of CPI is derived using (52), (53) and (54). Figure 8 plots the variances of CPI and output and presents the same exercises as those presented in Figure 4. A number of interesting facts emerge. First, the variance of CPI inflation is higher than the variance of domestic inflation.[14] Second, as in Figure 4, the variances of CPI and output increase with the level of domestic trend inflation. Third, foreign trend inflation increases the variance of CPI almost as much as the increases in domestic trend inflation do. These results appear to

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[13]They however, change the quantitative nature of the results: recall that increasing γ and ε and reducing σ and ρ, reduces the controllability of economy (since $Y\left(\Pi^h\right)$ falls), thereby "worsening" the variance trade-off.

[14]This is a standard result (see, for example, Gali and Monacelli (2005)).

be robust to changes the model's parameters γ, σ, ρ, and ε (figures not shown but available on request).

4.2 CPI inflation targeting

In this case, not considered by CGG (2002), the policy maker targets CPI inflation, π_t. We assume the same policy problem for the foreign policy maker as in the previous section. The loss function for the domestic policy maker is:

$$
\begin{aligned}
L_t^{CPI} &= \frac{1}{2} \left[\chi (y_t)^2 + \left((1 - \gamma) \pi_t^h + \gamma \pi_t^f \right)^2 \right] \tag{55} \\
&= \frac{1}{2} \left[\chi (y_t)^2 + \left((1 - \gamma) \pi_t^h + \gamma \left(\pi_t^{f,*} + \Delta e_t \right) \right)^2 \right]. \tag{56}
\end{aligned}
$$

The policy maker now needs to consider not only domestic but foreign inflation expressed in the local currency, π_t^f. Since foreign inflation in the local currency is equal to foreign inflation in the foreign currency plus the appreciation/depreciation of the nominal exchange rate, the problem for the domestic policy maker is to minimise (55) subject to:

$$
\begin{aligned}
y_t &\simeq E_t y_{t+1} - \frac{1}{\rho_0} E_t \left(r_t - \pi_{t+1}^h \right) + \frac{\kappa_0}{\rho_0} \Xi_y^d \phi^* \upsilon_t^* - \frac{\kappa_0}{\rho_0} \Xi_y^d \upsilon_t^* \tag{57} \\
\pi_t^h &\simeq C_1 y_t + C_2 \Xi_y^d \upsilon_t^* - C_3 a_t + C_4 \Omega_t + C_5 E_t \pi_{t+1}^h + \upsilon_t \tag{58} \\
\Omega_t &\simeq C_6 y_t - C_7 \Xi_y^d \upsilon_t^* + C_8 E_t \pi_{t+1}^h + C_9 E_t \Omega_{t+1} \tag{59} \\
\Xi_y^d \upsilon_t^* &\simeq \Xi_y^d \phi^* \upsilon_t^* - \frac{1}{\rho_0^*} \left(\Xi_r^d - \Xi_\pi^d \phi^* \right) \upsilon_t^* + \frac{\kappa_0^*}{\rho_0^*} E_t y_{t+1} - \frac{\kappa_0^*}{\rho_0^*} y_t \tag{60} \\
\Xi_\pi^d \upsilon_t^* &\simeq C_1^* \Xi_y^d \upsilon_t^* + C_2^* y_t - C_3^* a_t^* + C_4^* \Omega_t^* + C_5^* \Xi_\pi^d \phi^* \upsilon_t^* + \upsilon_t^* \tag{61} \\
\Omega_t^* &\simeq C_6^* \Xi_y^d \upsilon_t^* - C_7^* y_t + C_8^* \Xi_\pi^d \phi^* \upsilon_t^* + C_9^* E_t \Omega_{t+1}^* \tag{62} \\
e_t &\simeq \Xi_r^d \upsilon_t^* - r_t \tag{63}
\end{aligned}
$$

where the last equation is the UIP condition and where we have substituted the solutions for y_t^*, $\pi_t^{f,*}$ and r_t^*. The solution for this problem is:

$$
y_t = -\frac{\left(\overbrace{(1 - \gamma) (C_1 + C_4 C_6)}^{\text{dom. comp.}} + \overbrace{\gamma \rho_0}^{\text{exch. comp}} \right)}{\underbrace{\chi}_{=F}} \pi_t. \tag{64}
$$

The solution is similar to the solution for domestic inflation targeting. However, in this case, there are two components in this solution: the first is consistent with the solution for domestic inflation targeting (see (51)) and the second component comes through the exchange rate channel (UIP condition). Thus, the policy maker smooths both movements in domestic inflation and the exchange rate. Note that the solution for this problem does not depend on the coefficients of any of the foreign equations and therefore it would appear, as was the case in the domestic inflation targeting exercise, that foreign trend inflation does not matter. Nonetheless, as we show below, this is not the case.

The solutions used to compute the variances of interest in terms of the domestic and foreign cost push shocks are:

$$e_t = \Theta_v^e v_t + \Theta_{v*}^e v_t^* + \Theta_e^e e_{t-1}, \tag{65}$$

$$y_t = \Theta_v^y v_t + \Theta_{v*}^y v_t^* + \Theta_e^y e_{t-1}, \tag{66}$$

$$\pi_t = -\frac{\Theta_v^y}{F} v_t - \frac{\Theta_{v*}^y}{F} v_t^* - \frac{\Theta_e^y}{F} e_{t-1}, \tag{67}$$

$$\pi_t^h = \underbrace{-\left(\frac{\Theta_v^y + F\gamma\Theta_v^e}{(1-\gamma)F}\right) v_t}_{\pi_v^h} \underbrace{- \left(\frac{\Theta_{v*}^y + F\gamma\Theta_{v*}^e + F\gamma\Xi_\pi^d}{(1-\gamma)F}\right) v_t^*}_{\pi_{v*}^h} \underbrace{- \left(\frac{\Theta_e^y + F\gamma\Theta_e^e - F\gamma}{(1-\gamma)F}\right) e_{t-}}_{\pi_e^h} \tag{68}$$

Because there is no analytical solution for the coefficients $\Theta_v^e, \Theta_{v*}^e, \Theta_e^e, \Theta_v^y, \Theta_{v*}^y, \Theta_e^y$, numerical methods were used to find the solution (the equations used to solve for those coefficients given in Appendix B).

The variances of output, domestic inflation and CPI inflation are computed using (65)-(68). Figure 9 presents a similar exercise to that one presented in Figure 4. The message that emanates from this figure is similar to those emanating from previous figures for domestic trend inflation: higher domestic trend inflation rates increase the variances of output and of domestic inflation. Moreover, the variance of domestic inflation is higher when the policy maker follows CPI inflation compared to the case where it follows domestic inflation targeting. However, unlike the case where there is domestic inflation targeting, the variances of domestic inflation and output are affected by foreign trend inflation. Nonetheless, the impact of foreign trend inflation is lower than the impact that domestic trend inflation has on these variances. These results are robust to changes in the model's parameters γ, σ, ρ, and ε (results not shown but available on request).

Figure 10 plots the variances of output and CPI inflation and presents a similar exercise to that one presented in Figure 8. A number of noteworthy results emerge. First, the

variance of CPI inflation under CPI inflation targeting is markedly smaller than the same variance under domestic inflation targeting. Second, the variance of CPI is lower than the variance of domestic inflation under CPI targeting. Third, as domestic and foreign trend inflation rates increase, the variances of output and CPI inflation increase too. Fourth, the impact of domestic trend inflation appears to be marginally stronger on these variances than the impact of foreign trend inflation. Figures 11 and 12 demonstrate that these results are fairly robust to changes in the model's parameters (with the exception that when γ increases to 0.5, the impact of foreign trend inflation on the variances of CPI inflation and output is greater than the impact of domestic trend inflation).

