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of Interest Rates**

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# An Empirical Analysis of the Mexican Term Structure of Interest Rates\*

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## Abstract

We study the dynamics of the term-structure of interest rates in Mexico. Specifically, we investigate time variation in bond risk premia and the common factors that have influenced the behavior of the yield curve. We find that term-premia in government bonds appear to be time-varying. We then estimate a principal components model. We find that over 95% of the total variation in the yield curve can be explained by two factors. The first factor captures movements in the level of the yield curve, while the second one captures movements in the slope. Moreover, we find that the level factor is positively correlated with measures of long-term inflation expectations and that the slope factor is negatively correlated with the overnight interest rate.

**Keywords:** Term-Structure, Time-Varying Risk Premia, Principal Components.

**JEL Classification:** C13, E43, G12

## Resumen

Se estudia la dinámica de la estructura temporal de tasas de interés en México. En particular, se investiga la variación en las primas de riesgo implícitas en los bonos gubernamentales y los factores comunes que afectan el comportamiento de la curva de rendimientos. La evidencia sugiere que las primas de riesgo varían a través del tiempo. Posteriormente se estima un modelo de componentes principales. Se encuentra que más del 95% de la variación total en la curva de rendimientos puede ser explicada por 2 factores. El primer factor captura movimientos en el nivel de la curva de rendimientos, mientras que el segundo captura movimientos en la pendiente. Adicionalmente, se encuentra que el primer factor muestra una correlación positiva con indicadores de expectativas de inflación de largo plazo, mientras que la correlación del segundo factor con la tasa de interés de corto plazo es negativa.

**Palabras Clave:** Estructura-Temporal, Primas de Riesgo Variables, Componentes Principales.

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# 1 Introduction

In contrast to the large and growing theoretical and empirical dynamic term-structure literature examining the behavior of the yield curve in developed government bond markets, there is virtually no systematic study of the behavior of the yield curve or the performance of dynamic term-structure models in emerging markets. In this paper we study the information contained in the term-structure of interest rates in Mexico. More specifically, we study the dynamics of the Mexican term structure of interest rates between 2001 and 2008. Following Campbell (1995), we first study time variation in excess bond returns to test for the existence of time-varying risk premia in the Mexican bond market. Secondly, we present an estimation of the yield curve in Mexico using a principal components model in the tradition of Litterman and Scheinkman (1991). This approach allows us to analyze the common factors that have influenced the behavior of the yield curve over time and to summarize the information contained in the term structure of interest rates in a small number of principal components. In contrast to most term structure models, the factors that drive the dynamics of the term structure are linked to observable macroeconomic variables; namely, inflation expectations and the overnight interest rate. This paper is part of a research project that studies the joint dynamics of bond yields and macroeconomic variables in Mexico.

Understanding what drives the term structure of interest rates is important in finance and in economics for different reasons. A first reason is forecasting. When adjusted for risk, yields of long-maturity bonds represent expected values of average future short-term yields. Therefore, the yield curve contains information about the expected future path of the economy. In particular, yield spreads have been useful for forecasting not only future short yields and risk premia (Campbell and Shiller 1991, Cochrane and Piazzesi 2002), but also real activity (Harvey 1988, Estrella 1991, Ang, Piazzesi and Wei 2002) and inflation (Mishkin 1990, Fama 1990). These forecasts provide a basis for investment decisions of firms, saving decisions for consumers, and policy decisions. A second reason is closely connected to monetary policy. Central banks are only able to move the short end of the yield curve via their interest rate decisions. However, aggregate demand also depends on long-term yields. Thus, it is important to understand how movements at the short end translate into long-term yields

(for example, Balduzzi, Bertola and Foresi 1996, Piazzesi 2001, Evans and Marshall 1998, 2001). Debt policy constitutes a third reason. When issuing new debt, governments need to decide about the maturity of the new bonds. For example, Cochrane (2001) characterizes the dependence of the nominal term structure on debt policy in a frictionless economy, Missale (1997) considers distortionary taxation, while Angeletos (2002) assumes that markets are incomplete. Derivative pricing and hedging behavior provide a fourth reason. Prices of complex securities, such as swaps, caps and floors, options on interest rates, and futures can be computed from a given model of the yield curve (e.g. Duffie, Pan and Singleton 2000). Furthermore, banks need to manage the risk of paying short-term interest rates on deposits while receiving long-term interest rates on loans. Hedging strategies involve contracts that are contingent on future short rates, such as swap contracts. To compute appropriate strategies (e.g. Litterman and Scheinkman 1991), banks need to know how the price of derivative securities depends on the risk factors that drive the dynamics of expected future short rates and risk premia.

The paper is structured as follows. We begin our analysis in section 2, where we present some statistical indicators that illustrate the behavior of the yield curve over time. A first inspection of the data suggests the presence of time-varying risk premia. In section 3 we examine excess holding-period returns for bonds of various maturities in the spirit of Campbell (1995). Similar to the studies on bond markets in developed economies, we find that term-premia in government bonds appear to be time-varying. In section 4 we conduct a principal-components analysis to identify the common factors that drive the dynamics of the Mexican term-structure of interest rates. As in the literature for developed economies, we find that over 95% of the total variation in the yield curve can be explained by two factors. The first factor is shown to capture movements in the level of the yield curve, while the second one is shown to capture movements in the slope of the curve. Moreover, we find that the level factor is positively correlated with measures of long-term inflation expectations and that the slope factor is negatively correlated with the overnight interest rate (the monetary policy instrument). This statistical evidence suggests that shocks that affect long-term inflation expectations tend to have an effect on the level of the yield curve, while shocks that induce the central bank to move the short-term interest rate affect the slope of the yield

curve. Finally, in section 5 we present the conclusions.

## **2 Description of the Term-Structure of Interest Rates in Mexico**

This section is divided in two parts. The first one presents a brief description of the evolution of the Mexican yield curve over time. The second provides descriptive statistics to analyze some empirical regularities of the term structure of interest rates.

