

Banco de México
Documentos de Investigación

Banco de México
Working Papers

N° 2007-07

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March 2007

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Optimal Fiscal Policy in a Small Open Economy and the Structure of International Financial Markets

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Abstract This paper characterizes the behavior of debt and tax rates in a small open economy under both complete and incomplete markets. First, I show that when the government follows an optimal fiscal policy and agents have access to complete markets, the value of the government's debt portfolio is negatively correlated with government spending, and positively correlated with productivity and output, while output, labor, consumption and the tax rate are uncorrelated with government spending shocks. The stochastic processes followed by these variables inherit the serial-correlation properties of the stochastic process of the productivity shock. Second, I show that if agents can only buy and sell one-period risk-free bonds, public debt shows more persistence than other variables, and it is negatively correlated with productivity and output, and positively correlated with government spending. Moreover, the tax rate is positively correlated with government spending, while consumption is negatively correlated.

Keywords: Complete markets, Incomplete markets, Optimal fiscal policy.

JEL Classification: E60, F34, F41, G15 and H21.

Resumen El presente artículo caracteriza el comportamiento de la deuda pública y de la tasa impositiva bajo mercados completos e incompletos en una economía pequeña y abierta. En primer lugar, se demuestra que cuando el gobierno sigue una política fiscal óptima y los agentes tienen acceso a mercados completos, el valor del portafolio de deuda del gobierno está negativamente correlacionado con el gasto público y positivamente correlacionado con la productividad y el producto, mientras que el producto, el trabajo, el consumo y la tasa impositiva no están correlacionados con el gasto público. Los procesos estocásticos que siguen estas variables heredan las propiedades estadísticas del proceso estocástico que sigue la productividad. En segundo lugar, se demuestra que si los agentes tienen acceso a mercados incompletos, la deuda pública es más persistente que la otras variables y está negativamente correlacionada con la productividad y el producto, y positivamente correlacionada con el gasto público. Adicionalmente, la tasa impositiva está positivamente correlacionada con el gasto público, mientras que el consumo está negativamente correlacionado.

Palabras Clave: Mercados completos, Mercados incompletos, Política fiscal óptima.

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1 Introduction

In this paper, I characterize the behavior of debt and tax rates in a small open economy under both complete and incomplete markets using Ramsey's approach to optimal taxation. Since the seminal work of Lucas and Stokey (1983), an extensive literature characterizing optimal fiscal policy based on Ramsey's approach to dynamic optimal taxation has emerged. Most of the existing work, however has limited attention to closed economy environments. This paper instead studies optimal fiscal policy in a small open economy under both incomplete and complete markets. I follow the Ramsey approach in characterizing the optimal fiscal policy. In this approach the Ramsey planner chooses an allocation that maximizes the household's utility subject to the condition that this allocation be implementable as a competitive equilibrium. I also abstract, as it is standard in the literature of optimal taxation, from issues of time inconsistency.

The main contributions of the paper are the following: First, I show that when the government in a small open economy follows an optimal fiscal policy and agents have access to international complete asset markets, the value of the government's debt portfolio is a time invariant function of the underlying shocks. As a consequence, the value of the government's debt portfolio inherits the serial correlation structure of the shocks. Moreover, under complete markets the value of the government's debt portfolio is negatively correlated with government spending, and positively correlated with productivity and output, while output, labor, consumption and the tax rate are uncorrelated with government spending shocks. The stochastic processes followed by output, labor, consumption and the tax rate inherit

the serial- correlation properties of the stochastic process of the productivity shock. The Ramsey planner finances all innovations to government spending with state-contingent payments from the rest of the world. Second, I show that if agents in a small open economy can only buy and sell one-period risk-free bonds, public debt shows more persistence than other variables, and it is negatively correlated with productivity and output, and positively correlated with government spending, since the government uses debt to smooth tax distortions over time. Additionally, the tax rate is positively correlated with government spending, while consumption is negatively correlated. The negative correlation between consumption and government spending illustrates the limited insurance role played by non-contingent debt.

The paper proceeds as follows. Section 2 discusses some of the literature on optimal fiscal policy. Section 3 presents the complete markets model and analyzes the dynamic properties of the optimal fiscal policy under complete markets. Section 4 presents the incomplete markets model and analyzes the dynamic properties of the optimal fiscal policy under incomplete markets . Section 5 concludes.

2 Related Literature

This paper is related to several studies about optimal fiscal policy. An extensive literature on optimal fiscal policy has emerged since the seminal work of Lucas and Stokey (1983). Most of the existing work, however has limited attention to closed economy environments. This paper instead studies optimal fiscal policy in a small open economy with incomplete

markets.

In a closed economy environment with complete markets, Lucas and Stokey (1983) used the Ramsey approach of optimal taxation to study the properties of optimal fiscal policy. They found that it is optimal to respond to fiscal shocks by appropriately altering the state-contingent return on government debt and keeping the tax rate roughly constant, so state-contingent debt serves as an instrument to smooth tax distortions over time and states of nature. They also show that tax rates and debt inherit the serial correlation structure of the underlying shocks. Chari, Christiano and Kehoe (1994) analyzed the quantitative features of optimal fiscal policy in a standard real business cycle model with complete markets as in Lucas and Stokey (1983). They showed that another way to keep tax rates stable over the business cycle is to have non-state contingent debt with taxes on interest income that vary with the shocks, in this case state-contingent taxes on interest income should be used to provide insurance against adverse shocks. They found that in calibrated models to the U.S., the standard deviation of optimal income taxes is close to zero while taxes on interest income are highly volatile and serially uncorrelated.

Aiyagari et al (2002) restricted the government to issue only one-period non-contingent debt. They showed that optimal fiscal policy under this environment imposes a near random walk behavior on taxes and debt irrespective of the degree of autocorrelation of the underlying shocks. They also found that the level of debt permanently increases after a fiscal shock, and that the response of the tax rate is a weighted average of a random walk and a serially uncorrelated process. Their results affirm partially the random walk hypothesis

of Barro (1979) . Angeletos (2002), and Buera and Nicolini (2002) considered governments restricted to trading non-contingent real debt of different maturities. They showed that governments could use the maturity structure of non-contingent public debt to replicate the complete markets optimal allocation. However, Buera and Nicolini showed that the government might need to take extremely large long and short positions in debt of different maturities. Marcet and Scott (2000) compare the empirical implications of the model with complete markets, the model with just one period risk free debt and US data. They show that the one-period risk-free bond economy replicates the qualitative features of the data better.

In an open economy setting, Riascos and Vegh (2004) consider an environment in which government spending is determined endogenously. They show that when markets are complete, the correlation between public consumption and output is zero, while if markets are incomplete, the correlation between public consumption and output is large and positive.

In terms of the existing literature, this paper is closest to Riascos and Vegh (2004). Like them, I study optimal fiscal policy in a small open economy. However, this paper differs in two key respects from their paper. First, the goal of the present paper is to characterize the behavior of optimal tax rates and government debt under both complete and incomplete markets in a small open economy, while the goal of Riascos and Vegh (2004) is to analyze the procyclicality of fiscal policy in developing countries, so they do not analyze the optimal behavior of public debt under complete and incomplete markets. Second, these authors consider an endowment economy, while I consider a production economy with an elastic

labor supply, so movements in the tax rate affect the labor supply and output.

3 The Complete Markets Model

Consider a small open economy populated by an infinite number of identical, infinitely lived consumers. In each period $t = 0, 1, \dots$ the economy experiences one of finitely many events $s_t \in S = (1, 2, \dots, N)$. We denote by $s^t = (s_0, \dots, s_t)$ the history of events up to and including period t . The probability as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. Asset markets are complete, both the government and private agents have access to a complete set of Arrow-Debreu securities traded in world capital markets. The government finances an exogenous and stochastic sequence of unproductive public consumption by issuing state-contingent debt and by taxing income at the rate $\tau(s^t)$.

3.1 Households

Each household has preferences defined over consumption c_t and labor h_t . The representative agent's lifetime utility is given by:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), h(s^t)) \quad (1)$$

where $0 < \beta < 1$ denotes the subjective discount factor, $c(s^t)$ and $h(s^t)$ denote consumption and labor conditional on the history of events s^t , and the single-period utility function U is strictly increasing in consumption, decreasing in labor, strictly concave, and satisfies the Inada conditions.

Each period t , households have access to a set of N one-period state-contingent bonds $d(s^t, s_{t+1})$, which pay one unit of consumption in a particular state of period $t + 1$. The variable $D(s^t) = (d(s^t, s_{t+1}))_{s_{t+1} \in S}$ denotes the portfolio of bonds of the representative agent at time t , conditional on history s^t . Let $q(s_{t+1} | s^t)$ be the period t price conditional on history s^t of an asset that promises to pay one unit of consumption in period $t + 1$ in the event that s_{t+1} is realized. The value at t of the portfolio of state-contingent bonds purchased in period t conditional on history s^t is:

$$v^d(s^t) = \sum_{s_{t+1}} q(s_{t+1} | s^t) d(s^t, s_{t+1})$$

The variable $\sum_{s_{t+1}} q(s^t | s_{t+1})$ is the period t price of an asset that pays one unit of consumption in every state in period $t + 1$, therefore, this variable represents the inverse of the risk-free gross real interest rate. Letting $R(s^t)$ denote the gross risk-free real interest rate, we have

$$R(s^t) = \frac{1}{\sum_{s_{t+1}} q(s_{t+1} | s^t)}$$

In each period t , households have access to a concave technology to transform labor into output. The period-by-period budget constraint is given by

$$c(s^t) + \sum_{s_{t+1}} q(s_{t+1} | s^t) d(s^t, s_{t+1}) \leq d(s^t) + (1 - \tau(s^t)) z(s^t) f(h(s^t)) \quad (2)$$

where τ denotes the income tax rate imposed by the government, f the production function, and $z(s^t)$ a technology shock. The production function is increasing in labor, concave and homogeneous of degree $\eta < 1$.

