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Abstract

We propose a new approach to model high and low frequency components of equity correlations. Our framework combines a factor asset pricing structure with other specifications capturing dynamic properties of volatilities and covariances between a single common factor and idiosyncratic returns. High frequency correlations mean revert to slowly varying functions that characterize long-term correlation patterns. We associate such term behavior with low frequency economic variables, including determinants of market and idiosyncratic volatilities. Flexibility in the time varying level of mean reversion improves the empirical fit of equity correlations in the US and correlation forecasts at long horizons.

Keywords: Factor models, Low frequency volatilities and correlations, Dynamic conditional correlation, Spline-GARCH, Idiosyncratic volatility, Long-term correlation forecasts.

JEL Classification: C22, C32, C51, C53, G11, G12, G32

Resumen

Proponemos una nueva metodología para modelar componentes de alta y baja frecuencia de las correlaciones de valores financieros. Nuestra estructura combina un modelo de factores para la valuación de activos con otras especificaciones que capturan la dinámica de las volatilidades y correlaciones entre un factor común y los rendimientos idiosincráticos. Las correlaciones de alta frecuencia se regresan hacia funciones que varían lentamente en el tiempo y que caracterizan el comportamiento de las correlaciones de largo plazo. Asociamos este comportamiento con variables económicas de baja frecuencia, incluyendo determinantes de la volatilidad de mercado y de las volatilidades idiosincráticas. Flexibilidad en el nivel de media variable al que el proceso retorna mejora el ajuste empírico de las correlaciones de valores en el mercado de Estados Unidos y los pronósticos de dichos comovimientos a horizontes largos.

Palabras Clave: Modelos de Factores, Volatilidades y correlaciones de baja frecuencia, Correlación condicional dinámica (DCC), Spline-GARCH, Volatilidad idiosincrática, Pronósticos de correlaciones de largo plazo.

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I. Introduction

Understanding the dynamics of correlations in financial markets is crucial to many important issues in finance. Optimal portfolio decisions, assessments of risks, hedging and pricing derivatives are examples of questions in financial decision making and financial regulation that require accurate measures and competent forecasts of comovements between asset returns. This paper introduces a new approach to characterize high and low frequency variation in equity correlations and describe short- and long-term correlation behavior. By separating short-term from long-term components, our method not only facilitates the economic interpretation of changes in the correlation structure but achieves improvements over leading methods in terms of fitting and forecasting equity correlations.

A number of multivariate time series models have been proposed in the last two decades to capture the dynamic properties in the comovements of financial returns. As natural generalizations, multivariate versions of the well known univariate GARCH and Stochastic Volatility (SV) models guided the initial specifications (e.g., Bollerslev, Engle and Wooldridge (1988) and Harvey, Ruiz and Shephard (1994)). These initial generalizations showed limitations because they were heavily parameterized and/or difficult to estimate. Simplified versions, such as constant conditional correlation models (e.g., Bollerslev (1990), Alexander (1998), Harvey et al. (1994)), were also unattractive because they had problems in describing empirical features of the data.[1] Only recently, Engle (2002) introduced the Dynamic Conditional Correlation (DCC) model as an alternative approach to achieve parsimony in the dynamics of conditional correlations maintaining simplicity in the estimation process. However, none of the aforementioned models associate correlation dynamics with features of fundamental economic variables. Moreover, since they return to a constant mean in the long run, their forecasting

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[1] Recent surveys of multivariate GARCH and SV models are provided in Bauwens, Laurent, and Rombouts (2003), Shephard (2004) and McAleer (2005). Multivariate SV models are scant; recent developments include Chib, Nardari, and Shephard (2006) and Asai and McAleer (2005).

implications for long horizons do not take into account changing economic conditions. Thus, they produce the same long-term forecast at any point in time.

Financial correlation models, on the other hand, have only recently introduced time variation in the correlation structure (e.g., Ang and Bekaert, 2002, Ang and Chen, 2002, Bekaert, Hodrick, and Zhang, 2008). Although these models are linked to asset pricing frameworks, which facilitate the association of correlation behavior with financial and economic variables, a substantial part of the variation in the correlation structure remains unexplained and the implementation of these models for forecasting correlations appears difficult.

This paper presents a new model that captures complex features of the comovements of financial returns and allows us to empirically associate economic fundamentals with the dynamic behavior of variances and covariances. Thus, by exploiting recent developments in time series methods, this model incorporates within a single framework the attractive features of the two approaches mentioned above. Specifically, based on the correlation structure suggested by a simple one-factor CAPM model, and short- and long-term dynamic features of market and idiosyncratic volatilities, we derive a correlation model that allows conditional (high frequency) correlations to mean revert toward smooth time varying functions, which proxy the low frequency component of correlations.[2] This property not only represents a generalization of multivariate GARCH models that show mean reversion to a constant covariance matrix, but also gives flexibility to the long-term level of correlations to adapt to the changing economic environment. Therefore, within this framework we can associate low frequency correlation behavior with changes in economic variables that are only observed at low frequencies such as macroeconomic aggregates.

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[2] Factors models have been used in multivariate settings to characterize dynamics in second moments. See for example Engle, Ng, and Rothschild (1990) and King, Sentana, and Wadhwani (1994) in the GARCH context, Diebold and Nerlove (1989), Harvey et al. (1994), and Chib, Nardari, and Shephard (2006) in the SV framework, and Andersen et al. (2001) in the realized variance context.

To achieve this goal and characterize term variation in equity correlations, we take a semi-parametric approach to specify the dynamics of our correlation components. The factor asset pricing structure provides a framework to separate systematic and idiosyncratic terms, and to characterize the covariance structure of excess returns. The semi-parametric Spline-GARCH approach of Engle and Rangel (2008) is used to model high and low frequency dynamic components of both systematic and idiosyncratic volatilities. We include these volatility components in the specification of correlations. As a result, a slow-moving low frequency correlation part is separated from the high frequency part. Moreover, the effects of time varying betas and unobserved latent factors are incorporated into the high frequency correlation component by adding dynamic patterns to the correlations between the market factor and each idiosyncratic component, as well as between each pair of idiosyncratic risks.[3] These high frequency patterns are modeled using a dynamic conditional correlation (DCC) process. Therefore, the resulting "Factor-Spline-Garch" (FSG-DCC) model blends Spline-GARCH volatility dynamics with DCC correlation dynamics within a factor asset pricing framework.

From the empirical perspective, this study analyzes high and low frequency correlation patterns in the US market by considering daily returns of stocks in the DJIA over a period of seventeen years. We find that, in addition to the recently documented economic variation in market volatility at low frequencies (e.g., Engle and Rangel (2008) and Engle, Ghysels, and Sohn (2008)), average idiosyncratic volatility shows also substantial variation in its long-term component. We find that this variation is highly correlated with low frequency economic variables including an inter-sectoral employment dispersion index based on Lilien (1982). Since this variable measures the intensity of shifts in product demand across sectors, we use it to proxy changes in the intensity of idiosyncratic news. For instance, a technological change (or any other driver of demand shifts) can induce large movements of production factors from declining to growing sectors and lead to increases in the intensity of firm-specific news. Consistent with this intuition, we find that this variable is positively related to idiosyncratic volatility.

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[3] Factor models with time varying betas have been studied in Bos and Newbold (1984), Ferson and Harvey (1991, 1993, 1999), and Ghysels (1998), among others.

Moreover, since the intensity of sectoral reallocation is associated with the same sources that lead to variation in productivity (or profitability) across firms and sectors, our empirical findings are also consistent with the positive relationship between idiosyncratic volatility and the volatility of firm profitability suggested by Pastor and Veronesi (2003).

To explore whether these findings hold for sectoral idiosyncratic volatility, we have also analyzed sectoral portfolios that incorporate a broader set of companies and found the same results. This evidence highlights the contribution of our framework, which lies in incorporating low frequency economic effects into the dynamic behavior of the correlation structure. Moreover, in terms of empirical fit, we find evidence that among the class of one-factor CAPM models, specifications with more flexible dynamics in the second moments of idiosyncratic components provide a better fit of the data.

We also investigate the forecast performance of the FSG-DCC model focusing on long horizons (within four and six months). Based on an economic loss function and following the approach of Engle and Colacito (2006), we perform in-sample and out-of-sample comparisons between the FSG-DCC model and a number of competitors. The in-sample exercise compares this model with the standard DCC (that mean reverts toward the sample correlation) and a restricted version of the FSG-DCC model that shares its high frequency features, but mean reverts to a constant level determined by a static one-factor model. The results favor the unrestricted FSG-DCC model. We next perform a sequential out-of-sample exercise that enhances the set of competing models by adding to it a single-index covariance estimator, the sample covariance, and an optimal shrinkage covariance estimator. Again, we find significant evidence that the FSG-DCC outperforms its competitors at long horizons. Therefore, given the scope of competing models and their long-term forecasting properties, our results indicate that the strong performance of the FSG-DCC is associated with the flexibility in its level of mean reversion to capture variations in the economic environment. Although we take a time series approach here, our model introduces a framework that permits us to directly incorporate economic variables into the construction of correlation forecasts. This suggests promising extensions to achieve further forecast improvements.

The paper is organized as follows: Section two provides a description of a number of correlation specifications associated with different assumptions in the factor setup. Section three introduces the FSG-DCC model and discusses estimation issues. Section four presents an empirical analysis of correlations in the US market. It also presents empirical evidence of economic variation in aggregated idiosyncratic volatility and includes an empirical evaluation of correlation specifications derived from different factor models. Section five examines the forecast performance of the FSG-DCC model, and Section six concludes.

II. A Single Factor Model and Return Correlations

In this section, we use a simple one-factor version of the APT asset pricing model of Ross (1976) and we describe how modifying its underlying assumptions changes the specification of the correlation structure of equity returns. Suppose that there is a single market factor that enters linearly in the pricing equation such as in the Sharpe's (1964) CAPM model. Under this specification and measuring returns in excess of the risk free rate, the excess return of asset i is generated by:

$$r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}, \tag{1}$$

where r_{mt} denotes the market excess return. The first term characterizes asset i's systematic risk and the second describes its idiosyncratic component. Absence of any arbitrage assures $\alpha_i = 0$ should hold and $E(r_{it}) = \lambda \beta_i$, where λ denotes the risk premium per unit of systematic risk.[4] The standard APT structure assumes constant betas, idiosyncrasies uncorrelated with the factor(s), and idiosyncrasies uncorrelated with each other:

$$E\left(r_{mt} u_{it}\right) = 0, \quad \forall i, \tag{2}$$

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[4] In our empirical exercise, we allow $\alpha_i \neq 0$ and, for simplicity, we assume a constant risk premium. However, the econometric specification in Equation (11) can be modified to account for time variation in λ. For instance, under a conditional one-factor CAPM structure, a GARCH-in-mean term can be added into Equation (11) to capture the effect of time variation in the risk premium. In results not reported, we find that such effect is small and its inclusion does not affect the conclusions of this paper.

$$E\left(u_{it}u_{jt}\right)=0, \qquad \forall i \neq j \tag{3}$$

Thus, the assumptions in the factor structure impose a restriction in the covariance matrix of returns. Under these standard assumptions a typical element of the unconditional covariance and correlation matrices can be respectively characterized as:

$$\text{cov}(r_{it},r_{jt})=\beta_i\beta_j\sigma_m^2+\begin{cases}\sigma_i^2 & i=j \\ 0 & i\neq j\end{cases} \tag{4}$$

$$corr(r_{it},r_{jt})=\frac{\text{cov}(r_{it},r_{jt})}{\sqrt{V(r_{it})}\sqrt{V(r_{jt})}}=\frac{\beta_i\beta_j\sigma_m^2}{\sqrt{\beta_i^2\sigma_m^2+\sigma_i^2}\sqrt{\beta_j^2\sigma_m^2+\sigma_j^2}}, \tag{5}$$

where V denotes the variance operator, σ_m^2 and the σ_i^2's are the variances of the factor and the idiosyncratic terms, respectively. Now, from the definition of conditional correlation,

$$\rho_{i,j,t}\equiv corr_t(r_{it},r_{jt})=\frac{\text{cov}_{t-1}\left((\beta_i r_{mt}+u_{it})(\beta_j r_{mt}+u_{jt})\right)}{\sqrt{V_{t-1}\left(\beta_i r_{mt}+u_{it}\right)}\sqrt{V_{t-1}\left(\beta_j r_{mt}+u_{jt}\right)}}, \tag{6}$$

and assuming that the moment restrictions in (2) and (3) hold conditionally, we obtain:

$$\rho_{i,j,t}=\frac{\beta_i\beta_j V_{t-1}\left(r_{mt}\right)}{\sqrt{\beta_i^2 V_{t-1}\left(r_{mt}\right)+V_{t-1}\left(u_{it}\right)}\sqrt{\beta_j^2 V_{t-1}\left(r_{mt}\right)+V_{t-1}\left(u_{jt}\right)}} \tag{7}$$

This expression suggests that the dynamic behavior of conditional correlations is determined exclusively by dynamic patterns in the conditional variances of market and idiosyncratic risks.[5] The betas determine sign and location.

In this single factor model, if the restriction in (2) holds conditionally, then the betas are constant and correctly estimated from simple time series regressions of excess returns on the market portfolio. Restriction (3) rules out correlation between idiosyncratic innovations, which precludes the possibility of missing pricing factors in the model. As

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[5] This specification is the basis of dynamic versions of one-factor CAPM correlation models that incorporate time varying variances (e.g., the Factor ARCH model of Engle, Ng, and Rothschild (1990) and the Factor Double ARCH of Engle (2007)).

suggested by Engle (2007), these restrictions are empirically unappealing and limit importantly the dynamic structure of correlations. Allowing for temporal deviations from such conditions increases substantially the ability of the resulting correlation models to capture empirical features of the data without affecting the economic essence of the factor model. Based on the approach of Engle (2007), the following proposition characterizes changes in the specification of correlations when such restrictions are relaxed.

Proposition 1: *Consider the model specification in Equation (1) and let $\mathfrak{I}_t$ denote the set of current and past information available in the market. a) Suppose that the assumption in (3) holds, but $E_{t-1}\left(u_{it}u_{jt}\right) \neq 0, \ \forall i \neq j$, then the correlation structure corresponds to a model with latent unobserved factors and the conditional correlation takes the following form:*

$$\rho_{i,j,t} = \frac{\beta_i\beta_j V_{t-1}\left(r_{mt}\right) + E_{t-1}\left(u_{it}u_{jt}\right)}{\sqrt{\beta_i^2 V_{t-1}(r_{mt}) + V_{t-1}(u_{it})}\sqrt{\beta_j^2 V_{t-1}(r_{mt}) + V_{t-1}(u_{jt})}}. \tag{8}$$

Moreover, b) suppose that the assumption in (2) is satisfied, but $E_{t-1}(r_{mt}u_{it}) \neq 0$, then the correlation structure is consistent with that of a single factor model with latent unobserved factors and time varying betas,

$$r_{it} = \tilde{\alpha}_i + \beta_{it}r_{mt} + \tilde{u}_{it}, \tag{9}$$

where each of these betas mean reverts toward a constant and the following conditions are satisfied: [6]

i) $E(\tilde{u}_{it}r_{mt}) = E(\tilde{u}_{it}) = 0, \ \forall i, \ and \ E(\tilde{u}_{it}\tilde{u}_{jt}) = 0, \ \forall i \neq j$.

ii) $\beta_{it} = \beta_i + w_{it}, \forall i$.

iii) $w_t = \{w_{1t}, w_{2t}, ..., w_{Nt}\}$ is a zero mean covariance-stationary process.

iv) $\mathrm{cov}(w_{it}r_{mt}, r_{mt}) = 0, \ \forall i$.

Such a correlation structure is described as follows:

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[6] To simplify notation (and without loss of generality), we omit the alphas from the excess returns equations. However, note that the constant terms in (9) differ in general from those in (1).

$$\rho_{i,j,t} = \frac{\beta_i\beta_jV_{t-1}\left(r_{mt}\right)+\beta_jE_{t-1}\left(r_{mt}u_{it}\right)+\beta_iE_{t-1}\left(r_{mt}u_{jt}\right)+E_{t-1}\left(u_{it}u_{jt}\right)}{\sqrt{\beta_i^2V_{t-1}(r_{mt})+V_{t-1}(u_{it})+2\beta_iE_{t-1}(r_{mt}u_{it})}\sqrt{\beta_j^2V_{t-1}(r_{mt})+V_{t-1}(u_{jt})+2\beta_jE_{t-1}(r_{mt}u_{jt})}} \ (10)$$

The proof is given in Appendix A1. Equation (8) describes a case where the factor loadings are constant, but latent factors can have temporal effects on conditional correlations. The second part of Proposition 1 considers the case of time varying betas. Under assumption *i)*, this new specification satisfies standard constraints equivalent to (2) and (3). In such a case, assumptions *iii)* and *iv)* guarantee that any factor loading is covariance-stationary and mean reverts toward a constant level given by $E(\beta_{it}) = \beta_i = \dfrac{\text{cov}(r_{it}, r_{mt})}{V(r_{mt})}$. Equation (10) provides a more general specification for conditional correlations that simultaneously incorporates the effects of time variation in the betas and latent unobserved factors. For this reason, we take this form as the basis of our econometric approach that we describe in the following section. In addition, as illustrated in Section IV, Proposition 1 can be used as a guidance to evaluate the empirical importance of each assumption in the simple one-factor model specified in (1).

III. An Econometric Model for a Factor Correlation Structure

Different assumptions on the factor framework imply different correlation structures and modeling approaches. This section begins by motivating our econometric specification. We then present a new model for high and low frequency correlations and discuss the estimation strategy.

The Factor-Spline-GARCH Model

The evolution of equity volatilities over time shows different patterns at different frequencies. Short-term volatilities are mainly determined by fundamental news arrivals, which induce price changes at very high frequencies. Longer term volatilities show patterns governed by slow-moving structural economic variables. Engle and Rangel

(2008) analyze such determinants and characterize the dynamic behavior of equity volatilities at low frequencies.[7] They find economically and statistically significant variation in the US low frequency market volatility as well as in most cases in developed and emerging countries. We introduce this effect in our factor correlation model by including an equation that describes the dynamic behavior of this low frequency market factor volatility.

In regard to idiosyncratic volatilities, incorporating their low frequency variation in the correlation structure is also appealing from the empirical and theoretical perspective since these low frequency components describe long-term patterns of idiosyncratic volatilities. The importance of such term behavior was highlighted by the influential study of Campbell, Lettau, Malkiel and Xu (2001) who find evidence of a positive trend in idiosyncratic firm-level volatility over the period from 1962 to 1997. Moreover, they find that the market volatility does not observe such increasing trend, which suggests a declining long-term effect in the correlations among individual stocks. Theoretical explanations for the upward trend in idiosyncratic volatilities have been associated with different firm features such as the variance of total firm profitability, uncertainty about average profitability, age, institutional ownership, and the level and variance of growth options available to managers (see Pastor and Veronesi (2003), Wei and Zhang (2006), and Cao, Simin and Zhao (2008)). At an aggregated level and using low frequency returns, Guo and Savickas (2006) find high correlation between idiosyncratic volatility and the consumption-wealth ratio proposed by Lettau and Ludvigson (2001). They also find a strong correlation between these variables and popular market liquidity measures, and suggest that idiosyncratic volatility could measure an omitted risk factor or dispersion of opinion. Overall, these empirical and theoretical results motivate our approach of incorporating long-term patterns of both systematic and idiosyncratic volatilities into a model for correlation dynamics, and emphasize the relevance of relating such dynamics with low frequency economic variables.

