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# Wealth Financial Intermediation and Growth

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## Abstract

This paper presents empirical support for the existence non-linear effects in the contribution of financial intermediation to economic growth, and offers a theoretical explanation for these effects. We show that the exogenous contribution of financial development to economic growth has different effects for different levels of income per capita. We find that this contribution is generally increasing with the level of income per capita of the economy, up to a relatively high level of income. This contribution is consistently lower for poor countries, and for some low levels of income per capita it can be negative. We provide a growth model of financial intermediation to account for these wealth effects. We show that at early stages of economic development, a bank can increase the welfare of its depositors only at the cost of lowering investment and growth. However, once the economy has crossed a certain wealth threshold, the liquidity role of banks becomes unambiguously growth enhancing. However, for high levels of wealth, growth generated by financial intermediation declines as the economy attains the optimal level of consumption risk sharing.

**Keywords:** Growth models, Liquidity, Financial intermediation, Financial fragility, Banking crises.

**JEL Classification:** E44, G21, O16, O40

## Resumen

Este artículo presenta evidencia empírica sobre la existencia de efectos no lineales en la contribución del desarrollo financiero sobre el crecimiento económico y ofrece una explicación teórica sobre estos efectos. La contribución es generalmente creciente con el nivel de ingreso per capita de la economía, hasta un nivel relativamente alto de ingreso. Sin embargo, esta contribución es consistentemente menor para países pobres, e incluso puede ser negativa. Se propone un modelo de crecimiento con intermediarios financieros para explicar estos efectos. En las etapas tempranas de desarrollo económico, el sistema bancario puede incrementar el bienestar de sus depositantes a costa de reducir la inversión y el crecimiento. Sin embargo, existe un umbral de riqueza a partir del cual las funciones de liquidez del sistema bancario se vuelven inequívocamente positivas para el crecimiento. Finalmente, para niveles altos de riqueza, la contribución al crecimiento de la intermediación financiera se reduce conforme la economía tiende hacia el nivel máximo de riesgo compartido.

**Palabras Clave:** Modelos de crecimiento, Liquidez, Intermediación financiera, Fragilidad financiera, Crisis bancaria.

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# 1 Introduction.

In recent years the relationship between financial development and economic growth has received close attention in the literature. A large number of empirical studies supports the existence of a positive relationship between financial intermediation and growth.<sup>1</sup> King and Levine (1993 a,b), show that financial development is a good predictor of future growth. More recently, Levine, et al. (2000) and Beck et al. (2000) use an external instrumental variables estimation, and a GMM dynamic panel data estimation to overcome potential problems of endogeneity of financial development, and confirm the existence of a strong, positive contribution of the exogenous component of financial development to economic growth. However, most of these studies assume a uniform financial intermediation-growth relationship and, although panel data estimations, such as Levine et al. (2000), Benhabib and Spiegel (2000), etc., control for unobserved country-specific effects, there can be country-group characteristics that generate an asymmetric effect of financial intermediation on growth. De Gregorio and Guidotti (1995), for example, show that in a panel regression for Latin American countries financial intermediation is negatively linked with economic growth, and suggest that this result may reflect the negative effect of the financial crises experienced by several countries in the region.

The data on financial development provides initial evidence that the growth benefits of financial intermediation differ with the level of wealth of the economy.<sup>2</sup> Table 1 summarizes data on financial development and growth; it orders countries by quartiles according to their real per capita GDP growth over the period 1970-2000. For each group, it displays the percent variation of financial depth, as measured by the annual percent growth rates of private credit and liquid liabilities, respectively, as well as the mean of initial and final income. Table 1 suggests a positive relationship between growth performance and financial development. Moreover, those countries that grew faster had a joint high performance of growth and financial development, and are typically middle-income economies which have "emerged" during the period. At the other end, in the first quartile, we find countries that have experienced declines in per capita income during the period along with poor financial development. This suggest that financial intermediation plays a crucial role in the growth performance of middle-income, emerging economies, and the possibility of wealth effects on this relationship.

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<sup>1</sup>See for example Levine and Zervos (1996) and Benhabib and Spiegel (2000). Levine (1997) provides a good summary of the financial development-growth literature.

<sup>2</sup>This differential effect of financial intermediation on growth depending on the level of GDP per capita (used here as a proxy for wealth) is what we term the "wealth effect." It should be distinguished from the "wealth effect" arising from price changes that affect endowments and, thus, demand for a given good, a topic pertaining consumer demand.

**Table 1: Financial Development and Real Per Capita Growth (1970-2000)**

Growth Quartile	Average Annual Growth	Financial Development Private Credit Growth <sup>2</sup> (%)	Financial Development Liquid Liabilities Growth <sup>2</sup> (%)	Initial Income per Capita <sup>1</sup>	Final Income per Capita <sup>1</sup>
<b>Q1</b>	-0.90	0.15	0.60	1215.8	1042.3
<b>Q2</b>	1.16	1.04	1.63	4585.2	6544.1
<b>Q3</b>	2.07	1.90	1.20	8987.3	16444.1
<b>Q4</b>	3.33	3.28	2.24	6221.5	15090.3
<i>Average</i>	<i>1.41</i>	<i>1.59</i>	<i>1.42</i>	<i>5252.4</i>	<i>9780.2</i>
Number of Countries	72				

<sup>1</sup> Real GDP per capita, constant US\$. Source: World Development Indicators WB

<sup>2</sup> Source: International Financial Statistics IMF

In this paper we test for the existence of wealth effects in the contribution of financial intermediaries development to economic growth. It is important that the study of wealth effects on this relationship separates the effect of financial intermediation from other country and country-group characteristics that may have an asymmetric effect on growth for different income groups. We thus include country-specific effects and a set of control variables that can mitigate this problem. However, it is difficult to separate the growth-promoting effects of financial intermediation from the increase in financial vulnerability that financial development brings about, because financial crises can generate large and persistent output losses. Domestic credit of financial intermediaries to the private sector is one of the most common proxies for financial intermediation development in the growth literature, but its rate of growth is also among the best predictors for banking crises (see for example Demirguc-Kunt and Degatriache, 1998 and 2000; Kaminsky and Reinhart, 1999). Using a pooled mean group estimator, Loayza and Ranciere (2002) show that a long-run positive relationship between financial intermediation and growth can coexist with a negative short-run relationship due to financial crises.<sup>3</sup>

We use a GMM dynamic panel data system estimator, as in Levine et al. (2000), for a sample of 83 countries for the period 1960-2000. This estimation technique addresses the issue of joint endogeneity of financial development and the other explanatory variables, by using "internal" instruments (lagged values of the explanatory variables). After controlling for the episodes and duration of financial crises, we analyze how the level of income per capita modifies the estimated effect of financial intermediation on growth performance. We use two standard measures to proxy financial intermediaries development: the ratio of liquid liabilities to GDP, and the ratio of domestic credit to the private sector to GDP. For both indicators the results of the estimations show the existence of wealth effects, and suggest that in economies with low levels of per capita income, financial development has a consistently smaller growth

<sup>3</sup>Gaytan and Ranciere (2005) provide a theoretical model to integrate the short-run analysis of financial crises and the long-run positive contribution of financial intermediation to economic growth.

effect, and that this effect may even be negative for sufficiently low levels of wealth. For higher per capita income levels there is a positive and increasing contribution and after reaching a maximum, it starts to decrease.<sup>4</sup>

On the theoretical side, it has been recognized that financial intermediaries play several roles that can foster economic growth. Among these roles, financial intermediaries: *(i)* provide an efficient mechanism that channels savings to those investments with the highest rates of returns; *(ii)* are efficient suppliers of liquidity (i.e., can transform illiquid assets into liquid liabilities); *(iii)* provide liquidity insurance that reduces idiosyncratic liquidity risk; *(iv)* allow an efficient pooling of risk among different investment projects; *(v)* ameliorate information asymmetries (i.e., are efficient institutions for screening and monitoring investment projects).<sup>5</sup> The model developed in this paper focuses only on the relationship between the first three roles stated above and economic growth.

We use an intertemporal model of financial intermediaries to analyze the dynamics of wealth, capital and consumption. The model embeds a modified version of the Diamond and Dybvig (1983) model of liquidity provision (DD henceforth) into an overlapping generations framework.<sup>6</sup> There are two technologies available: a storage technology, and a long-term technology. The long-term technology is characterized by a Cobb-Douglas production function with capital and labor as inputs. This technology constitutes the channel of investment and growth over time and among generations.

As a benchmark of zero financial development, we study financial autarky, where agents have no access to credit markets. This benchmark is useful to study the effect on output, consumption and welfare of introducing financial intermediaries. Financial intermediaries provide liquidity insurance to maximize the welfare of the current generation of depositors. However, this insurance may introduce costs in terms of economic growth. For low levels of wealth, the provision of liquidity insurance may force an early liquidation of long-term

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<sup>4</sup>Odedokun (1996) finds that the positive effect of financial development on growth is higher for low income than for high income LDCs, an opposite result than the one presented here. However, this author estimates time series equations, one per each country using OLS-GLS with potential biases due to the endogeneity of explanatory variables (including factor accumulations rates), and it does not take into account potential country specific effects, and the higher recurrence of financial crises episodes in high income LDCs.

<sup>5</sup>See for example Greenwood and Jovanovic (1990), Saint Paul (1992), Acemoglu and Zilibotti (1997). Levine (1997) presents an excellent review of the growth-promoting effects of financial intermediation found in the literature.

<sup>6</sup>In this paper we are interested in the long-run effect of financial intermediation on growth. Consequently, the theoretical model abstracts from financial crises, and in the empirical estimation we control for the effect of banking crises. In Gaytan and Ranciere (2005) we present a model that includes the possibility of bank run equilibria and show what are the growth and welfare consequences of having financial crises with positive probability.

projects that, although efficient in terms of the welfare of the current generation, may reduce the rate of capital accumulation. Nevertheless, once the economy has crossed certain threshold of economic development, this trade-off disappears and financial intermediaries can provide liquidity insurance and attain higher levels of growth and output. In this case, the role of banks as liquidity providers is always growth enhancing. However, above some high level of wealth, once the economy has exhausted the consumption risk sharing possibilities, the growth-enhancing effect of financial intermediation declines.

Bencivenga and Smith (1991), Ennis and Keister (2003), Qi (1994) and Fulghieri and Rovelli (1998) have studied the DD model in overlapping generations frameworks. Qi (1994) and Fulghieri and Rovelli (1998) focus on intergenerational transfers and not on growth; their models have technologies with constant returns to capital and are endowment economies without capital accumulation or any wealth dynamics. Ennis and Keister (2003) and Bencivenga and Smith (1991) are closer to our paper, since they investigate the relationship between financial intermediation and growth. However, their models are endogenous growth models with constant returns to capital; under this assumption, the effect of financial intermediation is no longer wealth dependent. In our model, the use of a Cobb-Douglas technology for the long asset makes the returns to investment endogenous, the banking solution (the banking investment portfolio, liquidation policy and deposit contract) becomes dependent on the level of wealth of the economy, and introduces the possibility of a trade-off between short-run liquidity and economic growth.

The rest of the paper is organized as follows. Section 2 presents the data, methodology and the estimation results of wealth effects in the financial intermediation-growth relationship. Section 3 presents the general set up of the model: the structure of overlapping generations, preferences and the technologies available. Section 4 presents the portfolio choice and the growth implications of financial autarky. Section 5 presents the optimal risk sharing solution of financial intermediation and its consequences for economic growth. Section 6 analyzes the consequences of financial intermediation in terms of welfare and growth. Finally, section 7 concludes.

## **2 Wealth Effects in the Financial Intermediation-Growth Relationship**

In this section we study how the exogenous component of financial intermediation affects economic growth for different levels of income per capita. For this purpose we use the methodology of GMM dynamic panel data system estimator developed in Arellano and Bond (1991),

Arellano and Bover (1995) and Blundell and Bond (1998).<sup>7</sup> This methodology addresses the issues of joint endogeneity of all explanatory variables in a dynamic formulation, and of potential biases induced by country-specific effects. First, we describe the methodology and data used in the estimation, and then we show that the estimation results support the existence of wealth effects in the contribution of financial development to economic growth.

## 2.1 Data and Methodology

The data set consists of a panel of observations for 83 countries for the period 1960-2000. To smooth the series from business cycle fluctuations and concentrate on long run-growth effects of financial intermediation, we use low frequency data by averaging, for each country the information on growth and the explanatory variables in at most 8 non-overlapping five-year periods. The resulting panel of country and time-period observations is unbalanced. Table 2 in Appendix B presents the lists of countries included in the sample.

The dependent variable is average income per capita growth. We use two indicators of financial development: liquid liabilities as a proportion of GDP, and the ratio of domestic credit to the private sector to GDP. Liquid liabilities is a standard measure of financial depth, which measures the size of the financial sector. Private credit goes beyond a size effect of financial intermediation, since it provides more information on the level of financial services and the growth-promoting activities of financial intermediaries. In addition to the financial development variables, the regressions include a set of control variables. As is standard in the growth literature, initial income per capita is included to capture either the distance from the steady state, for a Solow-type growth model, or the catching up effect of new technologies, for an endogenous growth model. To control for human capital we use average years of secondary schooling. Inflation and the size of the government (government expenditure as proportion of GDP) enter the regression to control for the extent of macroeconomic instability. Finally, the degree of trade openness (exports plus imports as proportion of GDP) is presumed to have a positive impact on growth. Table 3 in Appendix B presents the definitions and sources of the variables used in the estimation.

We are interested in estimating the following equation:

$$y_{i,t} - y_{i,t-1} = (\alpha - 1) y_{i,t-1} + \beta' CV_{i,t} + \gamma FD_{i,t} + \mu_t + \eta_i + \varepsilon_{i,t}, \quad (1)$$

where  $y$  is the logarithm of real per capita GDP,  $CV$  is the set of control variables other than initial income,  $FD$  is the financial development measure,  $\mu_t$  is the time-specific effect,  $\eta_i$  is the country-specific effect, and  $\varepsilon_{i,t}$  is the error term.

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<sup>7</sup>Applications of this methodology to the growth-financial development relationship are: Levine et. al. (2000) and Beck et. al. (2000).

As a first approximation, we test for the existence of wealth effects in the relationship between financial intermediation and growth by introducing slope dummies for financial development variables for different income groups. We substitute the financial intermediation part in equation (1) by:  $\sum_{j \in J} \gamma^j FD_{i,t}^j$ , where  $FD_{i,t}^j$  is a slope dummy that takes the value of  $FD_{i,t}$  if country  $i$  belongs to the income group  $j$ , and zero otherwise. We use two different classifications; the first set of income groups is defined by the classification of the World Bank for low, middle-low, middle-high, and high-income OECD countries. The income groups using this classification are  $J = \{\gamma_L, \gamma_{ML}, \gamma_{MH}, \gamma_{OECD}\}$ .<sup>8</sup> However, the World Bank classification does not necessarily provides the appropriate thresholds for wealth effects on the finance-growth relationship. Thus, we also use a more flexible classification. We define income groups using deciles for average initial income per capita for the country data used in the estimation, and the income groups are  $J = \{\gamma_1, \dots, \gamma_{10}\}$ . Table 2 in Appendix B presents the list of countries and their respective classifications.<sup>9</sup>

Another way to capture the existence of wealth effects on the relationship is through the use of a non-linear specification in which the initial level of income per capita interacts with financial development. As we will see, the estimation results for the decile classification suggest that the contribution of financial development to growth,  $\gamma$ , can be approximated by the following linear quadratic specification:  $\gamma = \delta_0 + \delta_1 y_{i,t-1} + \delta_2 y_{i,t-1}^2$ . Thus, equation 1 can be rewritten as:

$$y_{i,t} - y_{i,t-1} = (\alpha - 1) y_{i,t-1} + \beta' CV_{i,t} + \delta_0 FD_{i,t} + \delta_1 FD_{i,t} y_{i,t-1} + \delta_2 FD_{i,t} y_{i,t-1}^2 + \mu_t + \eta_i + \varepsilon_{i,t}. \quad (2)$$

This specification has the advantage that it does not rely on fixed groups of countries, and ideally it would allow us to identify: (i) whether there is, within the sample, a low threshold of wealth below which the overall contribution of financial development to growth is negative; (ii) the level of wealth at which this contribution starts to decrease, that is, the level at which the marginal effect becomes negative; and (iii) whether there is a within-sample large level of wealth above which further financial development is detrimental for growth.