In addition to this budget constraint, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes

$$\lim_{j \rightarrow \infty} \sum_{s^{t+j}|s^t} Q^t(s^{t+j}) d(s^{t+j}) \geq 0 \text{ for all } t, s^t \quad (3)$$

where $Q^t(s^{t+j})$ is the price of an asset that promises to pay one unit of consumption in period $t + j$ conditional on history s^{t+j} being realized, the Arrow-Debreu price $Q^t(s^{t+j})$ is denominated in units of the date t history s^t consumption good.

$$Q^t(s^{t+j}) = q(s_{t+1} | s^t) q(s_{t+2} | s^{t+1}) \dots q(s_{t+j} | s^{t+j-1})$$

$$Q^t(s^t) = 1$$

The assumptions on the utility function imply that households will always choose allocations such that constraints (2) and (3) hold with equality. These two constraints holding with equality imply that the following intertemporal budget constraint must hold

$$d_{-1} + \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) (1 - \tau(s^t)) z(s^t) f(h(s^t)) = \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) c(s^t) \quad (4)$$

where $Q(s^t)$ is the price of an asset that promises to pay one unit of consumption in period t conditional on history s^t being realized. The Arrow-Debreu price $Q(s^t)$ is denominated in units of the date zero consumption good.

$$Q(s^t) = q(s_1 | s_0) q(s_2 | s^1) \dots q(s_t | s^{t-1})$$

$$Q(s_0) = 1$$

Expression (4) states that total wealth in period zero, which consists of the sum of initial financial wealth and the present discounted value of after-tax income must equal the present

discounted value of consumption. Given the stochastic processes $\{z(s^t), \tau(s^t), Q(s^t)\}_{t=0}^{\infty}$ and the initial condition d_{-1} , the household chooses state-contingent sequences $\{c(s^t), h(s^t)\}_{t=0}^{\infty}$ to maximize (1) subject to (4). The first-order conditions associated with the household's maximization problem are (4), and:

$$\beta^t u_c(s^t) \mu(s^t) = \lambda Q(s^t) \quad (5)$$

where λ is the multiplier on the intertemporal budget constraint, and u_c is the marginal utility of consumption

$$-\frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) z(s^t) f'(h(s^t)) \quad (6)$$

First-order condition (6) shows that the tax rate introduces a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor.

3.2 The Government

The government sets the tax rate on income and issues one-period state-contingent bonds to finance the exogenous sequence of government consumption, which is stochastic and unproductive. In each period t , the government issues one-period state-contingent bonds $b(s^t, s_{t+1})$, which pay one unit of consumption in a particular state of period $t + 1$. The variable $B(s^t) = (b(s^t, s_{t+1}))_{s_{t+1} \in \mathcal{S}}$ denotes the debt portfolio of the government at time t , conditional on history s^t . The value of the government's debt portfolio in period t conditional on history s^t is:

$$v^b(s^t) = \sum_{s_{t+1}} q(s_{t+1} | s^t) b(s^t, s_{t+1})$$

The government's period-by-period budget constraint is given by:

$$g(s^t) + b(s^t) \leq \sum_{s^{t+1}} q(s_{t+1} | s^t) b(s^t, s_{t+1}) + \tau(s^t) z(s^t) f(h(s^t)) \quad (7)$$

In addition to this budget constraint, the government is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j \rightarrow \infty} \sum_{s^{t+j} | s^t} Q^t(s^{t+j}) b(s^{t+j}) \leq 0 \text{ for all } t, s^t \quad (8)$$

A benevolent government seeking to maximize the welfare of private agents will always choose asset processes such that (7) and (8) hold with strict equality. These two constraint holding with equality imply the following intertemporal budget constraint

$$b_{-1} + \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) g(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \tau(s^t) z(s^t) f(h(s^t)) \quad (9)$$

where b_{-1} is the initial level of debt.

The fiscal policy consists in the announcement of a state-contingent plan for the tax rate $\{\tau(s^t)\}_{t=0}^{\infty}$.

3.3 Equilibrium Conditions

In this section, we characterize the equilibrium conditions of the small open economy. If we combine the household's and the government's sequential budget constraints we get an expression that describes the evolution of the net foreign asset position of the economy as a whole

$$\sum_{s^{t+1}} q(s_{t+1} | s^t) [d(s^t, s_{t+1}) - b(s^t, s_{t+1})] = d(s^t) - b(s^t) + z(s^t) f(h(s^t)) - g(s^t) - c(s^t)$$

Also, if we combine the household's and the government's intertemporal budget constraints, we obtain the economy's resource constraint

$$d_{-1} - b_{-1} = \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) [g(s^t) + c(s^t)] - \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) z(s^t) f(h(s^t))$$

We assume like in Schmitt-Grohé and Uribe (2003), that foreign agents have access to the same state-contingent bonds as in the domestic economy. First-order condition (5), for the foreign households, assuming the same discount factor is:

$$\beta^t u_{c^*}(s^t) \mu(s^t) = \lambda^* Q(s^t) \quad (10)$$

The domestic marginal rate of substitution between consumption at t , s^t and consumption at date zero is

$$\frac{\beta^t u_c(s^t) \mu(s^t)}{u_c(s_0)} = Q(s^t)$$

and the foreign marginal rate of substitution between consumption at t , s^t and consumption at date zero is

$$\frac{\beta^t u_{c^*}(s^t) \mu(s^t)}{u_{c^*}(s_0)} = Q(s^t)$$

combining domestic and foreign equations we get:

$$\frac{u_c(s^t)}{u_c(s_0)} = \frac{u_{c^*}(s^t)}{u_{c^*}(s_0)} \quad (11)$$

This expression holds at all dates and states. The domestic marginal utility of consumption is proportional to the foreign marginal utility of consumption

$$u_c(s^t) = \chi u_{c^*}(s^t) \quad (12)$$

where $\chi = \frac{u_c(s_0)}{u_{c^*}(s_0)}$

Since the domestic economy is small, $u_{c^*}(s^t)$ is exogenous. Additionally, we assume as in Schmitt-Grohé and Uribe (2003) that the foreign marginal utility of consumption is constant and equal to u_{c^*} . Therefore, equation (12) becomes

$$u_c(s^t) = \xi \quad (13)$$

where $\xi = \chi u_{c^*}$ is a constant. Evaluating (5) at $t = 0$, we get that $\lambda = u_c(s_0) = \xi$. Since we assumed that $u_{c^*}(s^t)$ is constant and equal to u_{c^*} , the Arrow-Debreu prices satisfy the following equation

$$q(s_{t+1} | s^t) = \beta \mu(s^{t+1} | s^t) \quad (14)$$

$$Q(s^t) = \beta^t \mu(s^t)$$

3.4 Competitive Equilibrium with Income Taxes

Given the initial condition b_{-1} , the parameter ξ , and the stochastic processes $\{g(s^t), z(s^t)\}_{t=0}^{\infty}$, a competitive equilibrium is a set of state-contingent sequences $\{c(s^t), h(s^t), Q(s^t), \lambda\}_{t=0}^{\infty}$, and a fiscal policy $\{\tau(s^t)\}_{t=0}^{\infty}$ satisfying the following conditions:

$$\beta^t u_c(s^t) \mu(s^t) = \lambda Q(s^t) \quad (15)$$

$$-\frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) z(s^t) f'(h(s^t)) \quad (16)$$

$$b_{-1} + \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) g(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) \tau(s^t) z(s^t) f(h(s^t)) \quad (17)$$

$$u_c(s^t) = \xi \quad (18)$$

$$Q(s^t) = \beta^t \mu(s^t) \quad (19)$$

The definition of the competitive equilibrium involves neither the variable d_{-1} nor the household's intertemporal budget constraint (4). The reason is that in equilibrium d_{-1} adjusts across the different states of nature in period zero to guarantee that (4) holds for a given value of ξ . That is, given the equilibrium values for the state-contingent sequences, we can find the value of d_{-1} that is associated with the competitive equilibrium from equation (4). The optimal fiscal policy is the process $\{\tau(s^t)\}_{t=0}^{\infty}$ associated with the competitive equilibrium that yields the highest level of utility to the representative household, that is, the process that maximizes (1). To find the optimal policy, it is convenient to use a simpler representation of the competitive equilibrium known as the primal form. Finding the primal form involves the elimination of all prices and tax rates from the equilibrium conditions, so that the resulting reduced form involves only real variables.

3.4.1 The Primal Form

Proposition 1 *Given the initial condition b_{-1} , the parameter ξ and the stochastic processes $\{g(s^t), z(s^t)\}_{t=0}^{\infty}$, the state-contingent sequences $\{c(s^t), h(s^t)\}_{t=0}^{\infty}$ satisfy:*

$$u_c(s^t) = \xi$$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} + z(s^t) f(h(s^t)) - g(s^t) \right\} \mu(s^t) = b_{-1}$$

if and only if they satisfy (15), (16), (17), (18) and (19)

Proof. *See the Appendix.* ■

3.5 The Ramsey Problem

It follows from Proposition 1 that the Ramsey problem can be stated as choosing state-contingent plans $\{c(s^t), h(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), h(s^t))$$

subject to

$$u_c(s^t) = \xi$$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} + z(s^t) f(h(s^t)) - g(s^t) \right\} \mu(s^t) = b_{-1}$$

The Lagrangian of the Ramsey planner's problem is

$$L = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \begin{array}{l} U(c(s^t), h(s^t)) + \gamma \left[\frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} + z(s^t) f(h(s^t)) - g(s^t) \right] \\ + \Omega(s^t) (u_c(s^t) - \xi) \end{array} \right\} \mu(s^t) - \gamma b_{-1}$$

where γ and Ω are the Lagrange multipliers on the implementability constraints. The first-order conditions of this problem are:

$$u_c(s^t) + \gamma \frac{h(s^t)}{\eta} \left[\frac{u_{ch}(s^t)}{u_c(s^t)} - \frac{u_h(s^t) u_{cc}(s^t)}{(u_c(s^t))^2} \right] = -\Omega(s^t) u_{cc}(s^t) \quad (20)$$

$$u_h(s^t) + \gamma \left\{ \frac{h(s^t)}{\eta} \left[\frac{u_{hh}(s^t)}{u_c(s^t)} - \frac{u_h(s^t) u_{ch}(s^t)}{(u_c(s^t))^2} \right] + \frac{u_h(s^t)}{u_c(s^t) \eta} + z(s^t) f'(h(s^t)) \right\} = -\Omega(s^t) u_{ch}(s^t) \quad (21)$$

$$u_c(s^t) = \xi \quad (22)$$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} + z(s^t) f'(h(s^t)) - g(s^t) \right\} \mu(s^t) = b_{-1} \quad (23)$$

Proposition 2 *If the stochastic processes on $s = (z, g)$ are Markov, then there exist functions c, h and τ such that the Ramsey consumption allocations, labor allocations and income tax rates are time invariant functions only of the productivity shock.*

$$c(s^t) = c(z_t), \quad h(s^t) = h(z_t), \quad \tau(s^t) = \tau(z_t)$$

Proof. *See the Appendix.* ■

Proposition 2 says that the allocations and the income tax rate are uncorrelated with government spending shocks, these variables depend only on the current realization of the productivity shock. The stochastic processes followed by all of these variables inherit the properties of the stochastic process of the productivity shock z_t . For example, if productivity shocks are i.i.d., then the allocations and the income tax rate are i.i.d. If the productivity shocks are persistent, then the allocations and the income tax rate are also persistent.