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[7] For different approaches on low frequency economic determinants of stock market volatility see, for example, Officer(1973), Schwert(1989), and Engle, Ghysels and Sohn (2006).

From the econometric point of view, the Spline-GARCH model of Engle and Rangel (2008) provides a semi-parametric framework to separate high and low frequency components of volatilities. Following this approach, we model the market factor in Equation (1) as:

$$r_{mt} = \alpha_m + \sqrt{\tau_{mt} g_{mt}}\, \varepsilon_t^m, \text{ where } \varepsilon_t^m \mid \Phi_{t-1} \sim (0,1)$$

$$g_{mt} = \left(1 - \theta_m - \phi_m - \frac{\gamma_m}{2}\right) + \theta_m \frac{(r_{mt-1} - \alpha_m)^2}{\tau_{mt-1}} + \gamma_m \frac{(r_{mt-1} - \alpha_m)^2}{\tau_{mt-1}} I_{r_{mt-1}<0} + \phi_m g_{mt-1}$$

$$\tau_{mt} = c_m \exp\left(w_{m0} t + \sum_{i=1}^{k_m} w_{mi} \left((t - t_{i-1})_+ \right)^2 \right), \tag{11}$$

where g_{mt} and τ_{mt} characterize the high and low frequency market volatility components, respectively. The term $I_{r_{mt}<0}$ is an indicator function of negative market returns. The high frequency volatility component is normalized to have an unconditional mean of unity leaving the low frequency term to describe unconditional volatilities. Different from Engle and Rangel (2008), the high frequency component is modeled as an asymmetric unit GARCH process following Glosten, Jagannathan, and Runkle (1993).[8] In this fashion, we capture the well documented leverage effect where bad news (negative returns) raises the future high frequency volatility more than good news (positive returns).[9]

The market variance in (11) describes multiplicatively the interaction between a dynamic term associated with high frequency news events and a slowly varying component that can capture the effect of changes in the economic environment. Intuitively, this specification allows systematic news events to have different impacts on the stock market when economic conditions change. It also captures variations in the news intensity that may arise in response to such changes in the economy. The term τ_{mt} approximates non-parametrically the unobserved low frequency market volatility that responds to low

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[8] The parameters of g_{mt} satisfy the standard stationarity conditions.
[9] This feature was first analyzed by Black (1976) and Christie (1982). They hypothesized that a firm's stock price decline causes an increase in the firm's debt to equity ratio (financial leverage), which results in an increase in future volatility. Campbell and Hentschel (1992) suggest the direction of causality runs opposite and explain this phenomenon by changes in risk premium and volatility feedback effects.

frequency fundamental variables, such as macroeconomic aggregates, which characterize the variation in the economic environment. This component is modeled using an exponential quadratic spline with equally spaced knots.[10] The number of knots, k_m, can be selected optimally using an information criterion. As in Engle and Rangel (2008), we will use the Schwarz Criterion (BIC) to control the degree of smoothness in the low frequency component.[11] The term g_{mt} describes transitory volatility behavior that, despite its persistency, does not have long term impacts on the levels of market volatility.

Similarly, we model the idiosyncratic part of returns in (1) as:

$$u_{it} = \sqrt{\tau_{it} g_{it}}\, \varepsilon_{it}, \text{ where } \varepsilon_{it} \mid \Phi_{t-1} \sim (0,1)$$

$$g_{it} = \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2}\right) + \theta_i \frac{(r_{it-1} - \alpha_i - \beta_i r_{mt-1})^2}{\tau_{it-1}} + \gamma_i \frac{(r_{it-1} - \alpha_i - \beta_i r_{mt-1})^2}{\tau_{it-1}} I_{r_{it-1}<0} + \phi_i g_{it-1}$$

$$\tau_{it} = c_i \exp\left(w_{i0}t + \sum_{r=1}^{k_i} w_{ir}\left((t - t_{r-1})_+\right)^2\right), \quad \forall i. \tag{12}$$

The g_{it}'s characterize the high frequency component of idiosyncratic volatilities associated with transitory effects, whereas the τ_{it}'s describe long-term variation in idiosyncratic volatilities. The $I_{r<0}$'s are indicator functions of negative returns that allow for firm-specific leverage effects. As before, we take a multiplicative error model to describe interactions between firm-specific news arrivals and low frequency state variables measuring firm- and industry-specific conditions. The intuition here is that a firm-specific news event will have a bigger effect, for example, when the firm is close to bankruptcy or when a major technological change is affecting the firm's industry. In such a context, bad news about the firm's fundamentals (e.g., lower than expected earnings) may increase the uncertainty about its future profits and, from the results of Pastor and Veronesi (2003), we may expect an increase in idiosyncratic volatility. The τ_{it}'s approximate non-parametrically the unobserved long-term idiosyncratic volatilities that

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[10] We follow the same notation as in Engle and Rangel (2008), where $(t - x)_+ = (t - x)$ if $t > x$, and it is zero otherwise. We refer to the original paper for further details on the spline specification.

[11] Improvements to this framework might be obtained by exploring theoretically and empirically the performance of alternative spline basis, different penalties, and specifications with non-equally-spaced knots. These are interesting extensions that require further statistical analyses outside the scope of this paper.

are functions of low frequency economic variables, which affect the magnitude and intensity of high frequency idiosyncratic shocks. They are modeled as exponential quadratic splines following the approach described above.

In addition, following the discussion in the previous section and Proposition 1, we incorporate time varying correlations between the market factor and idiosyncratic returns, as well as among the idiosyncratic terms themselves. Specifically, we assume that the vector of innovations in Equations (11) and (12), $\left(\varepsilon_t^m, \varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt} \right)'$, follows the DCC model of Engle (2002). Note that all the elements in this vector have unit conditional variance. Thus from the second stage in the standard DCC model, these correlations can be written as:[12]

$$\rho_{i,j,t}^{\varepsilon} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}}\sqrt{q_{j,j,t}}},$$

$$q_{i,j,t} = \bar{\rho}_{i,j}^{\varepsilon} + a_{DCC}(\varepsilon_{i,t-1}\varepsilon_{j,t-1} - \bar{\rho}_{i,j}^{\varepsilon}) + b_{DCC}(q_{i,j,t-1} - \bar{\rho}_{i,j}^{\varepsilon}), \quad \forall i,j \in \{1,...,N\},$$

$$q_{m,i,t} = \bar{\rho}_{m,i}^{\varepsilon} + a_{DCC}(\varepsilon_{t-1}^m \varepsilon_{i,t-1} - \bar{\rho}_{m,i}^{\varepsilon}) + b_{DCC}(q_{m,i,t-1} - \bar{\rho}_{m,i}^{\varepsilon}), \quad \forall i \in \{m,1,...,N\}, \quad (13)$$

where $\bar{\rho}_{i,j}^{\varepsilon} = E(\varepsilon_{it}\varepsilon_{jt})$ and $\bar{\rho}_{i,i}^{\varepsilon} = 1$, for all $i=m,1,2,...,N$.[13] Moreover, given the time variation in betas described in Proposition 1, we assume that $\bar{\rho}_{m,i}^{\varepsilon} = 0$, for all $i=1,2,...,N$.[14]

The specifications above, along with the factor structure presented in Section II, constitute the full Factor-Spline-GARCH (FSG-DCC) model and its correlation structure is described in the following proposition.

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[12] We consider a mean reverting DCC model. The parameters satisfy conditions to guarantee positive definiteness (they are positive, their sum is less than one, and the intercept matrix is positive definite). See Engle and Sheppard (2005a) for details.

[13] This DCC specification has *2+N(N+1)/2* parameters. To reduce the dimensionality problem, Engle (2002) and Engle and Sheppard (2005a) suggest estimating the *N(N+1)/2* constant terms ($\bar{\rho}_{i,j}^{\varepsilon}$'s) using the sample correlations. This follows the "variance targeting" approach of Engle and Mezrich (1996).

[14] Idiosyncratic innovations can be seen as residuals from regressions of returns on the market factor. Under the unconditional CAPM, they will be unconditionally uncorrelated with the market factor.

Proposition 2: *Given a vector of returns* $(r_{1t}, r_{2t}, ..., r_{Nt})'$ *satisfying the factor structure in Equation (1), suppose that the common market factor* r_{mt} *is described by (11), the idiosyncratic term* u_{it} *follows the process in (12), for all* $i=1,2,...N$, *and the vector of innovations* $(\varepsilon_t^m, \varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ *follows the DCC process in (13) and its assumptions, then the high frequency (conditional) correlation between* r_{it} *and* r_{jt} *is given by:*

(14)

$$\rho_{i,j,t} = \frac{\beta_i \beta_j \tau_{mt} g_{mt} + \beta_i \sqrt{\tau_{mt} g_{mt}} \sqrt{\tau_{jt} g_{jt}} \rho_{m,j,t}^\varepsilon + \beta_j \sqrt{\tau_{mt} g_{mt}} \sqrt{\tau_{it} g_{it}} \rho_{m,i,t}^\varepsilon + \sqrt{\tau_{it} g_{it}} \sqrt{\tau_{jt} g_{jt}} \rho_{i,j,t}^\varepsilon}{\sqrt{\beta_i^2 \tau_{mt} g_{mt} + \tau_{it} g_{it} + 2\beta_i (\tau_{mt} g_{mt} \tau_{it} g_{it})^{\frac{1}{2}} \rho_{m,i,t}^\varepsilon} \sqrt{\beta_j^2 \tau_{mt} g_{mt} + \tau_{jt} g_{jt} + 2\beta_i (\tau_{mt} g_{mt} \tau_{jt} g_{jt})^{\frac{1}{2}} \rho_{m,j,t}^\varepsilon}},$$

and, the low frequency component of this correlation is time varying and takes the following form:

$$\bar{\rho}_{i,j,t} = \frac{\beta_i \beta_j \tau_{mt} + \sqrt{\tau_{it}} \sqrt{\tau_{jt}} \bar{\rho}_{i,j}^\varepsilon}{\sqrt{\beta_i^2 \tau_{mt} + \tau_{it}} \sqrt{\beta_j^2 \tau_{mt} + \tau_{jt}}}. \qquad (15)$$

Moreover, assuming that $E_t(\tau_{k,t+h}) = \tau_{k,t}, \forall h > 0, k = 1,2,...,N,$ *then Equation (15) is the long horizon forecast of* $\rho_{i,j,t}$:

$$\lim_{h \to \infty} \rho_{i,j,t+h|t} = \bar{\rho}_{i,j,t} \qquad (16)$$

The proof is given in Appendix A2. Note that Equation (14) is a parametrized version of Equation (10) in Proposition 1. Equation (15) approximates the slow-moving component of correlation, which can be associated with long-term correlation dynamics. Indeed, the high frequency correlation parsimoniously mean reverts toward this time varying low frequency term. This approximation may be improved by either adding more factors or allowing for time variation in the $\bar{\rho}_{i,j}^\varepsilon$'s (the unconditional correlations across idiosyncratic innovations in the DCC equations), or both. The first alternative can be easily implemented once we have selected the new factors. Indeed, we can use the Spline-GARCH framework to estimate jointly the low frequency dynamics of the new factors and the corresponding loadings, as in Equations (11) and (12), and then we can add their innovations into the vector that follows the DCC process in (13). Therefore, the main issue reduces to the economics of a factor selection problem. The second extension

would capture long-term effects of excluded factors, but it is methodologically challenging because it requires the exploration of alternative functional forms and/or restrictions to guarantee positive-definiteness of the covariance matrix, which complicates the estimation process. Both extensions increase the number of parameters and the question of their relevance is mainly an empirical issue. We focus on the simplest case in *(15)* and leave the analysis of such extensions for future research.

Equation (15) also provides a simple approach to forecast long-term correlations using economic variables. Specifically, we can use forecasts of low frequency market and idiosyncratic volatilities, which can be obtained from univariate models that incorporate economic variables. For instance, the results of Engle and Rangel (2008) permit us to construct forecasts of the market volatility using macroeconomic and market information; indeed, we can use such economic market volatility forecasts to obtain long run correlation forecasts based on macroeconomic information. The last part of Section V presents an example that illustrates this application.

Moreover, from a time series perspective, Equation (16) presents a useful forecasting relation in which, due to the mean reversion properties of the model, the time varying low frequency correlation can be interpreted as the long run correlation forecast under the assumption that the low frequency market and idiosyncratic volatilities stay constant during the forecasting period. This provides a time series approach where long-term forecasts are constructed using Equation (15).

Another interesting case imbedded in Proposition 2 occurs when the low frequency components of volatility are constant over both the estimation and forecasting periods. This restricted version corresponds to the Factor DCC (FG-DCC) model of Engle (2007) and it is derived by assuming $\tau_{m,t} = \sigma_m{}^2$ and $\tau_{i,t} = \sigma_i{}^2, \forall i$, in (11) and (12). The corresponding variance specifications for the factor and the idiosyncrasies become standard mean reverting asymmetric GARCH(1,1) processes, which can be respectively written as:

$$h_{mt} \equiv \sigma_m^{\,2} g_{mt} = \sigma_m^{\,2}(1-\theta_m-\phi_m-\frac{\gamma_m}{2})+\theta_m(r_{mt-1}-\alpha_m)^2+\gamma_m(r_{mt-1}-\alpha_m)^2 I_{r_{mt-1}<0}+\phi_m h_{mt-1}, (17)$$

and

$$h_{it} \equiv \sigma_i^{\,2} g_{it} = \sigma_i^{\,2}(1-\theta_i-\phi_i-\frac{\gamma_i}{2})+\theta_i(r_{it-1}-\alpha_i-\beta_i r_{mt-1})^2+$$
$$\gamma_i(r_{it-1}-\alpha_i-\beta_i r_{mt-1})^2 I_{r_{it-1}<0}+\phi_i h_{it-1}, \forall i=1,2,...,N. \tag{18}$$

Hence, the conditional correlation in Equation (14) becomes:

$$\rho_{i,j,t}=\frac{\beta_i\beta_j h_{mt}+\beta_j\sqrt{h_{mt}}\sqrt{h_{it}}\rho_{m,i,t}^{\varepsilon}+\beta_i\sqrt{h_{mt}}\sqrt{h_{jt}}\rho_{m,j,t}^{\varepsilon}+\sqrt{h_{it}}\sqrt{h_{jt}}\rho_{i,j,t}^{\varepsilon}}{\sqrt{\beta_i^2 h_{mt}+h_{it}+2\beta_i(h_{mt}h_{it})^{\frac{1}{2}}\rho_{m,i,t}^{\varepsilon}}\sqrt{\beta_j^2 h_{mt}+h_{jt}+2\beta_i(h_{mt}h_{jt})^{\frac{1}{2}}\rho_{m,j,t}^{\varepsilon}}}, \tag{19}$$

and the low frequency correlation is the following constant:

$$\bar{\rho}_{i,j}=\frac{\beta_i\beta_j\sigma_m^{\,2}+\sigma_i\sigma_j\bar{\rho}_{i,j}^{\varepsilon}}{\sqrt{\beta_i^2\sigma_m^{\,2}+\sigma_i^{\,2}}\sqrt{\beta_j^2\sigma_m^{\,2}+\sigma_j^{\,2}}}. \tag{20}$$

This equation also represents the long run correlation forecast associated with the FG-DCC model. In section V, we will evaluate the forecast performance at long horizons of the FSG-DCC relative to the restricted FG-DCC model and other popular competitors.

Estimation

To facilitate the exposition of our estimation approach, it is useful to rewrite the FSG-DCC model using matrix notation. Suppose we have a vector of returns in excess of the risk free rate, including the market factor, at time t: $r_t=(r_{mt},r_{1t},...,r_{Nt})'$. The system of equations in the factor setup can be written as:

$$r_t=\alpha+Bu_t, \tag{21}$$

where u_t contains the market factor and the idiosyncrasies, $u_t=(r_{mt},u_{1t},u_{2t},...,u_{Nt})'$,

$B=\begin{pmatrix}1 & 0_{1\times N}\\ \beta & I_{N\times N}\end{pmatrix}$, $\beta=(\beta_1,\beta_2,...,\beta_N)'$, $0_{1\mathrm{x}N}$ is an N-dimensional row vector of zeros, $I_{N\mathrm{x}N}$ is

the N-dimensional identity matrix, and α is a vector of intercepts. The market factor is assumed to be weakly exogenous based on the definition of Engle, Hendry and Richard

(1983). Therefore, the parameters in (11) are considered variation free and they do not affect the inference in the conditional model given in (12).

In addition, the covariance matrix of u_t can be written as follows:

$$H_t^u = D_t R_t D_t, \tag{22}$$

where $D_t = diag\left\{\sqrt{\tau_{kt}h_{kt}}\right\}$, for $k=m,1,...,N$, $R_t = diag(Q_t)^{1/2}Q_t diag(Q_t)^{1/2}$ is a correlation matrix, and the typical element of Q_t is defined by Equation (13). The standardized innovations in Proposition 2 are the elements of $D_t^{-1}u_t = \left(\varepsilon_t^m, \varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt}\right)'$. Moreover, going back to the original vector of returns, we have that $r_t \mid \mathfrak{T}_{t-1} \sim (\alpha, H_t)$, where the full covariance matrix takes the following form[15]:

$$Var(r_t) = H_t = B D_t R_t D_t B' \tag{23}$$

As in the standard DCC model, the estimation problem can be formulated following Newey and McFadden (1994) as a two-stage GMM problem. Under this framework, we can write the vector of moment conditions in the form $g(r_t, \psi, \eta) = \left(m_1(r_t, \eta), m_2(\varepsilon_t, \psi, \eta)\right)'$, where $\varepsilon_t \equiv D_t^{-1}u_t$ is a vector of devolatized returns, η is a vector of parameters containing the alphas, the betas, and the volatility parameters in Equations (11) and (12), and ψ is a vector containing the DCC parameters in (13). The first vector of moment conditions, m_1, has the scores of individual asymmetric Spline-GARCH models as its components. The second set of moment conditions, m_2, contains the functions that involve the correlation parameters, which come from the assumed likelihood and the moment conditions used for correlation targeting in the DCC model.[16] If the first optimization problem involving the moment conditions in m_1 gives consistent estimates of the volatility and mean parameters, then the optimization of m_2 in the second step will give consistent estimates.

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[15] The correlation matrix of r_t is $R_t = diag\left\{B D_t R_t D_t B'\right\}^{-1} B D_t R_t D_t B' diag\left\{B D_t R_t D_t B'\right\}^{-1}$, and its typical element, ignoring the first row and first column that contains correlations between the factor and the idiosyncrasies, is the expression in Equation (14).

[16] Following the discussion in footnote 12, the constant terms in (13) are estimated using the correlation targeting constraint of Engle and Mezrich (1996). Thus, our moment conditions for correlation targeting equalize each unconditional covariance term $(E(\varepsilon_{it}\varepsilon_{jt}), \ i \neq j)$ to its sample analog.