5 Conclusions

In this paper we have investigated the implications of non-zero trend inflation rates in an Open Economy New Keynesian model. We have shown that trend inflation affects the dynamics of the model, both for the domestic and foreign economies. In terms of the variances for output and inflation we examined a number of policy problems and we showed that higher domestic trend inflation always increases the variances of output, domestic inflation and CPI inflation as was shown by Ascari and Ropele (2006), Kiley (2004) and Blake and Fernandez-Corugedo (2006) for the closed economy case. This suggests that aiming for low and stable domestic inflation rates is consonant with low output and inflation volatility. We also showed that higher foreign trend inflation rates also increase the variances of output and CPI inflation (and in some cases domestic inflation too). However, the impact of foreign trend inflation on the variances of output and inflation appears to be lower than that of the impact of domestic trend inflation with the exception of very open economies where the impact appears to be of a similar order of magnitude. Our results suggest that, although importing low inflation rates from the rest of the world, (due to globalization or better policy elsewhere, say) helps to improve welfare in the domestic economy, it is paramount for the domestic economy to follow sensible monetary policy. Finally, our results are broadly consistent with the data presented in Table 1 where it can be seen that lower levels of inflation are consistent with lower inflation volatility. The periods of lower inflation rates are consistent across many countries and with periods where policy makers had a commitment to fighting inflation.

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A   The steady-state and trend inflation

The steady-state for the domestic economy is considered in two steps: goods and labour market equilibrium.[15]

A.1   Goods market equilibrium

We start with goods market equilibrium. In the steady-state:

$$Y = CS^\gamma$$
$$Y = C^*S^\gamma$$

since $C = C^*$. For the foreign country the equivalent expression is:

$$Y^* = C^*S^{\gamma-1}.$$

Thus,

$$Y = C^*S^\gamma = Y^*S^\gamma S^{1-\gamma} = Y^*S. \tag{69}$$

Since in closed economy models trend inflation affects output (see Ascari (2004) and Blake and Fernandez-Corugedo (2006)), it is therefore conceivable that foreign inflation could affect domestic output via Y^* (that is if S does not change). We must therefore consider how the terms of trade move in the steady-state to validate that assertion. To do that we examine the labour market equilibrium conditions.

---

[15]The foreign economy's steady-state is not presented here.

A.2 Labour market equilibrium

This is given by:

$$CN(i)^{\sigma_n} = \frac{W}{P}$$

$$C\left(\underbrace{\left(X^h\right)^{-\varepsilon}\frac{Y}{A}}_{=N(i)}\right)^{\sigma_n} = \frac{W}{P^h}\frac{P^h}{P}$$

$$= \underbrace{A \times MC}_{=\frac{W}{P^h}}\frac{P^h}{P}$$

$$= A \times MCS^{-\gamma}. \tag{70}$$

An equivalent expression exists for the foreign country so that substituting $C = C^*$ into (70) yields:

$$C^*\left(\left(X^h\right)^{-\varepsilon}\frac{Y}{A}\right)^{\sigma_n} = AMCS^{-\gamma}$$

$$\frac{C^*S^\gamma\left(\left(X^h\right)^{-\varepsilon}\frac{Y}{A}\right)^{\sigma_n}}{A} = MC. \tag{71}$$

The next step is to obtain an expression for the marginal cost using the pricing decision of firms. Start with the optimal pricing decision and evaluate it at the steady-state:

$$X^h = \frac{\overline{P}^h(j)}{P^h} = \frac{\varepsilon}{\varepsilon - 1}\frac{1 - \theta\beta\left(\Pi^h\right)^{\varepsilon-1}}{1 - \theta\beta\left(\Pi^h\right)^{\varepsilon}}MC.$$

Since the domestic price index evolves as

$$1 = \theta\left(\Pi^h\right)^{\varepsilon-1} + (1 - \theta)\left(X^h\right)^{\varepsilon-1}$$

we have

$$X^h = \left[\frac{1 - \theta\left(\Pi^h\right)^{\varepsilon-1}}{1 - \theta}\right]^{\frac{1}{\varepsilon-1}} = \frac{\varepsilon}{\varepsilon - 1}\frac{1 - \theta\beta\left(\Pi^h\right)^{\varepsilon-1}}{1 - \theta\beta\left(\Pi^h\right)^{\varepsilon}}MC.$$

A.3   Putting all together

Substituting (71) and using $C^* = Y^* S^{1-\gamma}$, and (69) allows us to obtain an expression for Y:

$$Y = A \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \left[\frac{1 - \theta \left(\Pi^h \right)^{\varepsilon - 1}}{1 - \theta} \right]^{\frac{1 + \varepsilon \sigma_n}{\varepsilon - 1}} \left(\frac{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon}}{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon - 1}} \right) \right]^{\frac{1}{1 + \sigma_n}}.$$

Thus, domestic output is not affected by foreign (and domestic) trend inflation rates. Since $S = \frac{Y}{Y^*}$ we have:

$$S = \frac{A \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{1 - \theta \left(\Pi^h \right)^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1 + \varepsilon \sigma_n}{\varepsilon - 1}} \left(\frac{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon}}{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon - 1}} \right) \right]^{\frac{1}{1 + \sigma_n}}}{A^* \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{1 - \theta \left(\Pi^{f,*} \right)^{1 - \varepsilon}}{1 - \theta} \right)^{\frac{1 + \varepsilon \sigma_n}{1 - \varepsilon}} \left(\frac{1 - \theta \beta \left(\Pi^{f,*} \right)^{\varepsilon}}{1 - \theta \beta \left(\Pi^{f,*} \right)^{\varepsilon - 1}} \right) \right]^{\frac{1}{1 + \sigma_n}}}.$$

In Gali and Monacelli (2005) where $\Pi^h = \Pi^{f,*} = 1$:

$$S = \frac{A \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{1 + \sigma_n}}}{A^* \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\frac{1}{1 + \sigma_n}}} = \frac{A}{A^*}.$$

Assuming $A = A^*$ we clearly have $Y = Y^*$ which implies that $S = 1$. Moreover, if $\Pi^h = \Pi^{f,*}$, $A = A^*$, then $Y = Y^*$ also implying that $S = 1$. However if $\Pi^h \neq \Pi^{f,*}$:

$$S = \frac{A}{A^*} \left[\frac{\left[1 - \theta \left(\Pi^h \right)^{\varepsilon - 1} \right]^{\frac{1 + \varepsilon \sigma_n}{\varepsilon - 1}} \left(\frac{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon}}{1 - \theta \beta \left(\Pi^h \right)^{\varepsilon - 1}} \right)}{\left[1 - \theta \left(\Pi^{f,*} \right)^{1 - \varepsilon} \right]^{\frac{1 + \varepsilon \sigma_n}{1 - \varepsilon}} \left(\frac{1 - \theta \beta \left(\Pi^{f,*} \right)^{\varepsilon}}{1 - \theta \beta \left(\Pi^{f,*} \right)^{\varepsilon - 1}} \right)} \right]^{\frac{1}{1 + \sigma_n}}$$

that is, terms of trade are a function of the inflation rates in the different countries as well as the productivity component. As inflation increases in the foreign country, foreign output decreases but is compensated by a terms of trade improvement. Thus foreign inflation does not seem to reduce output in the domestic economy, as the foreign output loss is compensated by a terms of trade improvement (this is the result of the assumptions made about preferences). Nonetheless, we see that domestic inflation reduces domestic output.