Proposition 3 *Given the functions c, h and τ and condition (14), the equilibrium government's debt portfolio at time t $B(s^t) = (b(s^t, s_{t+1}))_{s_{t+1} \in S}$ is independent of the realization of the state $s_t = (z_t, g_t)$.*

Proof. The government's sequential budget constraint is given by

$$b(s^t) = \tau(z_t) z_t f(h(z_t)) - g_t + \sum_{s_{t+1}} q(s_{t+1} | s^t) b(s^t, s_{t+1})$$

This expression is equal to

$$b(s^t) = \tau(z_t) z_t f(h(z_t)) - g_t + \beta E_t [b(s^t, s_{t+1})]$$

$$b(s^t) = E_t \sum_{j=0}^{\infty} \beta^j [\tau(z_{t+j}) z_{t+j} f(h(z_{t+j})) - g_{t+j}]$$

The first equality uses (14) and the definition of conditional expectation, and the second equality is obtained by recursive substitution and (8). Since h and τ are stationary functions of the productivity shock, and s_t is Markov, the expectation on the right-hand side of the second equality is only a function of s_t . Therefore, $b(s^t) = b(s_t)$ is a time invariant function of the state s_t , and the government's debt portfolio is constant

$$B(s^t) = (b(s_{t+1}))_{s_{t+1} \in S} = B$$

■

Proposition 3 says that the government's debt portfolio does not respond neither to a productivity shock nor to a government spending shock. The government always issues the same amount of each security in equilibrium regardless of the current period and state of nature. Therefore, the government's debt portfolio is constant for all t, s^t .

Proposition 4 *The value of the government's debt portfolio of contingent bonds $v^b(s^t)$ is given by a time invariant function V^b such that:*

$$v^b(s^t) = V^b(s_t) = \beta E_t[b(s_{t+1})]$$

for all t and s^t

Proof. The value of the government's debt portfolio in period t conditional on history s^t is

$$v^b(s^t) = \sum_{s_{t+1}} q(s_{t+1} | s^t) b(s^t, s_{t+1})$$

This expression is equal to

$$v^b(s^t) = \sum_{s_{t+1}} q(s_{t+1} | s^t) b(s^t, s_{t+1}) = \beta E_t[b(s_{t+1})] = V^b(s_t)$$

■

The second equality uses (14) and proposition (3). The Markov assumption, and proposition (3) imply that V^b is time invariant. This proposition says that the value of the government's debt portfolio is affected by current shocks. We will show in the next section, that

the value of the government's debt portfolio decreases in response to a positive government spending shock, and that it increases in response to a positive productivity shock.

3.6 Dynamic Properties of the Optimal Fiscal Policy under Complete Markets

In this section we carry out some simulations to study the dynamic properties of the model economy under the Ramsey policy with complete markets. First, we describe the calibration of the model. Second, we show the impulse response functions of the model. Finally, we present the moments of the simulated time series.

3.6.1 Calibration

We calibrate the model so as to make it consistent with some of the empirical regularities that reflect the structure of a typical emerging economy. The time unit is one quarter, and the time endowment, which can be divided between labor and leisure is normalized to one.

We assume that the period utility function is Cobb-Douglas between consumption and leisure.

$$U(c, h) = \frac{(c^\alpha (1 - h)^{1-\alpha})^{1-\sigma} - 1}{1 - \sigma}$$

The parameter σ , the coefficient of relative risk aversion, is set equal to 2, which is a standard value. We calibrate α so that households devote on average 1/3 of their time to work in the steady-state. We also assume that the average real interest rate $r = 10\%$, therefore

$\beta = \left(\frac{1}{1+r}\right)^{1/4}$. The production function takes the following form

$$zf(h) = zh^\eta$$

The labor share in GDP is 67% , so $\eta = 0.67$. Additionally, we assume that the public-debt to GDP ratio is 20% in steady-state and that the share of government spending in GDP is also 20%. These values imply that the income tax rate in steady-state is equal to:

$$\bar{\tau} = \frac{\bar{g}}{\bar{y}} + (1 - \beta) \frac{\bar{b}}{\bar{y}}$$

We assume that the foreign-debt to GDP ratio is 40% in steady-state, so we can find the share of consumption in GDP in steady-state by combining the household's and the government's budget constraints in steady state.

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{g}}{\bar{y}} - (1 - \beta) \frac{\bar{b} - \bar{d}}{\bar{y}}$$

We calibrate ξ from equation (18) evaluated at the steady-state, and α , the consumption share in the utility function, from equation (6) evaluated in the steady-state.

We assume as in Schmitt-Grohé and Uribe (2003) that government spending and productivity shocks follow independent two-state Markov processes. Specifically, z_t can take on the values $z^h = 1 + \sigma_z$ or $z^l = 1 - \sigma_z$. We assume that z_t has a standard deviation of 0.04 and a first order serial correlation of 0.82. Similarly, g_t takes on the values $g^h = 1 + \sigma_g$ or $g^l = 1 - \sigma_g$. We assume that on average g_t is 20% of GDP and that it has a standard deviation of 0.00382 and a first-order serial correlation of 0.9. The deep structural parameters values are shown in table 1.

Table 1

Parameter	Value
α	0.2716
β	0.9765
ξ	1.6855
η	0.67
g^l	0.0646
g^h	0.0722
ϕ^g	0.95
z^l	0.96
z^h	1.04
ϕ^z	0.91
σ	2

3.6.2 Impulse Response Functions and Moments

Figure 1 displays the impulse response of the economy under the optimal fiscal policy to a one-standard deviation increase in the productivity shock, z_t . Productivity shocks induce movements in labor, output, consumption and the tax rate. Employment, output and consumption increase after a positive productivity shock. In response to a positive productivity shock, the government finds it optimal to increase the income tax rate since it makes state-contingent payments to the rest of the world in good states to smooth the marginal utility of consumption over time and states of nature. The primary surplus and the value of the

government's debt portfolio also increase, because as we mentioned above, the government makes state-contingent payments to the rest of the world in good states. The increase in productivity leads to an improvement to the trade balance, since output increases more than consumption and government spending remains constant. The agents in this economy use the trade balance to smooth out consumption over time and states of nature.