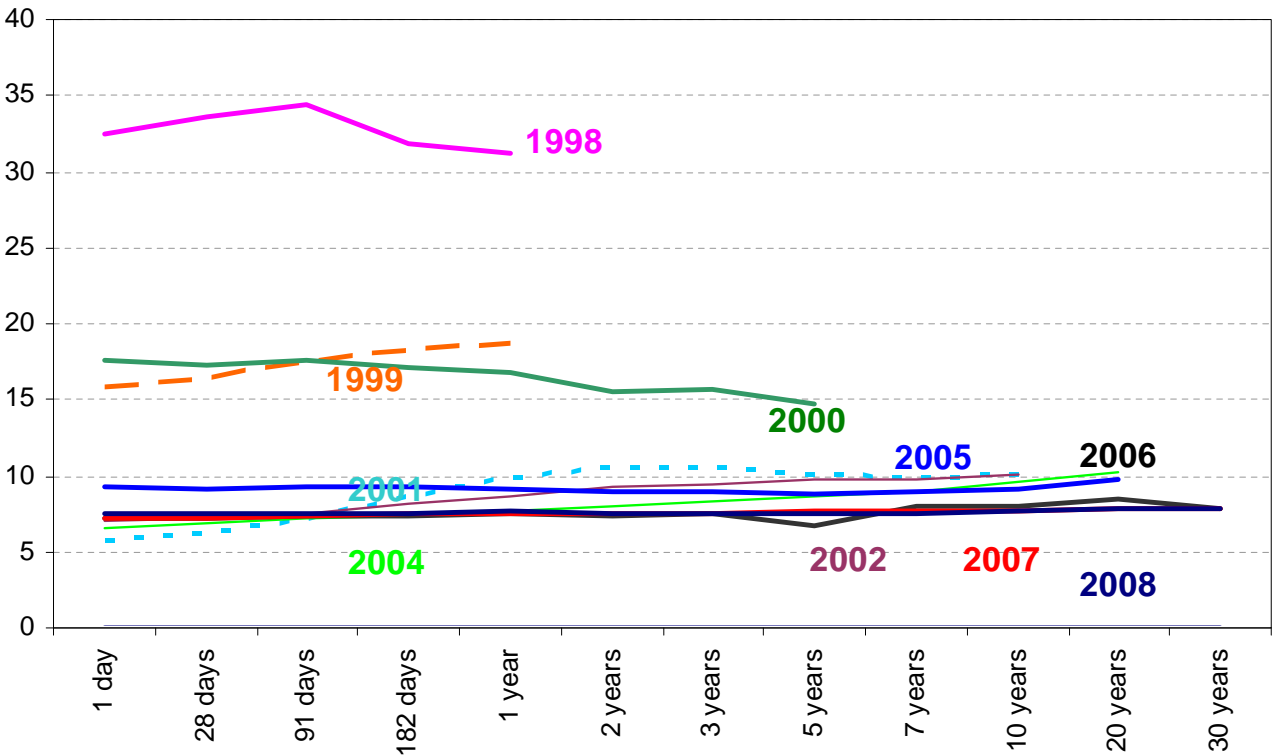
### **2.1 The Yield Curve in Mexico**

During the last years, Mexico has converged to a low, stable inflation equilibrium.<sup>1</sup> Consequently, the macroeconomic environment has become stable. Macroeconomic stability, along with important regulation developments, have been key to promote the development of the financial sector and, in particular, the government bond market. Over the last decade, both the primary and secondary markets for public sector debt of different maturities have developed substantially. The Mexican government has issued 3-month fixed rate bonds since 1978. In recent years it has been able to issue fixed-rate bonds for longer maturities. Following the 1995 crisis, bonds with maturity of more than 1 year were first issued in 2000, while 30-year bonds were first issued in October 2006. Figure 1 plots the evolution of the yield curve in Mexico.

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<sup>1</sup>Chiquiar, Noriega and Ramos Francia (2007) find that inflation in Mexico seems to have switched from a non-stationary process to a stationary process around the end of 2000 or the beginning of year 2001.

Figure 1  
Government Yield Curve



Source: Banco de México. Annual Average. For 2008, January-March Average.

## 2.2 Descriptive Statistics

To describe the dynamics of the yield curve we use zero-coupon bonds. The zero-coupon yield curve is of great importance both in concept and in practice. From a conceptual perspective, the zero-coupon yield curve determines the value that investors place today on nominal payments at all future dates, and constitutes a fundamental determinant of almost all asset prices and economic decisions. Zero-coupon bond yields represent the fundamental building blocks of fixed income markets. For example, coupon bonds can be valued as portfolios of zero-coupon bonds with payoffs and maturities that match the coupon payments.

The full sample consists of daily observations between July 26, 2001 and March 20, 2008 of zero-coupon bond yields for the following maturities: 1-day, 1, 3, and 6 months, and 1, 2, 3, 5, 7 and 10-year securities.<sup>2</sup> We use this sample for two main reasons. The first is that there

<sup>2</sup>We use data of zero-coupon bond yields corresponding to bonds published by Valmer. Valmer is a firm

is evidence that inflation in Mexico seems to have switched from a non-stationary process to a stationary process in the months previous to the beginning of the sample (for example, see Chiquiar, Noriega and Ramos Francia 2007). That is, it seems reasonable to assume that inflation in Mexico currently follows a stationary process, where inflation fluctuates around a well-defined mean.<sup>3</sup> The second reason is that the Mexican government has been able to issue fixed-rate bonds for long horizons (10 years) since 2001.<sup>4</sup> In this section we seek to analyze how the yield curve has evolved over the period under examination. Specifically, we examine the evolution of the level of key interest rates and yield-curve measures over time, including the distributional properties of those levels, and examine the first differences (or daily changes) of these key interest rates and yield-curve measures, again including the distributional properties.

### **2.2.1 Descriptive Statistics: Levels**

Before analyzing the dynamics of the zero coupon-bond yield curve, it is worth to illustrate the behavior of the yield curve over the full sample. Figure 2 plots the zero-coupon bond yields over the sample period considered. As can be seen from Figure 2, the overall level of zero-coupon bond yields decreased over the sample.<sup>5</sup> In addition, since long-term yields decreased more than short-term yields, the slope of the yield curve also decreased. Long-term yields have declined since long-term inflation expectations as well as risk premia have fallen.

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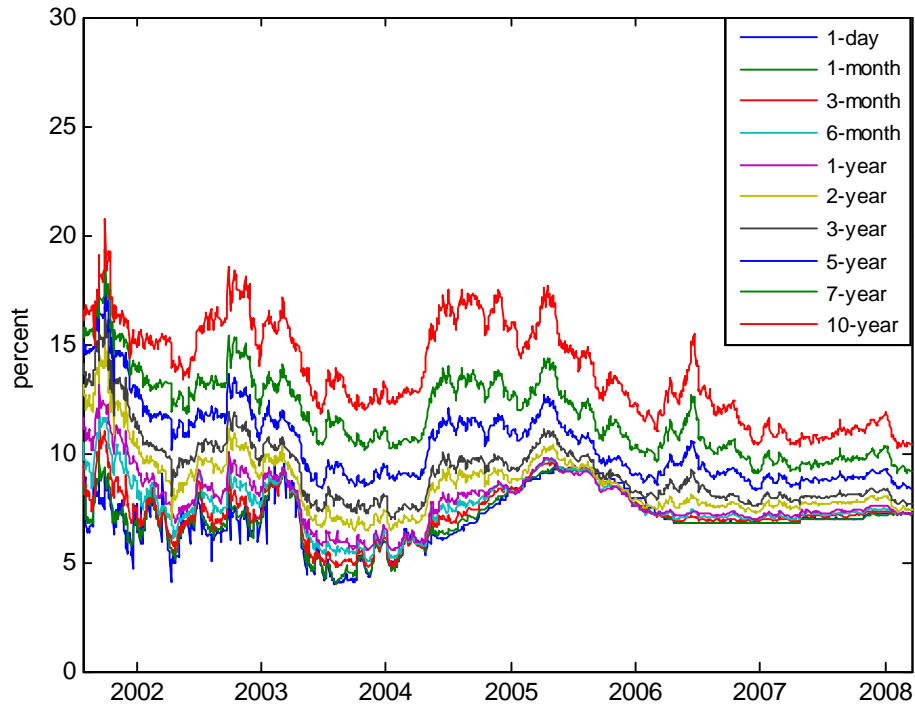
that provides daily prices for the valuation of financial instruments and other services for analysis and risk management.

<sup>3</sup>We conducted the Bai-Perron test for structural changes in inflation, and we did not find any change in mean or in trend after 2001.

<sup>4</sup>Data for 10-year zero-coupon bond yields are available since July 26, 2001.

<sup>5</sup>During the sample period low frequency movements in interest rates appear to be explained by inflation expectations and inflation risk premia.