B Expressions of interest

B.1 Solution for the foreign policy maker used in sections 4.1 and 4.2

The appropriate expressions for Ξ_y^d, Ξ_π^d and Ξ_r^d mentioned in the text are:

$$
\begin{aligned}
\Xi_y^d &= -\frac{(C_1^* + C_4^* C_6^*)}{\chi^f (1 - C_5^* \phi^*) + C_1^* (C_1^* + C_4^* C_6^*)} \\
\Xi_\pi^d &= \frac{\chi^f}{\chi^f (1 - C_5^* \phi^*) + C_1^* (C_1^* + C_4^* C_6^*)} \\
\Xi_r^d &= \left(\rho_0^* \Xi_y^d (\phi^* - 1) + \Xi_\pi^d \phi^* \right).
\end{aligned}
$$

B.2 Expression for the exchange rate under domestic inflation targeting

The solution for the exchange rate in terms of the fundamental shocks postulated in the text was:

$$
e_t = E_v v_t + E_{v^*} v_t^*
$$

where:

$$
\begin{aligned}
E_v &= -\frac{\rho_0 \left[y_v (\phi - 1) + \frac{1}{\rho_0} \pi_v^h \phi \right]}{(1 - \phi)}, \\
E_{v^*} &= \frac{\Xi_r^d - \rho_0 \left[y_{v^*} (\phi^* - 1) + \frac{1}{\rho_0} \left(\pi_{v^*}^h + \kappa_0 \Xi_y^d \right) \phi^* - \frac{\kappa_0}{\rho_0} \Xi_y^d \right]}{(1 - \phi^*)}.
\end{aligned}
$$

B.3 CPI targeting

B.3.1 The problem

The Lagrangian is given by:[16]

$$
\begin{aligned}
\Upsilon &= \frac{1}{2} \left[\chi (y_t)^2 + \left((1 - \gamma) \pi_t^h + \gamma \left(\Xi_\pi^d v_t^* + \Delta e_t \right) \right)^2 \right] + \phi_t^1 \left(\pi_t^h - (C_1 + C_4 C_6) y_t - v_t \right) \\
&\quad + \phi_t^2 \left(y_t + \frac{1}{\rho_0} r_t \right) + \phi_t^3 \left(e_t - \Xi_r^d v_t^* + r_t \right).
\end{aligned}
$$

---

[16] That is, we have ignored future values of variables and the variables that the policy maker cannot control such as exogenous variables and the foreign output gap.

The first order conditions are:

$$\pi^h \quad : \quad \left(\overbrace{(1-\gamma)\,\pi^h_t + \gamma\left(\Xi^d_\pi v^*_t + \Delta e_t\right)}^{=\pi_t} \right)(1-\gamma) + \phi^1_t = 0$$

$$y \quad : \quad \chi y_t - \phi^1_t\left(C_1 + C_4 C_6\right) + \phi^2_t = 0$$

$$r \quad : \quad \frac{\phi^2_t}{\rho_0} + \phi^3_t = 0$$

$$e \quad : \quad \pi_t \gamma + \phi^3_t = 0$$

plus the conditions for the lagrange multipliers, ϕ^1_t, ϕ^2_t, and ϕ^3_t.

B.3.2 Equations used to solve the CPI targeting problem

The equations used to solve for the coefficients $\Theta^e_v, \Theta^e_{v^*}, \Theta^e_e, \Theta^y_v, \Theta^y_{v^*}, \Theta^y_e$ (65), (66) and (67) are:

$$\Theta^e_v = \frac{\left[B\left(1-\phi\right) + \frac{1}{F}\phi\right]}{\left[1 + A\Theta^y_e - \left(\Theta^e_e + \phi\right)\right]}\Theta^y_v$$

$$\Theta^e_{v^*} = \frac{\Xi^d_r + \left[B\left(1-\phi^*\right) + \frac{1}{F}\phi^*\right]\Theta^y_{v^*} + \rho\gamma\Xi^d_y\left(1-\phi^*\right)}{\left[1 + A\Theta^y_e - \left(\Theta^e_e + \phi^*\right)\right]}$$

$$\Theta^e_e = \frac{B}{\left[1 + A\Theta^y_e - \Theta^e_e\right]}\Theta^y_e$$

$$\frac{\Theta^y_v}{C} = \frac{D}{(1-\gamma)}\left(\frac{\phi\Theta^y_v + \Theta^y_e\Theta^e_v}{F} + \gamma\left[\phi\Theta^e_v + \left(\Theta^e_e - 1\right)\Theta^e_v\right]\right) - \frac{\gamma\Theta^e_v}{(1-\gamma)} - E\Theta^y_v - 1$$

$$\frac{\Theta^y_{v^*}}{C} = G\Xi^d_y - \frac{\gamma\left(\Xi^d_\pi + \Theta^e_{v^*}\right)}{(1-\gamma)} - E\Theta^y_{v^*} + \frac{D\left(\phi^*\Theta^y_{v^*} + \Theta^y_e\Theta^e_{v^*}\right)}{(1-\gamma)F} - \frac{D\gamma\left(\left(1-\phi^* - \Theta^e_e\right)\Theta^e_{v^*} + \phi^*\Xi^d_\pi\right)}{(1-\gamma)}$$

$$\frac{\Theta^y_e}{C} = \frac{D\Theta^y_e\Theta^e_e}{C} - \frac{\gamma\left(\Theta^e_e - 1\right)}{(1-\gamma)} - E\Theta^y_e + \frac{\gamma D\Theta^e_e\Theta^e_e}{(1-\gamma)} - \frac{\gamma D\Theta^e_e}{(1-\gamma)}$$

where $A = \left(\rho\left(1-\gamma\right) - \frac{1}{F}\right)$, $B = \rho\left(1-\gamma\right)$, $C = \left(1-\gamma\right)F$, $D = \left(C_5 + C_4 C_8\right)$, $E = \left(C_1 + C_4 C_6\right)$, $G = \left(C_2 - C_4 C_7\right)$.
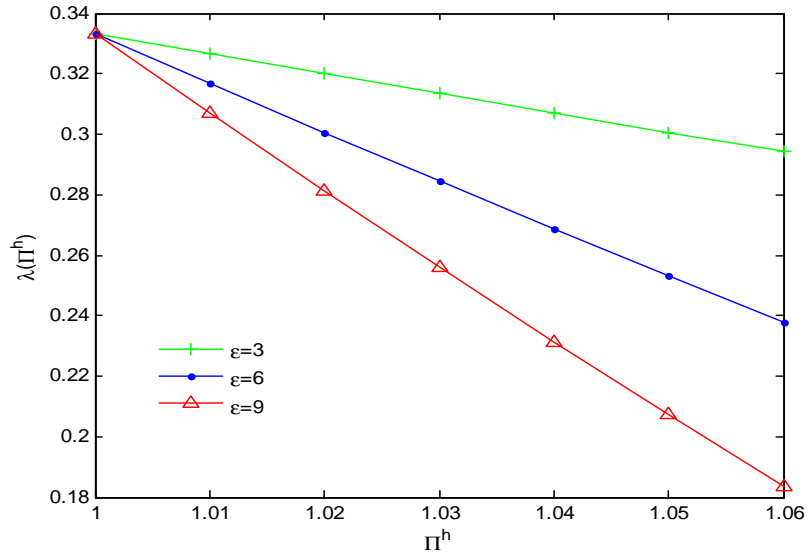
Figure 1: Value of $\lambda\left(\Pi^h\right)$ for different values of ε and Π
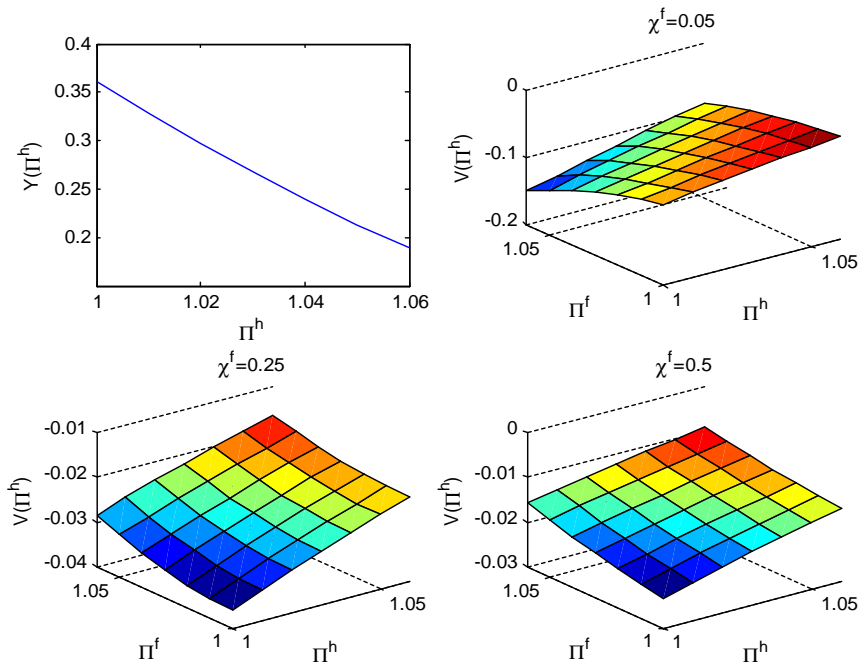