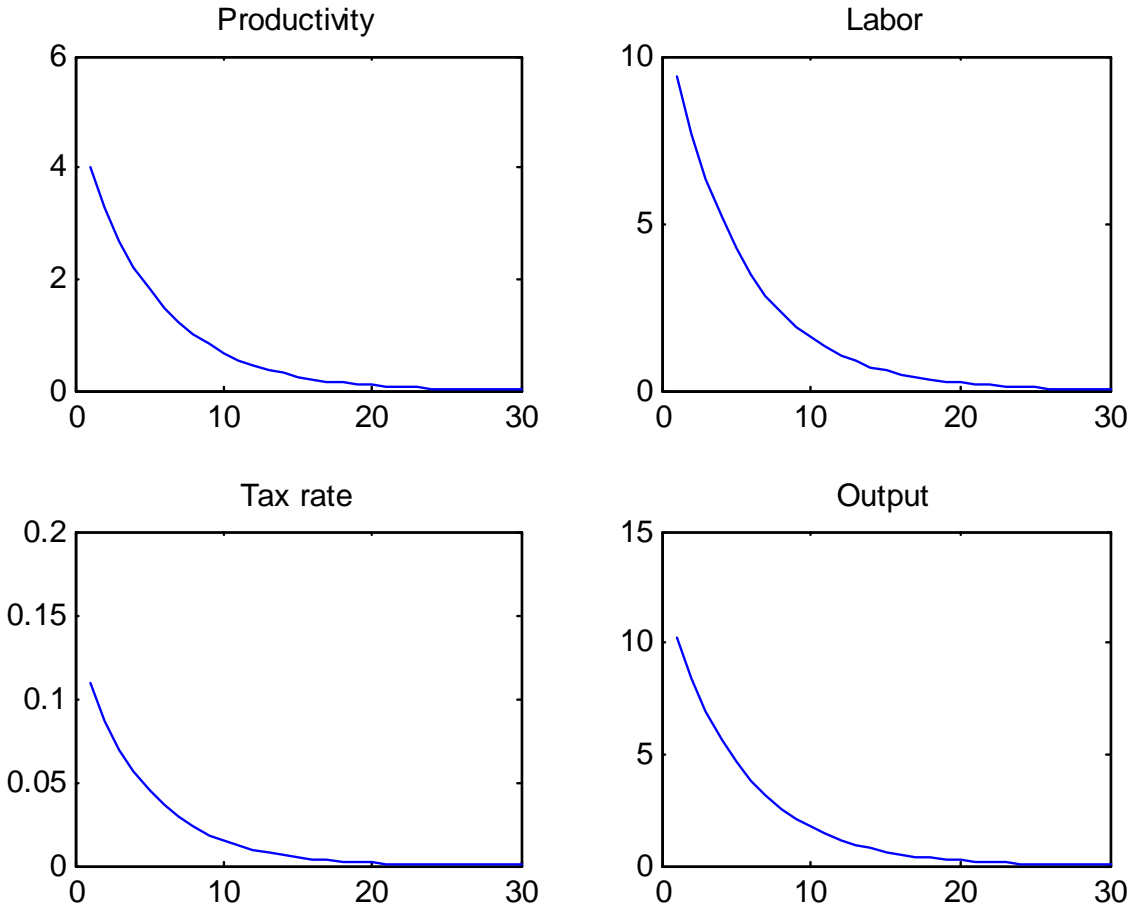


Figure 1. Impulse Response to a one-standard deviation increase in Productivity

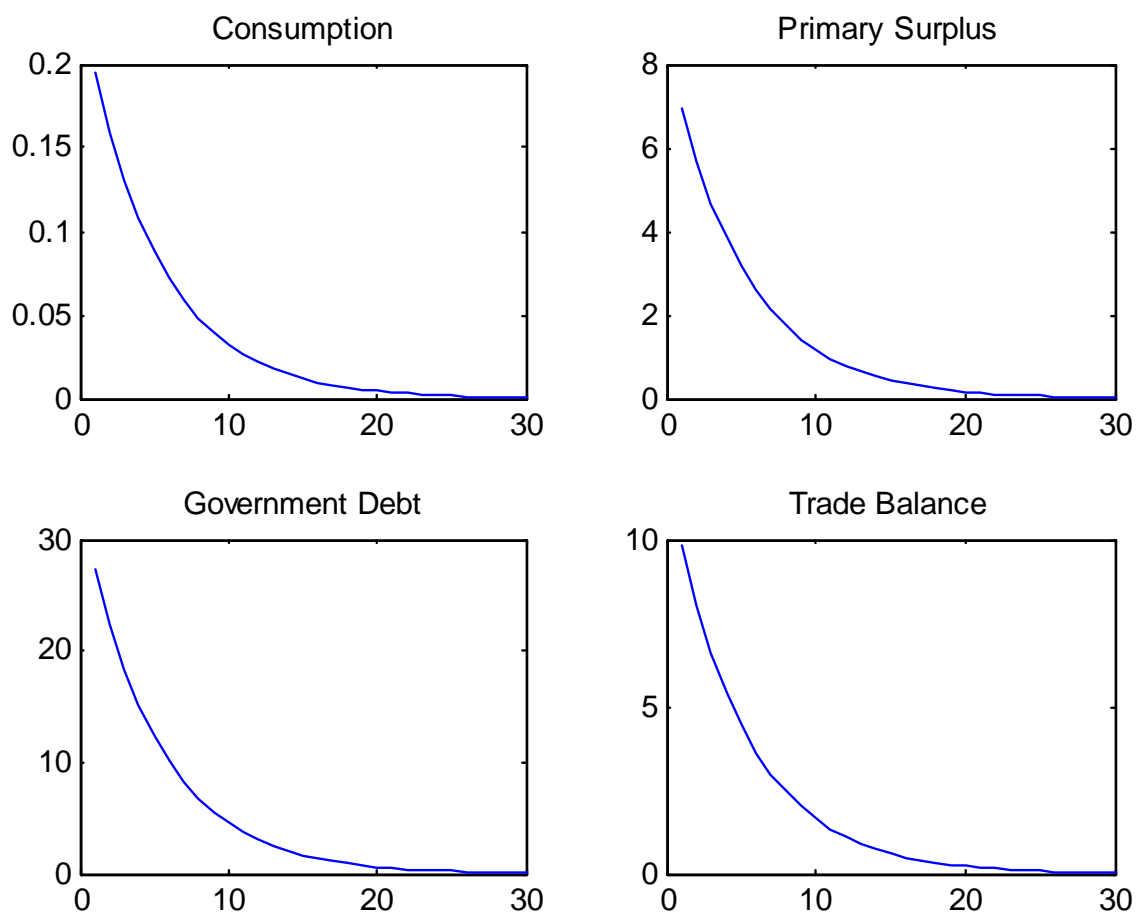


Figure 1 (cont.). Impulse Response to a one-standard deviation increase in Productivity

Figure 2 displays the impulse response functions of some of the variables to a one-standard deviation increase in government spending. Movements in government spending do not affect labor, output and consumption. This is because households are fully insured against government spending shocks via international financial markets. The government is able to maintain the tax rate constant in response to a government spending shock because it can fully finance the resulting changes in its budget through state-contingent debt. The primary

surplus and the value of the government’s debt portfolio decrease, since the government receives state-contingent payments from the rest of the world in bad states. The increase in government spending leads to a deterioration in the trade balance, since the economy uses the trade balance to smooth out consumption.

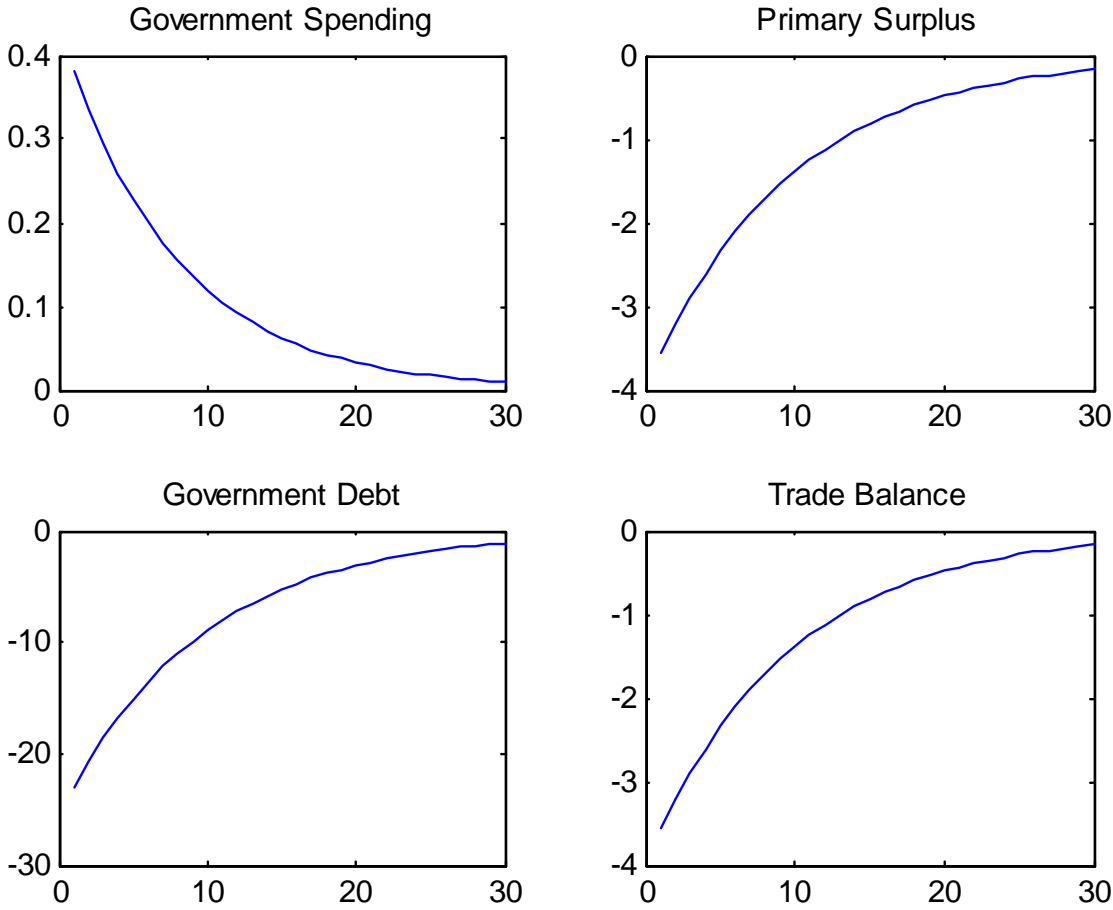


Figure 2. Impulse Response to a one-standard deviation increase in Government Spending

Table 2 displays a number of moments of key macroeconomic variables. Labor, output, consumption and the tax rate are uncorrelated with government spending under the optimal

fiscal policy with complete markets. The reason is that government spending shocks have only wealth effects, therefore, as agents have access to complete markets, they can fully insure against these shocks in international financial markets. After a positive government spending shock, the public deficit increases and the value of the debt portfolio with which the government ends the period decreases, thus the government finances the public deficit with state-contingent payments from the rest of the world. Under complete markets, the government constructs a debt portfolio that insures it against unanticipated variations in government spending and productivity. The government receives state-contingent payments from the rest of the world in bad states, and makes state-contingent payments in good states.

As we will see later, the neutrality of government spending shocks disappears when agents cannot hedge against such shocks, which is the case when markets are incomplete. Productivity shocks, on the other hand, affect labor, output consumption and the tax rate. This is because productivity shocks affect the marginal product of labor, so agents work more to take advantage of the temporary increase in productivity. The stochastic processes followed by all of these variables inherit the stochastic properties of the productivity shock. We can see in table 2 that the autocorrelation of labor, output, consumption and the tax rate is the same as the autocorrelation of the productivity shock, and that all these variables are positively correlated with the productivity shock. The trade balance and the value of the government's debt portfolio are negatively correlated with government spending, and positively correlated with productivity and output. This is because the government finances all innovations to government spending with state-contingent payments from the rest of the

world, and the trade balance is used to smooth out consumption.

Variable	Std. Dev. %	Autocorr.	Corr(x,z)	Corr(x,g)	Corr(x,y)
Productivity	4.00	0.82	1.00	0.00	0.99
Gov. Spending	0.38	0.90	0.00	1.00	0.00
Tax rate	0.10	0.82	0.99	0.00	0.99
Output	1.07	0.82	0.99	0.00	1.00
Labor	0.32	0.82	0.99	0.00	1.00
Consumption	0.05	0.82	0.99	0.00	1.00
Public Debt	4.84	0.85	0.76	-0.65	0.76
Trade Balance	31.10	0.82	0.99	-0.12	0.99

Table 2. Moments under Complete Markets

4 The Incomplete Markets Model

Suppose now that markets are incomplete in the sense that the small open economy has access only to risk-free debt. In this economy, agents can only borrow and lend issuing and buying non-contingent one-period discount bonds. Otherwise the economy is the same as the one described above for the complete markets case.

4.1 Households

The preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), h(s^t)) \quad (24)$$

The period-by-period budget constraint is given by

$$c(s^t) + a(s^{t-1}) + \psi(a(s^t)) \leq p(s^t) a(s^t) + (1 - \tau(s^t)) z(s^t) f(h(s^t)) \quad (25)$$

where $p(s^t)$ is the period t price of a bond that pays one unit of consumption in every state in period $t + 1$, therefore, this variable represents the inverse of the risk-free gross real interest rate. Letting $R(s^t)$ denote the gross risk-free real interest rate, we have

$$R(s^t) = \frac{1}{p(s^t)}$$

$a(s^t)$ denotes the quantity of bonds issued by the household at date t , conditional on history s^t , and the function $\psi(\cdot)$ captures a convex cost of adjusting the household's debt portfolio.

In addition to the budget constraint, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes

$$\lim_{j \rightarrow \infty} \prod_{i=0}^j p(s^{t+i}) a(s^{t+j}) \leq 0 \text{ for all } t, s^t \quad (26)$$

The assumptions on the utility function imply that households will always choose allocations such that constraints (25) and (26) hold with equality. The household's problem is then to choose state-contingent plans $\{c(s^t), h(s^t), a(s^t)\}_{t=0}^{\infty}$ to maximize (24) subject to (25) and (26) given the stochastic processes $\{z(s^t), \tau(s^t), p(s^t)\}_{t=0}^{\infty}$, and the initial condition a_{-1} . The first-order conditions associated with the household's maximization problem are (25), and (26) holding with equality for all t, s^t and:

$$-\frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) z(s^t) f'(h(s^t)) \quad (27)$$

$$u_c(s^t) [p(s^t) - \psi'(a(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t) \quad (28)$$

First-order condition (27) shows that the tax rate introduces a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor. First-

order condition (28) is the stochastic Euler equation. This equation show that at the optimum, the marginal benefit of issuing an additional unit of debt must equal its marginal cost.

4.2 The Government

In each period t , the government issues one-period non-contingent bonds $A(s^t)$, which pay one unit of consumption in every state in period $t + 1$. The government's period-by-period budget constraint is given by

$$g(s^t) + A(s^{t-1}) + \psi(A(s^t)) \leq p(s^t) A(s^t) + \tau(s^t) z(s^t) f(h(s^t)) \quad (29)$$

where $\psi(\cdot)$ captures a convex cost of adjusting the government's debt portfolio.