In a Gaussian quasi-likelihood (QML) framework, assuming multivariate normality leads to consistent estimates under mild regularity conditions as long as the mean and the covariance equations are correctly specified.[17] This distributional assumption gives a useful decomposition of the log-likelihood into the sum of two components that take the familiar Gaussian form: one involves the "mean and volatility" part and the other the "correlation" part, just as in the DCC model of Engle (2002).[18] In this case, the components of m_2 are the scores of the following likelihood:

$$L(\eta, \psi) = -\frac{1}{2} \sum_{t=1}^{T} \left(\log |R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t \right), \tag{24}$$

whereas the components of m_1 are the scores of log-likelihoods associated with univariate Spline-GARCH models with Gaussian innovations that can be estimated separately:

$$L(\eta_m) = -\frac{1}{2} \sum_{t=1}^{T} \log(\tau_{mt} g_{mt}) + \frac{(r_{mt} - \alpha_m)^2}{\tau_{mt} g_{mt}},$$

$$L(\eta_i) = -\frac{1}{2} \sum_{t=1}^{T} \log(\tau_{it} g_{it}) + \frac{(r_{it} - \alpha_i - \beta_i r_{mt})^2}{\tau_{it} g_{it}}, \quad i = 1, 2, ..., N. \tag{25}$$

Despite the convenience of the Gaussian QML approach, choosing the number of knots for the spline functions introduces an additional procedure in the estimation process that does not necessarily provide QML estimates of such quantities. Inaccurate choices for the number of knots might introduce some biases in the procedure due to misspecification of volatilities. Since distributional assumptions might have an effect on this part of the estimation, we depart from normality and use distributional assumptions that more realistically describe empirical features of excess returns. This can improve the accuracy of the knot selection criterion and reduce the misspecification problem. Therefore, for the first-stage of the estimation process, we consider likelihoods from the Student-t distribution because this distribution is better to capture the fat-tails typically observed in financial time series and it diminishes the effect of influential outliers. Thus, the following log-likelihoods correspond to the Asymmetric Spline-GARCH model with

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[17] See Bollerslev and Wooldridge (1992) and Newey and Steigerwald (1997) for details on consistency of QML estimators.
[18] Consistency and asymptotic normality are satisfied under standard regularity conditions. See Engle (2002) and Engle and Sheppard (2005a) for details.

Student-t innovations and determine moment conditions associated with the first stage of the GMM estimation process:[19]

$$L(\eta_m, v_m) = \log\left(\frac{\Gamma((v_m+1)/2)}{\Gamma(v_m/2)((v_m-2)g_{mt}\tau_{mt})^{1/2}}\right) - \frac{v_m+1}{2}\log\left(1 + \frac{(r_{mt}-\alpha_m)^2}{\tau_{mt}g_{mt}(v_m-2)}\right)$$

$$L(\eta_i, v_i) = \log\left(\frac{\Gamma((v_i+1)/2)}{\Gamma(v_i/2)((v_i-2)g_{it}\tau_{it})^{1/2}}\right) - \frac{v_i+1}{2}\log\left(1 + \frac{(r_{it}-\alpha_i-\beta_i r_{mt})^2}{\tau_{it}g_{it}(v_i-2)}\right), \quad i=1,...,N,$$

(26)

where Γ denotes the gamma function and the v's refer to the corresponding degrees of freedom.[20]

Regarding the second stage, the estimation can be performed in the usual way, which jointly estimates the whole correlation matrix. However, although DCC is very parsimonious in its parameterization, it is biased and slow for large covariance matrices. Alpha is biased downward and may approach zero. Thus, estimated correlations are less variable in big systems than when estimated for subsets even for simulated data (see Engle and Sheppard (2005b)). Since our empirical analysis includes a moderately large number of assets, we are interested in a simpler strategy to estimating large covariance matrices.

In this regard, two approaches have recently been suggested in the literature. Engle (2007) introduces the MacGyver method to reduce the bias problems and simplify the estimation process for large systems. This method is easily implemented by fitting all bivariate models and deriving a single estimator from all the estimated pairs. Montecarlo experiments favor the median of these estimators as a good candidate. However, an important limitation of this approach is the difficulty to conduct inference. In the same spirit, Engle, Shephard, and Sheppard (2008) introduce the composite likelihood (CL) method as another alternative to overcome the computational problems of estimating large systems and correct the mentioned biases. This approach constructs a CL function

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[19] This specification extends the Student-t GARCH model of Bollerslev (1987).

[20] An earlier version of this paper showed results using the Gaussian QML approach. Although all our empirical results in Sections IV and V are maintained, the Student-t distributional assumption leads to a superior forecast performance in all the DCC-type models compared in Section V. Results of the Gaussian case are available upon request.

as the sum of quasi-likelihoods of pairs of assets (i.e., the sum of all the unique bivariate log-likelihoods).[21] Under this strategy, only a single optimization of the CL function is needed and inference can be easily conducted. In the following section, we present results using the standard multivariate method and these two novel strategies. In Section V, we perform a sequential out-of-sample forecasting exercise using the CL method in the second step of the estimation process.

IV High and Low Frequency Correlations in the US Market.

Data

We use daily returns on the DJIA stocks from December 1988 to December 2006. The data is obtained from CRSP. During this period, there have been a number of changes in the index, including additions, deletions, and mergers. We include all the stocks in the 2006 index and those in the 1988 index that could be followed over the sample period.[22] As a result, we obtain a sample of 33 stocks. Regarding the market factor, we use daily returns on the S&P500. We use the one-month T-bill rate as the time varying risk free rate.

Individual stocks are described in Table 1, which include company names, market tickers, average excess returns and average annualized excess return volatility over the whole sample period. The most volatile stocks in the sample are INTC and HPQ, whereas the least volatile are XOM, CVX, and 3M.[23] The stocks with largest average daily return are MSFT and INTC and those with the smallest values are GM and IP.

Description of Estimation Results

We estimate the FSG-DCC model following the two stage GMM approach described in Section III and present the results in Table 2. The parameters estimated in the first step

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[21] If all the pairs are independent, the CL becomes the exact QML. Engle, Shephard, and Sheppard (2008) do not require independence. Moreover, they suggest little efficiency loss if the CL is constructed from a subset of bivariate systems that involve only contiguous pairs.

[22] Those include Chevron (CVX), Goodyear (GT), and International Paper (IP).

[23] We use the ticker name to identify individual stocks. The full company names are presented in the first column of Table 1.

are those in Equations (11), (12), and (26). The second step involves the DCC parameters in Equation (13). For reasons of space, we do not report the estimated coefficients of the spline functions. We only present the optimal number of knots selected by the BIC.

The first column shows the alphas, which are in general not significantly different from zero, as suggested by the CAPM framework. The only exceptions are MO, with a positive value, and GM and GT, with negative values. The second column reports the estimated unconditional market betas associated with each stock. All of them are highly significant and their values go from 0.66 to 1.52 (see Figure 1). CVX, PG, and XOM show the lowest factor loadings and also low levels of realized volatility over the sample period (see Table 1), whereas INTC, JPM, and HPQ show the largest betas as well as high levels of realized volatilities. The third column presents the estimates of the ARCH volatility coefficients. They take values between 0.004 and 0.13. They are in general significant, except for the market factor and IBM, and their median is 0.05. Column four presents the estimates of the GARCH volatility effects. They are all significant and their median is 0.84. In addition, they take values between 0.18 and 0.99, with only three cases lying below 0.50. For the market factor, a GARCH effect of 0.90 was estimated. Column five shows estimates of the leverage effects, which exhibit a median of 0.03. They are statistically significant for about half of the stocks and, in such a case, they are positive, which is consistent with the leverage theory of Black (1976) and Christie (1982). However, this effect is substantially higher and significant for the market factor, which provides stronger support for the volatility feedback hypothesis.[24] The cross-sectional variation in the GARCH and ARCH effects indicates variation in persistence across the high frequency idiosyncratic volatilities. For example, the first panel of Figure 2 illustrates the familiar highly persistent case associated with a small ARCH coefficient and a GARCH effect close to 1, whereas the last panel in this figure shows a noisier case where the ARCH affect is 0.08 and the GARCH effect is only 0.63.

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[24] Bekaert and Wu (2000) also find stronger support for the volatility feedback hypothesis using portfolios from Nikkei 225 stocks.

The sixth column of Table 2 shows the degrees of freedom of the univariate Student-t distributions. They fluctuate between 4 and 10, and the median value is 6. These values are in line with the traditional evidence of non-normality and excess of kurtosis in equity returns. The last column of Table 2 presents the optimal number of knots in the spline functions according to the BIC. These numbers reflect changes in the curvature of the long-term trend of idiosyncratic volatilities. Examples of such patterns are illustrated in Figure 2. For instance, the first graph has a smoother low frequency component associated with a single knot. As we move to the bottom of the figure, the number of knots increases and more cyclical effects are observed. Four knots characterize the low frequency component of the market volatility. Its dynamic behavior is illustrated in Figure 3.

The bottom part of Table 2 presents the estimation results of the second stage. The first column reports the standard DCC estimates from the multivariate traditional approach along with their standard errors. The second column presents the MacGyver DCC coefficients, which are the median values of all DCC estimates from all the unique bivariate systems, as described in Section III, and the third column presents the CL estimates obtained from the optimization of the sum of all the unique bivariate quasi-log-likelihoods. The estimator of a_{DCC} increases from 0.0027, using the standard multivariate method, up to 0.004 (or 0.005) when using the CL (or the MacGyver) method. This suggests that both methods deliver a correction in the downward bias of the traditional DCC estimator.

Figures 4 and 5 illustrate the time series properties of the FSG-DCC with a number of examples that show the high and low frequency correlation components along with model-free rolling correlations. The high frequency component mean reverts toward the slow-moving low frequency component. It is visually clear by looking at the model-free rolling correlations that the model characterizes fairly well trend behavior in correlations. These examples also show the interaction between the factor pricing structure and the low frequency variation of market and idiosyncratic volatilities. For instance, focusing on the last two years of the sample, where the market volatility shows a declining trend, the

model suggests that stocks with increasing low frequency idiosyncratic volatilities will have pronounced declining correlations. Figure 4 illustrates this pattern using the stocks in Figure 2 that show increasing idiosyncratic volatilities. In contrast, Figure 5 presents examples where market and idiosyncratic volatilities have opposite effects on low frequency correlations and mixed patterns during the last years of the sample are observed. While these figures only exhibit a description of a few particular cases, the dimensionality of our results renders it difficult to analyze them at a disaggregated level. Instead, we present an aggregated analysis of volatilities and correlations linking their patterns to economic variables.

Aggregated Volatility Components and Average Correlations

The most distinguishing feature of the FSG-DCC model is its ability to characterize dynamic long-term correlation behavior exploiting the structure of a factor asset pricing model, and the low frequency variation in systematic and idiosyncratic volatilities. The empirical results of Engle and Rangel (2008) and Engle, Ghysels, and Sohn (2008) provide evidence that the low frequency market volatility responds to changes in the slow-moving macroeconomic environment. In addition, the estimation results presented earlier show evidence of substantial low frequency variation in idiosyncratic volatilities. A natural question is whether such idiosyncratic behavior is reflecting changes in fundamentals. This subsection presents evidence that the aggregated low frequency component of idiosyncratic volatility systematically varies with low frequency economic variables. This emphasizes the importance of incorporating such a feature into the dynamic behavior of the correlation structure, which in this model is flexible enough to adapt its long-term level to economic conditions that typically change slowly. Besides this interpretational advantage, Section V shows that this flexibility pays out when we forecast correlations at long horizons.

As mentioned earlier, cross-sectional aggregation facilitates the exposition to illustrate the effect of our volatility components on correlations. We construct aggregates by taking

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the cross sectional average of our dynamic components at each point in time.[25] Figure 6 shows averages over biannual sub-periods (from the whole sample) of low frequency market and idiosyncratic volatilities. Before 1997-1998, while the average low frequency idiosyncratic volatility shows an increasing pattern, the low frequency market volatility is declining. This is consistent with the findings of Campbell et al. (2001) and suggests a decline in correlations, which is confirmed by both the aggregated model-free rolling correlations and the aggregated FSG-DCC correlations in Figure 7. After 1997, market and idiosyncratic volatilities seem to move in a similar fashion having opposite effects on correlations, which at the aggregated level show a non-monotonic pattern. An interesting effect is observed in the last period where, although the market volatility reached historical lows, correlations decreased only moderately due to the low levels of idiosyncratic volatility. Indeed, the average low frequency market volatility during 2005-2006 was as low as during the period 1993-1996, but the average correlation was almost doubled relative to that of the earlier period, whereas the average idiosyncratic volatility in 2005-2006 was almost half of the corresponding 1993-1996 value.

We have illustrated that low frequency variation in idiosyncratic volatilities is not negligible and can have big effects on the level of correlations. We now focus on the economic sources of such variation, keeping the analysis at the aggregated level. As mentioned in Section III, the trend behavior in idiosyncratic volatility has received important attention in the literature following the results of Campbell et al. (2001).[26] At a micro level, the theoretical framework of Pastor and Veronesi (2003) suggests a positive relationship between idiosyncratic volatility and both the variance of firm profitability and uncertainty about the average level of firm profitability. In this regard, changes in the

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[25] The aggregated average correlation associated with model m over a specific period p is defined as:

$$\bar{\rho}_{T_p}^m = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{T_p} \frac{1}{(N-1)} \sum_{j \neq i} \sum_{t=1}^{T_p} \rho_{i,j,t}^m \right\}, \text{ where } T_p \text{ denotes the number of daily observations over such}$$

period, and $\rho_{i,j,t}^m$ is the time varying correlation (from model m) between assets i and j at time t. The

average low frequency idiosyncratic volatility over period p is defined as $Ivol_{T_p} = \frac{1}{T_p} \frac{1}{N} \sum_{t=1}^{T_p} \sum_{i=1}^{N} \tau_{it}^{1/2}$

[26] Moreover, its relationship with returns recently analyzed by Ang, Hodrick, Xing, and Zhang (2006) and Spiegel and Wang (2006) have opened a debate on whether idiosyncratic risk is priced.

profits of a firm are associated with two main causes. The first is changes in the firm's productivity, which can be explained by supply-side aspects such as technological change (e.g., Jovanovic (1982)) and/or by demand-side features such as product substitutability (e.g., Syverson (2004)). The other is price variations (of products and factors), which are also related to interactions between idiosyncratic demand shocks and the level of competition within the relevant industries. Therefore, fluctuations in demand (which could be related to changes in taste, technological changes, new trade liberalization policies, and new regulations, among other factors) can have an impact on these two fundamentals that affect the firm's future cash-flows, as well as on their volatility. Moreover, we may expect these effects to be accompanied by changes in the intensity of both firm- and industry-specific news because such intensities vary in response to the same factors.

At a more aggregated level, the theory of sectoral reallocation that followed the work of Lilien (1982) offers an explanation to link random fluctuations in sectoral demand with sectoral shifts in the labor market. The employment dispersion index (EDI) suggested by Lilien (1982) proxies the intensity of such sectoral shifts and it can therefore be used as an indicator of idiosyncratic news intensity. For example, sources triggering demand shifts, such as a technological change, can induce important movements of labor and other production factors from declining to growing sectors. Such sectoral reallocation of resources can be accompanied by a higher intensity of idiosyncratic shocks and therefore by increases in firm-specific volatility. Following this intuition, we associate the measure of Lilien with low frequency variation of aggregated idiosyncratic volatility.[27] In addition, we control for the economic variables that Guo and Savickas (2006) associate with the aggregated behavior of idiosyncratic volatility, such as the consumption-wealth

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[27] Following Lilien (1982), the Employment Dispersion Index (EDI) is defined as:

$$EDI_k = \left\{ \sum_{i=1}^{11} \frac{x_{ik}}{X_k} \left(\Delta \log x_{ik} - \Delta \log X_k \right)^2 \right\}^{\frac{1}{2}}$$, where x_{ik} is employment in industry i (among 11 industry sectors) at quarter k, and X_k denotes aggregate employment. To construct the index, we use sectoral employment data from the Bureau of Labor Statistics.

ratio (CAY), the market volatility, and the market liquidity.[28] As in this study, all the variables are aggregated at a quarterly-frequency.

We use two model-based measures of aggregated low frequency idiosyncratic volatility: 1) the cross sectional average of low frequency idiosyncratic volatilities, aggregated at a quarterly level, and 2) the cross-sectional average of rolling moving averages (based on a 100-day window) of squared idiosyncratic returns from Equation (12), aggregated at a quarterly level. Figure 8 shows a graph of these two measures of aggregated idiosyncratic volatility and Lilien's EDI from 1990 to 2006. The visual high-correlation is confirmed in Table 3 that reports the sample Pearson's correlation across our idiosyncratic volatility measures and the economic explanatory variables mentioned above. As expected, idiosyncratic volatility is positively correlated with the EDI and the market volatility. The rolling measure shows correlation coefficients of 0.48 and 0.72 respectively, whereas the spline measure shows correlations of 0.34 and 0.64 respectively. Moreover, idiosyncratic volatility is negatively correlated with both CAY and the market liquidity. Overall, the signs of these correlations are consistent with the results of Guo and Savickas (2006) and with the expected effects of sectoral reallocation on the volatility of firms' fundamentals. To explore further these relationships, we project separately our two measures of idiosyncratic volatility on the explanatory variables over our sample period using a linear regression framework. Due to the nature of the idiosyncratic volatility aggregates, especially the spline measure, the regressions will be affected by a severe serial correlation problem in the residuals. To lessen this problem and address endogeneity issues associated with simultaneous causality, we use the Generalized Method of Moments (GMM) with robust Newey and West (1987) standard errors, and four lags of the explanatory variables as instruments. Table 4 reports the estimated coefficients associated with the two linear projections. The two regressions suggest the same effects

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[28] The CAY variable is based on the measure of Lettau and Ludvigson (2001). The S&P 500 excess-returns volatility is used to proxy the market volatility. Regarding market liquidity, we use the quarterly average of the quoted spread (QSPR) as defined in Chordia, Roll, and Subrahmanyam (2001).

and, as in the sample correlation analysis, the estimated coefficients show the expected sign. Nevertheless, only the EDI and the market volatility are statistically significant.[29]

Sectoral Idiosyncrasies

The previous results are based on measures of low frequency idiosyncratic volatility at a firm level incorporating only large cap stocks. To explore whether these results can be generalized to industry sectors including a broader group of companies, we construct two model-based measures of sectoral idiosyncratic volatility using excess returns on 48 equally-weighted versions of the industry portfolios defined in Fama and French (1997).[30] We estimate rolling and spline idiosyncratic volatilities using the one-factor CAPM specification in (1) and the Spline-GARCH model in (12).[31] As before, we take cross-sectional averages of rolling means of squared returns and spline functions, respectively, and we construct quarterly aggregates. Figure 9 illustrates such idiosyncratic volatility aggregates along with the EDI over the period 1990-2006. The graph shows again a visual high correlation between these sectoral measures of idiosyncratic volatility and the EDI. Moreover, Table 5 confirms these positive correlations, which are within the same order of magnitude than those for large cap stocks. Moreover, the other explanatory variables are also correlated in the expected direction. As before, we project separately the idiosyncratic volatility variables on the explanatory variables. Table 6 reports the new GMM estimation results, which are fully consistent with our previous findings. Moreover, CAY becomes significant with the expected negative sign. Overall, these results show evidence that aggregated idiosyncratic volatility shows low frequency variation driven by economic variables. The FSG-DCC model non-parametrically accounts for such variation, as well as for the low frequency movements of the market factor volatility, to characterize changes in the long-term level of equity correlations. The

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[29] It is important to point out that these results are sensitive to the sample period. The EDI series are highly noisy before mid 80's. Thus structural breaks should be taken into account for analyses incorporating longer sample periods.