Figure 2  
Zero-Coupon Nominal Bond Yields

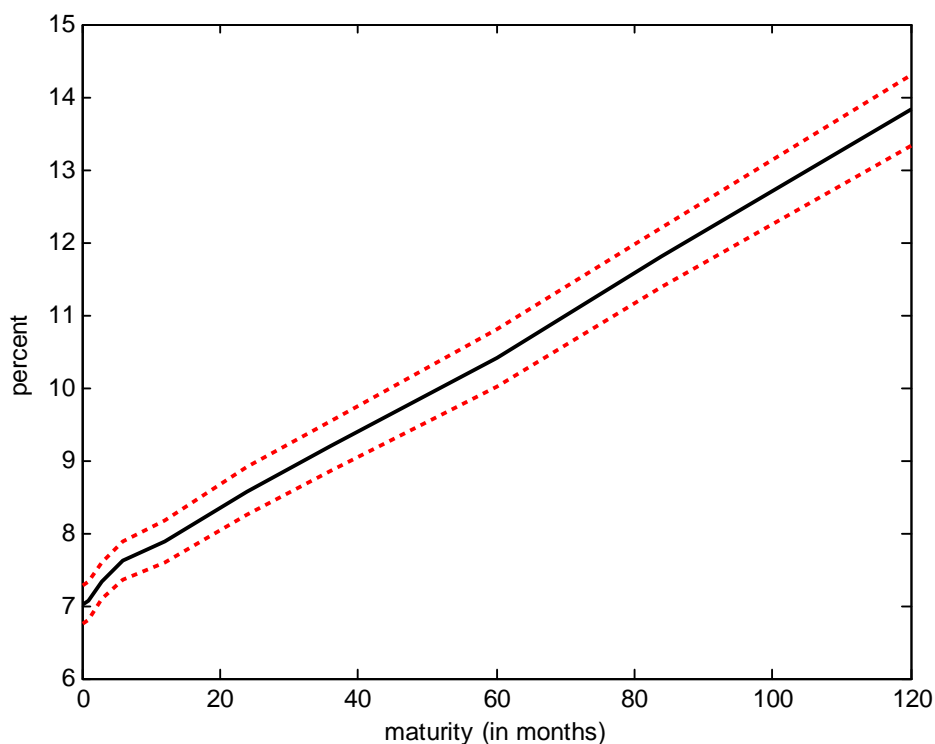


Source: VALMER.

As a first step in examining the results, Figure 3 depicts what the average yield curve looked like over the whole sample. Yields of bonds with longer maturities were on average higher than those of bonds with shorter maturities. This means that the yield curve was on average upward sloping.



Figure 3  
Average yield curve with 2 x standard error bounds



The average yield curve is shown together with dotted approximate 95% confidence bounds (two times Newey-West standard errors). The plot shows that the shortest yield was significantly lower than the longest yield on average. The yield curve contains both information about market expectations of future short-term interest rates and about risk premia. If the yield curve is upward sloping, either people expect interest rates to rise in the future or there are risk premia in long term bonds. The fact that interest rates did not rise on average over the sample suggests the presence of risk premia on long-term bonds. Since bond prices fluctuate over time, there is uncertainty regarding the return from holding a long-term bond over the next period. Moreover, as we will show in section 3, the amount of uncertainty increases with the maturity of the bond.

Figure 4 plots the slope of the yield curve over the whole sample. We use the difference between the 10-year yield and the 1-day yield, and the difference between the 10-year yield

and the 3-month yield as proxies for the slope of the yield curve.<sup>6</sup>

Figure 4  
Yield Curve Slope (Basis Points)

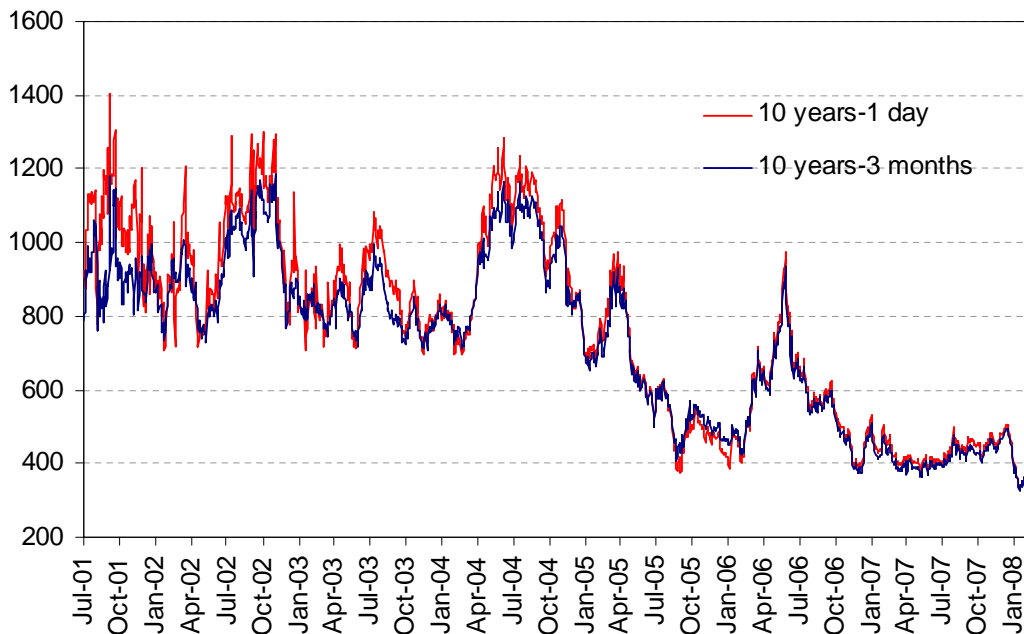


Figure 4 shows that the slope of the zero-coupon bond yield curve decreased over the sample. As we can see from this figure, there are low frequency and high frequency movements in the slope of the yield curve. The low frequency shows a gradual reduction in the slope of the yield curve which is mainly explained by a reduction in inflation and inflation expectations over the sample. As explained below, the high frequency movements in the slope result mainly from variations in risk premia and in expected future short-term interest rates.

The yield curve is forward looking by construction, and contains information about market expectations of future short-term interest rates and about risk premia. If the risk premia that are required by investors as compensation for holding long-term bonds were constant, then changes in the slope of the yield curve would forecast changes in future short-term interest rates. However, in the next section we present evidence that suggests that bond risk

<sup>6</sup>Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006) have used these proxies, among others.

Table 1  
Descriptive Statistics-Levels

<b>maturity (months)</b>	0.033	1	3	6	12	24	36	60	84	120
<b>mean</b>	7.02 (0.13)	7.07 (0.13)	7.34 (0.13)	7.62 (0.13)	7.88 (0.14)	8.58 (0.16)	9.20 (0.18)	10.40 (0.19)	11.79 (0.20)	13.82 (0.24)
<b>std dev</b>	1.24 (0.09)	1.19 (0.09)	1.19 (0.09)	1.20 (0.09)	1.30 (0.12)	1.47 (0.17)	1.62 (0.19)	1.76 (0.17)	1.84 (0.13)	2.23 (0.10)
<b>skewness</b>	-0.18 (0.13)	-0.17 (0.13)	0.11 (0.18)	0.37 (0.25)	0.80 (0.25)	1.41 (0.26)	1.49 (0.25)	1.18 (0.23)	0.63 (0.22)	0.17 (0.17)
<b>kurtosis</b>	2.94 (0.32)	3.01 (0.35)	3.17 (0.44)	3.52 (0.58)	4.18 (0.68)	5.68 (1.02)	5.73 (1.06)	4.39 (0.76)	2.94 (0.48)	1.86 (0.16)

premia appear to be time-varying. In particular, we analyze the high frequency movements in nominal interest rates and in risk premia. Movements in risk premia over time are responsible for a sizable fraction of the movements of the slope of the yield curve. When risk premia decrease, so does the slope, even though expectations of future short-term interest rates are unchanged.