In addition to this budget constraint, the government is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes

$$\lim_{j \rightarrow \infty} \prod_{i=0}^j p(s^{t+i}) A(s^{t+j}) \leq 0 \text{ for all } t, s^t \quad (30)$$

Constraint (30) is a requirement for the existence of a well defined Ramsey equilibrium. The no-Ponzi game constraint cannot be ignored because without it the first best allocation is feasible. A benevolent government seeking to maximize the welfare of private agents will always choose state-contingent allocations such that (29) and (30) hold with equality. The fiscal policy consists in the announcement of state-contingent plans for $\{\tau(s^t), A(s^t)\}_{t=0}^{\infty}$

4.3 Competitive Equilibrium with Income Taxes

Given the initial conditions a_{-1} , A_{-1} , and the stochastic processes $\{g(s^t), z(s^t), p(s^t)\}_{t=0}^{\infty}$, a competitive equilibrium is a set of state-contingent sequences $\{c(s^t), h(s^t), a(s^t)\}_{t=0}^{\infty}$, and a fiscal policy $\{\tau(s^t), A(s^t)\}_{t=0}^{\infty}$ satisfying the following conditions *for all* t, s^t

$$c(s^t) + a(s^{t-1}) + \psi(a(s^t)) = p(s^t) a(s^t) + (1 - \tau(s^t)) z(s^t) f(h(s^t)) \quad (25)$$

$$\lim_{j \rightarrow \infty} \prod_{i=0}^j p(s^{t+i}) a(s^{t+i}) = 0 \quad (26)$$

$$-\frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) z(s^t) f'(h(s^t)) \quad (27)$$

$$u_c(s^t) [p(s^t) - \psi'(a(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t) \quad (28)$$

$$g(s^t) + A(s^{t-1}) + \psi(A(s^t)) = p(s^t) A(s^t) + \tau(s^t) z(s^t) f(h(s^t)) \quad (29)$$

$$\lim_{j \rightarrow \infty} \prod_{i=0}^j p(s^{t+i}) A(s^{t+i}) = 0 \quad (30)$$

Since the domestic economy is small, $p(s^t)$ is exogenous. We assume that $p(s^t)$ is constant and equal to β , therefore, the risk-free gross real interest rate $R(s^t)$ is also constant and equal to $1/\beta$. The household's debt $a(s^t)$ plus the government's debt $A(s^t)$ represent the economy's net foreign debt at the end of period t . If we combine the household's and the government's budget constraints, we obtain an expression that describes the evolution of the economy's net foreign debt

$$p(s^t) [a(s^t) + A(s^t)] = A(s^{t-1}) + a(s^{t-1}) - TB(s^t) \quad (31)$$

where $TB(s^t)$ denotes the trade balance of the economy, and is given by

$$TB(s^t) = z(s^t) f(h(s^t)) - c(s^t) - g(s^t) - \psi(a(s^t)) - \psi(A(s^t)) \quad (32)$$

4.3.1 The Primal Form

Proposition 5 *Given the initial conditions a_{-1}, A_{-1} , and the exogenous stochastic processes $\{g(s^t), z(s^t), p(s^t)\}_{t=0}^{\infty}$, state-contingent plans $\{c(s^t), h(s^t), a(s^t), A(s^t)\}_{t=0}^{\infty}$ satisfy (28)*

$$p(s^t) a(s^t) - \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} = c(s^t) + a(s^{t-1}) + \psi(a(s^t)) \quad (33)$$

$$p(s^t) [a(s^t) + A(s^t)] + z(s^t) f(h(s^t)) = A(s^{t-1}) + a(s^{t-1}) + c(s^t) + g(s^t) + \psi(a(s^t)) + \psi(A(s^t)) \quad (34)$$

if and only if they satisfy (25), (27), (28) and (29)

Proof. *See the Appendix.* ■

4.4 Ramsey Problem

It follows from Proposition 5 that the Ramsey problem can be stated as choosing state-contingent plans $\{c(s^t), h(s^t), a(s^t), A(s^t)\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), h(s^t))$$

subject to

$$u_c(s^t) [p(s^t) - \psi'(a(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t) \quad (28)$$

$$p(s^t) a(s^t) - \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} = c(s^t) + a(s^{t-1}) + \psi(a(s^t)) \quad (33)$$

$$p(s^t) [a(s^t) + A(s^t)] + z(s^t) f(h(s^t)) = A(s^{t-1}) + a(s^{t-1}) + c(s^t) + g(s^t) + \psi(a(s^t)) + \psi(A(s^t)) \quad (34)$$

This problem is not recursive because constraint (28) involves a conditional expectation of future control variables. Therefore, the usual Bellman equation is not satisfied, and the optimal choice at time t is not a time invariant function of the state variables $\{g_t, z_t, a_{t-1}, A_{t-1}\}$ as in standard dynamic programming, so the whole history of shocks can matter for today's optimal decision. Nevertheless Marcet and Marimon (1998) show that when the original maximization problem is not recursive because implementability constraints depend on plans for future variables, an equivalent saddle point problem can be constructed leading to a recursive formulation. The resulting saddle point problem expands the state space by including new state variables that summarize the evolution of the lagrange multipliers of the original problem. To solve the Ramsey problem, we need to write the problem in a recursive framework. The first step in this approach is to transform the original problem into a recursive saddle point problem.

The corresponding Lagrangian is

$$\Gamma = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left\{ \begin{array}{l} U(c(s^t), h(s^t)) + \vartheta(s^t) \\ \left[u_c(s^t) [p(s^t) - \psi'(a(s^t))] - \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t) \right] \end{array} \right\}$$

subject to (33) and (34), where $\beta^t \mu(s^t) \vartheta(s^t)$ is the Lagrange multiplier of constraint

(28). Using the law of iterated expectations and reordering terms, one can show that the function H defined as

$$H = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \{U(c(s^t), h(s^t)) + \vartheta(s^t) u_c(s^t) [p(s^t) - \psi'(a(s^t))] - \zeta(s^t) u_c(s^t)\} \quad (35)$$

$$\zeta(s^{t+1}) = \vartheta(s^t) \text{ for all } t \geq 0 \quad (36)$$

$$\zeta_0 = 0$$

is such that, for all feasible sequences $\Gamma = H$

Therefore, any solution to the original Ramsey problem must also be a solution to the problem of maximizing (35) subject to (36), (33) and (34).

Here $\zeta(s^t)$ acts as a co-state variable. Notice that this saddle point problem does not have any future variables in the constraints, and that all the functions in the constraints are known. If we include $\zeta(s^t)$ in the set of state variables, the problem becomes recursive in the sense that the optimal solution to the Ramsey problem

$$(c(s_t), h(s_t), a(s_t), A(s_t), \vartheta(s_t)) = \varsigma(a(s_{t-1}), A(s_{t-1}), \zeta(s_t), g(s_t), z(s_t))$$

for all t , and $\zeta_0 = 0$, where ς is a time-invariant function.

The Lagrangian for this problem, after substituting $\zeta(s^t) = \vartheta(s^{t-1})$ in the objective function, is given by:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} U(c_t, h_t) + \vartheta_t u_c(t) [p - \psi'(a_t)] - \vartheta_{t-1} u_c(t) \\ + \lambda_t \left[p a_t - \frac{u_h(t) h_t}{u_c(t) \eta} - c_t - a_{t-1} - \psi(a_t) \right] \\ + \phi_t [p(a_t + A_t) + z_t f(h_t) - A_{t-1} - a_{t-1} - c_t - g_t - \psi(a_t) - \psi(A_t)] \end{array} \right\}$$

where $\beta^t \lambda_t$ and $\beta^t \phi_t$ are the Lagrange multipliers on constraints (33) and (34) respectively.

The first-order conditions are given by

$$u_c(t) + \vartheta_t u_{cc}(t) [p - \psi'(a_t)] - \vartheta_{t-1} u_{cc}(t) = \lambda_t \left[1 + \frac{h_t}{u_c(t) \eta} \left(u_{ch}(t) - \frac{u_h(t) u_{cc}(t)}{u_c(t)} \right) \right] + \phi_t \quad (37)$$

$$u_h(t) + \vartheta_t u_{ch}(t) [p - \psi'(a_t)] - \vartheta_{t-1} u_{ch}(t) = \lambda_t \left[\frac{u_{hh}(t) h_t}{u_c(t) \eta} + \frac{u_h(t)}{u_c(t) \eta} \left(1 - \frac{h_t u_{ch}(t)}{u_c(t)} \right) \right] - \phi_t z_t f'(h_t) \quad (38)$$

$$-\vartheta_t u_c(t) \psi''(a_t) + (\lambda_t + \phi_t) [p - \psi'(a_t)] = \beta E_t (\lambda_{t+1} + \phi_{t+1}) \quad (39)$$

$$\phi_t [p - \psi'(A_t)] = \beta E_t (\phi_{t+1}) \quad (40)$$

$$u_c(t) [p - \psi'(a_t)] = \beta E_t (u_c(t+1)) \quad (41)$$

$$p a_t - \frac{u_h(t) h_t}{u_c(t) \eta} = c_t + a_{t-1} + \psi(a_t) \quad (33)$$

$$p [a_t + A_t] + z_t f(h_t) = A_{t-1} + a_{t-1} + c_t + g_t + \psi(a_t) + \psi(A_t) \quad (34)$$

$$\lim_{j \rightarrow \infty} p^{j+1} a_{t+j} = 0 \quad (42)$$

$$\lim_{j \rightarrow \infty} p^{j+1} A_{t+j} = 0 \quad (43)$$

$$\lim_{j \rightarrow \infty} p^{j+1} \vartheta_{t+j} = 0 \quad (44)$$

a_{-1}, A_{-1} given, and $\vartheta_{-1} = 0$

4.5 Dynamic Properties of the Optimal Fiscal Policy under Incomplete Markets

In this section we carry out some simulations to study the dynamic properties of the model economy under the Ramsey policy with incomplete markets. We compute the equilibrium dynamics by solving a linear approximation to the Ramsey planner's optimality conditions. We assume that the adjustment cost functions for the household and the government are respectively:

$$\psi(a(s^t)) = \frac{\psi}{2}(a(s^t) - \bar{a})$$

$$\psi(A(s^t)) = \frac{\psi}{2}(A(s^t) - \bar{A})$$

where \bar{a} and \bar{A} are the steady-state values of the household's debt and the government's debt respectively. The parameter ψ is chosen so that the costs are minimal and do not affect the short-run properties of the model, therefore, ψ is set to the minimum value that guarantees that the equilibrium solution is stationary. For all the other parameter values we use the same calibration and parameterization as in the model with complete markets. First, we show the impulse response functions of the model, and second we present the moments of the simulated time series.