[30] The Fama-French portfolio data are available from Kenneth French's web page. The 48 portfolios are based on a four-digit SIC classification (see Fama and French (1997) for details).

[31] This specification allows for different loadings across sectors. However, it restricts the loadings to be the same within each sector.

analyses that follow in this paper illustrate the importance of such features to fit and forecast equity correlations.

Empirical Fit of Factor Correlation Models

This subsection evaluates a range of one-factor models with varying dynamic components in terms of their empirical fit using our sample of 33 Dow stocks. The process follows a simple-to-general strategy. We start from the simplest case, labeled FC-C, where factor and idiosyncratic volatilities are constant over the sample period (at high and low frequencies), and restrictions in (2) and (3) apply. We estimate the correlations across Dow stocks from this model and use a Gaussian metric to compute the quasi-likelihood. We then consider subsequent models that relax one or more assumptions of the initial factor model, and compute their quasi-likelihoods. The last step consists in comparing the empirical fit of this range of factor correlation models based on such Gaussian metric. This allows us to assess which restrictions in the factor structure are the most important to describe correlation behavior. The assumptions to be weakened are the following:

1. Constant volatility of the market factor.
2. Constant volatility of the idiosyncratic components of returns.
3. One single common factor.
4. Constant betas.

For example, adding high frequency variation into the volatility of the factor through a standard GARCH process, and keeping the idiosyncratic volatilities constant, we obtain a specification called FG-C. Similarly, when high and low frequency spline-GARCH dynamics is added to the market factor volatility holding the idiosyncratic volatilities constant, we obtain the FSG-C model. These models and their correlations, along with a range of specifications derived from adding dynamics to the previous assumptions, are described in Table 7. Their quasi-likelihoods are constructed from the general factor structure in Equation (21) assuming that $r_t \mid \Im_{t-1} \sim N(0, H_t)$. A mapping of each correlation specification in Table 7 with a specific covariance matrix provides the inputs

to compute the quasi-likelihood of each model. We estimate the models in the first panel of Table 7 using QMLE. For the other models, we follow the two-stage GMM approach with Gaussian moment conditions described in Section III.

Table 8 presents the log quasi-likelihoods of the factor models in Table 7, along with likelihood ratio tests that compare each model with the biggest FSG-DCC model. The results indicate that the FSG-DCC model dominates the other specifications. Close to this model is the FSG-IDCC model, which is a model with constant betas that accounts for the effect of latent unobserved factors and both, high and low frequency dynamics in market and idiosyncratic volatilities. The FG-DCC model follows in the list. In addition, the three biggest models show the best empirical fit even when they are penalized by the BIC (see the last column of Table 8). In contrast, the model with poorest empirical fit is the constant correlation model (FC-C). Overall, we find that specifications with low frequency dynamics dominate those with only high frequency dynamics.

In addition, the results indicate large improvements in the quasi-likelihoods when we relax the assumptions of "constant idiosyncratic volatilities" and "only one common factor". This confirms that, besides the importance of modeling market behavior, adding dynamic features to the second moments of idiosyncratic components improves substantially the empirical performance of this class of one-factor CAPM models.

V Forecast Performance of the Factor-Spline-GARCH Model

This section investigates the forecasts performance at long horizons of the FSG-DCC model using in-sample and out-of-sample exercises. We consider our sample of 33 Dow stocks. We first review some forecasting properties of the model and then we evaluate its performance with respect to other competitive specifications using an economic loss function.

Forecasting Features

In terms of forecasting properties, we mentioned earlier that the conditional correlation in the FSG-DCC model mean reverts toward the smooth low frequency correlation component rather than toward a constant level. Hence, given the empirical patterns in low frequency correlations discussed in the previous section, long horizon forecasts from the FSG-DCC model might differ considerably from those based on models that mean revert to constant levels. This characteristic is illustrated in the examples of Figure 10, which show multiple-step ahead forecasts associated with different correlation specifications. Here, we compare forecasting features of the FSG-DCC correlations described in Equations (14) and (15) with those of two competitors: 1) the standard DCC model and 2) the restricted FG-DCC model in Equation (19). The first case is interesting because it provides a non-factor reference to evaluate the importance of imposing a factor structure. The second case is relevant to evaluate the implications of allowing time varying low frequency components. The forecasts are constructed out-of-sample at horizons that go from 1 to 130 steps ahead. The low frequency volatility forecasts in the FSG-DCC model are constructed under the assumption that $\tau_{i,t+k|t} = \tau_{i,t}$, for all $i=$m,1,2,…,N, and k=1,2,…120.

The examples in Figure 10 correspond to the dynamic correlations between AIG and DIS, and between CVV and INTC, respectively. The vertical line separates the sample estimation period from the out of sample forecasting period. The DCC model's long-term forecasts approximate the sample covariance. In contrast, the FSG-DCC forecasts approach the low frequency correlation forecast. The long-term forecasts of the FG-DCC model tend to the constant in Equation (20). The first graph shows a case in which the long horizon forecasts from the three models are similar since the low frequency correlation at the end of the estimation period is approximately flat and close to the sample correlation. In contrast, the second graph presents an example in which long-term forecasts from the FSG-DCC considerably differ from those of the other two models due to a large discrepancy between the sample correlation and the trend in the low frequency correlation at the end of the estimation period. The top graph also illustrates that, contrary to the DCC model, the two factor correlation models might show non-monotonic mean

reversion since their individual components might be associated with different long memory features.

Evaluation Criterion and Economic Loss Function

Our forecast comparison follows the approach of Engle and Colacito (2006) by using an economic loss function to assess the performance of each model. Different from them, however, we use forward portfolios based on forecasts of the covariance matrix associated with each of the models to be compared. Specifically, we focus on a portfolio problem where an investor wants to optimize today her forward asset allocation given a forward conditional covariance matrix. In the classical mean variance setup, this problem can be formulated as:

$$\min_{w_{t+k|t}} w'_{t+k|t} H_{t+k|t} w_{t+k|t}$$

$$s.t. \ w'_{t+k|t} \mu = \mu_0,$$

(27)

where μ is the vector of expected excess returns, $w_{t+k|t}$ denotes a portfolio at time $t+k$ that was formed using the information at time t, and $H_{t+k|t}$ is a k steps ahead forecast (made at time t) of the conditional covariance matrix of excess returns. So, the solution to (27) is:

$$w_{t+k|t} = \frac{H_{t+k|t}^{-1} \mu}{\mu' H_{t+k|t}^{-1} \mu} \mu_0,$$

(28)

and it represents optimal forward portfolio weights given the information at time t.

Each covariance forecast $H_{t+k|t}$ implies a particular forward portfolio $w_{t+k|t}$ given a vector of expected returns. However, an important issue arises if we do not know the true vector of expected returns. Engle and Colacito (2006) point out that a direct comparison of optimal portfolio volatilities can be misleading when we use a particular estimate of expected excess returns, such as their realized mean, to compute such volatilities. Their framework isolates the effect of covariance information by using a wide range of alternatives for the vector of expected excess returns and the asymptotic properties of sample standard deviations of optimized portfolio returns. We follow their approach and consider different vectors of expected excess returns associated with a variety of

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multivariate hedges.[32] We then compare the standard deviations of returns on long-term forward hedge portfolios formed with each model's covariance matrix forecast.

In-Sample Evaluation

We now proceed to evaluate in-sample the forecast performance of the FSG-DCC in comparison with the FG-DCC and the standard DCC models.[33] This exercise focuses on their long run forecast performance; specifically, we consider a period of 100-days as our fixed long horizon.[34] We acknowledge the potential in-sample biases in favor of the FSG-DCC model that may arise since the spline volatilities will know ex-ante their future paths. To account for this problem, the in-sample exercise rules out such foresight advantage by constructing long horizon correlation forecasts at each point in the sample keeping the spline functions fixed during the forecasting period. This implies mean reversion of the long run forecasts according to Equation (16).

At each point in the sample and for each model, we construct the corresponding long horizon (*100-days* ahead) covariance forecast; then we use it to form optimized forward hedge portfolios according to Equation (28) using a variety of vectors of expected returns associated with different hedges and a required return normalized to one ($\mu_0=1$). Thus, given a sample of size *T*, the in-sample standard deviations of returns on long-term optimized forward hedge portfolios are given by:

$$\sigma_{p,IS}^{(j)} = \sqrt{\frac{\sum_{t=1}^{T-100} \left\{ w_{p,t+100|t}'^{(j)} (r_{t+100} - \overline{r}) \right\}^2}{T-100}}, \qquad \begin{array}{l} j = FSG, FG-DCC, DCC, \\ p = 1, 2, ..., 33, \end{array} \qquad (29)$$

where $\overline{r}$ denotes the sample mean of daily excess returns, r_{t+100} is the vector of one-day excess returns at day *t+100*, and $w_{p,t+100|t}^{(j)}$ corresponds to a 100-days forward hedge

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[32] A hedge is constructed by holding one asset for return and using the other assets for hedge. Thus, we have vectors of expected returns with one positive entry (normalized to one) and everything else set to zero.
[33] All the models are estimated using the two stage-GMM approach described in Section III, including asymmetric Student-t GARCH volatilities and the composite likelihood (CL) method.
[34] Given that all the estimations are based on daily data, a horizon of 100 days can be seen as a reasonable "long-term" period. However, horizons from four to six months are also considered in the out-of-sample exercise presented in the following subsection.

portfolio constructed from covariance model j where asset p is hedged against the other assets.[35]

Table 9 presents the results for this in-sample exercise. It is found that the FSG-DCC model outperforms the other models for the considered horizon. The FSG-DCC model obtains smaller volatilities for twenty seven cases whereas the FG-DCC model is superior only for six cases. The standard DCC model is always inferior. Moreover, our results indicate that, on average, using long horizon FSG-DCC correlation forecasts to hedge reduces the standard deviation of the corresponding optimized portfolios by approximately 6 basis points, when it is compared with that of FG-DCC portfolios, and by approximately 125 basis points, when it is compared with that of DCC portfolios.

Out-of-Sample Evaluation: A Sequential Forecasting Exercise

In this subsection, we examine the out-of-sample forecast performance of the FSG-DCC model. Our analysis implements a sequential forecasting exercise that is described in Figure 11 and can be characterized by iterations of the following simple steps: 1) a set of models is estimated based on an initial estimation period with T_0 daily observations, and multi-horizon covariance forecasts from 1 to 126-days (six months) ahead are computed out-of-sample, 2) The mentioned 126-days forecasting period is incorporated into a new estimation period with $T_1=T_0+126$ daily observations, the models are re-estimated, and a new set of out-of-sample forecasts is constructed for the following six months. We iterate these two steps several times starting with a sample period from December 1988 to June 1995. As illustrated in Figure 11, the process is repeated 22 times (up to December 2006) and none of the out-of-sample forecast blocks overlap. In this exercise, we also enhance the scope of models to be compared. Specifically, we include three new covariance estimators: 1) the sample covariance (*SCOV*), 2) the static one-factor beta covariance (*BCOV*), and 3) the optimal shrinkage covariance of Ledoit and Wolf (2003) (*LCOV*).[36] All the models are re-estimated at each iteration.

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[35] Each of the 33 assets in our sample is associated with one hedge.

[36] Jagannathan and Ma (2003) use these covariance matrix estimators to examine the effect of portfolio weight constraints on the out-of-sample performance of minimum variance portfolios.

For the FSG-DCC model, we extrapolate the spline functions by keeping them constant at their last values during the forecasting periods and restricting them to have a zero-slope at the last observation of the sample. The zero-slope condition in the boundary is imposed in the estimation process and provides a conservative approach that underestimates the behavior of the time series near the end of the sample.[37] This differs from the approach taken in the previous in-sample exercise since in such a case the spline functions were fitted using data at both sides of the "last" data point (thus no boundary-bias problems were present), whereas in the out-of-sample case the bandwidth covers only data on the left side of the last observation in the estimation period. The zero-slope condition helps to reduce potential anomalies caused by outliers near the right boundary.[38]

As in the in-sample exercise, we focus on long horizons. Thus, among the multi-horizon forecasts generated at each iteration, we only consider the last 40 days (i.e., out-of-sample forecast from 87 to 126 days ahead). As before, the standard deviations of returns on out-of-sample optimized forward hedge portfolio are computed according to:

$$\sigma_{p,OS}^{(j)} = \sqrt{\frac{\sum_{i=1}^{22} \sum_{k=87}^{126} \left\{ w_{p,T_i+k|T_i}'^{(j)} \left(r_{T_i+k} - \overline{r} \right) \right\}^2}{22 \times 40}}, \quad \begin{array}{l} j = FSG, FG-DCC, DCC, SCOV, \\ BCOV, LCOV, \\ p = 1, 2, ..., 33, \end{array} \quad (30)$$

where T_i is the last day of the estimation period associated with iteration i.

For each model and hedge portfolio, Table 10 reports these standard deviations. The FSG-DCC specification produces smallest volatilities for fifteen hedges; the sample covariance is preferred for five hedges; the shrinkage covariance and the FG-DCC models dominate in four cases each; the DCC model is preferred in three cases; and the static beta covariance model dominates in only one case. The average across all the hedges is shown in the last row of the table. According to this number, the best performer is the FSG-DCC specification followed by the FG-DCC model. The sample and the

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[37] An unrestricted approach would impose less smoothness. Silverman (1984) and Nychka (1995) illustrate the boundary issues associated with smoothing splines.
[38] More elaborated boundary correction methods for smoothing splines are examined in Rice and Rosenblatt (1981) and Eubank and Speckman (1991).

shrinkage covariance models come next followed by the DCC model. The worst performer is the static beta covariance model.

To explore further the significance of these results, we perform joint Diebold and Mariano-style tests following Engle and Colacito (2006). The objective is to test the equality of the FSG-DCC model with respect to each of its competitors.[39] This approach is based on statistical inference about the mean of the difference between square returns on optimized portfolios generated by the FSG-DCC specification and a competitor model m. For every iteration i (with last observation $= T_i$), a vector of difference series associated with the hedge p is defined as:

$$u_{T_i}^{p,m} = \left\{ \left(w'^{FSG-DCC}_{p,T_i+k|T_i} (r_{T_i+k} - \overline{r}) \right)^2 - \left(w'^{m}_{fh,T_i+k|T_i} (r_{T_i+k} - \overline{r}) \right)^2, k = 86,...,126 \right\}, \qquad (31)$$

where m=FG-DCC, DCC, SCOV, LCOV, BCOV, and p=1,2,...,33. Using these hedge difference vectors, we construct a joint difference vector that stacks all of them as follows:

$$U_{T_i}^{FSG-DCC,m} = \left(u_{T_i}^{1,m}, u_{T_i}^{2,m}, ..., u_{T_i}^{33,m} \right), \quad i = 1, 2, ..., 22. \qquad (32)$$

The null of equality of covariance models tests the mean of $U_t^{FSG-DCC,m}$ equals zero. Therefore, for each comparison, the test is performed by running a regression of $U_t^{FSG-DCC,m}$ on a constant. The regressions are estimated by GMM using robust HAC covariance matrices. Table 11 reports the t-statistics for the Diebold-Mariano tests. The competitor models are named in each column. A negative value suggests that the FSG-DCC model dominates the column-model since the former is associated with smaller average volatility of optimal portfolios than the latter. At a 5% confidence level, the results indicate again that the FSG-DCC dominates its competitors at long horizons.

An Example of Forecasting Correlations Using Macroeconomic Variables

In the previous forecasting exercises, we have not used any economic variables to construct the correlation forecasts. Following the discussion is Section III, this subsection illustrates with a simple example that low frequency macroeconomic information can be

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[39] See Diebold and Mariano (1995) for details on the general test.

incorporated to forecast correlations within the FSG-DCC framework. The idea is to change the approach used to extrapolate the spline associated with the low frequency market volatility. A simple strategy to incorporate macroeconomic information into this extrapolation consists in forecasting the average low frequency market volatility using macroeconomic information and then taking such a value as an out-of-sample target for the spline function.[40] We apply this strategy to forecast correlations during the first half of 2007 conditional on the information in our sample period (up to 2006). To keep this example as simple as possible, we use the same macro variables and estimation results reported in Engle and Rangel (2008).[41]

Table 12 shows such estimates along with updated values of the macroeconomic variables for the years 2006 and 2007. Under perfect foresight, these estimates predict an increase of six basis points in the annual low frequency market volatility. Therefore, after calibrating the levels according to the observed volatility in 2006, the predicted annual low frequency market volatility increases from 9.3% in 2006 to 10% in 2007.[42] Although this increase is small compared to the big jump observed in the annual level of realized volatility in 2007, due mostly to the August events, the predicted value changed in the right direction.[43]

As before, we can compute out-of-sample forecast of the covariance matrix restricting, for example, the spline of the market factor to be flat at 10% in the mid point of 2007 (or to cross such target at this point). This is only an example to illustrate the application of simple forecasting strategies that use macroeconomic information and can be implemented within the FSG-DCC framework. A formal evaluation about the gains of

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[40] Here the spline function is forced to cross the economic target in a pre-specified point out-of-sample. If we are forecasting the annual low frequency level of the market volatility, a natural choice for such point is the middle of the forecasting year. A zero slope condition in the target point can also be imposed.

[41] Updated values of the explanatory variables are obtained from the same sources used in Engle and Rangel (2008). Note that the estimation period in this study is from 1990 to 2003. Therefore, the estimates are suboptimal for the period of our example.

[42] Comparing the estimated low frequency market volatility from the macro variables in 2006 (7.3%) with the realized value in this year (9.4%), we obtain an estimate of the time fixed effect. We use this estimate to calibrate the prediction in 2007.

[43] The annual realized volatility of the S&P 500 (excess returns) was 16% in 2007.

using macroeconomic variables and idiosyncratic volatility predictors to forecast correlations is a promising extension that we leave for future work.

VI Concluding Remarks

This paper develops a new model for asset correlations that characterizes dynamic patterns at high and low frequencies in the correlation structure of equity returns. Exploiting a factor asset pricing structure and dynamic properties of low frequency volatilities associated with systematic and idiosyncratic terms, we introduce a slow-moving correlation component that proxies low frequency changes in correlations. Our semi-parametric approach generalizes dynamic conditional correlation (DCC) and other multivariate GARCH models by allowing the high frequency correlation component to mean revert toward a time varying low frequency component. This framework permits the level to which conditional correlations mean revert to adapt to varying economic conditions.

At high frequencies, our model incorporates dynamic effects that arise from relaxing assumptions in the standard one-factor CAPM model. Such effects account for time varying betas and missing pricing factors. At low frequencies, the long-term trends of the market and idiosyncratic volatilities govern the dynamics of low frequency correlations. We provide evidence that, in addition to the recently documented economic variation of market volatility at low frequencies, average idiosyncratic volatility shows substantial variation in its long-term trend. We find that this variation is highly correlated with low frequency economic variables including an inter-sectoral employment dispersion index that proxies the intensity of sectoral reallocation of resources in the economy and, since such movements are mainly driven by shocks that are specific to either individual firms or sectors, it serves as an indicator of idiosyncratic news intensity. Moreover, we find the same results for sectoral idiosyncratic volatility.