Even if the use of five-year averages reduces the potential effects of unaccounted short-term fluctuations, the lasting recessions associated with financial crises may still contaminate the

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<sup>8</sup>The groups are classified according to their GNI per capita: low income, \$735 or less; lower-middle income, \$736 - \$2,935; upper-middle income, \$2,936 - \$9,075; and high income, which are those countries that belong to the OECD and have a GNI greater than \$9,076; this excludes Mexico and Turkey from the OECD members. With this classification, 29 percent of the countries included in the sample are classified as low-income economies, 29 percent as lower-middle, 14 percent as upper-middle, and 28 percent as high income.

<sup>9</sup>There is an important drawback with these two classifications: they cannot account for changes over time across income groups. The estimation procedure uses first differences of the explanatory variables, which does not allow for variations among income groups.



finance-growth relationship. Accounting for the effect of banking crises in a low frequency data set presents some conceptual complications. The first complication is that frequently banking crises are preceded by lending booms with high growth and, after the usual recession associated with the crisis, economic growth can be lower than average. In addition, the duration of crises differ for each episode. Within a five year period several combinations are possible. We choose to account for the effects of banking crises on economic growth only for the duration of the crises as reported in Caprio and Klingebiel (1999, 2003). For this purpose we construct a frequency-of-crises dummy variable that takes the value of the proportion of the five-year period with banking crisis. For example, for the 1976-1985 Israeli banking crises, the dummy variable will take the value of 0.8 in the period 1976-1980, and 0.6 for the period 1981-1985. This dummy variable is then included to control for the growth losses generated by the crises.<sup>10</sup>

### ***Methodology***

The estimation of equation (1) poses some challenges. The first is the presence of time and country specific effects. Time effects can be accounted for by including period-specific dummies; however, the common methods to account for individual effects (within or difference estimators) are not valid since the resulting error term is correlated with initial income. The second problem is the likely joint endogeneity of most explanatory variables and economic growth. Simultaneous or reverse causation can generate biases in the estimates of the contribution of financial development to growth. In the following paragraphs we outline the econometric methodology used to control for unobserved country-specific effects and joint endogeneity in a dynamic model of panel data.

We use the Generalized-Method-of-Moments (GMM) estimators developed for dynamic models of panel data, introduced by Holtz-Eakin, Newey, and Rosen (1988), Arellano and Bond (1991), and Arellano and Bover (1995). Taking advantage of the panel nature of the data, these estimators are based on: (i) differencing regressors and/or instruments to control for unobserved effects, and; (ii) using lagged observations of the explanatory variables in levels as instruments (called “internal” instruments). After accounting for the time-specific effects and grouping all explanatory variables in a vector  $X$ , equation (1) can be rewritten as:

$$y_{i,t} = \alpha y_{i,t-1} + \beta' X_{i,t} + \eta_i + \varepsilon_{i,t}. \quad (3)$$

In order to eliminate the country-specific effect, we take first-differences of equation (3):

$$y_{i,t} - y_{i,t-1} = \alpha(y_{i,t-1} - y_{i,t-2}) + \beta'(X_{i,t} - X_{i,t-1}) + \varepsilon_{i,t} - \varepsilon_{i,t-1}. \quad (4)$$

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<sup>10</sup>It is also possible to hypothesize that further financial expansion in a context of financial vulnerability can be detrimental to economic growth, and include an interactive crises-financial development dummy. However this variable is consistently non significant.

The use of instruments must deal with the two challenges of the estimation: (i) the likely endogeneity of the explanatory variables, and (ii) the problem that, by construction, the new error term,  $\varepsilon_{i,t} - \varepsilon_{i,t-1}$ , is correlated with the lagged dependent variable,  $y_{i,t-1} - y_{i,t-2}$ . The instruments consist of previous observations of the explanatory and lagged dependent variables. Given that the estimation uses past values of the regressors, the estimation procedure is valid only under the assumption of weak exogeneity of explanatory variables; this is, they can be correlated with current and past realizations of the growth rate, but they are assumed to be uncorrelated with future realizations of the error term.<sup>11</sup>

Under the assumptions that (a) the error term,  $\varepsilon_{i,t}$ , is not serially correlated, and (b) the explanatory variables  $X$  are weakly exogenous, the GMM dynamic panel estimator uses the following moment conditions:

$$E[y_{i,t-s} \cdot (\varepsilon_{i,t} - \varepsilon_{i,t-1})] = 0 \text{ for } s \geq 2, t = 3, \dots, T, \quad (5)$$

$$E[X_{i,t-s} \cdot (\varepsilon_{i,t} - \varepsilon_{i,t-1})] = 0 \text{ for } s \geq 2, t = 3, \dots, T. \quad (6)$$

The GMM estimator based on these conditions is known as the difference estimator. Although this estimator has clear advantages over the classic difference estimator for panel data, it also has some statistical shortcomings. As shown by Alonso-Borrego and Arellano (1999) and Blundell and Bond (1998) when the explanatory variables are persistent over time, the use of lagged levels of these variables make weak instruments for the equations in differences.<sup>12</sup> In small samples, Monte Carlo experiments show that this weakness can produce biased coefficients. In addition, it influences the consistency of the estimator, because asymptotically the variance of the coefficient estimates rises.

Arellano and Bover (1995) and Blundell and Bond (1997) propose an estimator that reduces potential biases and imprecisions associated with the difference estimator. This new estimator combines in a system the regression in differences with the regression in levels. The instruments for the regression in differences are the same as above. The instruments for the regression in levels are the lagged differences of the corresponding variables. These lagged differences are valid instruments under the additional assumption of no correlation between the change in the explanatory variables and the country-specific effect (which does not rule out the possibility of correlation between the level of these variables and the country-specific effect). This assumption results from the following mean stationarity property:

$$E(y_{i,t+q} \cdot \eta_i) = E(y_{i,t+p} \cdot \eta_i) \text{ and } E(X_{i,t+q} \cdot \eta_i) = E(X_{i,t+p} \cdot \eta_i) \text{ for all } p \text{ and } q. \quad (7)$$

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<sup>11</sup>As Levine et al. (2000) mention, the assumption of weak exogeneity does not imply that expectations of future growth do not have an effect on current financial development, but only that unanticipated future shocks to economic growth do not influence financial development.

<sup>12</sup>Persistence in banking balances make this shortcoming particularly important for the financial intermediation variables.

The additional moment conditions for the second part of the system (the regression in levels) are:<sup>13</sup>

$$E[(y_{i,t-1} - y_{i,t-2}) \cdot (\eta_i + \varepsilon_{i,t})] = 0, \quad (8)$$

$$E[(X_{i,t-1} - X_{i,t-2}) \cdot (\eta_i + \varepsilon_{i,t})] = 0. \quad (9)$$

Using the moment conditions given by equations (5), (6),(8) and (9), we employ a Generalized Method of Moments (GMM) procedure to generate consistent estimates of the parameters of interest and their asymptotic variance-covariance matrix (Arellano and Bond 1991, and Arellano and Bover 1995). The equations for the system estimator are:

$$\hat{\theta} = \left( \bar{X}'Z\hat{\Omega}^{-1}Z\bar{X} \right)^{-1} \bar{X}'Z\hat{\Omega}^{-1}Z\bar{y}, \quad (10)$$

where  $\theta$  is the vector of coefficients  $(\alpha, \beta)$ ,  $\bar{y}$  is a vector of growth stacked first in differences and then in levels,  $\bar{X}$  is a matrix of explanatory variables (including the lagged dependent variable, that is,  $[y_{t-1}, X]$ ) stacked first in differences and then in levels,  $Z$  is the matrix of instruments derived from the moment conditions, and  $\hat{\Omega}$  is a consistent estimate of the variance-covariance matrix.<sup>14</sup>

The consistency of the GMM estimators depends on whether lagged values of the explanatory variables are valid instruments in the growth regression. We address this issue by considering two specification tests suggested by Arellano and Bond (1991) and Arellano and Bover (1995). The first is a Sargan test of over-identifying restrictions, which tests the overall validity of the instruments. Failure to reject the null hypothesis gives support to the model. The second test examines the null hypothesis that the error term is not serially correlated. As with the Sargan test, the model specification is supported when the null hypothesis is not rejected. In the system specification we test whether the differenced error term (that is, the residual of the regression in differences) is second-order serially correlated.<sup>15</sup> Second-order

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<sup>13</sup>Given that lagged levels are used as instruments in the differences specification, only the most recent difference is used as instrument in the levels specification. Using other lagged differences would result in redundant moment conditions. (see Arellano and Bover 1995).

<sup>14</sup>Arellano and Bond (1991) suggest the following two-step estimation procedure to improve the consistency and efficiency of the GMM estimates. The first step: assuming that the residuals  $\varepsilon_{i,t}$ , are independent and homoskedastic both across countries and over time, it is possible to obtain consistent estimators of the coefficients using an specific weighting matrix to determine the variance covariance matrix  $\hat{\Omega}^{-1}$ . The residuals obtained from the consistent estimated coefficient of the first step are used to provide a new consistent estimate of the variance-covariance matrix of the moment conditions. This matrix is then used to re-estimate the parameters of interest (i.e. second-step estimates). Asymptotically, the second-step estimates are superior to the first-step ones insofar as efficiency is concerned.

<sup>15</sup>By construction, first-order serial correlation of the differenced error term is expected even if the original error term  $\varepsilon_{i,t}$  is uncorrelated, unless the latter follows a random walk.

serial correlation of the differenced residual would indicate that the original error term  $\varepsilon_{i,t}$  is serially correlated and follows a moving average process at least of order one. This would reject the appropriateness of the proposed instruments (and would call for higher-order lags to be used as instruments).

## 2.2 Estimation Results, Wealth Effects.

For comparison purposes, we first estimate the model without allowing for wealth effects in the financial development growth relationship. The results of this estimation are summarized in Table 4 in Appendix B. The coefficients of the control variables, with the exception of inflation, are significant and have the expected sign. As in previous studies, the results show that financial depth has positive and significant growth effects. Moreover, when we control for the output loss generated by banking crises, there is an important increase in the contribution of financial intermediation to income per capita growth.

Table 5 presents the estimation results with wealth effects using the World Bank classification of countries. Financial development exerts a positive and significant effect on growth for all income groups; the exception is when financial development is proxied using liquid liabilities, in the low-income group of countries this contribution is statistically insignificant. Moreover, for both indicators of financial depth, there is evidence that low-income countries have a smaller benefit of further financial development than middle-income and high-income countries; when we control for banking crises, this benefit in low income countries is around a third of the effect in the other income groups. In addition, Table 5 contains the results of hypothesis tests of whether there are significant differences or not in the growth effects of financial intermediation for the other income groups. When credit to the private sector is used, we cannot reject that for middle-income and high-income groups the effect is the same. However, when we use liquid liabilities, high-income OECD economies enjoy a significantly higher growth gain from financial intermediation than middle-income countries.

Although the previous estimation provides some evidence of wealth effects in the finance-growth relationship, the income thresholds defined by the World Bank classification are not necessarily the appropriate ones for this relationship. We group countries into deciles according to their average initial income. Table 6 reports the growth regression results using this classification. These results depict a clearer picture of the existence of wealth effects in the contribution of financial development to economic growth: (i) for the two lowest deciles of income per capita, this contribution is negative; (ii) this contribution appears to be increasing in the level of income (except for a drop in the middle of the distribution). Figures 1 and 2 in Appendix B summarize the coefficients of financial development in the growth equation, along with their 95 percent confidence interval.

The results of the decile classification suggest that we can approximate the effect of financial intermediation on growth by a linear quadratic interaction between financial development and the initial level of income per capita. Table 7 presents the results of these estimations. The parameters of financial development are highly significant for both indicators of financial depth, and their signs suggest the existence of three thresholds of income per capita: (i) a low level of income, below which financial development is detrimental to income growth ( $y_o^1$ ); (ii) a threshold when the effect is maximum, and further increases in financial depth have a lower contribution to growth ( $y_o^2$ ); and (iii) a high-income threshold where the contribution becomes negative ( $y_o^3$ ). Table 7 also reports these three thresholds for both proxies of financial development. The low income threshold  $y_o^1$  for private credit is \$131.2;<sup>16</sup> 4 countries have observations below this threshold, for a total of 10 observations (represent only 1.5% of the sample). For liquid liabilities the threshold  $y_o^1$  is \$167.7; there are 7 countries with a total of 22 observations (3.3% of the sample). The second thresholds  $y_o^2$  are \$3,835.6 for private credit and \$4,711.5 for liquid liabilities. In the region defined between the first and the second threshold there are 61 countries and 428 observations (64.5% of the sample) for private credit, and 61 countries and 441 observations (66.4% of the sample) for liquid liabilities. The third thresholds  $y_o^3$  are well above the maximum level of income per capita in the sample (\$112,155.1 and \$132,382.9 for private credit and liquid liabilities, respectively, while the maximum observation is \$45,952). Thus, there cannot be any inference from the third threshold  $y_o^3$ .

In all the different specifications used, the Sargan tests and the second-order correlation tests indicate that we cannot reject the validity of the moment conditions assumed for the estimation.

In the rest of the paper we develop a model of financial intermediation to offer an explanation of the wealth effects observed in the estimation.

### 3 The Basic Model

The economy consists of an infinite sequence of overlapping generations. In each period, a generation, composed by a continuum of ex-ante identical agents with unit mass, is born; there is no population growth.

Agents live for two periods. They have an endowment of one unit of labor during the first period of their lives, which they supply inelastically. Agents do not value consumption when they are young. During the second period of their life they are subject to a time preference shock. With probability  $\pi$ , an agent only values consumption when middle aged (the beginning of her second period), and becomes an *early consumer*. With probability  $(1 - \pi)$ , she only

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<sup>16</sup>All figures are measured in purchasing power parity at 1995 international dollars.

values consumption when old (the end of her second period) and becomes a *late consumer*. The shock is stochastically independent across agents, and is private information to the agent. Preferences of an agent that belongs to generation  $t$  are:

$$U(c_t^t, c_E^t, c_L^t) = \theta u(c_E^t) + (1 - \theta)u(c_L^t) \quad (11)$$

$$\text{with } \theta = \begin{cases} 1 & \text{with probability } \pi \\ 0 & \text{with probability } (1 - \pi) \end{cases}, \text{ and} \\ u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln(c) & \text{if } \sigma = 1 \end{cases} \quad (12)$$

where  $c_E^t, c_L^t \geq 0$ , are the levels of early and late consumption at  $t + 1$  of an agent born at  $t$ .