While Figures 3 and 4 depict the general shape of the yield curve over the horizon of the database, Table 1 presents some sample statistics.

This evidence shows that our data are characterized by some standard stylized facts. The average yield curve is on average upward sloping, since average yields increase with maturity. The standard deviations of yields decrease with maturity at first, but then they rise with maturity. The yield levels show mild excess kurtosis at medium-term maturities, and positive skewness at medium and long-term maturities.

### 2.2.2 Descriptive Statistics: First Differences

The first differences (or daily changes) in the level and shape of the yield curve drive the short-term risk and return behaviour for zero-coupon bonds. Since a zero-coupon has no interest payments, its return is entirely driven by price changes. These price changes can arise from two sources. The first is the simple accretion of price towards the maturity value that happens over time (zero-coupon bonds are issued at discount and mature at par). The second source is a change in yield. Over relatively short time horizons, the second source is by far the most significant. Table 2 presents some descriptive statistics of the first differences (or daily changes) of the yields over the sample period considered. Three key observations

Table 2  
Descriptive Statistics-first differences

<b>maturity (months)</b>	0.033	1	3	6	12
<b>mean</b>	-0.0007 (0.004)	-0.0009 (0.004)	-0.001 (0.004)	-0.002 (0.003)	-0.003 (0.003)
<b>std dev</b>	0.26 (0.03)	0.13 (0.01)	0.14 (0.02)	0.13 (0.02)	0.12 (0.02)
<b>skewness</b>	1.48 (0.63)	1.64 (0.71)	1.70 (1.06)	2.26 (1.07)	1.58 (0.64)
<b>kurtosis</b>	32.63 (5.45)	34.01 (10.76)	53.07 (19.73)	51.33 (19.03)	41.45 (14.80)
<b>maturity (months)</b>	24	36	60	84	120
<b>mean</b>	-0.003 (0.003)	-0.004 (0.003)	-0.004 (0.003)	-0.004 (0.004)	-0.004 (0.005)
<b>std dev</b>	0.16 (0.03)	0.17 (0.03)	0.17 (0.02)	0.18 (0.02)	0.23 (0.02)
<b>skewness</b>	-2.48 (1.04)	-2.83 (0.96)	-1.22 (0.86)	-0.34 (0.85)	-0.30 (0.79)
<b>kurtosis</b>	61.23 (9.96)	66.96 (12.95)	37.08 (4.62)	30.45 (6.02)	25.55 (6.85)

can be made. First, not surprisingly, the average change in the various yields was very small, essentially zero for all maturities. Given that these represent daily changes, this small size is to be expected. Second, the uncertainty surrounding the average measure was very high, with standard deviations that were very large relative to the mean value. Third, the distribution of yield changes is clearly not normal. Rather, the distributions are all highly leptokurtic.

### 3 Time-Varying Risk Premia

To provide evidence of time variation in bond market risk premia, we examine excess holding-period returns for bonds of various maturities in the spirit of Campbell (1995). Variation in excess returns suggests the presence of time-varying risk premia in government bonds. The expectations hypothesis, that long yields are the average of expected future short yields plus a constant term premium, implies that excess returns should be constant. We will use historical yield series to answer two questions related to this hypothesis. First, have bonds of different maturities provided equivalent returns for a given holding period. Second, were the

returns earned from holding longer-term instruments riskier than they were for shorter-term bonds?

### 3.1 The Discount Function and Zero-Coupon Bond Yields

The starting point for pricing any fixed-income asset is the discount function, or the price of a zero-coupon bond. This represents the value today to an investor of a \$1 nominal payment  $n$  years hence. We denote this as  $P_t(n)$ . The continuously compounded yield on this zero-coupon bond can be written as:

$$y_t(n) = -\frac{1}{n} p_t(n) \quad (1)$$

where  $p_t(n) = \ln P_t(n)$  and, conversely, the discount function can be written in terms of the yield as:

$$P_t(n) = \exp(-y_t(n)n) \quad (2)$$

The yield curve shows the yields across a variety of maturities. The next step is to define holding-period returns. An  $m$ -day holding period return beginning at time  $t$  on  $n$ -year bonds is defined as the net percentage return that is realized from the following hypothetical strategy: i) at a given date  $t$ , purchase a risk-free zero-coupon bond maturing in  $n$  years (i.e., at date  $t+n$ ). The price of this bond at date  $t$  is given by  $P_t(n)$ ; ii) hold the bond for  $m$  days; iii) on date  $t+m/364$ , sell the bond. Note that as of date  $t+m/364$ , the bond will have a time-to-maturity of  $(n-m/364)$  years. The price of this bond when it is sold is  $P_{t+m/364}(n-m/364)$ ; iv) defining  $d = m/364$ , the log holding period return to the strategy is

$$r_{t+d}(n) = p_{t+d}(n-d) - p_t(n) \quad (3)$$

Expressed in terms of zero-coupon yields rather than prices, we get

$$r_{t+d}(n) = ny_t(n) - (n-d)y_{t+d}(n-d) \quad (4)$$

Note that, since the holding-period return depends on the bond price at  $t+d$ ,  $r_{t+d}(n)$  is

not known at time  $t$ . Our analysis focuses on the concept of excess holding period returns. The excess yield is defined as the excess of the holding period return compared with some risk-free reference rate. The risk-free reference rate is defined as the yield on a zero-coupon bond with  $d$  years to maturity. This yield is risk-free in that the investor does not need to sell the bond at time  $t + d$ , but rather the bond matures with a known terminal value in this date. As a result, the realized yield is known at time  $t$  with certainty. The excess holding-period return is

$$r_{n,t+1}^x = d(y_t(n) - y_t(d)) - (n - d)(y_{t+d}(n - d) - y_t(n)) \quad (5)$$

### 3.2 Results for Excess Holding-Period Returns

Holding-period returns are calculated for a holding period of  $m = 91$  days and using zero-coupon instruments with maturities of  $n = 0.5, 1, 2, 3, 5, 7$  and 10 years. To calculate excess returns, these returns are compared with the yield on a zero-coupon instrument with a 91-day maturity. Figure 5 plots the excess holding-period returns for the whole sample. As Figure 5 shows, excess returns were very volatile during the sample period. For example, investors that sold long-term bonds in June 2006 suffered substantial capital losses (negative excess returns), while investors that sold long-term bonds in September 2006 had capital gains (positive excess returns). Table 3 shows the summary results for the sample period. It is immediately evident that excess returns get both larger and more volatile as the maturity of the bonds held increases. The results conform with the notion of longer-term assets being riskier, and therefore demanding a positive risk premium. It appears that longer-dated assets carry a positive risk premium to compensate for the additional volatility of their returns.<sup>7</sup>