4.5.1 Impulse Response Functions and Moments

Figure 3 displays the impulse response of the economy under the optimal fiscal policy to a one-standard deviation increase in the productivity shock, z_t . All the variables are expressed

in percentage deviations from their steady-state values. For a given tax rate, hours worked, output and consumption increase after a positive productivity shock. Since the tax rate is distortionary, the government decreases the income tax rate, which increases the incentive to work by increasing the after tax marginal product of labor causing people to substitute leisure for consumption. Consequently, output and consumption increase even more. Tax revenues also increase, even though the tax rate decreases, since the tax base increases significantly. The primary surplus increases as well since tax revenues increase and government spending remains constant, so the government uses the additional income to repay debt. The impulse response function of the primary surplus changes sign after some periods since a lower debt interest will have to be serviced in the future in response to a decrease in debt today. Since consumption responds less than output and government spending remains constant, the trade balance improves, but it changes sign after a few periods, because a lower debt interest on foreign debt will have to be serviced in the future. Public debt decreases after a positive productivity shock, while under complete markets the value of the government's debt portfolio increases after a productivity shock. When the government can not borrow contingent on the state of nature, it uses debt to smooth tax distortions over time. In the long-run all variables converge to the steady-state.

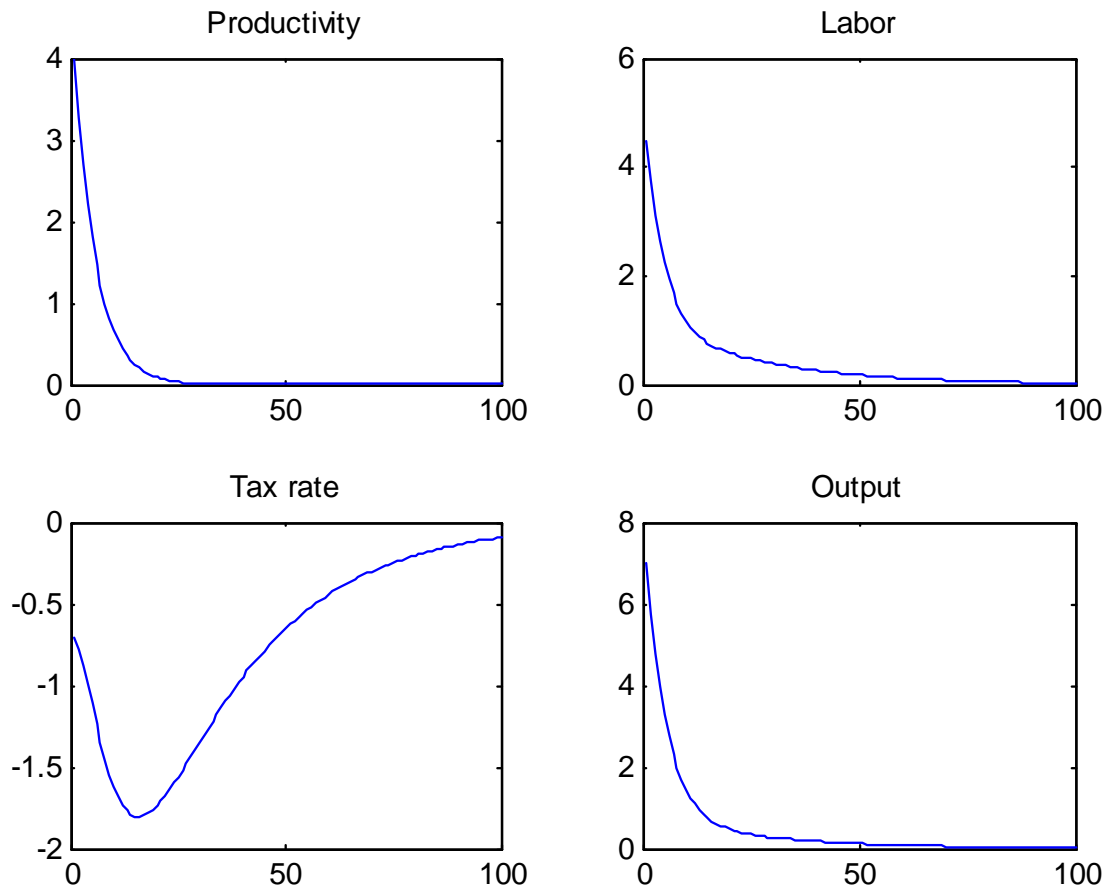


Figure 3. Impulse Response to a one-standard deviation increase in Productivity

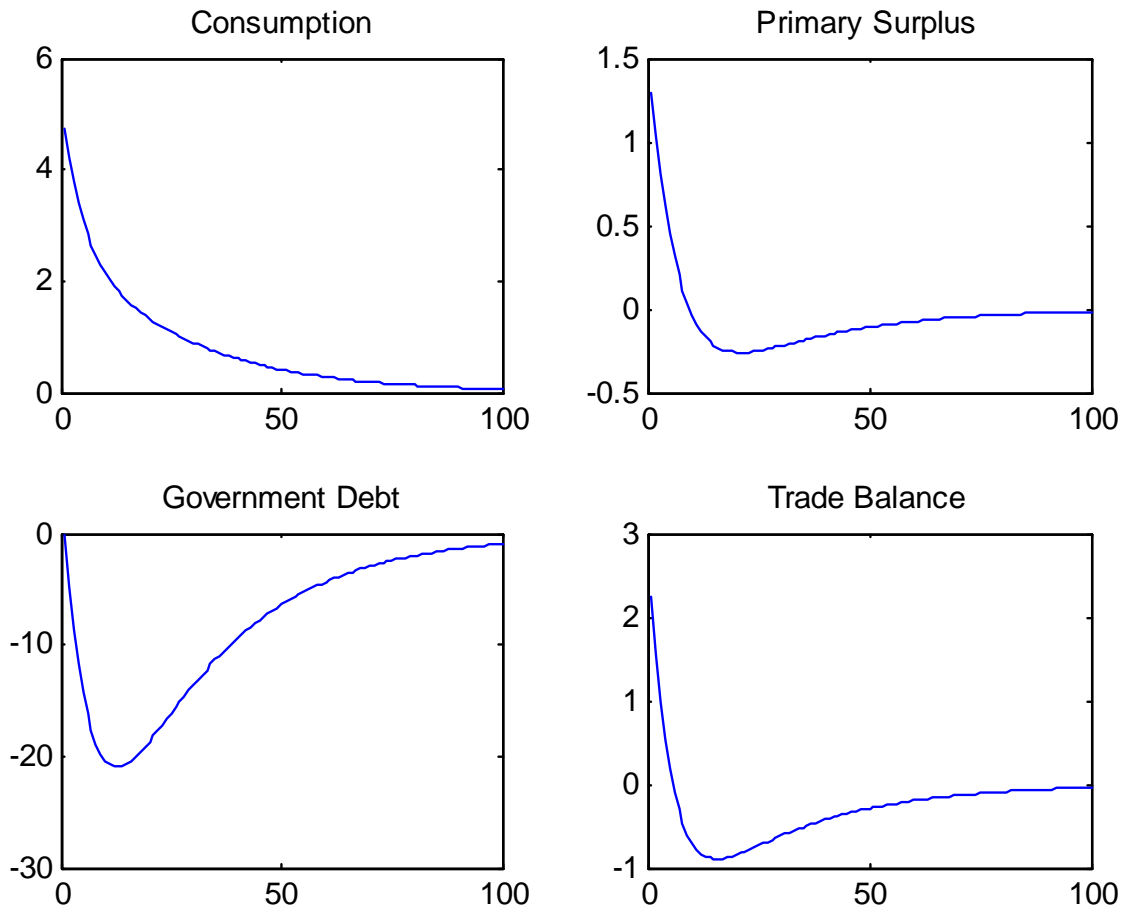


Figure 3 (cont.). Impulse Response to a one-standard deviation increase in Productivity

Figure 4 displays the impulse response of the economy under the optimal fiscal policy to a one-standard deviation increase in government spending. For a given tax rate, the household's demand for consumption and leisure are unaffected by government spending shocks, therefore the government needs to finance the increase in its expenditure by the least distortionary combination of tax rates and government debt. In response to a positive and persistent increase in government spending the government finds it optimal to increase

its debt, and to have a small but persistent increase in the tax rate that will pay off the increase in the stock of debt gradually over time, so the primary surplus decreases. The impulse response function of the primary surplus changes sign after some periods to pay for the additional debt service and to prevent debt from exploding.

Since government spending shocks are financed with debt and distortionary taxes, these shocks have income and substitution effects on consumption and leisure, while under complete markets, they only have income effects. Consumption decreases after a positive shock to government spending, since these shocks have negative income and substitution effects on this variable. For leisure the substitution effect is positive, since an increase in the tax rate reduces the incentives to work by lowering the after tax marginal product of labor, while the income effect is negative. For the benchmark parameterization, the substitution effect dominates, thus hours worked decrease after a positive government spending shock. Since technology remains constant, and labor decreases, output also decreases after a positive government spending shock.

The trade balance deteriorates after a positive government spending shock because agents use the trade balance as a shock absorber to smooth consumption over time. The impulse response function of the trade balance changes sign after some periods because a higher debt interest on foreign debt will have to be serviced in the future. In the long run all variables converge to the steady state.