There is one good, used for consumption and investment. There are two technologies available. First, there is a storage technology that uses the good as the only input and, for each unit invested at  $t$ , gives a return of one unit in any sub-period of  $t + 1$ . There is also a long-term technology with a Cobb-Douglas production function, which uses labor  $l$  and capital  $k$  as inputs. Capital fully depreciates after being used in production. If the technology is left until full maturity (the end of the period), it gives the return:  $F(k, l) = Ak^\beta l^{1-\beta}$ . Since the unit of labor is supplied inelastically, the capital intensive production function can be expressed as:

$$f(k) \equiv F(k, 1) = Ak^\beta \quad (13)$$

This production can be prematurely liquidated, with a liquidation cost. In this case, the product generated is a fraction  $\gamma$ ,  $0 < \gamma < 1$ , of the full return at maturity,  $\gamma f(k)$ . Hence, this liquidation cost is expressed in terms of output and not in terms of capital as  $\gamma f(k)$ .<sup>17</sup>

Factor markets are competitive, and each input is paid its realized marginal product. However, the realized marginal product depends on the financial arrangements in the economy because it depends on the proportion of long-term projects liquidated.

Wages received at the end of period  $t$  represent the unique source of wealth for members of the generation. After receiving wages, agents make their investment decisions before observing the realization of their liquidity shock.<sup>18</sup> There is an initial generation endowed with  $w_0 > 0$  units of the consumption good.

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<sup>17</sup>This assumption makes the relative marginal returns of a long project left until maturity and one liquidated prematurely a constant ( $\frac{f'(k)}{\gamma f'(k)} \equiv \frac{1}{\gamma}$ ).

<sup>18</sup>We abstract from the consumption-saving decision to stress the choice among assets with different liquidity. Agents do not value consumption when young, the consumption-saving decision at  $t$  is trivial, and they will invest their full wealth either directly in the two technologies (autarky), or as bank deposits (financial intermediation).

## 4 Financial Autarky

In this section we find the benchmark solution of financial autarky. This benchmark will be used to analyze the growth and welfare costs and benefits of financial intermediation.

Under financial autarky, young agents make their investment decision between storing goods and investing in capital on their own. We assume that each worker supplies her unit of labor to a continuum of representative firms with mass  $m \in (0, 1]$ .<sup>19</sup> Under this assumption, young workers are paid a wage equal to the expected marginal product of labor  $w_{t+1} = (1 - \beta)[\pi\gamma + 1 - \pi]f(k_t)$  and, at the same time, the investors (old agents) receive the marginal product of their investment-liquidation decision ( $\gamma\beta f(k)$  if early consumer and  $\beta f(k)$  if late).

### 4.1 The Optimal Individual Investment Decision

We start by analyzing the investment problem of a representative member of an arbitrary generation, and find her optimal portfolio decision as a function of her level of wealth ( $w$ ).

Define the return of holding the long asset as the function  $h(k) \equiv \beta f(k)$ . The marginal return of the long investment is  $h'(k)$  if the investment is maintained until full maturity, and  $\gamma h'(k)$  if it is liquidated prematurely. Let's define the two following capital levels:

$$\begin{aligned} \underline{k} & \text{ such that } \gamma h'(\underline{k}) = 1 \\ \bar{k} & \text{ such that } h'(\bar{k}) = 1 \end{aligned}$$

Since labor is inelastically supplied, the long-term asset presents diminishing returns to capital. For low levels of capital ( $k < \underline{k}$ ), the marginal return of the long-term asset, even when it is prematurely liquidated, exceeds the marginal return of the storage technology ( $\gamma h'(k) \geq 1$ ). Beyond some level of investment in the long asset ( $k > \bar{k}$ ), its marginal return is lower than one ( $h'(k) \leq 1$ ).

Under financial autarky, investment in capital is risky because its return depends on the realization of the liquidity shock. At the end of their first period, for any given level of wealth  $w > 0$ , a typical agent of generation  $t$  chooses investment in the long technology  $k$  to maximize:

$$\pi u(c_E) + (1 - \pi)u(c_L) \tag{14}$$

$$\text{subject to } 0 \leq k \leq w \tag{15}$$

where  $c_E = w - k + \gamma h(k)$ ,  $c_L = w - k + h(k)$ , and the difference between wealth and capital ( $w - k$ ) is investment in the storage technology.

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<sup>19</sup>This mass  $m$  can be arbitrarily close to zero; however, this is equivalent to assuming that every worker works for all firms. This assumption is not essential for the results of the paper, and it helps to avoid heterogeneity in the size of firms, something which would unnecessarily complicate the dynamics of the model.

The following proposition characterizes the optimal solution for members of any given generation under financial autarky:

**Proposition 4.1** *For every level of wealth  $w$ , the unique solution  $(k_{opt}(w), c_E(w), c_L(w))$  to the agent's problem under autarky is characterized by the following conditions:*

*There exists a unique level of wealth  $w^* \in (\underline{k}, \bar{k})$  defined by  $\frac{u'(\gamma h(w^*))}{u'(h(w^*))} = \frac{(1-\pi)(h'(w^*)-1)}{\pi(1-\gamma h'(w^*))}$  such that:*

$$(i) \text{ if } w \leq w^* \text{ then } \begin{cases} k_{opt}(w) = w \\ c_E = \gamma h(w) \text{ (corner solution)} \\ c_L = h(w) \end{cases}$$

$$(ii) \text{ if } w > w^* \text{ then } \begin{cases} 0 < k_{opt}(w) < w \\ c_E = w - k_{opt} + \gamma h(k_{opt}) \text{ (interior solution)} \\ c_L = w - k_{opt} + h(k_{opt}) \end{cases}$$

where  $k_{opt}(w)$  in (ii) is defined by:  $\frac{u'(c_E)}{u'(c_L)} = \frac{(1-\pi)(h'(k_{opt})-1)}{\pi(1-\gamma h'(k_{opt}))}$

**Proof.** See Appendix A. ■

The intuition for this result is the following: the expected marginal benefit of investing in capital is  $(1 - \pi) u'(c_L) (h'(k) - 1)$ , while the expected marginal cost is  $\pi u'(c_E) (1 - \gamma h'(k))$ . For low levels of wealth it is possible that  $\gamma h'(w) > 1$ , making a corner solution with full investment in capital optimal. The threshold  $w^*$  is defined as the level of wealth at which full investment in capital equates marginal benefit and marginal cost. This threshold defines two regions of the optimal portfolio under financial autarky. When wealth is lower than  $w^*$ , agents invest their full wealth in capital; above this threshold they invest in both assets.<sup>20</sup> The optimal capital choice  $k_{opt}(w)$  is strictly increasing and concave in  $w$ . The proportion of wealth invested in the long asset,  $\frac{k_{opt}(w)}{w}$ , is constant for  $w \leq w^*$ , and it is strictly decreasing in  $w$  for  $w > w^*$ .<sup>21</sup>

Investment under autarky is inefficient. The first inefficiency arises because in poor economies, self-insured agents invest, as precautionary savings, their full wealth in capital beyond the point where it is efficient to do so. When the marginal return of the short asset exceeds the marginal liquidation value of the long asset,  $(\gamma h'(w) < 1)$ , it would be efficient to start investing in the short asset. However,  $w^* > \underline{k}$  means that for levels of wealth between  $\underline{k}$  and  $w^*$  agents are over-investing in the long asset ( $k = w$ ). The second inefficiency occurs for levels of wealth greater than  $w^*$ . Early consumers are forced to liquidate productive investments to cover their liquidity needs, while late consumers finance some of their consumption

<sup>20</sup>The threshold level of wealth  $w^*$  is defined by:  $h'(w^*) = \frac{\pi + (1-\pi)\gamma^\sigma}{\gamma\pi + (1-\pi)\gamma^\sigma}$

<sup>21</sup>See appendix A for the derivation of the properties of the optimal capital choice function



using the less productive liquid investment. The impossibility of receiving insurance through financial markets generates an inefficient liquidation of the long investment.<sup>22</sup> This liquidation implies that capital is bounded above by  $k_{\max}$  ( $h'(k_{\max}) = \frac{1}{\pi\gamma + (1-\pi)} > 1$ ), while it is efficient to invest up to the higher level  $\bar{k}$  ( $h'(\bar{k}) = 1$ ).<sup>23</sup>

For low levels of wealth, when agents are investing only in the long technology, liquidity self insurance is constant ( $\frac{c_E}{c_L} = \gamma$ ). For higher levels of wealth, when agents are investing in both assets ( $w > w^*$ ), an increase in wealth reduces the gap between early and late consumption. Nevertheless, full liquidity risk insurance is not possible under financial autarky.

## 4.2 The Dynamics of Wealth, Capital and Consumption Under Autarky

In this part, we characterize the steady state of the economy and study the evolution of wealth, capital and consumption towards this stationary equilibrium. Since capital fully depreciates after it is used, the connection between the individual problem and the dynamics of the intertemporal model is given by wages of the next generation.

At  $t = 1$  an initial old generation exists, endowed with  $w_0 > 0$ , and we assume that this wealth has already been invested according with the optimal conditions stated above. The timing convention is that a generation born at  $t$  makes its optimal choice of capital  $k_{t+1}$ , therefore:

$$\begin{aligned} w_t &= F^a(w_{t-1}) = (1 - \beta)(\pi\gamma + 1 - \pi)f(k(w_{t-1})) \\ k_t &= k(w_{t-1}) = k_{opt}(w_{t-1}) \end{aligned} \tag{16}$$

Since  $f(\cdot)$ ,  $k(\cdot)$  are continuous, increasing and concave, and  $k_{opt}(0) = 0$  and  $\lim_{w \rightarrow \infty} k_{opt}(w) = k_{\max}$ ,  $F^a(w)$  is a continuous, increasing and concave function. Moreover,  $F^a(w) = w$  admits a unique stable non-zero root. For any positive initial wealth  $w_0 > 0$ , this implies the following dynamics:

**Proposition 4.2** *Convergence and the steady state under financial autarky.*

(i) *The economy converges monotonically and asymptotically towards a unique stable steady state  $\bar{w}^a > 0$  and  $k(\bar{w}^a)$ .*

(ii) *The steady state has investment in both assets ( $\bar{w}^a > w^*$ ) if and only if  $\frac{1-\beta}{\beta^2} \frac{f'(w^*)}{f'(k_{\max})} > 1$ .<sup>24</sup>*

<sup>22</sup>This is the inefficiency of autarky that in DD motivates a role for banks as efficient liquidity providers.

<sup>23</sup>A risk neutral consumer will invest in a risky asset up to the point where the actuarial marginal return of the risky asset equates the marginal return of the safe asset; a risk averse consumer will invest always strictly less than this level and  $\lim_{w \rightarrow \infty} k_{opt}(w) = k_{\max}$

<sup>24</sup>The position of the steady state between the two regions, defined by  $w^*$ , gives a qualitative characterization of the steady state but does not determine its level, because the threshold  $w^*$  is a function of the parameters.

**Proof.** See Appendix A. ■

Figure 4 in Appendix C presents the dynamics of wealth.<sup>25</sup> It shows how the economy converges to its stationary equilibrium  $\bar{w}^a$ . Beyond the threshold  $w^*$ , the rate of growth decreases rapidly, overinvestment in the previous region has already exhausted the marginal returns on capital, and the constant level of liquidation  $\pi$  becomes more and more costly in terms of growth. Finally, as both consumptions ( $c_E$ ,  $c_L$ ) are monotonically increasing in wealth, their dynamics follow the dynamics of wealth.

## 5 Intra-generational Risk-Sharing and Financial Intermediaries.

Uncertainty in this economy pertains to the liquidity needs of individuals and is idiosyncratic. Therefore, welfare gains are possible via a mechanism of liquidity preference insurance. Diamond and Dybvig (1983) show that, in the absence of banking crises, the first-best solution can be implemented through a competitive banking system with standard deposit contracts. Nevertheless, if banks offer the first-best contract, the first-best equilibrium is one of two possible equilibria. In the second equilibrium there is a coordination of beliefs that banks will not be able to service all the deposits at the late sub-period, driving a total run on the bank at the beginning of  $t + 1$ . In this paper we abstract from these bank-run equilibrium, and assume that banks can offer optimal risk sharing.<sup>26</sup>

### 5.1 Generation $t$ Optimal Risk-Sharing

In this model, a financial intermediary is a mutual, in which members of the same generation pool their resources. After having worked in the first period, agents deposit their full wealth  $w$  in the bank. On the aggregate, all uncertainty disappears: by the law of large numbers, the bank knows that a proportion  $\pi$  of agents will demand their deposits in the early sub-period, and a proportion  $(1 - \pi)$  in the late sub-period. Therefore, the bank can select the investment portfolio ( $k$ ), the optimal proportion of the long asset that it will liquidate prematurely ( $\lambda$ ), as well as the fixed payoffs for early and late consumers to form the most efficient match between liquidity needs of agents and the highest returns of the assets. In addition, the bank centralizes production and pays a wage to the following generation ( $w'$ ) equal to the realized marginal product of labor  $w' = (1 - \beta)(\lambda\gamma + 1 - \lambda)f(k)$ .

At any period  $t$ , and for any given level of deposits (wealth  $w > 0$ ), a representative bank chooses  $k$ ,  $\lambda$ ,  $c_E$  and  $c_L$  to maximize expected utility of a representative current depositor:

<sup>25</sup>The parameters used in the simulation are presented in Appendix C.

<sup>26</sup>This contract is equivalent to the Diamond and Dybvig solution, or unconstrained optimal risk sharing in which either types are observable or the probability of the sunspot equilibrium is zero. In Gaytan and Ranciere (2005) we explore the consequences of having bank runs with positive probability.

$$\pi u(c_E) + (1 - \pi)u(c_L) \text{ subject to} \quad (17)$$

$$0 \leq k \leq w \quad (18)$$

$$0 \leq \lambda \leq 1 \quad (19)$$

$$\pi c_E \leq w - k + \lambda \gamma h(k) \quad (20)$$

$$\pi c_E + (1 - \pi)c_L \leq (1 - \lambda)h(k) + w - k + \lambda \gamma h(k) \quad (21)$$

$$c_E \leq c_L \quad (22)$$

Equation (20) is the resource constraint at the early sub-period of  $t + 1$ ; for serving agents with early liquidity needs, the bank can liquidate the short asset ( $w - k$ ) and a proportion  $\lambda$  of the long-term technology. Equation (21) is the resource constraint at the late sub-period of  $t + 1$ ; the bank uses all its remaining assets to service late consumers. Agents still have access to the storage technology; therefore, the bank must offer a higher return to patient consumers so that they do not withdraw their funds at the early stage and store it up to the late stage. This is reflected in the incentive compatibility constraint (22).

The bank provides the optimal level of liquidity (given by the liquidation policy of the long asset ( $\lambda$ ) and investment in the short technology ( $w - k$ )) and by the provision of optimal liquidity insurance ( $\frac{c_E}{c_L}$ ). Liquidity provision can be measured by the share of liquidity requirements of early consumers covered using the short asset ( $\frac{w-k}{\pi c_E}$ ).

The complete characterization of the optimal solution of the problem is stated in the following proposition (see Appendix A for proofs and details).