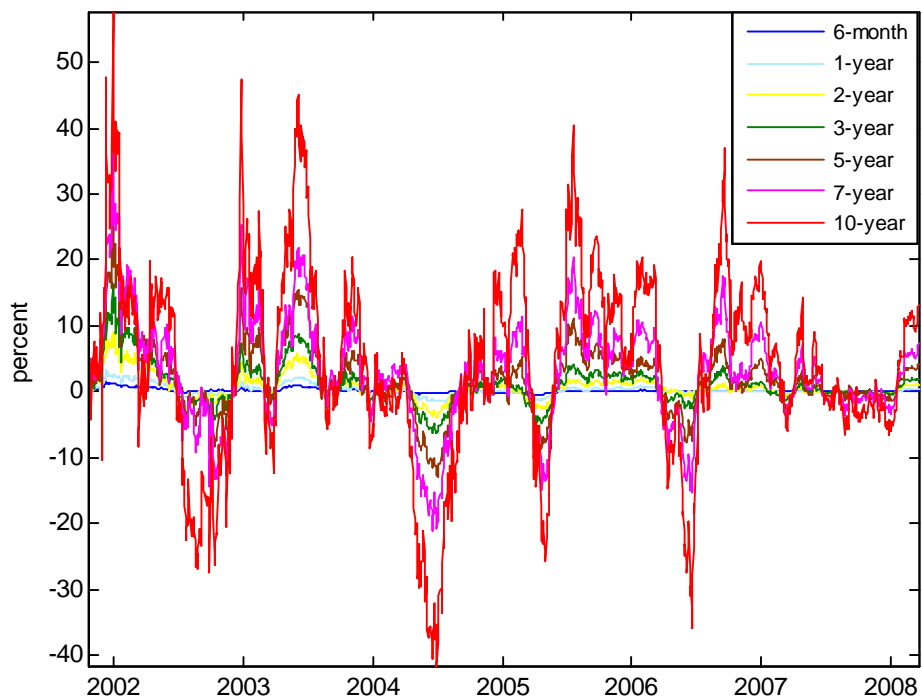
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<sup>7</sup>Holding period returns were also calculated for holding periods of 30, 60, 180 and 360 days. We found similar results for these holding periods. Mean excess returns get both larger and more volatile as the maturity of the bonds held increases.

Table 3  
Excess Holding-Period Returns

maturity (years)	0.5	1	2	3	5	7	10
<b>mean</b>	0.16 (0.03)	0.26 (0.08)	0.70 (0.2)	1.17 (0.33)	2.06 (0.57)	2.94 (0.85)	4.18 (1.57)
<b>std dev</b>	0.30 (0.03)	0.75 (0.07)	1.84 (0.21)	3.02 (0.33)	5.29 (0.51)	7.97 (0.66)	14.74 (1.14)
<b>skewness</b>	1.58 (0.28)	0.88 (0.22)	1.14 (0.31)	1.02 (0.32)	0.58 (0.31)	0.05 (0.27)	-0.24 (0.21)
<b>kurtosis</b>	5.77 (1.38)	4.41 (0.80)	5.76 (1.15)	5.64 (1.09)	4.61 (0.73)	3.85 (0.48)	3.48 (0.37)

Figure 5  
Excess Holding-Period Returns



The excess return data supports two main conclusions: i) term-premia are time-varying; ii) the expected risk and the expected return increase as the time to maturity of the bond examined increases. These results imply that, to provide an adequate characterization of the Mexican term-structure of interest rates, one should consider models that allow for time-varying risk premia.

## 4 Principal-Components Analysis

This section presents evidence of the relationship between the term structure of interest rates and some macroeconomic variables. The first part of this section studies the dynamics of the yield curve using principal components analysis. The second part relates the common factors that affect the yield curve to some macroeconomic variables. To describe the behavior of the yield curve over time, we use the principal-components analysis proposed by Litterman and Scheinkman (1991). This approach has several advantages: i) it allows us to summarize all the information contained in the yield curve into a small number of factors; ii) it delivers some intuition for what drives the dynamics of zero-coupon bond yields.

Since the seminal work of Litterman and Scheinkman (1991), several authors have recognized the importance of identifying the common factors that affect the term-structure of interest rates. To explain the variation in these rates, it is critical to distinguish the systematic risks that have a general impact on the yield curve from the specific risks that influence individual bonds. Principal components can be computed from levels and changes in yields, so we will do both.

### 4.1 Yield-Curve Dynamics

Principal-component analysis describes the behavior of correlated random variables in terms of a small number of uncorrelated variables called principal components.<sup>8</sup> The main idea is that the dynamics of the original variables can be described by a small number of these components.<sup>9</sup> Moreover, principal component-analysis delivers some intuition about the factors that drive the dynamics of zero-coupon bond yields. To begin, we denote by  $Y$  the matrix of observations for each maturity over time, where each column represents a different bond yield, and each row a different point in time. The first step in the analysis is to calculate

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<sup>8</sup>Litterman and Scheinkman (1991) were the first to use principal components analysis to describe the behavior of the yield curve over time.

<sup>9</sup>Alemán and Treviño used this methodology on data from the yield curve in México. Their results are similar to those presented in this section.



the variance-covariance matrix of the zero-coupon bond yields:

$$\Sigma = cov(Y) \tag{6}$$

Note that  $\Sigma$  is a square symmetric matrix of dimension  $n \times n$ , where  $n$  is the number of yields used in the analysis. The diagonal elements of  $\Sigma$  are the variances of the bond yields, while the off-diagonal elements correspond to the covariances between yields of different maturities. As long as none of the yields is an exact linear combination of the others,  $\Sigma$  will be positive definite. If  $\Sigma$  is a positive definite matrix, it has a complete set of  $n$  distinct and strictly positive eigenvalues, and there exists an orthogonal matrix  $\Omega$  (which means it satisfies  $\Omega' = \Omega^{-1}$ ) consisting of the eigenvectors of  $\Sigma$  such that:

$$\Sigma = \Omega \Lambda \Omega' \tag{7}$$

where  $\Lambda$  is the  $n \times n$  diagonal matrix of eigenvalues of  $\Sigma$ , and  $\Omega$  is the corresponding  $n \times n$  matrix of eigenvectors. The principal components of the yield curve at time  $t$  are obtained as follows:

$$pc_t = \Omega' (Y_t - \bar{Y}) \tag{8}$$

where  $Y_t$  is a column vector that contains the  $n$  different yields at time  $t$  and  $\bar{Y}$  is the sample mean of the yields. The same procedure can be repeated for yield changes by replacing  $Y_t$  with  $\Delta Y_t$  and  $\bar{Y}$  with 0 in the above formulas. Each column of the matrix  $\Omega$  measures how a change in each associated principal component affects the whole yield curve. For example, the first column of  $\Omega$  is the eigenvector associated to the first eigenvalue of  $\Sigma$ , and each entry corresponds to how a change in the first principal component affects each maturity along the yield curve. The second column of  $\Omega$  measures the effect of a change in the second principal component on the yield curve. Lets denote by  $pc$  the matrix of principal components over time, where each column represents a principal component, and each row a different point time. Hence, principal components are defined as follows:

$$pc = \tilde{Y} \times \Omega \tag{9}$$

where  $\tilde{Y}$  is the matrix of demeaned yields. The covariance matrix of  $pc$  is given by:

$$var(pc) = \Omega' \Sigma \Omega = \Omega' \Omega \Lambda \Omega' \Omega = \Lambda \quad (10)$$

Thus, by making the transformation  $pc = \tilde{Y} \times \Omega$  we have constructed a set of uncorrelated random variables. The variance of the  $k^{th}$  principal component is just equal to  $\Lambda_k$ , the  $k^{th}$  eigenvalue of  $\Sigma$ . It is also true that the total variation in yields  $trace(\Sigma)$  is equal to the total variation of principal components  $trace(\Lambda)$ . We define the percentage variation explained by the  $i^{th}$  principal component as:

$$100 \times \frac{\Lambda_i}{trace(\Lambda)} \quad (11)$$

Then, the percentage explained indicates how large a given eigenvalue is relative to the rest. The percentage variation explained by the first  $k$  principal components can be computed as:

$$100 \times \frac{\sum_{i=1}^k \Lambda_i}{trace(\Lambda)}$$

If the last  $n - k$  eigenvalues are small, it means that only the first  $k$  principal components are needed to adequately describe the variation of zero-coupon bond yields. In other words, there are only  $k$  driving forces that govern the dynamics bond yields.