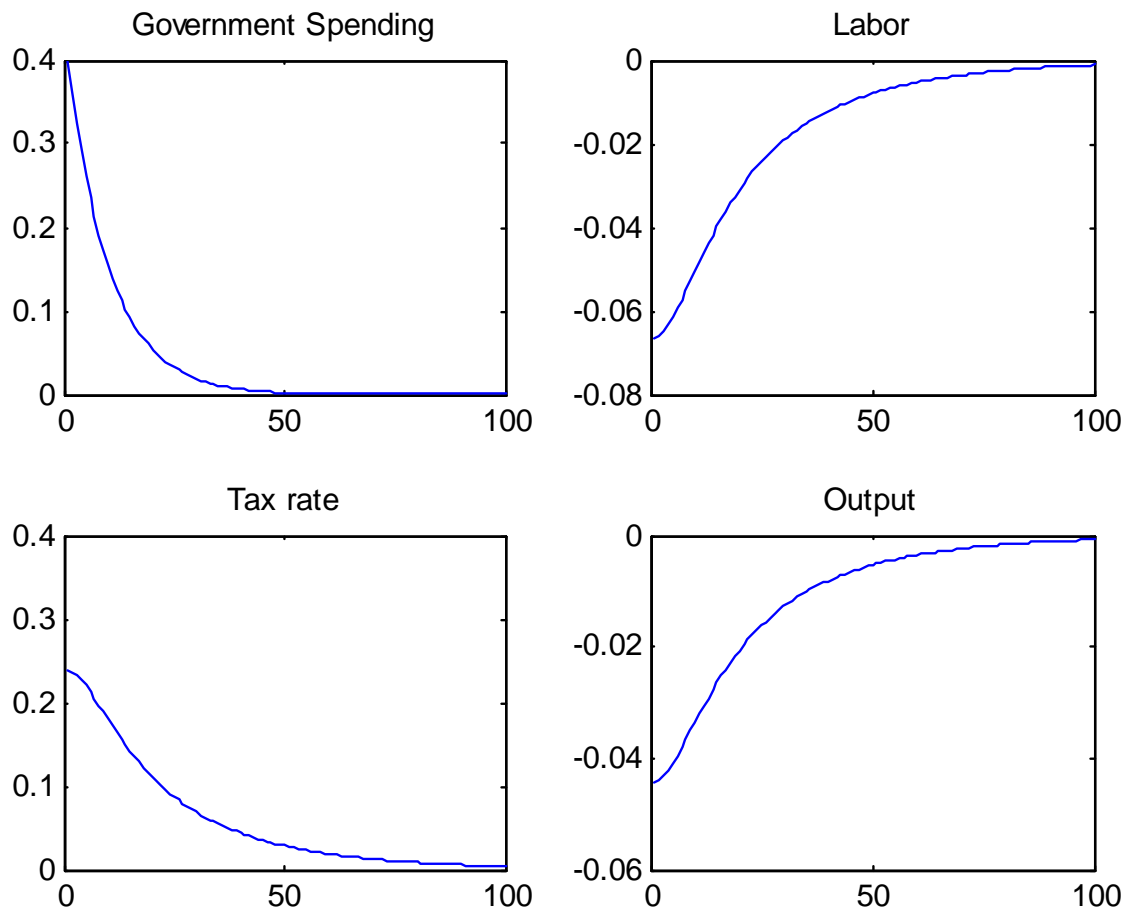


Figure 4. Impulse Response to a one standard deviation increase in Government Spending

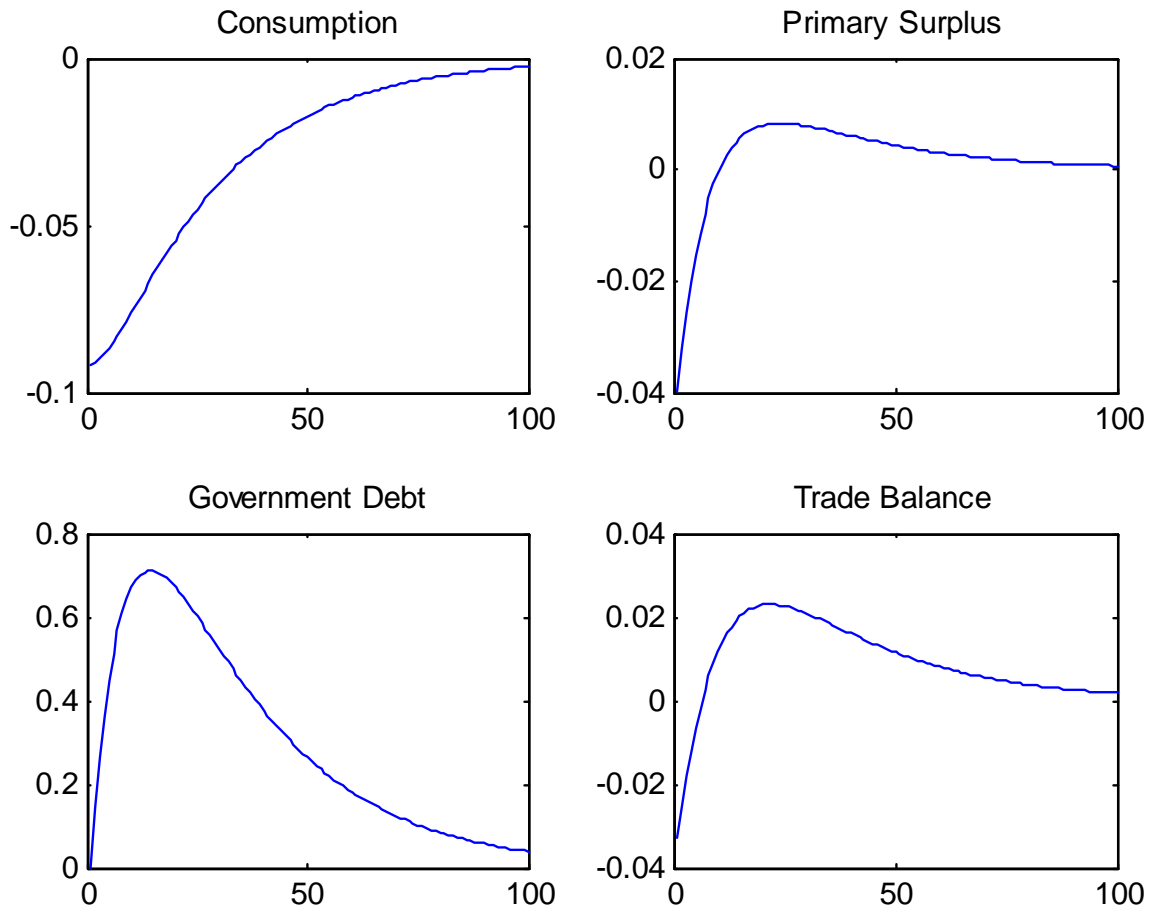


Figure 4 (cont.). Impulse Response to a one-standard deviation increase in Government Spending

The following table displays a number of moments of key macroeconomic variables under the Ramsey policy with incomplete markets. Table 3 reports the volatilities, correlations and autocorrelations of these variables.

Variable	Std. Dev. %	Autocorr.	Corr(x,z)	Corr(x,g)	Corr(x,y)
Productivity	4.00	0.82	1.00	0.00	0.99
Gov. Spending	0.38	0.90	0.00	1.00	-0.01
Tax rate	0.99	0.82	-0.38	0.24	-0.41
Output	6.91	0.60	0.99	-0.01	1.00
Labor	4.38	0.60	0.99	-0.02	0.99
Consumption	4.72	0.63	0.97	-0.02	0.98
Public Debt	14.33	0.93	-0.20	0.08	-0.22
Trade Balance	2.42	0.60	0.94	-0.02	0.93

Table 3. Moments under Incomplete Markets

Some interesting facts emerge from this table:

1. The income tax rate and specially public debt are very persistent. The reason is that the planner finances an increase in government spending or a decrease in the tax base partly by increasing public debt and partly by increasing the tax rate. In order to avoid a large distortion at the time of the shock, the planer smooths the tax distortion over time.
2. The Ramsey planner smooths tax distortions over the business cycle; the standard deviation of the tax rate is just 0.99%, which is smaller than the standard deviations of the other endogenous variables in the economy. Moreover, public debt is the most volatile variable because when the government cannot borrow contingent on the state of nature, it uses public debt to smooth tax distortions over time.
3. The standard deviations of consumption, output and labor are higher than in the model with complete markets. The increase in volatility relative to the case of complete mar-

kets is costly from a welfare perspective. While I do not provide quantitative estimates of this welfare costs, recent research suggests that the welfare costs of macroeconomic volatility in developing countries are substantial.

4. Public debt is negatively correlated with productivity and output, and positively correlated with government spending. By contrast, when agents have access to complete markets, the value of the government's debt portfolio is positively correlated with productivity and output, and negatively correlated with government spending. In an economy where agents only have access to one-period risk-free debt, the market value of outstanding debt is completely independent of the state of nature, therefore, the government needs to adjust the tax rate and the public debt in response to shock that affects government spending or the tax base.
5. The tax rate is positively correlated with government spending, while consumption, hours worked and output are negatively correlated. By contrast, in the model with complete markets, the real allocation and the tax rate are uncorrelated with government spending, since the government can insure completely against these shocks by borrowing and lending contingent on the state of nature. The negative correlation between consumption and government spending illustrates the limited insurance role played by non-contingent debt.

5 Conclusions

I have characterized optimal fiscal policy in a small open economy under both complete and incomplete markets. I have shown that when the government in a small open economy follows an optimal fiscal policy and agents have access to international complete asset markets, the value of the government's debt portfolio is a time invariant function of the underlying shocks. As a consequence, the value of the government's debt portfolio inherits the serial correlation structure of the shocks. Moreover, under complete markets the value of debt is negatively correlated with government spending, and positively correlated with productivity and output, while output, labor, consumption and the tax rate are uncorrelated with government spending shocks. The stochastic processes followed by output, labor, consumption and the tax rate inherit the serial-correlation properties of the stochastic process of the productivity shock. The Ramsey planner finances all innovations to government spending with state-contingent payments from the rest of the world.

By contrast, if agents in a small open economy can only buy and sell one-period risk-free bonds, public debt shows more persistence than other variables, and it is negatively correlated with productivity and output, and positively correlated with government spending, since the government uses debt to smooth tax distortions over time. Moreover, the tax rate is positively correlated with government spending, while consumption is negatively correlated. The negative correlation between consumption and government spending illustrates the limited insurance role played by non-contingent debt. In addition, since non-contingent one-period debt is not as good as contingent debt for smoothing purposes, the volatilities of

consumption and labor increase when the economy does not have access to complete markets. Hence, from a policy point of view, this paper stresses the importance of providing a richer menu of financial assets for developing countries, since several authors like Angeletos (2002) and Buera and Nicolini (2004) have shown that the government can use the maturity structure of non-contingent debt to replicate the complete markets optimal allocation as long as it has access to a sufficiently rich maturity structure.

6 References

1. Aiyagari, S. Rao., A. Marcet, T.J. Sargent, and J. Seppala. "Optimal Taxation without State-Contingent Debt" *Journal of Political Economy* (2002) Vol 110 (6), pp 1220-1254.
2. Angeletos, G.M., "Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure" *Quarterly Journal of Economics*, 117 (2002), 3 1105-31.
3. Barro, Robert J. "On the determination of public debt" *Journal of Political Economy* (1979) , Vol 87, pp. 940-971.
4. Bohn, Henning, "Tax Smoothing with Financial Instruments" *American Economic Review*, December, (1990), pp. 1217-1230.
5. Buera, Fransisco, and J.P. Nicolini. "Optimal Maturity of Government Debt without State Contingent Bonds" *Journal of Monetary Economics* (2004), 51, 531-554.
6. Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe. "Optimal Fiscal Policy in a Business Cycle Model". *Journal of Political Economy* (1994) Vol 102, No 4 (1994), 617-

652.

7. Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe (1995). "Policy Analysis in Business Cycle Models" In Thomas Cooley (ed), *Frontiers of Business Cycle Research* Princeton University Press: Princeton, New Jersey.
8. Chari, V.V. and Patrick J. Kehoe (1999). "Optimal Fiscal and Monetary Policy", *Handbook of Macroeconomics*, vol. III, edited by J. Taylor and M. Woodford, 1999.
9. Correia, Isabel., Neves, Joao C., Rebelo, Sergio. "Business Cycles in a small open economy" *European Economic Review* 39 (1995) 1089-1113.
10. Lane, P.R. and G.M. Milesi-Ferretti. "The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries" *Journal of International Economics* 55 (2001), 263-294.
11. Lucas, Robert E. Jr. and Nancy L. Stokey (1983). "Optimal Fiscal and Monetary Policy in an Economy without Capital". *Journal of Monetary Economics*, Vol 12, pp. 55-93.
12. Marcet, A., and R. Marimon, "Recursive Contracts". *Universitat Pompeu Fabra, Barcelona* (1998), Mimeo.
13. Marcet, A, and A, Scott. "Debt Fluctuations and the Structure of Bond Markets" *Universitat Pompeu Fabra, (2000)*, Mimeo.
14. Mendoza, Enrique "Real Business Cycles in a Small Open Economy" *American Economic Review* (1991), vol 81, 797-818.
15. Neumeyer, PA., Perri,F. "Business cycles in emerging economies: the role of interest rates" *Journal of Monetary Economics* Volume 52, Issue 2 March 2005 345-380.

16. Riascos, Alvaro and Carlos A. Vegh. “Procyclical Fiscal Policy in Developing Countries: The Role of Capital Market Imperfections” UCLA and Banco de la Republica, Colombia (2004), Mimeo.
17. Schmitt-Grohé, Stephanie and M. Uribe. (2003a) “Closing small open economy models” *Journal of International Economics* 61 163-185.
18. Schmitt-Grohé, Stephanie and M. Uribe. (2003b) “Anticipated Ramsey Reform and the Uniform Taxation Principle: The Role of International Financial Markets”, *European Central Bank Working Paper* 210.
19. Schmitt-Grohé, Stephanie and M. Uribe. “Optimal Fiscal and Monetary Policy Under Sticky Prices” *Journal of Economic Theory* (2004), vol 114, 198-230.
20. Siu, H. “Optimal Fiscal and Monetary Policy with Sticky Prices” *Journal of Monetary Economics* (2004), vol 51, 575-607.
21. Talvi, Ernesto, and Carlos A. Vegh, “Tax Base Variability and Procyclical Fiscal Policy” *Journal of Development Economics* (2005), forthcoming.

7 Appendix

Proposition 1. Given the initial condition b_{-1} , the parameter ξ and the stochastic processes $\{g(s^t), z(s^t)\}_{t=0}^{\infty}$, state-contingent sequences $\{c(s^t), h(s^t)\}_{t=0}^{\infty}$ satisfy

$$u_c(s^t) = \xi \tag{18}$$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} + z(s^t) f(h(s^t)) - g(s^t) \right\} \mu(s^t) = b_{-1} \quad (23)$$

if and only if they satisfy (15), (16), (17), (18) and (19) ■

Proof. First I show that if state-contingent plans $\{c(s^t), h(s^t)\}_{t=0}^{\infty}$ satisfy (15), (16), (17), (18) and (19), then they also satisfy (18) and (23). To this end, solve for the Arrow-Debreu price from equation (19), and for the tax rate from equation (16). Second, use the resulting expressions to eliminate $Q(s^t)$ and $\tau(s^t)$ from equation (17), which is the government's intertemporal budget constraint. Finally, reorder terms to obtain equation (23). Second, I show that if state-contingent plans $\{c(s^t), h(s^t)\}_{t=0}^{\infty}$ satisfy (18) and (23), then they also satisfy (15), (16), (17), (18) and (19). To this end, set $Q(s^t)$ such that (19) holds, $\tau(s^t)$ such that (16) holds, and λ such that (15) holds. Therefore, (15), (16) and (19) are satisfied by construction. Finally, substituting the constructed state-contingent sequences $\{Q(s^t), \tau(s^t)\}_{t=0}^{\infty}$ in equation (23), and reordering terms, we obtain equation (17) ■

Proposition 2. If the stochastic processes on $s = (z, g)$ are Markov, then there exist functions c, h and τ such that the Ramsey consumption allocations, labor allocations and income tax rates are time invariant functions only of the productivity shock.

$$c(s^t) = c(z_t), h(s^t) = h(z_t), \tau(s^t) = \tau(z_t) \quad \blacksquare$$

Proof. First, we assume that z_t and g_t follow independent 2-state symmetric Markov processes. Let z_t take on the values z^h and z^l and g_t the values g^h and g^l . Let $\phi^z = \text{Pr ob}(z_{t+1} = z^i | z_t = z^i)$ and $\phi^g = \text{Pr ob}(g_{t+1} = g^i | g_t = g^i)$ for $i = h, l$. Then the possible

states of the economy are described by the 4×1 state vector S , where

$$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \begin{bmatrix} (z^h, g^h) \\ (z^l, g^h) \\ (z^h, g^l) \\ (z^l, g^l) \end{bmatrix}$$

Let Φ denote the transition matrix of the state vector S

$$\Phi = \begin{bmatrix} \phi^z \phi^g & (1 - \phi^z) \phi^g & \phi^z (1 - \phi^g) & (1 - \phi^z) (1 - \phi^g) \\ (1 - \phi^z) \phi^g & \phi^z \phi^g & (1 - \phi^z) (1 - \phi^g) & \phi^z (1 - \phi^g) \\ \phi^z (1 - \phi^g) & (1 - \phi^z) (1 - \phi^g) & \phi^z \phi^g & (1 - \phi^z) \phi^g \\ (1 - \phi^z) (1 - \phi^g) & \phi^z (1 - \phi^g) & (1 - \phi^z) \phi^g & \phi^z \phi^g \end{bmatrix}$$

Second, we choose an initial state $s_0 = (z_0, g_0)$. Third, we guess a value for γ , there is one equilibrium value of γ for each possible initial state.

Fourth, we can check from the first-order conditions of the Ramsey problem that in each period $t \geq 0$, given a value of γ and a realization of the state of the economy s_i , equations (20) – (22) form a static system that can be solved numerically for c, h and Ω as functions of γ and z_i . The government spending shock g_t only enters in the intertemporal implementability constraint, but it does not enter in equations (20) – (22). Therefore, the realization of the government spending shock in period t does not affect that period's real allocation. Since there are only two possible values for z_i , given γ , the variables c, h and Ω

take only two different values. Thus, for $t \geq 0$ and a given value of γ , the solution to the Ramsey conditions can be written as $c(\gamma, z^i)$, $h(\gamma, z^i)$ and $\Omega(\gamma, z^i)$. Fifth, having computed the values taken by c , h and Ω at every state and date for a given guess of γ , we now check whether this guess of γ is the correct one by evaluating the intertemporal implementability constraint, equation (23). For $t \geq 0$ the expression $\frac{u_h(c(\gamma, z^i), h(\gamma, z^i))}{u_c(c(\gamma, z^i), h(\gamma, z^i))} \frac{h(\gamma, z^i)}{\eta} + z^i f(h(\gamma, z^i)) - g^i$ on the left-hand side of the implementability constraint can be written as $x(\gamma, s_i)$. Define the vector $x(\gamma)$ as

$$x(\gamma) = \begin{bmatrix} x(\gamma, s_1) \\ x(\gamma, s_2) \\ x(\gamma, s_3) \\ x(\gamma, s_4) \end{bmatrix}$$

Thus, the left-hand side of (23), which we denote by $X(\gamma, s_0)$ can be written as

$$X(\gamma, s_0) = x_0(\gamma, s_0) + \beta \Phi(s_0) (I - \beta \Phi)^{-1} x(\gamma)$$

where $\Phi(s_0)$ is the row of the transition matrix Φ corresponding to the state s_0 , and $x_0(\gamma, s_0)$ is the value of $x(\gamma, s_i)$ in the initial state s_0 . The right-hand side of (23) is equal to b_{-1} . Finally, compute the difference $y(\gamma, s_0) = X(\gamma, s_0) - b_{-1}$. This is a nonlinear equation, which can be solved numerically, we need to find a value for γ such that $y(\gamma, s_0) = 0$. This yields the equilibrium value of γ , and with it the equilibrium processes c , h and Ω for a given initial state s_0 . Moreover, it follows from first-order condition (16) that if c and h are time

invariant functions only of the productivity shock, then the income tax rate is also a time invariant function of the productivity shock. ■

Proposition 5. Given the initial conditions a_{-1} , A_{-1} , and the exogenous stochastic processes $\{g(s^t), z(s^t), p(s^t)\}_{t=0}^{\infty}$, state-contingent plans $\{c(s^t), h(s^t), a(s^t), A(s^t)\}_{t=0}^{\infty}$ satisfy (28)

$$p(s^t) a(s^t) - \frac{u_h(s^t) h(s^t)}{u_c(s^t) \eta} = c(s^t) + a(s^{t-1}) + \psi(a(s^t)) \quad (33)$$

$$p(s^t) [a(s^t) + A(s^t)] + z(s^t) f(h(s^t)) = A(s^{t-1}) + a(s^{t-1}) + c(s^t) + g(s^t) + \psi(a(s^t)) + \psi(A(s^t)) \quad (34)$$

if and only if they satisfy (25), (27), (28) and (29) ■

Proof. First, I show that if state-contingent plans $\{c(s^t), h(s^t), a(s^t), A(s^t)\}_{t=0}^{\infty}$ satisfy (25), (27), (28) and (29), then they also satisfy (28), (33) and (34). To this end, solve for $\tau(s^t)$ from equation (27) and substitute this expression in equation (25), reordering terms we obtain equation (33). Second, to obtain equation (34) combine equations (25) and (29). Next, I show that if state-contingent plans $\{c(s^t), h(s^t), a(s^t), A(s^t)\}_{t=0}^{\infty}$ satisfy (28), (33) and (34), then they also satisfy (25), (27), (28) and (29). To this end, set $\tau(s^t)$ such that equation (27) holds, therefore, (27) is satisfied by construction. Second, substitute the constructed state-contingent sequence $\{\tau(s^t)\}_{t=0}^{\infty}$ in (33), and reorder terms to obtain (25). Finally, to obtain (29) combine (33) and (34), and substitute $\{\tau(s^t)\}_{t=0}^{\infty}$ in the resulting expression. ■