**Proposition 5.1** *For each level of wealth, the unique solution  $(k_{opt}(w), \lambda_{opt}(w), c_E(w), c_L(w))$  to the optimal risk sharing problem is characterized by the following conditions:*

	$w$	$F.O.C$	$k_{opt}$	$\lambda_{opt}$	$c_E$	$c_L$
<b>A</b>	$0 < w \leq \underline{k}$	$\frac{u'(c_E)}{u'(c_L)} = \frac{1}{\gamma}$	$w$	$\lambda^*$ constant	$\frac{\lambda^* \gamma h(w)}{\pi}$	$\frac{(1-\lambda^*)h(w)}{1-\pi}$
<b>B</b>	$\underline{k} \leq w \leq \tilde{w}$	$\frac{u'(c_E)}{u'(c_L)} = \frac{1}{\gamma}$ $\gamma h'(k) = 1$	$\underline{k}$	$0 \leq \lambda(w) \leq \lambda^*$	$\frac{w-\underline{k}+\lambda\gamma h(\underline{k})}{\pi}$	$\frac{(1-\lambda)h(\underline{k})}{1-\pi}$
<b>C</b>	$\tilde{w} \leq w \leq \hat{w}$	$\frac{u'(c_E)}{u'(c_L)} = h'(k)$ $1 < h'(k) < \frac{1}{\gamma}$	$\underline{k} < k_{opt} < \bar{k}$	$\lambda = 0$	$\frac{w-k_{opt}}{\pi}$	$\frac{h(k_{opt})}{1-\pi}$
<b>D</b>	$w \geq \hat{w}$	$\frac{u'(c_E)}{u'(c_L)} = 1$	$\bar{k}$	$\lambda = 0$	$w - \bar{k} + h(\bar{k})$	$= c_E$

where  $\underline{k}$  and  $\bar{k}$  were defined in section 3.1;  $\tilde{w}$ ,  $\hat{w}$ ,  $\lambda^*$  and  $\lambda_{opt}(w)$  are defined by:

$$\tilde{w} = \underline{k} \left( 1 + \frac{\pi \gamma^{1/\sigma}}{(1-\pi)\gamma\beta} \right) \quad (23)$$

$$\hat{w} = \bar{k} \left( 1 + \frac{\pi}{\beta(1-\pi)} \right) \quad (24)$$

$$\lambda^* = \frac{\pi \gamma^{\frac{1}{\sigma}}}{\pi \gamma^{\frac{1}{\sigma}} + (1-\pi)\gamma} \quad (25)$$

$$\lambda(w) = \lambda^* - (1-\lambda^*)\beta \frac{w - \underline{k}}{\underline{k}} \quad (26)$$

The optimal portfolio and liquidation policy define four regions (**A** to **D**) delimited by the income thresholds  $\underline{k}$ ,  $\tilde{w}$  and  $\hat{w}$ :

**Region A: No investment in short-term technology, no liquidity provision and constant level of liquidity insurance.**

Region A is defined as the range of wealth ( $w \leq \underline{k}$ ) for which investing in capital dominates investment in the short asset. The optimal portfolio is the same as under autarky, and all wealth is invested in the long-term asset. In this case, the bank is not providing extra liquidity, but only acting as a liquidity insurance provider. The bank is liquidating prematurely a fixed proportion of the long technology  $\lambda^*$ . Optimal liquidation increases with the degree of risk aversion, which reflect agents' preference for liquidity insurance. For sufficiently high risk aversion ( $\sigma > 1$ ), optimal liquidation is higher than aggregate liquidation under autarky  $\pi$ . In that case, higher liquidity insurance imposes the cost of lower output and rate of growth than under autarky.

**Region B: Constant level of long-term investment, reduction of early liquidation, increasing liquidity provision and constant level of liquidity insurance**

For  $\underline{k} \leq w \leq \tilde{w}$  the financial intermediary invests in both assets and provides extra liquidity. Investment in capital is kept fixed at  $\underline{k}$ , the bank keeps the marginal return of the long asset fixed at a high level, so that its marginal return, when liquidated prematurely, equals the marginal return of storing the good.<sup>27</sup> Even if the capital stock is fixed, output can grow because the bank is liquidating a decreasing proportion of the long asset. This effect would be reflected as an increase in total factor productivity. The bank starts using the liquid asset as a source of liquidity to pay out early consumers, reducing premature liquidation of the long asset. Late consumers are served using an increasing proportion of the fully-matured output. However, since the bank is still using premature liquidation to serve early agents, liquidity insurance is kept fixed at the same level as in region A.

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<sup>27</sup>Two assets can be used to serve the same type of consumption only if their marginal returns are the same at the required moment of liquidation.

**Region C: No liquidation of long term investment, increasing investment in both assets, constant provision of liquidity and increasing level of liquidity insurance.**

When wealth has crossed certain threshold ( $w \geq \tilde{w}$ ), the financial intermediary stops using the long asset to serve early consumers. All the long technology is left until full maturity ( $\lambda = 0$ ) to serve late consumers, and investment in capital can increase again. Increasing investment in capital over this region implies that the marginal return of the asset used to serve late consumers decreases relative to the marginal return of the asset used for early consumption, making optimal to increase liquidity insurance ( $\frac{u'(c_E)}{u'(c_L)} = h'(k)$  implies that  $\frac{c_E}{c_L}$  is increasing in  $w$ ).<sup>28</sup>

**Region D: Full insurance, provision of liquidity for late consumers.**

For high levels of wealth ( $w > \hat{w}$ ), investment in the long asset is kept fixed at the level ( $\bar{k}$ ) at which the marginal return of both assets is equalized ( $h'(\bar{k}) = 1$ ), and it is optimal for the bank to provide full insurance by offering the same level of consumption for early and late withdrawers.<sup>29</sup> Early consumption is always served using the storage technology, yet, when wealth is strictly greater than the threshold  $\hat{w}$ , late consumption is served using both assets and implies a transfer of the liquid asset to the late subperiod.

The regions of the optimal risk sharing solution imply that liquidity and liquidity insurance provided by the bank are endogenous, and they increase with the level of deposits (wealth) received in each period. A larger economy will have higher liquidity, less inefficient liquidation, and better insurance among different liquidity needs. In the following section we analyze the dynamics of this economy.

## 5.2 The Dynamics of Wealth, Capital and Consumption

Financial intermediaries can implement welfare-improving allocations for consumers of any given generation. In this section we analyze the dynamic implications of this financial arrangement. We show that the economy has a unique stationary equilibrium ( $\bar{w}^b$ ) and analyze the dynamics of wealth, capital and consumption.

At any period, wages  $w_t$ , capital  $k_t$  and optimal liquidation  $\lambda_t$  are defined by the following

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<sup>28</sup>The faster the increase in capital, the faster is the improvement in liquidity insurance.

<sup>29</sup>Perfect insurance is possible because there are decreasing returns to capital. When the returns to capital are constant, the provision of perfect insurance require some form of intergenerational transfer, see Fulgueri and Rovelli (1998).

relations:

$$\begin{aligned}
w_t &= F^b(w_{t-1}) = (1 - \beta)(\lambda(w_{t-1})\gamma + 1 - \lambda(w_{t-1}))f(k(w_{t-1})) \\
k_t &= k_{opt}(w_{t-1}) \\
\lambda_t &= \lambda_{opt}(w_{t-1})
\end{aligned} \tag{27}$$

The properties of  $F^b(w_{t-1})$  (see Appendix A) imply the following properties of the dynamics of wealth.

**Proposition 5.2** *Convergence and the steady state under financial intermediation.*

- (i) *For any initial wealth  $w_0 > 0$ , the economy with financial intermediaries has a unique, stable steady state  $\bar{w}^b > 0$  and  $k^{ss} = k(\bar{w}^b)$ . The steady state is defined by  $F^b(\bar{w}^b) = \bar{w}^b$ .<sup>30</sup>*
- (ii) *For any positive level of initial wealth  $w_0 > 0$ ,  $w_0 \neq \bar{w}^b$ , the economy converges monotonically towards its steady state  $\bar{w}^b$ :*
  - *if  $\bar{w}^b < \hat{w}$  this convergence is asymptotic.*
  - *if  $\bar{w}^b > \hat{w}$  the economy converges to the steady state in finite time.*

**Proof.** See Appendix A ■

The dynamics of early and late consumption under financial intermediation follow the dynamics of wealth. When wealth is increasing (decreasing) over time, both types of consumption are increasing (decreasing). In regions A,B, and D, keeping liquidity insurance constant implies that both early and late consumptions must be growing at the same rate. By contrast, in Region C, liquidity insurance is increasing in capital  $\frac{c_E}{c_L} = \left(\frac{1}{h'(k(w))}\right)^{\frac{1}{\sigma}}$ . Thus, if wealth is growing over region C, the rate of growth of early consumption must be greater than the rate of growth of late consumption. Positive growth over this region implies improving insurance.

## 6 The Consequences of Financial Intermediation.

In this section, we analyze the implications of the presence of financial intermediaries by comparing the consequences of the autarkic and banking solutions for welfare and economic growth (the variables of these solutions are indexed by  $\{a, b\}$ , respectively).

The fundamental source of inefficiency under financial autarky is the absence of a mechanism for pooling liquidity risk, making necessary that each agent insures herself against such risk. Since the bank can implement optimal risk sharing among current depositors, for a given

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<sup>30</sup>The position of the steady state among the different regions, as in autarky, does not determine its level.

level of wealth, welfare for the current generation is necessarily higher than under financial autarky.

Under banking, liquidation of the long technology is optimally chosen and it allows to distribute a fraction of the high returns of this asset to early consumers. Once the economy has attained certain level of wealth, it gradually reduces this liquidation to zero. Conversely, under autarky aggregate liquidation of the long term technology remains a constant fraction  $\pi$ . Since the marginal products of capital and labor depend inversely on the level of liquidation, its suppression is central to explain why financial intermediaries can attain a higher steady state level of wealth.

The following proposition compares the optimal capital choice under financial autarky and financial intermediation.

**Proposition 6.1** *A unique wealth level  $m \in (\max\{w^*, \tilde{w}\}, \hat{w})$  exists, such that the optimal capital choice under financial intermediation and the optimal choice under autarky are equal for  $w = m$ . For  $\underline{k} < w < m$ , the capital choice is higher under autarky than under financial intermediation (**overinvestment**). For  $w > m$ , the capital choice is lower under autarky than under financial intermediation (**underinvestment**)*

**Proof.** see Appendix A ■

Figure 3 compares the optimal capital choice and liquidation policy of both regimes for each level of wealth. In autarky, over-investment for low levels of wealth is a direct consequence of precautionary saving behavior of agents that face idiosyncratic liquidity risk. At low levels of wealth  $w \in (\underline{k}, w^*)$ , agents overinvest in the long asset when it would be efficient to start investing a fraction of wealth in the short asset and reduce liquidation. In region B,  $w \in (\underline{k}, \tilde{w})$ , the marginal return of capital under financial autarky decreases, whereas, under financial intermediation it increases because capital is kept fixed and liquidation is reduced. Consequently, investment under financial intermediation increases faster in Region C ( $w \in [\underline{k}, w^*]$ ). Even when both economies invest the same level of capital ( $w = m$ ), growth and the marginal returns of factors are higher for financial intermediation since there is no costly liquidation. After this point, aggregate underinvestment in capital is a direct consequence of the low marginal return to capital due to inefficient liquidation.<sup>31</sup>

We now compare the dynamics of wealth under financial autarky and financial intermediation. We focus on the dynamics of wealth rather than on the dynamics of capital, because

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<sup>31</sup>Notice that our analysis of the inefficiency of financial autarky echoes the literature on precautionary savings in presence of uninsured idiosyncratic risks and liquidity constraints (Aiyagari (1994), Jappelli and Pagano (1994), Calvet-Angeletos (2001)).

wealth reflects both the optimal capital choice and the optimal liquidation policy of the previous generation. For any level of wealth  $w$ , it is possible to decompose the growth consequences of the financial regimes using the ratio of output of the long term technologies:

$$\frac{F^a(w)}{F^b(w)} = \frac{1 - \pi(1 - \gamma)}{1 - \lambda(w)(1 - \gamma)} \left( \frac{k^a(w)}{k^b(w)} \right)^\beta \quad (28)$$

The expression above can be written in terms of growth rates as:

$$g^a(w) - g^b(w) \approx \underbrace{\ln(1 - \pi(1 - \gamma)) - \ln(1 - \lambda(w)(1 - \gamma))}_{\text{Liquidation Effect (A)}} + \beta \underbrace{\left[ \ln k^a(w) - \ln k^b(w) \right]}_{\text{Investment Effect (B)}}$$

The relative growth performance depends on the combination of a liquidation effect (A), which reflects the different level of liquidation ( $\lambda(w)$  vs.  $\pi$ ), and an investment effect (B), which reflects the difference in capital choice. In terms of growth accounting, the first effect expresses a “total factor productivity” gap and the second effect an investment gap. On the one hand, autarkic agents are overinvesting in capital on  $(\underline{k}, m)$ ; nevertheless, this investment effect vanishes gradually and it changes sign afterwards (see Fig 4a). On the other hand, the liquidation effect will eventually favor the economy with financial intermediaries as they reduce liquidation to zero (at  $\tilde{w}$ ). The strength and timing of these effects will crucially depend on the level of risk aversion of agents.

For  $w \leq \underline{k}$ , investment is identical in both regimes, and any difference in growth must come from differences in liquidation. For high risk aversion ( $\sigma > 1$ ), financial intermediaries provide high liquidity insurance through higher liquidation than autarky ( $\lambda^* > \pi$ ), and as a consequence the economy exhibits lower growth. By contrast, when agents are less risk averse ( $\sigma < 1$ ), the liquidation level is lower under financial intermediation ( $\lambda^* < \pi$ ), allowing a higher growth rate than under autarky.

For  $\underline{k} < w < m$ , there is a sufficiently low level of risk aversion ( $\sigma < \underline{\sigma} < 1$ ) below which lower liquidation by banks completely offsets the effect of autarkic precautionary savings, and the economy with financial intermediaries grows faster. For higher risk aversion, there will always be a range of wealth for which overinvestment in the autarkic economy, as precautionary savings, will translate into a higher growth performance.

For  $w \geq m$ , growth under financial intermediation is unambiguously higher, as investment is higher and no liquidation is optimal. The comparison in growth performance is summarized in the following proposition:



**Proposition 6.2** *it exists a  $\underline{\sigma} \in (0, 1)$*

- i) For high risk aversion ( $\sigma > 1$ ), there exists a unique level of wealth  $w_u \in (\underline{k}, m)$  such that for  $w > w_u$  ( $w < w_u$ ), the growth rate under financial intermediation is higher (lower) than the growth rate under autarky.*
- ii) For low risk aversion ( $0 < \sigma < \underline{\sigma}$ ), the growth rate under financial intermediation is strictly higher than the growth rate under autarky for all wealth levels.*
- iii) For intermediate risk aversion ( $\underline{\sigma} \leq \sigma \leq 1$ ), the growth rate under financial intermediation is higher than under financial autarky for  $w \in \mathfrak{R} - [w_L, w_u]$  with  $[w_L, w_u] \subset [\underline{k}, m)$*

**Proof.** *see Appendix A* ■

Figure 4 in Appendix C presents a comparison of the dynamics under both financial arrangements for a simulation of the economy (the parameters used in the simulation are presented in Table 8). From any level of wealth  $w_{t-1}$ , we can read the transitional dynamics towards the steady states ( $\bar{w}^a = F^a(\bar{w}^a)$  and  $\bar{w}^b = F^b(\bar{w}^b)$ ). This graph illustrates the growth cost of liquidity insurance: at early stage of economic development, the economy with financial intermediaries has a lower rate of growth. After the economy has crossed the threshold  $w_u$ , financial intermediation has a strictly growth enhancing effect. Figure 4 also illustrates the stage at which the development of a banking system starts to have crucial long-run effects. When the economy has enough resources to keep an increasing number of long-term projects until full maturity, financial intermediation has an increasing contribution to growth. This result replicates the empirical importance of financial intermediation for the growth perspectives of middle income or emerging economies, and can explain why they are willing to undertake the risk of increasing financial vulnerability by developing their financial systems (see Gaytan and Ranciere, 2005). For very rich economies  $w > \hat{w}$ , financial intermediation implements full liquidity insurance, which is obviously unattainable under financial autarky.