Looking at principal components reveals that much of the variance in yields is explained by the first principal components. Table 4 computes the cumulative percentage in the variation of yields changes and levels explained by the principal components. The table shows that the first  $k = 3$  principal components already explain over 99% of the total variation in yields. In the case of yield changes, the first  $k = 3$  principal components explain over 85% of their total variation.

The results in table 4 are interesting, because they indicate that, similar to Litterman and Scheinkman's results, 99% of the variation in the Mexican zero-coupon yield curve can be explained in terms of only three uncorrelated principal components. These results

Table 4  
Variation in yield changes and levels

P.C.	1	2	3	4	5	6	7	8	9	10
% explained in $Y_t$	78.56	95.01	99.31	99.64	99.77	99.87	99.92	99.96	99.99	100
% explained in $\Delta Y_t$	53.49	75.61	85.45	91.89	94.61	96.65	97.75	98.77	99.45	100

indicate that there are three major sources of aggregate risk driving the dynamics of the term-structure of interest rates in Mexico.

To use only  $k \leq n$  principal components, we define the  $n \times k$  matrix  $\tilde{\Omega}$  by:

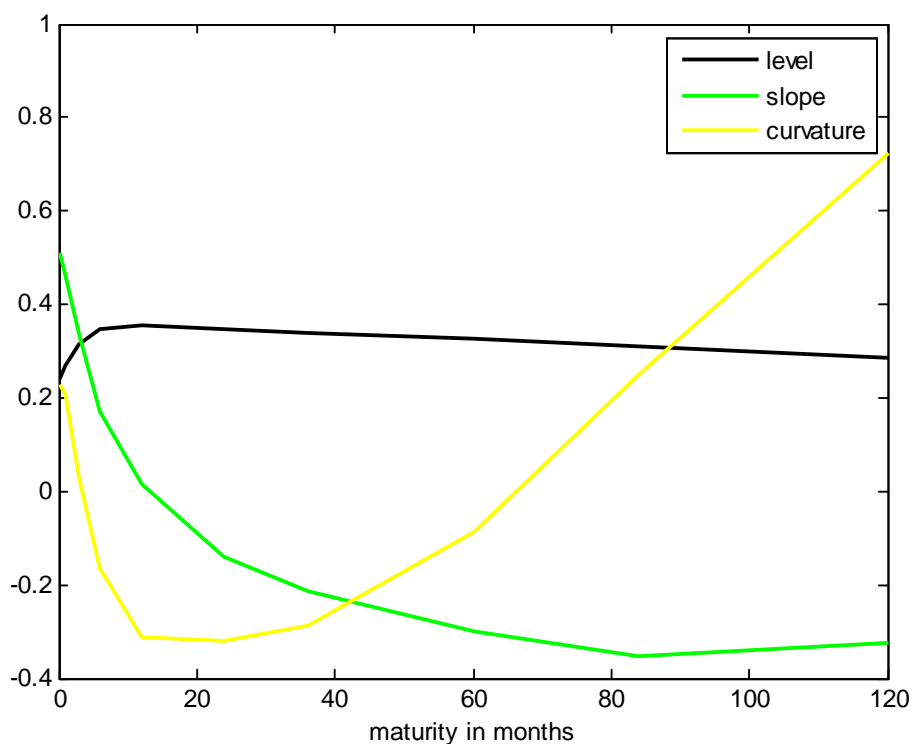
$$\tilde{\Omega}_{ij} = \Omega_{ij} \text{ for } j \leq k$$

and compute the  $k = 3$  principal components of yield levels as

$$\tilde{p}c_t = \tilde{\Omega}' (Y_t - \bar{Y}) \tag{12}$$

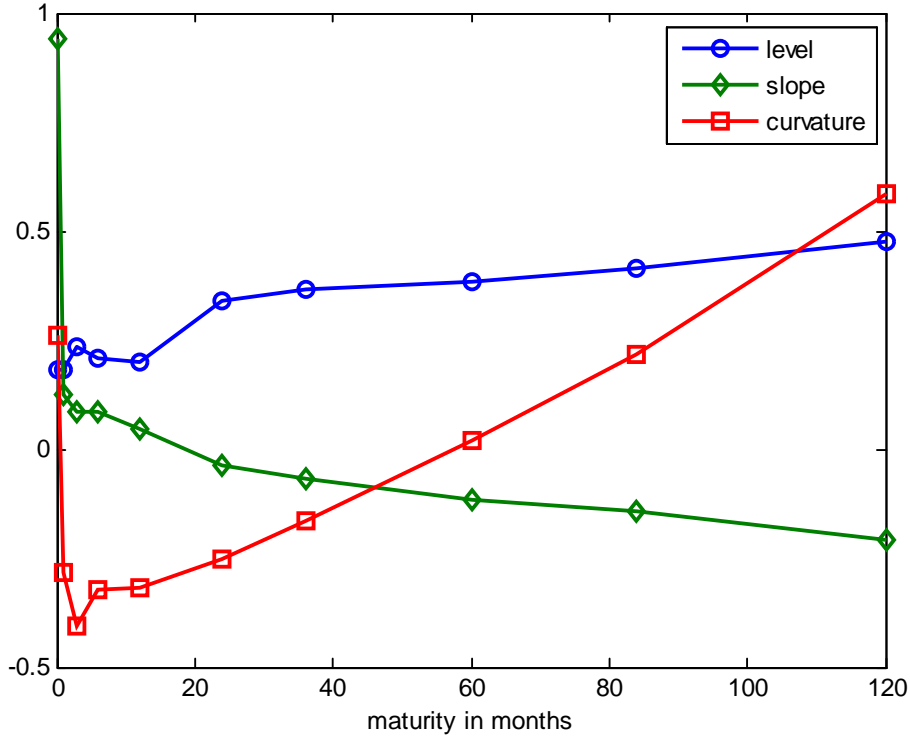
The  $k$  principal components are linear combinations of  $n = 10$  yields. We refer to the sensitivity of a bond's yield to a common factor as the loading of the bond yield on that factor. Using the information in  $\tilde{\Omega}$  it is possible to plot each eigenvector against the maturity of the yields. This allows to identify how a shock to each of the  $k$  factors affects the yield curve. Figure 6 plots the coefficients of these linear combinations (or loadings), which are the  $k = 3$  columns of  $\tilde{\Omega}$ , as function of the maturity of the yields in months. In other words, a given curve in the graph plots the components of the eigenvectors corresponding to the first three factors.

Figure 6  
Loadings of yields on principal components



The loadings of the first principal component are almost horizontal. This pattern means that changes in the first principal component correspond to parallel shifts in the yield curve. This principal component is therefore called the level factor. The loadings of the second principal component are downward sloping. Changes in the second principal component thus rotate the yield curve. This means that the second component is a slope factor. A positive change in this component will induce a rise of the short-end of the yield curve, and a fall of the long-end of the curve. This slope factor will cause the yield curve to flatten (positive change), or to steepen (negative change). The third principal component corresponds to the curvature factor, because it causes the short and long ends to increase, while decreasing medium-term yields. The third principal component therefore affects the curvature of the yield curve. Figure 7 looks similar for the loadings of principal components of yield changes.