The ability of our correlation model to incorporate non-parametrically such low frequency features not only produces improvements in terms of the empirical fit of equity

correlations and their association with economic conditions, but also leads to an improvement for forecasting applications. Indeed, in-sample and out-of-sample forecasting experiments indicate that, at long horizons, this new model with time varying long-term trends outperforms standard models that mean revert to fixed levels. This result is explained by the model's flexibility in adjusting the level of mean reversion to varying economic conditions. Although this comes at a cost of estimating more parameters, we can apply the methods that have been recently developed to estimate high-dimensional dynamic covariance models and keep the estimation process tractable even for a large number of assets.

The results in this paper motivate interesting extensions in terms of economic modeling and forecasting: a detailed analysis of the cross-sectional determinants of idiosyncratic volatilities would provide a richer firm-specific context to analyze the long-term behavior of such volatilities; analyzing the nature of common components in idiosyncratic volatility can shed more light on the continuing debate about pricing idiosyncratic risk; an analysis of implications for international markets are important for international asset allocation and the evaluation of global financial risk. Also, the favorable forecasting results shown in the paper could be improved further by incorporating economic variables and finding more efficient strategies to forecast the long-term volatility components.

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Appendix A1

Proof of Proposition 1: Consider the specification in (1) and assumptions (2) and (3). Allowing $E_{t-1}(u_{it}u_{jt}) \neq 0$, we have $\text{cov}_{t-1}(r_i r_j) = \beta_i \beta_j V_{t-1}(r_{mt}) + E_{t-1}(u_{it}u_{jt})$, and Equation (8) follows. For the second part, Lets consider a single factor model with time varying betas satisfying assumptions *i)-iv)*. The equation for excess returns can be written as:

$$r_{it} = \tilde{\alpha}_i + \beta_{it} r_{mt} + \tilde{u}_{it}, \tag{33}$$

where $\beta_{it} = \beta_i + w_{it}$. If we define $u_{it} \equiv w_{it} r_{mt} + \tilde{u}_{it} - E(w_{it} r_{mt}), \forall i = 1, ..., N$, then we can rewrite (33) as:

$$r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}. \tag{34}$$

where $E(u_{it}) = 0, \forall i$ and, by assumptions *i)* and *iv)*, $E(u_{it} r_{mt}) = 0$. Moreover, from *iii)* and *iv)*, $E(\beta_{it}) = \beta_i = \dfrac{\text{cov}(r_{mt}, r_{it})}{V(r_{mt})}$, β_{it} is covariance stationary and mean reverts toward β_i. However, the new errors will be conditionally correlated with the market factor since $\text{cov}_{t-1}(w_{it} r_{mt}, r_{mt})$ is in general different from zero. Thus,

$$\text{cov}_{t-1}(r_i r_j) = \beta_i \beta_j V_{t-1}(r_{mt}) + \beta_i E_{t-1}(r_{mt} u_{jt}) + \beta_j E_{t-1}(r_{mt} u_{it}) + E_{t-1}(u_{it} u_{jt}), \tag{35}$$

and

$$E_{t-1}(r_{kt}^2) = \beta_k V_{t-1}(r_{mt}) + V_{t-1}(u_{kt}) + 2\beta_k E_{t-1}(r_{mt} u_{kt}), \ \ k = i, j. \tag{36}$$

Also, note that $E_{t-1}(u_{it} u_{jt})$ is in general different from zero for $i \neq j$ since it includes covariance terms such as $\text{cov}_{t-1}(w_{it} r_{mt}, w_{jt} r_{mt})$, $\text{cov}_{t-1}(w_{it} r_{mt}, u_{jt})$, and $E_{t-1}(\tilde{u}_{it} \tilde{u}_{jt})$, $i \neq j$, which may deviate from zero (for example, due to temporal comovements across the time varying beta components as well as to the effect of latent unobserved factors). Thus, we obtain Equation *(10)* by combining (35) and (36).∎

Appendix A2

Proof of Proposition 2: Consider the following vectors of returns, factor loadings, and innovations: $r_t = (r_{1t}, r_{2t}, ..., r_{Nt})'$, $\beta = (\beta_{1t}, \beta_{2t}, ..., \beta_{Nt})'$, and $u_t = (u_{1t}, u_{2t}, ..., u_{Nt})'$. Given a vector F_t of common factor(s) and omitting the constant terms, without loss of generality, we can rewrite the model in Equation (1) as:

$$r_t = \beta F_t + u_t. \tag{37}$$

Thus, given the t-1 information set $\Im_{t-1}$, the conditional covariance matrix is:

$$E_{t-1}(r_t r_t') = \beta E_{t-1}(F_t F_t')\beta' + \beta E_{t-1}(F_t u_t') + E_{t-1}(u_t F_t')\beta' + E_{t-1}(u_t u_t'). \tag{38}$$

In particular, for the one-factor CAPM case, $F_t = r_{mt}$ and (38) takes the following form:

$$E_{t-1}(r_t r_t') = E_{t-1}(r_{mt})\beta\beta' + \beta E_{t-1}(r_{mt} u_t') + E_{t-1}(u_t r_{mt})\beta' + E_{t-1}(u_t u_t'). \tag{39}$$

From Equations (10) and (11), the typical (i,j) element of the first term on the RHS of (39) is:

$$\beta_i \beta_j E_{t-1}(r_{mt}^2) = \beta_i \beta_j \tau_{mt} g_{mt}. \tag{40}$$

Similarly, from Equations (11), (12), and (13), the typical (i,j) element of the second term is:

$$\beta_i E_{t-1}(r_{mt} u_{jt}) = \beta_i \sqrt{\tau_{mt} g_{mt}} \sqrt{\tau_{jt} g_{jt}} \rho_{m,j,t}^\varepsilon, \tag{41}$$

the typical (i,j) element of the third term is:

$$\beta_j E_{t-1}(r_{mt} u_{it}) = \beta_j \sqrt{\tau_{mt} g_{mt}} \sqrt{\tau_{it} g_{it}} \rho_{m,i,t}^\varepsilon, \tag{42}$$

and the typical (i,j) element of the last term is:

$$E_{t-1}(u_{it} u_{jt}) = \sqrt{\tau_{it} g_{it}} \sqrt{\tau_{jt} g_{jt}} \rho_{i,j,t}^\varepsilon. \tag{43}$$

Equation (14) follows from substituting these conditional expectations in (10).

The unconditional version of (10) is then used to derive the low frequency correlation.

$$\bar{\rho}_{i,j} = \frac{\beta_i \beta_j E(r_{mt}^2) + \beta_j E(r_{mt} u_{it}) + \beta_i E(r_{mt} u_{jt}) + E(u_{it} u_{jt})}{\sqrt{\beta_i^2 E(r_{mt})^2 + E(u_{it})^2 + 2\beta_i E(r_{mt} u_{it})} \sqrt{\beta_j^2 E(r_{mt})^2 + E(u_{jt})^2 + 2\beta_j E(r_{mt} u_{jt})}}.$$

Under the assumption that the factor(s) and the idiosyncrasies are unconditionally uncorrelated, we have:

$$\bar{\rho}_{i,j} = \frac{\beta_i\beta_j E\left(r_{mt}^2\right) + E(u_{it}u_{jt})}{\sqrt{\beta_i^2 E(r_{mt})^2 + E(u_{it})^2}\sqrt{\beta_j^2 E(r_{mt})^2 + E(u_{jt})^2}}. \tag{44}$$

Now, from (11), (12), and the LIE:

$$E(r_{mt}) = \tau_{mt}E(g_{mt}) = \tau_{mt},$$

and

$$E(u_{it}^2) = \tau_{it}E(g_{it}) = \tau_{it}, \quad \forall i = 1, 2, ..., N.$$

Also,

$$\frac{E(u_{it}u_{jt})}{\sqrt{\tau_{it}}\sqrt{\tau_{jt}}} = \frac{E(g_{it}^{1/2}\varepsilon_{it}g_{jt}^{1/2}\varepsilon_{jt})}{\sqrt{E(g_{it})}\sqrt{E(g_{jt})}} = corr(g_{it}^{1/2}\varepsilon_{it}, g_{jt}^{1/2}\varepsilon_{jt}) \equiv \bar{\rho}_{i,j}^{\tilde{\varepsilon}}.$$

Note that $\rho_{i,j,t}^{\tilde{\varepsilon}} = \rho_{i,j,t}^{\varepsilon}$, $\forall t$, thus we approximate $\bar{\rho}_{i,j}^{\tilde{\varepsilon}}$ with the sample correlation, $\bar{\rho}_{i,j}^{\varepsilon}$, from Equation (12). Plugging in the previous expressions into (44), we obtain the time varying low frequency correlation in Equation (15).

Moreover, if we assume $E_t(\tau_{k,t+h}) = \tau_{k,t}, \forall h > 0, k = 1, 2, ..., N$, then the long horizon forecast of (14) can be constructed using the mean reversion properties of the GARCH and DCC equations. Indeed, the GARCH dynamics implies $\lim_{h \to \infty} g_{k,t+h|h} = 1$, $\forall k = m, 1, 2, ..., N$. Also, the long horizon correlation forecasts associated with the vector of innovations are given by the terms targeting correlations (see Equation (13)):

$$\lim_{h \to \infty} \rho_{i,j,t+h}^{\varepsilon} = \lim_{h \to \infty} \frac{q_{i,j,t+h|t}}{\sqrt{q_{i,i,t+h|t}}\sqrt{q_{j,j,t+h|t}}} = \frac{\bar{\rho}_{i,j}^{\varepsilon}}{\sqrt{\bar{\rho}_{i,i}^{\varepsilon}}\sqrt{\bar{\rho}_{j,j}^{\varepsilon}}} = \bar{\rho}_{i,j}^{\varepsilon}, \tag{45}$$

$\forall i, j \in \{1, 2, ..., N\}$. In addition, from our assumption that the idiosyncrasies are unconditionally uncorrelated with the factor, $\bar{\rho}_{m,i}^{\varepsilon} = 0$ and $\lim_{h \to \infty} \rho_{m,i,t+h|t} = 0$, $\forall i = 1, ..., N$. Hence, substituting the long run forecasts of each term into (14), we obtain:

$$\lim_{h \to \infty} \bar{\rho}_{i,j,t+h|t} = \frac{\beta_i\beta_j\tau_{mt} + \sqrt{\tau_{it}}\sqrt{\tau_{jt}}\bar{\rho}_{i,j}^{\varepsilon}}{\sqrt{\beta_i^2\tau_{mt} + \tau_{it}}\sqrt{\beta_j^2\tau_{mt} + \tau_{jt}}}, \tag{46}$$

which coincides with the low frequency correlation.∎

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Table 1

Description of Individual Stock Returns (Dec 1988-Dec 2006)			
Company Name	Ticker	Average Daily Return	Annualized Sample Volatility
Alcoa	AA	0.05%	0.52
American Intl. Gp.	AIG	0.07%	0.36
American Express	AXP	0.07%	0.54
Boeing	BA	0.06%	0.48
Citigroup	C	0.10%	0.57
Caterpillar	CAT	0.06%	0.49
Chevron-Texaco	CVX	0.05%	0.27
Du Pont E I De Nemours	DD	0.04%	0.38
Walt Disney	DIS	0.06%	0.49
General Electric	GE	0.06%	0.34
General Motors	GM	0.02%	0.56
Goodyear	GT	0.03%	0.75
Home Depot	HD	0.10%	0.60
Honeywell Intl.	HON	0.06%	0.55
Hewlett-Packard	HPQ	0.08%	0.85
International Bus. Mach.	IBM	0.04%	0.47
Intel	INTC	0.11%	0.94
International Paper	IP	0.02%	0.42
Johnson & Johnson	JNJ	0.07%	0.30
JP Morgan Chase & Co.	JPM	0.06%	0.63
Coca Cola	KO	0.06%	0.32
McDonalds	MCD	0.06%	0.38
3M	MMM	0.05%	0.27
Altria Group Inco.	MO	0.07%	0.45
Merck & Co.	MRK	0.05%	0.40
Microsoft	MSFT	0.12%	0.64
Pfizer	PFE	0.07%	0.44
Procter & Gamble	PG	0.07%	0.32
AT&T	T	0.04%	0.39
United Technologies	UTX	0.07%	0.39
Verizon Comms.	VZ	0.03%	0.37
Wal Mart Stores	WMT	0.07%	0.45
Exxon Mobil	XOM	0.05%	0.25

Source: Daily returns from CRSP. The annualized sample volatility is the square root of the sample average of annualized daily squared returns.

Table 2

Estimation Results US Market

$$r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}, \quad r_{mt} = \alpha_m + \sqrt{\tau_{mt} g_{mt}} \, \varepsilon_t^m,$$

$$u_{it} = \sqrt{\tau_{it} g_{it}} \, \varepsilon_{it}, \text{ where } \varepsilon_{it} \mid \Phi_{t-1} \sim Student-t(v_i), \forall i = m, 1, 2, ..., N$$

$$g_{mt} = \left(1 - \theta_m - \phi_m - \frac{\gamma_m}{2}\right) + \theta_m \frac{(r_{mt-1} - \alpha_m)^2}{\tau_{mt-1}} + \gamma_m \frac{(r_{mt-1} - \alpha_m)^2}{\tau_{mt-1}} I_{r_{mt-1}<0} + \phi_m g_{mt-1}$$

$$g_{it} = \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2}\right) + \theta_i \frac{(r_{it-1} - \alpha_i - \beta_i r_{mt-1})^2}{\tau_{it-1}} + \gamma_i \frac{(r_{it-1} - \alpha_i - \beta_i r_{mt-1})^2}{\tau_{it-1}} I_{r_{it-1}<0} + \phi_i g_{it-1}$$

$$\tau_{it} = c_i \exp\left(w_{i0} t + \sum_{r=1}^{k_i} w_{ir} \left((t - t_{i-1})_+\right)^2 \right), \quad \forall i = m, 1, ..., N \text{ [a]}$$

	1		2		3		4		5		6		7
	α_i		β_i		θ_i		ϕ_i		γ_i		v_i		k_i
	First Stage												
	Coeff	T-Stat	Coeff	T-Stat	Coeff	T-Stat	Coeff	T-Stat	Coeff	T-Stat	Coeff	T-Stat	
Market(m)	0.028%	2.48			-0.00	-0.83	0.90	67.6	0.122	8.33	8.4	10.2	4
AA	-0.025%	-1.20	0.99	44.9	0.02	4.2	0.96	144.1	0.008	1.00	6.3	11.9	1
AIG	-0.004%	-0.30	1.01	58.5	0.07	5.0	0.82	30.8	0.034	1.62	7.1	11.7	4
AXP	-0.018%	-1.05	1.17	61.4	0.05	4.0	0.84	32.8	0.063	2.96	6.5	11.0	4
BA	-0.005%	-0.26	0.93	41.6	0.11	4.1	0.41	4.3	0.056	1.41	6.3	13.0	4
C	0.002%	0.12	1.25	70.4	0.07	4.7	0.83	32.2	0.019	0.98	6.8	12.5	4
CAT	0.020%	1.03	1.01	49.6	0.01	2.8	0.99	322.2	0.003	0.94	4.9	16.0	1
CVX	0.022%	1.28	0.66	37.5	0.05	5.9	0.94	110.8	-0.015	-1.46	7.9	9.2	1
DD	-0.019%	-1.19	0.98	53.5	0.05	3.6	0.85	20.0	-0.019	-1.17	5.9	12.1	4
DIS	-0.004%	-0.23	1.04	50.3	0.12	4.4	0.52	6.6	0.033	0.90	5.0	14.9	4
GE	-0.006%	-0.49	1.13	77.9	0.03	3.2	0.84	27.8	0.065	3.48	9.0	8.3	5
GM	-0.063%	-3.01	1.03	48.8	0.02	3.8	0.96	174.1	0.028	3.80	5.5	13.2	1
GT	-0.071%	-3.08	0.93	37.4	0.01	2.7	0.98	278.8	0.017	3.91	4.4	15.4	3
HD	0.030%	1.56	1.23	54.8	0.01	1.6	0.91	65.1	0.059	4.37	6.1	12.9	5
HON	-0.031%	-1.73	1.06	51.8	0.11	4.6	0.65	14.8	0.091	2.63	5.1	15.4	4
HPQ	0.000%	-0.02	1.30	50.9	0.01	4.0	0.99	347.6	0.004	1.11	4.3	18.6	1
IBM	-0.028%	-1.87	0.95	55.3	0.00	1.4	0.97	216.3	0.028	5.20	4.5	17.2	2
INTC	0.038%	1.55	1.53	55.4	0.02	2.7	0.95	112.2	0.022	2.69	5.4	15.7	2
IP	-0.036%	-1.92	0.91	46.8	0.08	4.2	0.63	8.1	0.029	1.08	10.4	7.0	7
JNJ	-0.009%	-0.61	0.78	45.7	0.05	4.1	0.83	32.6	0.088	3.79	5.9	12.1	4
JPM	-0.016%	-0.98	1.26	64.1	0.03	3.4	0.90	52.2	0.054	3.76	6.2	12.6	5
KO	-0.006%	-0.41	0.81	49.0	0.07	3.8	0.63	10.9	0.113	3.46	6.2	12.1	4
MCD	0.003%	0.17	0.80	38.7	0.02	4.3	0.97	182.7	0.009	1.12	5.6	12.4	1
MMM	-0.002%	-0.11	0.78	55.3	0.13	4.4	0.18	1.7	0.001	0.02	5.2	16.4	7
MO	0.039%	2.43	0.76	39.9	0.13	4.9	0.58	13.2	0.127	3.01	4.0	18.7	4
MRK	0.016%	0.95	0.93	51.0	0.11	4.1	0.46	6.0	0.043	1.13	5.0	17.6	6
MSFT	0.010%	0.53	1.16	55.4	0.08	4.3	0.76	20.1	0.039	1.54	4.9	16.3	5
PFE	0.005%	0.29	0.94	45.9	0.09	4.7	0.78	21.1	-0.001	-0.07	5.8	13.4	4

PG	0.019%	1.36	0.73	44.5	0.04	3.8	0.91	56.8	0.028	1.95	6.1	13.7	4
T	-0.003%	-0.17	0.86	44.6	0.07	4.5	0.83	26.4	0.017	0.92	7.6	9.6	6
UTX	-0.001%	-0.08	0.93	51.3	0.09	3.5	0.53	7.0	0.093	2.48	5.9	14.1	7
VZ	-0.016%	-1.00	0.84	45.4	0.07	5.1	0.83	28.9	0.016	0.88	6.8	11.2	5
WMT	0.002%	0.11	1.07	56.2	0.04	4.9	0.95	133.8	0.003	0.38	6.8	10.3	2
XOM	0.018%	1.22	0.74	44.2	0.08	5.2	0.84	30.8	-0.019	-1.07	9.1	9.0	4

$$\left(\varepsilon_t^m, \varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt}\right)' \sim DCC,$$

$$\rho_{i,j,t}^{\varepsilon} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}}\sqrt{q_{j,j,t}}},$$

$$q_{i,j,t} = \bar{\rho}_{i,j}^{\varepsilon} + a_{DCC}(\varepsilon_{i,t-1}\varepsilon_{j,t-1} - \bar{\rho}_{i,j}^{\varepsilon}) + b_{DCC}(q_{i,j,t-1} - \bar{\rho}_{i,j}^{\varepsilon}), \quad \forall i,j \in \{1,...,N\},$$

$$q_{m,i,t} = \bar{\rho}_{m,i}^{\varepsilon} + a_{DCC}(\varepsilon_{t-1}^m\varepsilon_{i,t-1} - \bar{\rho}_{m,i}^{\varepsilon}) + b_{DCC}(q_{m,i,t-1} - \bar{\rho}_{m,i}^{\varepsilon}), \quad \forall i \in \{m,1,...,N\}$$

Second Stage (DCC)

Param	Std. DCC		Median Bivariate DCC Systems	Composite Likelihood	
	Coeff	T-Stat	Median	Coeff	T-Stat
a_{DCC}	0.0024	4.8	0.0047	0.0043	50.8
b_{DCC}	0.9919	142.7	0.9856	0.9906	4531.2

a) The returns are in excess of the risk free rate. The sample is the 33 DJIA stocks described in Table 1.
The one-month T-bill rate is used as the time varying risk free rate
b) The optimal number of knots was selected based on the BIC.