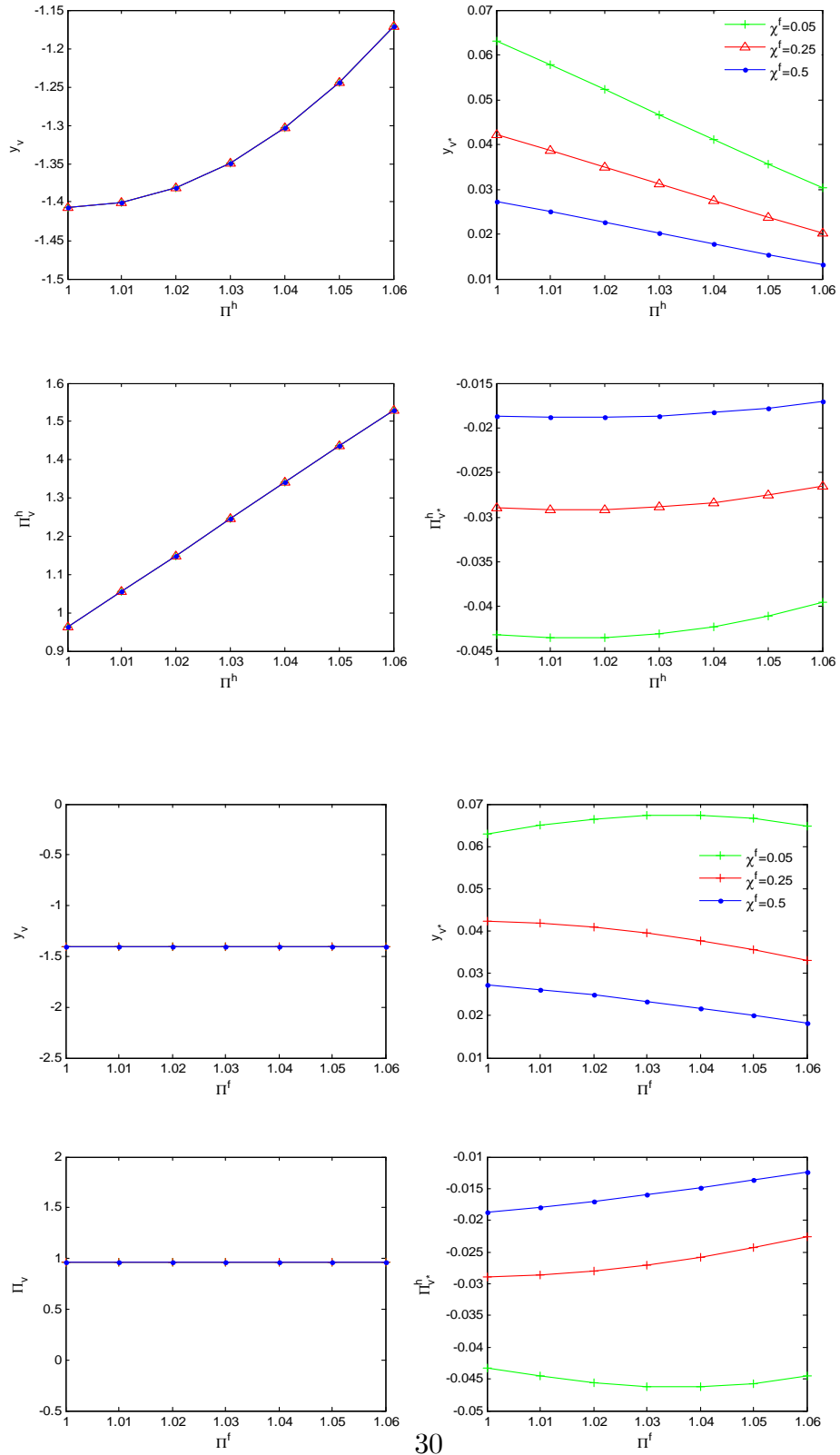Figure 2: Multipliers in the NKPC, equation(48)

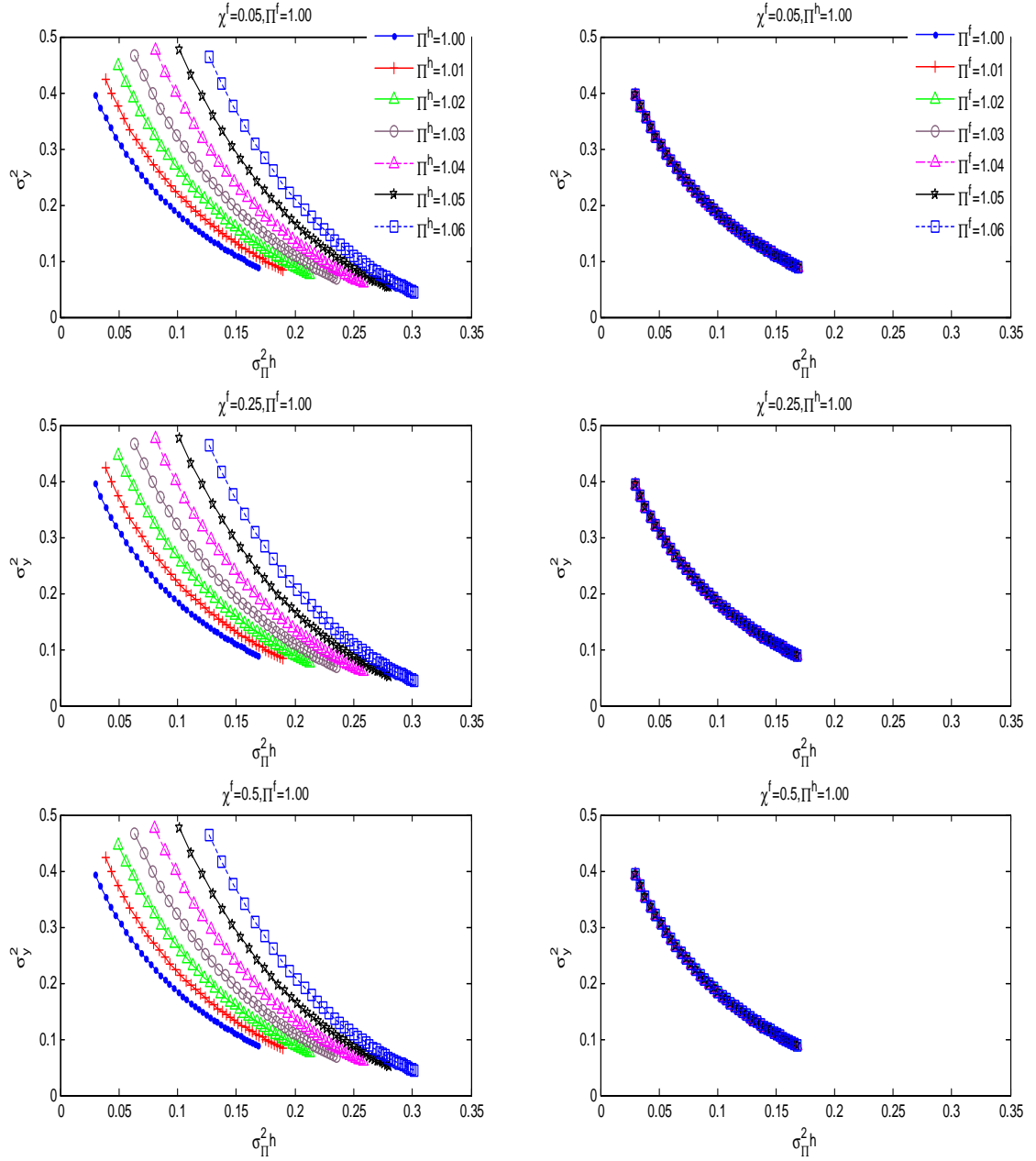Figure 3: Solution coefficients for domestic inflation targeting