Figure 7  
 Loadings of yields on principal components - changes



The interpretation of these principal components in terms of level, slope and curvature goes back to Litterman and Scheinkman (1991). These labels have turned out to be extremely useful in thinking about the driving forces of the yield curve. The latent factors implied by estimated affine models typically behave like principal components.

#### 4.1.1 Cross-sectional performance

Traditional factor models provide a natural benchmark for the cross-sectional fit. Factor models based on  $k$  principal components predict all  $n$  yields in the cross-section as

$$\hat{Y}_t = \bar{Y} + \tilde{\Omega} \tilde{p}c_t \quad (13)$$

where  $\tilde{p}c_t$  is given by (12). This model implies fitting errors for yields which are defined as the difference between actual yields  $Y_t$  and model-predicted yields  $\hat{Y}_t$ . Table 5 computes the mean, standard deviation and maximum of the absolute value of these fitting errors for

Table 5  
Absolute value of fitting errors for yields

<b>maturity</b> <b>(months)</b>	<b>0.033</b>	<b>1</b>	<b>3</b>	<b>6</b>	<b>12</b>	<b>24</b>	<b>36</b>	<b>60</b>	<b>84</b>	<b>120</b>
<b>mean</b>	0.13	0.07	0.08	0.10	0.10	0.07	0.07	0.09	0.11	0.06
<b>std dev</b>	0.15	0.08	0.10	0.10	0.09	0.07	0.06	0.09	0.08	0.05
<b>max</b>	1.59	0.94	1.23	0.81	0.91	0.68	0.41	0.59	0.71	0.49

$k = 3$  principal components. The absolute fitting errors are less than 13 basis points for all yields in the dataset. This means that this low-dimensional factor model not only explains much of the variance in yields, but also performs extremely well according to this additional metric.

## 4.2 Term-Structure and Macroeconomic Dynamics

Since most of the variation of the yield curve in Mexico is explained by the first two principal components (these components explain 95.01 percent of the total variation, as shown in Table 4), we only analyze the dynamics of these components. It is possible to construct a time series for these components using the information in  $\Omega$  and the zero-coupon bond yields. This allows to compare these principal components with standard empirical proxies for level and slope. Let  $\Omega_i$  denote the  $i$ -th column of  $\Omega$ . Thus, we can calculate the  $i$ -th principal component over time as follows:

$$pc_{it} = \Omega'_i (Y_t - \bar{Y}) \quad (14)$$

where  $pc_{it}$  is the  $i$ -th principal component at time  $t$ , and  $\Omega'_i$  is the transpose of the  $i$ -th column of  $\Omega$ .

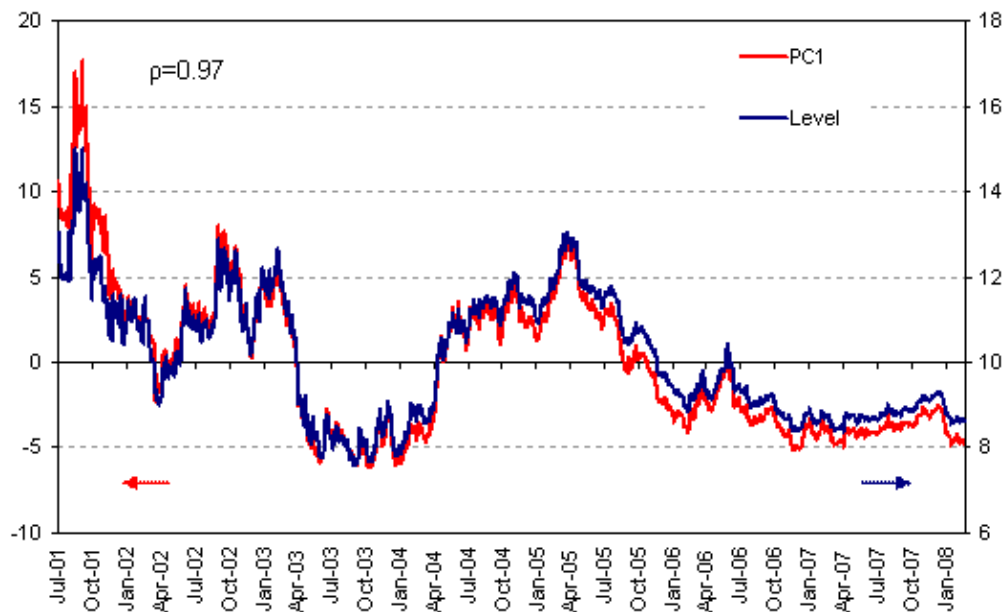
The principal components are linear combinations of all yields, and the coefficients are the eigenvectors of  $\Sigma$ . We can calculate the paths of all the principal components over time using the columns of  $\Omega$ . To confirm our assertion that the first two principal components in our model correspond to the level and slope of yield curve respectively, we plot in Figures 7 and 8 these principal components along with empirical proxies for level and slope.

In Figure 8 we show the first principal component and a common empirical proxy for level (namely, the average of the 1-day, 1-year and 10-year yields). The high 0.97 correlation

between these series supports our interpretation of the first principal component as a level factor.

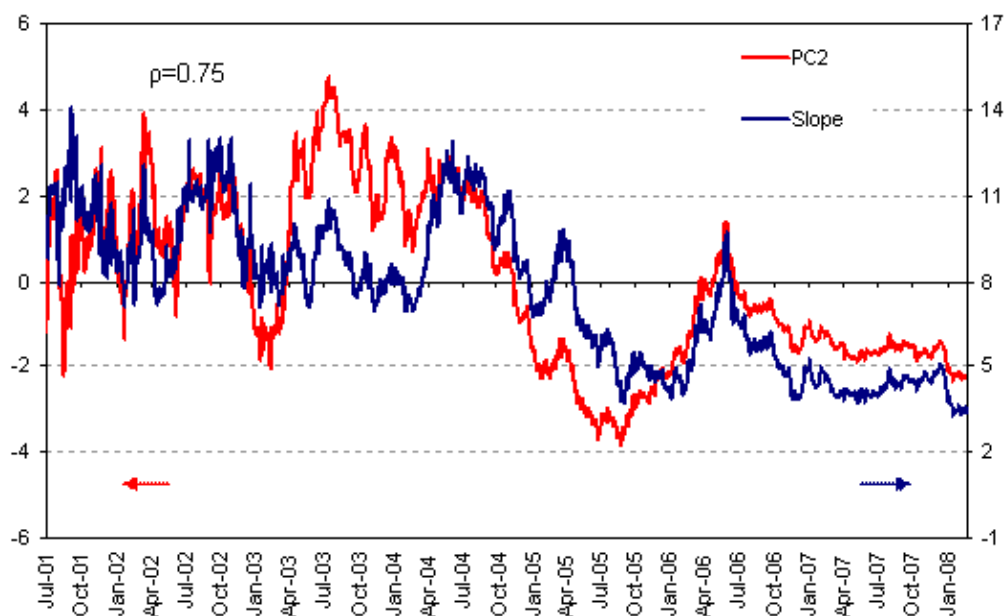
In Figure 9 we show the second principal component and a standard empirical slope proxy (the 10-year minus the 1-day yield). The 0.75 correlation between these series lends credibility to our interpretation of the second principal component as a slope factor.<sup>10</sup>

Figure 8  
Level and Principal Component 1



<sup>10</sup>Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006) have used these proxies, among others.