The mapping between the empirical results of Section 2 and the predictions of the theoretical model requires some warnings. In the first place, the model compares two extreme regimes: financial autarky and full financial intermediation. In reality, every country is located at some intermediate point between these regimes. In addition, the model concentrates only in the liquidity functions of financial intermediaries, while financial intermediaries perform several other growth-promoting activities. Nevertheless, the model can account for the observed wealth effects in the financial intermediation-growth relationship. Figure 5 summarizes the growth contribution of financial development in the model, by taking the difference between growth under financial intermediation and growth under autarky. Financial development has a negative effect on economic growth for low income economies because the optimal

intragenerational level of liquidity conflicts with the output maturity of productive investments. Middle-income countries enjoy higher growth rates, higher liquidity insurance and, at some point, higher growth than financial autarky. However, when the economy attains complete liquidity insurance, it is no longer optimal to increase investment in growth promoting technologies, and the positive contribution of financial intermediation declines.

## 7 Conclusion

The central empirical finding of the paper is that the effect of financial development on growth depends in a non-linear way on the level of income of countries. This effect is increasing and varies from negative in low-income countries to positive above a certain wealth threshold; it reaches a maximum at intermediate levels of wealth before declining for richer countries. This result is robust to the inclusion of controls for the growth costs associated with banking crises episodes.

We provide a theoretical explanation for this finding that relies on the growth consequences of the liquidity risk-sharing provided by financial intermediaries at different level of income in a neo-classical context. When countries are poor and returns to long-term projects high, even when liquidated, banks perform risk-sharing among agents by increasing the share of long-term projects. Risk-sharing implies then lower growth. This result is similar to the growth depressing effect of a reduction in precautionary saving in models with incomplete markets (e.g. Ayagari, 1994). As countries become richer it is optimal for banks to increase risk-sharing by using liquid investments to finance early consumption needs. In this case, risk-sharing implies lower liquidation, higher long-term investment and higher growth. In growth accounting terms, the reduction of liquidation corresponds to an increase in total factor productivity that result from bank's efficient match between the liquidity needs and preferences of agents and the timing of the highest returns of the technologies.

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# Appendix

## A The Model

### A.1 Financial Autarky

#### Proof of proposition 4.1 (The autarkic solution).

Let's define the value function :  $V(k, w) = \pi u(w - k + \gamma h(k)) + (1 - \pi)u(w - k + h(k))$ .

The optimal program is to find  $k_{opt}$  that maximizes  $V(k, w)$  subject to  $k \leq w$  (since  $\lim_{k \rightarrow 0} h'(k) = \infty$ ,  $k_{opt} > 0$ ).  $V(k, w)$  is twice continuous differentiable in  $k$  and strictly concave:

$$\frac{\delta^2 V(k, w)}{\delta k^2} = \left\{ \begin{array}{l} \pi (u''(c_E) (\gamma h'(k) - 1)^2 + u'(c_E) \gamma h''(k)) \\ + (1 - \pi) (u''(c_L) (h'(k) - 1)^2 + u'(c_L) h''(k)) \end{array} \right\} < 0 \quad (\text{A1})$$

We are maximizing a continuous strictly concave function over a compact set  $[0, w]$ , a maximum  $k_{opt}$  exists and is unique. Define  $A(k, w) = \frac{\delta V(k, w)}{\delta k}$  and  $\mu$ , the Lagrange multiplier associated with  $k \leq w$ .

- (i) First Order Conditions:  $A(k, w) = \frac{\delta V(k, w)}{\delta k} = \mu$ ;  $\left\{ \begin{array}{l} \text{if } k < w \text{ (interior solution) } A(k, w) = 0 \\ \text{if } k = w \text{ (corner solution), } A(k, w) > 0 \end{array} \right.$
- (ii)  $\frac{\delta A(k, w)}{\delta k} = \frac{\delta^2 V(k, w)}{\delta k^2} < 0$  and  $k \leq w \Leftrightarrow A(k, w) \geq A(w, w)$  (with strict inequality if  $k < w$ ).
- (iii)  $\frac{\delta A(w, w)}{\delta w} < 0$ , hence  $A(w, w) = 0$  admits a unique solution  $w = w^*$  defined by  $\frac{u'(\gamma h(w^*))}{u'(h(w^*))} = \frac{(1 - \pi)(h'(w^*) - 1)}{\pi(1 - \gamma h'(w^*))}$ .

Hence by (i),(ii) and (iii):

(interior and corner solution): for  $w = w^*$  :  $A(w^*, w^*) = 0$  and  $k_{opt} = w^*$ .

(corner solution): for  $w < w^*$  :  $A(w, w) > 0$  and  $k_{opt} = w$ .

(interior solution): for  $w > w^*$  :  $A(k_{opt}, w) = 0 > A(w, w)$  and  $k_{opt} < w$  (A2)

$$A(k_{opt}, w) = 0 \Leftrightarrow \frac{u'(c_E)}{u'(c_L)} = \frac{(1 - \pi)(h'(k) - 1)}{\pi(1 - \gamma h'(k))} \quad (\text{A3})$$

and the threshold  $w^*$  is defined by  $h'(w^*) = \frac{(1 - \pi)\gamma^\sigma + \pi}{(1 - \pi)\gamma^\sigma + \pi\gamma}$

#### Properties of the optimal capital choice function $k_{opt}(w)$

For  $w \leq w^*$ ,  $k_{opt}(w) = w$ , and the properties for  $w \leq w^*$  are obvious.

For  $w \geq w^*$ ,  $k_{opt}$  is uniquely defined by  $A(k_{opt}, w) = 0$

**P1:**  $k_{opt}(w)$  is continuous on  $[w^*, \infty)$ , differentiable in  $w$  on  $[0, w^*) \cup (w^*, \infty)$  and right and left differentiable at  $w^*$ .  $A(k, w)$  is continuously differentiable in  $k$  and  $w$ .

**P2:**  $k_{opt}(w)$  is strictly increasing in  $w$

$$\text{for } w \geq w^* : \frac{\delta k_{opt}(w)}{\delta w} = - \frac{\frac{\delta A(k, w)}{\delta w}}{\frac{\delta A(k, w)}{\delta k}} \Big|_{A(k, w) = 0} \quad (\text{A4})$$

$$\begin{aligned} \frac{\delta A(k, w)}{\delta k} &< 0 \text{ by (A1)} \\ \frac{\delta A(k, w)}{\delta w} &> 0 \Leftrightarrow \frac{u''(c_L)}{u''(c_E)} < \frac{u'(c_L)}{u'(c_E)} \Leftrightarrow c_L > c_E \end{aligned} \quad (\text{A5})$$

**P3:**  $k_{opt}(w)$  is concave in  $w$  and strictly concave for  $w \geq w^*$ .

**P4:**  $\frac{k_{opt}(w)}{w}$  is non increasing in  $w$ , and strictly decreasing for  $w \geq w^*$

$$\text{for } w \geq w^* : \frac{\delta^2 k_{opt}(w)}{\delta w^2} = \frac{\partial \left( -\frac{\frac{\delta A(k, w)}{\delta w}}{\frac{\delta A(k, w)}{\delta k}} \right)}{\partial w} \Big|_{A(k, w)=0} \quad (\text{A6})$$

Using (A3) and the properties of the utility function, (A4) can be rewritten as:

$$\frac{\delta k_{opt}(w)}{\delta w} = \frac{-\sigma(c_L - c_E)}{\frac{c_E c_L (1-\gamma) h''(k)}{(1-\gamma h'(k))(h'(k)-1)} - \sigma[c_L(1-\gamma h'(k)) + c_E(h'(k)-1)]} = \frac{-\sigma(c_L - c_E)}{D} \quad (\text{A7})$$

hence,

$$\frac{\delta^2 k_{opt}(w)}{\delta w^2} = \frac{\sigma(c_L - c_E) \left[ \frac{(c_E + c_L)(1-\gamma) h''(k)}{(1-\gamma h'(k))(h'(k)-1)} - \sigma(1-\gamma) h'(k) \right]}{D^2} < 0 \quad (\text{A8})$$

At  $w = w^*$ ,  $k'_{opt}(-)(w^*) = 1$ , while  $k'_{opt}(+)(w^*) = \frac{\sigma}{\sigma + \frac{\gamma(1-\beta)h(w^*)h'(w^*)}{w^*(1-\gamma h'(k))(h'(k)-1)}} < 1$

**P6**  $k_{opt}(w)$  is bounded by  $k_{\max} = h'^{-1}\left(\frac{1}{1-\pi+\pi\gamma}\right)$ . By construction  $c_L > c_E \Leftrightarrow \frac{u'(c_E)}{u'(c_L)} < 1 \Leftrightarrow k_{opt} < h'^{-1}\left(\frac{1}{1-\pi+\pi\gamma}\right) = k_{\max}$  by (A3). hence as  $w \rightarrow \infty$ ,  $c_E \rightarrow c_L$  and  $\frac{u'(c_E)}{u'(c_L)} \rightarrow 1 \Leftrightarrow h'(k) \rightarrow \frac{1}{1-\pi+\pi\gamma}$ .

### Proof of proposition 4.2 (Steady state and convergence under autarky).

Define the growth rate of wealth  $g(w_{t-1}) = \frac{F^a(w_{t-1})}{w_{t-1}}$ .

(i) The growth rate of wealth is decreasing.  $1 + g(w_{t-1}) = \frac{1-\beta}{\beta}(\pi\gamma + 1 - \pi)f'(k(w_{t-1}))\frac{k(w_{t-1})}{w_{t-1}}$ .  $\frac{k(w_{t-1})}{w_{t-1}}$  is non increasing,  $k(w_{t-1})$  is strictly increasing, hence  $f'(k(w_{t-1}))$  is strictly decreasing.

$\lim_{w \rightarrow \infty} \frac{k(w_{t-1})}{w_{t-1}} = 0$  and  $\lim_{w \rightarrow \infty} f'(k(w_{t-1})) = f'(k_{\max})$ , then  $\lim_{w \rightarrow \infty} 1 + g(w_{t-1}) = 0$ .

Using l'Hopital Rule,  $\lim_{w \rightarrow 0} \frac{f(k(w_{t-1}))}{w_{t-1}} = \lim_{w \rightarrow 0} f'(k(w_{t-1})) = \infty$ .

(ii) The steady state (uniqueness, stability and convergence).

From (i),  $\exists! \bar{w}^a$  such that  $g(\bar{w}^a) = 0 \Leftrightarrow F^a(\bar{w}^a) = \bar{w}^a$  has a unique non zero root.

Since by definition  $g(\bar{w}^a) = 0$  and  $g(\cdot)$  is strictly decreasing  $\Rightarrow \forall w_{t-1} < \bar{w}^a$ ,  $g(w_{t-1}) > 0$  and  $\forall w_{t-1} > \bar{w}^a$ ,  $g(w_{t-1}) < 0$ . Then, the unique steady state is stable.

Since  $F^a(w_{t-1})$  is strictly increasing:  $w_1 > w_0 \Rightarrow F(w_1) > F(w_0) \Leftrightarrow w_2 > w_1 \Leftrightarrow w_t > w_{t-1}$ ,  $\forall t \geq 0$  (by iteration), thus, convergence is asymptotic.

## A.2 Optimal Risk Sharing and Financial Intermediation

**Remark A.1** *We are maximizing a continuous function over a compact set, then a maximum exists.*

**Proof. of proposition 5.1** (The optimal risk sharing solution).

The maximization problem (eqns. 17-22) can be simplified before solving: (a)  $k_{opt} > 0$  since  $h'(0) = \infty$ ; (b)  $\lambda < 1$ , if  $\lambda = 1$  it would always be possible to increase both  $c_E$  and  $c_L$  by reducing liquidation of the long technology; (c) following Allen and Gale (1998), we solve the problem without the ICC (eqn 22), and then show that the optimal solution always satisfies it. The problem becomes to choose  $k, \lambda, c_L, c_E$  to maximize  $\pi u(c_E) + (1 - \pi)u(c_L)$  s.t.: (i)  $\lambda \geq 0$ ; (ii)  $k \leq w$  ( $\mu_1$ ); (iii)  $\pi c_E \leq w - k + \lambda\gamma h(k)$  ( $\mu_2$ ); and (iv)  $\pi c_E + (1 - \pi)c_L \leq (1 - \lambda)h(k) + w - k + \lambda\gamma h(k)$  ( $\mu_3$ ).

**First Order Conditions.** Based on the Kuhn-Tucker conditions:

$$u'(c_E) = \mu_2 + \mu_3 \quad (\text{A9})$$

$$u'(c_L) = \mu_3 \quad (\text{A10})$$

$$(\mu_2 + \mu_3)\gamma \leq \mu_3 \quad \text{with equality if } \lambda > 0 \quad (\text{A11})$$

$$(\mu_2 + \mu_3)(1 - \lambda\gamma h'(k)) + \mu_1 = \mu_3(1 - \lambda)h'(k) \quad (\text{A12})$$

(C) The incentive-compatibility constraint is satisfied by the optimal solution:(A9) and (A10) $\Rightarrow u'(c_E) = \mu_2 + u'(c_L)$ ,  $\mu_2 \geq 0 \Rightarrow c_E \leq c_L$

**The optimal solution.**

Region A:  $\lambda > 0, k = w$  :

$$\lambda > 0 \Rightarrow \frac{u'(c_E)}{u'(c_L)} = \frac{1}{\gamma} \quad (\text{A13})$$

$$\mu_1 \geq 0 \text{ and (A13)} \Rightarrow \gamma h'(w) \geq 1 \Rightarrow \text{case A applies for } : w \in [0, \underline{k}] \quad (\text{A14})$$

$$c_E = \frac{\lambda\gamma h(w)}{\pi} \text{ and } c_L = \frac{(1 - \lambda)h(w)}{(1 - \pi)} \quad (\text{A15})$$

$$\text{(A15) in (A13)} \Rightarrow \lambda_{opt} = \lambda^* \equiv \frac{\pi\gamma^{1/\sigma}}{\pi\gamma^{1/\sigma} + (1 - \pi)\gamma} \quad (\text{A16})$$

optimal liquidation in this region is constant. Notice that:  $\lambda^* \stackrel{\geq}{<} \pi \Leftrightarrow \sigma \stackrel{\geq}{<} 1$ .