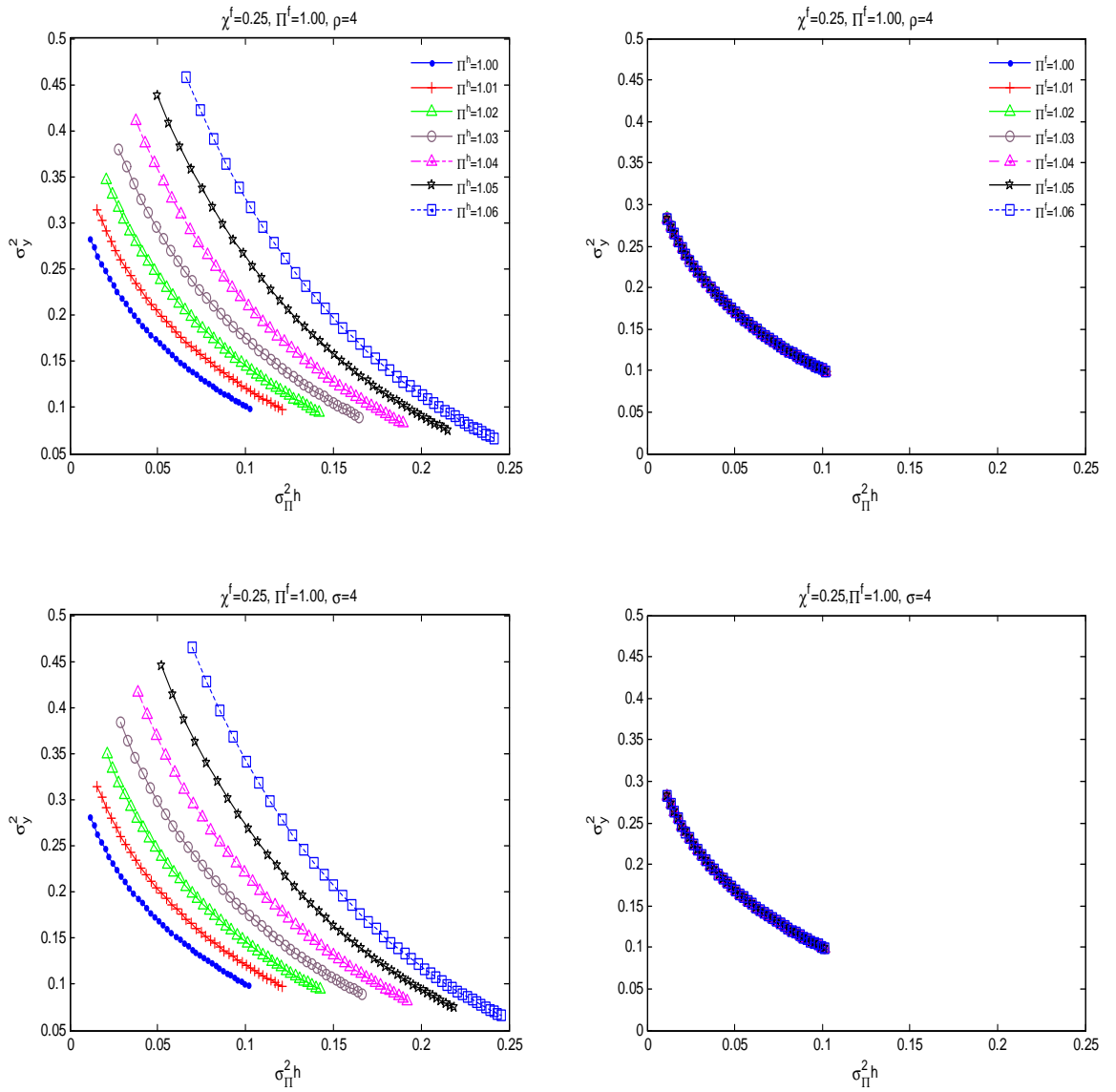Figure 4: Variances of output and domestic inflation

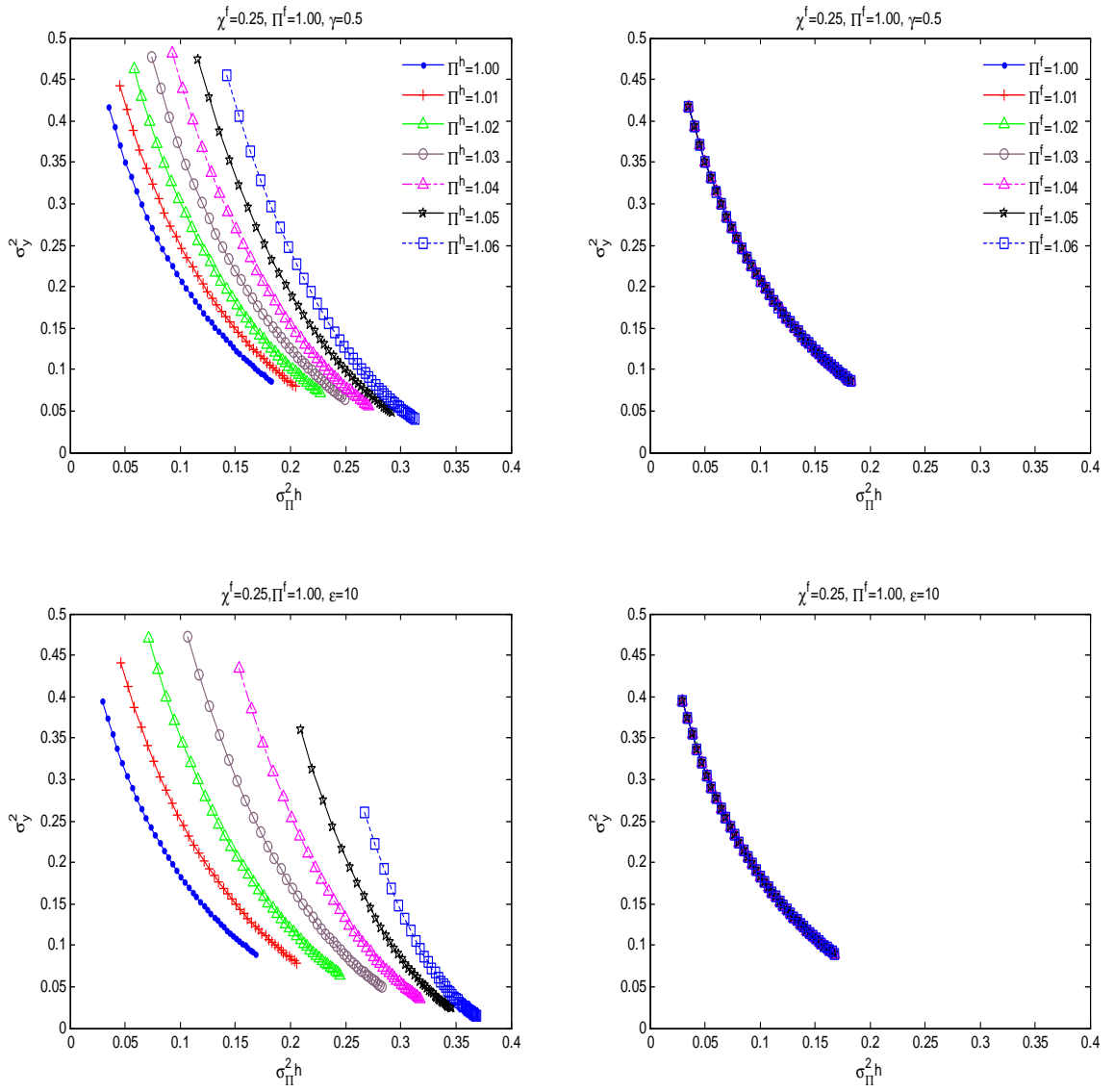Figure 5: Sensitivity Analysis to utility parameters