Figure 9  
Slope and Principal Component 2



Some interesting facts are worth mentioning. First, both the first principal component and the level of the yield curve fell sharply during the sample period. Second, both the slope of the yield curve and the second principal component also fell during the sample period, indicating a flattening of the yield curve over the sample period.

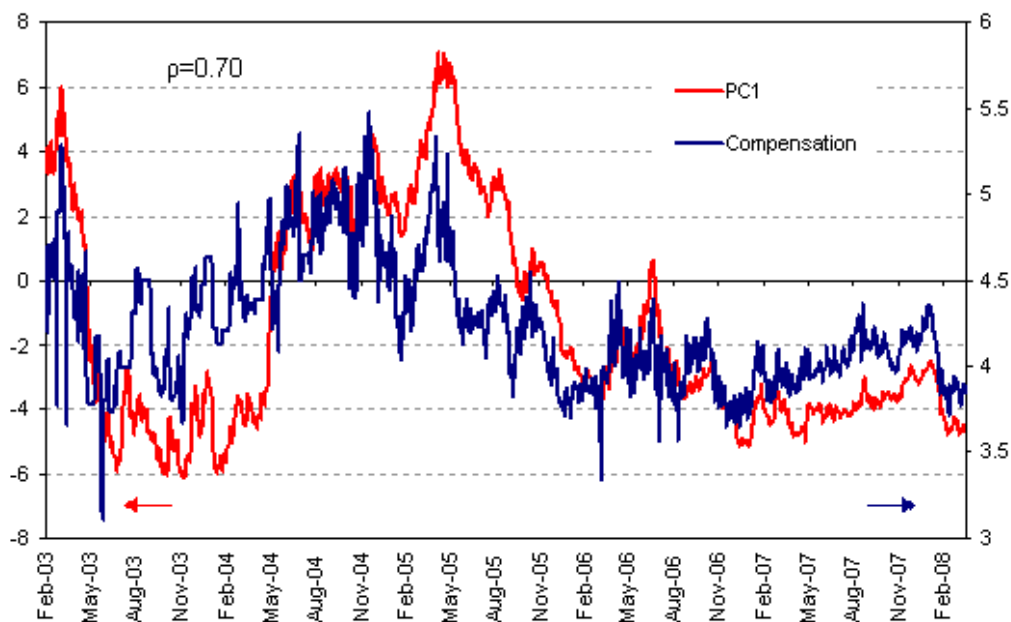
The next step is to provide some preliminary evidence on the relationship between the term-structure of interest rates and some macroeconomic variables.

The level of the yield curve has been associated in the term-structure literature with measures of long-term inflation expectations. For example, Rudebusch and Wu (2004) interpret the difference between nominal and inflation-linked yields as a measure of expected inflation. Figure 10 displays the first principal component and a measure of long-run inflation compensation. The last of these, is measured as the spread between 10-year yields on nominal and indexed securities. The first principal component appears to be closely linked to expected inflation. The correlation between this component and long-run inflation compensation, which is 0.70, is consistent with a link between the level of the yield curve and inflationary expectations, as suggested by the Fisher equation. This link is a common theme in the recent macro-finance literature, including Kozicki and Tinsley (2001), Dewachter and



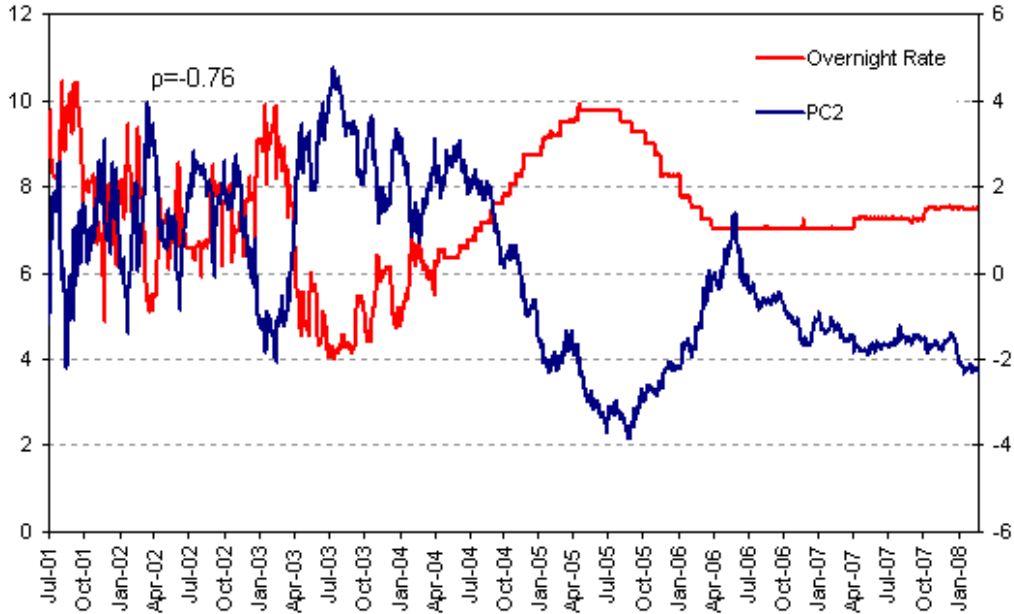
Lyrio (2002), and Hordahl et al. (2002).

Figure 10  
Principal Component 1 and Inflation Compensation



The term-structure literature has also shown that the yield curve slope is connected to the cyclical dynamics of the economy (e.g. Piazzesi 2005). The overnight interest rate is the key policy instrument under control of the central bank, that adjusts in response to macro shocks in order to achieve the economic stabilization goals of monetary policy. Therefore, the slope of the yield curve should be related to the policy rate. Figure 11 provides some evidence about the relationship between the slope of the yield curve and the overnight interest rate. The correlation between the second principal component and the overnight rate, which is  $-0.76$ , suggests that the yield curve slope is related to the cyclical response of the central bank. Since the second principal component captures movements in the slope of the yield curve, this empirical evidence suggests that shocks that induce the central bank to move the short-term interest rate move the slope of the yield curve in the opposite direction. This evidence is consistent with Ang and Piazzesi (2003), and Rudebusch and Wu (2004). These authors find that in the US the short-term interest rate and the slope factor are negatively correlated.

Figure 11  
Principal Component 2 and Overnight Interest Rate



We have shown that, similar to other bond markets, over 95% of the total variation in the yield curve can be explained by two factors. The first factor captures movements in the level of the yield curve, while the second one captures movements in the slope of the curve. Moreover, we find that the level factor is positively correlated with measures of long-term inflation expectations and that the slope factor is negatively correlated with the overnight interest rate (the monetary policy instrument). This empirical evidence suggests that shocks that affect long-term inflation expectations tend to have an effect on the level of the yield curve, while shocks that induce the central bank to move the short-term interest rate affect the slope of the yield curve.

## 5 Conclusions

We have analyzed the dynamics of the term-structure of interest rates in Mexico. Three predominant conclusions can be drawn from the results presented here. First, we found that term-premia in the Mexican government bond market appear to be time-varying. Second, we show that two principal components explain over 95% of the total variation in the Mexican yield curve. Finally, we found that the first principal component captures movements in the level of the yield curve and that it is positively correlated with long-term inflation expectations, while the second principal component captures movements in the slope of the yield curve and it is negatively correlated with the overnight interest rate.

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