Region B:  $\lambda > 0, k < w$  :

$$\lambda > 0, k < w \Rightarrow \frac{u'(c_E)}{u'(c_L)} = \frac{1}{\gamma} = \frac{(1 - \lambda)h'(k)}{1 - \lambda\gamma h'(k)} \quad (\text{A17})$$



implies a constant optimal choice of capital:  $\gamma h'(k) = 1 \Leftrightarrow k_{opt} = \underline{k}$ , and  $c_E = \frac{w - \underline{k} + \lambda \gamma h(\underline{k})}{\pi}$  and  $c_L = \frac{(1 - \lambda)h(\underline{k})}{(1 - \pi)}$ . Optimal liquidation policy is obtained by substituting the two consumptions in (A13):

$$\lambda_{opt}(w) = \lambda^* - (1 - \lambda^*) \beta \frac{w - \underline{k}}{\underline{k}} \quad (\text{A18})$$

$$\lambda(w) \geq 0 \Leftrightarrow w \leq \tilde{w} = \underline{k} \left( 1 + \frac{\pi \gamma^{\frac{1}{\sigma}}}{(1 - \pi) \gamma \beta} \right) \Rightarrow \text{case } \mathbf{B} \text{ applies for } w \in [\underline{k}, \tilde{w}] \quad (\text{A19})$$

Region C:  $\lambda = 0$ ,  $k < w$  and (iii) binding:

$$\lambda = 0, k < w \Rightarrow \frac{u'(c_E)}{u'(c_L)} = h'(k) \leq \frac{1}{\gamma} \quad (\text{A20})$$

$$\Rightarrow k \geq \underline{k} \text{ and } h'(k) \geq 1 \Rightarrow k \leq \bar{k}$$

$$h'(k) \rightarrow 1 \Rightarrow w \rightarrow \hat{w} = \bar{k} \left( 1 + \frac{\pi}{\beta(1 - \pi)} \right) \Rightarrow \text{case } \mathbf{C} \text{ applies for } w \in [\tilde{w}, \hat{w}] \quad (\text{A21})$$

$$k_{opt} \in [\underline{k}, \bar{k}] \text{ is defined by } \frac{u' \left( \frac{w - k_{opt}}{\pi} \right)}{u' \left( \frac{h(k_{opt})}{1 - \pi} \right)} = h'(k_{opt}) \quad (\text{A22})$$

Over region C the optimal capital choice is unique  $\left( \frac{dA(k, w)}{dk} \Big|_{\tilde{w} \leq w \leq \hat{w}} < 0 \right)$ :

$$A(k, w) = u' \left( \frac{h(k)}{1 - \pi} \right) h'(k) - u' \left( \frac{w - k}{\pi} \right) = 0 \Leftrightarrow k = k_{opt}$$

$$\frac{dA(k, w)}{dk} = u'(c_L) h''(k) + u''(c_L) \frac{h''(k)^2}{1 - \pi} + u''(c_E) \frac{1}{\pi} < 0$$

Region D:  $\lambda = 0$ ,  $k < w$  and (iii) not binding:

$$\lambda = 0, k < w \Rightarrow \frac{u'(c_E)}{u'(c_L)} = h'(k) \quad (\text{A23})$$

$$(iii) \text{ not binding} \Rightarrow c_E = c_L = c \Leftrightarrow h'(k) = 1 \Leftrightarrow k_{opt} = \bar{k} \quad (\text{A24})$$

and optimal consumption is:  $c = w - \bar{k} + h(\bar{k})$ .

**Properties of the optimal policy function  $k_{opt}(w)$ .**

The properties of optimal investment in capital are trivial for regions A, B and D ( $w \in \mathfrak{R}^+ / [\tilde{w}, \hat{w}]$ ), where  $k_{opt}(w)$  is either equal to  $w$ , or constant and equal to  $\underline{k}$  or  $\bar{k}$ . Thus, we focus on region C. For  $w \in [\tilde{w}, w^*]$ :

PC1  $k_{opt}(w)$  is strictly increasing.  $k_{opt}(w)$  is implicitly defined by:  $A(k, w) = h'(k)u' \left( \frac{h(k)}{1 - \pi} \right) - u' \left( \frac{w - k}{\pi} \right) = 0$ .  $A(k, w)$  is a continuous, decreasing differentiable function in  $k$ . Using the IFT:

$$k'_{opt}(w) \Big|_{\tilde{w} \leq w \leq \hat{w}} = \frac{\sigma k}{\sigma k + (w - k)(1 - \beta + \sigma \beta)} > 0$$

PC2  $k_{opt}(w)$  is strictly concave in  $w$  for  $w \in [\tilde{w}, \hat{w}]$

$$k_{opt}''(w) |_{\tilde{w} \leq w \leq \hat{w}} = \frac{-\sigma k(1 - \beta + \sigma\beta)}{[\sigma k + (w - k)(1 - \beta + \sigma\beta)]^2} < 0$$

■

**Dynamics with financial intermediaries.**

**Concavity of  $F^b(w)$ .**

The following properties of  $F^b(w_{t-1})$  follow directly from the properties of  $f(w_{t-1})$ ,  $k(w_{t-1})$  and  $\lambda(w_{t-1})$ . The function  $F^b(w_{t-1})$ :

- (i) is continuous on  $[0, \infty)$ .
- (ii) is strictly increasing on  $[0, \hat{w}]$  and constant on  $[\hat{w}, \infty)$
- (iii) is differentiable on  $[0, \underline{k}] \cup (\underline{k}, \tilde{w}) \cup (\tilde{w}, \hat{w}) \cup (\hat{w}, \infty)$ , left and right differentiable at  $k, \tilde{w}, \hat{w}$ .

As we shown below:

$F^b(w_{t-1})$  is strictly concave on  $[0, \tilde{w}]$ , strictly concave on  $[\tilde{w}, \hat{w}]$ , concave on  $[\hat{w}, \infty)$ .  $F^b(w_{t-1})$  is strictly concave on  $[0, \hat{w}]$  and concave on  $\mathfrak{R}^+$  iff  $\sigma \leq \frac{1-\beta}{\beta} \frac{1-\gamma}{\gamma} \lambda^*$

**Proposition A.1**  $F^b(w_{t-1})$  is concave on  $[0, \infty)$  if and only if  $\sigma \leq \frac{1-\beta}{\beta} \frac{1-\gamma}{\gamma} \lambda^*$ .

The proof consists in analyzing concavity within the different regions and the slopes of the function  $F^b(w_{t-1})$  at the thresholds  $(\underline{k}, \tilde{w}, \hat{w})$ .

Region A:  $F^{b'}(w_{t-1}) = (1 - \beta)(1 - \lambda^*(1 - \gamma)) f'(w_{t-1}) > 0$ , and

$F^{b''}(w_{t-1}) = (1 - \beta)(1 - \lambda^*(1 - \gamma)) f''(w_{t-1}) < 0$ . Hence,  $F^b(\cdot)$  is strictly concave.

At the threshold  $\underline{k}$  the slope is  $F^{b(-)}(\underline{k}) = \frac{1-\beta}{\beta} \frac{\pi\gamma^{\frac{1}{\sigma}} + 1 - \pi}{\pi\gamma^{\frac{1}{\sigma}} + (1-\pi)\gamma}$

Region B:  $F^{b'}(w_{t-1}) = \frac{1-\beta}{\beta} \frac{(1-\gamma)(1-\pi)}{\pi\gamma^{\frac{1}{\sigma}} + (1-\pi)\gamma}$  (constant). Then  $F^{b(-)}(\underline{k}) > F^{b(+)}(\underline{k})$ .

Region C:  $F^{b'}(w_{t-1}) = (1 - \beta) f'(w_{t-1}) k'(w_{t-1}) > 0$  and since  $k''(w_{t-1}) < 0$

$F^{b''}(w_{t-1}) = (1 - \beta) (f''(w_{t-1}) k'(w_{t-1}) + f'(w_{t-1}) k''(w_{t-1})) < 0$

At  $\tilde{w}$ :  $F^{b(+)}(\tilde{w}) = \frac{(1-\beta)}{\gamma\beta} \frac{\sigma \underline{k}}{\sigma \underline{k} + (\tilde{w} - \underline{k})(1-\beta+\beta\sigma)}$ , and at  $\hat{w}$ :  $F^{b(-)}(\hat{w}) = \frac{(1-\beta)}{\beta} \frac{\sigma \bar{k}}{\sigma \bar{k} + (\hat{w} - \bar{k})(1-\beta+\beta\sigma)} > 0$

Then  $F^{b(-)}(\tilde{w}) \geq F^{b(+)}(\tilde{w}) \iff \sigma < \frac{1-\beta}{\beta} \frac{1-\gamma}{\gamma} \lambda^*$

Region D:  $F'(w_{t-1}) = 0$

Then  $F^{b(-)}(\hat{w}) > F^{b(+)}(\hat{w}) = 0$  ■

If  $\sigma > \frac{1-\beta}{\beta} \frac{1-\gamma}{\gamma} \lambda^*$  the set defined by the curve  $F^b(w_{t-1})$  is non-convex. However, as we show in the following proof, the steady state equilibrium is unique.

**Proof of proposition 5.2 (The steady state and convergence).**

(i) The growth rate of wealth is decreasing (the continuity of  $F^b(w)$  implies the continuity of  $g^b(w)$ ).

$$\text{Region A: } 1 + g^b(w) = \frac{1-\beta}{\beta} (\lambda^* \gamma + 1 - \lambda^*) f'(w) \Rightarrow \frac{dg^b(w)}{w} < 0$$

$$\text{Region B: } 1 + g^b(w) = (1 - \beta) f(\bar{k}) \left\{ \left( 1 + \frac{\pi \gamma^{\frac{1}{\sigma}} + (1-\pi) \beta \gamma}{\pi \gamma^{\frac{1}{\sigma}} + (1-\pi) \gamma} \right) \frac{1}{w} + \frac{(1-\pi)(\gamma \beta)}{\pi \gamma^{\frac{1}{\sigma}} + (1-\pi) \gamma} \frac{1}{\bar{k}} \right\} \Rightarrow \frac{dg^b(w)}{w} < 0$$

$$\text{Region C: } 1 + g^b(w) = \frac{1-\beta}{\beta} f'(k) \frac{k}{w}, \text{ then } g'(w) = \frac{1-\beta}{\beta} f'(k) \left( \frac{wk'(w)-k}{w^2} - \frac{(1-\beta)k'(w)}{w} \right)$$

hence,  $g'(w) < 0 \iff \beta wk'(w) < k \iff k \left( \frac{\beta \sigma w}{\sigma k + (w-k)(1-\beta+\beta\sigma)} - 1 \right) < 0 \iff \sigma k + (w-k) > 0$  which is always satisfied.

Region D:  $1 + g^b(w) = (1 - \beta) \frac{f(\bar{k})}{w} \Rightarrow \frac{dg^b(w)}{w} < 0$ . By continuity  $g^b(w)$  is strictly decreasing.

$$\lim_{w_{t-1} \rightarrow \infty} \frac{(1-\beta)f(\bar{k})}{w_{t-1}} = 0 \Rightarrow \lim_{w_{t-1} \rightarrow \infty} g^b(w_{t-1}) = -1$$

$$\text{Using l'Hopital rule, } \lim_{w_{t-1} \rightarrow 0} \frac{F^b(w_{t-1})}{w_{t-1}} = \lim_{w_{t-1} \rightarrow 0} F^{b'}(w_{t-1}) = \infty$$

(ii) Uniqueness and stability of the steady state.

From (i),  $F^b(\bar{w}^b) = \bar{w}^b$  has a unique root.

Since by definition  $g(\bar{w}^b) = 0$  and  $g(\cdot)$  is continuous and strictly decreasing  $\Rightarrow \forall w_{t-1} < \bar{w}^b$ ,  $g(w_{t-1}) < 0$  and  $\forall w_{t-1} < \bar{w}^b$ ,  $g(w_{t-1}) > 0$ . Then, the unique steady state is stable.

(iii) Asymptotic convergence and convergence in finite time.

By part (i), for all  $w_{t-1} <_{(>)} \bar{w}^b$  then  $g^b(w_{t-1}) >_{(<)} 0$ .

$F^b(w_{t-1})$  is strictly increasing in  $w_{t-1}$  for  $0 < w_{t-1} < \hat{w}$ , hence,

Asymptotic convergence

- if  $w_0 < \bar{w}^b < \hat{w}$  then  $g(w_0) > 0$  and  $\forall t \geq 1$   $w_t > w_{t-1} \Rightarrow g(w_t) > 0$
- if  $w_0 > \bar{w}^b$ , and  $\bar{w}^b < \hat{w}$  then  $g(w_0) < 0$  and  $\forall t \geq 1$   $w_t < w_{t-1} \Rightarrow g(w_t) < 0$

Convergence in finite time

For  $\bar{w}^b > \hat{w}$ ,  $\bar{w}^b = (1 - \beta) f(\bar{k})$

- if  $w_0 < \hat{w} < \bar{w}^b \Rightarrow g(\bar{w}^b) = 0 < g(\hat{w}) < g(w_0)$

by (i) and (ii),  $\exists$  a unique  $t^* < \infty$ , such that  $\begin{cases} w_{t^*-1} < \hat{w} \\ w_{t^*} \geq \hat{w} \end{cases}$ , hence,  $w_{t^*+1} = (1-\beta) f(\bar{k}) =$

$\bar{w}^b$ , the economy will converge to the s.s. after  $t^* + 1$  periods.

- if  $w_0 > \hat{w} \Rightarrow w_1 = (1 - \beta) f(\bar{k}) = \bar{w}^b$ , the economy will converge after one period.

### A.3 Consequences of Financial Intermediation

**Proof of proposition 6.1** (Inefficient level of investment under Financial Autarky).

Let  $k^a(w)$  and  $k^b(w)$  be the optimal choice function under autarky and financial intermediation

For  $\sigma > 0$ ,  $\{w^*, \tilde{w}\} \subset ]\underline{k}, \hat{w}[$

(i) If  $w \in ]\underline{k}, w^*[$ ,  $k^a(w) = w > k^b(w)$ ; if  $w \in ]\underline{k}, \tilde{w}[$ ,  $k^a(w) > k^b(w) = \underline{k}$ , then for  $w \in ]\underline{k}, \max(w^*, \tilde{w})[$ ,  $k^a(w) > k^b(w)$ .

(ii) At  $w = \hat{w}$ ,  $k^a(w) < k^b(w) = \bar{k}$ ; therefore since  $k^a(w)$  and  $k^b(w)$  are strictly increasing strictly and concave over  $[\max(w^*, \tilde{w}), \hat{w}[$ , it exists a unique  $m \in ]\max(w^*, \tilde{w}), \hat{w}[$  such that  $k^a(m) = k^b(m)$ .

**Proof of proposition 6.2** (Growth under Financial Autarky and Financial Intermediation).

$\underline{\sigma} > 1$ . *Exists at least one  $w_u \in ]\underline{k}, m[$  such that  $F^a(w_u) = F^b(w_u)$*

(R1)  $w \in (0, \underline{k})$ :  $\frac{F^a(w)}{F^b(w)} = \frac{(\pi\gamma+1-\pi)}{(\lambda^*\gamma+1-\lambda^*)} > 1 \iff \lambda^* > \pi \iff \sigma > 1$

(R2) for  $w \geq m$  with  $m \in (\tilde{w}, \hat{w})$  as defined in Prop.6.1:  $\frac{F^a(w)}{F^b(w)} = \frac{(\pi\gamma+1-\pi)h(k^a(w))}{h(k^b(w))}$  Since  $w \geq m \Rightarrow k^a(w) \leq k^b(w) \Rightarrow F^a(w) < F^b(w)$

form (R1) and (R2) by continuity, it exists *at least* one level of wealth  $w_u \in ]\underline{k}, m[$  such that  $F^a(w_u) = F^b(w_u)$

**Uniqueness of  $w_u$ :**

(R3)  $F^a(w)$  is strictly increasing, strictly concave over  $]\underline{k}, m[$  and  $F^b(w)$  is strictly increasing and affine over  $]\underline{k}, \tilde{w}[$ . Therefore it can be at most one root of  $F^a(w) = F^b(w)$  over  $]\underline{k}, \tilde{w}[$ . If there is one root over  $]\underline{k}, \tilde{w}[$ ,  $F'^a(\tilde{w}) < F'^{(-)b}(\tilde{w})$ .