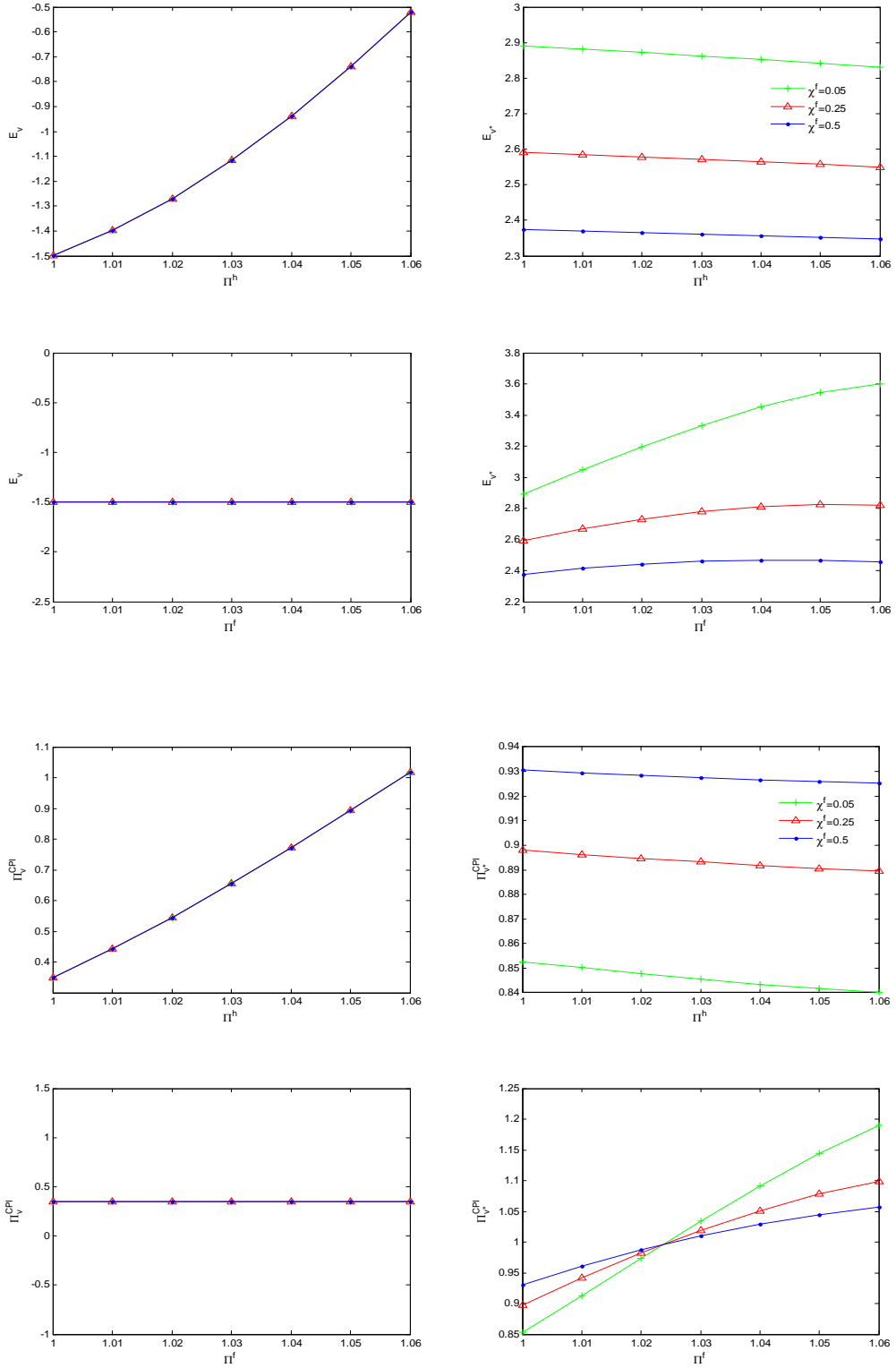Figure 6: Sensitivity Analysis to openness and mark-ups