Table 3

Sample Correlations: Idiosyncratic Volatilities and Economic Variables

Economic Variables[d]	Average Idiosyncratic Volatilities (Quarterly Frequency)[a]	
	Average Rolling Idiosyncratic Volatility[b]	Average Low Frequency Idiosyncratic Volatility[c]
Employment Dispersion Index	0.48	0.34
CAY	-0.40	-0.39
Market Factor Volatility	0.72	0.64
Illiquidity (QSPR)	-0.30	-0.23

a) Average idiosyncratic volatilities are based on daily DJIA stock returns. These measures are aggregated at a quarterly frequency over the period 1990-2006.

b) The Average Rolling Idiosyncratic Volatility is defined as $ARIV_t = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{100} \sum_{k=1}^{100} u_{i(t-k)}^2 \right)^{\frac{1}{2}}$, where u_{it} is the daily idiosyncratic return from Equation (1).

c) The Average Low Frequency Idiosyncratic Volatility is defined as $Ivol_t = \frac{1}{N} \sum_{i=1}^{N} \tau_{it}^{1/2}$, where the τ's are the daily low frequency volatilities estimated from Equation (12)

d) The Employment Dispersion Index follows the definition in Lilien (1982):

$$EDI_k = \left\{ \sum_{i=1}^{11} \frac{x_{ik}}{X_k} \left(\Delta \log x_{ik} - \Delta \log X_k \right)^2 \right\}^{\frac{1}{2}}$$, where x_{ik} is employment in industry i (among 11 industry

sectors) at quarter k, and X_k is aggregate employment; the consumption-wealth ratio (CAY) is defined in Lettau and Ludvigson (2001); the market factor volatility is estimated according to Equation (11); and the average quoted spread (QSPR) follows the definition in Chordia, Roll, and Subrahmanyam (2001).

Table 4

Idiosyncratic Volatility GMM Regressions

	Average Rolling Idiosyncratic Volatility	Average Low Frequency Idiosyncratic Volatility
Constant	**0.082**	0.080
	(0.037)	*(0.076)*
EDI	**104.581**	**117.298**
	(53.314)	*(67.166)*
CAY	-0.154	-0.031
	(0.562)	*(0.699)*
Market Volatility	**0.615**	**0.643**
	(0.233)	*(0.218)*
QSPR	-0.083	-0.002
	(0.073)	*(0.087)*
R-squared	0.533	0.365
J-statistic	0.051	0.049

Notes: The Average Rolling Idiosyncratic Volatility is defined as $ARIV_t = \dfrac{1}{N}\sum_{i=1}^{N}\left(\dfrac{1}{100}\sum_{k=1}^{100} u_{i(t-k)}^2\right)^{1/2}$,

where u_{it} is the daily idiosyncratic return from Equation (1). The Average Low Frequency Idiosyncratic

Volatility is defined as $Ivol_t = \dfrac{1}{N}\sum_{i=1}^{N}\tau_{it}^{1/2}$, where the τ's are the daily low frequency volatilities estimated

from Equation (12). These averages are based on daily DJIA stock returns. Aggregating the variables at a quarterly frequency from 1989 to 2006, the two measures of idiosyncratic volatility are regressed on the following set of variables: the employment dispersion index (EDI) of Lilien (1982), the consumption-wealth ratio (CAY) of Lettau and Ludvigson (2001), the market factor volatility estimated in Equation (11), and the average quoted spread (QSPR) of NYSE stocks (as defined in Chordia, Roll, and Subrahmanyam (2001)). The regressions are estimated using the Generalized Method of Moments (GMM) with Newey-West standard errors and four lags of the regressors as instruments.

Table 5

Sample Correlations: Idiosyncratic Volatilities based on 48 Industry Portfolios and Economic Variables

Economic Variables[d]	Average Idiosyncratic Volatilities (Quarterly Frequency)[a]	
	Average Rolling Idiosyncratic Volatility[b]	Average Low Frequency Idiosyncratic Volatility[c]
Employment Dispersion Index	0.51	0.27
CAY	-0.26	-0.66
Market Factor Volatility	0.61	0.69
Illiquidity (QSPR)	-0.32	-0.29

a) Average idiosyncratic volatilities are based on daily returns from the 48 equally-weighted industry portfolios of Fama and French (1997). These measures are aggregated at a quarterly frequency over the period 1990-2006.

b) The Average Rolling Idiosyncratic Volatility is defined as $ARIV_t = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{100} \sum_{k=1}^{100} u_{i(t-k)}^2 \right)^{\frac{1}{2}}$, where u_{it} is the daily idiosyncratic return from Equation (1).

c) The Average Low Frequency Idiosyncratic Volatility is defined as $Ivol_t = \frac{1}{N} \sum_{i=1}^{N} \tau_{it}^{1/2}$, where the τ's are the daily low frequency volatilities estimated from Equation (12).

d) The Employment Dispersion Index follows the definition in Lilien (1982):

$EDI_k = \left\{ \sum_{i=1}^{11} \frac{x_{ik}}{X_k} \left(\Delta \log x_{ik} - \Delta \log X_k \right)^2 \right\}^{\frac{1}{2}}$, where x_{ik} is employment in industry i (among 11 industry sectors) at quarter k, and X_k is aggregate employment; the consumption-wealth ratio (CAY) is defined in Lettau and Ludvigson (2001); the market factor volatility is estimated according to Equation (11); and the average quoted spread (QSPR) follows the definition in Chordia, Roll, and Subrahmanyam (2001).

Table 6

Regressions of Idiosyncratic Volatility Based on 48 Industry Portfolios

	Average Rolling Idiosyncratic Volatility	Average Low Frequency Idiosyncratic Volatility
Constant	**0.095**	**0.125**
	(0.008)	*(0.003)*
EDI	**33.820**	**28.924**
	(19.132)	*(5.786)*
CAY	-0.108	**-0.305**
	(0.192)	*(0.039)*
Market Volatility	**0.200**	**0.056**
	(0.091)	*(0.028)*
QSPR	-0.011	-0.007
	(0.020)	*(0.009)*
R-squared	0.376	0.264
J-statistic	0.097	0.127

Notes: The Average Rolling Idiosyncratic Volatility is defined as $ARIV_t = \dfrac{1}{N}\sum_{i=1}^{N}\left(\dfrac{1}{100}\sum_{k=1}^{100} u_{i(t-k)}^2\right)^{1/2}$,

where u_{it} is the daily idiosyncratic return from Equation (1). The Average Low Frequency Idiosyncratic

Volatility is defined as $Ivol_t = \dfrac{1}{N}\sum_{i=1}^{N}\tau_{it}^{1/2}$, where the τ's are the daily low frequency volatilities estimated

from Equation (12). These averages are based on daily returns from the 48 equally-weighted industry portfolios of Fama and French (1997). Aggregating the variables at a quarterly frequency from 1989 to 2006, the two measures of idiosyncratic volatility are regressed on the following set of variables: the employment dispersion index (EDI) of Lilien (1982), the consumption-wealth ratio (CAY) of Lettau and Ludvigson (2001), the market factor volatility estimated in Equation (11), and the average quoted spread (QSPR) of NYSE stocks (as defined in Chordia, Roll, and Subrahmanyam (2001)). The regressions are estimated using the Generalized Method of Moments (GMM) with Newey-West standard errors and four lags of the regressors as instruments.

Table 7

Correlation Models from Factor Assumptions

PANEL 1: DYNAMIC VOLATILITY COMPONENTS[a]

Constant Volatilities	Components with High and Low Frequency Dynamics	
	Factor Component	Factor and Idiosyncratic Components
FC-C: Constant factor and idiosyncratic volatilities	FG-C: GARCH factor volatility	FG-G: GARCH factor and Idiosyncratic volatilities

$$\rho_{i,j,t}^{FG-C} = \frac{\beta_i \beta_j h_{mt}}{\sqrt{\beta_i^2 h_{mt} + \sigma_i^2}\sqrt{\beta_j^2 h_{mt} + \sigma_j^2}}$$

$$\rho_{i,j,t}^{FG-G} = \frac{\beta_i \beta_j h_{mt}}{\sqrt{\beta_i^2 h_{mt} + h_{it}}\sqrt{\beta_j^2 h_{mt} + h_{jt}}}$$

FSG-C: Spline-GARCH factor volatility

FSG-SG: Spline-GARCH factor and Idiosyncratic volatilities

$$\rho_{i,j}^{FC-C} = \frac{\beta_i \beta_j \sigma_m^2}{\sqrt{\beta_i^2 \sigma_m^2 + \sigma_i^2}\sqrt{\beta_j^2 \sigma_m^2 + \sigma_j^2}}$$

$$\rho_{i,j,t}^{FSG-C} = \frac{\beta_i \beta_j \tau_{mt} g_{mt}}{\sqrt{\beta_i^2 \tau_{mt} g_{mt} + \sigma_i^2}\sqrt{\beta_j^2 \tau_{mt} g_{mt} + \sigma_j^2}}$$

$$\rho_{i,j,t}^{FSG-SG} = \frac{\beta_i \beta_j \tau_{mt} g_{mt}}{\sqrt{\beta_i^2 \tau_{mt} g_{mt} + \tau_{it} g_{it}}\sqrt{\beta_j^2 \tau_{mt} g_{mt} + \tau_{jt} g_{jt}}}$$

PANEL 2: OTHER DYNAMIC COMPONENTS[b]

Idiosyncratic Correlations (Latent Factors)	All Components
FG-IDCC: FG-G with latent factors	FG-DCC

$$\rho_{i,j,t}^{FG-IDCC} = \frac{\beta_i \beta_j h_{mt} + \sqrt{h_{it}}\sqrt{h_{jt}}\rho_{i,j,t}^{u}}{\sqrt{\beta_i^2 h_{mt} + h_{it}}\sqrt{\beta_j^2 h_{mt} + h_{jt}}}$$

See Equation **(19)**

FSG-IDCC: FSG-SG with latent factors

FSG-DCC

$$\rho_{i,j,t}^{FSG-IDCC} = \frac{\beta_i \beta_j \tau_{mt} g_{mt} + \sqrt{\tau_{it} g_{it}}\sqrt{\tau_{jt} g_{jt}}\rho_{i,j,t}^{u}}{\sqrt{\beta_i^2 \tau_{mt} g_{mt} + \tau_{it} g_{it}}\sqrt{\beta_j^2 \tau_{mt} g_{mt} + \tau_{jt} g_{jt}}}$$

See Equation **(14)**

a) σ denotes constant (C) volatilities, h and g refer to GARCH (G) and Spline-GARCH (SG) variances, respectively. b) These models are parametrizations of (8) and (10) in Proposition 1.

Table 8

Evaluation of Factor Correlation Models

Assumption[a]	Model	Quasi-log-likelihood	Parameters	Quasi-Likelihood Ratio[c]	BIC
	FC-C	-426550	67	45980*	-186.96
1	FG-C	-427110	69	44860*	-187.20
	FSG-C	-427160	75	44760*	-187.21
2	FG-G	-440670	135	17740*	-193.03
	FSG-SG	-442820	383	13440*	-193.51
3	FG-IDCC	-446930	663	5220*	-194.80
	FSG-IDCC	-449100	911	880*	-195.29
4[b]	FG-DCC	-447330	696	4420*	-194.91
	FSG-DCC	-449540	944		-195.42

a) This column corresponds to the assumptions that are weakened in the factor specifications:
1.   Constant volatility of the market factor.
2.   Constant volatility of the idiosyncratic components.
3.   One single common factor.
4.   Constant betas.
The models are described in Table 7 and the sample is the 33 DJIA stocks described in Table 1.
b) The last step is associated with the full FG-DCC and FSG-DCC models.
c) The quasi-likelihood-ratios compare the models on each row with the FSG-DCC model (see the last row).
*) Indicates that the quasi-likelihood-ratios are above the 1% critical value of a chi-square distribution with degrees of freedom given by the difference between 944 and the number of parameters associated with each model.

Table 9

In-Sample Evaluation: Standard Deviations of Optimized Forward Hedge Portfolios for a 100-Days Horizon

Forward Hedge Portfolios	FSG-DCC	FG-DCC	DCC
μ_{AA}	0.2563	0.2560	0.2646
μ_{AIG}	0.2010	0.2018	0.2312
μ_{AXP}	0.2385	0.2392	0.2443
μ_{BA}	0.2607	0.2605	0.2749
μ_{C}	0.2364	0.2376	0.2516
μ_{CAT}	0.2525	0.2541	0.2634
μ_{CVX}	0.1675	0.1658	0.1770
μ_{DD}	0.2087	0.2088	0.2127
μ_{DIS}	0.2640	0.2654	0.2728
μ_{GE}	0.1756	0.1771	0.1812
μ_{GM}	0.2805	0.2803	0.3162
μ_{GT}	0.3365	0.3368	0.3419
μ_{HD}	0.2586	0.2603	0.2765
μ_{HON}	0.2672	0.2680	0.3098
μ_{HPQ}	0.3376	0.3381	0.3488
μ_{IBM}	0.2528	0.2534	0.2570
μ_{INTC}	0.3279	0.3295	0.3357
μ_{IP}	0.2250	0.2260	0.2307
μ_{JNJ}	0.1873	0.1880	0.1952
μ_{JPM}	0.2574	0.2594	0.2654
μ_{KO}	0.2029	0.2045	0.2108
μ_{MCD}	0.2402	0.2399	0.2521
μ_{MMM}	0.1857	0.1858	0.1925
μ_{MO}	0.2760	0.2769	0.3129
μ_{MRK}	0.2160	0.2167	0.2323
μ_{MSFT}	0.2724	0.2728	0.2864
μ_{PFE}	0.2273	0.2278	0.2358
μ_{PG}	0.2081	0.2093	0.2147
μ_{T}	0.2087	0.2090	0.2166
μ_{UTX}	0.2157	0.2165	0.2321
μ_{VZ}	0.2022	0.2030	0.2075
μ_{WMT}	0.2313	0.2304	0.2393
μ_{XOM}	0.1547	0.1553	0.1645
All Portfolios	0.2374	0.2380	0.2499

Notes: Sample standard deviations of returns on optimized forward hedge portfolios constructed at each point in the sample using 100-days-ahead covariance forecasts from FSG-DCC, FG-DCC, and DCC models, respectively, and subject to a required return of 1. The sample is described in Table 1. The stock in the corresponding row is hedged against all other stocks.

Table 10

Out-of-Sample Evaluation: Standard Deviations of Optimized Forward Hedge Portfolios at Multiple Long Horizons

Forward Hedge Portfolios	FSG-DCC	FG-DCC	DCC	BCOV	SCOV	LCOV
μ_{AA}	0.2573	0.2569	0.2569	0.2787	0.2549	0.2550
μ_{AIG}	0.1914	0.1947	0.1917	0.1949	0.1925	0.1923
μ_{AXP}	0.2130	0.2138	0.2137	0.2293	0.2151	0.2152
μ_{BA}	0.2690	0.2702	0.2697	0.2744	0.2715	0.2714
μ_{C}	0.1979	0.1992	0.1985	0.2182	0.1979	0.1983
μ_{CAT}	0.2445	0.2461	0.2452	0.2532	0.2455	0.2451
μ_{CVX}	0.1593	0.1586	0.1575	0.2065	0.1584	0.1592
μ_{DD}	0.1997	0.2009	0.2013	0.2119	0.1981	0.1980
μ_{DIS}	0.2718	0.2695	0.2708	0.2708	0.2726	0.2723
μ_{GE}	0.1921	0.1947	0.1918	0.1874	0.1913	0.1909
μ_{GM}	0.3035	0.3061	0.3055	0.3069	0.3043	0.3040
μ_{GT}	0.3509	0.3504	0.3508	0.3645	0.3547	0.3546
μ_{HD}	0.2704	0.2707	0.2705	0.2837	0.2703	0.2704
μ_{HON}	0.2714	0.2756	0.2719	0.2812	0.2718	0.2718
μ_{HPQ}	0.3688	0.3735	0.3677	0.3925	0.3665	0.3668
μ_{IBM}	0.2363	0.2428	0.2367	0.2506	0.2388	0.2386
μ_{INTC}	0.3325	0.3394	0.3347	0.3794	0.3351	0.3357
μ_{IP}	0.2292	0.2296	0.2287	0.2531	0.2279	0.2283
μ_{JNJ}	0.1792	0.1808	0.1797	0.1972	0.1803	0.1797
μ_{JPM}	0.2169	0.2183	0.2190	0.2427	0.2187	0.2189
μ_{KO}	0.2161	0.2163	0.2149	0.2232	0.2150	0.2150
μ_{MCD}	0.2522	0.2541	0.2525	0.2531	0.2525	0.2523
μ_{MMM}	0.1937	0.1965	0.1938	0.1977	0.1925	0.1924
μ_{MO}	0.3172	0.3204	0.3174	0.3210	0.3176	0.3176
μ_{MRK}	0.2156	0.2162	0.2156	0.2417	0.2142	0.2141
μ_{MSFT}	0.2474	0.2510	0.2485	0.2711	0.2485	0.2481
μ_{PFE}	0.2439	0.2482	0.2438	0.2710	0.2440	0.2439
μ_{PG}	0.1827	0.1826	0.1824	0.1991	0.1820	0.1820
μ_{T}	0.2139	0.2161	0.2145	0.2522	0.2158	0.2168
μ_{UTX}	0.2158	0.2204	0.2166	0.2195	0.2157	0.2154
μ_{VZ}	0.1924	0.1953	0.1913	0.2237	0.1901	0.1904
μ_{WMT}	0.2351	0.2370	0.2349	0.2500	0.2364	0.2366
μ_{XOM}	0.1517	0.1506	0.1513	0.1966	0.1518	0.1528
All Portfolios	0.2374	0.23757	0.2393	0.2545	0.23764	0.2377

Notes: Sample standard deviations of returns on optimized forward hedge portfolios subject to a required return of 1, and based on 22 iterations of out-of-sample covariance forecasts at horizons from 87 to 126 days ahead. The forecasts are constructed from FSG-DCC, FG-DCC, DCC, BCOV (static one-factor beta covariance), SCOV (sample covariance), and LCOV (optimal shrinkage covariance of Ledoit and Wolf (2003)) models, respectively. The 22 sequential sample periods are described in Figure 11 and include the 33 DJIA stocks described in Table 1. The stock in the corresponding row is hedged against all other stocks.