Case (i):  $F^a(\tilde{w}) > F^b(\tilde{w})$ . By (R3),  $F^a(w) > F^b(w)$  over  $]\underline{k}, \tilde{w}[$ . Since  $F^a(w)$  and  $F^b(w)$  are strictly increasing and strictly concave over  $]\tilde{w}, m[$  and  $F^a(m) < F^b(m)$ ,  $F^a(w) = F^b(w)$  must have a unique root over  $]\tilde{w}, m[$

Case (ii):  $F^a(\tilde{w}) \leq F^b(\tilde{w})$ . By (R3),  $F^a(w) = F^b(w)$  as exactly one root over  $]\underline{k}, \tilde{w}[$ . Let's show that in this case cannot be a root over  $]\tilde{w}, m[$ . Consider the unique affine function  $G(w)$  with  $G'(w) = e$ , such that  $G(\tilde{w}) = F^b(\tilde{w})$  and  $G(m) = F^b(m)$ .

- If  $F'^{(-)b}(\tilde{w}) < e \Rightarrow F'^a(\tilde{w}) < F'^{(-)b}(\tilde{w}) < e$ .  $F^a$  is strictly concave  $\Rightarrow F^a(w) < G(w) \leq F^b(w)$  over  $]\tilde{w}, m[$ .
- If  $F'^{(-)b}(\tilde{w}) \geq e$ , consider the function  $L(w) = F^b(w)$  for  $w \leq \tilde{w}$ , and  $L(w) = G(w)$  for  $w > \tilde{w}$ .  $L(w)$  is continuous, strictly increasing and concave over  $]\underline{k}, m[$ . Then  $L(w) = F^a(w)$  has a exactly one root over  $]\underline{k}, \tilde{w}[$  as  $F^a(\tilde{w}) \leq L(w)$  and no root over  $]\tilde{w}, m[$ . As  $F^b(w) \geq L(w)$ ,  $F^b(w) = F^a(w)$  has exactly one root over  $]\underline{k}, m[$ .

$\sigma \leq 1$

**(R4)** Consider  $\sigma = 1$ . For  $w \leq \underline{k}$ ,  $F^a(w) = F^b(w)$ , and at  $w = \underline{k}$ ,  $F'^{(+a)}(\underline{k}) > F'^{(+b)}(\underline{k})$ .<sup>32</sup>

Then it exists a range  $]\underline{k}, w']$  over which  $F^a(w) > F^b(w)$ . Thus by the same reasoning as above, it exist a unique  $w_u \in ]\underline{k}, \tilde{w}[$  such that  $F^a(w_u) = F^b(w_u)$ . In this case  $F^a(w)$  and  $F^b(w)$  are intersecting continuously over  $[0, \underline{k})$ , and twice over  $[\underline{k}, m]$  at  $\underline{k}$  and  $w_u$ .

**(R5)** *Intermediate risk aversion.* By (R4) when  $\sigma \rightarrow 1^-$ ,  $F^a(\underline{k}) \nearrow F^b(\underline{k})$ . We know that  $F'^{(-b)}(\underline{k}) > F'^{(+b)}(\underline{k})$  and  $F'^{(-a)}(\underline{k}) = F'^{(-a)}(\underline{k})$ . Therefore for  $\sigma$  sufficiently close to one, it exists a point  $w_L > \underline{k}$  such that  $F^a(w_L) = F^b(w_L)$  and  $F'^a(w_L) \geq F'^b(w_L)$ . Since  $F^a(m) < F^b(m)$  by continuity (following a parallel argument to (R3)) there must exist a unique  $w_u$  such that  $F^a(w_u) = F^b(w_u)$  and  $F'^a(w_u) \leq F'^b(w_u)$ .

$\Rightarrow$ Then by continuity, it exists a range  $(\sigma_1, 1)$ , such that  $F^a$  and  $F^b$  are intersecting in  $w_L$  and  $w_u \in ]\underline{k}, m[$ ,  $w_L \leq w_u$  and  $w_L < w^*$ .

**(R6)** *Low risk aversion.* As  $\sigma \rightarrow 0$ :  $\lambda^* \rightarrow 0$ ,  $\tilde{w} \rightarrow \underline{k}$ ,  $k_{opt}^b(w)|_{w \leq \tilde{w}} \rightarrow w$ ,  $F^b(w)|_{w \leq \tilde{w}} \rightarrow (1 - \beta)f(w)$ , and  $F^a(w) = (1 - \beta)(\pi\gamma + 1 - \pi)f(k^b(w)) < (1 - \beta)f(w)$

$\Rightarrow$ by continuity it exists a range of  $[0, \sigma_o]$  such that growth with financial intermediation strictly dominates growth in autarky

**(R7)** *Relative Growth and risk aversion.* For all  $w \leq w^*$   $\frac{d \frac{F^a(w)}{F^b(w)}}{d\sigma}|_{0 < \sigma < 1} > 0$ .

$\frac{d^{(-)} \frac{F^a(w^*)}{F^b(w^*)}}{d\sigma} > \frac{d^{(+)} \frac{F^a(w^*)}{F^b(w^*)}}{d\sigma}$ .<sup>33</sup>  $\Rightarrow$ From (R5), (R6) and (R7) there exists a  $\underline{\sigma} \in [\sigma_o, \sigma_1]$  such that:

For  $0 \leq \sigma < \underline{\sigma}$   $F^a(w) < F^b(w) \forall w > 0$

For  $\underline{\sigma} < \sigma < 1$   $F^a(w) < F^b(w) \forall w \in \mathfrak{R}^+ - [w_u, w_L]$  with  $[w_u, w_L] \in ]\underline{k}, m[$

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$$\frac{32}{F^{b'}(\underline{k})} \frac{F^{a'}(\underline{k})}{F^b(\underline{k})} = \begin{cases} \frac{(\pi\gamma + 1 - \pi)}{(1 - \gamma)(1 - \lambda^*)} & \text{for } \sigma > 0 \\ \pi\gamma + 1 - \pi < 1 & \text{for } \sigma = 0 \end{cases}, \sigma = 1 \Rightarrow \frac{F^{a'}(\underline{k})}{F^{b'}(\underline{k})} = \frac{1}{1 - \gamma} \left( \frac{\gamma\pi}{1 - \pi} + 1 \right) > 1$$

$$33 \frac{dF^b}{d\sigma}|_{w \leq \tilde{w}} < 0, \frac{dF^b}{d\sigma}|_{w \leq \tilde{w}} \begin{cases} = 0 & \text{for } w \leq w^* \\ < 0 & \text{for } w > w^* \end{cases}$$

## B Estimation

**Table 2: Countries included in the estimation, and income classifications**

CODE	Country	World Bank Classification <sup>1</sup>	Decile <sup>2</sup>	CODE	Country	World Bank Classification <sup>1</sup>	Decile <sup>2</sup>
ARG	Argentina	MH	7	JPN	Japan	OECD	10
AUS	Australia	OECD	9	KEN	Kenya	L	2
AUT	Austria	OECD	10	KOR	Korea, Rep.	OECD	8
BEL	Belgium	OECD	9	LKA	Sri Lanka	ML	3
BFA	Burkina Faso	L	1	MAR	Morocco	ML	4
BGD	Bangladesh	L	2	MDG	Madagascar	L	1
BOL	Bolivia	ML	4	MEX	Mexico	MH	6
BRA	Brazil	ML	7	MWI	Malawi	L	1
BWA	Botswana	MH	6	MYS	Malaysia	MH	7
CAN	Canada	OECD	9	NER	Niger	L	1
CHE	Switzerland	OECD	10	NGA	Nigeria	L	1
CHL	Chile	MH	7	NIC	Nicaragua	L	3
CHN	China	ML	2	NLD	Netherlands	OECD	10
CIV	Cote d'Ivoire	L	3	NOR	Norway	OECD	10
COG	Congo, Rep.	L	4	NZL	New Zealand	OECD	9
COL	Colombia	ML	6	PAK	Pakistan	L	3
CRI	Costa Rica	MH	7	PAN	Panama	MH	6
DEU	Germany	OECD	10	PER	Peru	ML	6
DNK	Denmark	OECD	10	PHL	Philippines	ML	4
DOM	Dominican Republic	ML	5	PNG	Papua New Guinea	L	4
DZA	Algeria	ML	5	PRT	Portugal	OECD	8
ECU	Ecuador	ML	5	PRY	Paraguay	ML	5
EGY	Egypt, Arab Rep.	ML	4	SEN	Senegal	L	3
ESP	Spain	OECD	8	SGP	Singapore	MH	8
FIN	Finland	OECD	9	SLE	Sierra Leone	L	1
FRA	France	OECD	9	SLV	El Salvador	ML	5
GBR	United Kingdom	OECD	9	SWE	Sweden	OECD	10
GHA	Ghana	L	2	SYR	Syrian Arab Republic	ML	4
GMB	Gambia, The	L	2	TGO	Togo	L	2
GRC	Greece	OECD	8	THA	Thailand	ML	6
GTM	Guatemala	ML	4	TTO	Trinidad and Tobago	MH	7
HND	Honduras	ML	3	TUN	Tunisia	ML	5
HTI	Haiti	L	2	TUR	Turkey	ML	6
IDN	Indonesia	L	4	UGA	Uganda	L	1
IND	India	L	2	URY	Uruguay	MH	7
IRL	Ireland	OECD	8	USA	United States	OECD	10
IRN	Iran, Islamic Rep.	ML	5	VEN	Venezuela, RB	MH	7
ISL	Iceland	OECD	9	ZAF	South Africa	ML	7
ISR	Israel	MH	8	ZAR	Congo, Dem. Rep.	L	1
ITA	Italy	OECD	8	ZMB	Zambia	L	3
JAM	Jamaica	ML	6	ZWE	Zimbabwe	L	3
JOR	Jordan	ML	5				

1/ World Bank Atlas. Classification According with GNI per capita

2/ Deciles calculated using GDP per capita. Source: World Development Indicators, World Bank

**Table 3: Variables and Sources**

<b>Variable</b>	<b>Source</b>	<b>Definition</b>
GDP Level and Growth	World Development Indicators, and Penn World data set.	Real per capita GDP in 1995 US dollars.
Average years of schooling	Barro and Lee (1996)	Average years of schooling in the population over 25.
Government size	World Development Indicators	Ratio of government expenditure to GDP.
Inflation rate	International Financial Statistics	Log difference of Consumer Price Index.
Openness to trade	World Development Indicators	Sum of real exports and imports as share of real GDP.
Liquid Liabilities	International Financial Statistics	Ratio of liquid liabilities (average of year t and t-1) to year t GDP.
Private Credit	International Financial Statistics	Ratio of credit by deposit money banks and other financial institutions to the private sector (average of year t and t-1) to year t GDP.
Frequency of Crises	Caprio and Klingebiel (1999, 2003)	Number of years with banking crises within the period divided by the length of the period 5.

**Table 4: Financial Development and Growth, No Wealth Effects**

Regressors	Private Credit				Liquid Liabilities			
	Coefficient	p-val	Coefficient	p-val	Coefficient	p-val	Coefficient	p-val
Constant	-1.18	0.14	-1.70	0.03	-1.96	0.00	-1.67	0.00
Initial Income per Capita	-0.42	0.00	-0.47	0.00	-0.31	0.00	-0.37	0.00
Avg. secondary schooling	1.76	0.00	1.68	0.00	1.47	0.00	1.34	0.00
Government size	-0.46	0.02	-0.32	0.14	-0.69	0.00	-0.93	0.00
Inflation Rate	-0.08	0.17	0.07	0.52	-0.10	0.13	0.10	0.27
Trade Openness	1.31	0.00	1.15	0.00	1.22	0.00	1.35	0.00
Frequency of Crisis			-2.73	0.00			-2.94	0.00
Financial Development	1.09	0.00	1.23	0.00	1.40	0.00	1.72	0.00
Dummy 71-75	-0.98	0.00	-1.18	0.00	-0.72	0.00	-1.05	0.00
Dummy 76-80	-1.97	0.00	-2.11	0.00	-1.58	0.00	-1.88	0.00
Dummy 81-85	-4.00	0.00	-3.65	0.00	-3.65	0.00	-3.46	0.00
Dummy 86-90	-3.30	0.00	-3.06	0.00	-3.03	0.00	-2.86	0.00
Dummy 91-94	-3.78	0.00	-3.31	0.00	-3.36	0.00	-3.18	0.00
Dummy 96-2000	-3.87	0.00	-3.39	0.00	-3.56	0.00	-3.36	0.00
Sargan Test (P-value)		0.25		0.36		0.33		0.32
2nd Order Serial Corr.(P-Value)		0.33		0.25		0.46		0.32

**Table 5: Financial Development and Growth, World Bank Classification**

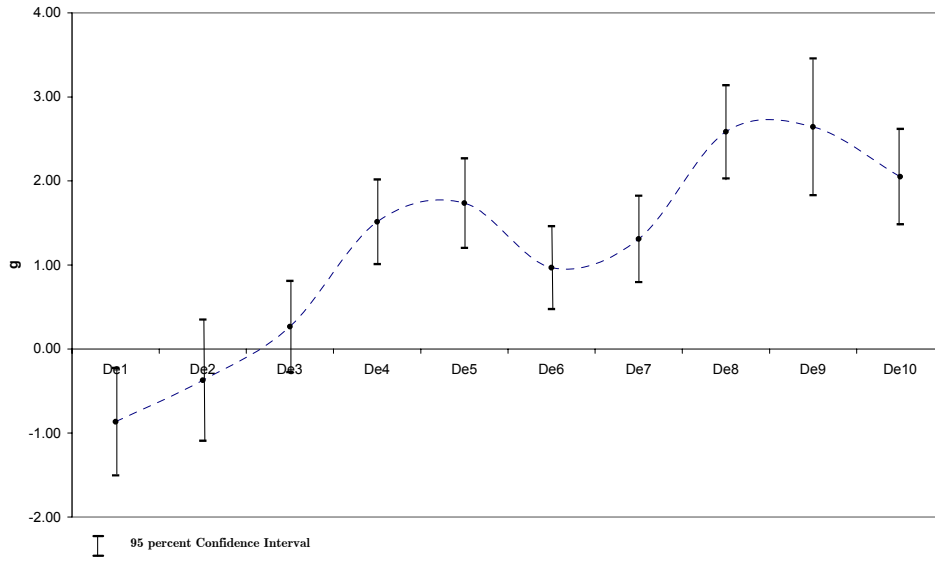
Regressors	Private Credit				Liquid Liabilities			
	Coefficient	p-val	Coefficient	p-val	Coefficient	p-val	Coefficient	p-val
Constant	1.94	0.23	4.42	0.01	8.99	0.00	11.01	0.00
Initial Income per Capita	-0.70	0.00	-1.13	0.00	-1.44	0.00	-1.65	0.00
Avg. secondary schooling	1.59	0.00	1.26	0.00	0.91	0.00	0.59	0.00
Government size	-0.62	0.01	-0.33	0.18	-0.84	0.00	-1.03	0.00
Inflation Rate	-0.10	0.19	-0.06	0.55	-0.08	0.31	-0.07	0.39
Trade Openness	1.26	0.00	1.11	0.00	1.44	0.00	1.31	0.00
Frequency of Crisis			-3.09	0.00			-3.58	0.00
Fin. Dev. Low Inc. Countries	0.65	0.00	0.58	0.01	0.24	0.44	0.34	0.34
Fin. Dev. Middle-Low Inc. Countries	1.15	0.00	1.55	0.00	1.32	0.00	1.62	0.00
Fin. Dev. Middle-High Inc. Countries	1.17	0.00	1.35	0.00	1.38	0.00	1.56	0.00
Fin. Dev. OECD Countries	1.15	0.00	1.69	0.00	1.94	0.00	2.30	0.00
Dummy 71-75	-0.86	0.00	-0.77	0.00	-0.37	0.07	-0.52	0.02
Dummy 76-80	-1.79	0.00	-1.66	0.00	-1.06	0.00	-1.05	0.00
Dummy 81-85	-3.66	0.00	-3.10	0.00	-2.78	0.00	-2.20	0.00
Dummy 86-90	-3.09	0.00	-2.48	0.00	-2.16	0.00	-1.58	0.00
Dummy 91-94	-3.57	0.00	-2.61	0.00	-2.44	0.00	-1.61	0.00
Dummy 96-2000	-3.51	0.00	-2.70	0.00	-2.35	0.00	-1.66	0.00
Sargan Test (P-value)		0.28		0.43		0.27		0.40
2nd Order Serial Corr.(P-Value)		0.36		0.31		0.56		0.39
Tests: Ho: $\beta_L = \beta_{ML}$		0.00		0.00		0.00		0.00
Ho: $\beta_{MH} = \beta_{ML}$		0.93		0.27		0.72		0.64
Ho: $\beta_{beed} = \beta_{ML}$		0.98		0.48		0.00		0.00