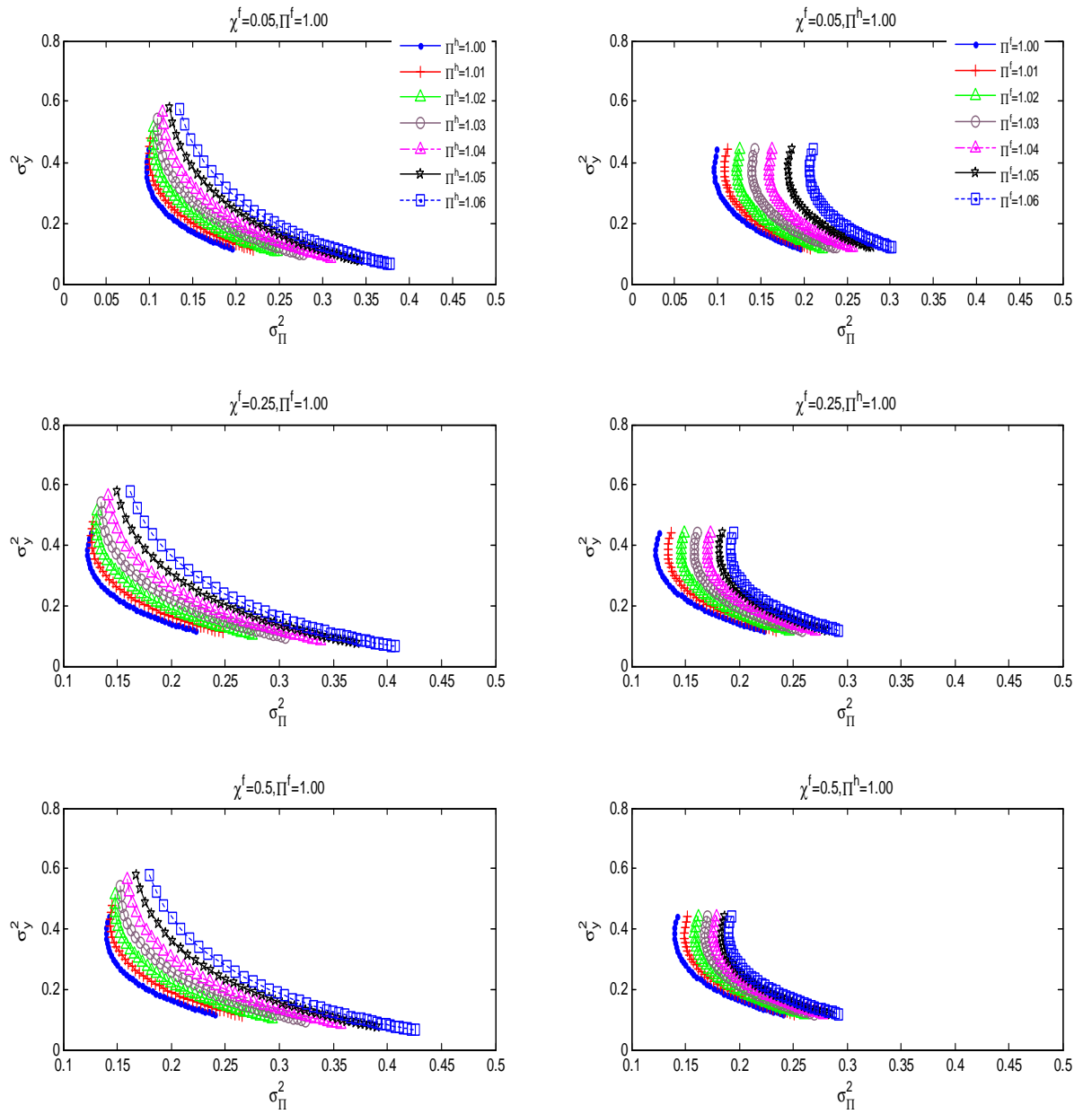Figure 7: Solution coefficients for domestic inflation targeting
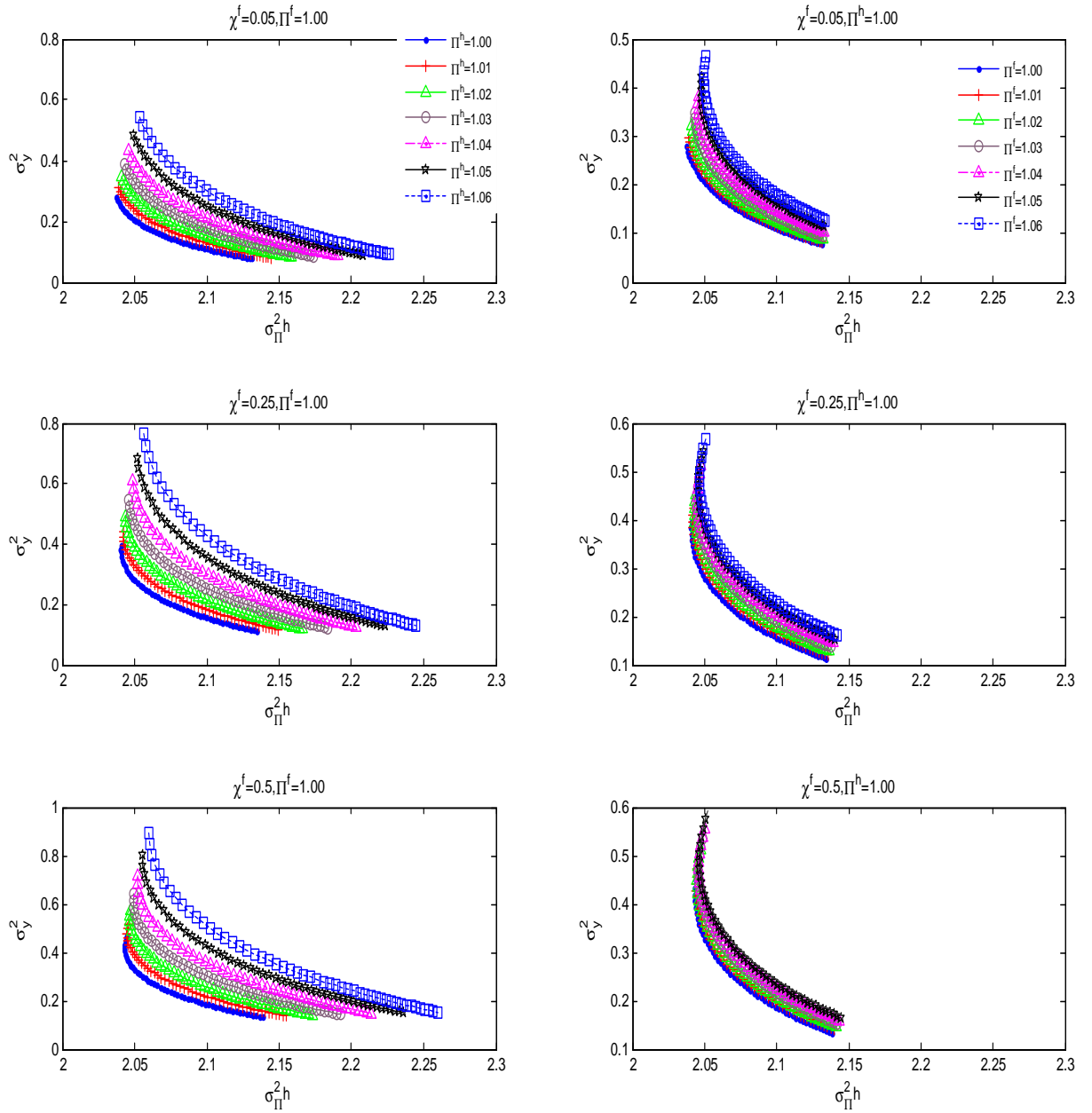
Figure 8: Variances of output and CPI
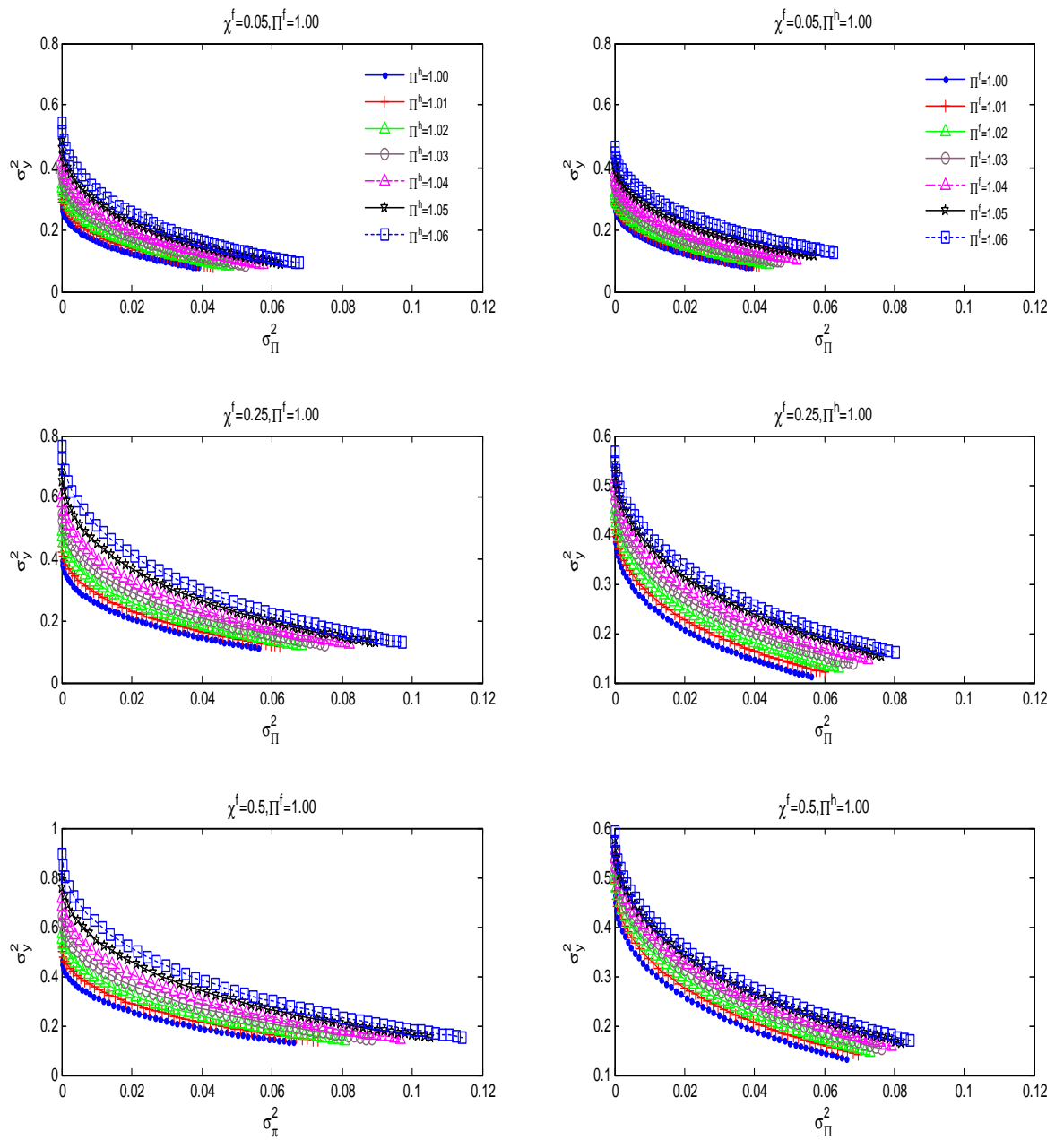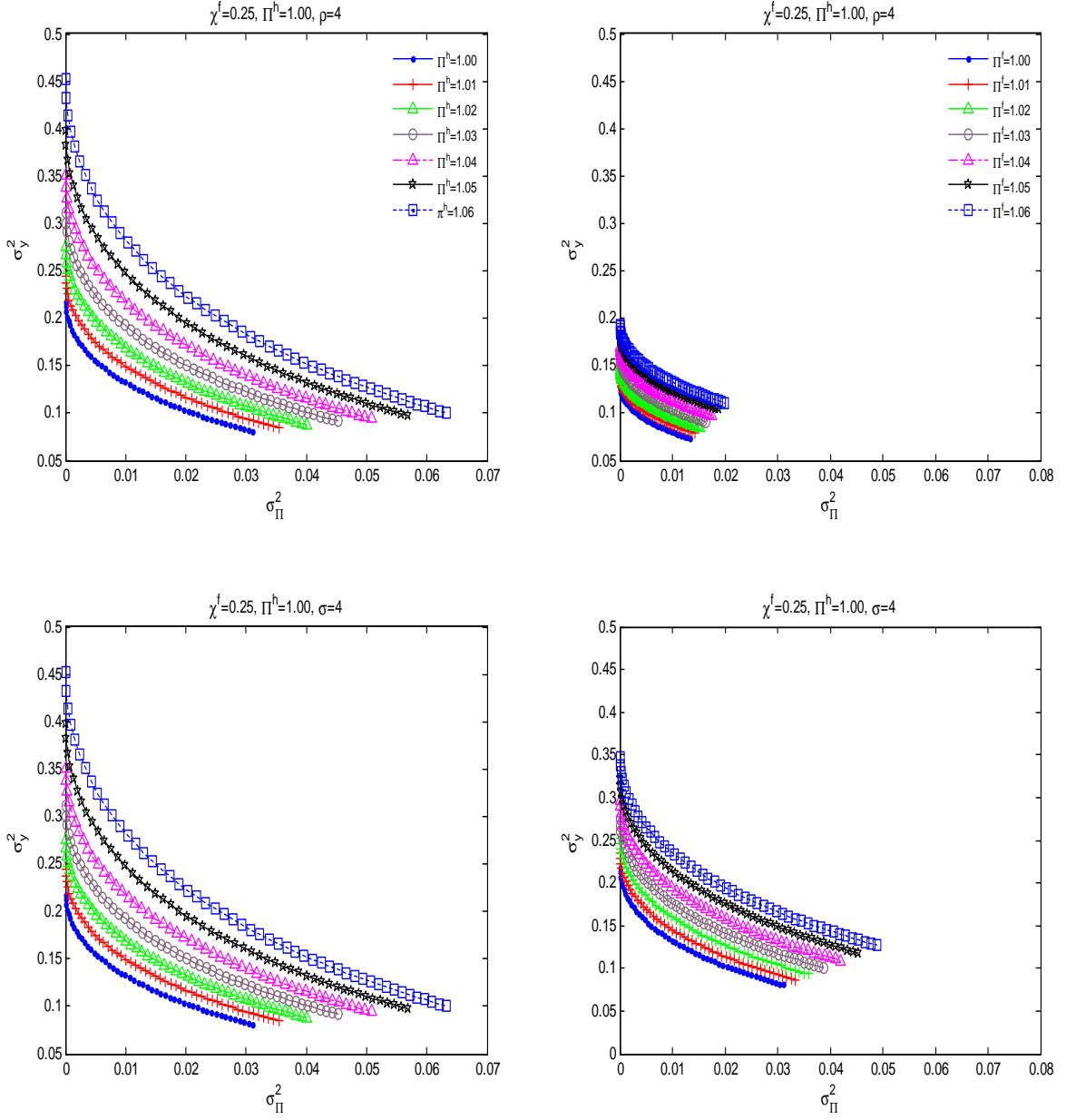
Figure 9: Variances of output and domestic inflation