Table 11

Joint Diebold-Mariano Tests to Compare the Long-Term Forecast Performance of the FSG-DCC Model Relative to Competing Models

	Competing Models				
FSG-DCC vs. Column Model	DCC	FG-DCC	LCOV	SCOV	BCOV
T-Statistics	-3.66	-2.96	-2.14	-2.07	-5.72

Notes: This table reports t-statistics for joint Diebold-Mariano tests that evaluate the forecast performance of the FSG-DCC model relative to the following competitors: DCC, FG-DCC, BCOV (static one-factor beta covariance), SCOV (sample covariance), and LCOV (optimal shrinkage covariance of Ledoit and Wolf (2003)). Each t-statistic is derived from estimating a regression of a vector of differences of squared realized returns of two models on a constant. The vector of differences is constructed using squared realized forward hedge portfolio returns associated with the FSG-DCC model and the competing column model. The forward hedge portfolios are constructed using the sample periods described in Figure 11 and the 33 DJIA stocks in Table 1. We include only long-term forward hedge portfolios (from 87 to 126 days forward). The vector regressions are estimated using the Generalized Method of Moments (GMM) with heteroskedasticity and autocorrelation consistent (HAC) covariance matrix. A negative value is evidence of better performance of the FSG-DCC model.

Table 12

Economic Variables Associated with Low Frequency Variation in the Market Volatility

Explanatory Variables	Regression Coefficients	Realizations of Explanatory Variables	
		2006	2007
log(mc/gdpus)	-0.0092	6.8717	6.8413
log(gdpus)	0.0181	9.4876	9.5287
nlc	-1.77E-05	2280	2297
grgdp	-0.1603	0.0256	0.0220
gcpi	0.3976	0.0192	0.0290
vol_irate	0.0020	0.2807	0.3910
vol_gforex	0.0222	0.0147	0.0129
vol_grgdp	0.8635	0.0034	0.0042
vol_gcpi	0.1532	0.0079	0.0091

Notes: The definition of the explanatory variables and the estimated regression coefficients are taken from Engle and Rangel (2008). The explanatory variables are defined as follows: MC=Market capitalization of the NYSE, GDPUS=Nominal GDP in US dollars, NLC=Number of listings in the NYSE, GRGDP=Real GDP growth rate, GCPI=Inflation rate (based on the consumer price index), VOL_IRATE=Volatility of the short-term interest rate, VOL_GFOREX=Volatility of the US exchange rate relative to the EURO, VOL_GRGDP=Volatility of real growth, VOL_GCPI=Volatility of the short-term interest rate. The estimated coefficients are taken from the first column of Table 9 in Engle and Rangel (2008). The same sources of this paper are used to update the values of the explanatory variables.

Figure 1

Betas
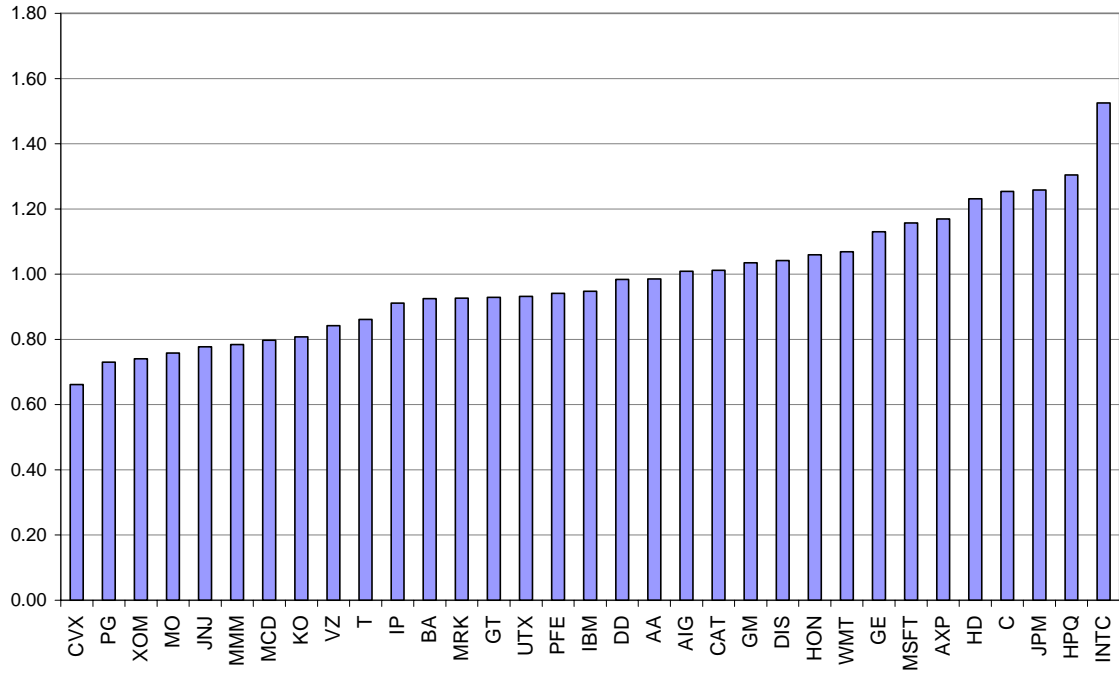
Notes: The estimation uses daily returns on the DJIA stocks from December 1988 to December 2006. The data is obtained from CRSP. The beta estimates correspond to the one-factor CAPM specification in Equation (1): $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$. They are estimated based on the moment conditions in Equation (26) and a two-stage GMM estimation approach.
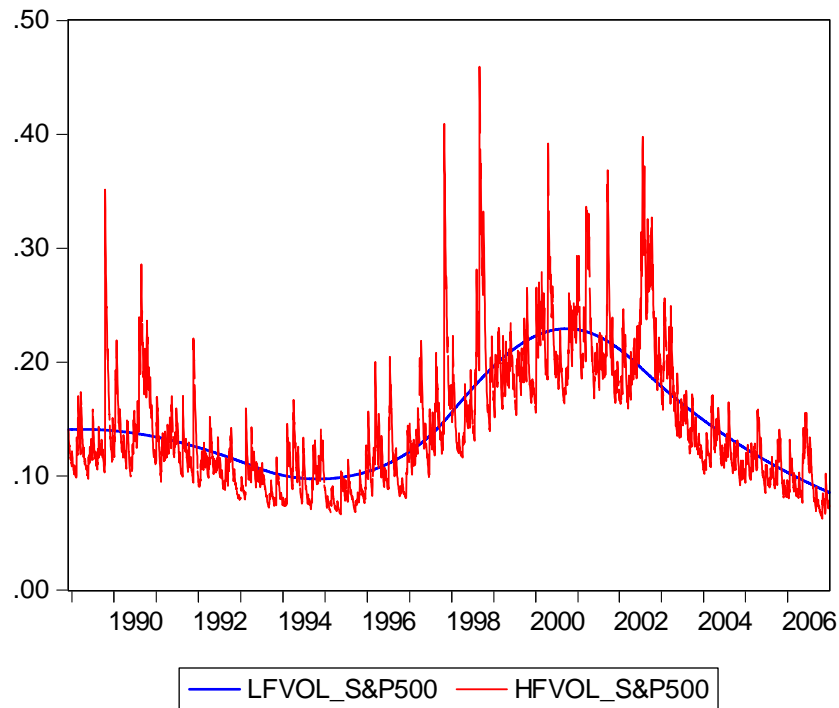
Figure 2

**Idiosyncratic Volatilities: High and Low Frequency
Components**
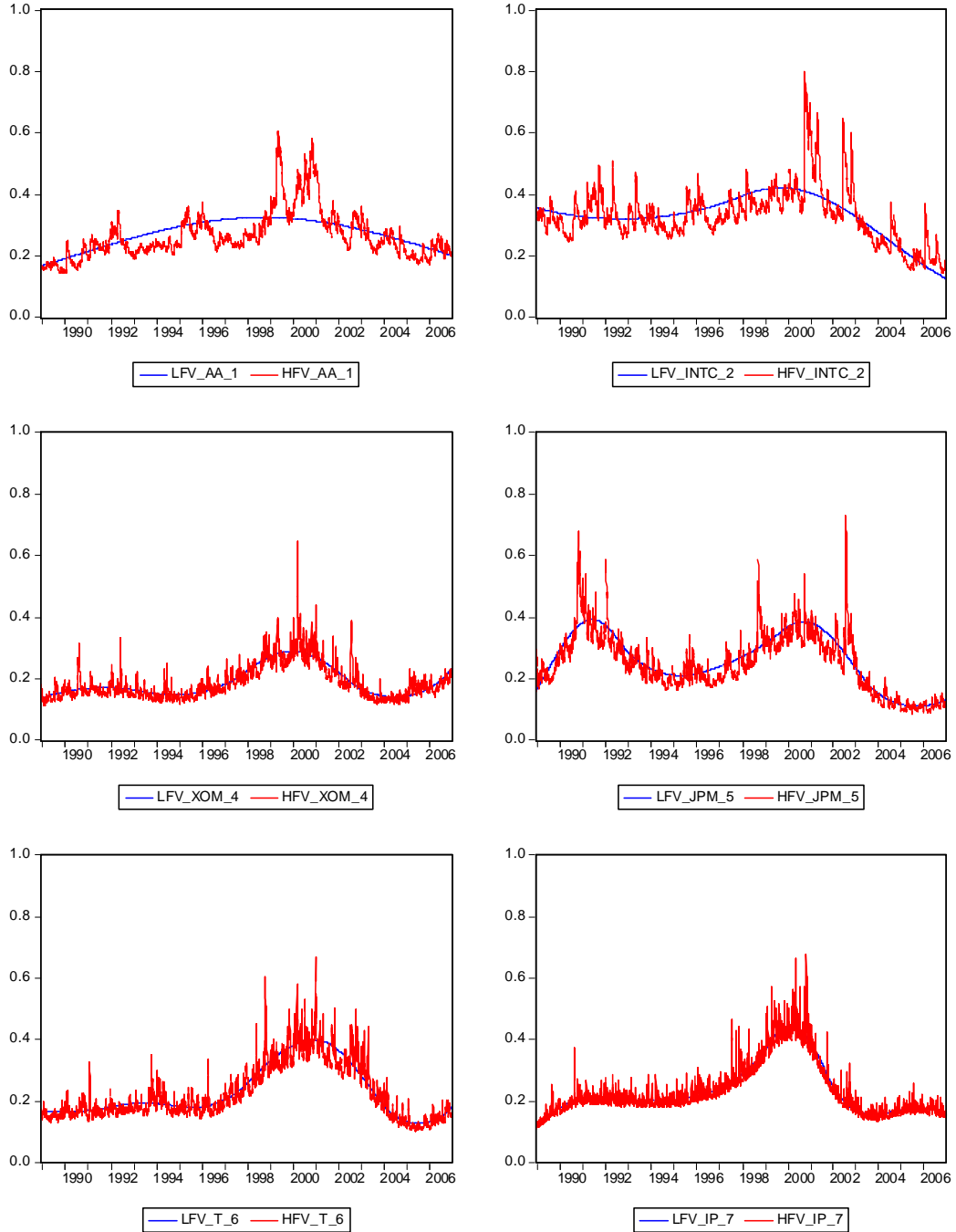
Notes: The estimation uses daily returns on the DJIA stocks from December 1988 to December 2006. The data is obtained from CRSP. Company names are referred by their tickers (AA=Alcoa, INTC=Intel, XOM=Exxon Mobil, JPM=JP Morgan Chase, T=AT&T, and IP=International Paper). HFV stands for "High frequency idiosyncratic volatility" (see second equation in (12)) and LFV refers to "Low frequency idiosyncratic volatility" (see third equation in (12)). The last number in the series labels denotes the optimal number of knots.

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Figure 3
High and Low Frequency Market Volatility

Notes: The estimation uses daily returns on the S&P500 from December 1988 to December 2006. HFVOL=High frequency market volatility (see second equation in (11)). LFVOL=Low frequency market volatility (see third equation in (11)).

Figure 4

Correlations for Stocks with Increasing Low Frequency Idiosyncratic Volatility at the End of the Sample
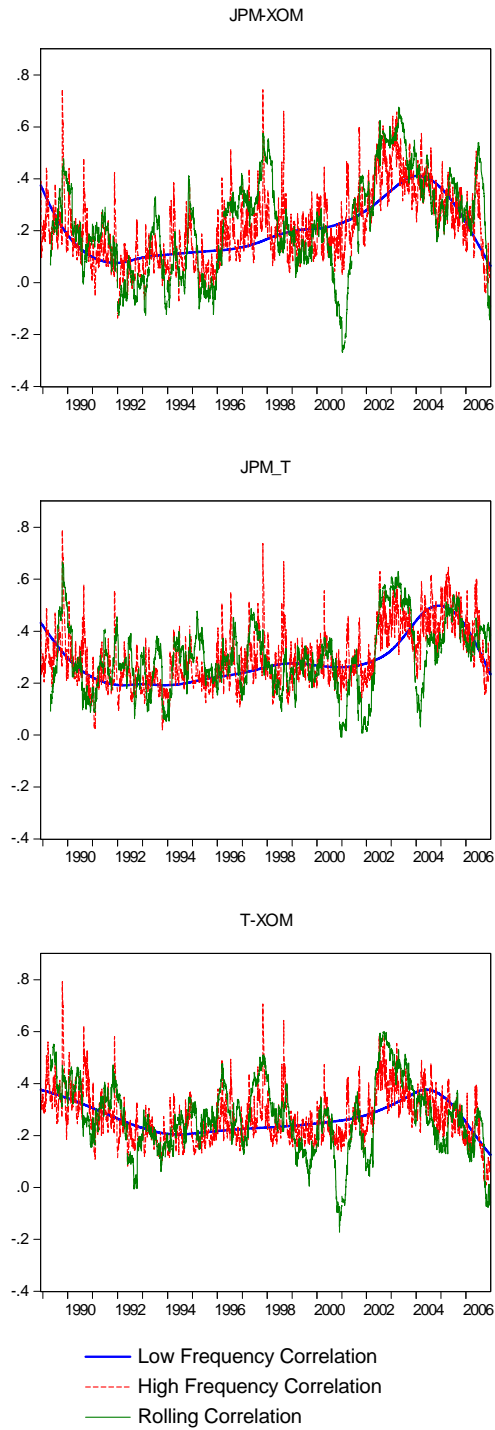
Notes: The estimation uses daily returns on the DJIA stocks from December 1988 to December 2006. The data is obtained from CRSP. Company names are referred by their tickers (see Table 1).

Figure 5

Correlations for Stocks with Decreasing Low Frequency Idiosyncratic Volatility at the End of the Sample
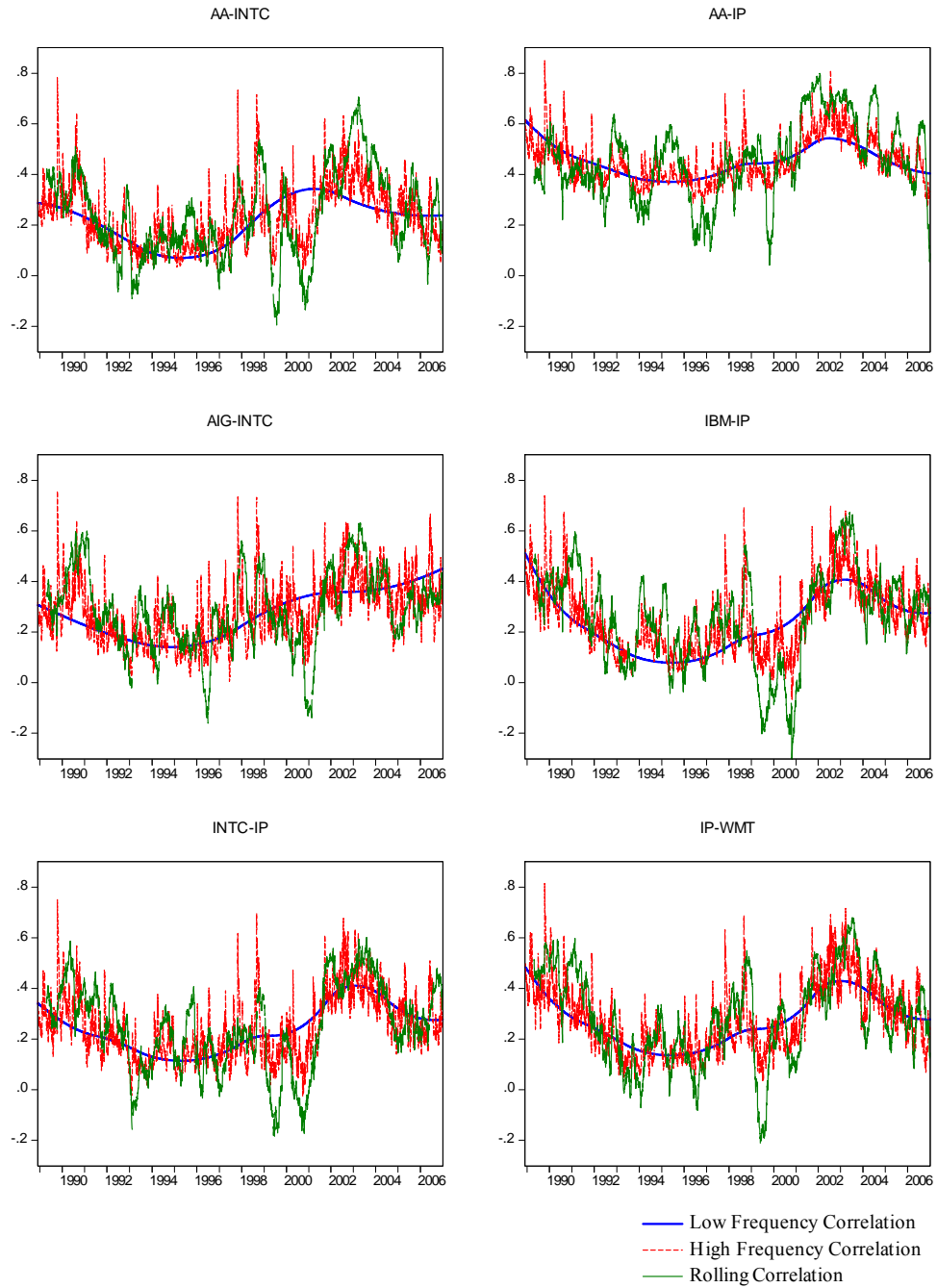
Notes: The estimation uses daily returns on the DJIA stocks from December 1988 to December 2006. The data is obtained from CRSP. Company names are referred by their tickers (see Table 1).