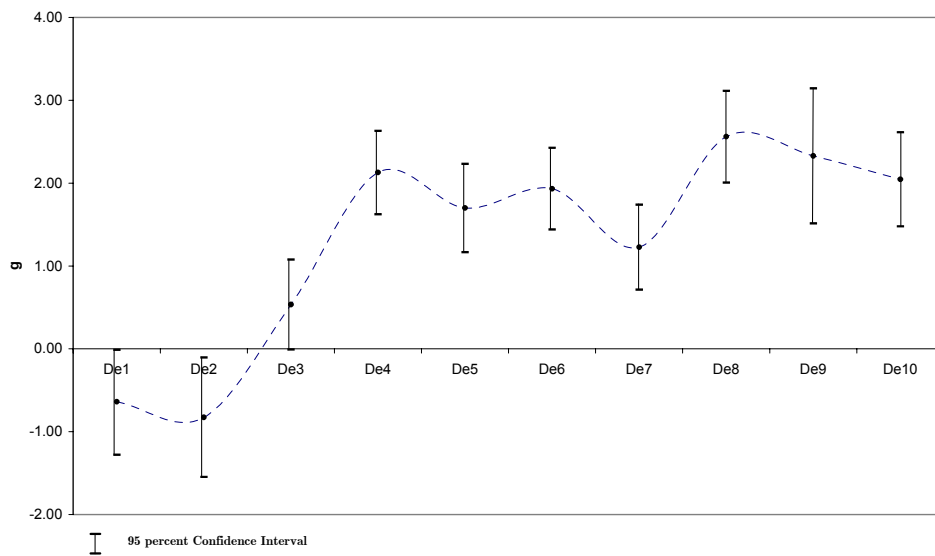
**Table 6: Financial Development and Growth, Decile Classification**

Regressors	Private Credit						Liquid Liabilities					
	Coefficient	Std.Err.	p-val	Coefficient	Std.Err.	p-val	Coefficient	Std.Err.	p-val	Coefficient	Std.Err.	p-val
<b>Constant</b>	19.50	3.49	0.00	14.42	3.02	0.00	18.75	3.49	0.00	11.64	3.02	0.00
<b>Initial Income per Capita</b>	-3.04	0.41	0.00	-2.30	0.37	0.00	-2.82	0.41	0.00	-1.82	0.37	0.00
<b>Avg. secondary schooling</b>	0.88	0.25	0.00	0.72	0.25	0.00	0.43	0.25	0.05	0.33	0.25	0.14
<b>Government size</b>	-0.44	0.34	0.19	-0.33	0.35	0.35	-0.64	0.34	0.11	-0.32	0.35	0.49
<b>Inflation Rate</b>	-0.12	0.10	0.24	-0.04	-0.03	0.22	0.09	0.10	0.42	-0.07	-0.04	0.11
<b>Trade Openness</b>	1.02	0.24	0.00	1.25	0.20	0.00	0.64	0.24	0.00	0.80	0.20	0.00
<b>Frequency of Crisis</b>				-2.83	0.53	0.00				-4.37	0.53	0.00
<b>Decile 1</b>	-1.13	0.38	0.00	-0.87	0.33	0.01	-1.02	0.38	0.00	-0.64	0.33	0.08
<b>Decile 2</b>	-0.28	0.43	0.50	-0.37	0.37	0.31	-0.64	0.43	0.07	-0.83	0.37	0.04
<b>Decile 3</b>	0.14	0.28	0.62	0.27	0.28	0.34	0.51	0.28	0.04	0.54	0.28	0.08
<b>Decile 4</b>	1.47	0.31	0.00	1.51	0.26	0.00	1.92	0.31	0.00	2.13	0.26	0.00
<b>Decile 5</b>	1.61	0.30	0.00	1.74	0.27	0.00	1.57	0.30	0.00	1.60	0.27	0.00
<b>Decile 6</b>	0.91	0.31	0.00	0.97	0.25	0.00	2.15	0.31	0.00	1.93	0.25	0.00
<b>Decile 7</b>	1.68	0.27	0.00	1.31	0.26	0.00	1.86	0.27	0.00	1.13	0.26	0.00
<b>Decile 8</b>	2.79	0.33	0.00	2.58	0.28	0.00	2.87	0.33	0.00	2.56	0.28	0.00
<b>Decile 9</b>	3.23	0.37	0.00	2.64	0.42	0.00	3.10	0.37	0.00	2.33	0.42	0.00
<b>Decile 10</b>	2.64	0.29	0.00	2.05	0.29	0.00	2.94	0.29	0.00	2.05	0.29	0.00
<b>Dummy 71-75</b>	-0.30	0.23	0.20	-0.60	0.23	0.01	-0.27	0.23	0.23	-0.50	0.23	0.05
<b>Dummy 76-80</b>	-0.89	0.32	0.01	-1.22	0.29	0.00	-0.89	0.32	0.00	-1.03	0.29	0.00
<b>Dummy 81-85</b>	-2.70	0.37	0.00	-2.81	0.30	0.00	-2.58	0.37	0.00	-2.39	0.30	0.00
<b>Dummy 86-90</b>	-2.19	0.28	0.00	-2.04	0.26	0.00	-2.07	0.28	0.00	-1.60	0.26	0.00
<b>Dummy 91-94</b>	-2.27	0.35	0.00	-2.26	0.33	0.00	-2.03	0.35	0.00	-1.67	0.33	0.00
<b>Dummy 96-2000</b>	-2.29	0.37	0.00	-2.03	0.34	0.00	-1.85	0.37	0.00	-1.49	0.34	0.00
<b>Sargan Test (P-value)</b>			0.54			0.55			0.36			0.42
<b>2nd Order Serial Corr.(P-Value)</b>			0.54			0.44			0.51			0.25

**Figure 1: Contribution of Financial Development to Economic Growth.  
Wealth Effects, Decile Classification.  
(Private Credit)**



**Figure 2: Contribution of Financial Development to Economic Growth.  
Wealth Effects, Decile Classification.  
(Liquid Liabilities)**



**Table 7: Financial Development and Growth, Linear-Quadratic Specification**

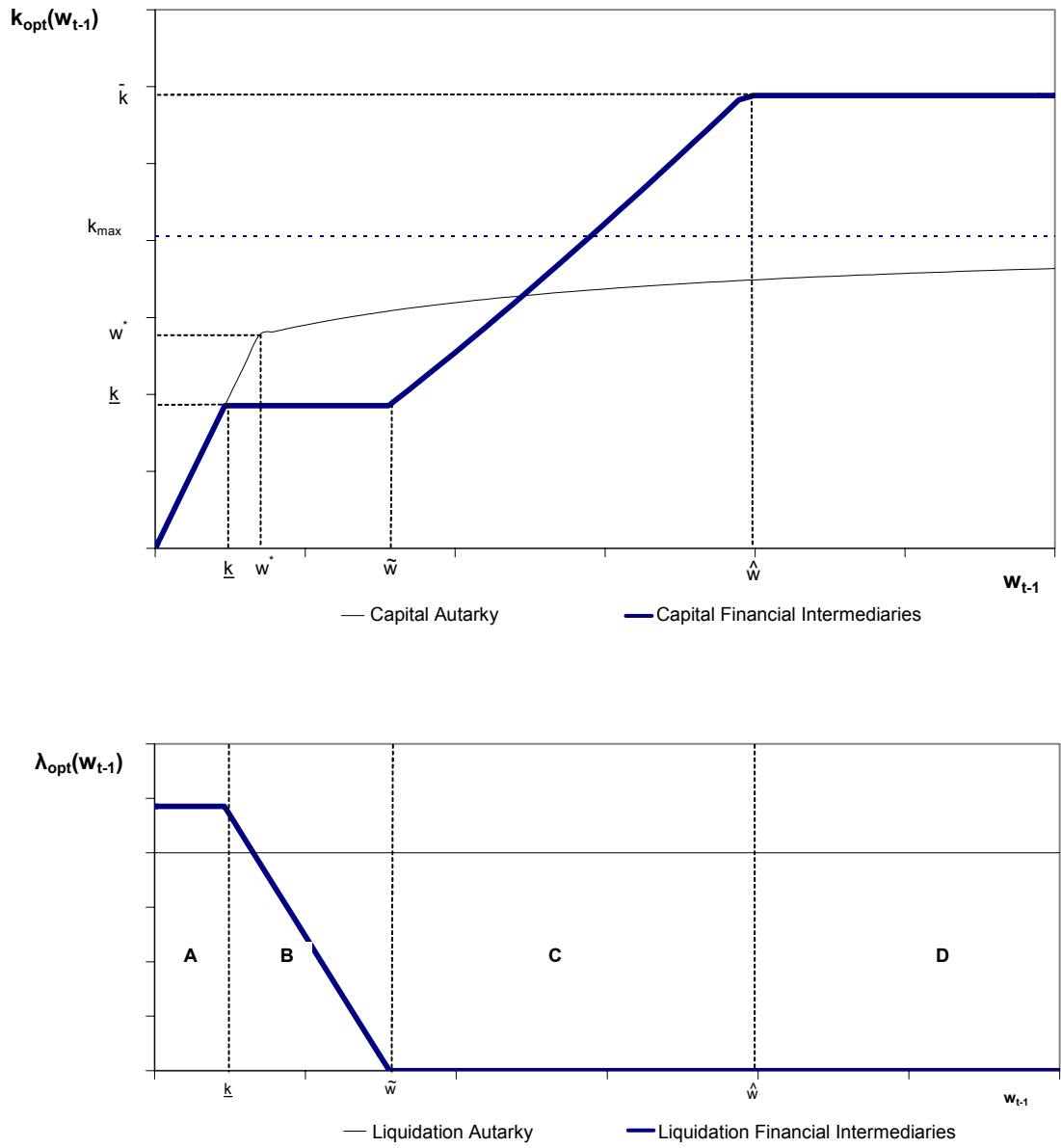
Regressors	Private Credit			Liquid Liabilities		
	Coefficient	Std.Err.	p-val	Coefficient	Std.Err.	p-val
Constant	0.97	2.91	0.74	3.71	4.34	0.39
Initial Income per Capita	-0.45	0.52	0.39	-0.59	0.66	0.37
Avg. secondary schooling	1.48	0.20	0.00	1.19	0.19	0.00
Government size	-0.78	0.30	0.01	-1.30	0.20	0.00
Inflation Rate	-0.09	0.11	0.40	-0.09	0.10	0.35
Trade Openness	1.45	0.22	0.00	1.67	0.20	0.00
Frequency of Crisis	-3.50	0.32	0.00	-3.86	0.37	0.00
Financial Development	-5.54	1.92	0.00	-7.36	2.02	0.00
Financial Dev.* $y_0$	1.61	0.41	0.00	2.06	0.39	0.00
Financial Dev.* $(y_0)^2$	-0.10	0.02	0.00	-0.12	0.02	0.00
Dummy 71-75	-0.87	0.17	0.00	-0.64	0.16	0.00
Dummy 76-80	-1.62	0.22	0.00	-1.34	0.18	0.00
Dummy 81-85	-3.05	0.26	0.00	-2.66	0.19	0.00
Dummy 86-90	-2.45	0.21	0.00	-2.10	0.17	0.00
Dummy 91-94	-2.84	0.18	0.00	-2.51	0.18	0.00
Dummy 96-2000	-2.85	0.21	0.00	-2.58	0.23	0.00
Sargan Test (P-value)			0.44	0.50		
2nd Order Serial Corr.(P-Value)			0.24	0.29		
<b>Income Thresholds in 1995 US dollars</b>						
$y_0^1$	131.2			167.7		
$y_0^2$	3,835.6			4,711.5		
$y_0^3$	112,155.1			132,382.9		
Min $\{y_0\}$ in sample	89.6					
Max $\{y_0\}$ in sample	45,951.9					

## C Simulation

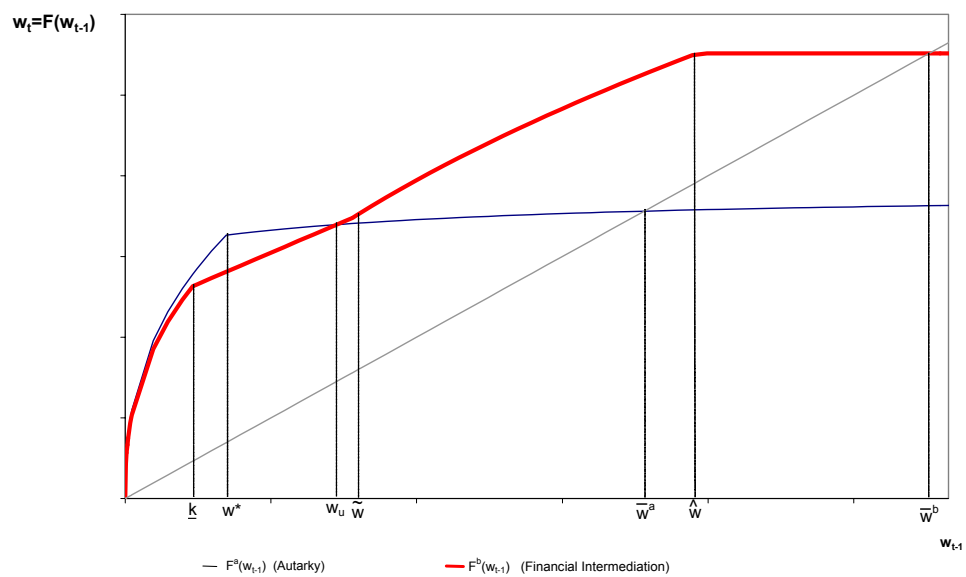
The parameters used in the simulation are:

Table 8: Simulation Parameters	
Factor Productivity	$A = 3$
Capital Share	$\beta = 0.4$
Liquidation Value	$\gamma = 0.5$
Probability of Liquidity Shock	$\pi = 0.4$
Coefficient of Risk Aversion	$\sigma = 2$
Initial wealth	$w_0 = 0.0000001$

**Figure 3: Optimal Investment in Capital  $k_{opt}(w)$ , and Optimal Liquidation  $\lambda_{opt}(w)$   
Autarky and Financial Intermediation**



**Figure 4: Comparison of the Dynamics under Autarky and Financial Intermediation**



**Figure 5: The Growth Consequences of Financial Intermediation**