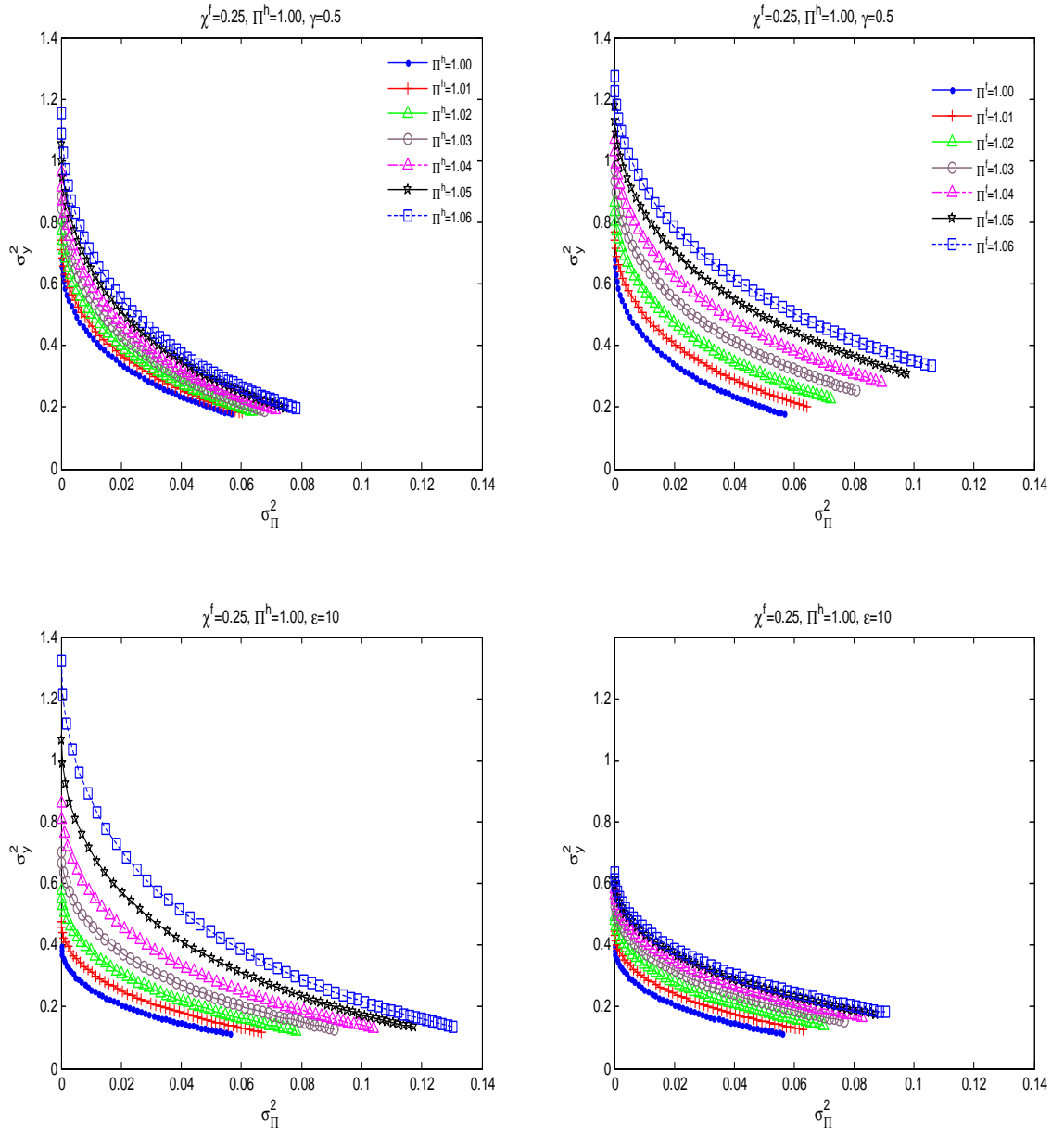Figure 10: Variances of output and CPI

Figure 11: Sensitivity Analysis to utility parameters

Figure 12: Sensitivity analysis to openness and mark-ups