Figure 6

Average Low Frequency Market and Idiosyncratic Volatilities Over Two-Year Periods
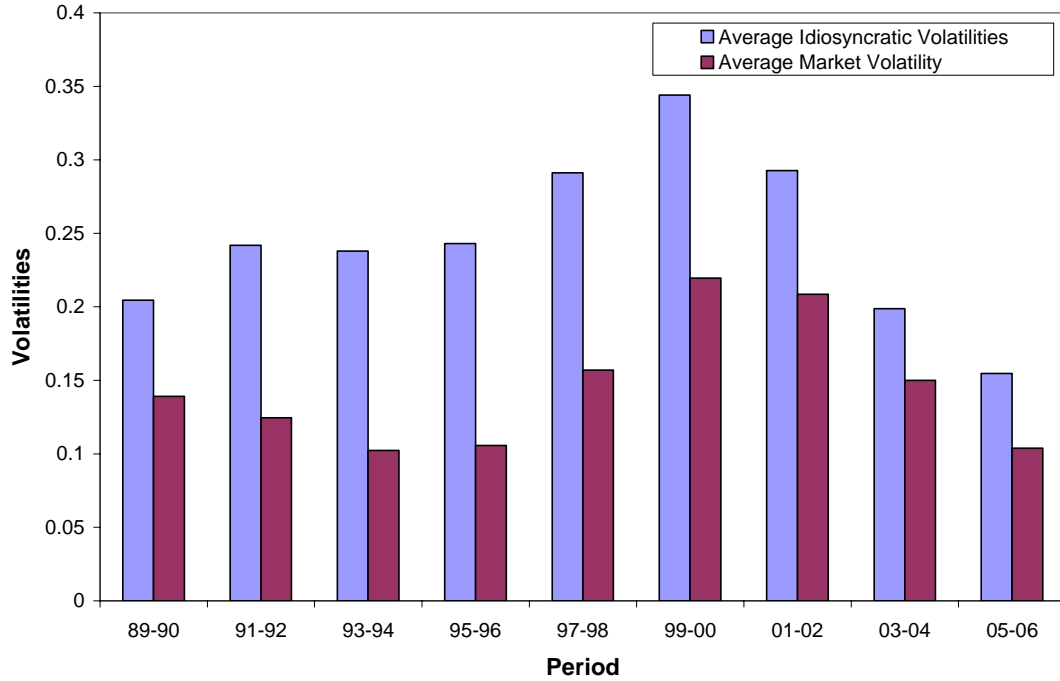
Notes: The average low frequency volatility over the period p is defined as follows:

Average Annualized Idiosyncratic Volatility of Asset $i = \dfrac{1}{T_p} \sum_{t=1}^{T_p} \tau_{it}^{1/2}$, where $\tau_{it}^{1/2}$ is the annualized low frequency volatility for asset i at time t, $\forall i = m, 1, 2, ..., N$, and T_p is the number of daily observations in period p .

Figure 7

Average Correlations
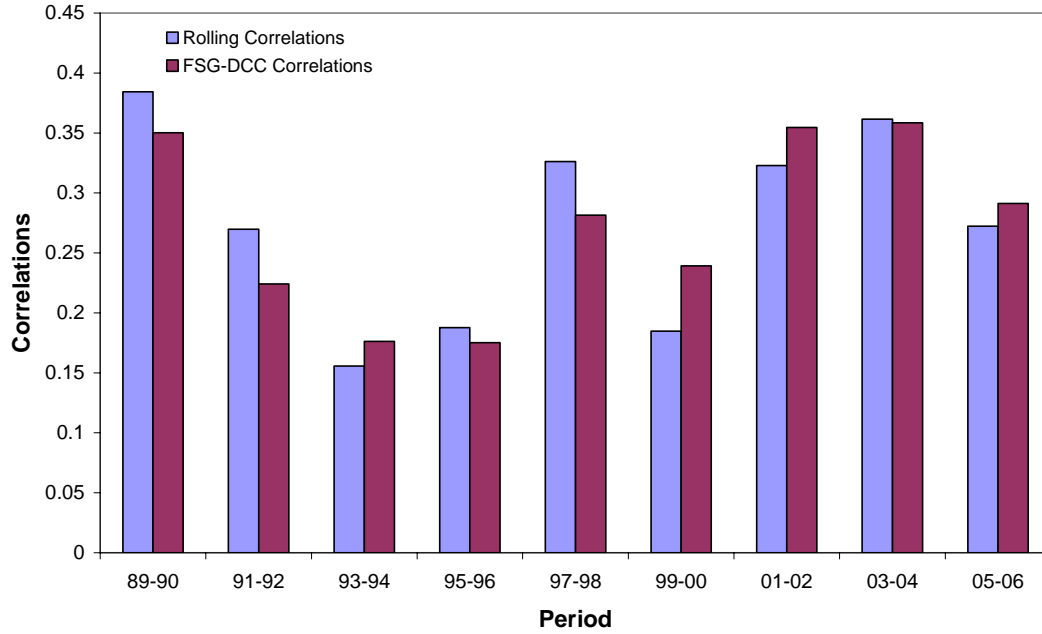
Notes: The average rolling correlation over the period p is defined as follows:

$$\overline{\rho}_{T_p}^{Rolling} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{T_p} \frac{1}{(N-1)} \sum_{j \neq i} \sum_{t=1}^{T_p} \rho_{i,j,t}^{Rolling} \right\},$$ where T_p denotes the number of daily

observations in period p, and $\rho_{i,j,t}^{Rolling}$ is the rolling correlation between assets i and j at

time t. Similarly, the average FSG-DCC correlations are constructed by using $\rho_{i,j,t}$ in

Proposition 2.

Figure 8

Employment Dispersion Index and Aggregated Idiosyncratic Volatilities
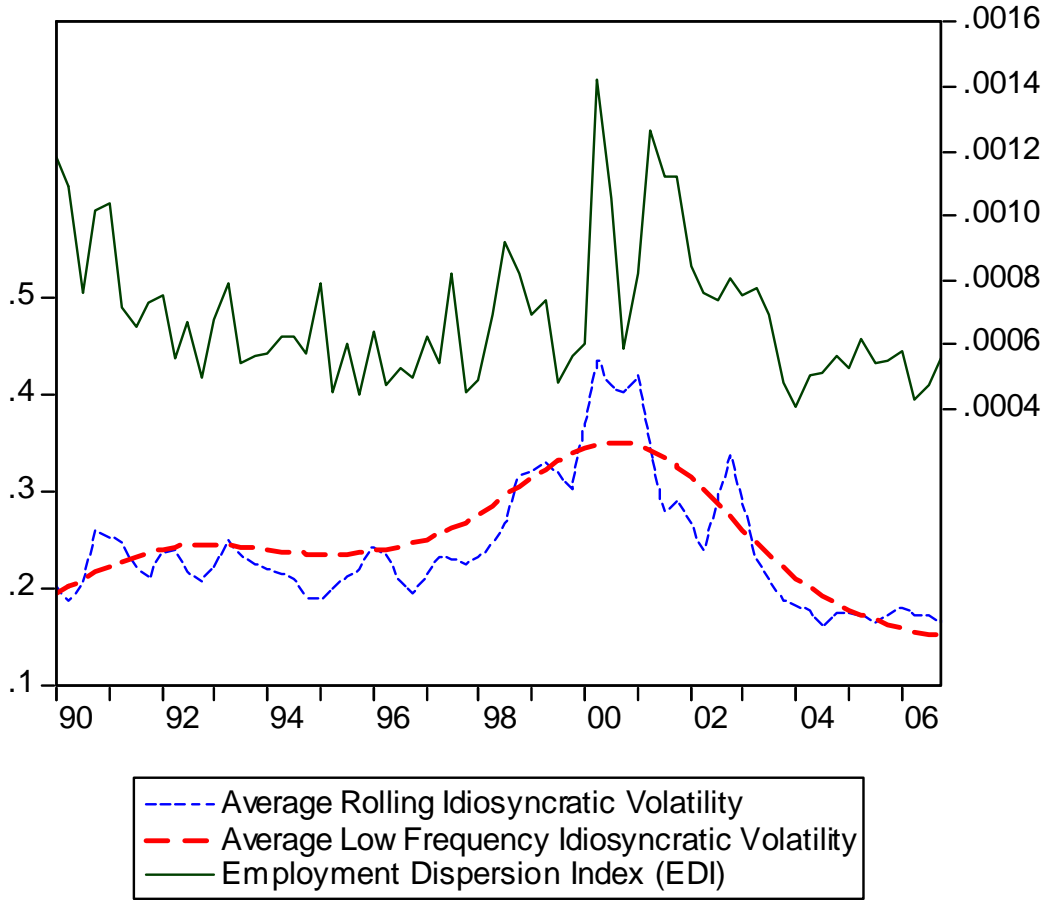
Notes: The graph shows quarterly aggregates of the following variables: Average Rolling Idiosyncratic

Volatility, defined as $ARIV_t = \dfrac{1}{N}\sum_{i=1}^{N}\left(\dfrac{1}{100}\sum_{k=1}^{100}u_{i(t-k)}^2\right)^{1/2}$, where u_{it} is the daily idiosyncratic return from

Equation (1); Average Low Frequency Volatility defined as $Ivol_t = \dfrac{1}{N}\sum_{i=1}^{N}\tau_{it}^{1/2}$, where the τ's are daily low

frequency idiosyncratic volatilities (see Equation (12)); and Employment Dispersion Index, which follows the

definition in Lilien (1982): $EDI_k = \left\{\sum_{i=1}^{11}\dfrac{x_{ik}}{X_k}\left(\Delta\log x_{ik} - \Delta\log X_k\right)^2\right\}^{1/2}$, where x_{ik} is employment in

industry i (among 11 industry sectors) at quarter k and X_k is aggregate employment.

Figure 9

Employment Dispersion Index and Aggregated Idiosyncratic Volatilities from 48 Industry Portfolios
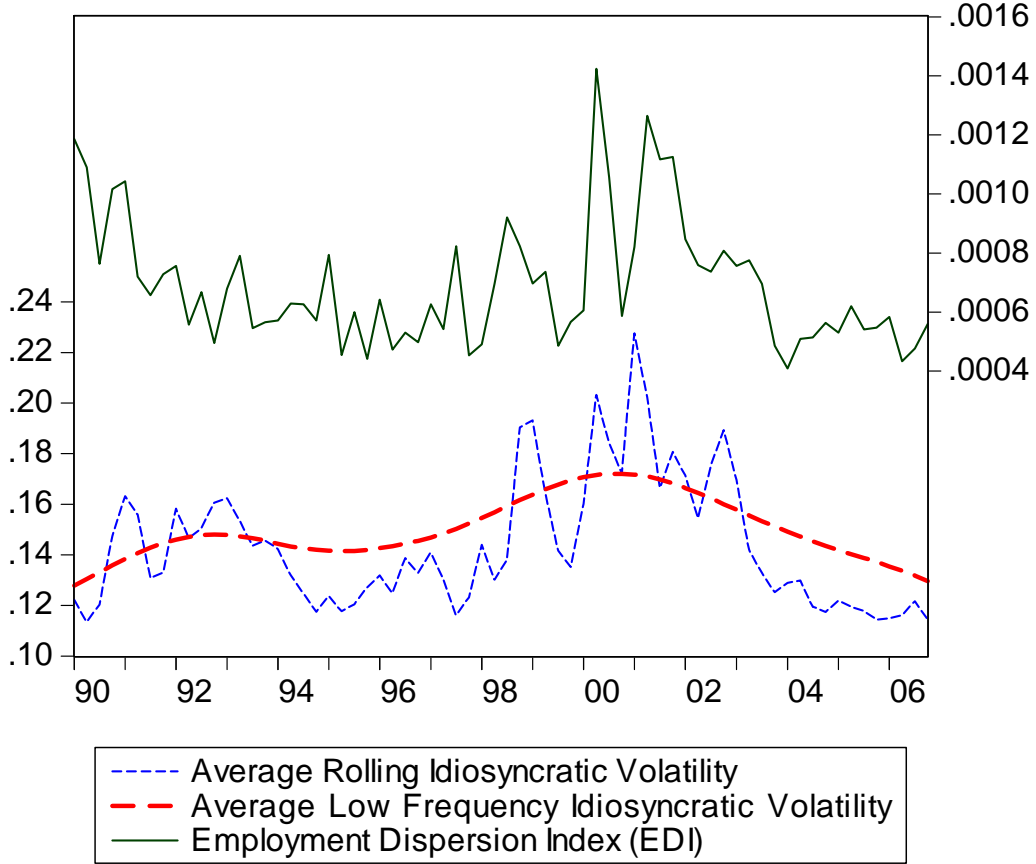
Notes: The graph shows quarterly aggregates of the following variables: Average Rolling Idiosyncratic Volatility, defined as $ARIV_t = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{100}\sum_{k=1}^{100}u_{i(t-k)}^2\right)^{1/2}$, where u_{it} is the idiosyncratic return from fitting Equation (1) to the 48 industry portfolios of Fama and French (1997); Average Low Frequency Volatility, defined as $Ivol_t = \frac{1}{N}\sum_{i=1}^{N}\tau_{it}^{1/2}$, where the $\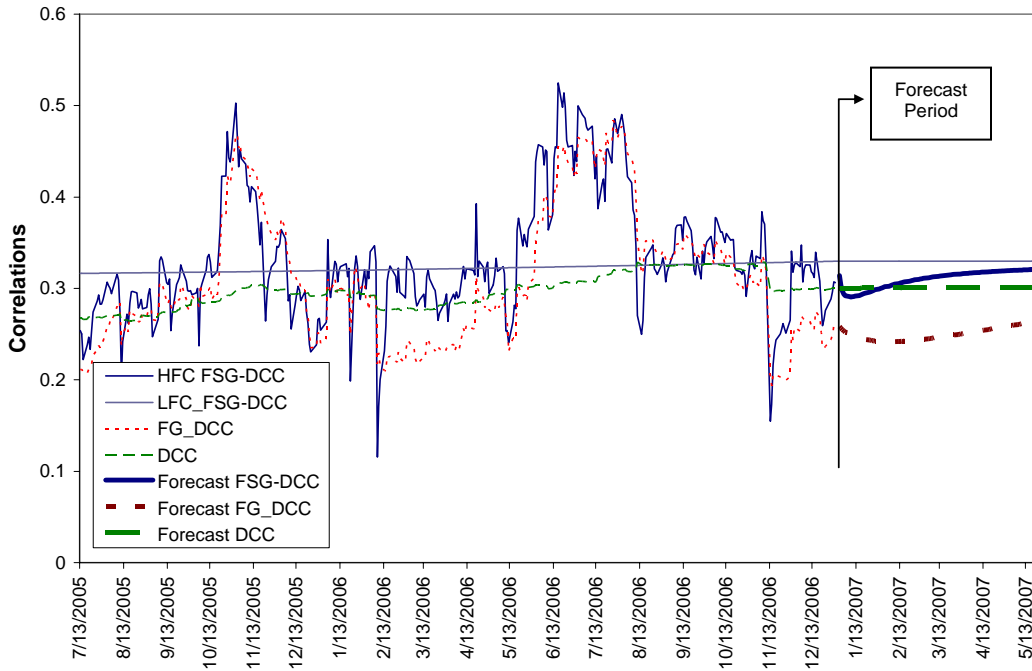tau$'s are the low frequency volatilities of such portfolios based on Equation (12); and Employment Dispersion Index, which follows the definition in Lilien (1982):

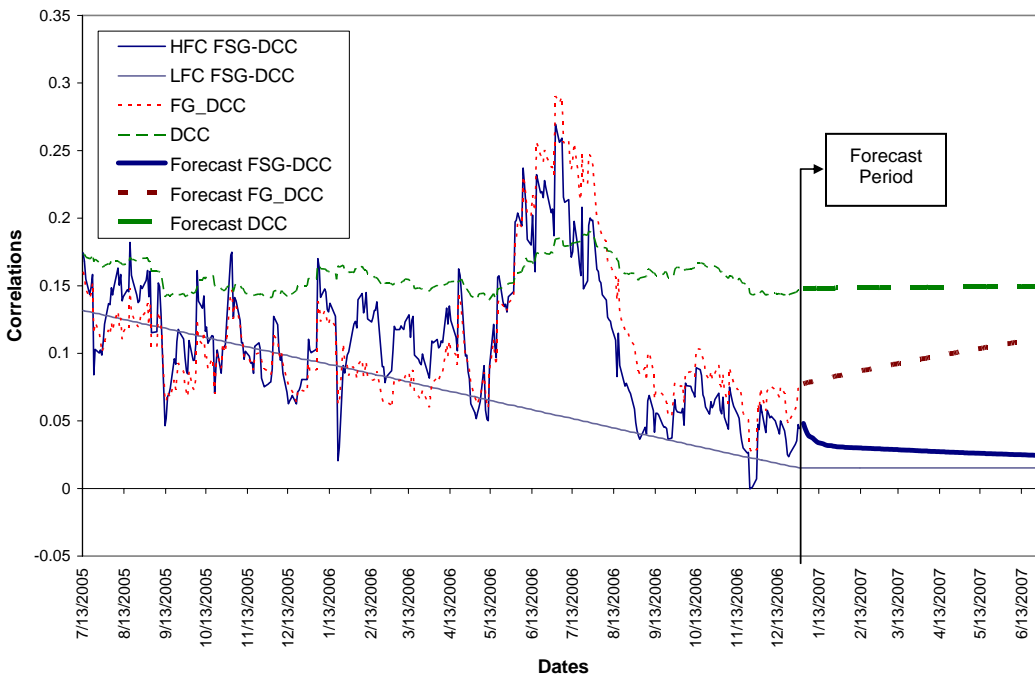$EDI_k = \left\{\sum_{i=1}^{11}\frac{x_{ik}}{X_k}\left(\Delta\log x_{ik} - \Delta\log X_k\right)^2\right\}^{1/2}$, where x_{ik} is employment in industry i (among 11 industry sectors) at quarter k and X_k is aggregate employment.

Figure 10

### Mutiple-Step Ahead Forecasts
AIG and DIS

CVX and INTC

Notes: Multi-step forecasts of the correlations between DIS and AIG. The estimation period goes from December 1988 to December 2006. The forecasting period goes from January to July 2007. HFC=High frequency correlation, LFC=Low frequency correlation. Forecast horizon: 130 days.

Figure 11

Sequential Forecasting Exercise
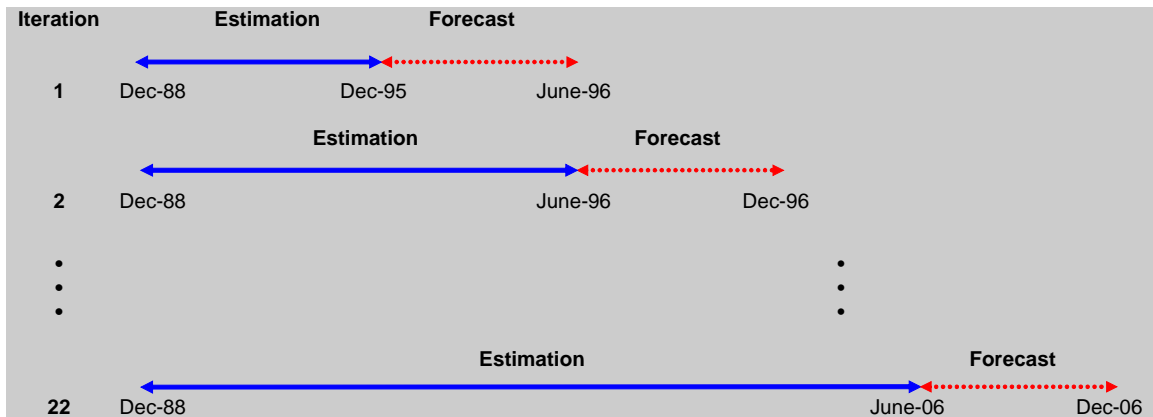
Notes: This figure describes the iterations of an out-of-sample sequential forecasting exercise. In the first iteration a number of covariance models are estimated using data from 12/01/1988 to 12/31/1995. Then multi-step out-of-sample forecasts are constructed for each model over a period of six months (from 01/01/1996 to 06/30/1996). In iteration 2 the estimation period is extended six months (from 12/01/1988 to 06/30/1996) and a new set of multi-step out-of-sample forecasts are generated for the following six months. At each iteration, the estimation period incorporates the previous iteration's forecasting period and new out-of-sample forecasts are generated. Proceeding in this manner, 22 iterations are completed ending at 12/31/2006. The forecasting periods do not overlap.